GW Signal Search
(중력파 신호 탐색)

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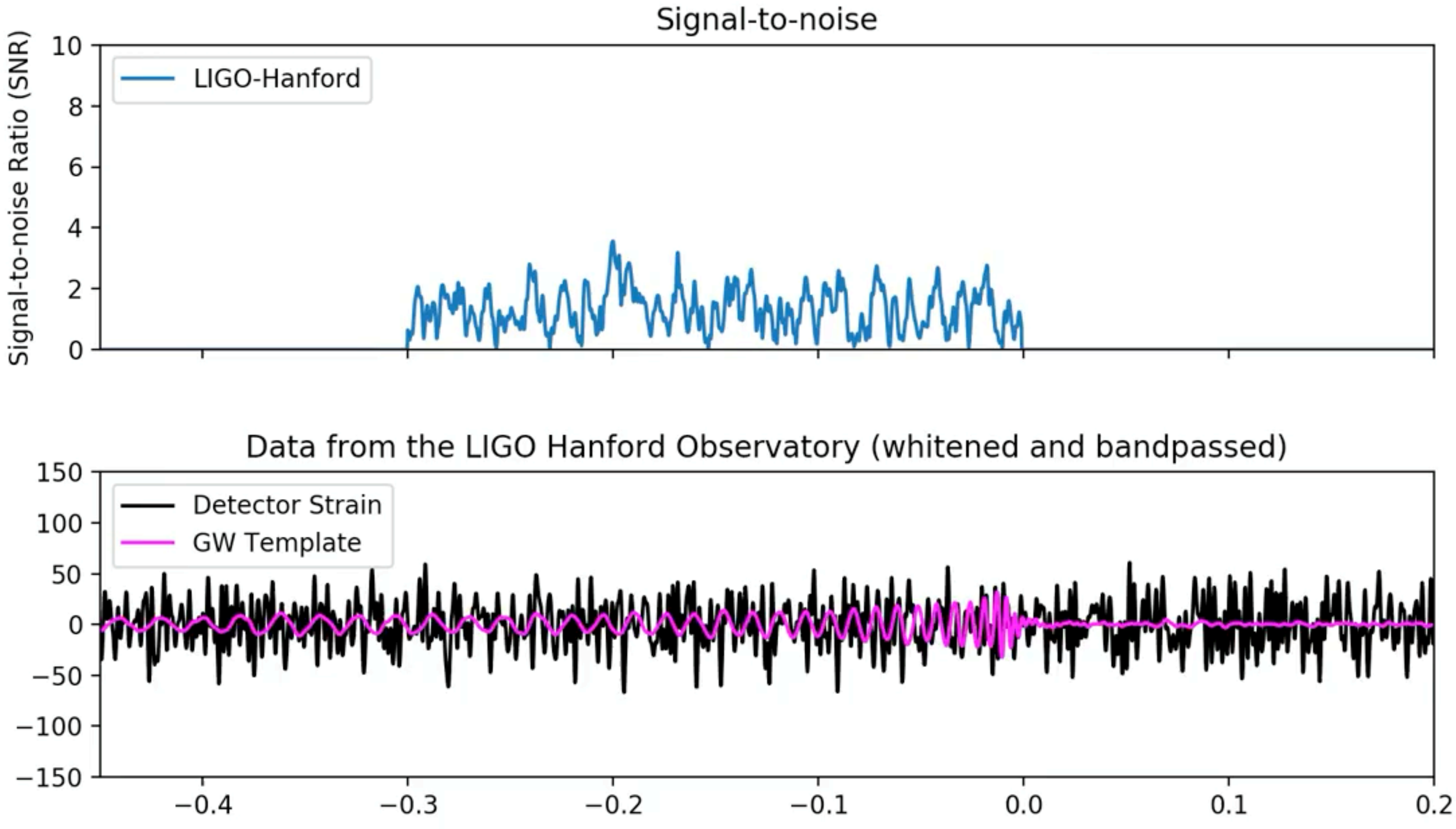
CBC Search in Practice w/ Python

- References
  - Main: GW Open Data Workshop (ODW) 2024 - Day 1 & 2 Tutorials
    - Homepage: https://www.gw-openscience.org/odw/odw2024
    - Github:
      https://github.com/gw-odw/odw-2024/tree/main/Tutorials/Day_1
      https://github.com/gw-odw/odw-2024/tree/main/Tutorials/Day_2
  - Additional: GW Open Science Center
    - "Signal processing with GW150914 Open Data"
    - https://www.gw-openscience.org/s/events/GW150914/GW150914_tutorial.html

- 2024 ODW Tutorial materials:
  - GWpy
    - Tuto 1.2 Open Data access with GWpy
    - Tuto 1.3 Q-transforms with GWpy
  - PyCBC
    - Tuto 2.1 Matched filtering introduction
    - Tuto 2.2 Matched filtering in action
    - Tuto 2.3 Signal consistency and significance

Introduction to matched filtering

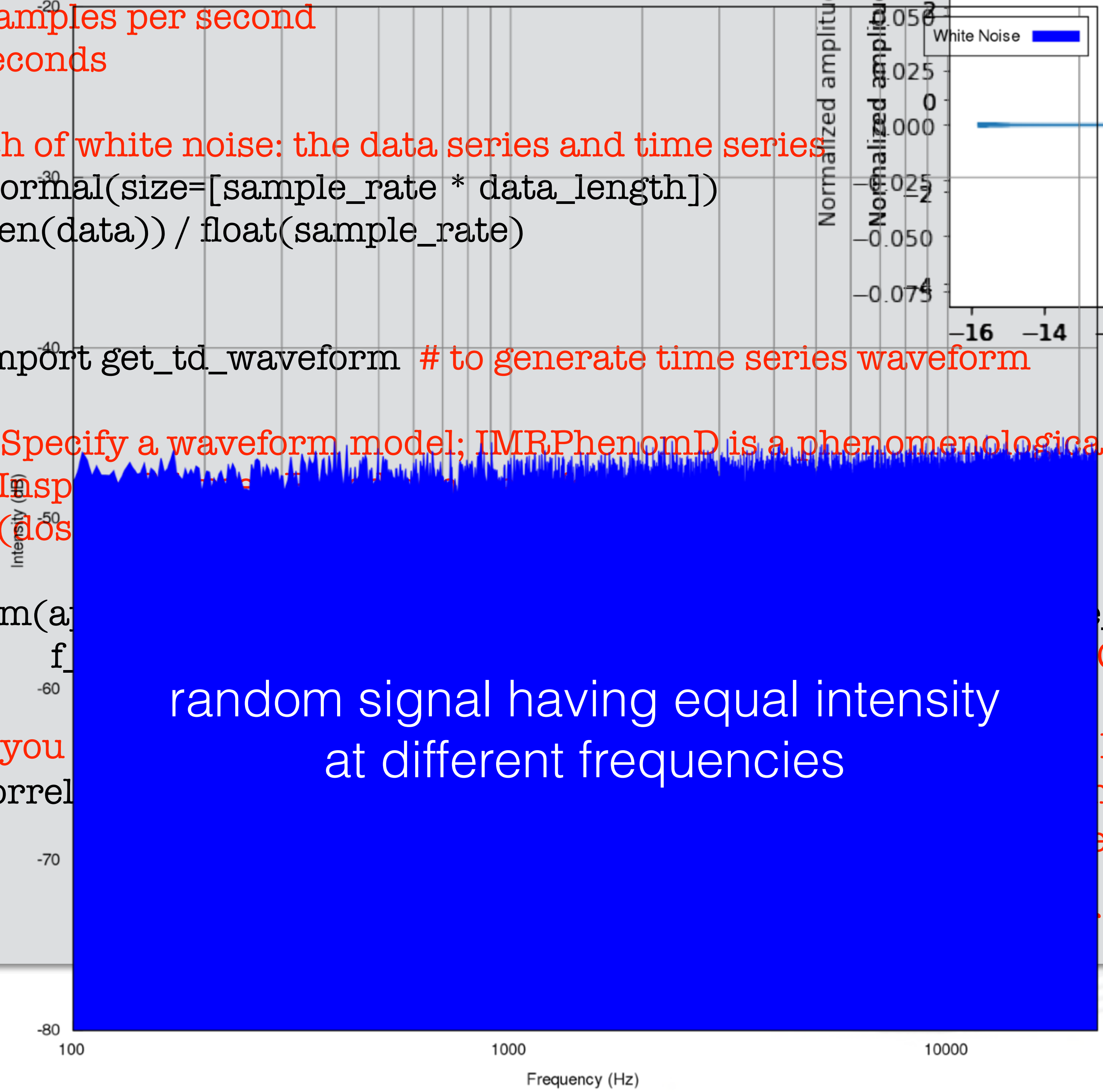
- Matched filtering
  - optimal method for “detecting” known signals in Gaussian noise via computing cross-correlation



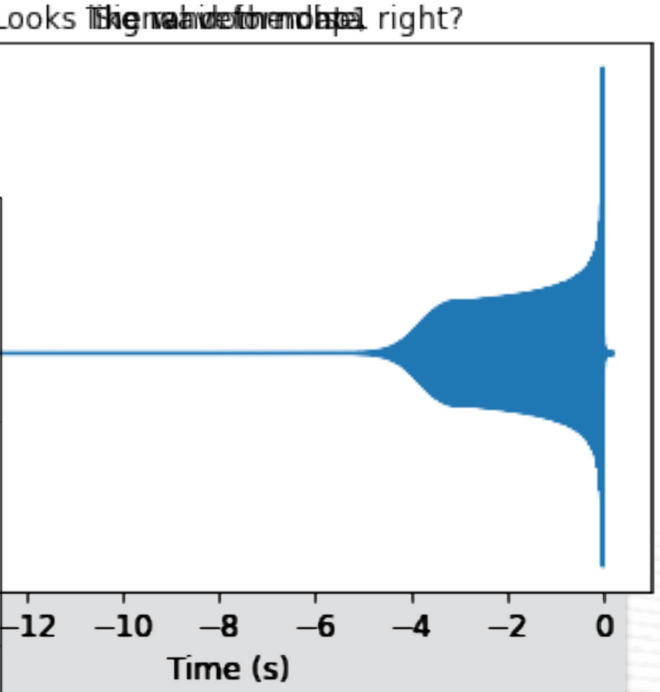
Introduction to matched filtering

- Let’s learn how matched filtering works.
- Start with an example waveform in white noise.

```
import numpy

sample_rate = 1024  # samples per second
data_length = 1024  # seconds

# Generate a long stretch of white noise: the data series and time series
data = numpy.random.normal(size=[sample_rate * data_length])
times = numpy.arange(len(data)) / float(sample_rate)


from pycbc.waveform import get_td_waveform  # to generate time series waveform

apx = ‘IMRPhenomD’  # Specify a waveform model; IMRPhenomD is a phenomenological
                     Insp
                     (dos                                                          modes)

hp, hx = get_td_waveform(a                                                        _rate,
                        f                                                         GW signal

# use h+ only for now. if you                                                     h = hp + hx.
hp = hp / max(numpy.correl                                                        hod on white noise
                                                                                  e normalize our signal
                                                                                  of the signal with
                                                                                  1.
```

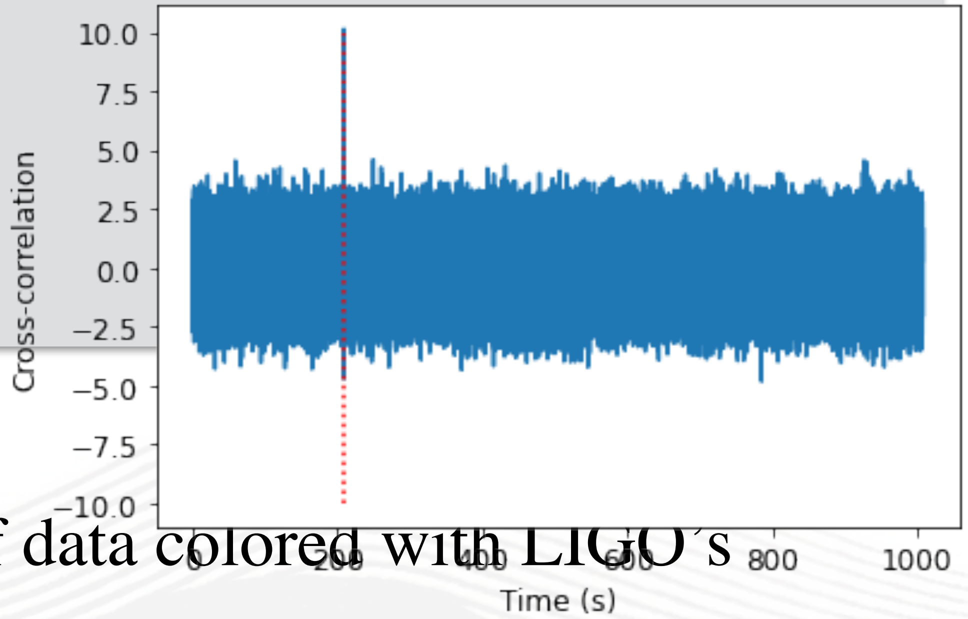


random signal having equal intensity at different frequencies

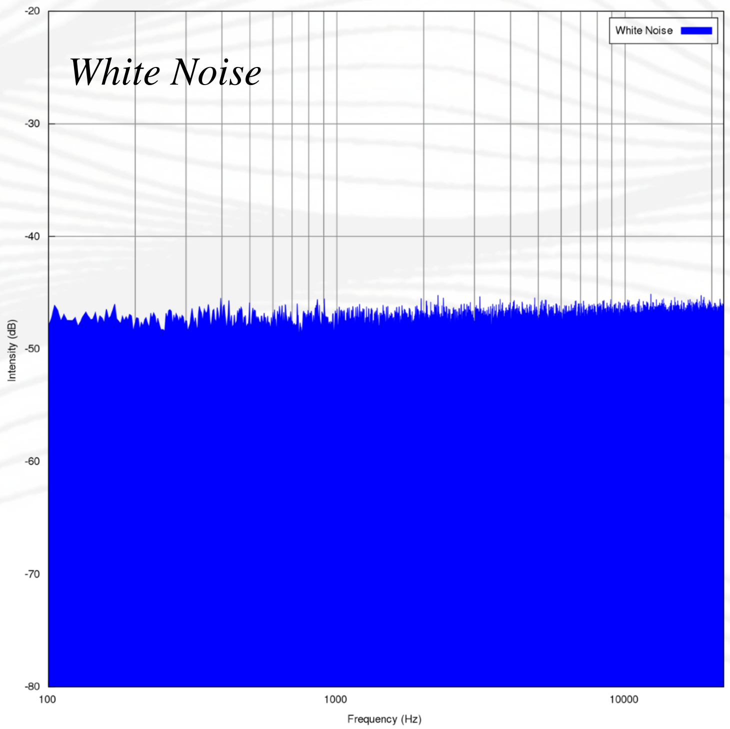


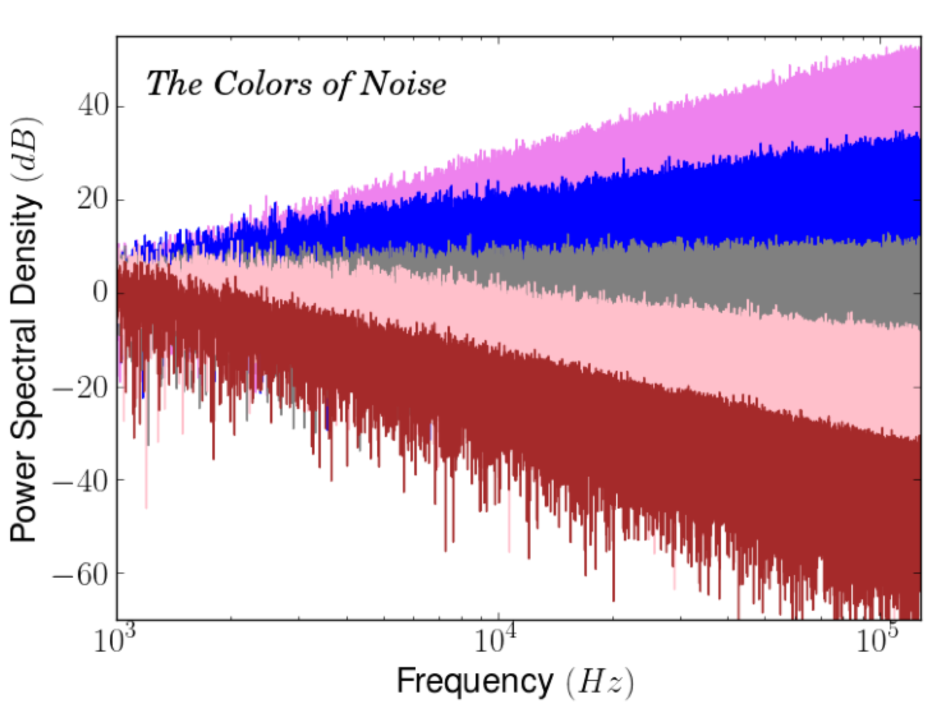
Introduction to matched filtering

```
# To search for this signal, we can cross-correlate the signal with the entire dataset.
# We do the cross-correlation in the time domain.

cross_correlation = numpy.zeros([len(data)-len(hp)])
hp_numpy = hp.numpy()
for i in range(len(data) - len(hp_numpy)):
   cross_correlation[i] = (hp_numpy * data[i:i+len(hp_numpy)]).sum()
```



- Detection in Colored Noise
  - Let’s repeat the same process, but generate a stretch of data colored with LIGO’s zero-detuned-high-power noise curve.

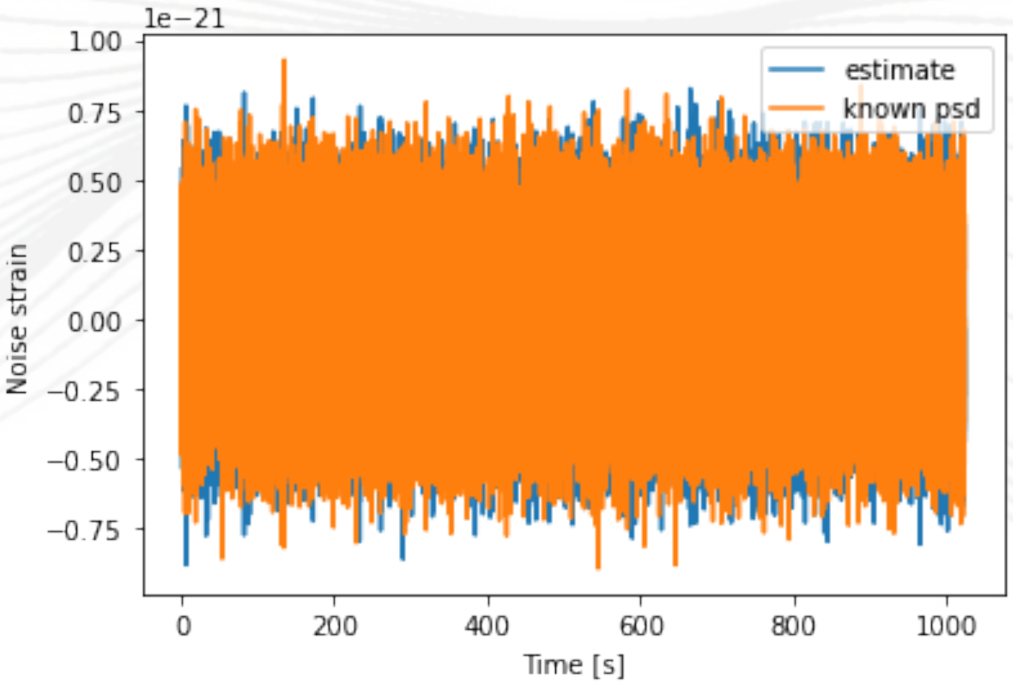


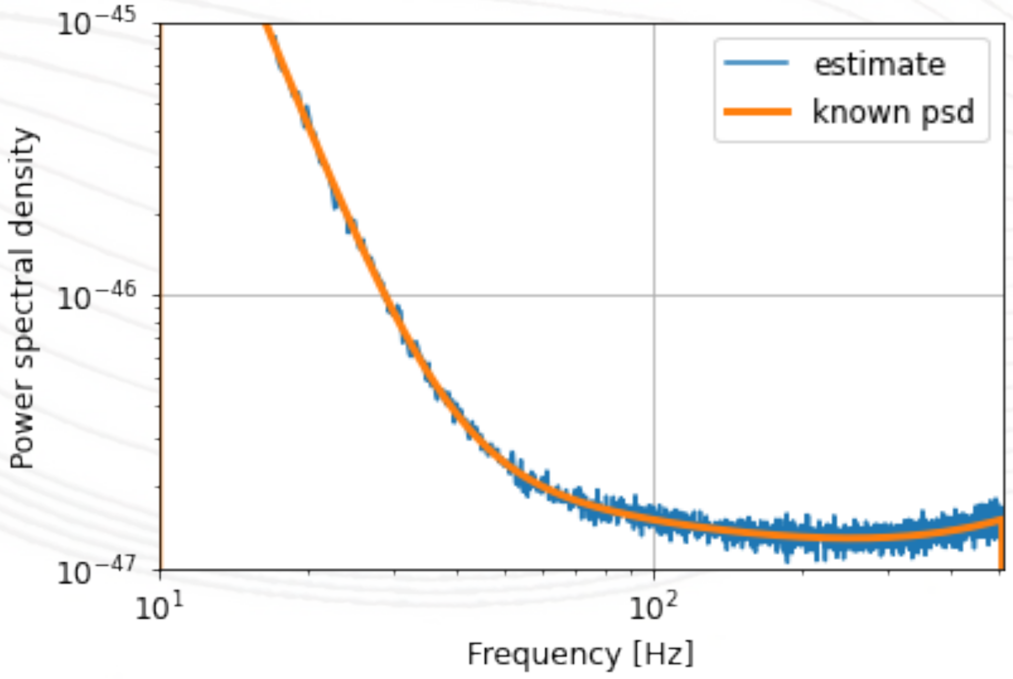


[Images:
from Wikipedia,
“Colors of noise”]

Introduction to matched filtering

```
import pycbc.noise, pycbc.psd

# The color of the noise matches a PSD which you provide, Advanced LIGO’s zero-detuned-high-power noise curve
flow = 10.0
delta_f = 1.0 / 128
flen = int(sample_rate / (2*delta_f)) + 1  # sample_rate = 1024 samples per second
psd = pycbc.psd.aLIGOZeroDetHighPower(flen, delta_f, flow)

# Generate colored noise
delta_t = 1.0 / sample_rate
ts = pycbc.noise.noise_from_psd(data_length*sample_rate, delta_t, psd, seed=127)

# Estimate the power spectral density for the noisy data using the “Welch” method.
# We’ll choose 4 seconds PSD samples that are overlapped 50%
# For more details about the “Welch” method, see arXiv:gr-qc/0509116 (Section VI)
seg_len = int(4 / delta_t)
seg_stride = int(seg_len / 2)
estimated_psd = pycbc.psd.welch(ts, seg_len=seg_len, seg_stride=seg_stride)
```





Introduction to matched filtering

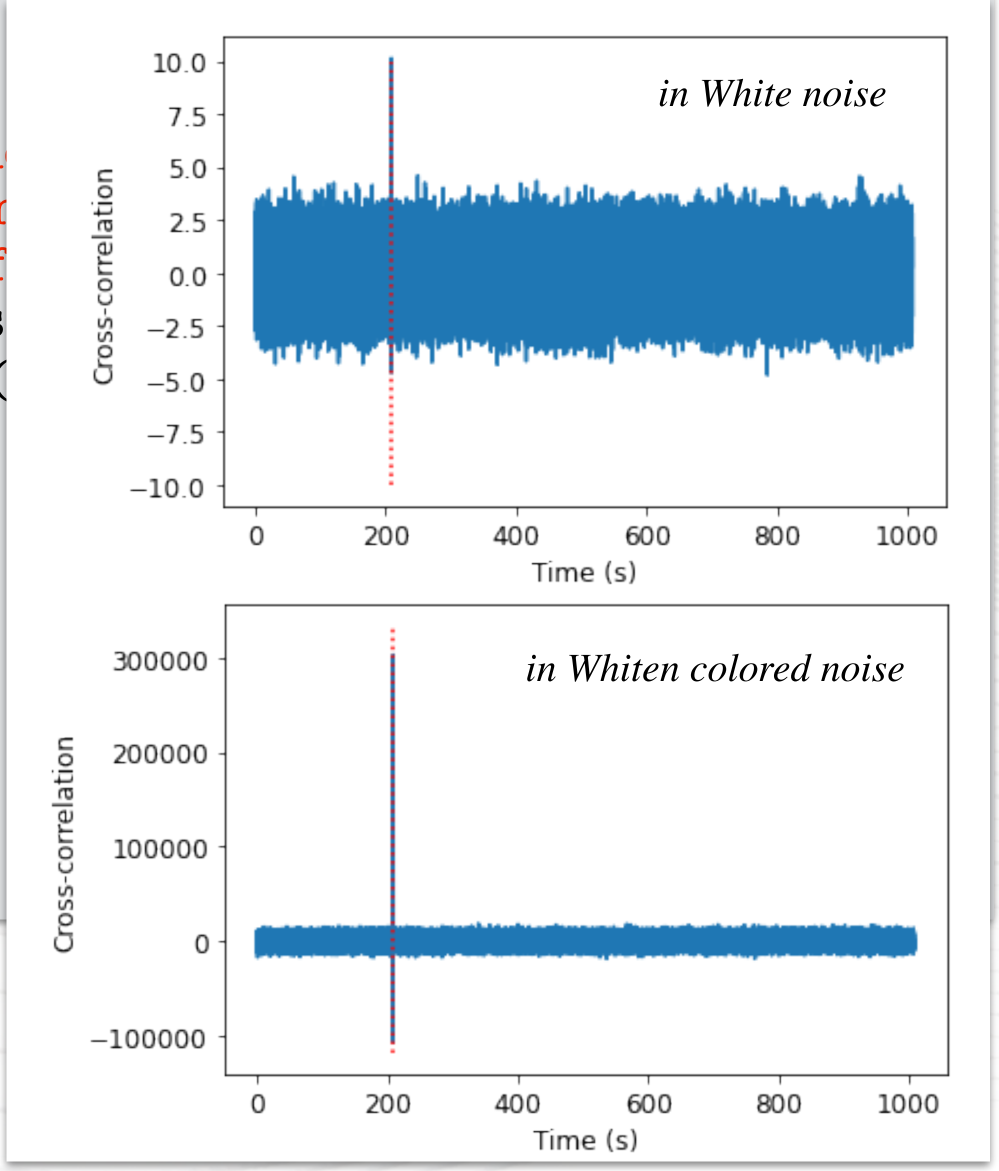
- Then, all we need to do is to “whiten” both the data and the template waveform.

- Why do we need whitening?
  - From the PSD, we can see that the data are very strongly “colored”.
  - We can “whiten” the data suppressing the extra noise at low frequencies to better see the weak signals in the most sensitive band.
  - Whitening is always one of the first steps in astrophysical data analysis.

- This can be done, in the frequency domain, by dividing by the PSD.
  (This can be done in the time domain as well, but it’s more intuitive in the frequency domain.)

Introduction to matched filtering

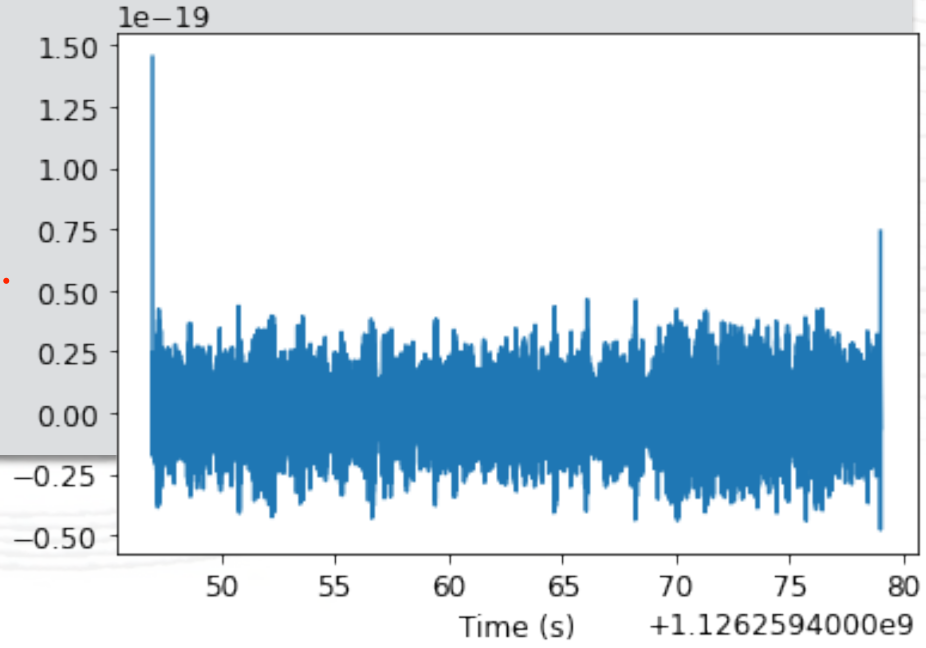
```
# The PSD, sampled properly for the noisy data
delta_f = 1.0 / data_length  # data_length = 1024 seconds
flen = int(sample_rate / (2*delta_f)) + 1  # sample_rate = 1024 samples per second
psd_td = pycbc.psd.aLIGOZeroDetHighPower(flen, delta_f, 0)

# The PSD, sampled properly for the signal
delta_f = sample_rate / float(len(hp))
flen = int(sample_rate / (2*delta_f)) + 1
psd_hp = pycbc.psd.aLIGOZeroDetHighPower(flen, delta_f, 0)

# Convert both noisy data and the signal to frequency domain, and divi
# then covert back to time domain. This “whitens” the data and the sign
# Multiplying the signal template by 1E-21 puts it into realistic units of
data_whitened = (ts.to_frequencyseries() / psd_td**0.5).to_timeseries
hp_whitened = (hp.to_frequencyseries() / psd_hp**0.5).to_timeseries(

# Now let’s re-do the correlation, in the time domain, but with
# whitened data and template.
cross_correlation = numpy.zeros([len(data)-len(hp1)])
hpn = hp_whitened.numpy()
datan = data_whitened.numpy()
for i in range(len(datan) - len(hpn)):
   cross_correlation[i] = (hpn * datan[i:i+len(hpn)]).sum()
```



Matched filtering in action

- Looking for a specific signal in the data
  - If you know what signal you are looking for in the data, then matched filtering is known to be the optimal method in Gaussian noise to extract the signal.
  - Even when the parameters of the signal are unknown, one can test any set of parameters interested in finding.

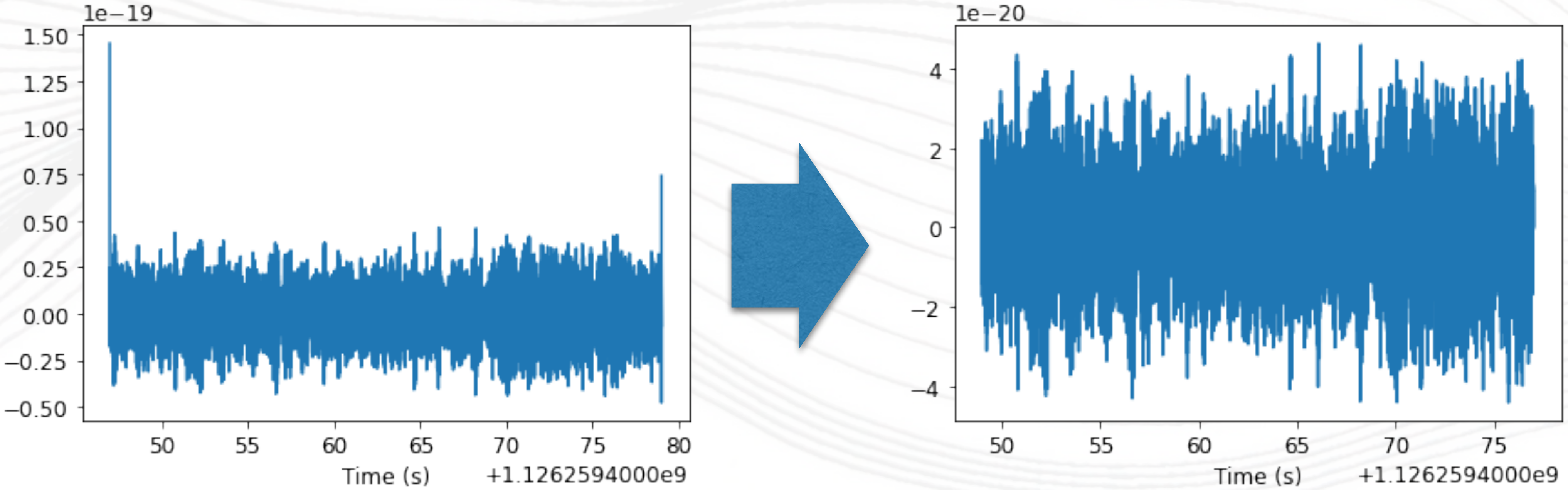
```
# Preconditioning the data.

# The purpose of preconditioning the data is to reduce the dynamic range of the data and to suppress low
frequency behavior that can introduce numerical artifacts. We may also wish to reduce the sample rate of the
data if high frequency content is not important.

from pycbc.catalog import Merger
from pycbc.filter import resample_to_delta_t, highpass

# As an example we use the GW150914 data
merger = Merger(“GW150914”)

# Get the data from the Hanford detector
strain = merger.strain(‘H1’)

# Remove the low frequency content and downsample the data to 2048 Hz.
strain = highpass(strain, 15.0)
strain = resample_to_delta_t(strain, 1.0/2048)
```



Matched filtering in action

- Filter wraparound
  - Note the spike in the data at the boundaries. This is caused by the highpass and resampling stages filtering the data. When the filter is applied to the boundaries, it wraps around to the beginning of the data. Since the data itself has a discontinuity (i.e. it is not cyclic) the filter itself will ring off for a time up to the length of the filter.
  - Even if a visible transient is not seen, we want to avoid filters that act on times which are not causally connected. To avoid this, we trim the ends of the data sufficiently to ensure that they do not wrap around the input. We will enforce this requirement in all steps of our filtering.

```
# Remove 2 seconds of data from both the beginning and end
conditioned = strain.crop(2, 2)
```



Matched filtering in action

- Calculate the power spectral density
  - Optimal matched filtering requires weighting the frequency components of the potential signal and data by the noise amplitude. We can view this as filtering the data with the time series equivalent of 1 / PSD. To ensure that we can control the effective length of the filter, we window the time domain equivalent of the PSD to a specific length.

```
from pycbc.psd import interpolate, inverse_spectrum_truncation

# We use 4 second samples of our time series in Welch method.
psd = conditioned.psd(4)

# Now that we have the psd we need to interpolate it to match our data and then limit the filter length of 1 / PSD.
psd = interpolate(psd, conditioned.delta_f)

# 1/PSD will now act as a filter with an effective length of 4 seconds.
# Since the data has been highpassed above 15 Hz, and will have low values below this, we need to inform the
function to not include frequencies below the frequency
psd = inverse_spectrum_truncation(psd, int(4*conditioned.sample_rate), low_frequency_cutoff=15)
```

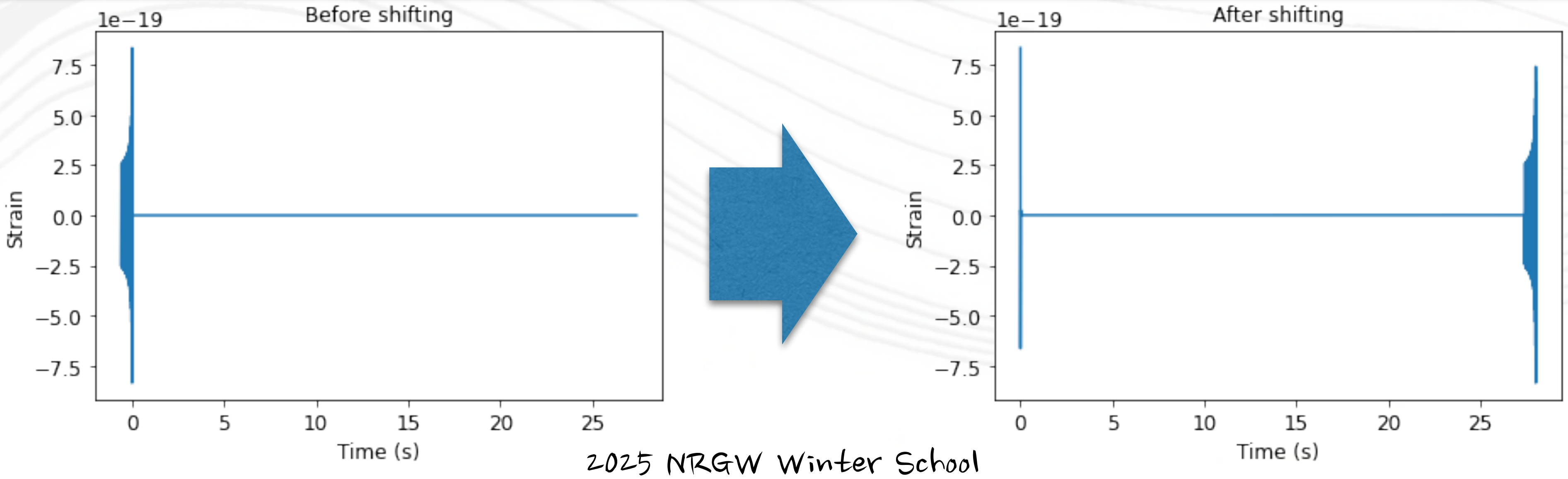
Matched filtering in action

- Make our signal model
  - In this case, we “know” what the signal parameters are. In a real search, we would grid over the parameters and calculate the SNR time series for each one.
  - We assume equal masses and non-rotating black holes.

```
from pycbc.waveform import get_td_waveform

m = 36  # Solar masses
hp, hc = get_td_waveform(approximant=“SEOBNRv4_opt”, mass1=m, mass2=m, delta_t=conditioned.delta_t,
                                              f_lower=20)

# Resize the vector to match our data
hp.resize(len(conditioned))
```

- The waveform begins at the start of the vector, so if we want the SNR time series to correspond to the approximate merger location (time), we need to shift the data so that the merger is approximately at the first bin of the data.

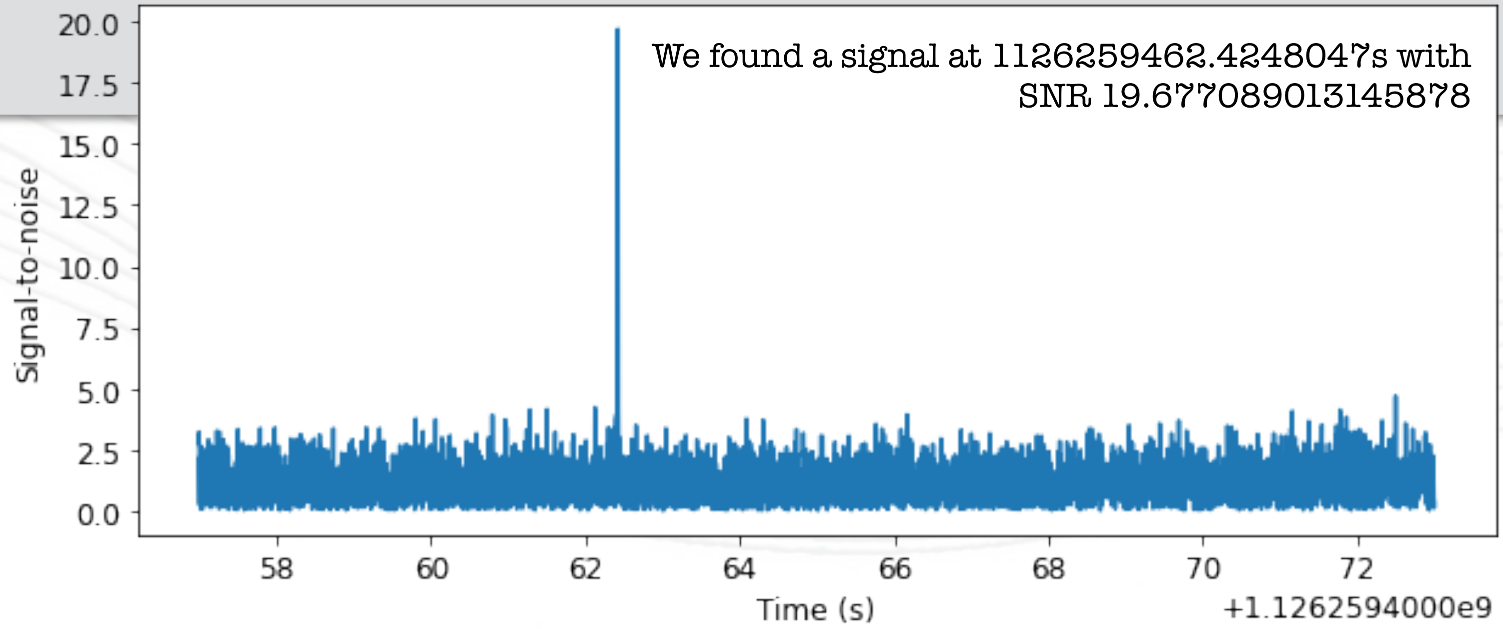
```
template = hp.cyclic_time_shift(hp.start_time)
```



Matched filtering in action

- Calculating the signal-to-noise time series
  - We’ll take care to handle issues of filter corruption / wraparound by truncating the output time series. We need to account for both the length of the template and 1/PSD.

```
from pycbc.filter import matched_filter
import numpy

snr = matched_filter(template, conditioned, psd=psd, low_frequency_cutoff=20)

# Remove time corrupted by the template filter and the psd filter. We remove 4 seconds at the beginning and end for the PSD filtering.
# And we remove 4 additional seconds at the beginning to account for the template length (this is somewhat generous for so short a template). A longer signal such as from a BNS, would require much more padding at the beginning of the vector
snr = snr.crop(4 + 4, 4)

peak = abs(snr).numpy().argmax(). # returns the index of peak SNR
snrp = snr[peak]
time = snr.sample_times[peak]
```

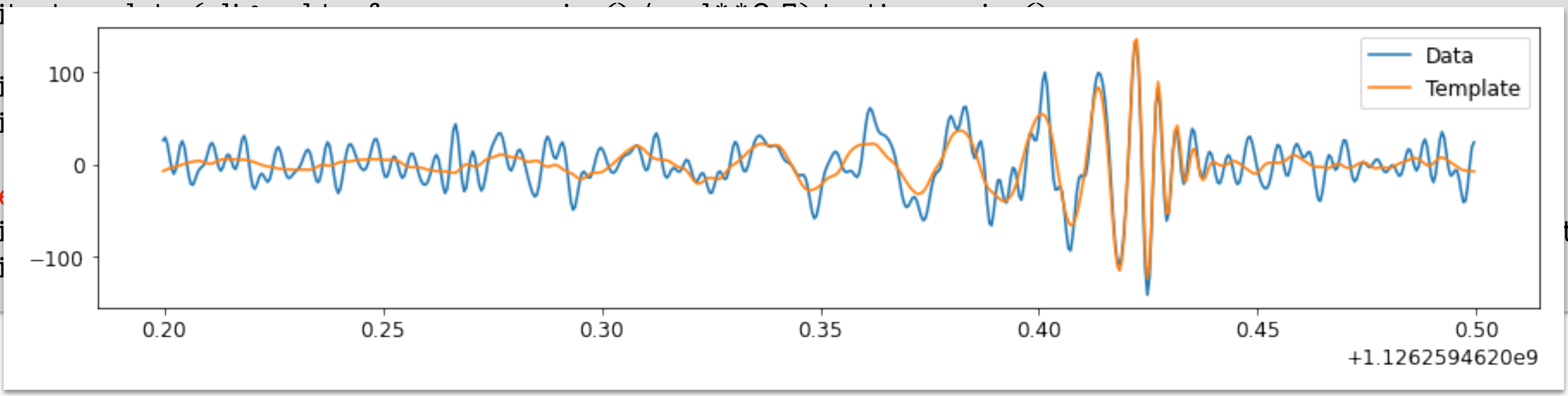


We found a signal at 1126259462.4248047s with SNR 19.677089013145878

Matched filtering in action

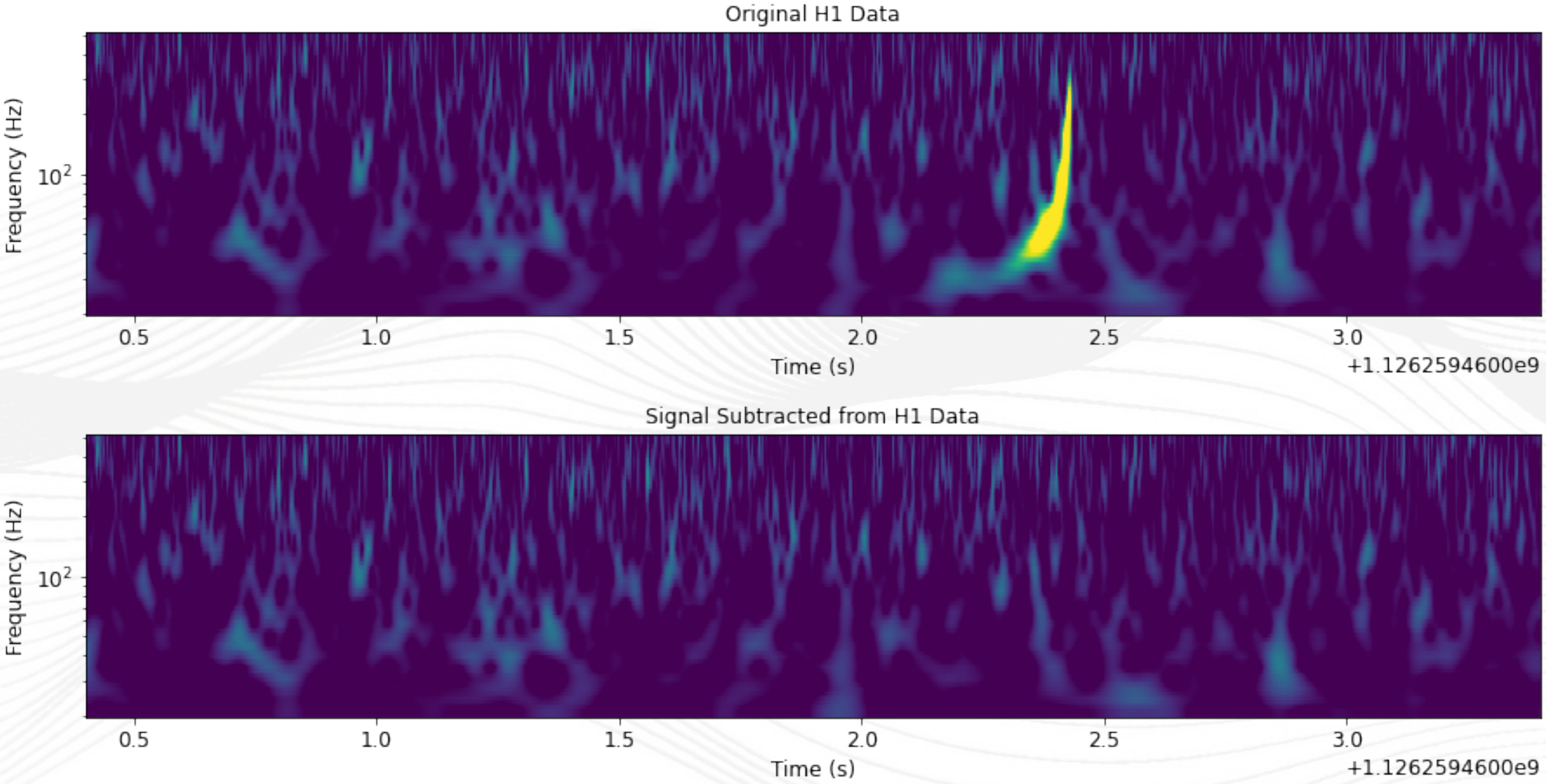
- Visualize the overlap between the signal and the data

```
from pycbc.filter import sigma

# Shift the template to the peak time
dt = time - conditioned.start_time
aligned = template.cyclic_time_shift(dt)

# Scale the template so that it would have SNR 1 in this data
aligned /= sigma(aligned, psd=psd, low_frequency_cutoff=20.0)

# Scale the template amplitude and phase to the peak value
aligned = (aligned.to_frequencyseries() * snrp).to_timeseries()
aligned.start_time = conditioned.start_time

# To compare the data and signal on equal footing, and to concentrate on the frequency range that is important,
we whiten both the template and the data.
# Then, bandpass both the data and template between 30-300 Hz. In this way, any signal that is in the data is
transformed in the same way that the template is.
white_data = (conditioned.to_frequencyseries() / psd**0.5).to_timeseries()
whi

whi
whi

# Se
whi                                                                                                   time
whi
```



Matched filtering in action

- Subtracting the signal from the data
  - Now that we’ve aligned the template we can simply subtract it. Let’s see it how that looks in the time-frequency plots, *a.k.a. spectrograms*.

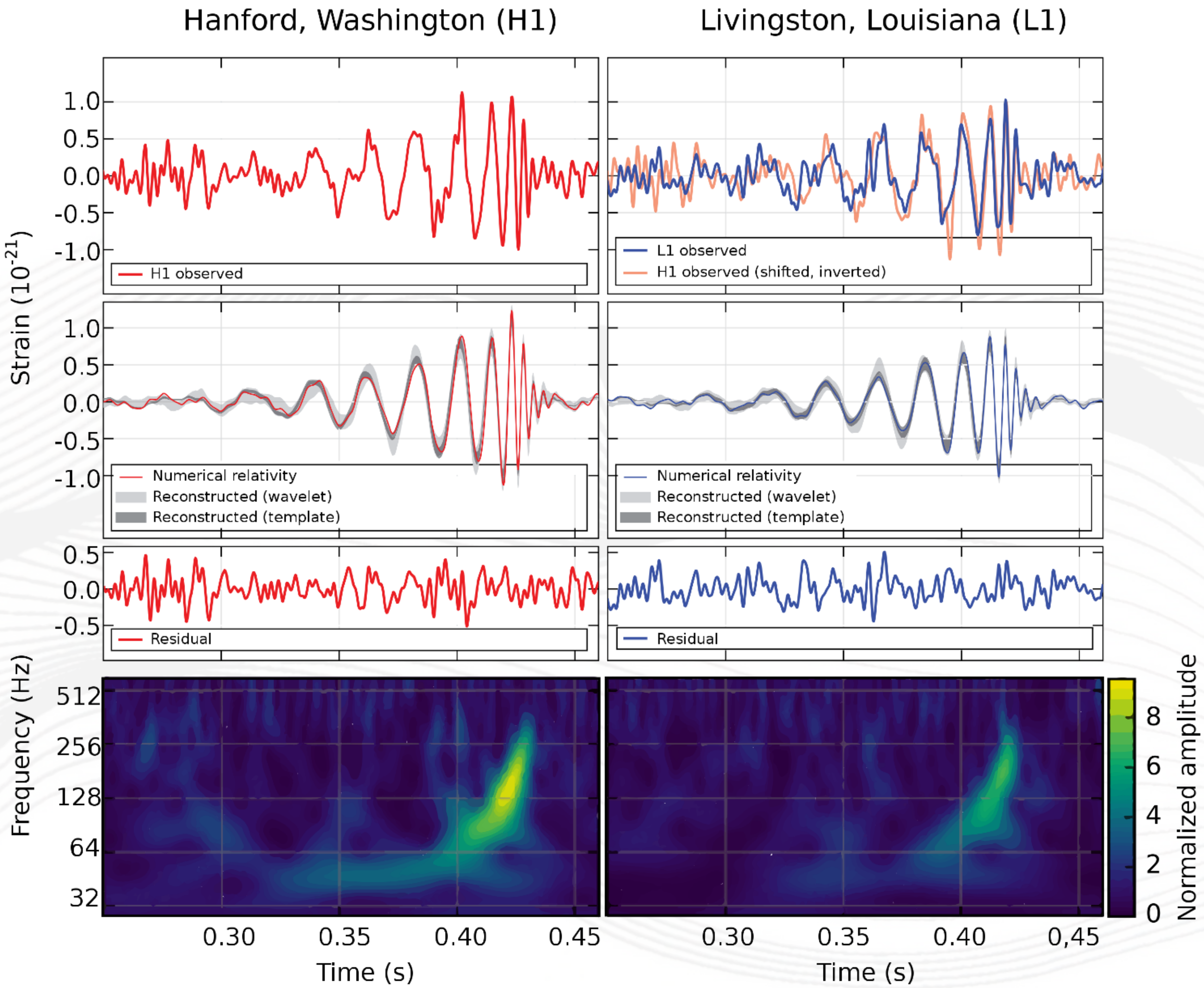


Original H1 Data

Signal Subtracted from H1 Data

Matched filtering in action

- Subtracting the signal from the data in reality.



[Figure from Abbott+ (2016, PRL)]

Visualization w/ Q-transform

- How to generate spectrogram?
  - Fundamental method: (inverse) Fourier transform
  - Advanced (and convenient) method: Q-transform [Chatterji+ (2004, CQG)]

- Q-transform
  - $Q = f_c / \sigma_f$: dimensionless quality factor, where

$$\text{central frequency } f_c = 2\int_0^{\infty} f \frac{|h(t)|^2}{\|h\|^2} df$$

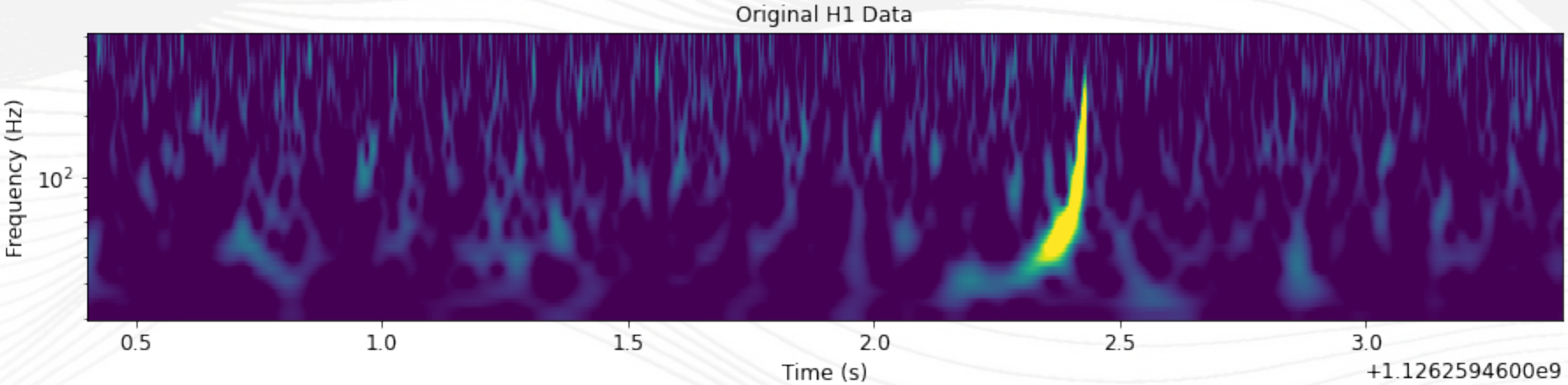
$$\text{bandwidth } \sigma_f^2 = 2\int_0^{\infty} (f - f_c) \frac{\|h(t)\|^2}{|h|^2} df$$

$$\|h\|^2 = \int_{-\infty}^{+\infty} |h(t)|^2 dt = \int_{-\infty}^{+\infty} |\tilde{h}(f)|^2 df$$

Visualization w/ Q-transform

- Use a built-in function!

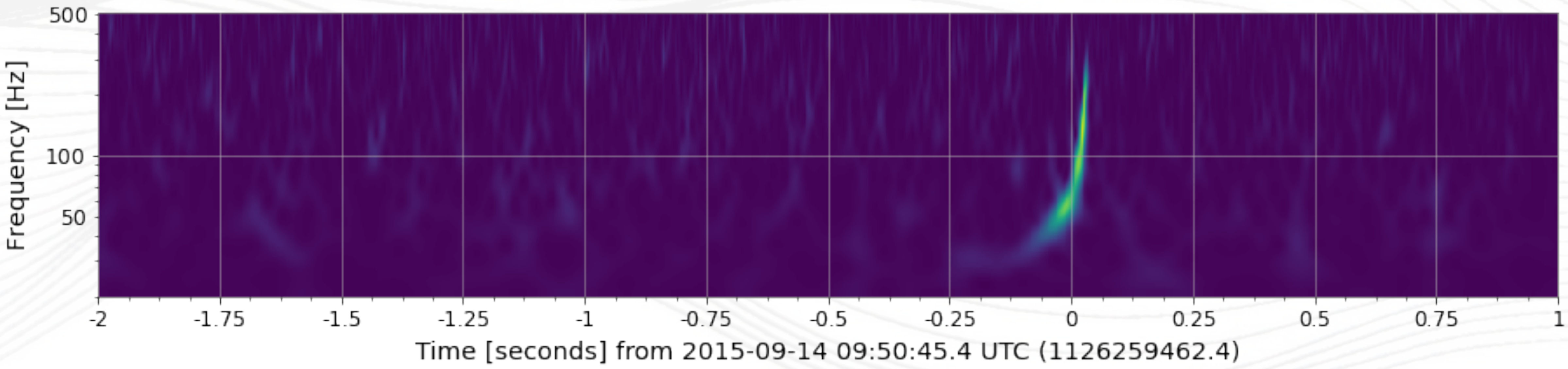
```
# delta_t: time resolution to interpolate to; logfsteps: number of steps for log interpolation;
# qrange: range of q; frange: range of frequency
# details: https://pycbc.org/pycbc/latest/html/_modules/pycbc/types/timeseries.html#TimeSeries.qtransform
t, f, p = conditioned.whiten(4, 4).qtransform(delta_t=.001, logfsteps=100, qrange=(8, 8), frange=(20, 512))
pylab.figure(figsize=[15, 3])
pylab.title(‘Original H1 Data’)
pylab.pcolormesh(t, f, p**0.5, vmin=1, vmax=6, shading='auto')
pylab.yscale('log')
pylab.xlabel('Time (s)')
pylab.ylabel('Frequency (Hz)')
pylab.xlim(merger.time - 2, merger.time + 1)
pylab.show()
```



Visualization w/ Q-transform

- Alternatively, if you prefer gwpy, you can generate spectrogram with gwpy too.

```
# This script is tested with gwpy=2.0.2
from gwpy.timeseries import TimeSeries
from gwosc.datasets import event_gps

gps = event_gps('GW150914')
segment = (int(gps) - 30, int(gps) + 2)
hdata = TimeSeries.fetch_open_data('H1', *segment, verbose=True, cache=True)
hq = hdata.q_transform(frange=(20, 512), qrange=(8,8), outseg=(gps-2,gps+1))

plot = hq.plot(figsize=[15, 3])
ax = plot.gca()
ax.set_epoch(gps)
ax.set_yscale('log')
ax.colorbar(label="Normalised energy")
```



Signal consistency and significance

- How well is the data actually fitting our model?
  - χ^2-based signal consistency test is a standard one for the purpose.

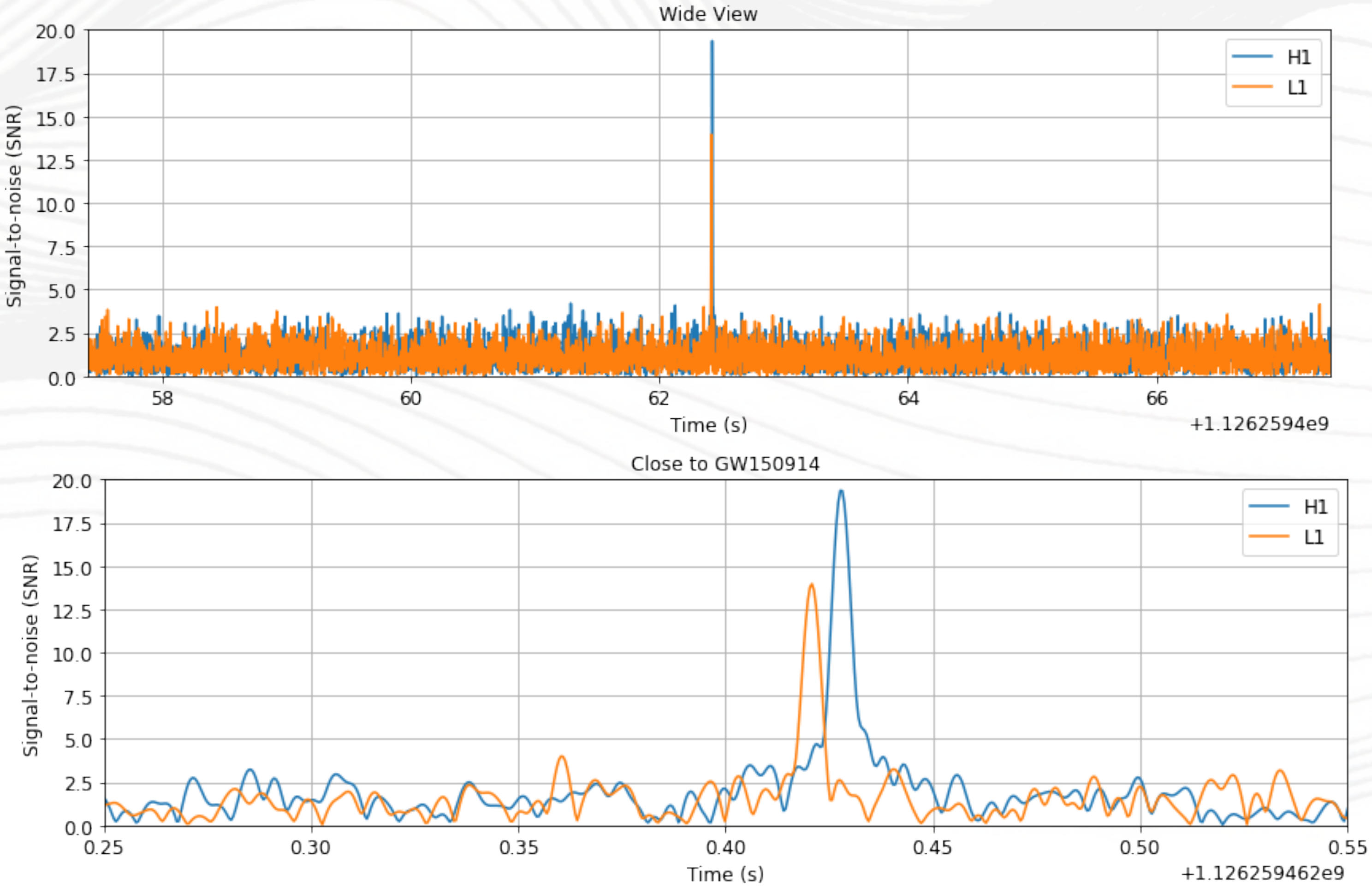
$$\chi^2 = \sum_{i=0}^{p} (\rho_i - \rho/p)^2$$

  - Schematically, we chop up our template into p number of bins and see how much each contributes to the SNR (ρ_i).
- Now, we use both LIGO-Hanford (H1) and LIGO-Livingston (L1) data of GW150914.

```
merger = Merger(“GW150914”)

ifos = [‘H1’, ‘L1’]
from pycbc.vetos import power_chisq
data = { }
psd = { }

for ifo in ifos:
   ts = merger.strain(ifo).highpass_fir(20, 512)
   data[ifo] = resample_to_delta_t(ts, 1.0/2048).crop(2, 2)

   # Estimate the power spectral density of the data
   p = data[ifo].psd(4)
   p = interpolate(p, data[ifo].delta_f)
   p = inverse_spectrum_truncation(p, int(2 * data[ifo].sample_rate), low_frequency_cutoff=20.0)
   psd[ifo] = p
```

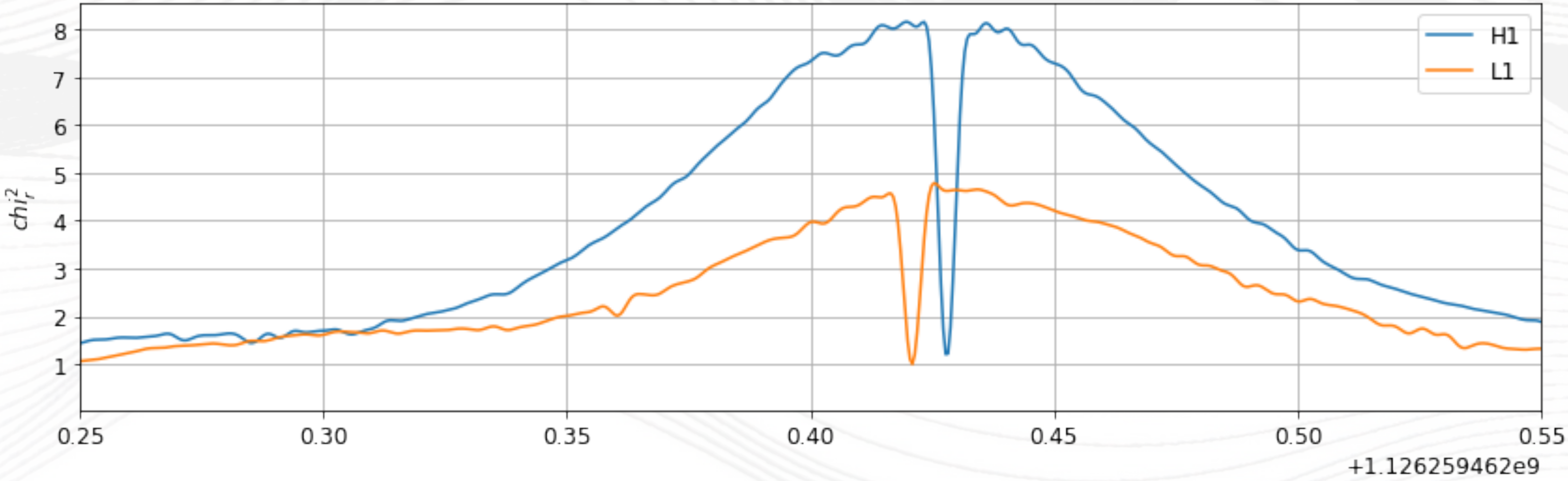
Signal consistency and significance

```
# Calculate the component mass of each black hole in the detector frame
cmass = (merger.median1d(“mass1”)+merger.meedian1d(“mass2”)) / 2  # This is in the source frame
cmass *= (1 + merger.median1d(“redshift”)). # apply redshift to get to the detector frame

hp, _ = get_fd_waveform(approximant=“IMRPhenomD”, mass1=cmass, mass2=cmass, f_lower=20.0,
                                    delta_f=data[ifo].delta_f)
hp.resize(len(psd[ifo]))

# For each observatory, use this template to calculate the SNR time series
snr = { }
for ifo in ifos:
   snr[ifo] = matched_filtering(hp, data[ifo], psd=psd[ifo], low_frequency_cutoff=20)
   snr[ifo] = snr[ifo].crop(4+4, 4)
```



Signal consistency and significance

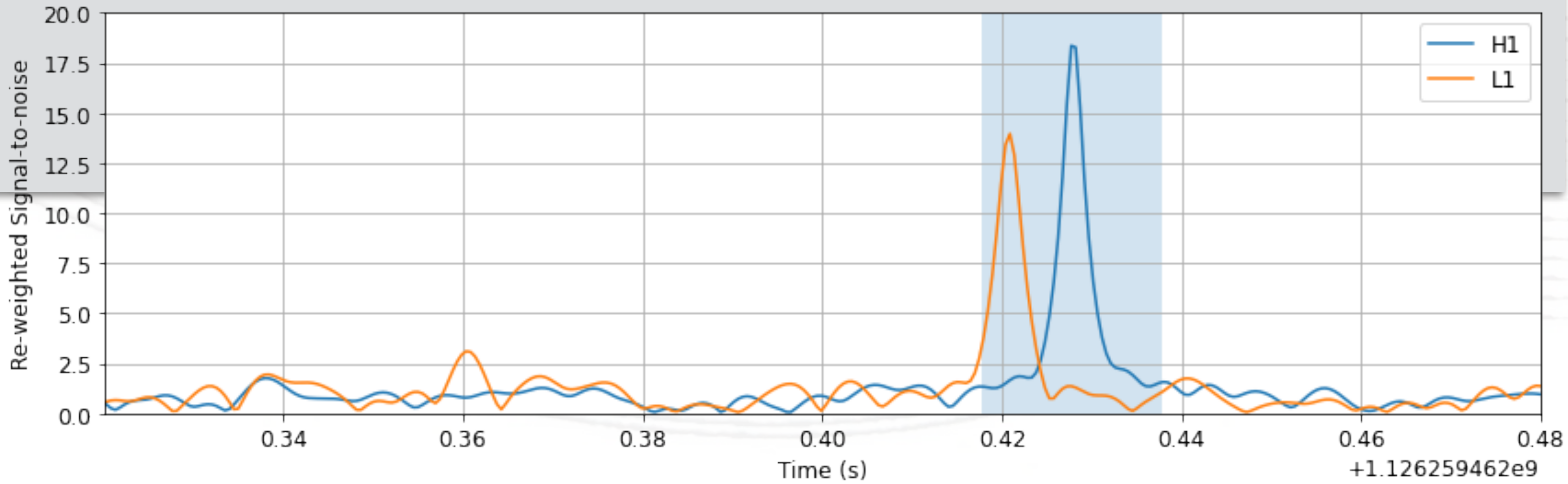
```
from pycbc.vetoes import power_chisq

chisq = { }
for ifo in ifos:
    # The number of bins to use. In principle, this choice is arbitrary. In practice, this is empirically tuned.
    nbins = 26
    chisq[ifo] = power_chisq(hp, data[ifo], nbins, psd[ifo], low_frequency_cutoff=20.0)
    chisq[ifo] = chisq[ifo].crop(4+4, 4)

    dof = nbins * 2 - 2
    chisq[ifo] /= dof
```



Signal consistency and significance

- We see the SNR of L1 is lower than that of H1. Let’s see the significance of L1 event.

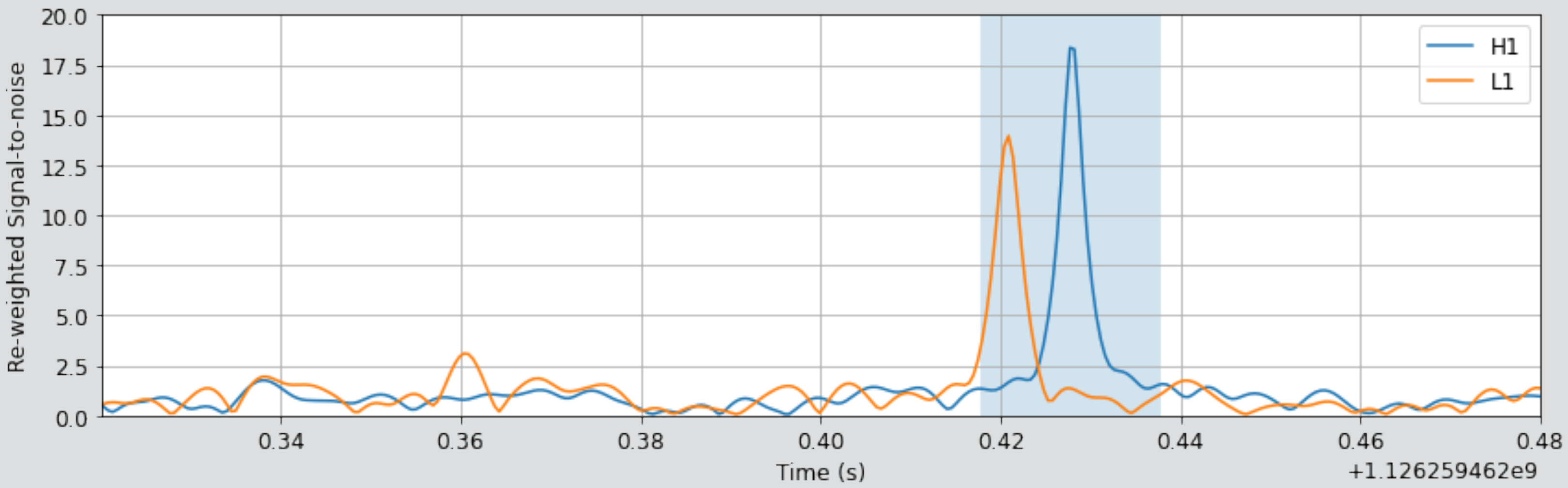
```
from pycbc.detector import Detector

# Calculate the time of flight between the LIGO-Livingston and LIGO-Hanford
d = Detector(“L1”)
tof = { }
tof[‘H1’] = d.light_travel_time_to_detector(Detector(“H1”))

# Record the time of the peak in the LIGO-Hanford
ptime = { }
ptime[‘H1’] = snr[‘H1’].sample_times[snr[‘H1’].argmax()]

# Calculate the span of time that LIGO-Livingston peak could in principle happen in from time of flight
considerations.
start = ptime[‘H1’] - tof[‘H1’]
end = ptime[‘H1’] + tof[‘H1’]

# convert the times to indices along with how large the region is in number of samples
window_size = int((end - start) * snr[‘L1’].sample_rate)
sidx = int((start - snr[‘L1’].start_time) * snr[‘L1’].sample_rate)
eidx = sidx + window_size

# Calculate the “on-source” peak
onsource = snr[‘L1’][sidx:eidx].max()
```

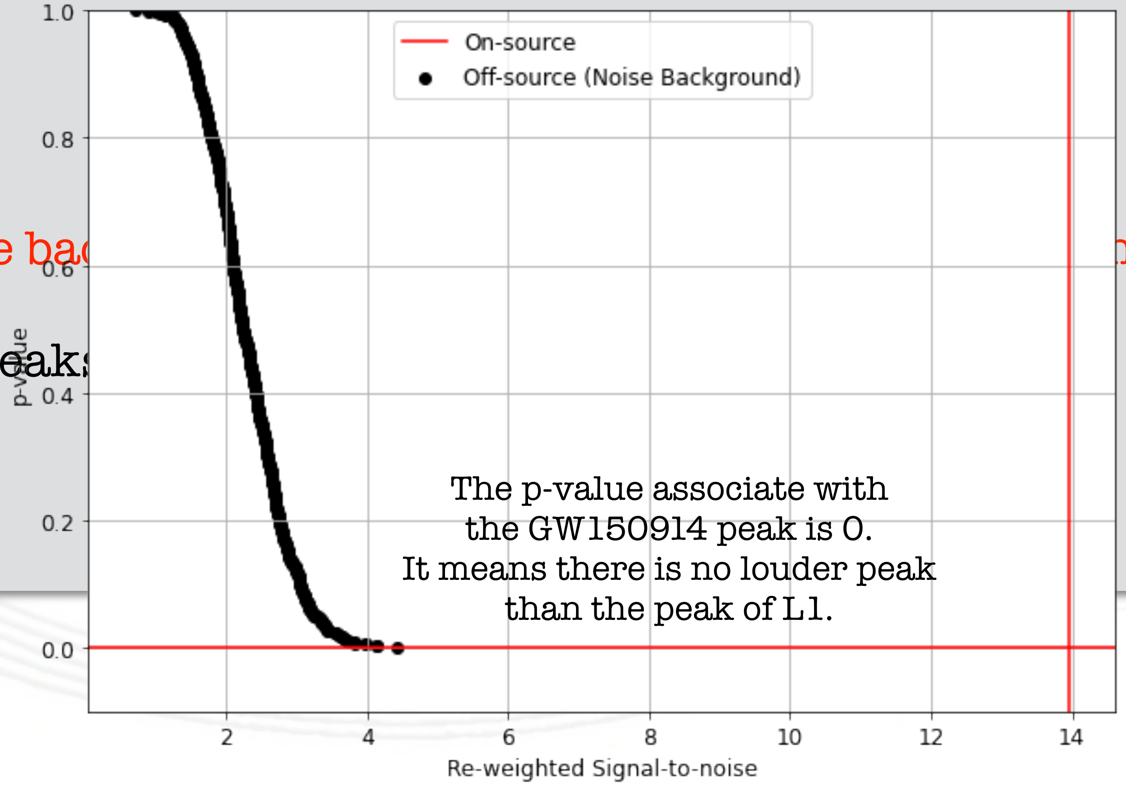


Signal consistency and significance

- Now that we’ve calculated the on-source peak, we should calculate the background peak values.
  - We do this by chopping up the time series into chunks that are the same size as our on-source and repeating the same peak finding (max) procedure.

```
import numpy

peaks = []
i = 0
while i + window_size < len(snr[‘L1’]):
   p = snr[‘L1’][i:i+window_size].max()
   peaks.append(p)
   i += window_size

   # skip past the onsource time
   if abs(i - sidx) < window_size:
      i += window_size * 2
peaks = numpy.array(peaks)

# The p-value is just the number of samples observed in the bac                                 he on-
source divided by the number of samples.
pcurve = numpy.arange(1, len(peaks)+1)[::-1] / float(len(peaks
peaks.sort()

pvalue = (peaks > onsource).sum() / float(len(peaks))
```

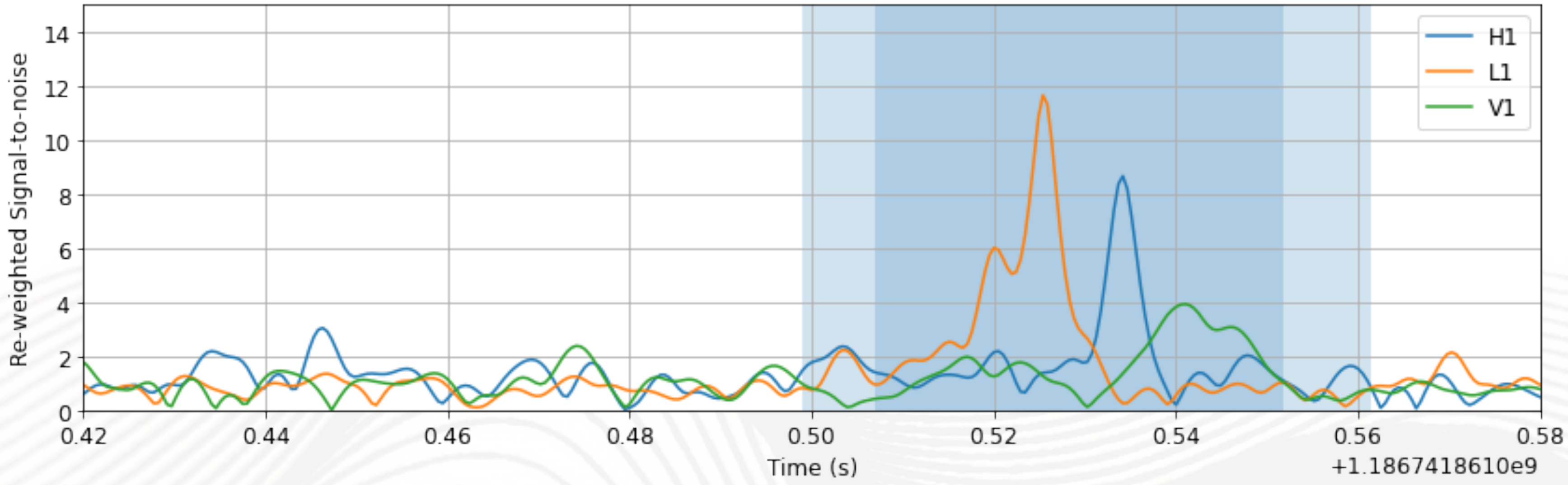


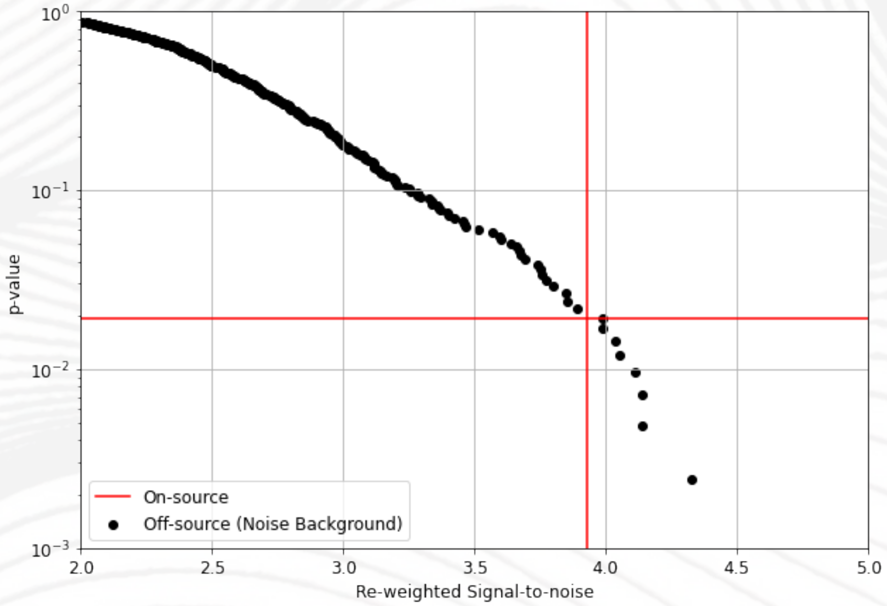


The p-value associate with the GW150914 peak is 0. It means there is no louder peak than the peak of L1.

Signal consistency and significance

- However, we may have $p > 0$ if a peak of any detector is not that much significant.
- Example: GW170814 observed by the LIGO observatories and Virgo





```
The p-value associate with the GW170814 peak of Virgo is
0.019277108433734494.
```

- We find a peak in Virgo as large as the observed one has an approximately 2% chance of occurring due to the noise alone.
- If $p < 0.05$, we may reject the null hypothesis that the observed peak is due to noise alone.

Summary

- We have demonstrated how to find a candidate GW signal from noisy data.

  (1) Estimating PSD from noisy data

  (2) Preparing template waveform

  (3) Whitening

  (4) Computing the cross-correlation (signal-to-noise ratio) between the template and the data

  (5) Testing consistency between the template and the data with χ^2 test

  (6) Evaluating significance with p-value estimation

Thank you!