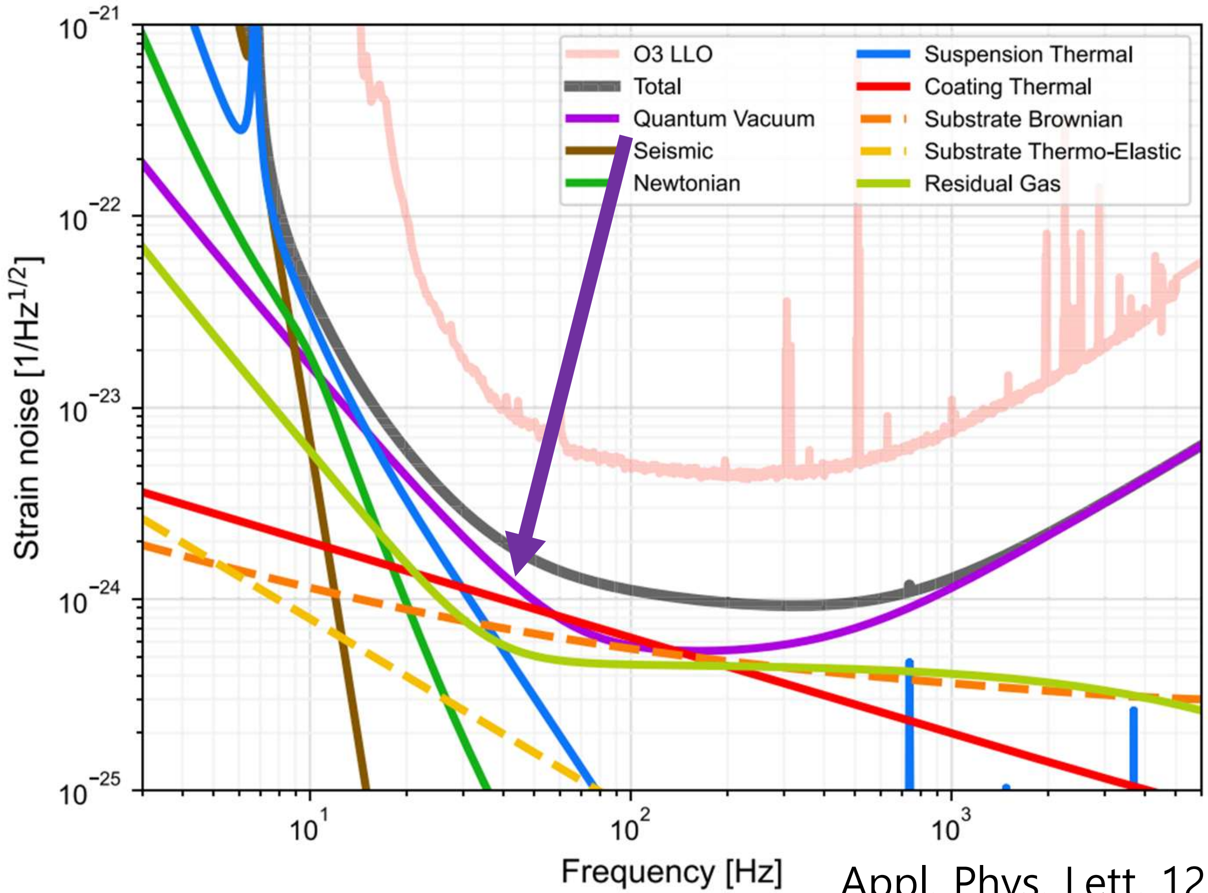
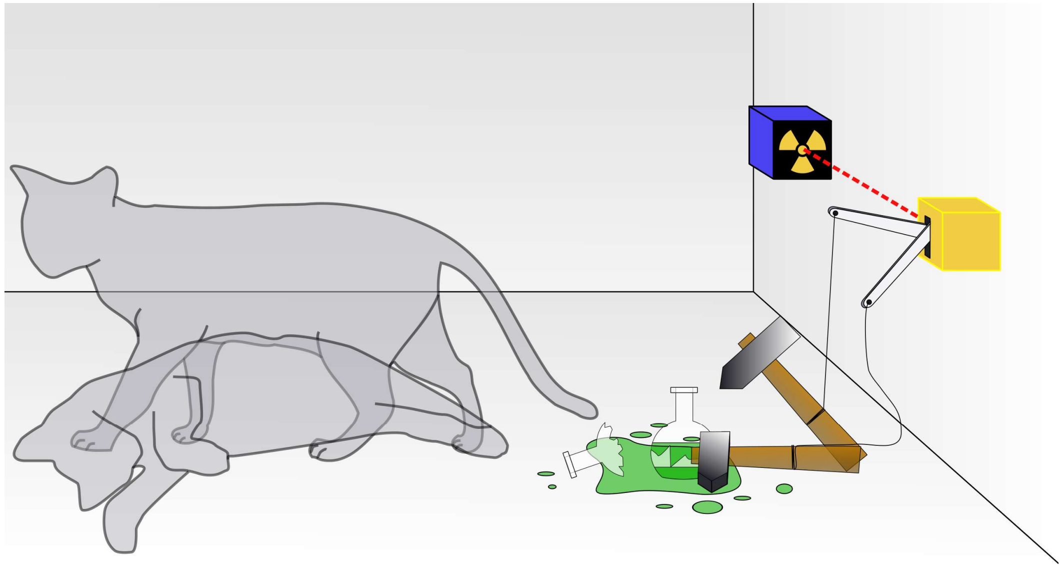
3. Quantum Noise of Gravitational Wave Detector

LIGO Sensitivity

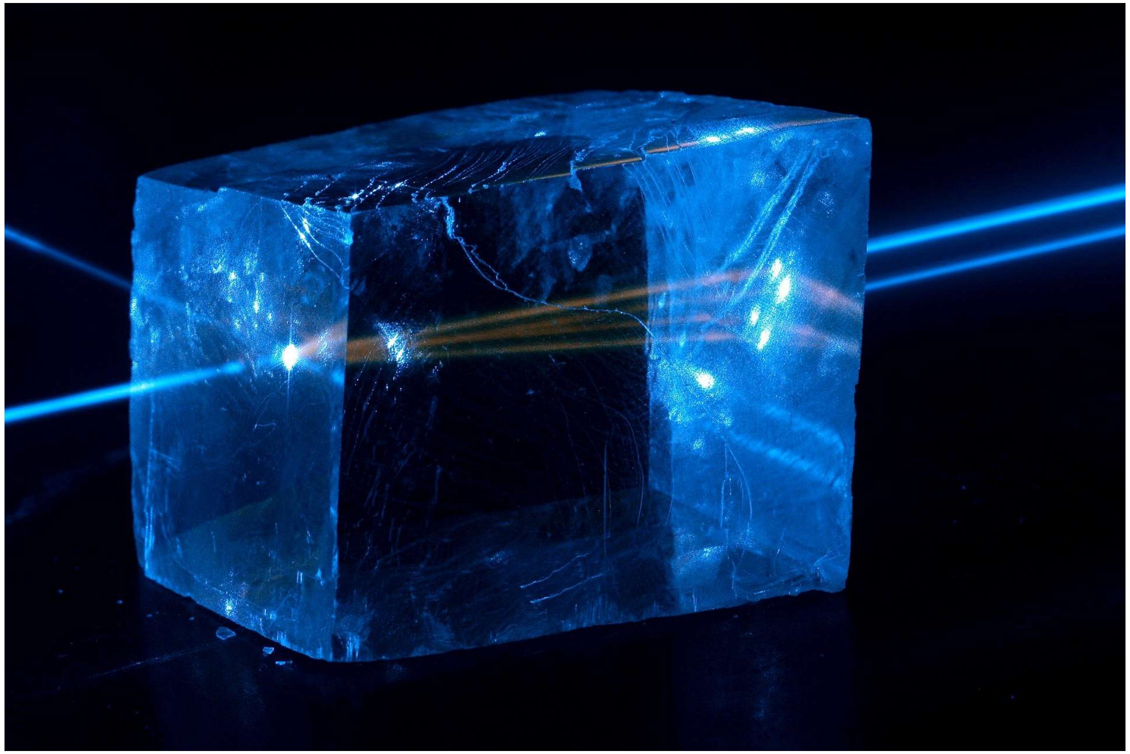


Appl. Phys. Lett. 122, 110502 (2023)

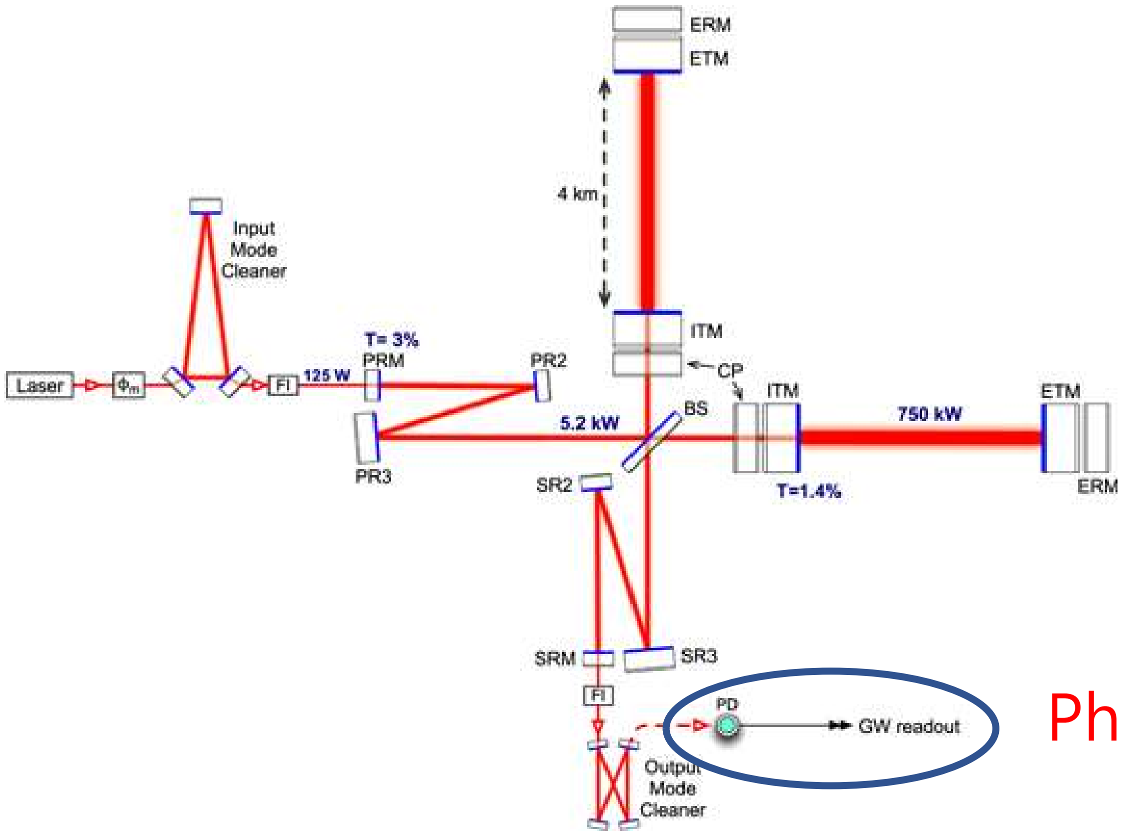
Quantum Noise??



Wave-Particle Duality

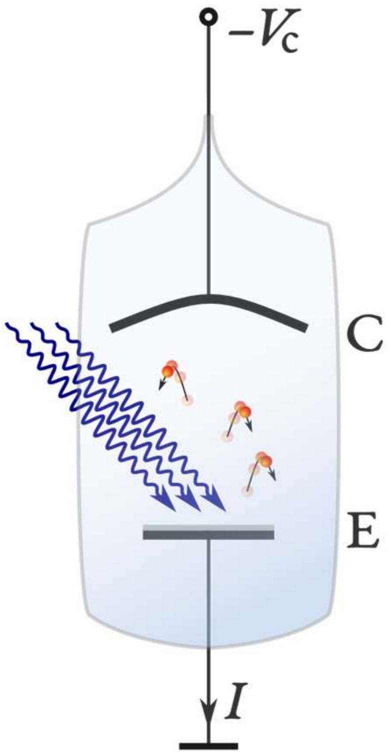


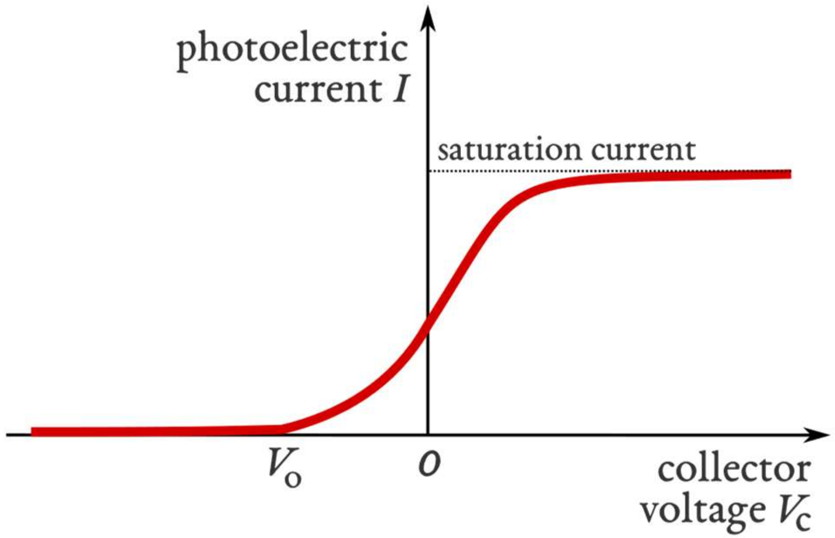
Gravitational Wave Detector



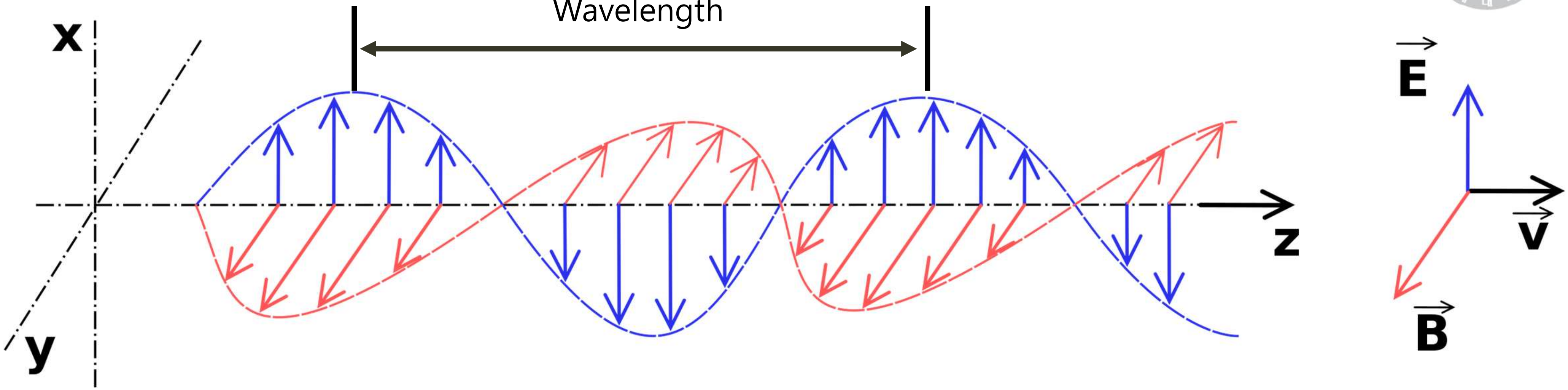
Photon counting

Photoelectric Effect



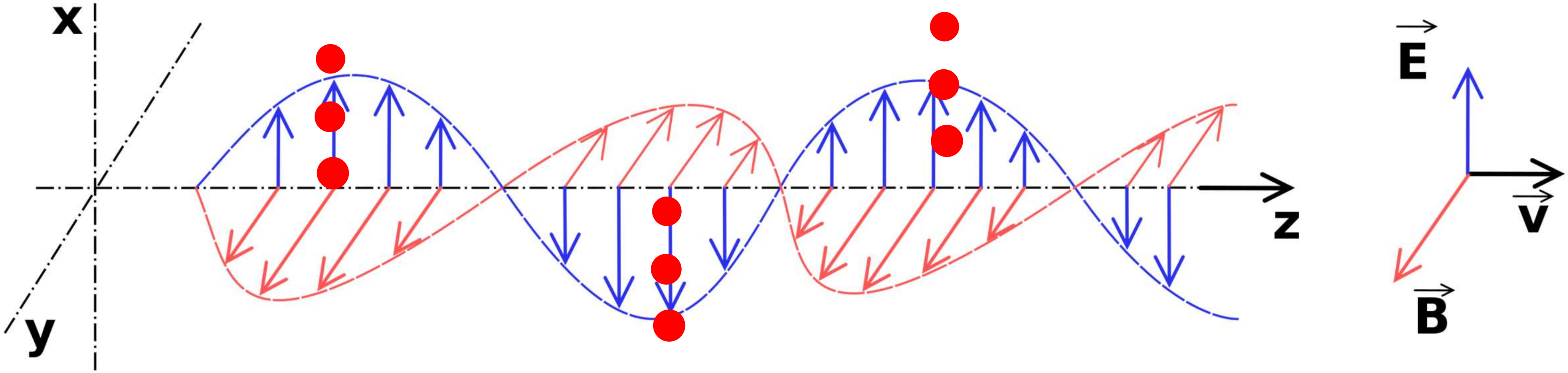


Light(Electromagnetic Wave)



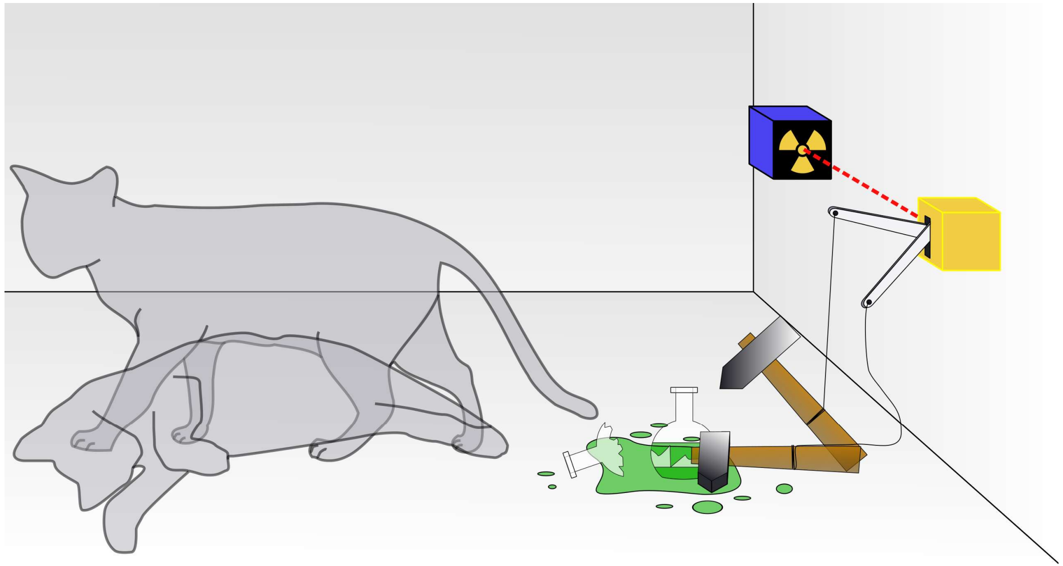
$$\left|\vec{E}\right| \propto Intensity\ of\ Light$$

Wave-Particle Duality



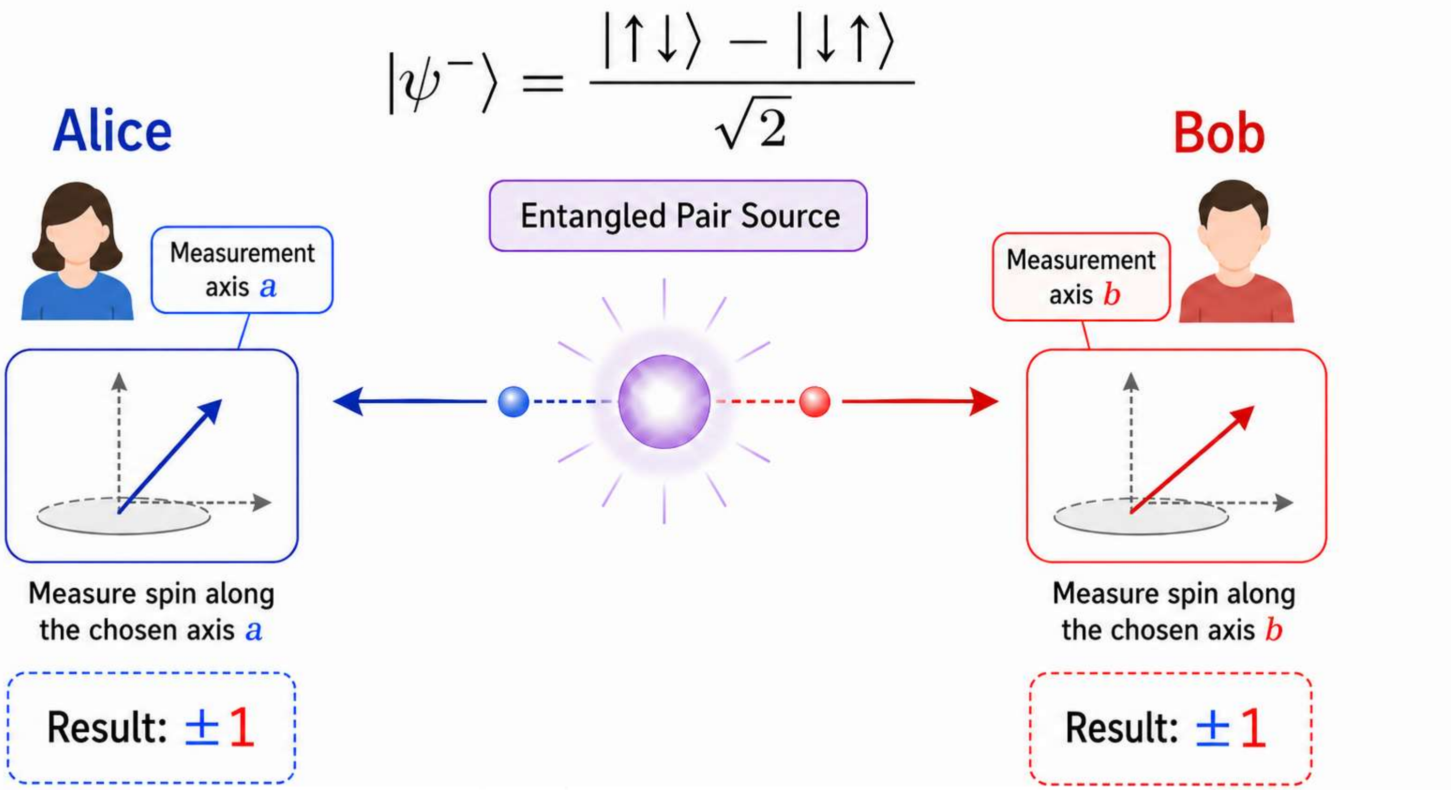
Observed Photon

Schrödinger's Cat



<Dead>
or
<Alive>

Bell's Inequality



$$|\langle A_0 B_0\rangle + \langle A_0 B_1\rangle + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle| \leq 2$$

CHSH inequality

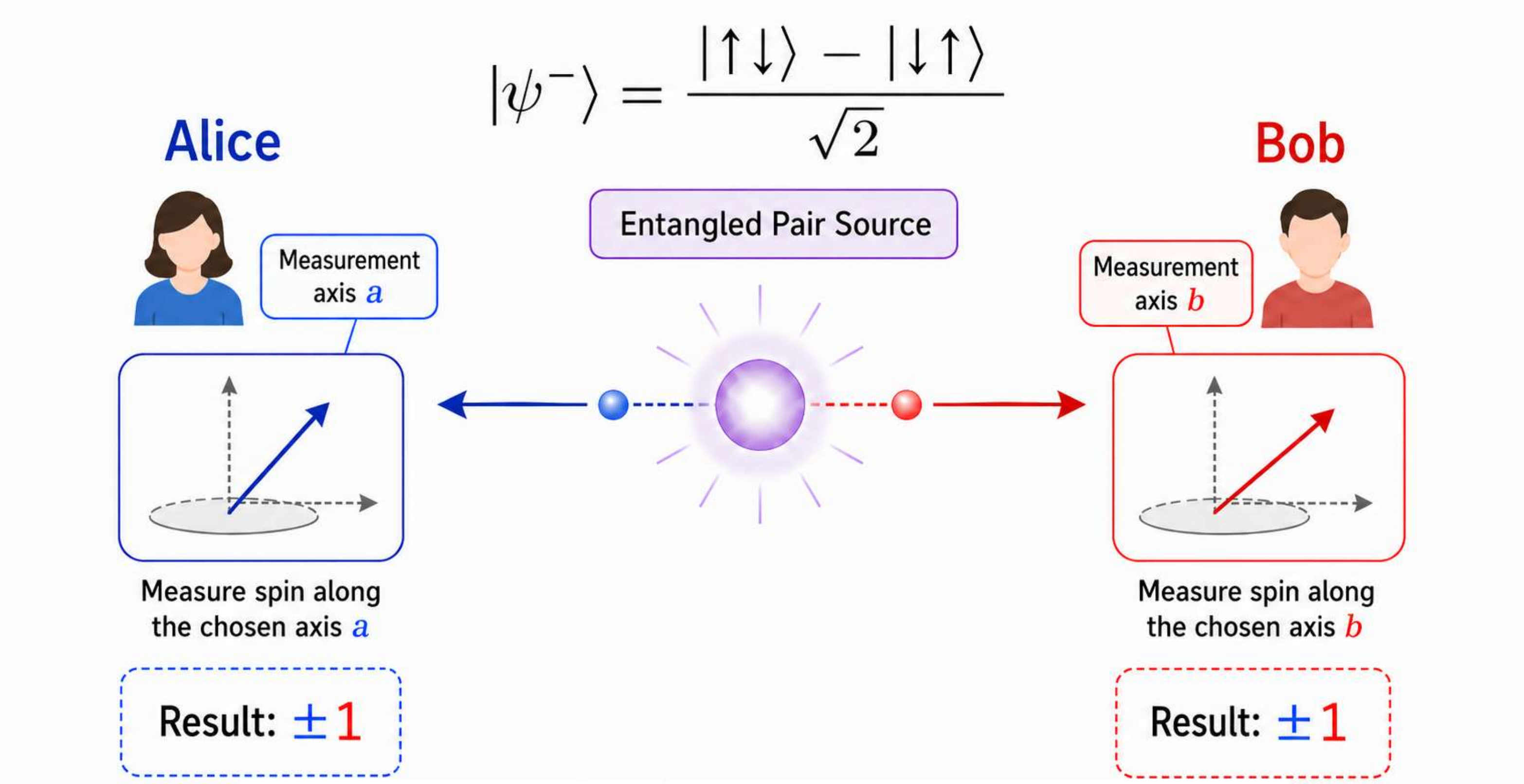
Bell's Inequality

$$|\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle| \leq 2$$

All deterministic local hidden-variable assignments

Case	A_0	A_1	B_0	B_1	A_0B_0	A_0B_1	A_1B_0	$-A_1B_1$	S
1	+1	+1	+1	+1	+1	+1	+1	−1	+2
2	+1	+1	+1	−1	+1	−1	+1	+1	+2
3	+1	+1	−1	+1	−1	+1	−1	−1	−2
4	+1	+1	−1	−1	−1	−1	−1	+1	−2
5	+1	−1	+1	+1	+1	+1	−1	+1	+2
6	+1	−1	+1	−1	+1	−1	−1	−1	−2
7	+1	−1	−1	+1	−1	+1	+1	+1	+2
8	+1	−1	−1	−1	−1	−1	+1	−1	−2
9	−1	+1	+1	+1	−1	−1	+1	−1	−2
10	−1	+1	+1	−1	−1	+1	+1	+1	+2
11	−1	+1	−1	+1	+1	−1	−1	−1	−2
12	−1	+1	−1	−1	+1	+1	−1	+1	+2
13	−1	−1	+1	+1	−1	−1	−1	+1	−2
14	−1	−1	+1	−1	−1	+1	−1	−1	−2
15	−1	−1	−1	+1	+1	−1	+1	+1	+2
16	−1	−1	−1	−1	+1	+1	+1	−1	+2

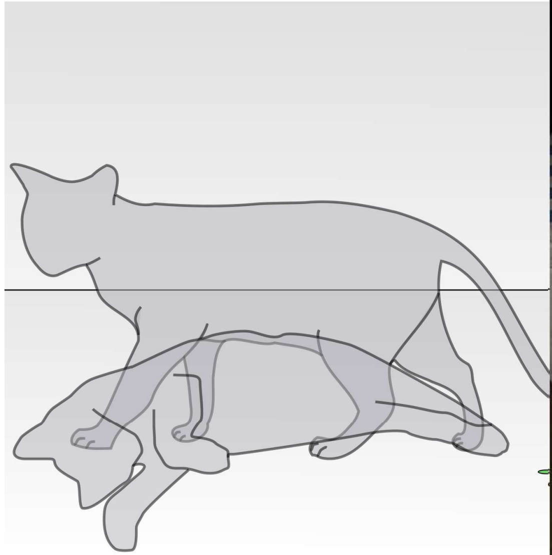
Bell's Inequality



$$|\langle A_0 B_0\rangle + \langle A_0 B_1\rangle + \langle A_1 B_0\rangle - \langle A_1 B_1\rangle| \leq 2$$

CHSH inequality

Schrödinger's Cat



Whether Schrödinger's cat is alive or dead is not predetermined.

Schrödinger Equation

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

Wave Eq

Measure
(Hamiltonian)

For particle

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

Kinetic + Potential

Schrödinger Equation

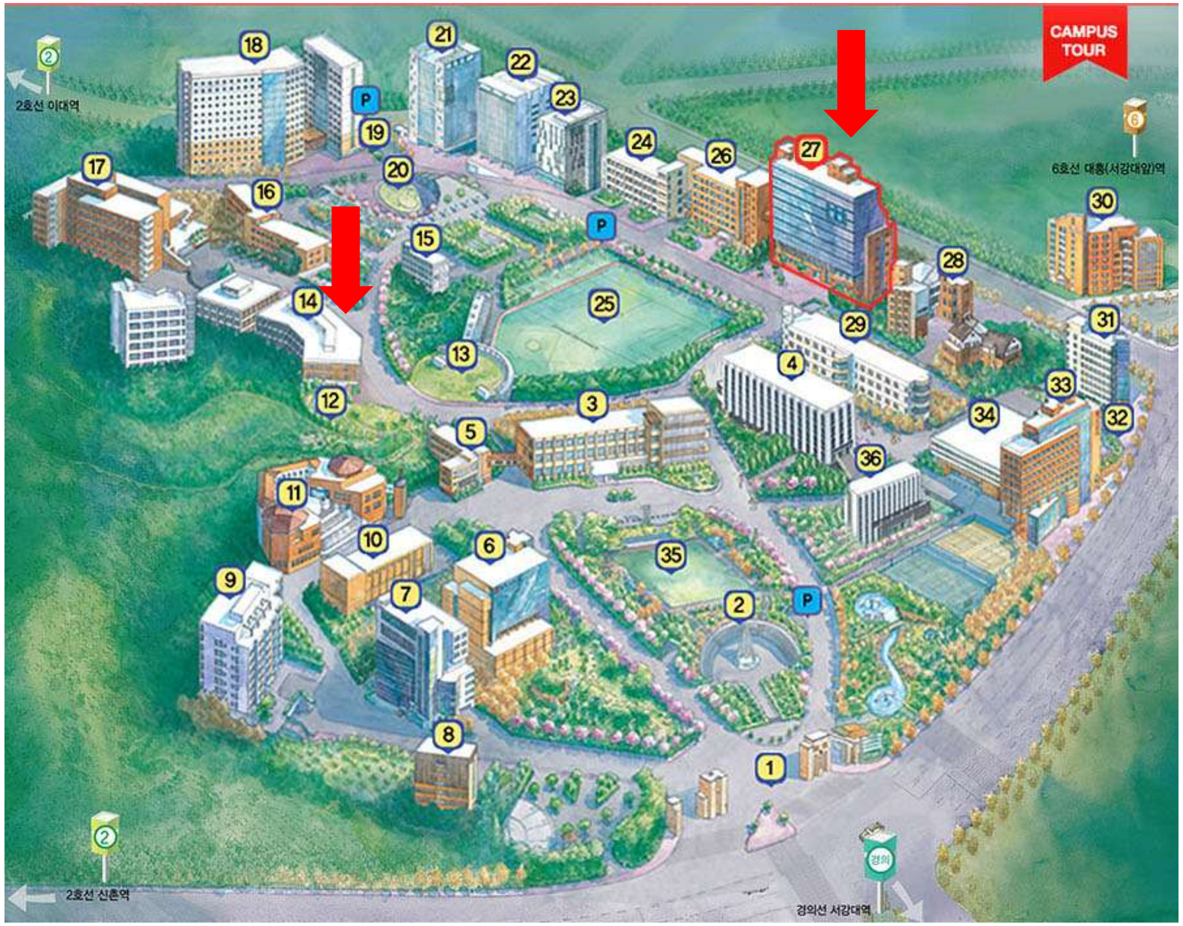
$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t)$$

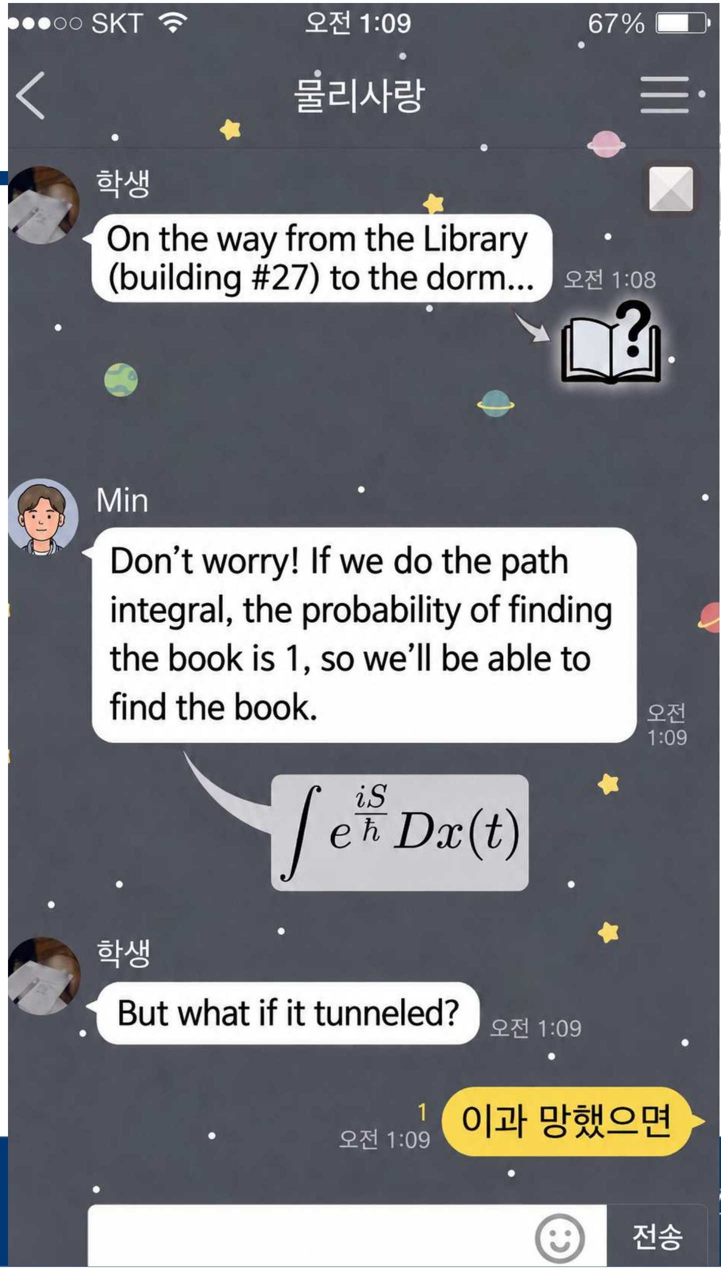
Kinetic + Potential

Coordinate(by symmetry)

- **Legendre**
- **Bessel**
- **Hermite**
- **....**

Path Integral



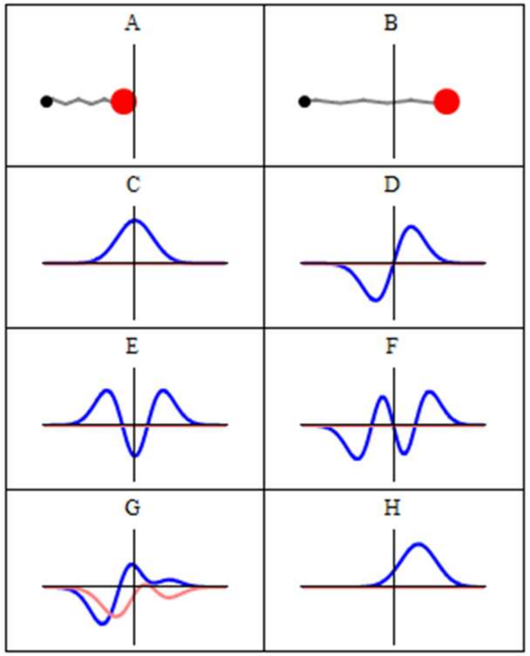


Wave Equation

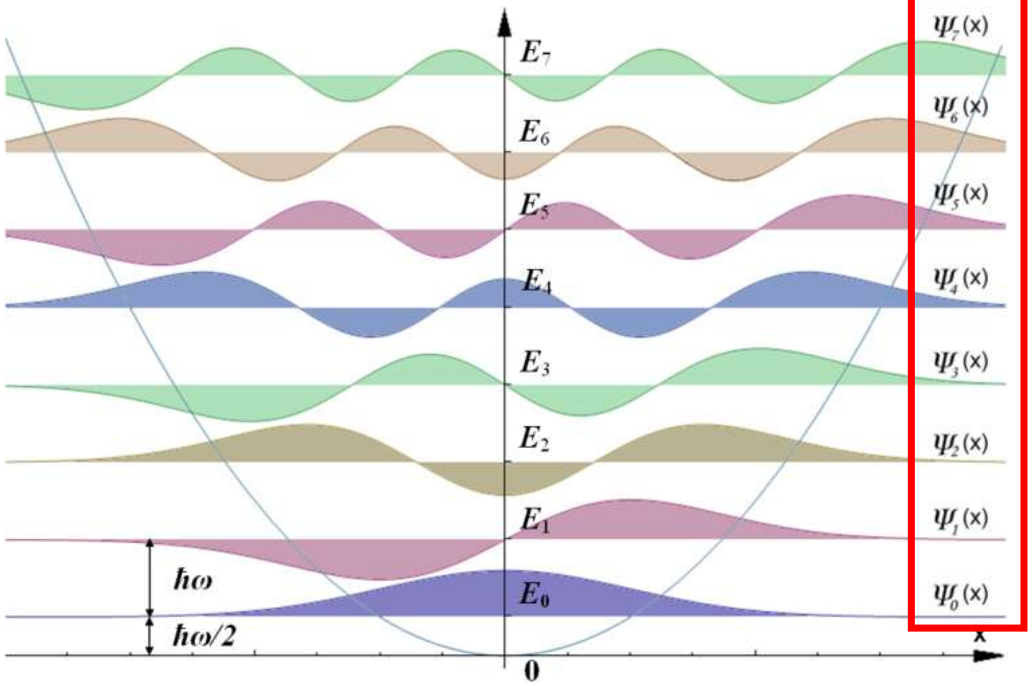
$$\psi = \psi_1 + \psi_2$$

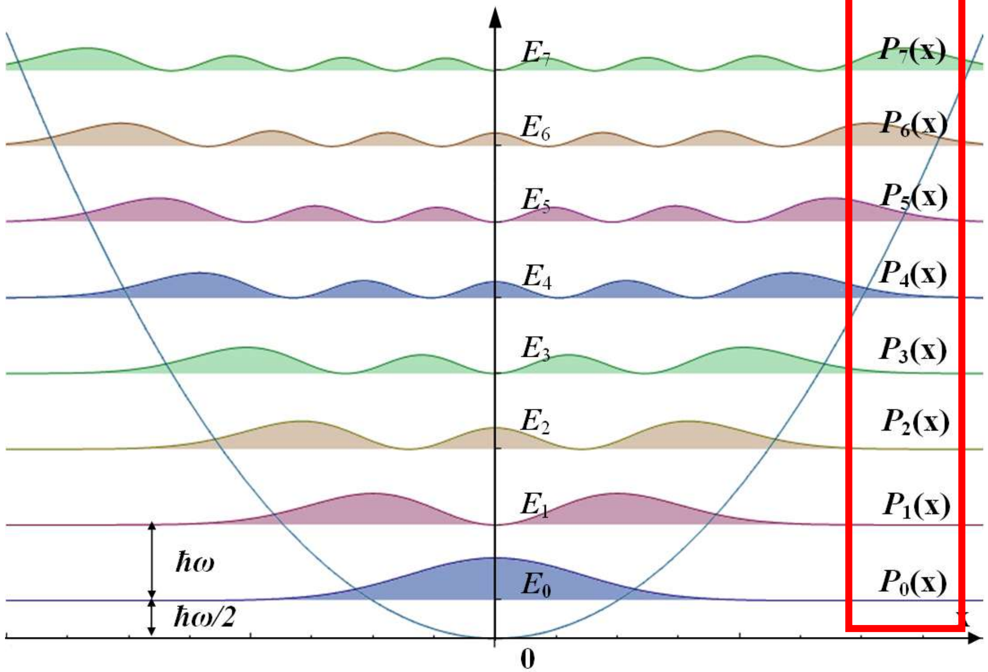
$$|\psi|^2 = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \underbrace{2\,\mathrm{Re}(\psi_1^*\psi_2)}_{\text{interference term}}$$

Quantum Harmonics Oscillator



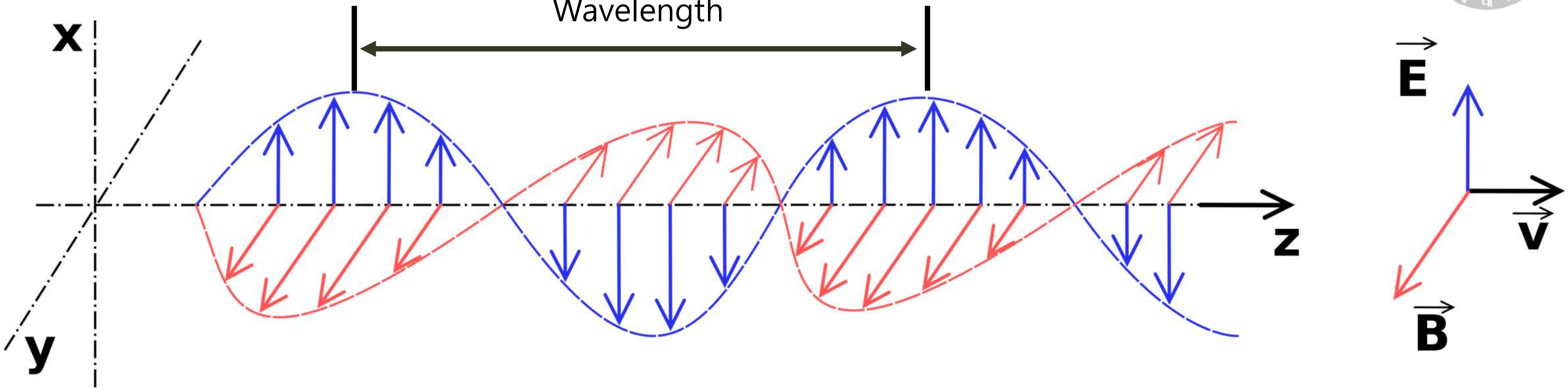
Quantum Harmonics Oscillator





Each state has an associated probability
However, we cannot know in advance which state will be realized

Light(Electromagnetic Wave)



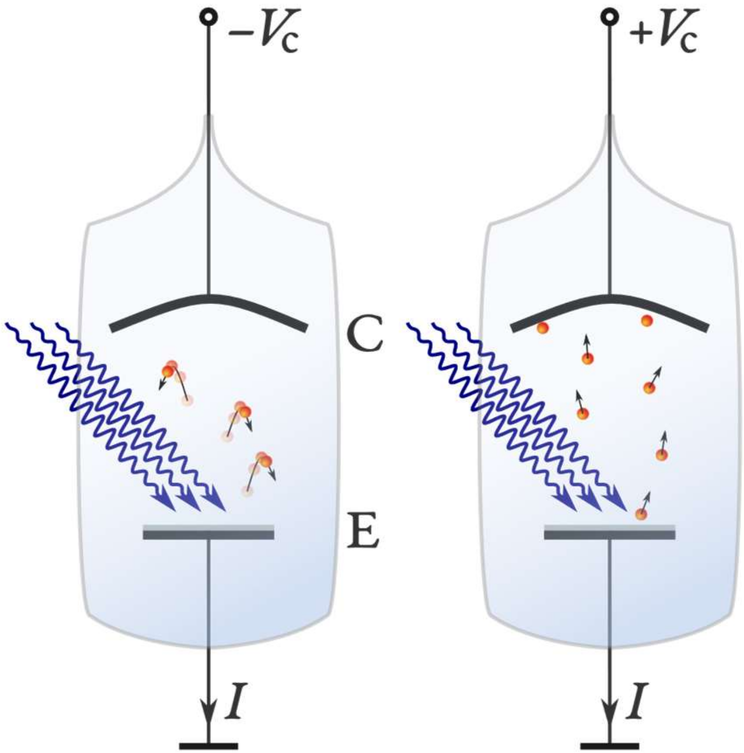
$$\left|\vec{E}\right| \propto Intensity\ of\ Light$$

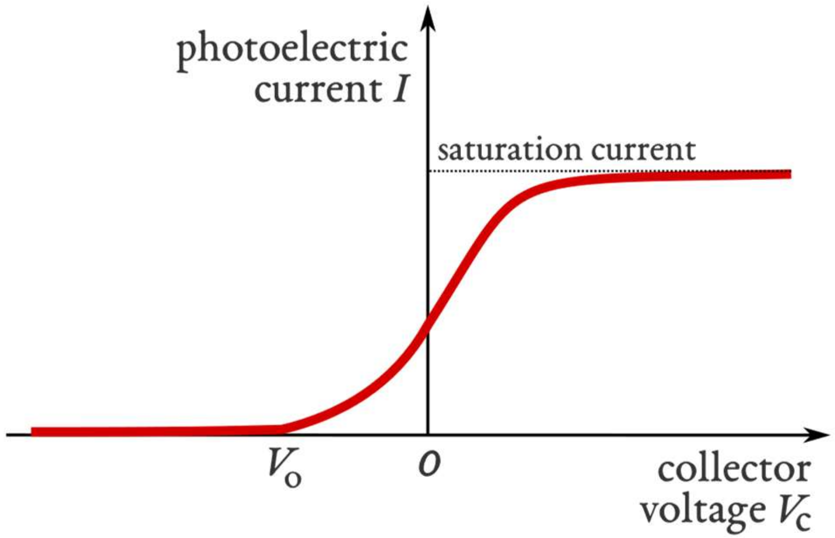
Photo Detector

Amplitude of electricfield
$\propto$ *Number of photon*

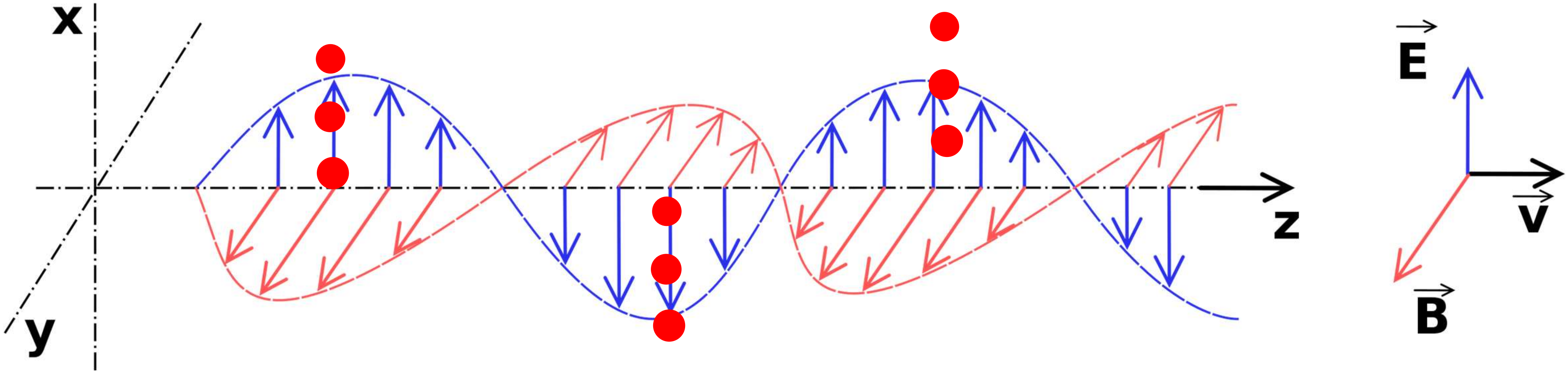
Measuring the intensity of light
= Counting the number of photon

Photoelectric Effect



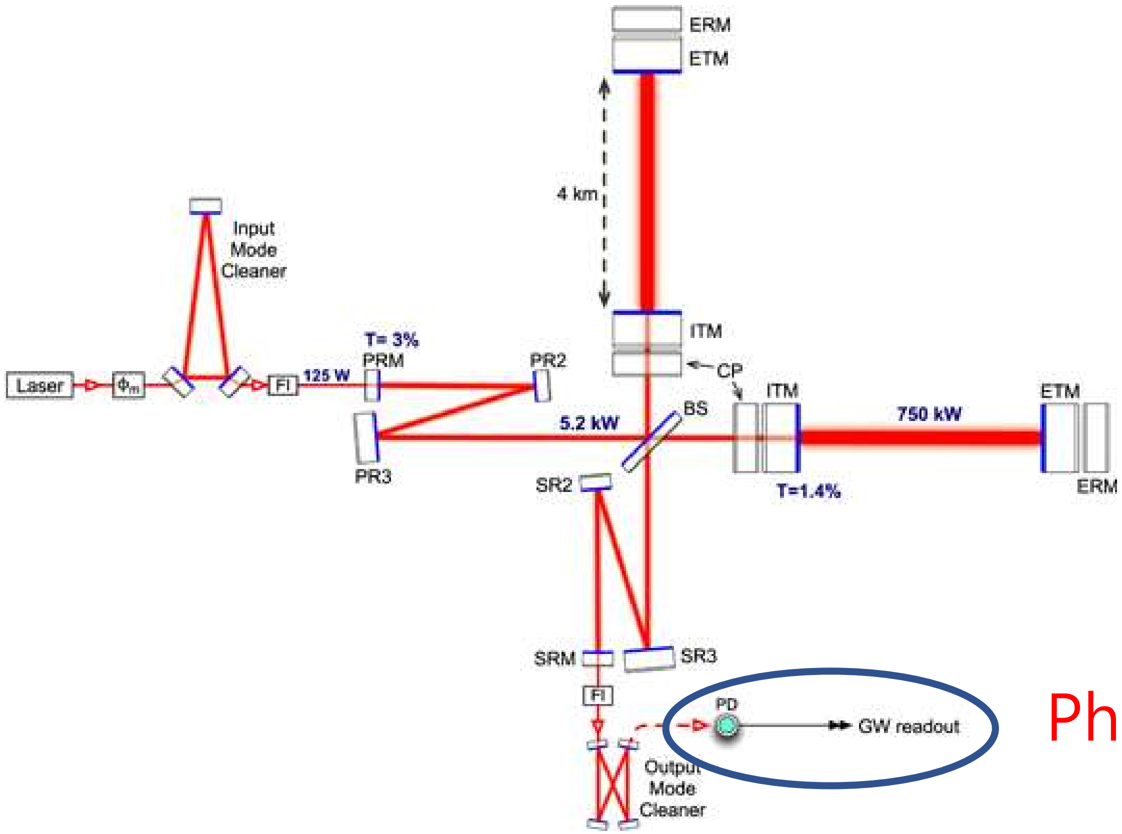


Wave-Particle Duality



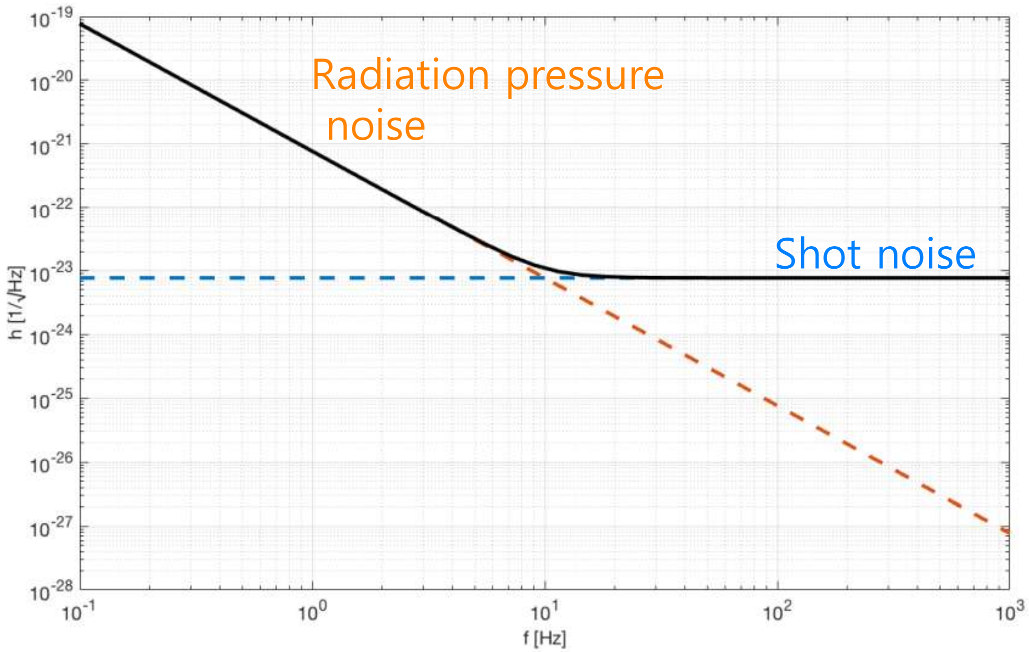
Observed Photon

Gravitational Wave Detector



Photon counting

Standard Quantum Limit of GW Detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

Shot Noise

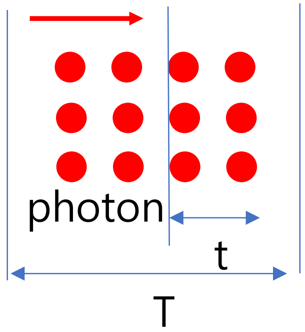
Shot Noise of Interferometer

$$E = h\nu$$

Photon energy

Laser power(W) = the number of photon(n) / time(s)

$nh\nu$ = / s



Shot noise

Shot Noise of Interferometer

$E = h\nu$

Photon energy

Laser power(W) = the number of photon(n) / time(s)

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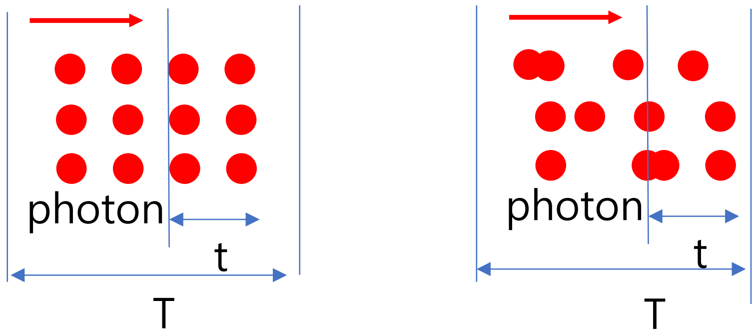
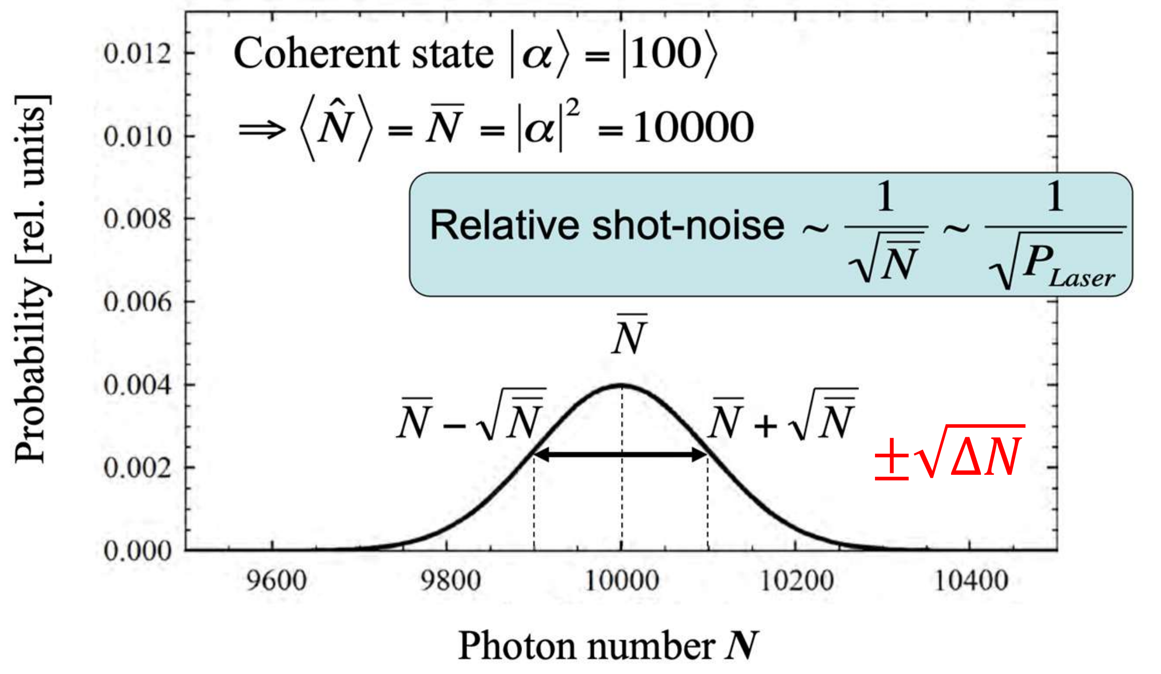


Shot noise

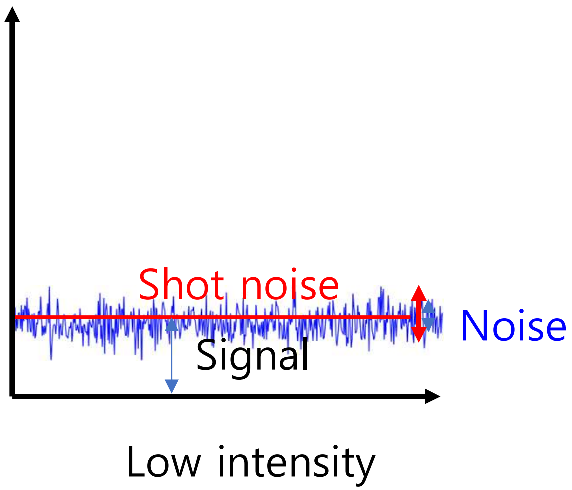
Shot Noise of Interferometer

Photon Counting Statistics

h = 6.62607015×10^{-34} J·Hz^{-1}

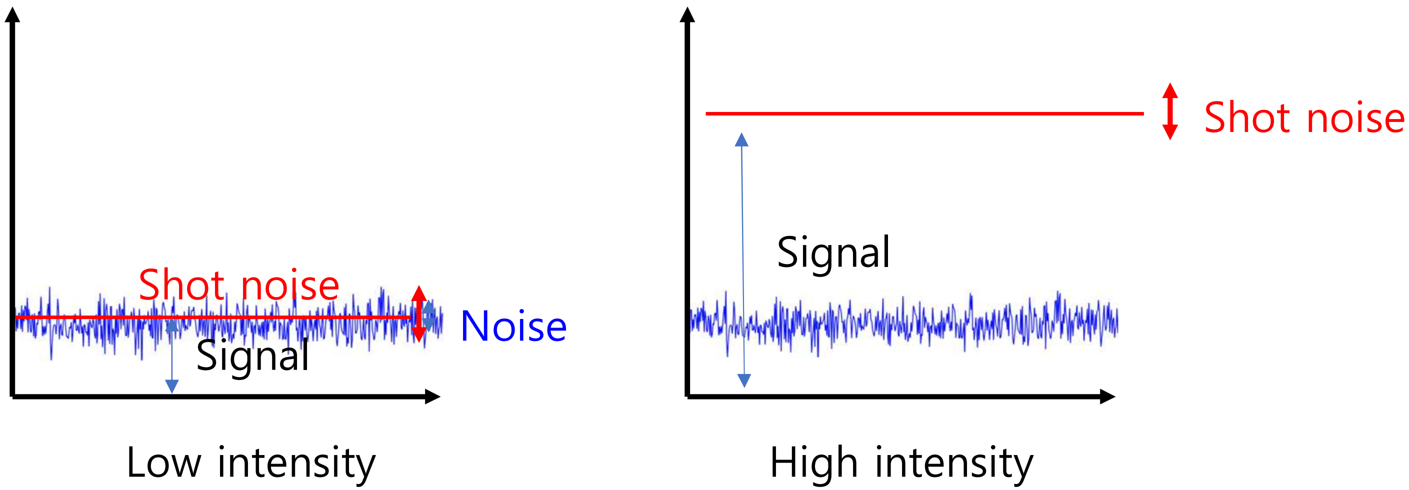


Shot Noise of Interferometer



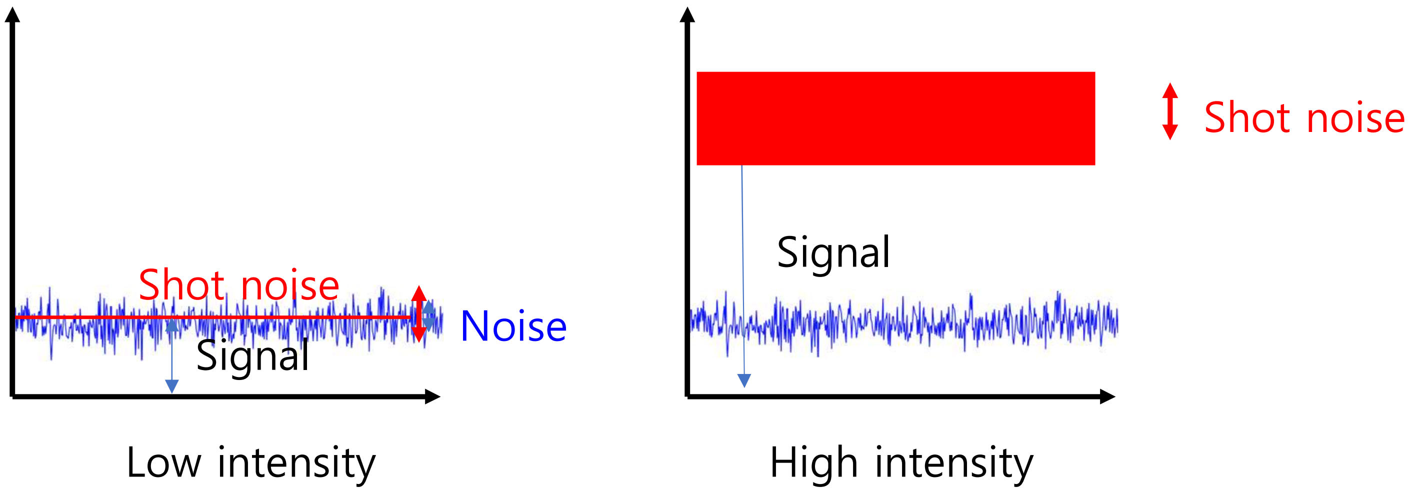
$$Signal\ to\ Noise\ ratio = \frac{N}{\sqrt{N}}$$

Shot Noise of Interferometer



$$Signal\ to\ Noise\ ratio = \frac{N}{\sqrt{N}}$$

Shot Noise of Interferometer

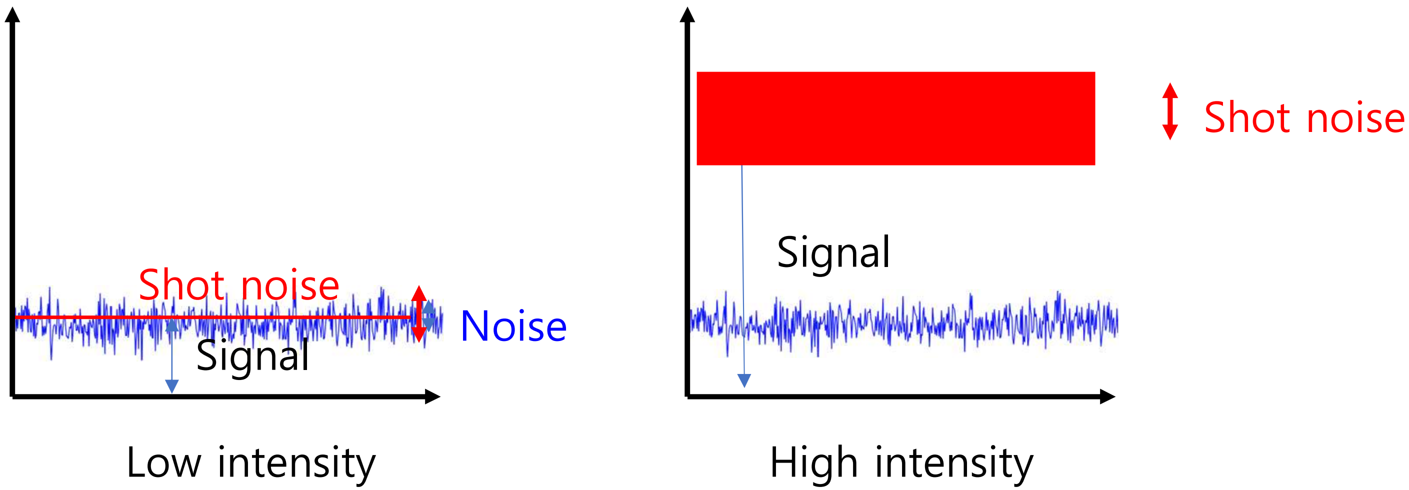


$$Signal\ to\ Noise\ ratio = \frac{N}{\sqrt{N}}$$

If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
We say that it has shot noise limit sensitivity

Shot Noise

Shot Noise of Interferometer

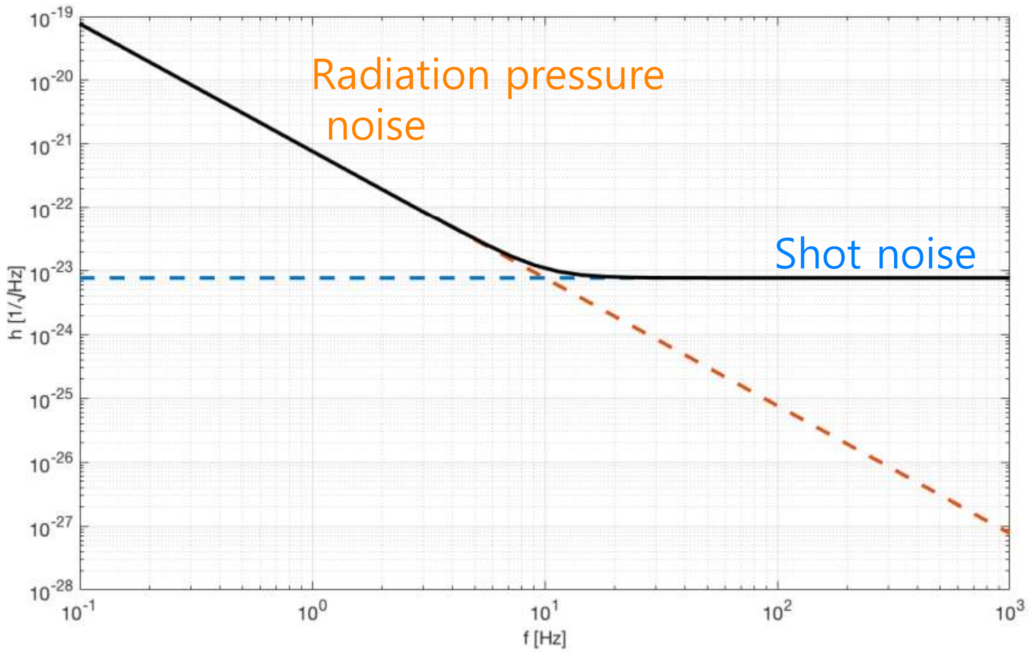


$$Signal\ to\ Noise\ ratio = \frac{N}{\sqrt{N}}$$

N=100 , SNR =10
N=10000 , SNR =100

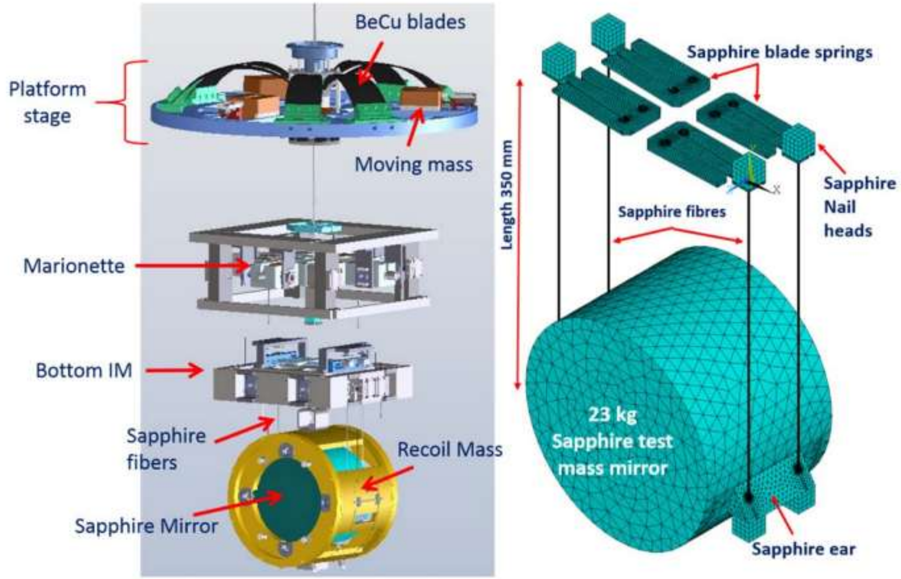
If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
We say that it has shot noise limit sensitivity

Standard Quantum Limit of GW Detector

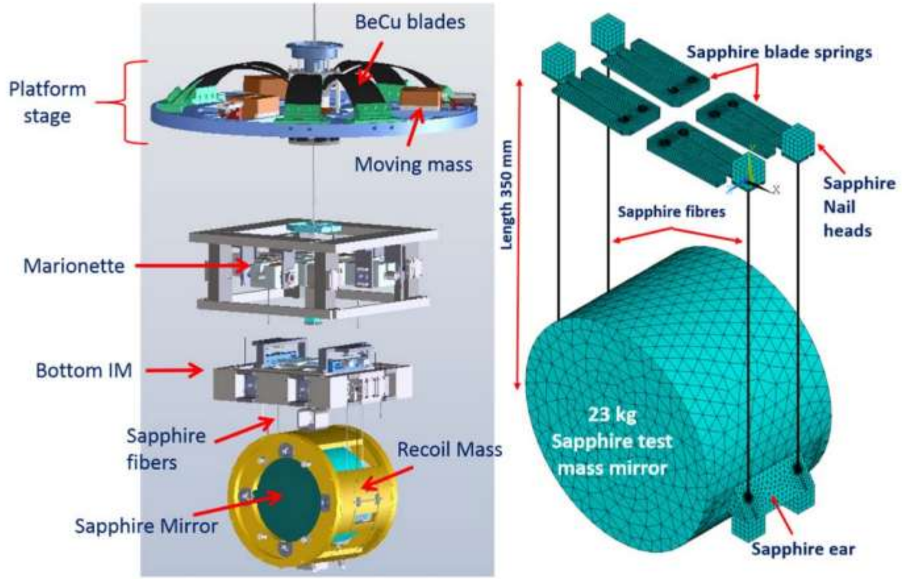


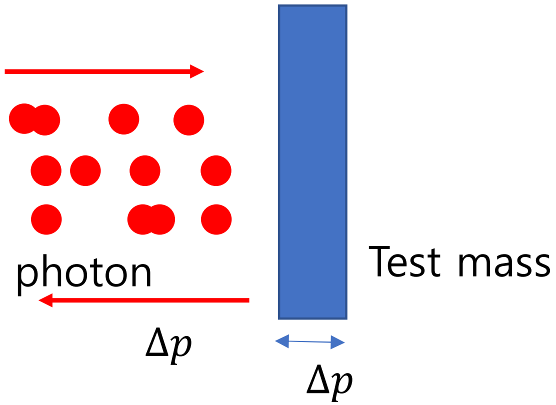
Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

Radiation Pressure Noise



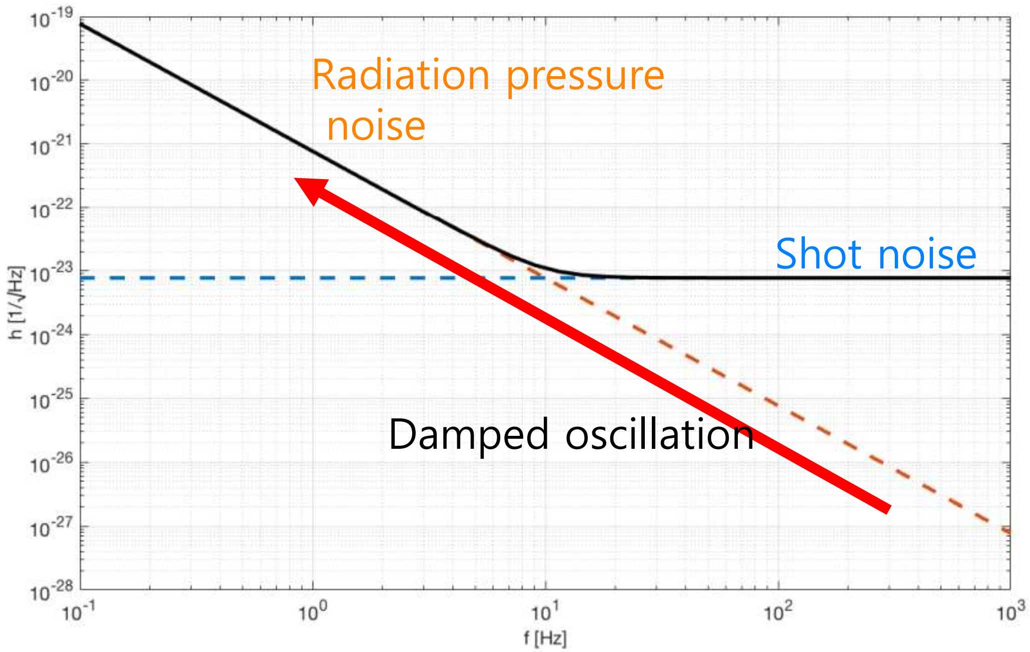
Radiation Pressure Noise





- Stored energy is very high (750 kW)
- Desired sensitivity is very high ($10^{-21} \sim 10^{-24}$)

Standard Quantum Limit of GW Detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

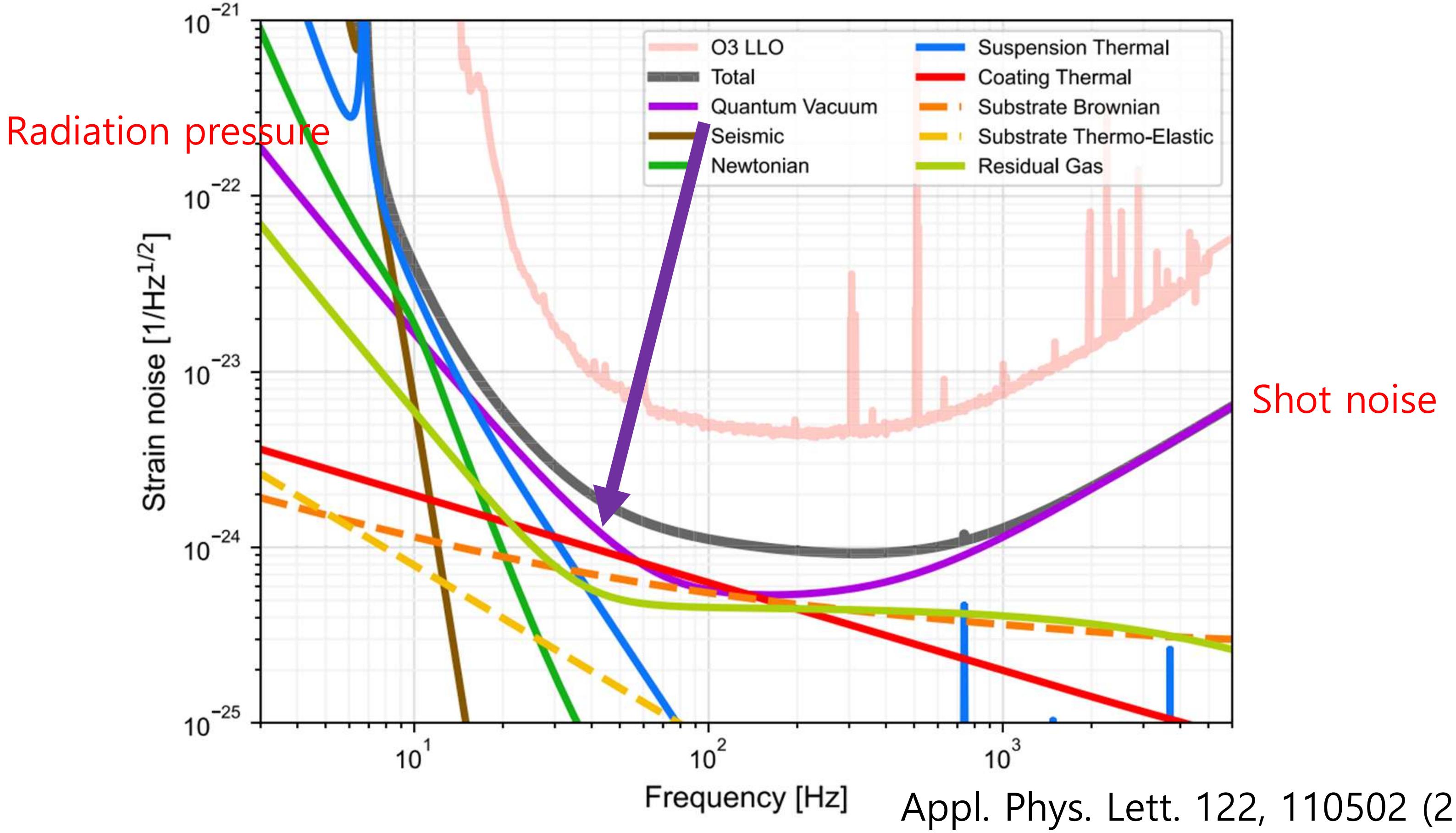
Radiation pressure noise
- Photon pressure fluctuation at **mirror**

-> Intensity noise

Shot noise
- Photon number fluctuation at **PD**

-> Phase noise

LIGO Sensitivity



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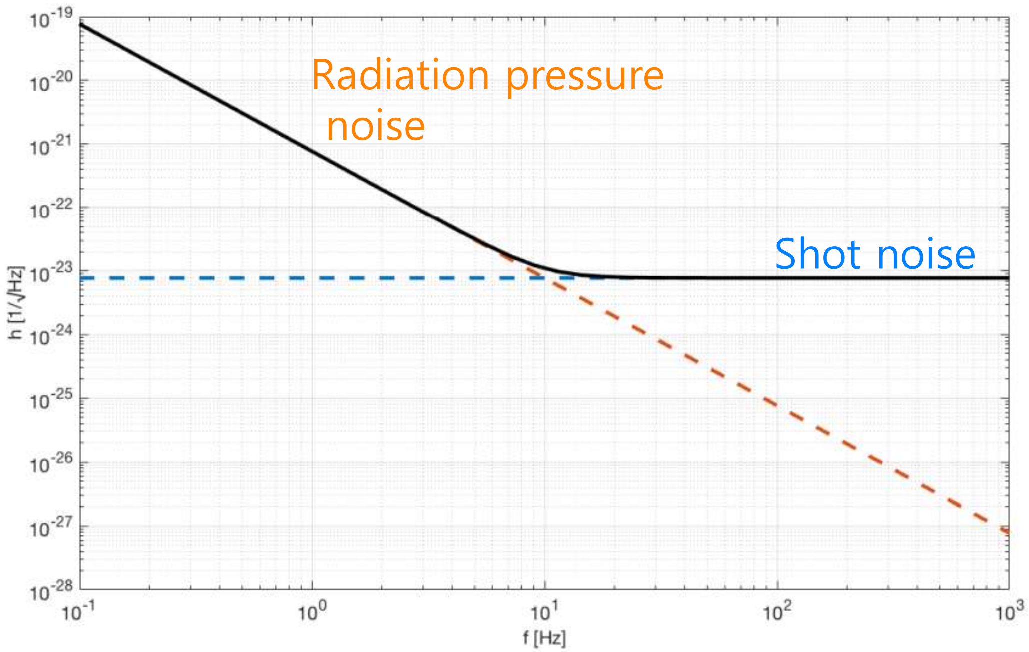
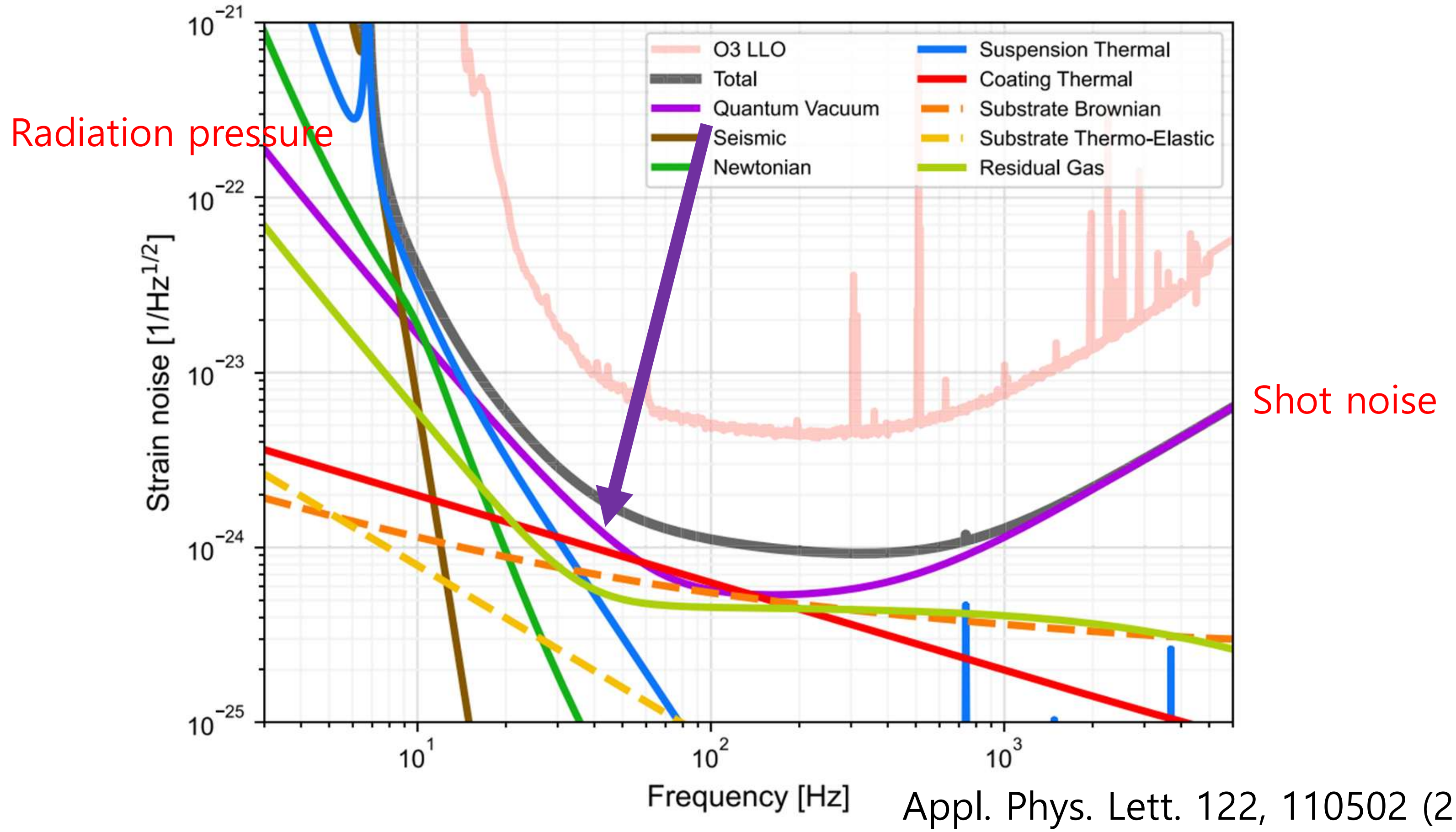
Standard Quantum Limit of GW Detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

LIGO Sensitivity



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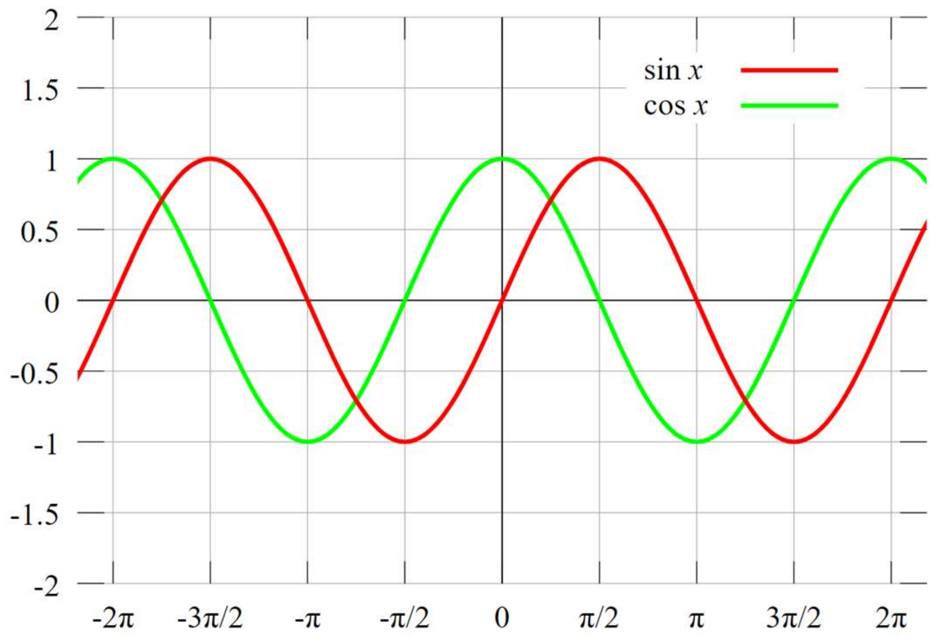
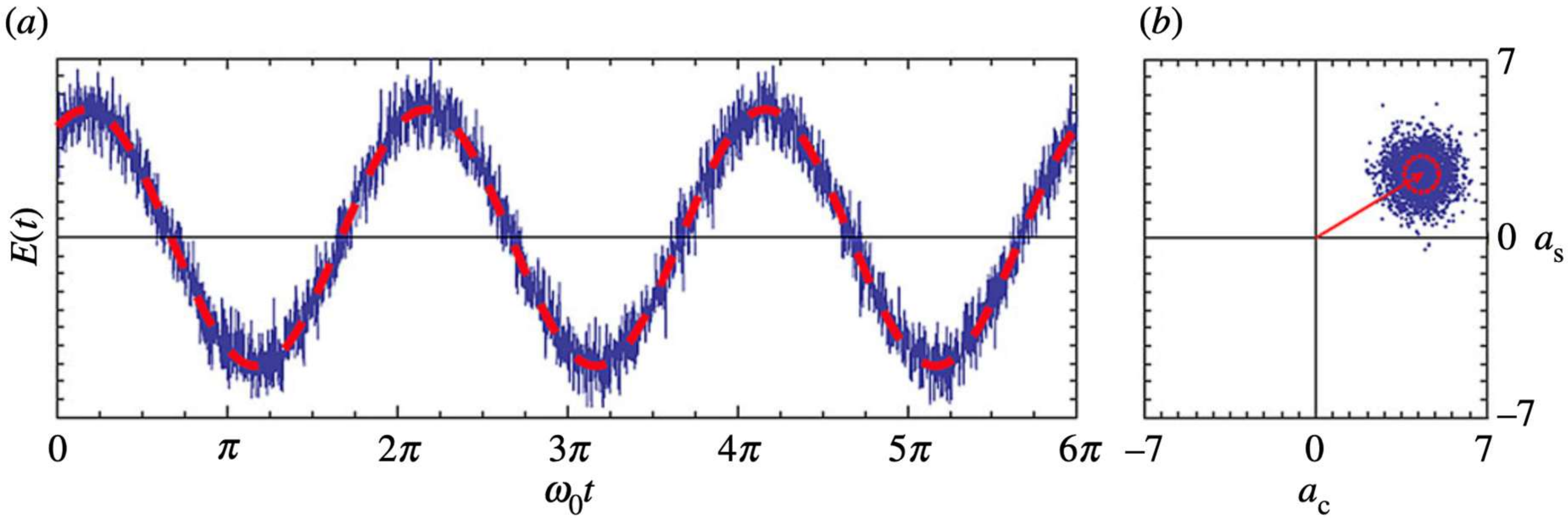
Classical Electromagnetic Wave



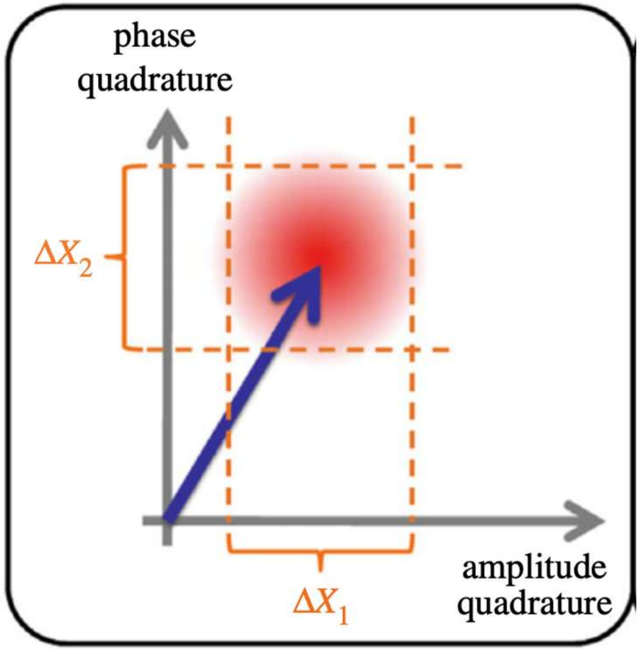
Quantum Noise of Coherent Light



Heurs M. 2018 Gravitational wave detection u
sing laser interferometry beyond the standard
quantum limit.Phil. Trans. R. Soc. A 376: 20170
289.

Phase and Amplitude Quadrature

(a) Shot noise



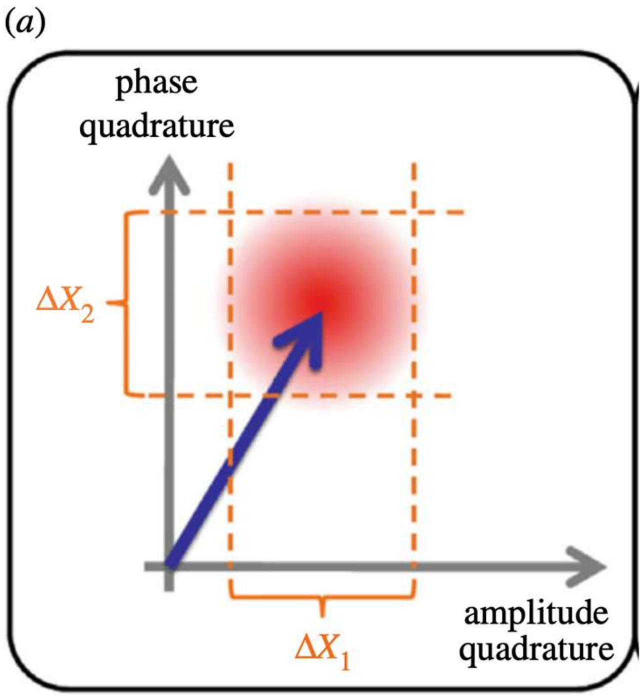
Radiation pressure noise

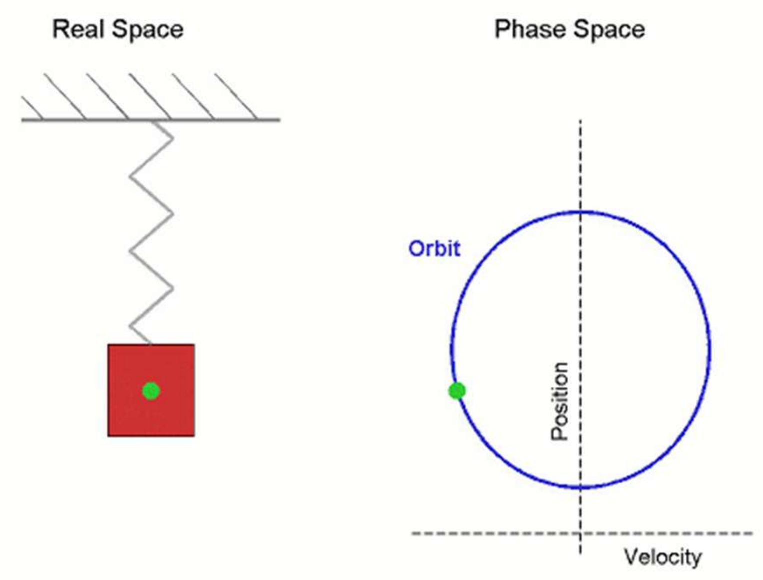
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Uncertainty principle

Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit.Phil. Trans. R. Soc. A 376: 20170289.

Phase and Amplitude Quadrature

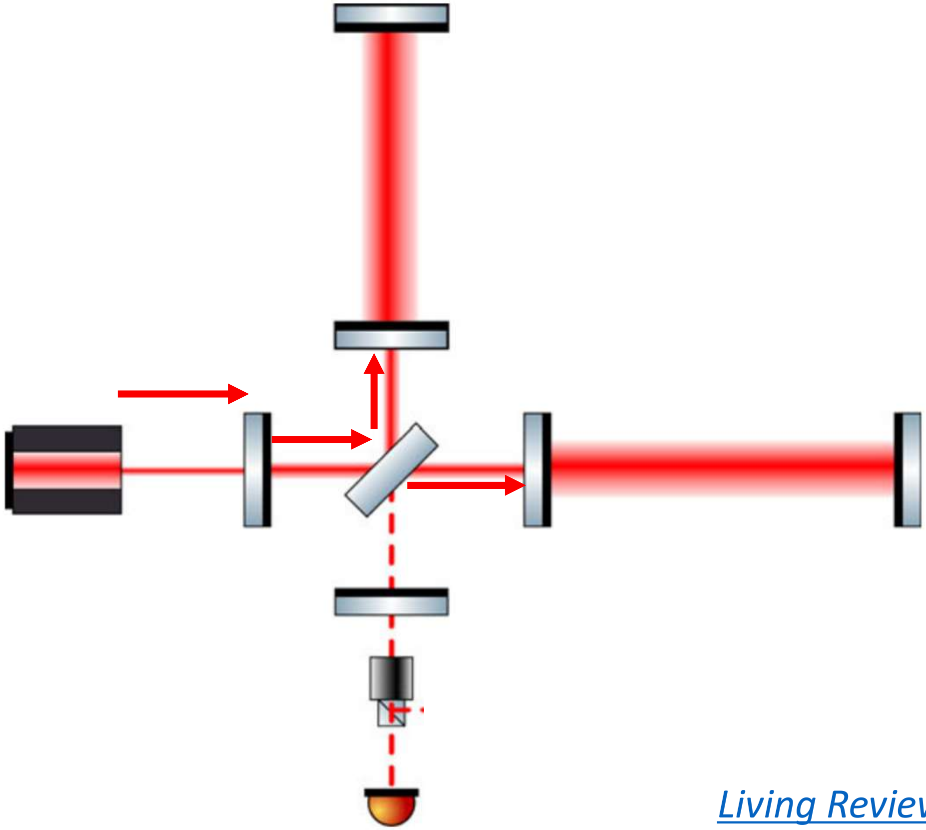


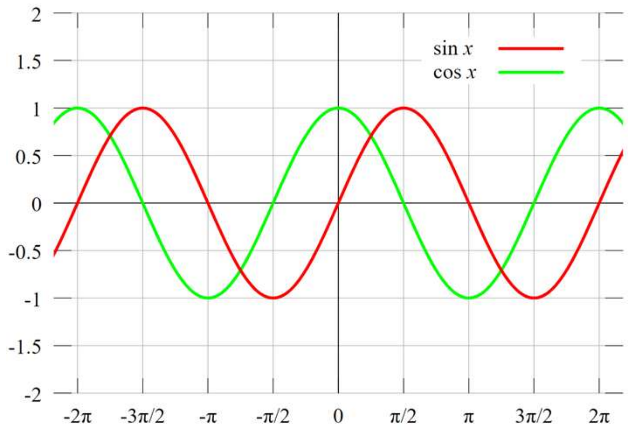


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Uncertainty principle

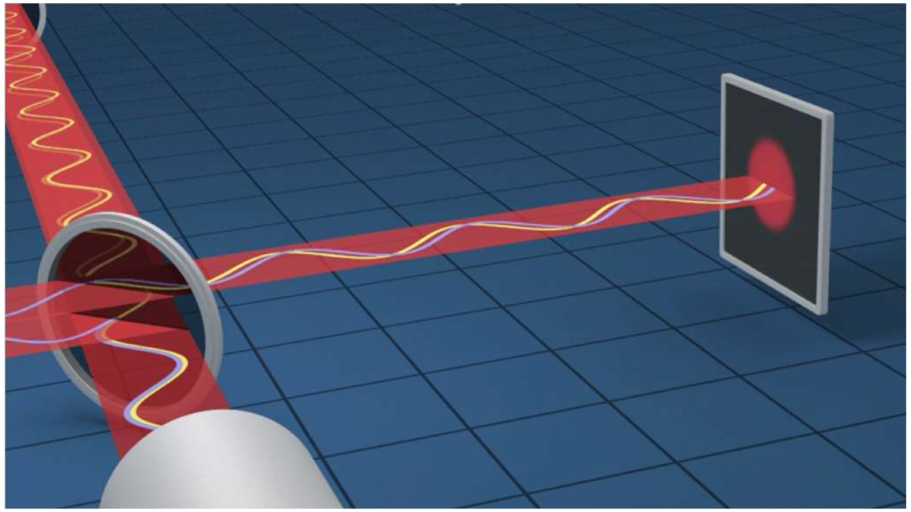
Gravitational Wave Detector



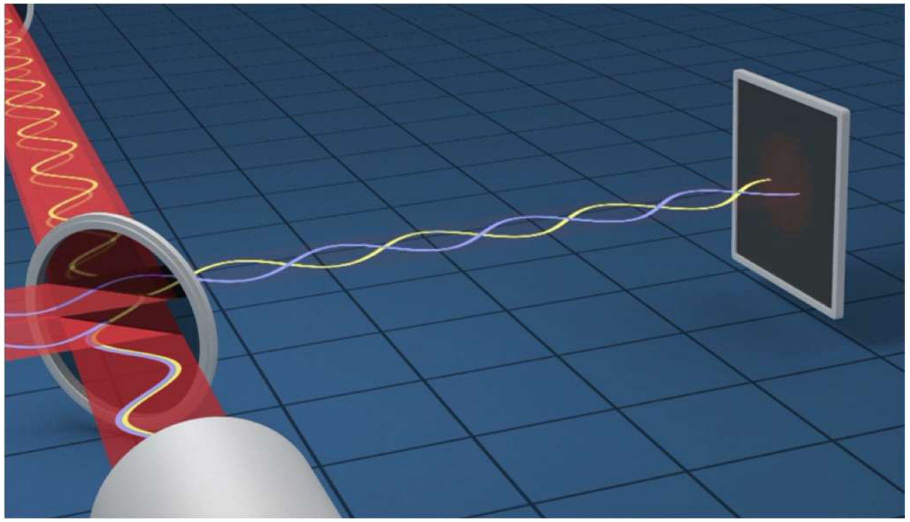


Living Reviews in Relativity **volume 22**, Article number: 2 (2019)

Bright Port and Dark Port

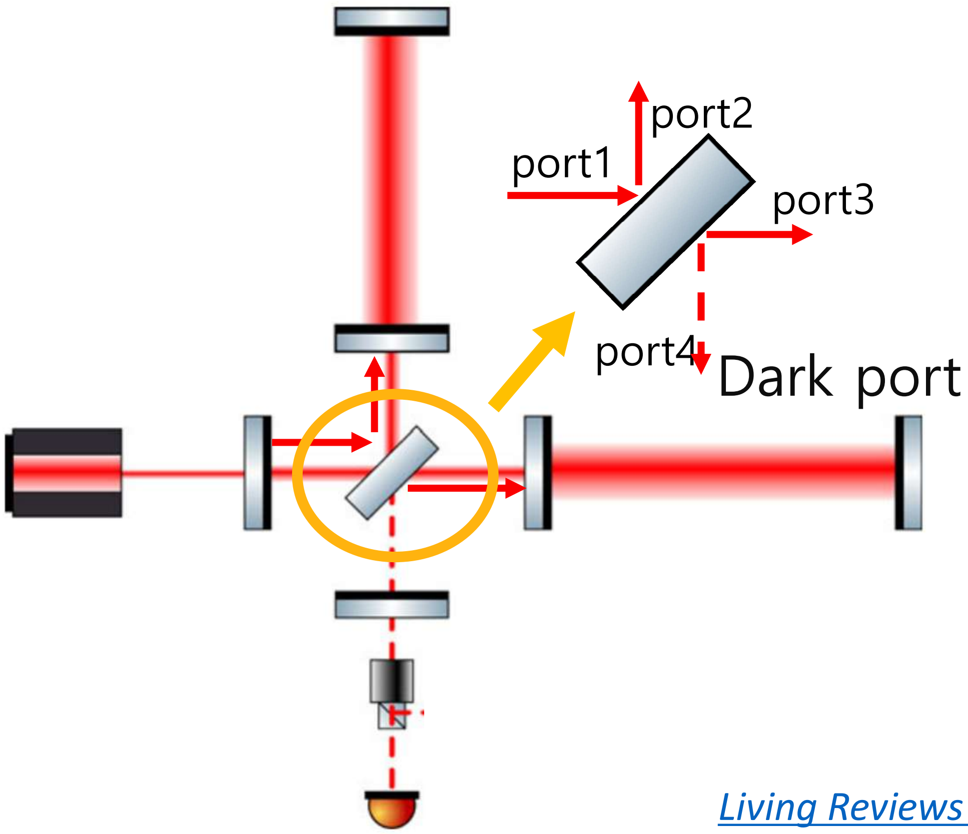


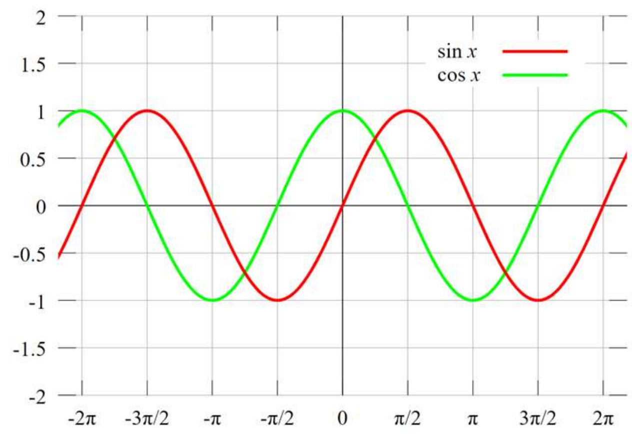
Bright



Dark

Gravitational Wave Detector





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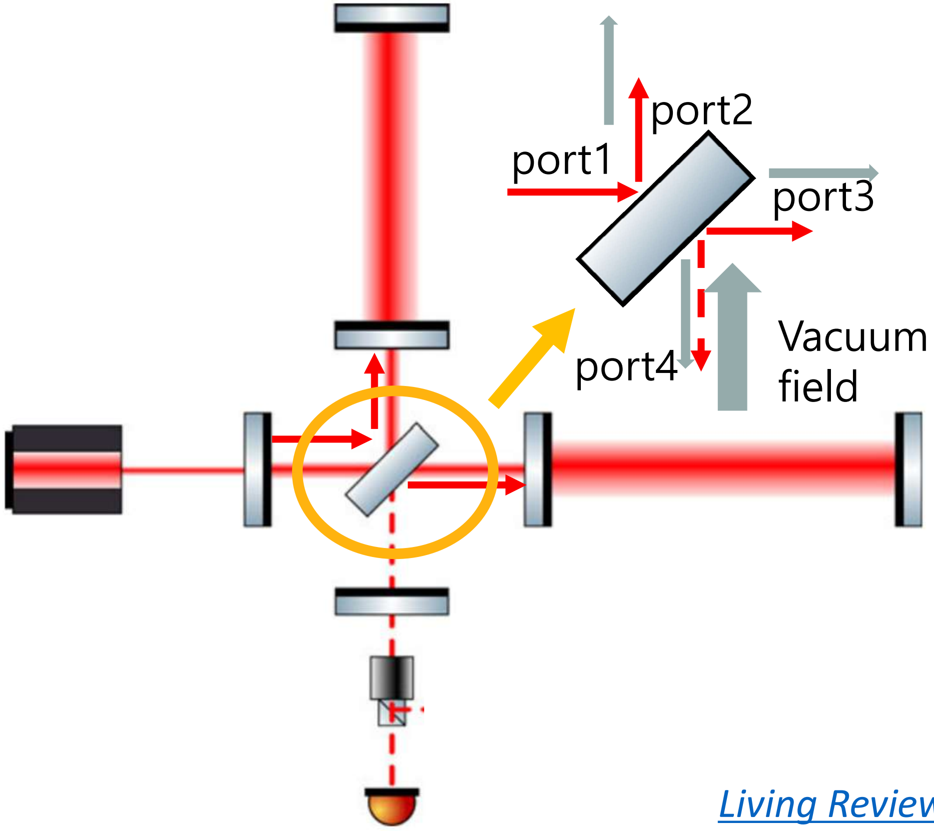
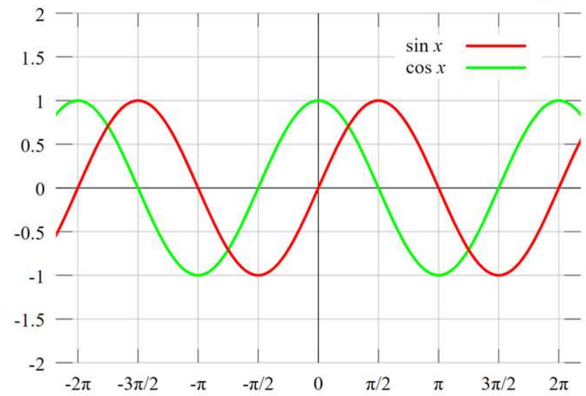
Quantum Noise of Gravitational Wave Detector





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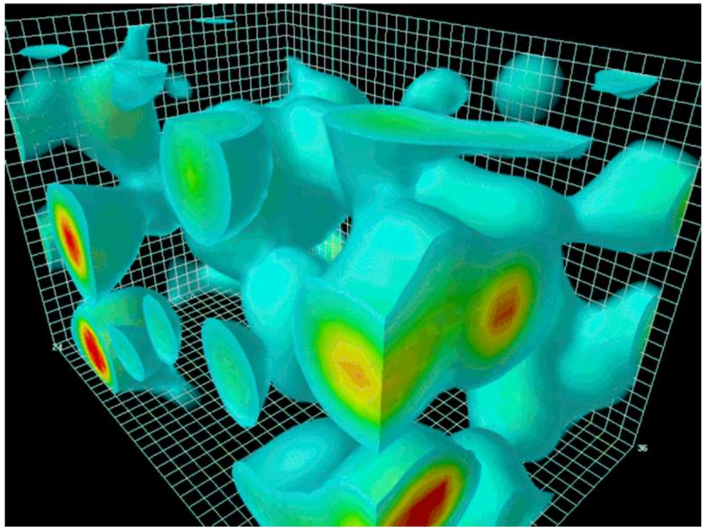
Quantum Vacuum Fluctuation

Vacuum ≠ empty

Vacuum

Transmission medium



Average energy = zeropoint energy

Quantum Vacuum Fluctuation

JOURNAL ARTICLE

Evidence for vacuum birefringence from the first optical-polarimetry measurement of the isolated neutron star RX J1856.5-3754 FREE

R. P. Mignani, V. Testa, D. González Caniulef, R. Taverna, R. Turolla, S. Zane, K. Wu

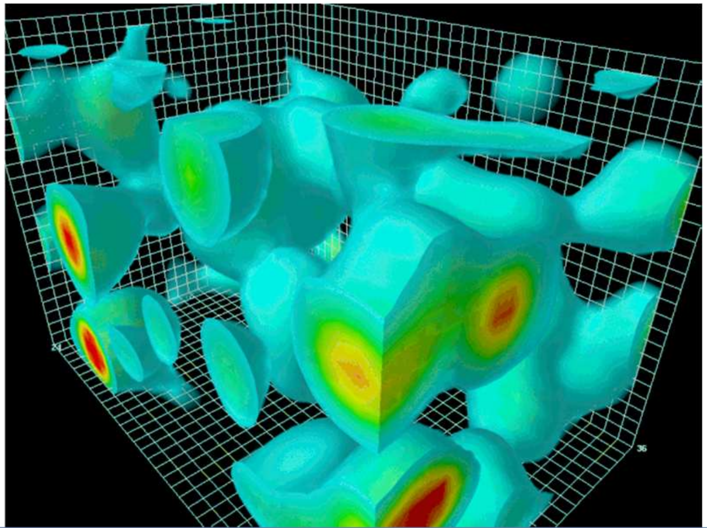
Monthly Notices of the Royal Astronomical Society, Volume 465, Issue 1, 11 February 2017, Pages 492–500, https://doi.org/10.1093/mnras/stw2798

Published: 02 November 2016 **Article history** ▾

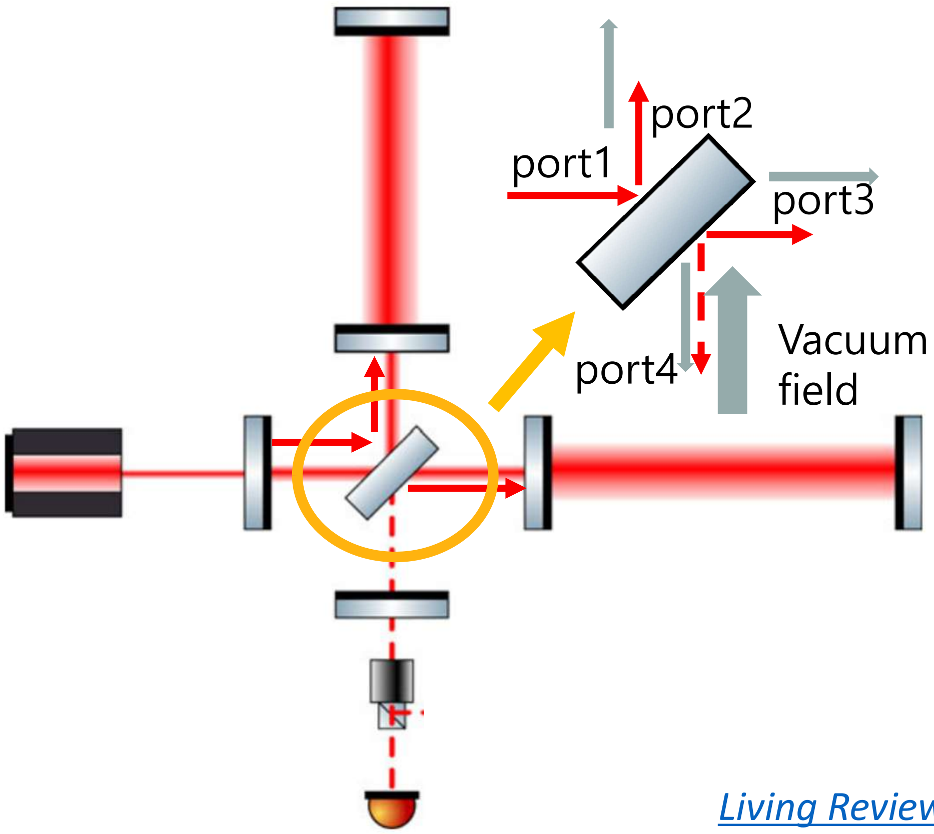
Vacuum ≠ empty

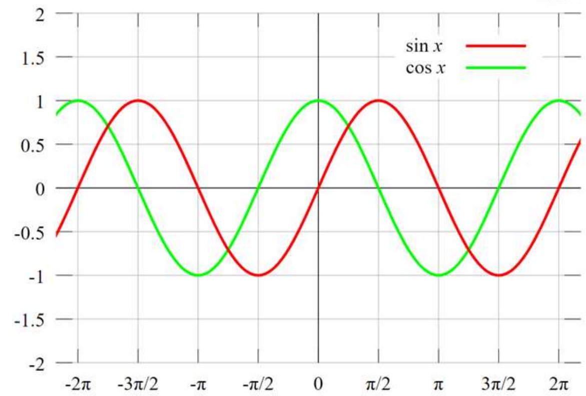
Transmission medium

Vacuum



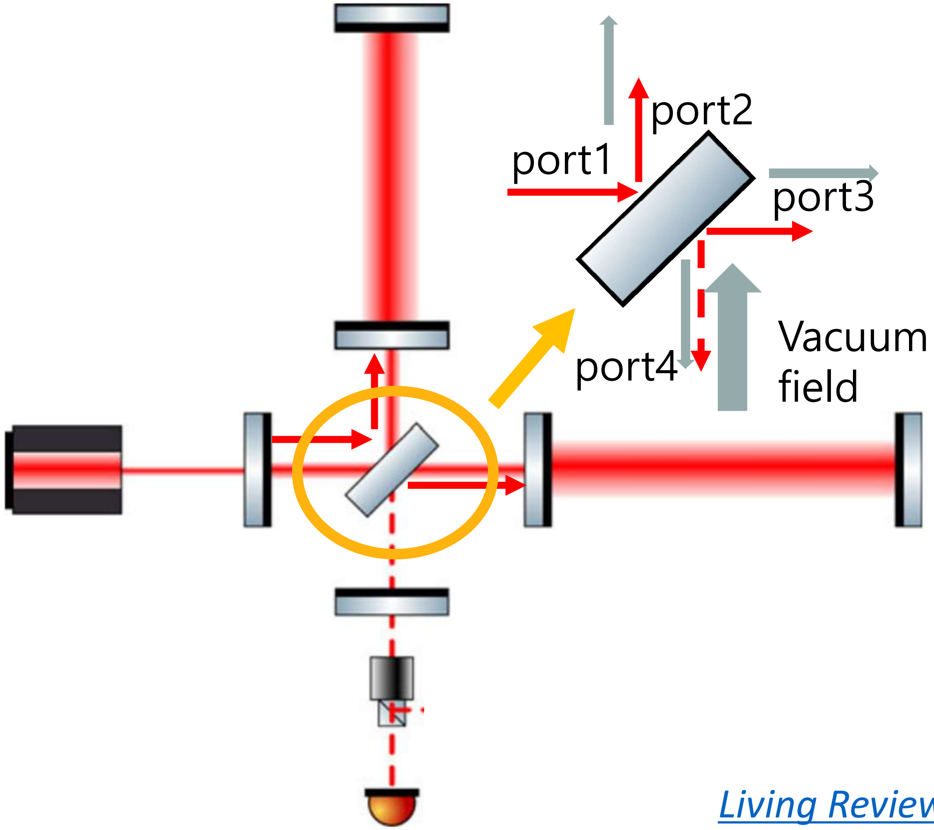
Quantum Noise of Gravitational Wave Detector

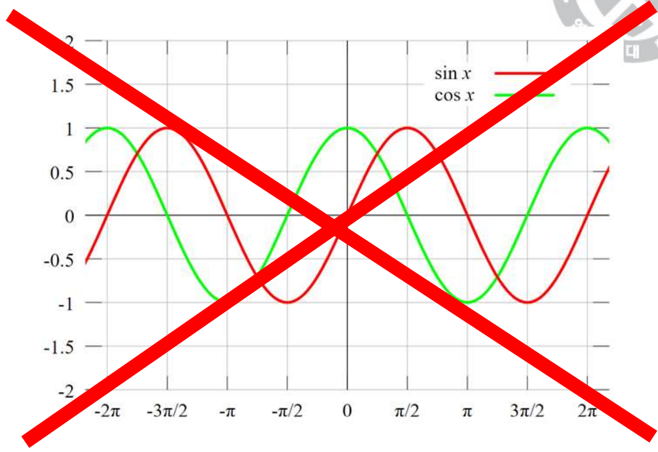


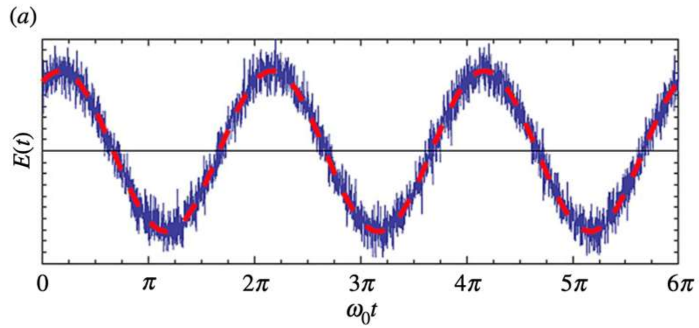


Living Reviews in Relativity **volume 22**, Article number: 2 (2019)

Quantum Noise of Gravitational Wave Detector

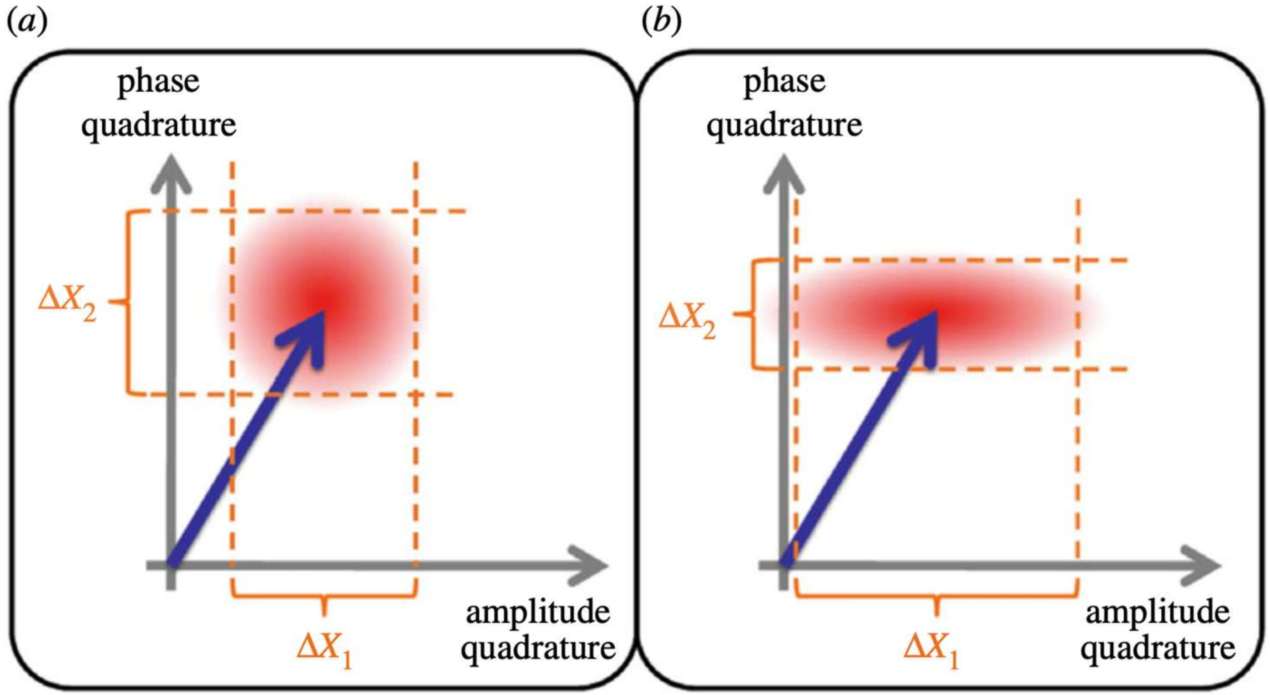






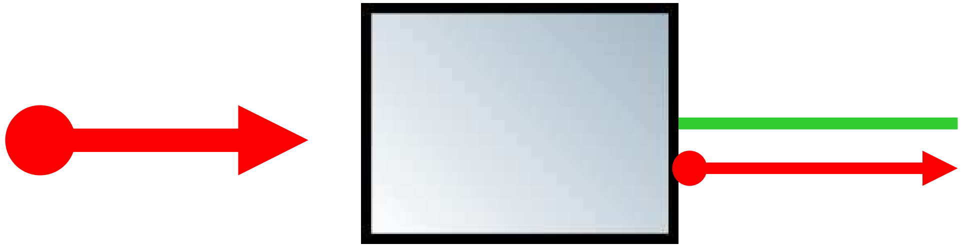
Living Reviews in Relativity **volume 22**, Article number: 2 (2019)

Squeezed Light

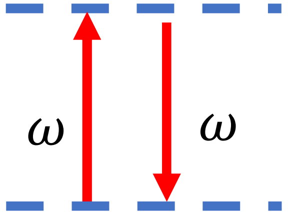


Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit.Phil. Trans. R. Soc. A 376: 20170289.

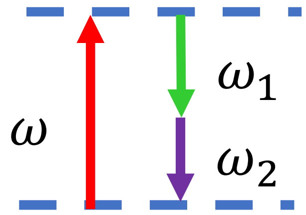
Non Linear Crystal



Non-linear crystal

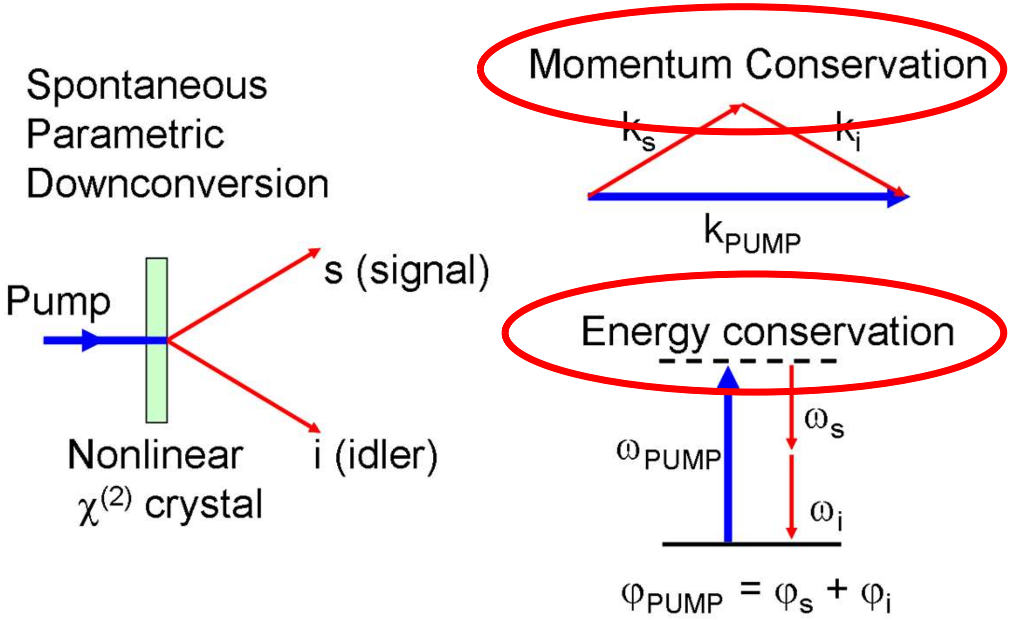


linear optics



Non linear optics

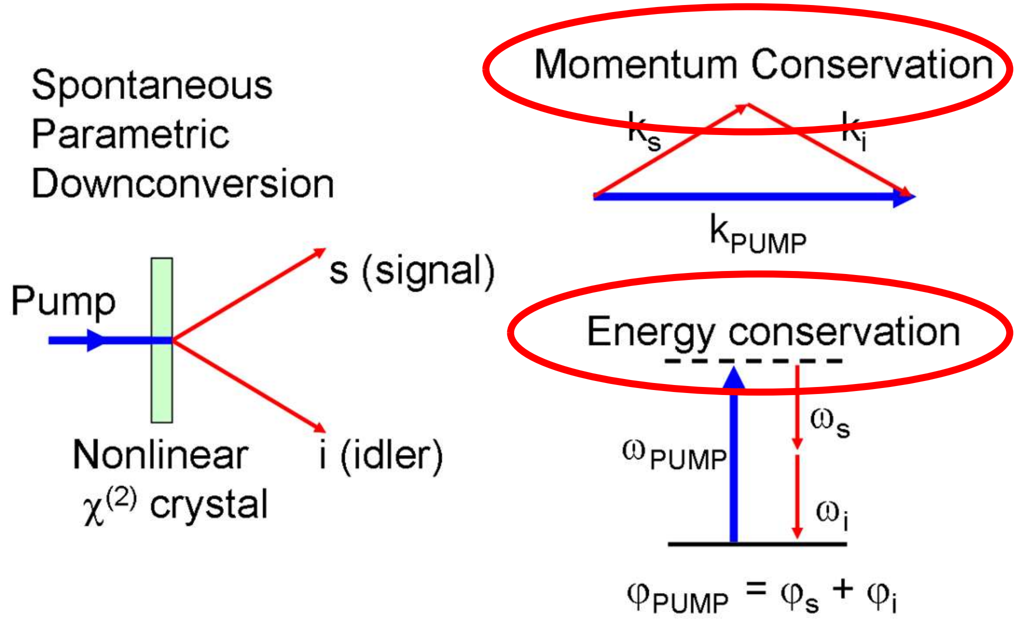
Parametric Down Conversion



Every filed which are participated in
Non-linear conversion process,
Must obey energy, momentum conservation.

J S Lundeen

Parametric Down Conversion

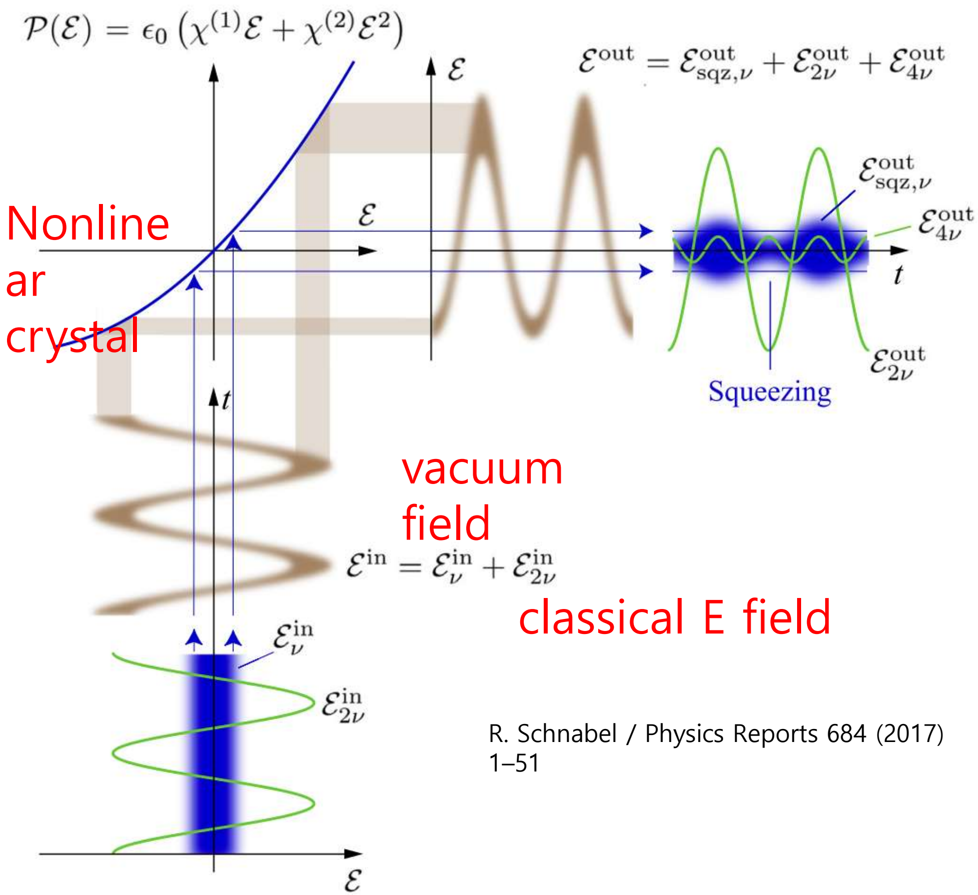


Every filed which are participated in
Non-linear conversion process,
Must obey energy, momentum conservation.

Even for vacuum field!!

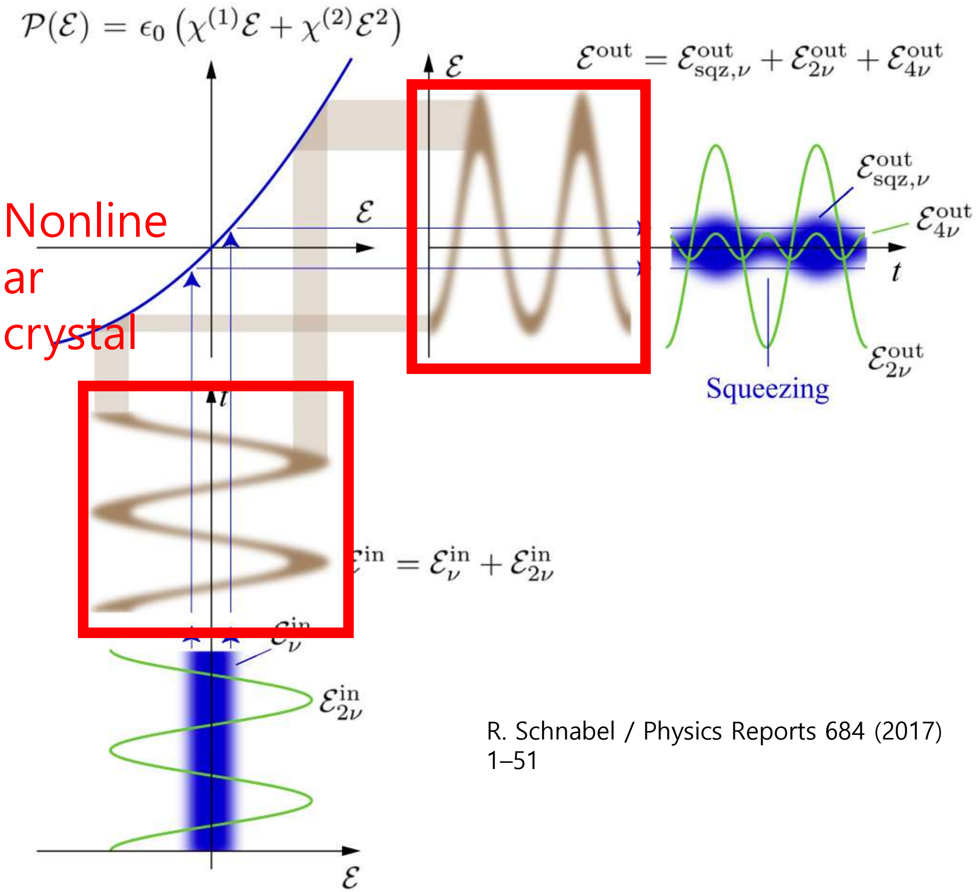
J S Lundeen

Parametric Down Conversion

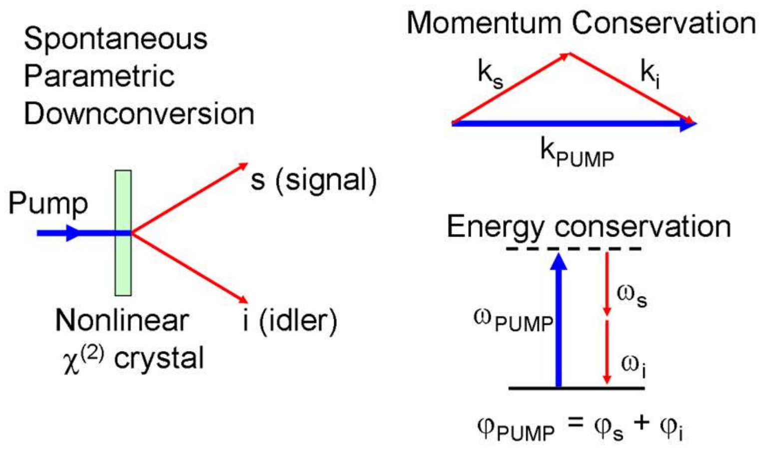


R. Schnabel / Physics Reports 684 (2017) 1–51

Parametric Down Conversion

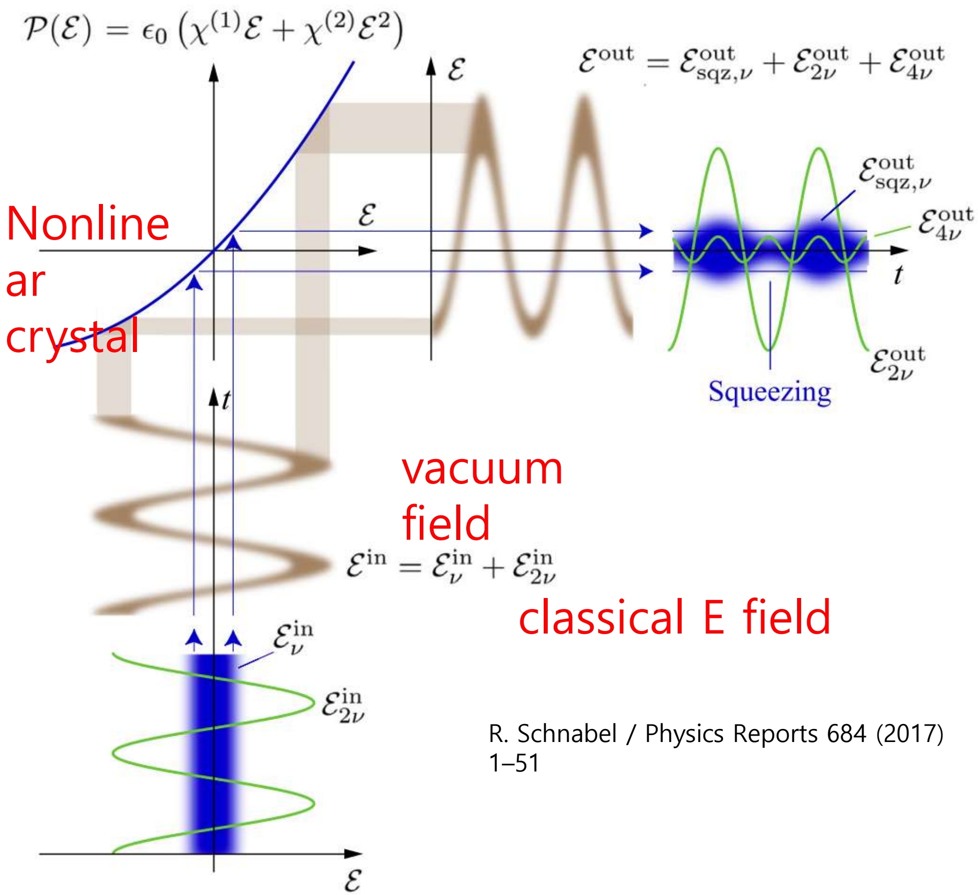


R. Schnabel / Physics Reports 684 (2017) 1–51



Phase matching condition

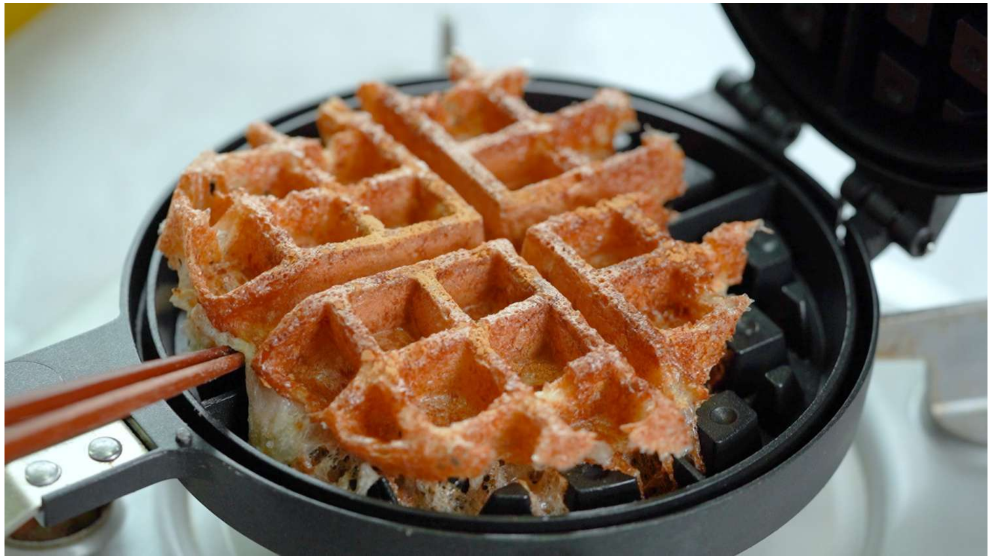
Parametric Down Conversion



R. Schnabel / Physics Reports 684 (2017) 1–51

State of vacuum is changed during conversion process
due to phase matching condition

Phase Matching Condition



Squeezed Vacuum

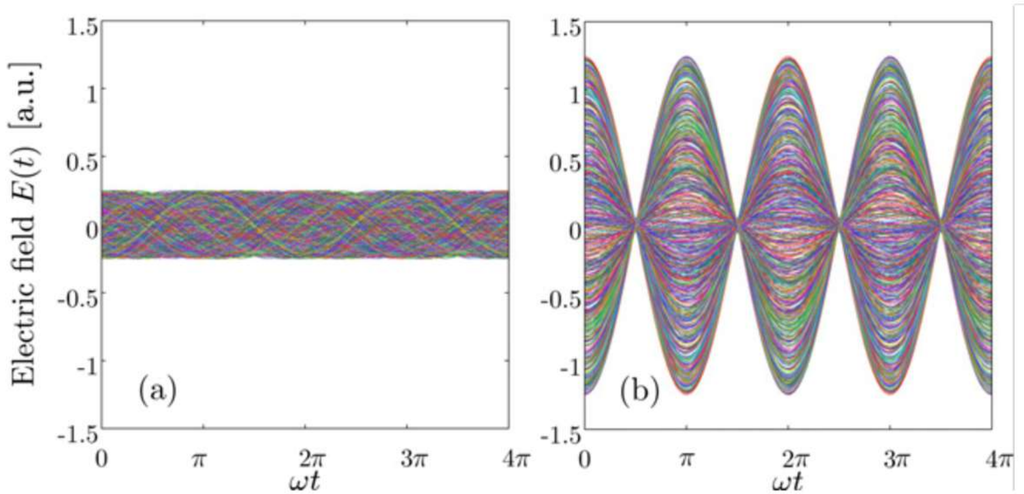
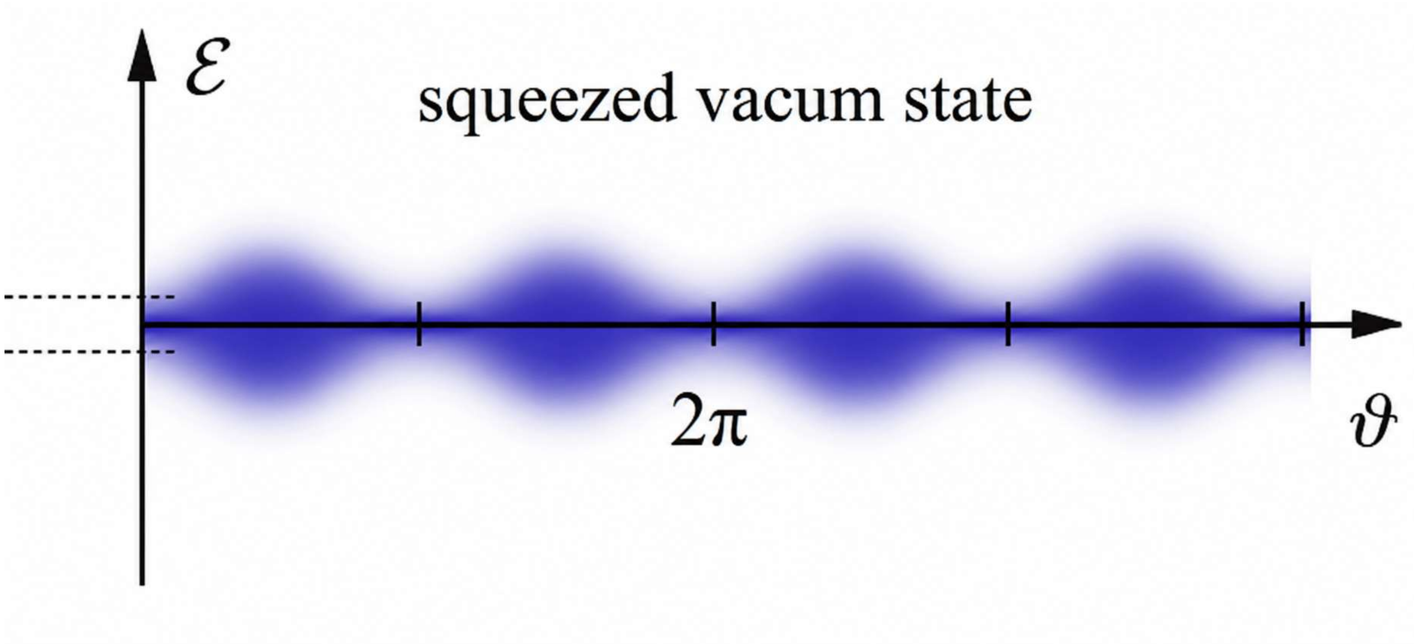
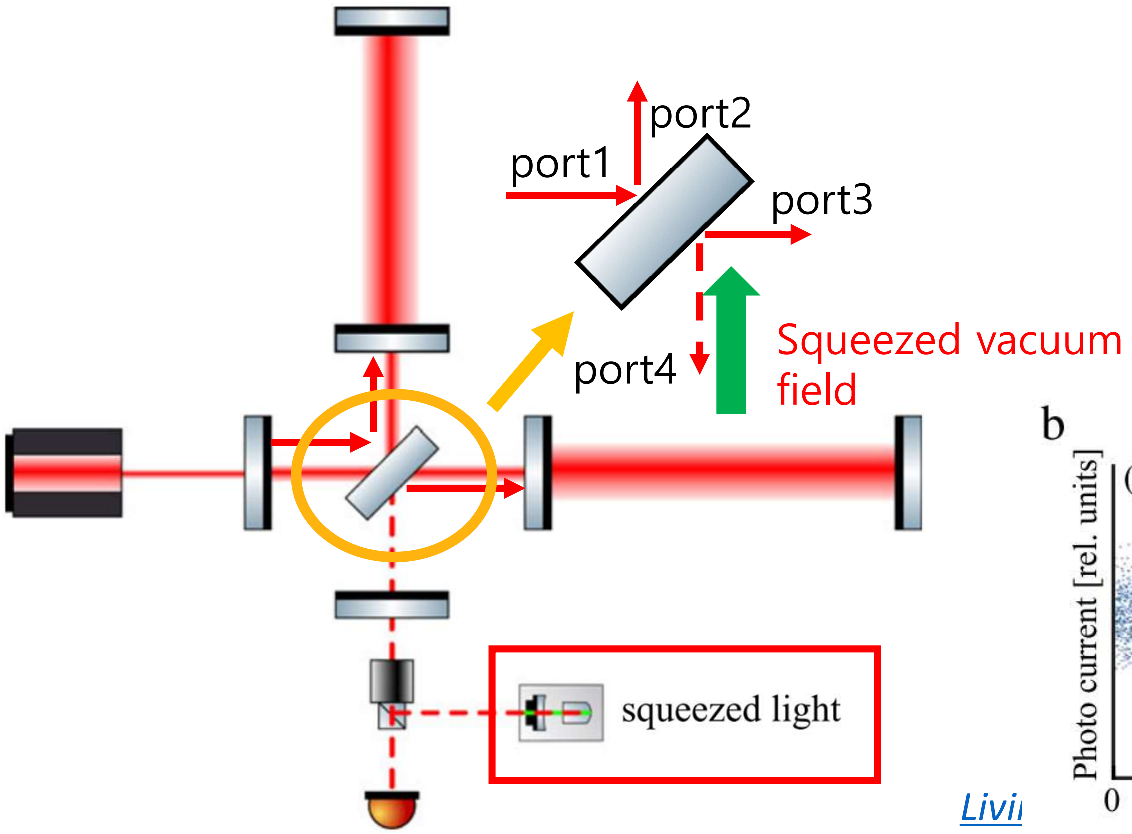


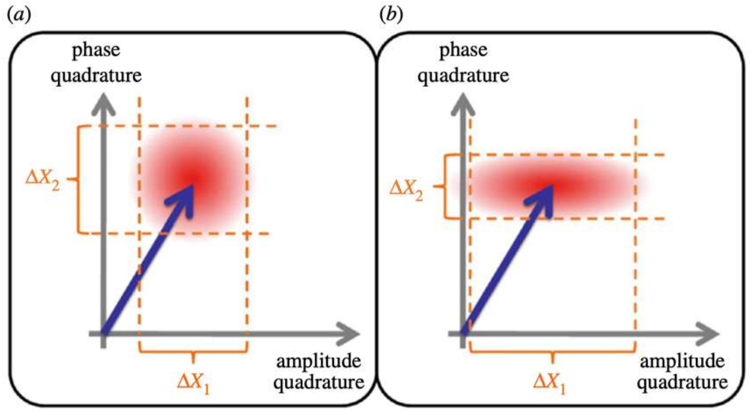
Figure 1.6: Simulation of electric field in time for (a) vacuum state and for (b) squeezed vacuum.

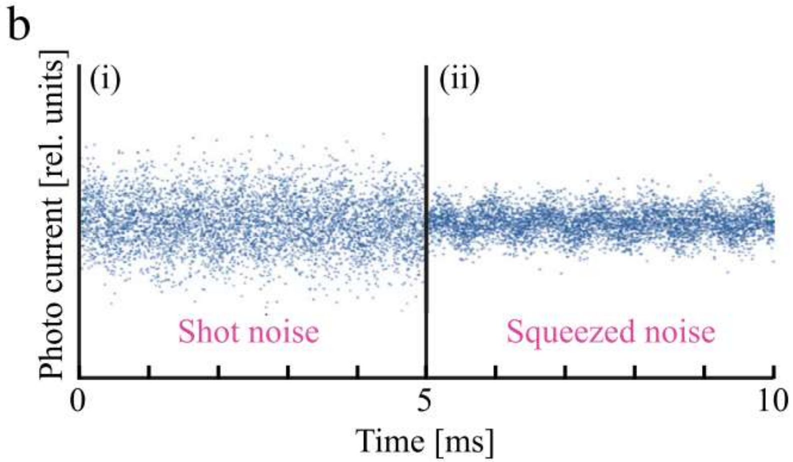
Squeezed Vacuum



Gravitational Wave Detector

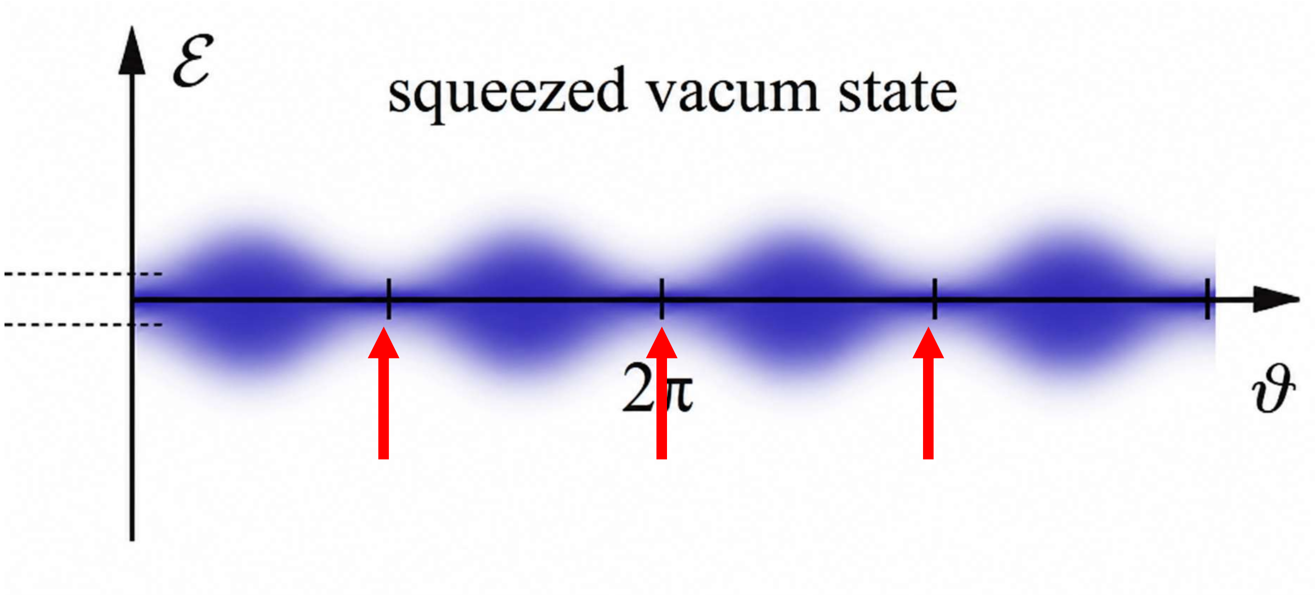




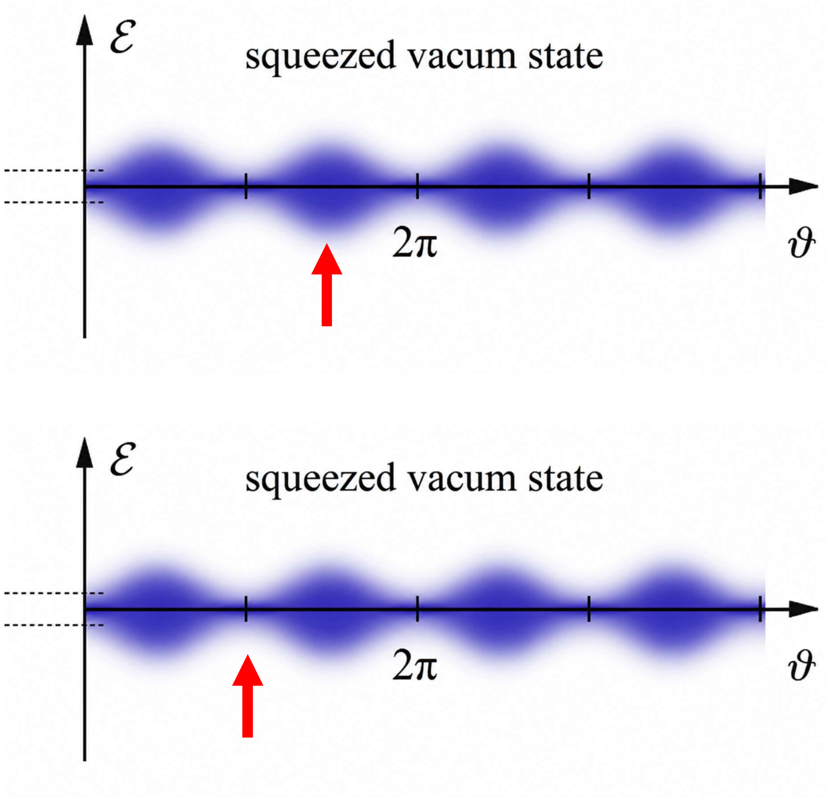


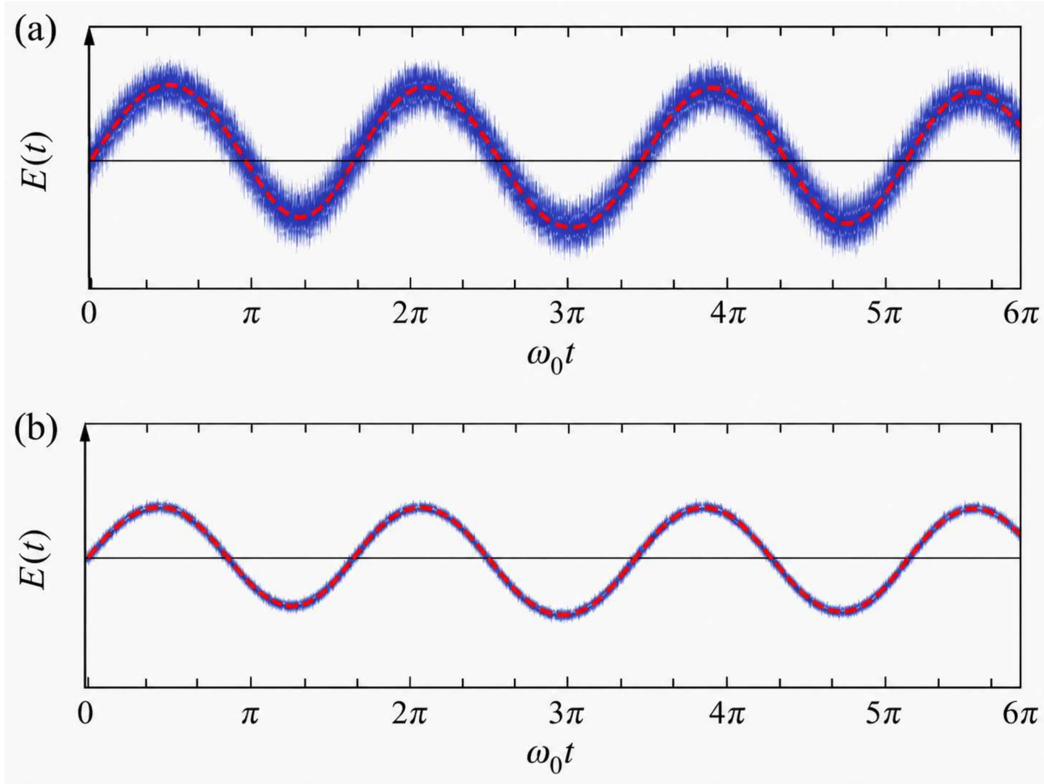
Livi … 2019)

Squeezed Vacuum

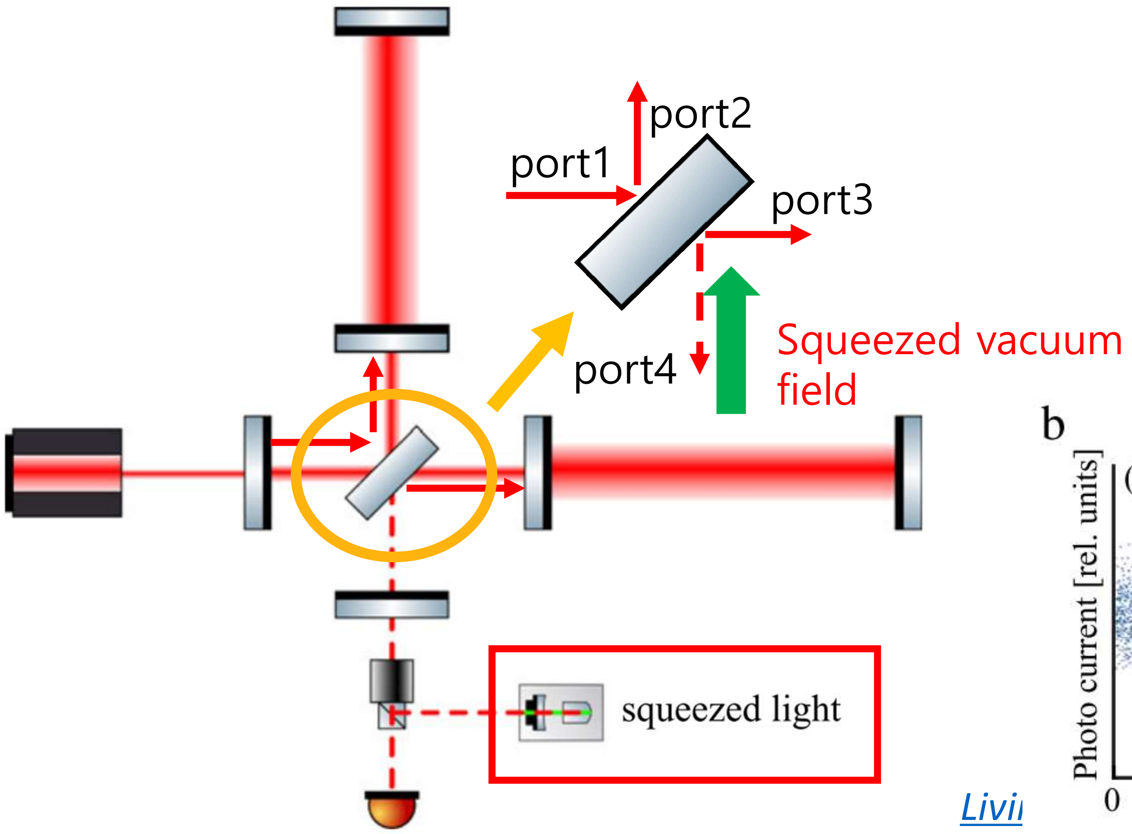


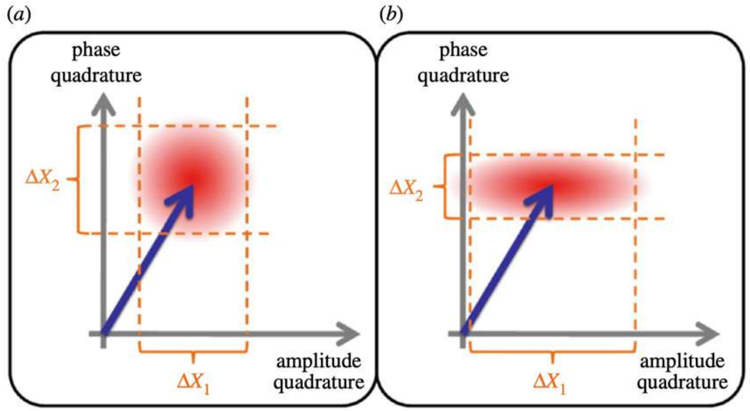
Squeezed Vacuum Injection

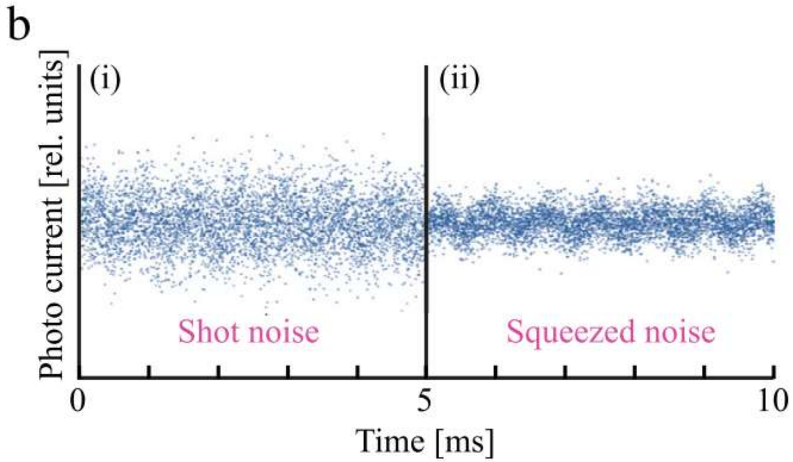




Gravitational Wave Detector

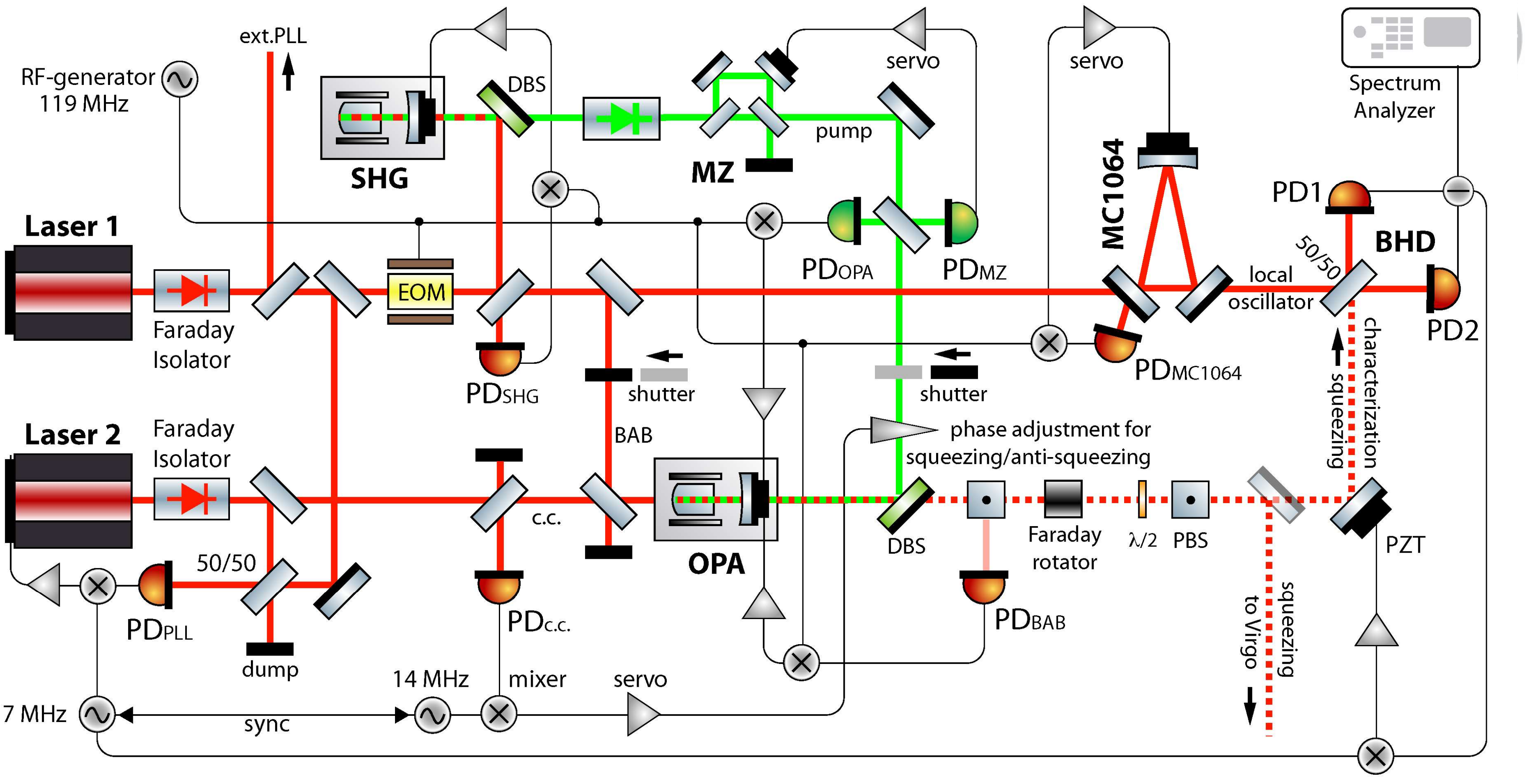




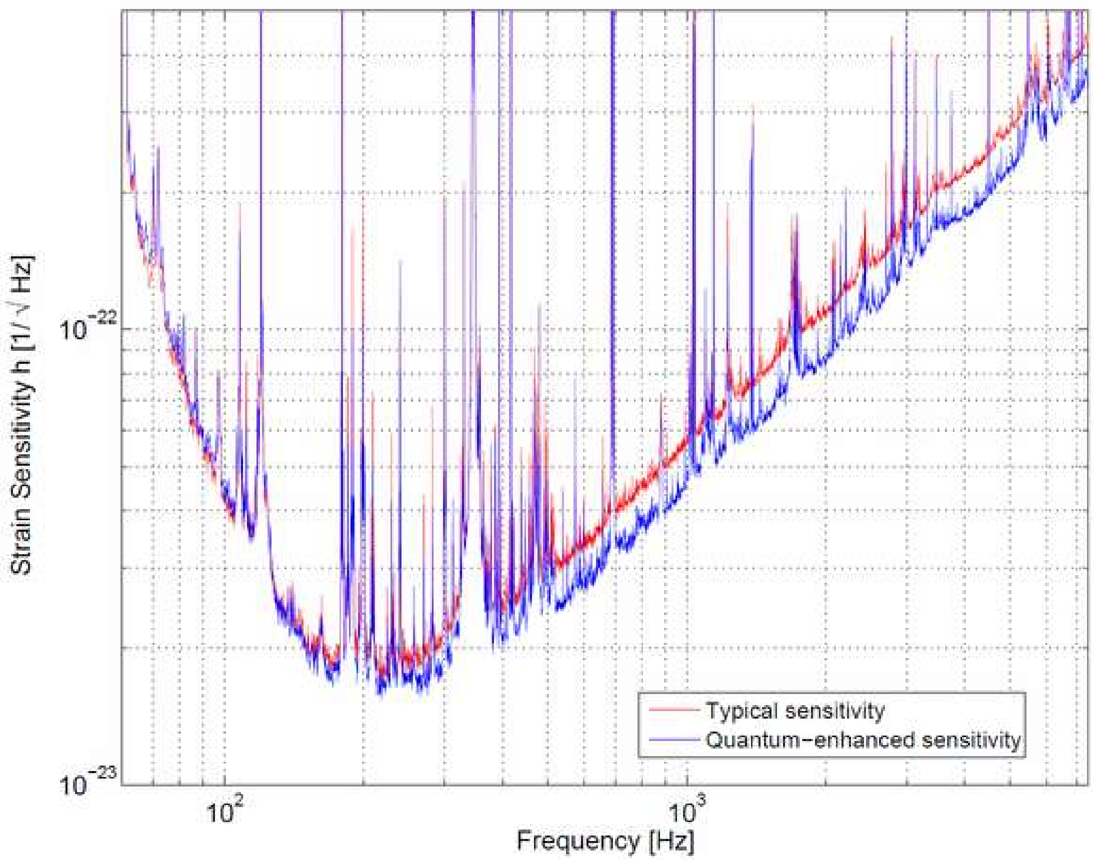


Livi

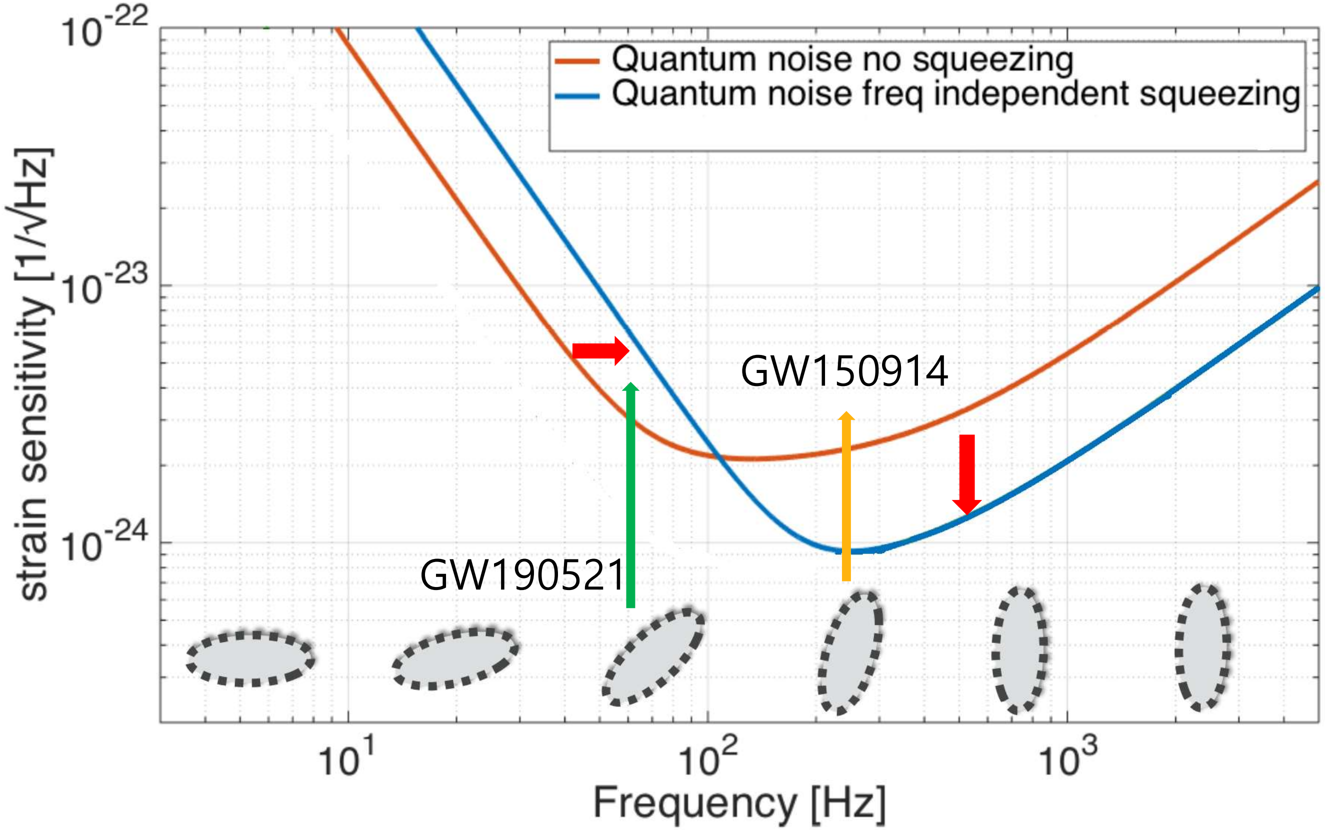
2019)


ext.PLL
RF-generator
119 MHz
SHG
DBS
MZ
pump
servo
servo
Spectrum
Analyzer
MC1064
PD1
BHD
50/50
local
oscillator
PD2
Laser 1
Faraday
Isolator
EOM
PDOPA
PDMZ
PDMC1064
PDSHG
shutter
shutter
characterization
squeezing
Laser 2
Faraday
Isolator
BAB
phase adjustment for
squeezing/anti-squeezing
c.c.
OPA
DBS
Faraday
rotator
λ/2
PBS
PZT
50/50
PDPLL
dump
PDc.c.
PDBAB
to Virgo
squeezing
7 MHz
sync
14 MHz
mixer
servo

Quantum Noise Enhancement of LIGO

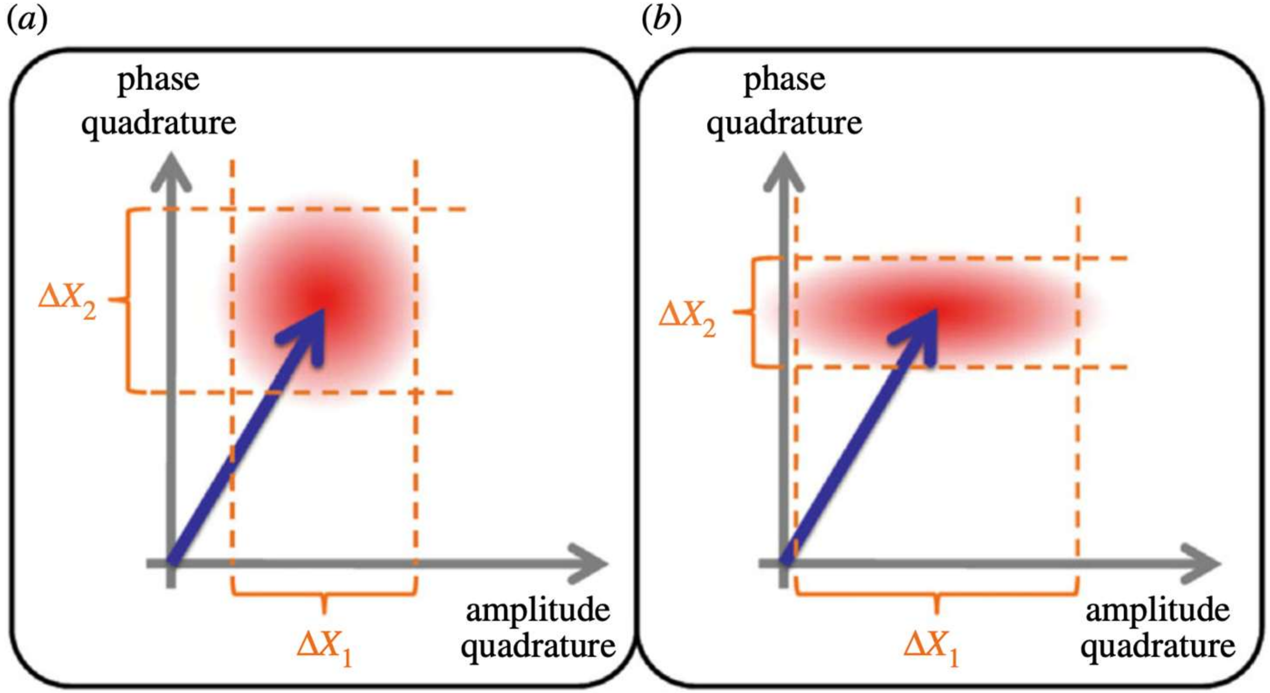


Frequency Independent squeezing



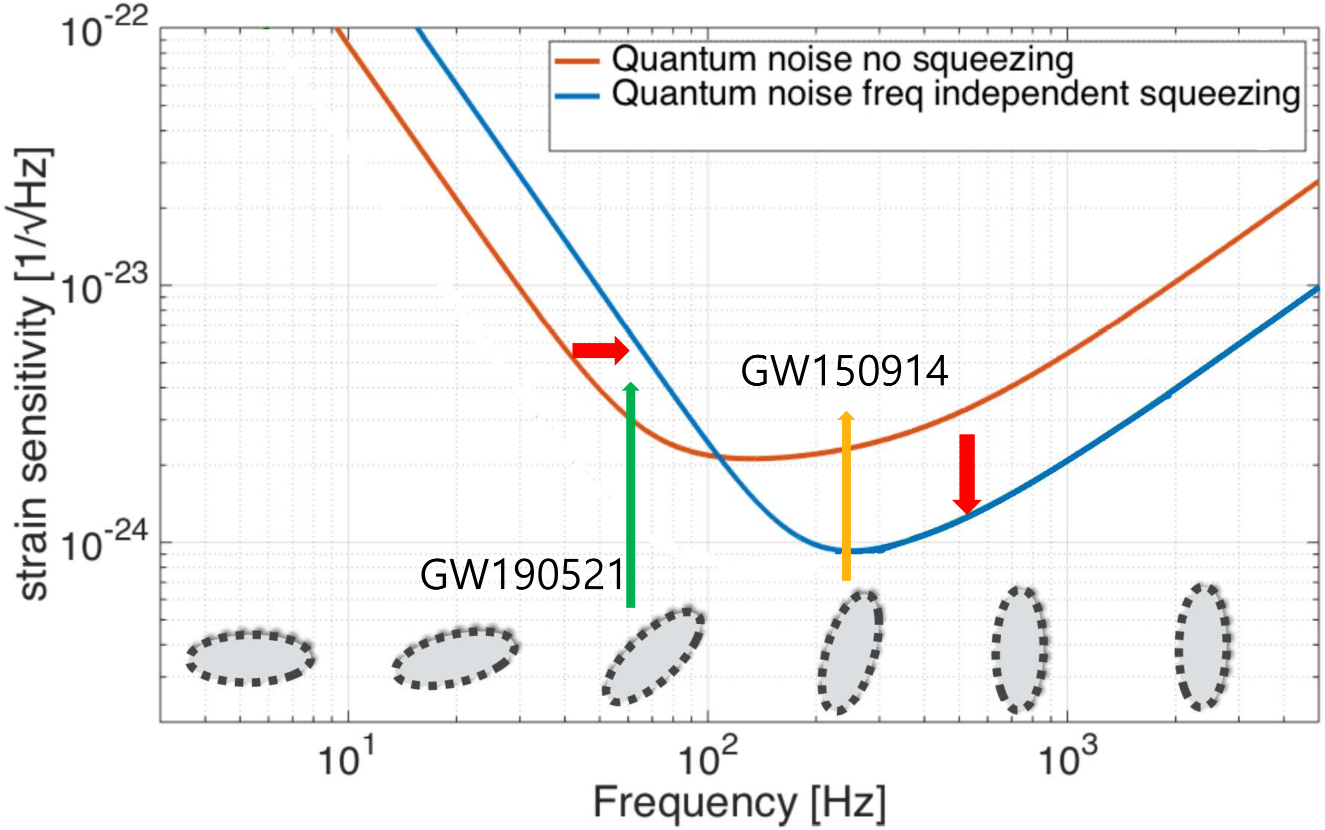
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Squeezed Light



Heurs M. 2018 Gravitational wave detection u
sing laser interferometry beyond the standard
quantum limit.Phil. Trans. R. Soc. A 376: 20170
289.

Frequency Independent Squeezing



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
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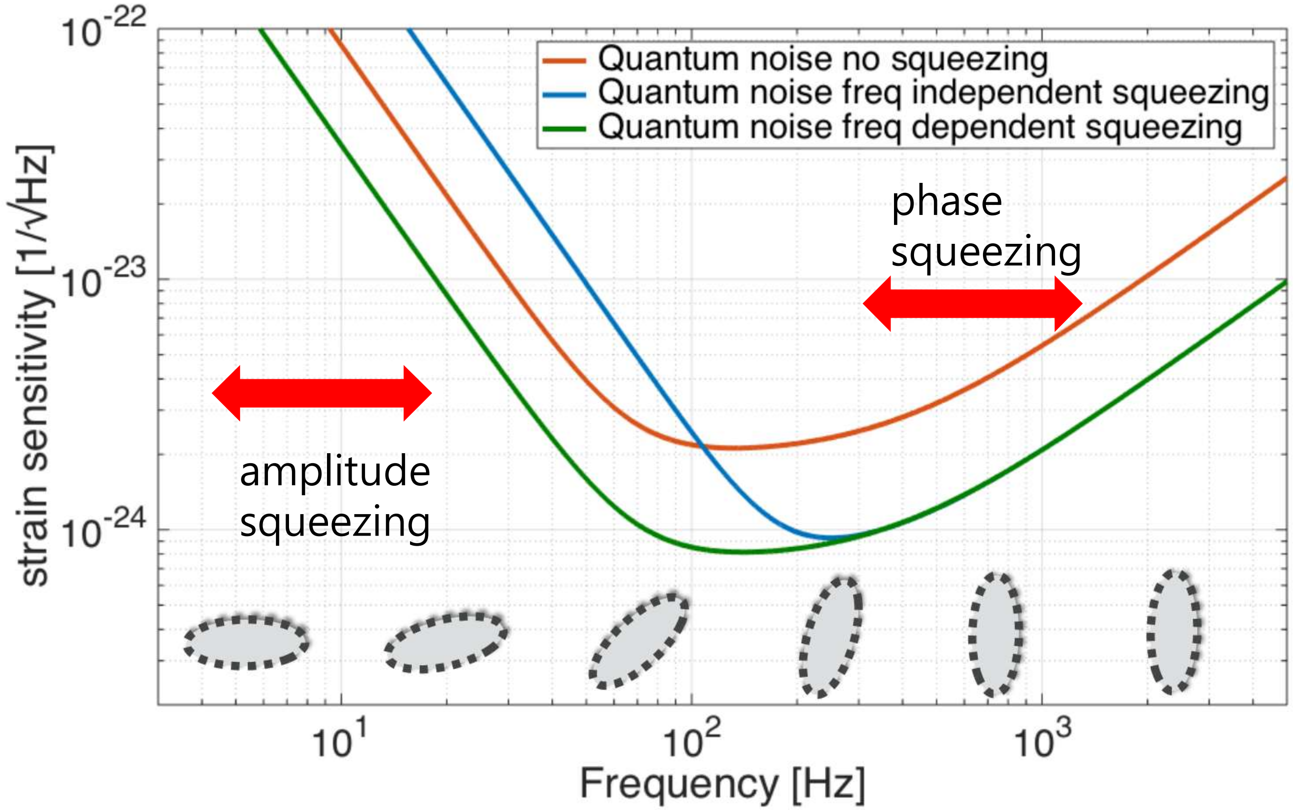
Frequency Dependent Squeezing(FDS)



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
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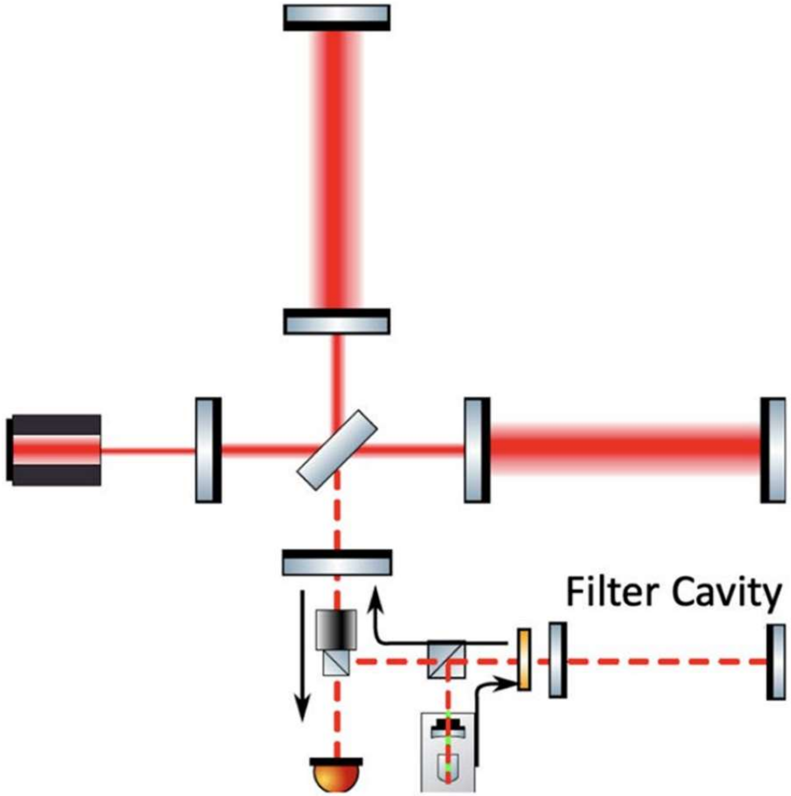
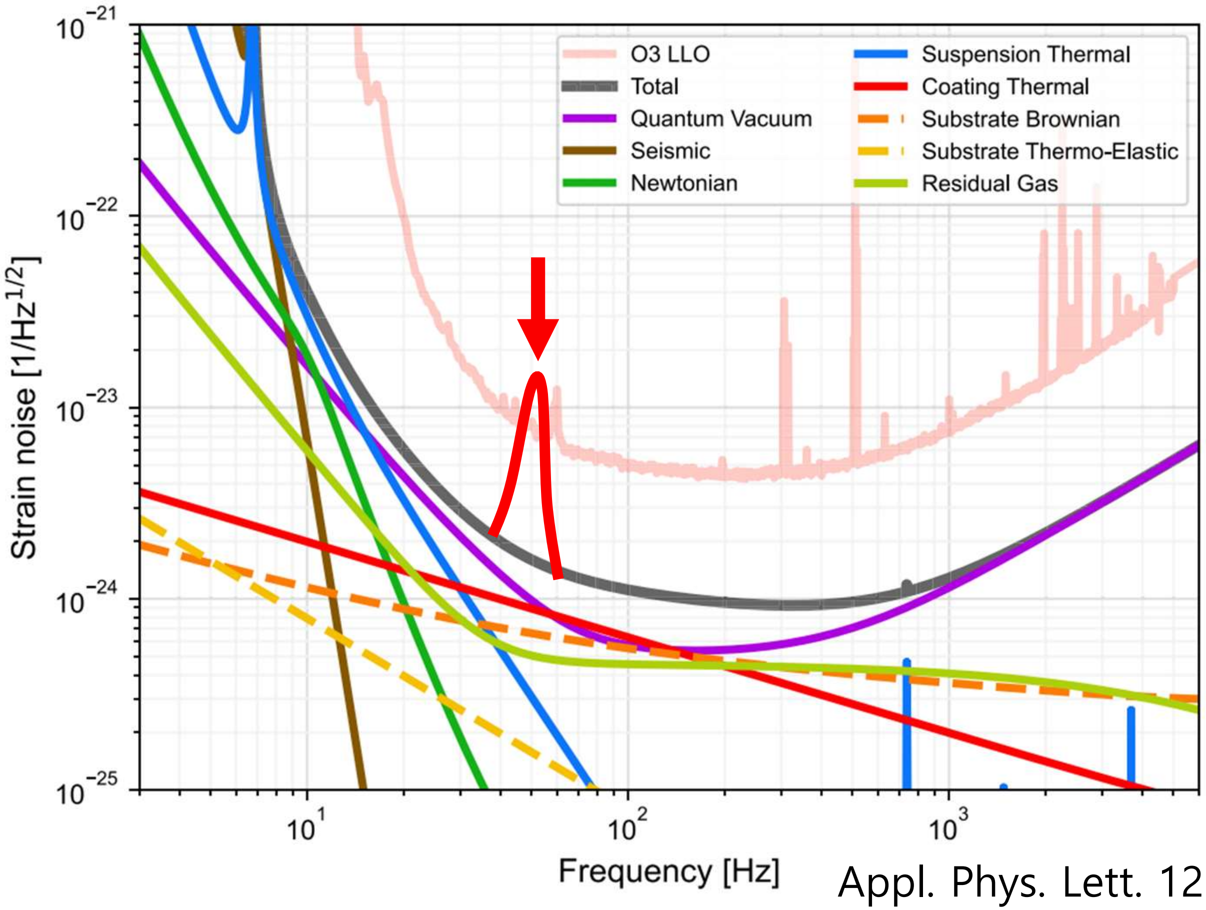
Frequency Dependent Squeezing(Filter Cavity)

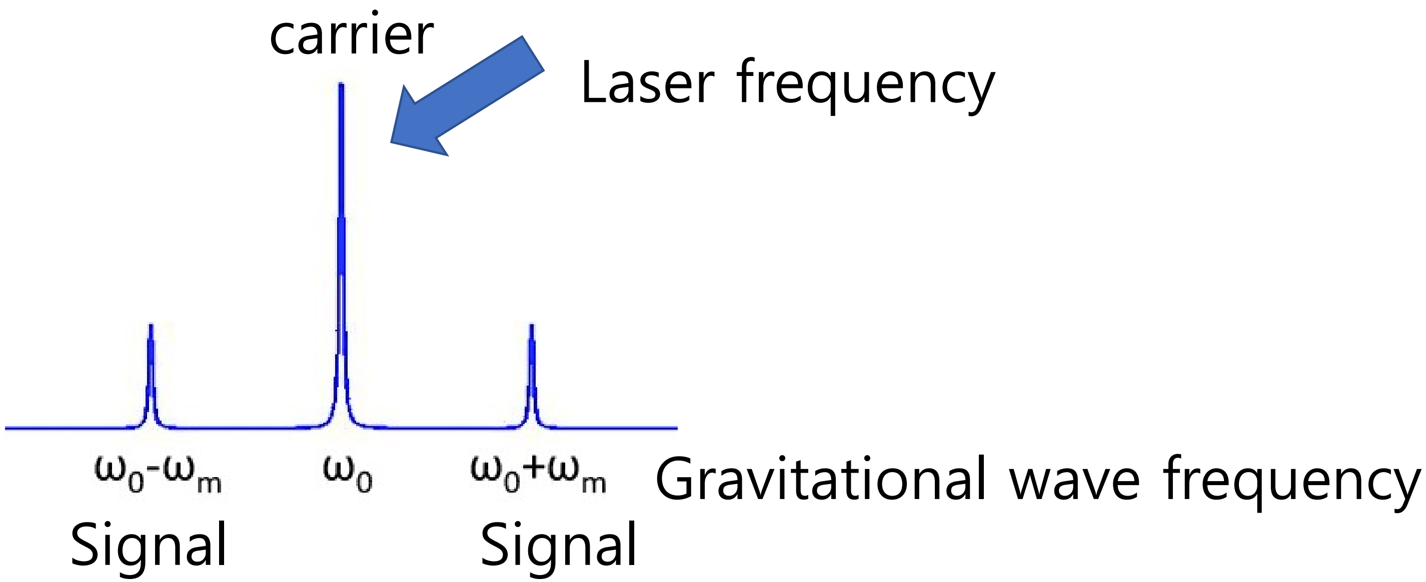


LIGO Sensitivity

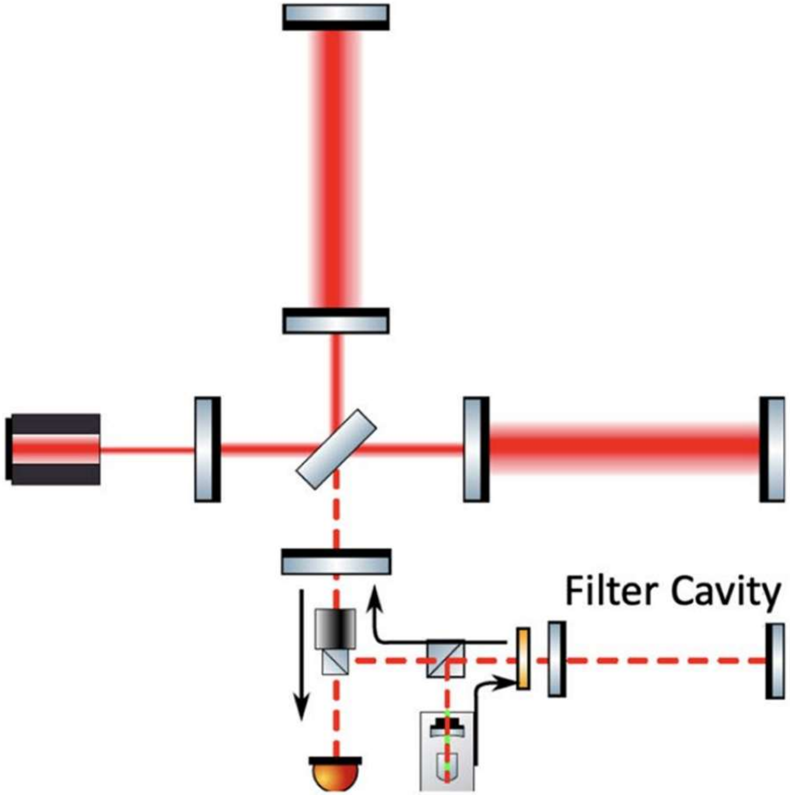


Appl. Phys. Lett. 122, 110502 (2023)

Side Band Figure

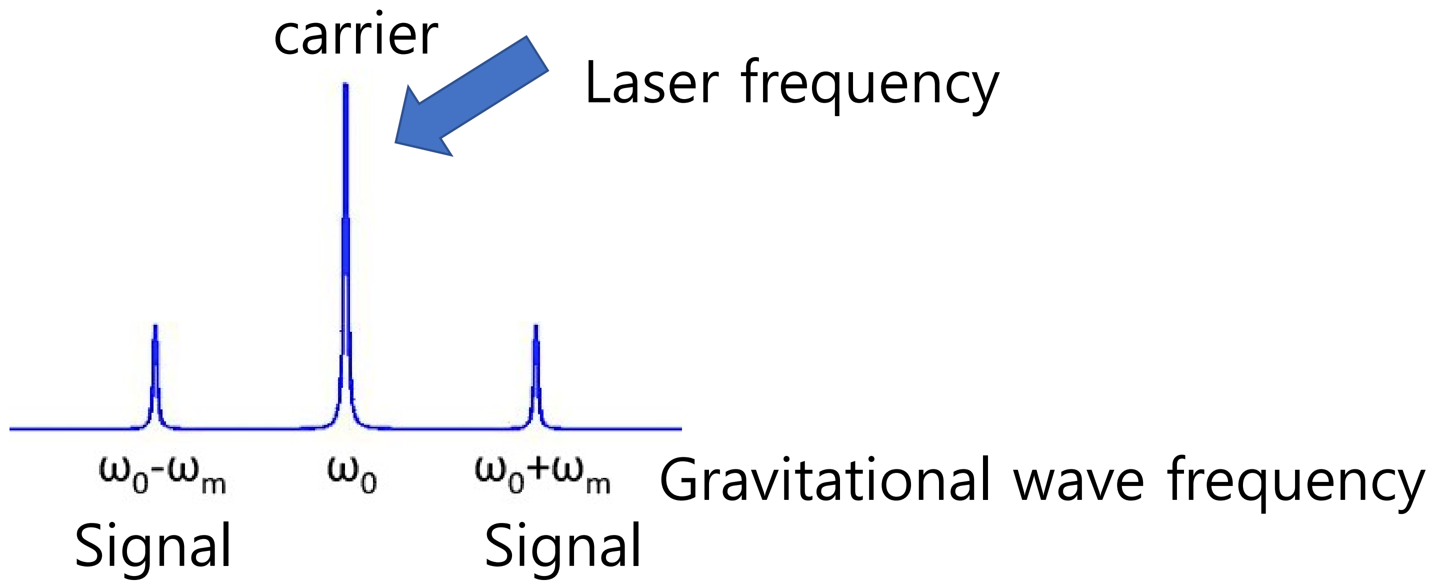


Frequency Dependent Squeezing(Filter Cavity)

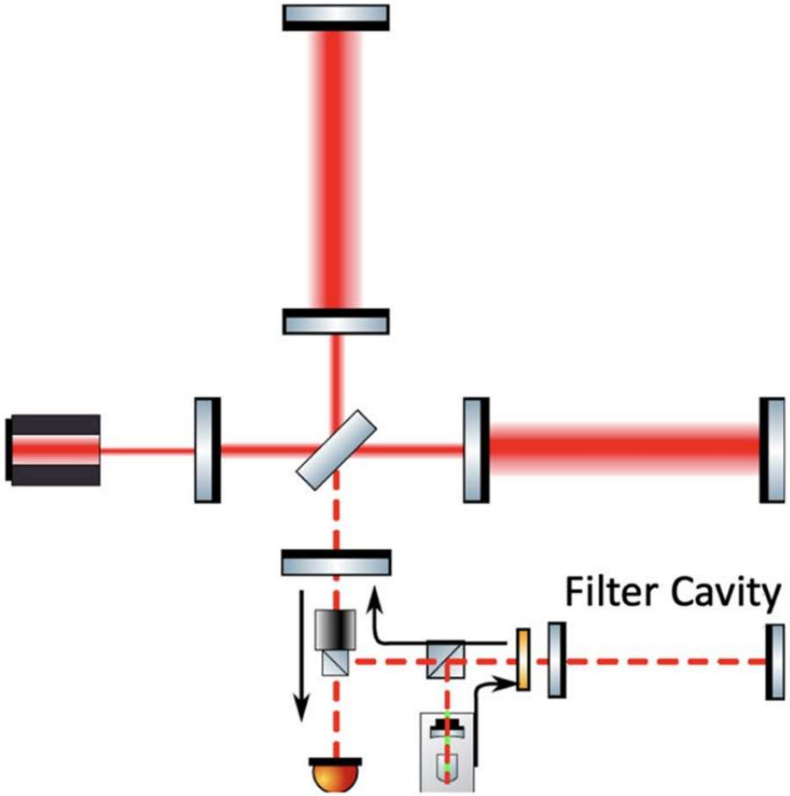


$$E^{(+)}(t) = \int_0^\infty E(\omega)e^{-i\omega t}\frac{d\omega}{2\pi}$$

Side Band Figure



Frequency Dependent Squeezing(Filter Cavity)



$$E^{(+)}(t) = \int_0^\infty E(\omega) e^{-i\omega t} \frac{d\omega}{2\pi}$$

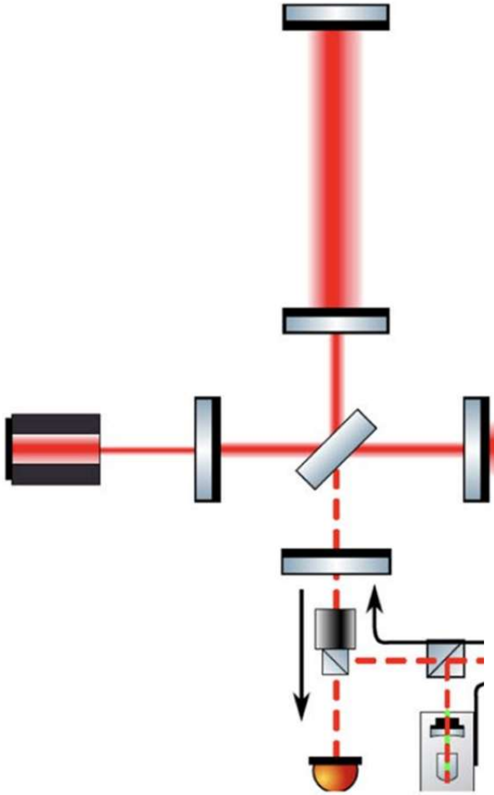
$$E^{(+)}(t) = e^{-i\omega_0 t} \int_0^\infty \left(E(\Omega) e^{-i\Omega t} + E(-\Omega) e^{+i\Omega t} \right) \frac{d\Omega}{2\pi}$$

$$\omega = \omega_0 + \Omega$$

Laser frequency

GW signal frequency

Frequency Dependent Squeezing(Filter Cavity)



$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\left[a_1(t)\cos(\omega_0 t) + a_2(t)\sin(\omega_0 t)\right]$$

$$\textbf{Amplitude Quadrature}: \ a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

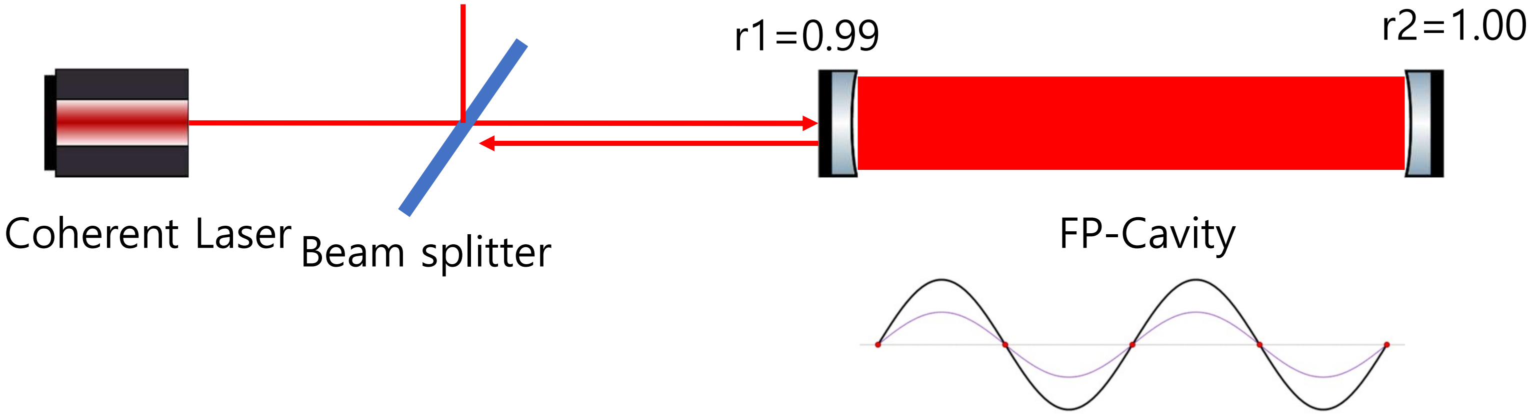
$$\textbf{Phase Quadrature}: \ a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$

$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}}\left[\cos(\omega_0 t)\int_0^\infty (a_1(\Omega)e^{-i\Omega t} + a_1^\dagger(\Omega)e^{+i\Omega t})\frac{d\Omega}{2\pi} + \sin(\omega_0 t)\int_0^\infty (a_2(\Omega)e^{-i\Omega t} + a_2^\dagger(\Omega)e^{+i\Omega t})\frac{d\Omega}{2\pi}\right]$$

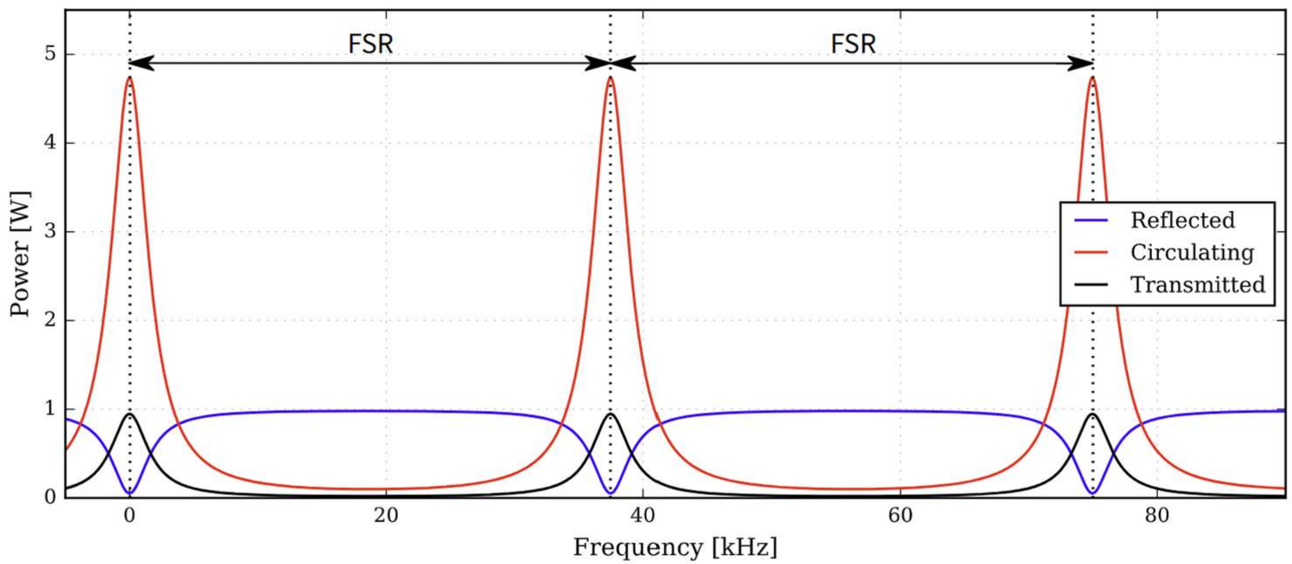
Fabry-perot cavity

Number of round trip > 250

4 km x 250 ~ 1000 km



Free Spectral Range of Cavity



Detuned cavity

Simple picture

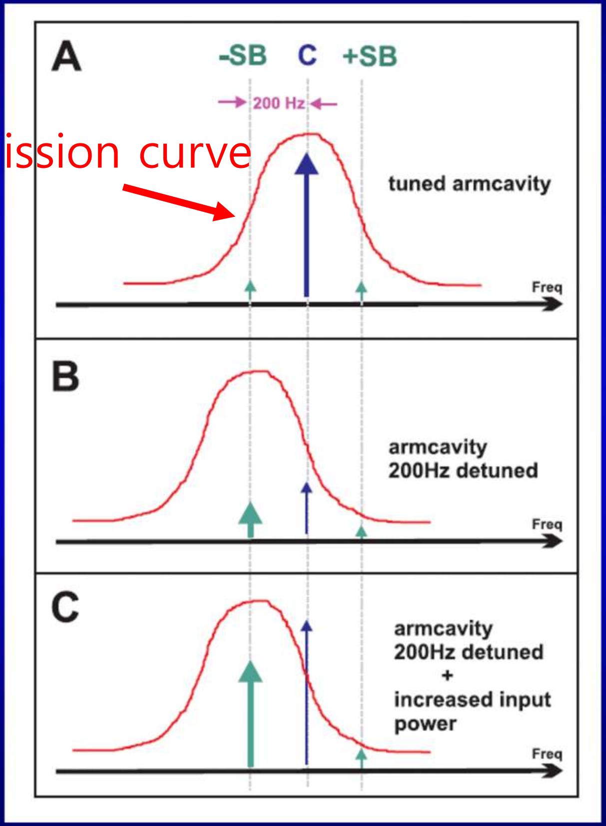
Stefan Hild et al, “Detuned arm cavities”, 3rd GEO simulation workshop, Hannover, June 2007

B:
- less carrier light in cavity => less GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced.

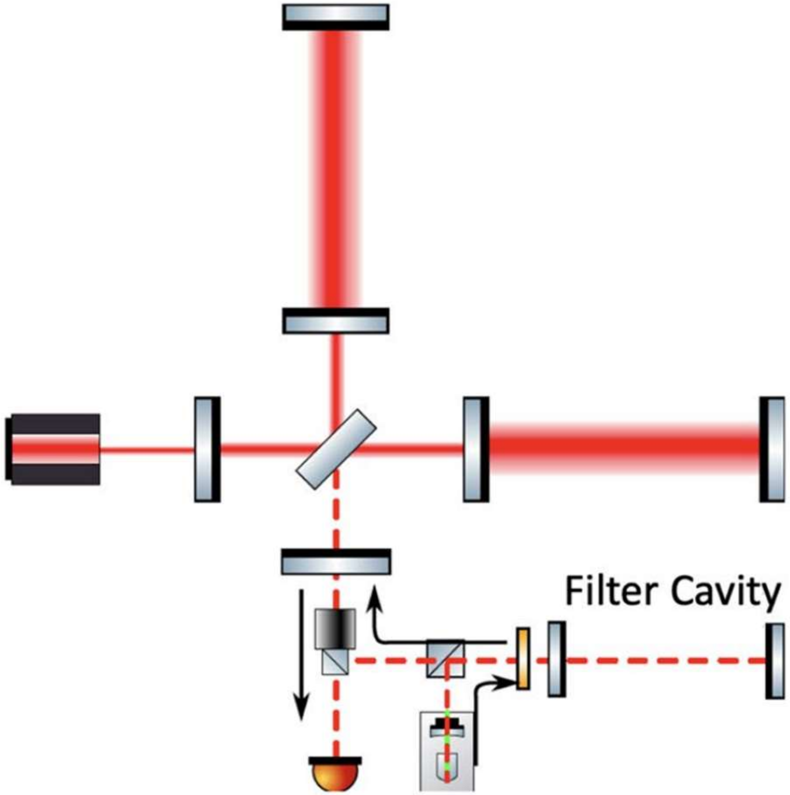
=> Smaller GW signal

C:
- optical power is restored in the cavity by larger PR-gain.
- Same amount of GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced. Overall we win GW signal.

=> Larger GW signal

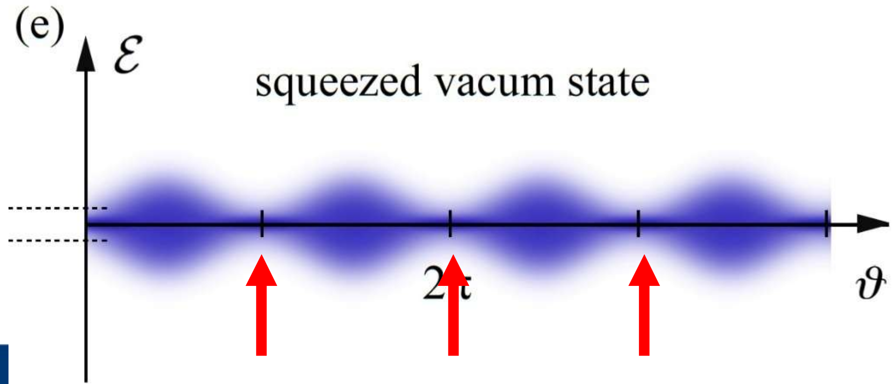


Frequency dependent squeezing

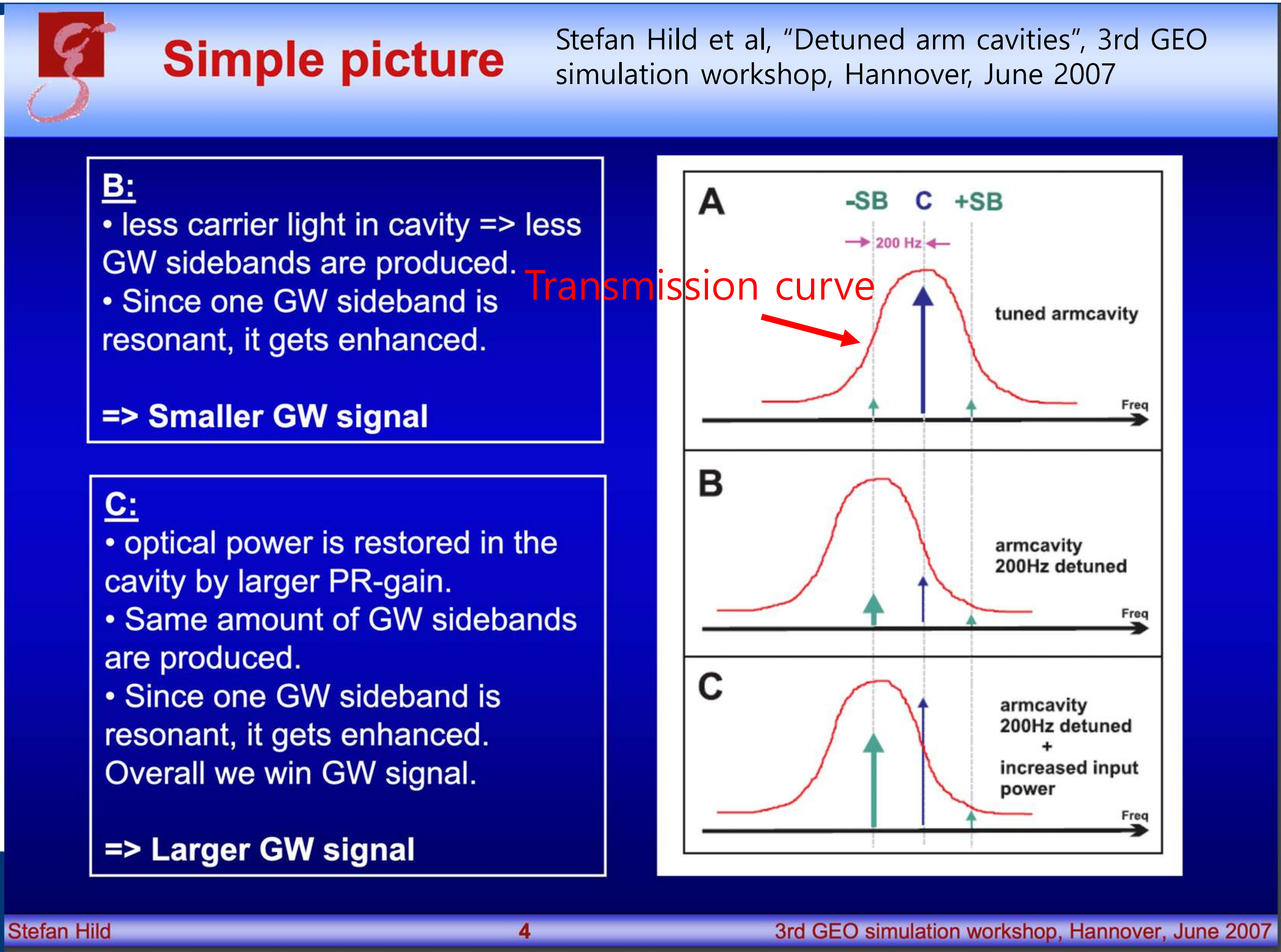


Phase of return field for Filter cavity

Reflection from FC
+ Transmission from inside FC

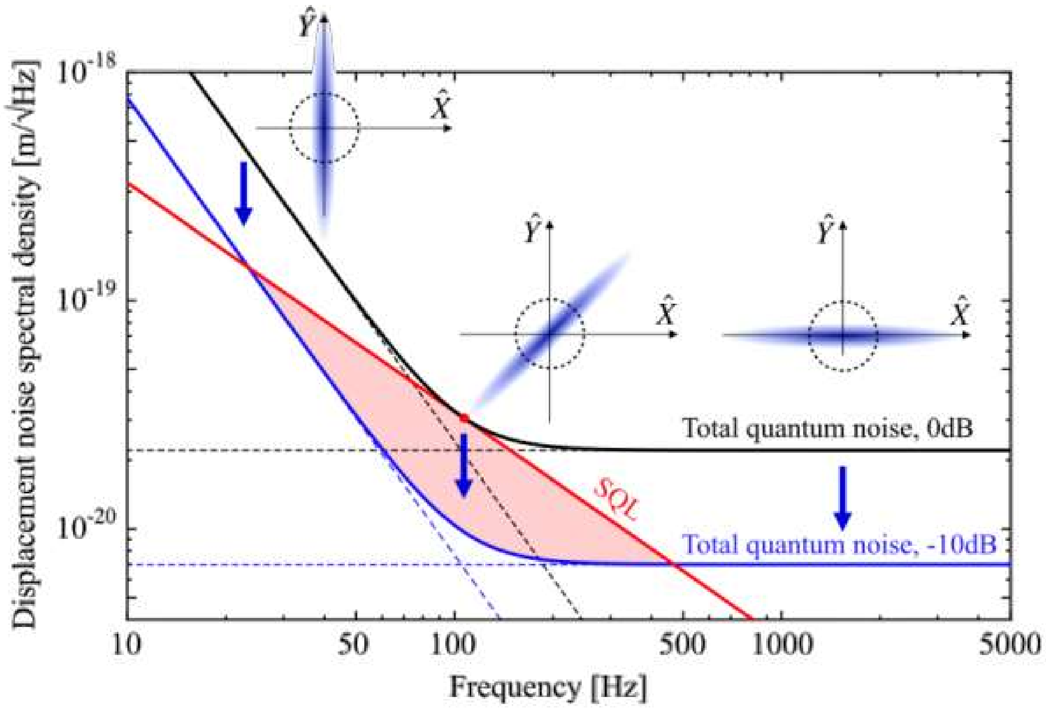


Detuned cavity



Stefan Hild et al, “Detuned arm cavities”, 3rd GEO simulation workshop, Hannover, June 2007

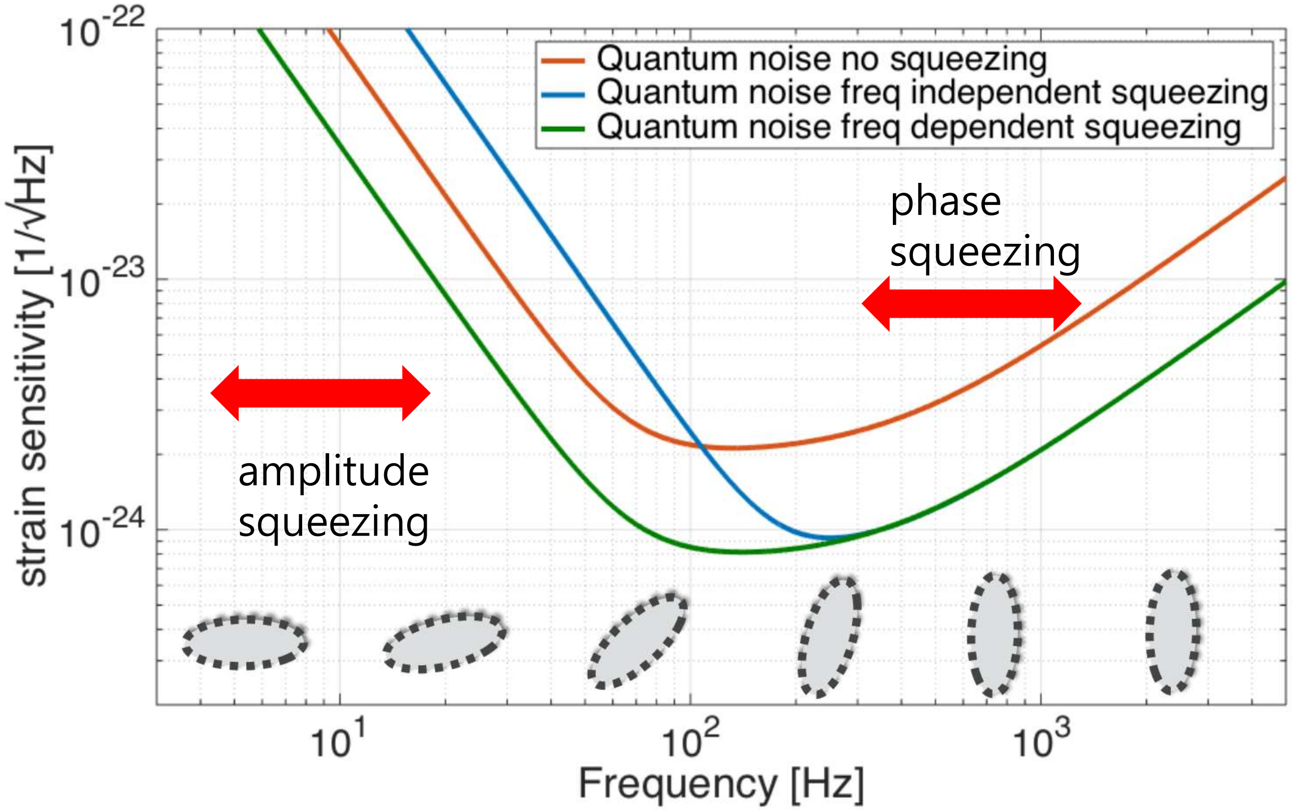
Squeeze angle rotation



$$\alpha_p = \arctan\left(\frac{2\gamma_{fc}\Delta\omega_{fc}}{\gamma_{fc}^2 - \Delta\omega_{fc}^2 + \Omega^2}\right)$$

$$t_{st} = \frac{1}{\gamma_{fc}} = \frac{\sqrt{2}}{\Omega_{SQL}} \simeq 3\,\text{ms}$$

Frequency Dependent Squeezing(FDS)



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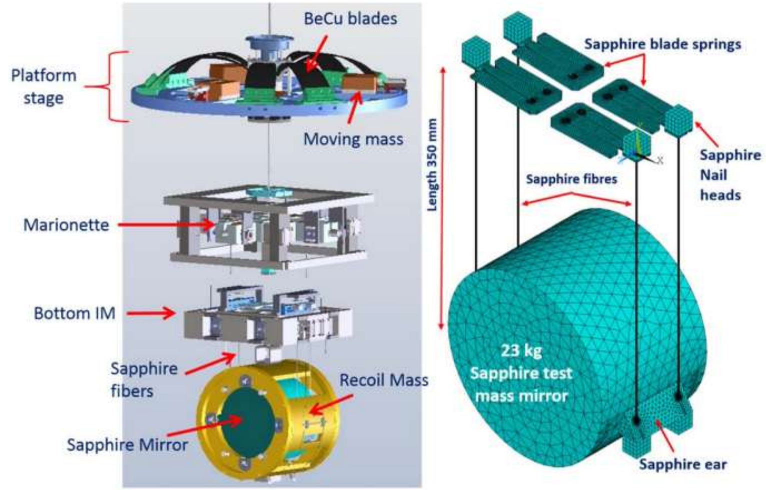
Filter Cavity of GWD



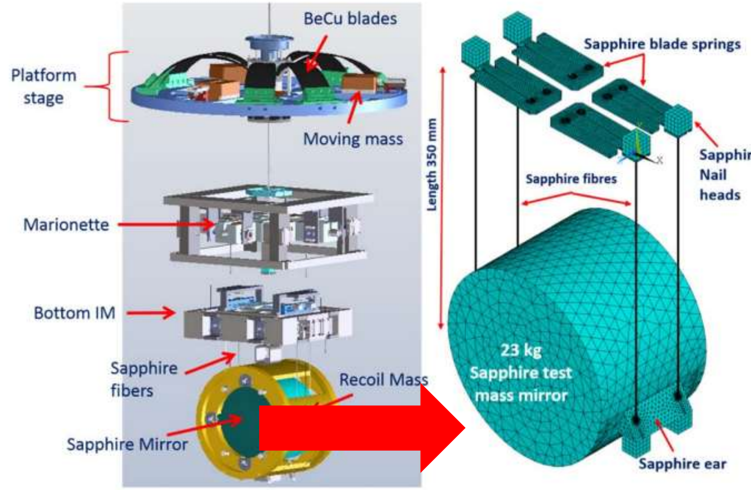
Optical spring by photon + Mechanical suspension

frequency-dependent effective spring constant

$$\boldsymbol{k_{\mathrm{eff}}(\Omega)}$$

Filter Cavity of GWD

FP-Cavity

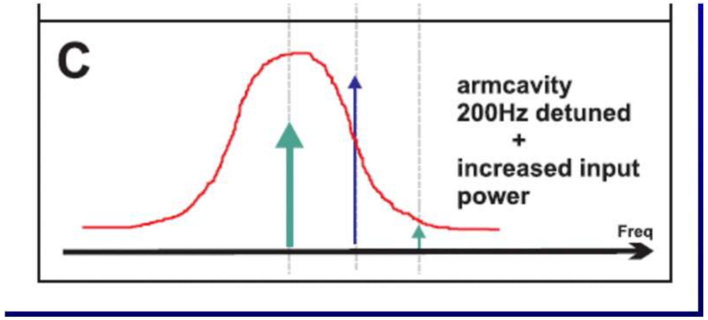


Optical spring by photon + Mechanical suspension

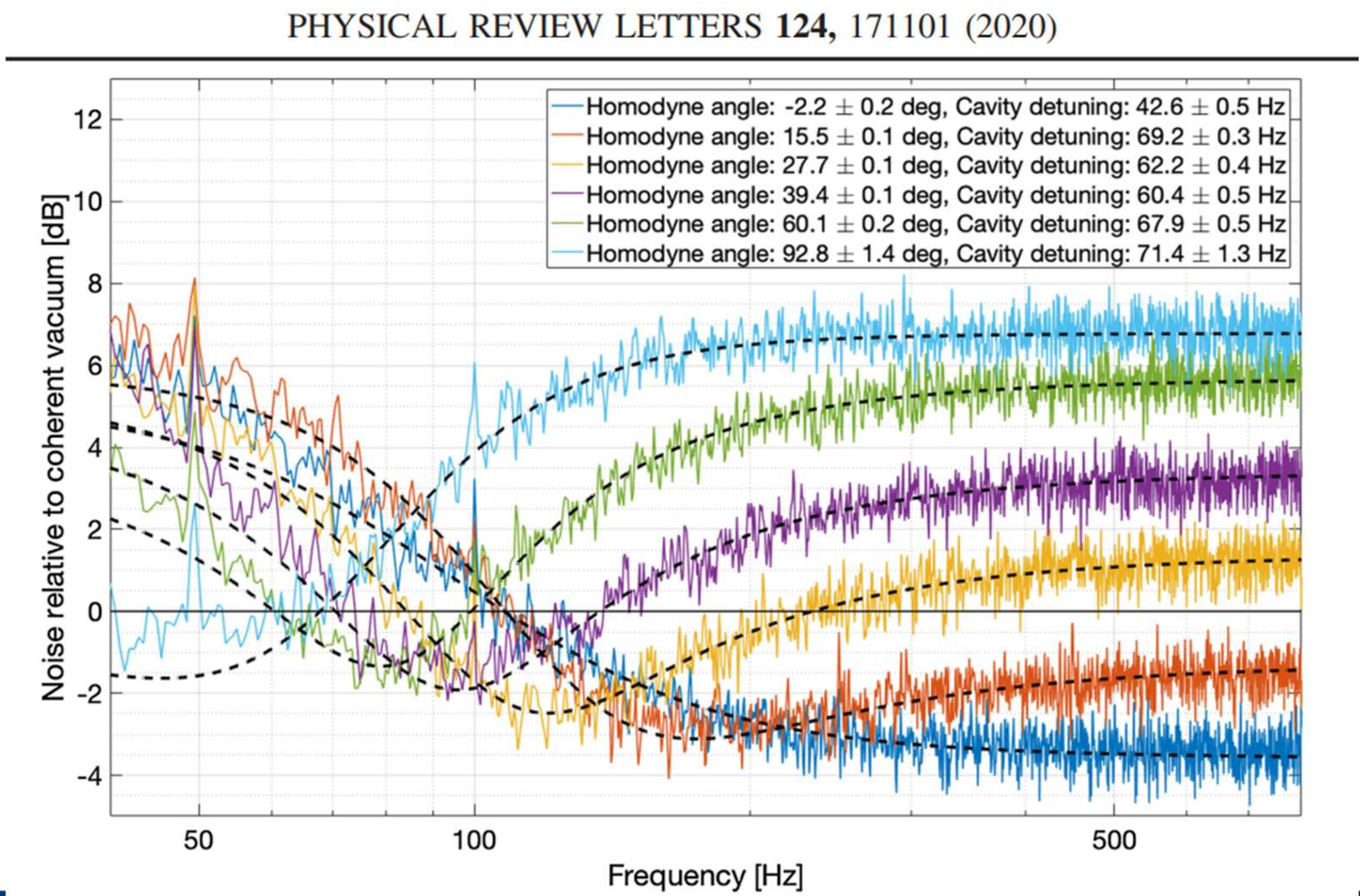
frequency-dependent effective spring constant

$$k_{\text{eff}}(\Omega)$$

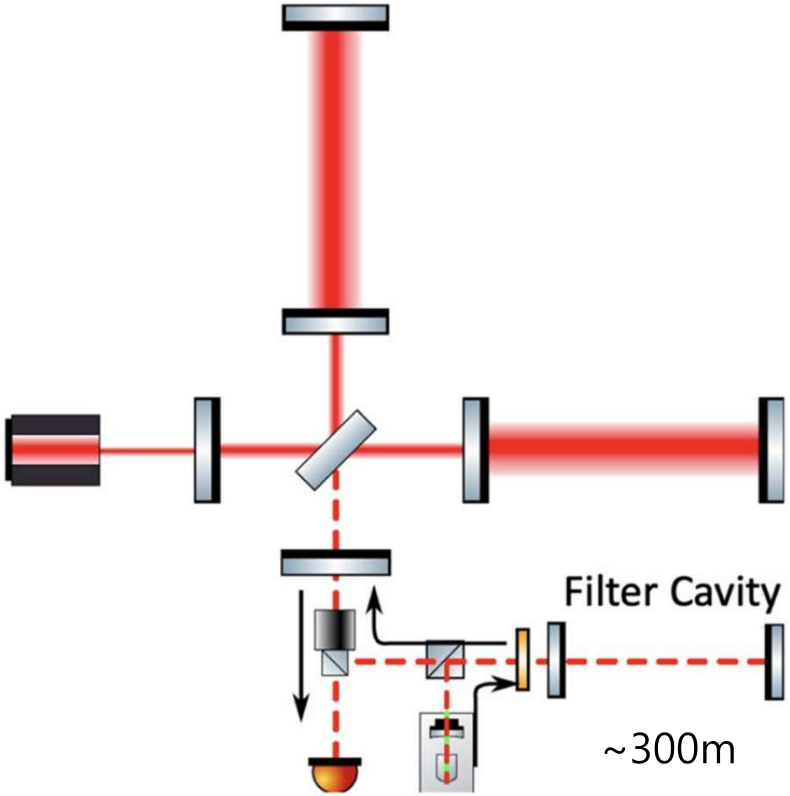
Rad pressure

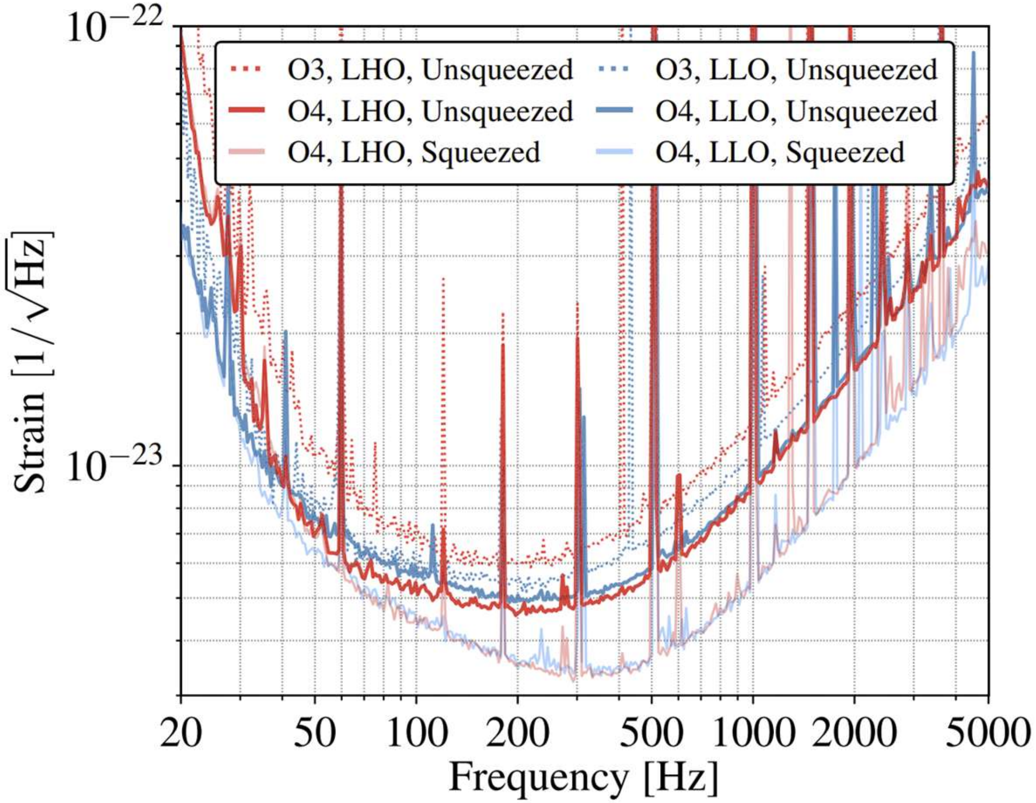


Frequency dependent squeezing - KAGRA



FDS using Filter Cavity



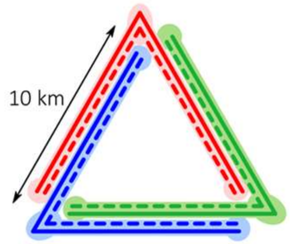


Future generation GW detector: Einstein Telescope (ET)

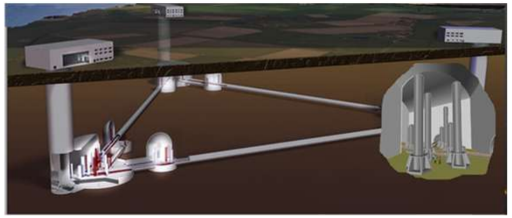
Target: 10^5 to 10^6 events/year

Expected detectable sources:
Merging BH throughout the whole universe and reconstruct BH demography
Investigate primeval universe and connections with particle physics

- **Configuration: 2L or Equilateral triangle**
- **Arm cavities 10 km long**
- **Underground (300 m)**
- **Cooled mirrors (10 K)**

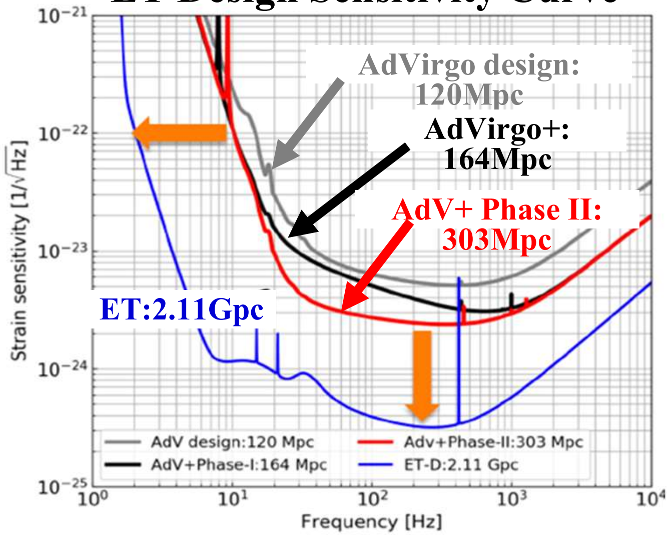


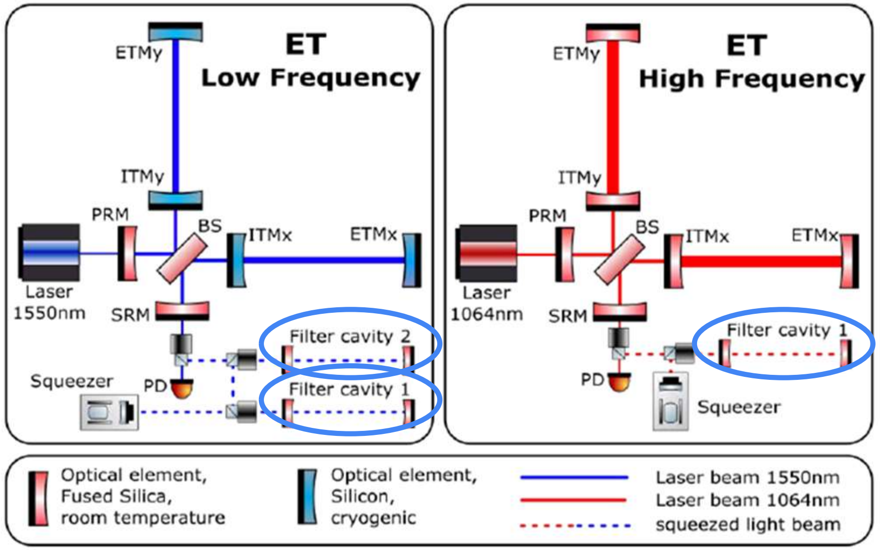
Candidate sites are: Sardinia, Euregio Meuse-Rhine, Lausitz



Quantum Noise limited in all frequency band

ET Design Sensitivity Curve





Required FC for ideal QN suppression:

- **L=5 km for ET-LF (x2)**
- **L=1 km for ET-HF**

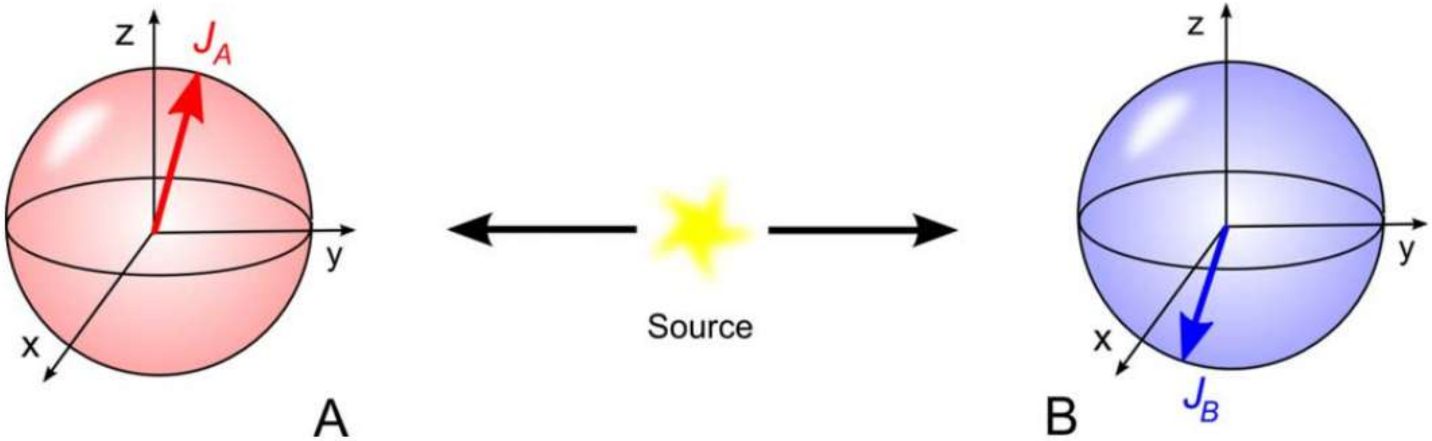
→ 9 FCs in triangle conf. each few km-long and underground!

→ More compact setups will reduce infrastructure impact and costs!

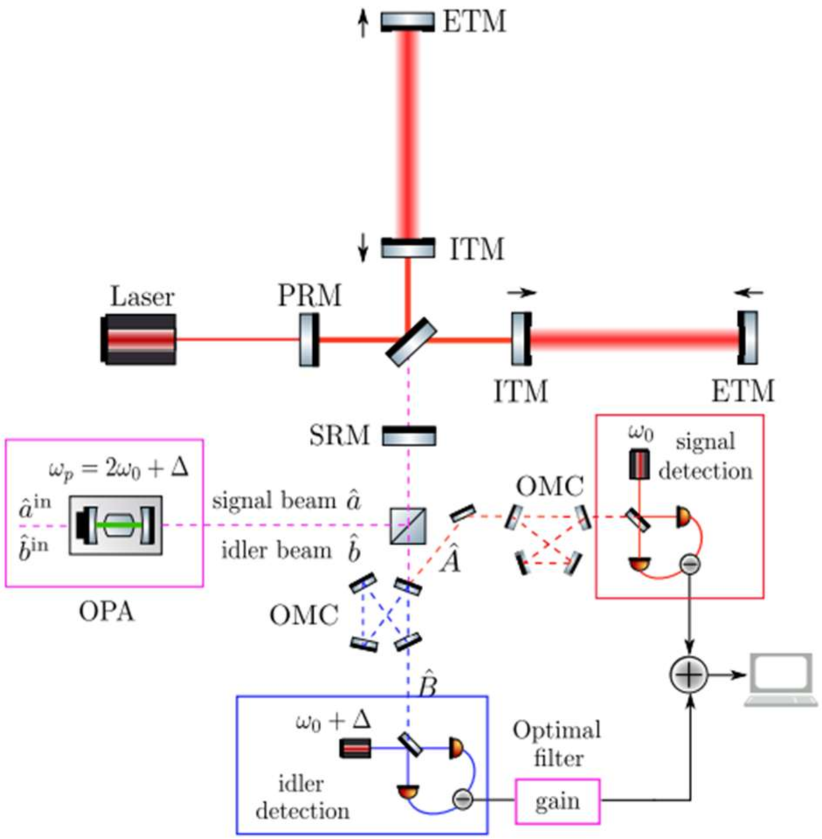
ET design report update 2020 ("long ESFRI document"), *ET-0007A-20*
Science Case for the Einstein Telescope, *https://doi.org/10.1088/1475-7516/2020/03/050*

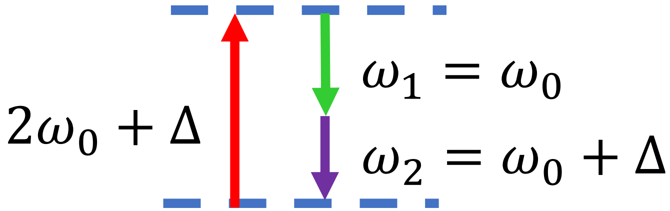
Important to develop more compact Quantum Noise Reduction Techniques

Einstein-Podolsky-Rosen paradox



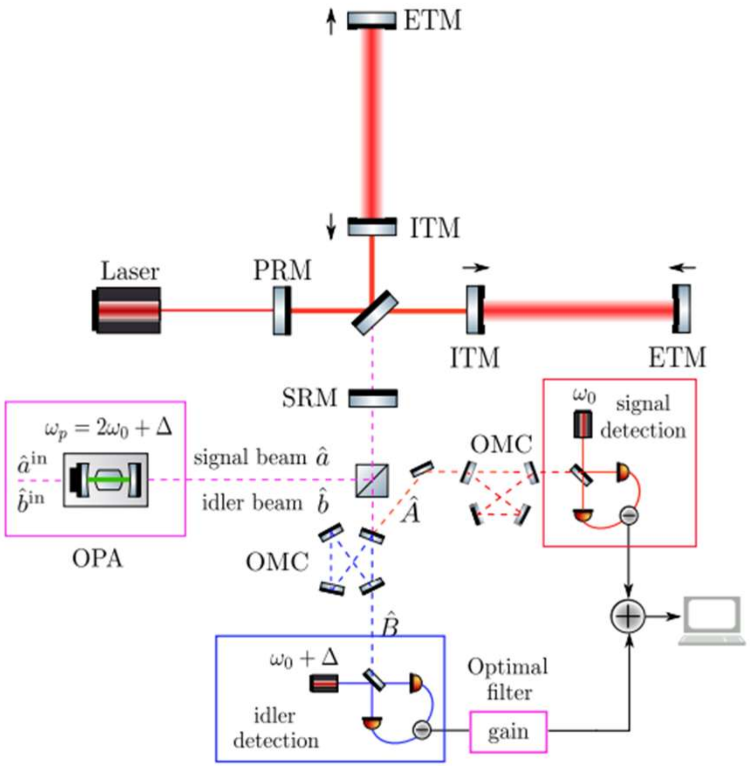
EPR squeezing for gravitational wave detector

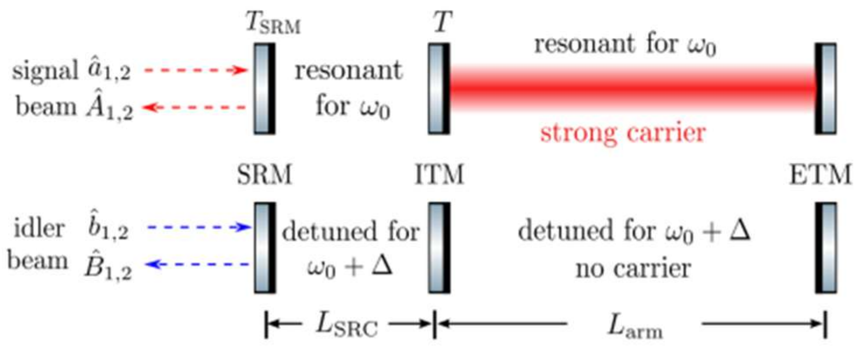


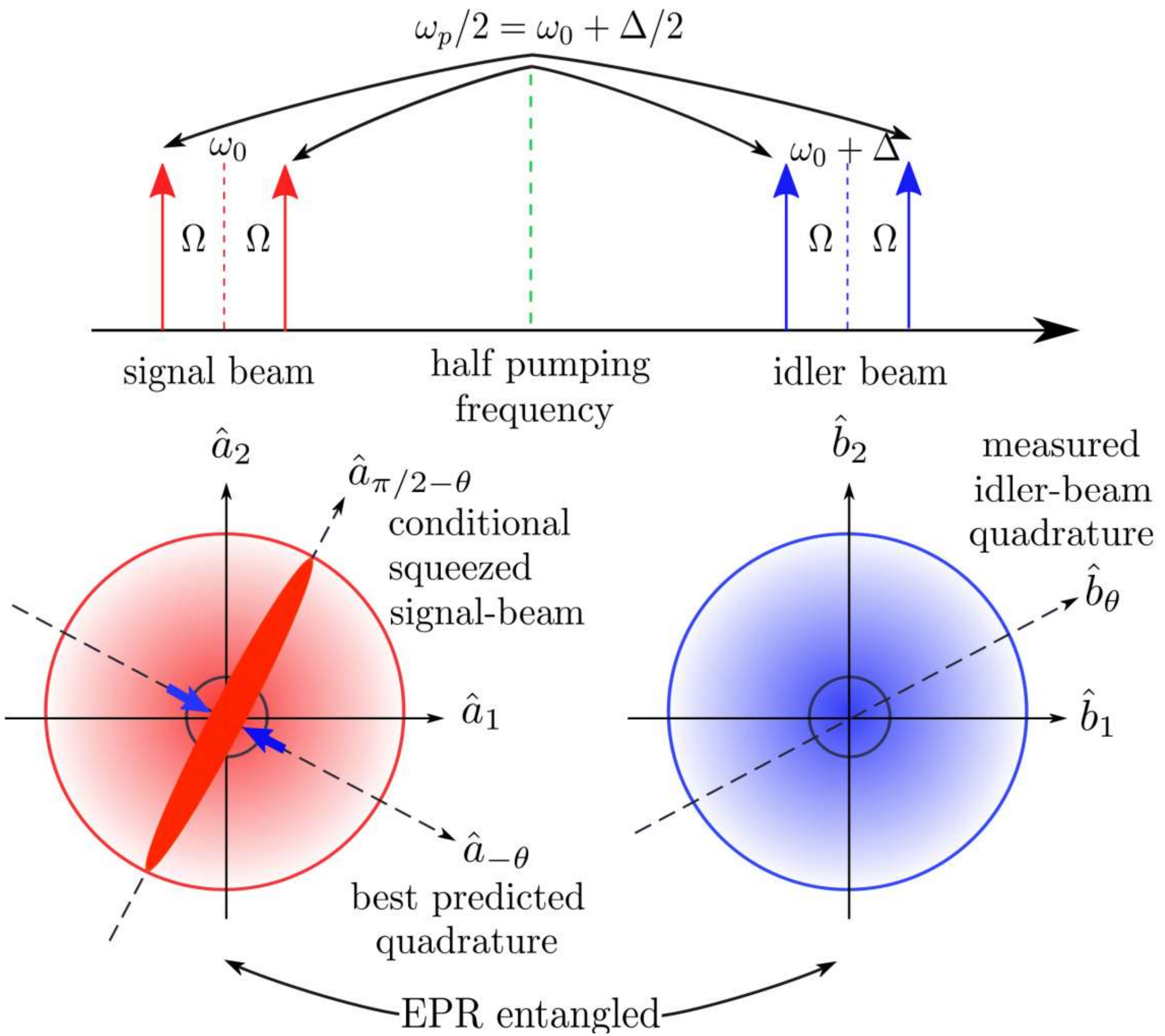


Non linear optics

EPR squeezing for gravitational wave detector







$\omega_p/2 = \omega_0 + \Delta/2$
ω_0
$\omega_0 + \Delta$
Ω
Ω
Ω
Ω
signal beam
half pumping frequency
idler beam
$\hat{a}_2$
$\hat{a}_{\pi/2-\theta}$
conditional squeezed signal-beam
$\hat{a}_1$
$\hat{a}_{-\theta}$
best predicted quadrature
$\hat{b}_2$
measured idler-beam quadrature
$\hat{b}_\theta$
$\hat{b}_1$
EPR entangled

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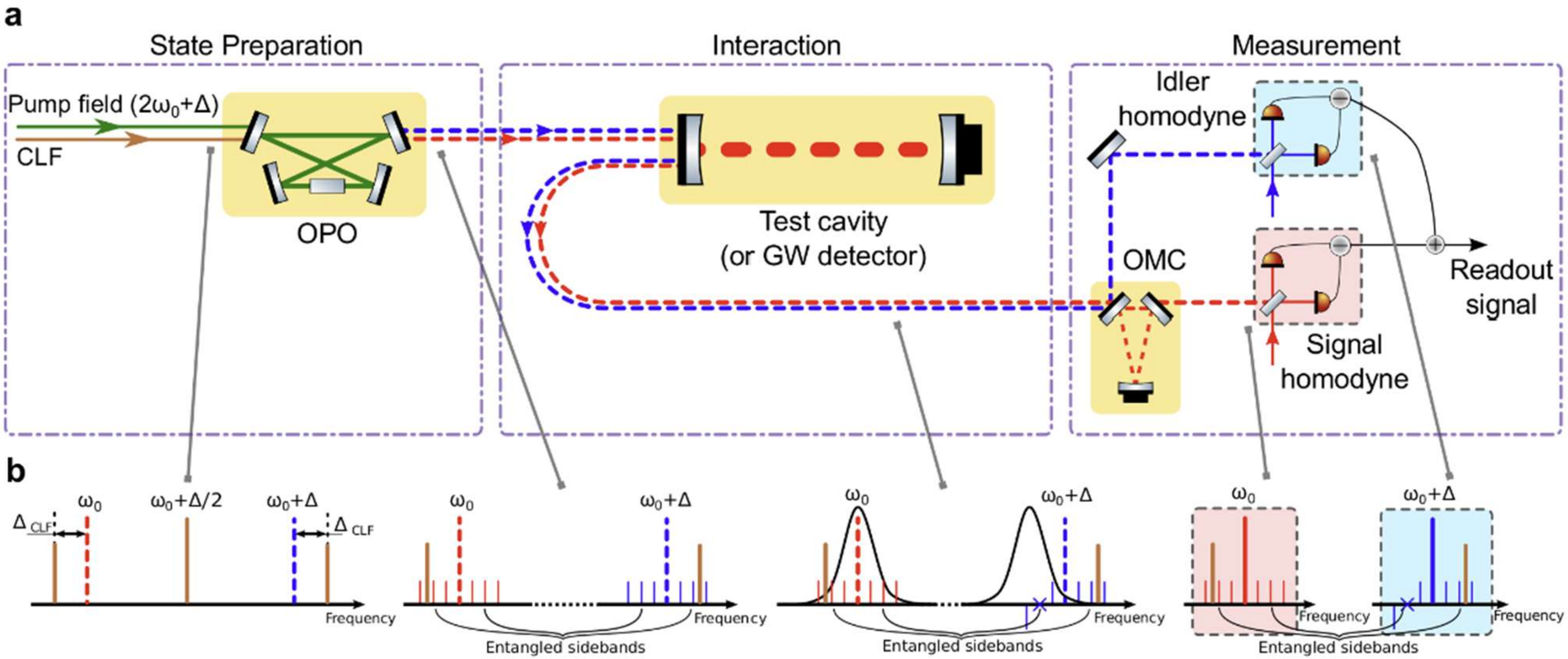
https://doi.org/10.1038/s41566-019-0582-4

Generation and control of frequency-dependent squeezing via Einstein–Podolsky–Rosen entanglement

Min Jet Yap*, Paul Altin, Terry G. McRae, Bram J. J. Slagmolen, Robert L. Ward and David E. McClelland



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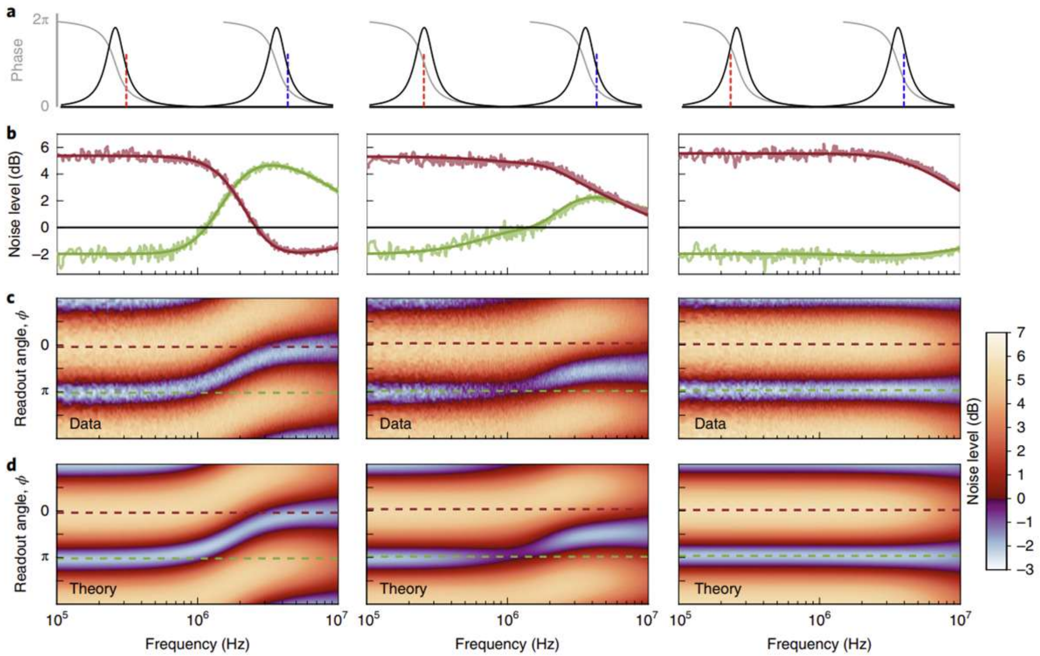

Fig. 2 | Noise spectrum for different test cavity detuning. a, Frequency spacing diagram of the signal (red) and idler (blue) fields with respect to test cavity resonances for $\delta_{sig} = \delta_{idl} = \gamma_{tc}$ (left), $\delta_{sig} = 0$, $\delta_{idl} = \gamma_{tc}$ (middle) and $\delta_{sig} = -\delta_{idl} = 0.5\gamma_{tc}$ (right). The grey trace shows the cavity phase response causing the quadrature rotation. **b**, Noise spectrum at the combined output measured at readout angles 0 and π (maroon and green, respectively). Solid dark colour lines are fits to the theoretical model. **c**, Measured noise spectrum as a function of readout angle. **d**, Associated modelled noise spectrum from the theoretical model. Dashed lines in **c** and **d** correspond to the spectra in **b**.

EPR squeezing for gravitational wave detector