Nuclear Equations of State for Neutron Stars

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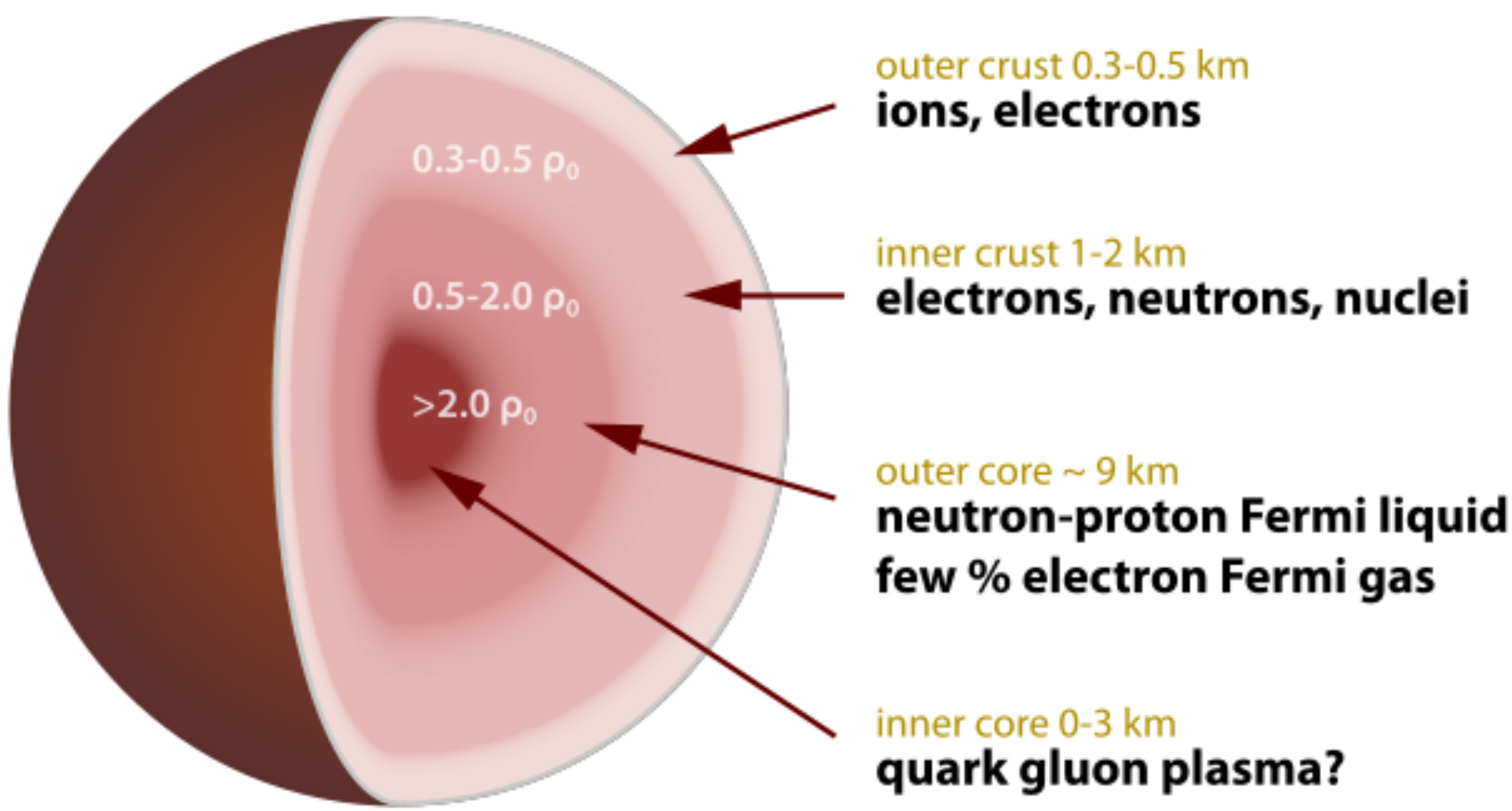
Equation of State (EoS)

1. Equation of state : a thermodynamic equation relating state variables (P,V,T, E) which describe the state of matter under a given set of physical conditions. - from Wikipedia

- NS is cold on the nuclear scale (1 MeV ~ 10^{10} K) : frozen in the point of view of hadronic and leptonic compositions.

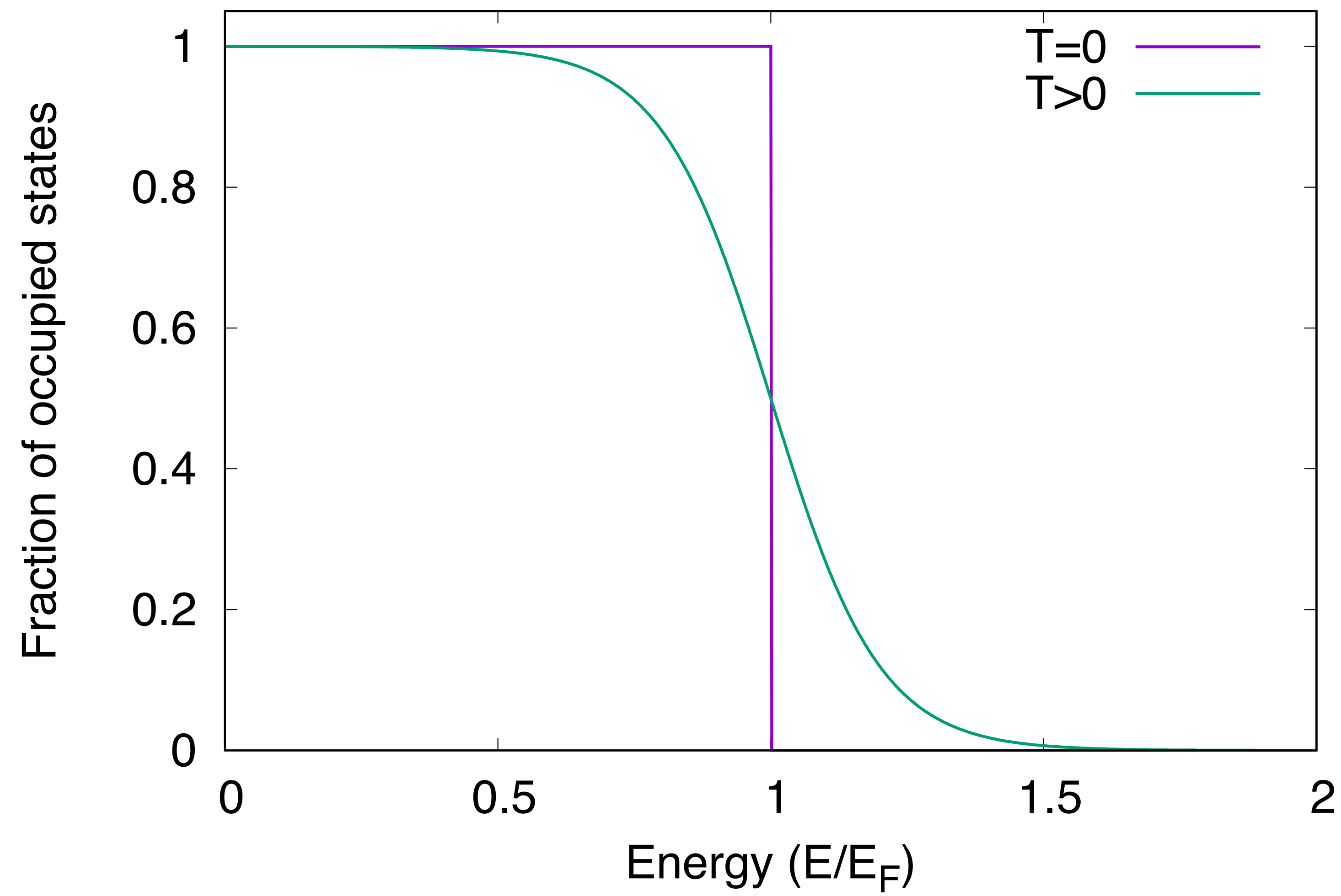


Degenerate Fermi Gas

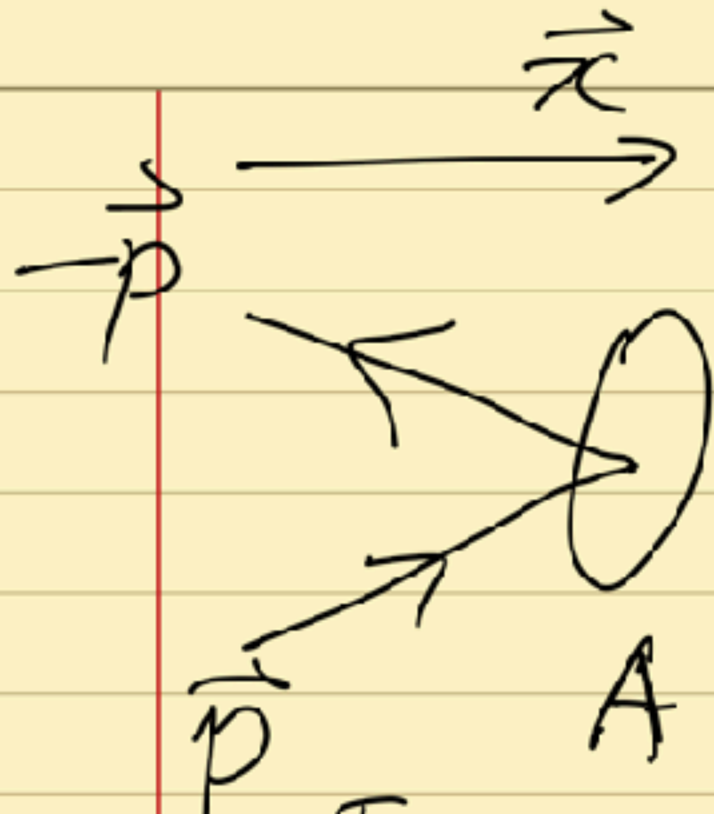
$$f(E) = \frac{1}{e^{(E-\mu)/kT} + 1}$$

$$T \to 0$$

$$f(E) = \begin{cases} 1, & E \leq E_F \\ 0, & E > E_F \end{cases}$$



Degenerate Pressure of electron gas (1)



$$\Delta P_x = 2P_x$$

$$v_x = \frac{dx}{dt}$$

$$\frac{dF_x}{dA} = \frac{2P_x}{dA\,dt} = \frac{2P_x}{dA\,(dx/v_x)} = \frac{2P_x v_x}{dV}$$

Degenerate Pressure of electron gas (2)

pressure
$$P_e = \int_0^{\infty} \frac{dN(p)}{2} \frac{2 p_x v_x}{dV} dp$$

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{3 v_x^2}$$

$$p = \sqrt{3 p_x^2}$$

$$v p = 3 p_x v_x$$

Degenerate Pressure of electron gas (3)

$$\Rightarrow \quad P_e = \frac{1}{3}\int_0^\infty n_e(p)\, p\, v\, dp$$

$$dN(p)\,dp = 2 \times \frac{d^3p\, dV}{h^3} \qquad \text{if } |\vec{p}| \le p_f$$

$$\Rightarrow \quad n_e(p)\,dp = \frac{8\pi}{h^3}\, p^2\, dp$$

Degenerate Pressure of electron gas (4)

$$\Rightarrow \quad P_e = \frac{8\pi}{3h^3} \int_0^{p_F} v\, p^3\, dp$$

$$p = \gamma m v, \quad E = \gamma m c^2$$

$$v = p/m\gamma = \frac{pc^2}{E} = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}}$$

$$\Rightarrow \quad P_e = \frac{8\pi}{3h^3} \int_0^{p_F} \frac{p^4 c^2}{\sqrt{p^2c^2 + m_e^2 c^4}}\, dp$$

Degenerate Pressure of electron gas (5)

$$x \equiv p/mc$$

$$P_e = \frac{8\pi m_e^4 c^5}{3h^3} \int_0^{x_F} \frac{x^4}{\sqrt{1+x^2}} dx$$

$$= \frac{\pi m_e^4 c^5}{h^3} \left[x_F (1+x_F^2)^{1/2} \left(\frac{2}{3} x_F^2 - 1 \right) + \ln\left[x_F + (1+x_F^2)^{1/2} \right) \right]$$

Degenerate Pressure of electron gas (6)

$x_F \ll 1$

$$P_e = \frac{8\pi m_e^4 c^5}{15 h^3}\left[x_F^5 - \frac{5}{14}x_F^7 + \frac{5}{24}x_F^9 + \cdots\right]$$

$$\rightarrow P_e \sim \frac{8\pi m_e^4 c^5}{15 h^3} x_F^5 \qquad \left(x_F = \frac{P_F}{m_e},\quad P_F \sim \rho^{1/3} \right.$$

non-relativistic

$$\Rightarrow P_e \sim \rho^{5/3} \Rightarrow \boxed{P = K_1 \rho^{5/3}}$$

Degenerate Pressure of electron gas (7)

$x_F \gg 1$

$$P_e = \frac{2\pi m_e^4 c^5}{3h^3}\left[x_F^4 - x_F^2 + \frac{3}{2}\ln(2x_F) + \cdots\right]$$

$$\Rightarrow P_e \sim \frac{2\pi m_e^4 c^5}{3h^3} x_F^4$$

like-wise

$$P_e \sim \rho^{4/3} \Rightarrow P = K_2 \rho^{4/3}$$

for fully relativistic.

Another way to obtain Pe

partition function

$$Q = \ln\left(1 + e^{-\beta(\epsilon - \mu)}\right)$$

Ideal Fermi Gas

$$PV = k_B T \sum_i g_i \ln\left[1 + e^{-\beta(\epsilon - \mu)}\right]$$

where $\beta \equiv 1/k_B T$

$\mu \equiv$ chemical potential

$$N \equiv \sum_i \langle n_e \rangle = \sum_{\epsilon_i} \frac{1}{1 + e^{\beta(\epsilon_i - \mu_i)}}$$

$v \rightarrow h^3$

$$\Rightarrow P = k_B T \int_0^\infty \frac{4\pi p^2 dp}{h^3} \cdot \ln[\quad]$$

$$N = \int_0^\infty \frac{4\pi V}{h^3} p^2 dp \cdot \frac{1}{1 + e^{\beta(\epsilon - \mu)}}$$

$$p = \frac{1}{3} \frac{N}{V} \langle p u \rangle$$

where $u \equiv \frac{d\epsilon}{dp}$

$$\equiv P_e = \frac{1}{3} \int_0^\infty n_e(p)\, p v\, dp$$

$$P = \frac{8\pi}{3h^3} \int_0^{p_F} \frac{(p^2/m)}{[1 + (p/mc)^2]^{1/2}} p^2 dp$$

$$\epsilon = mc^2 \left[\{1 + (p/mc)^2\}^{1/2} - 1\right]$$

$$p = mc \sinh\theta$$

$$u = c \tanh\theta$$

Nucleus and Strong force

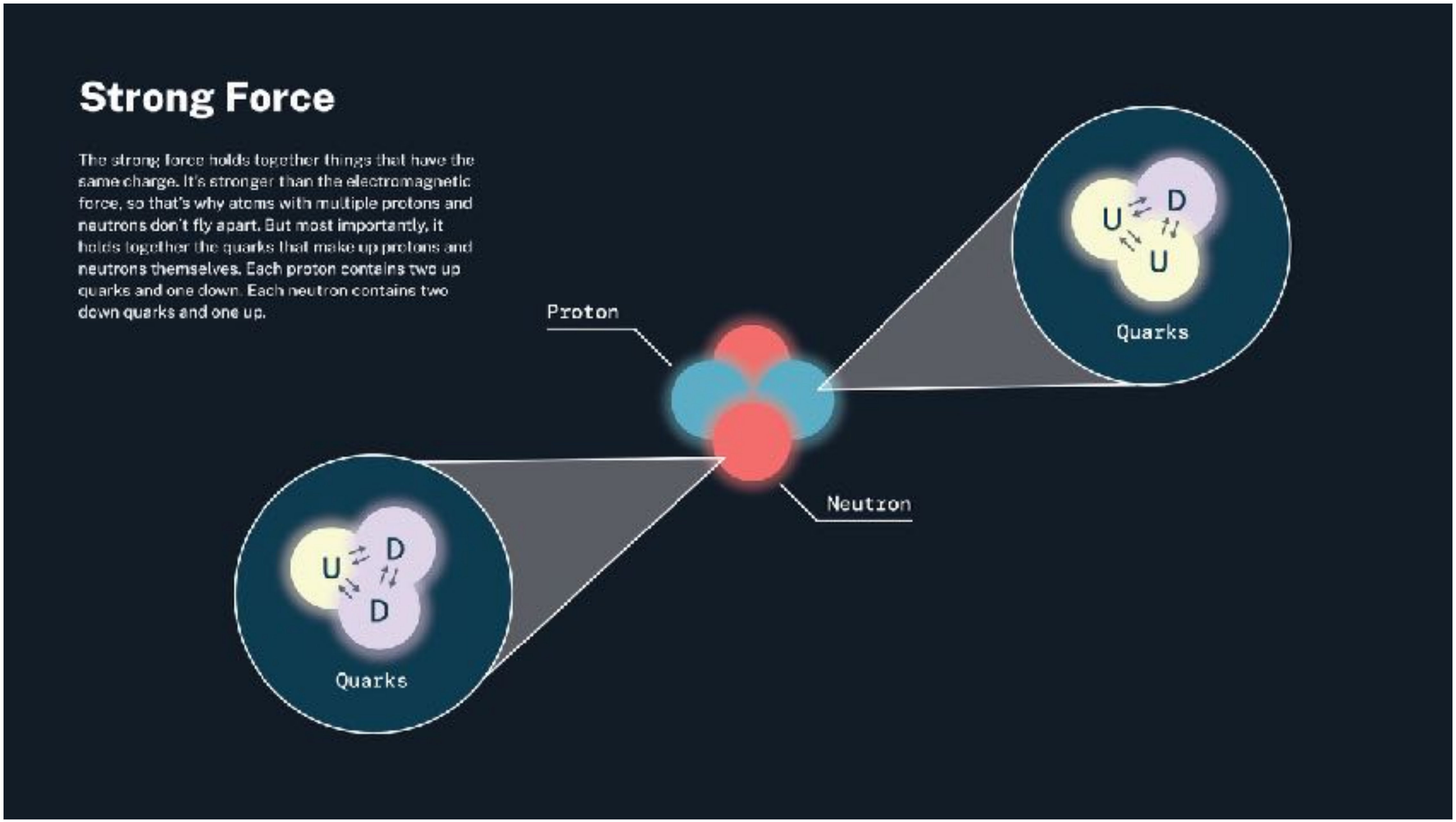
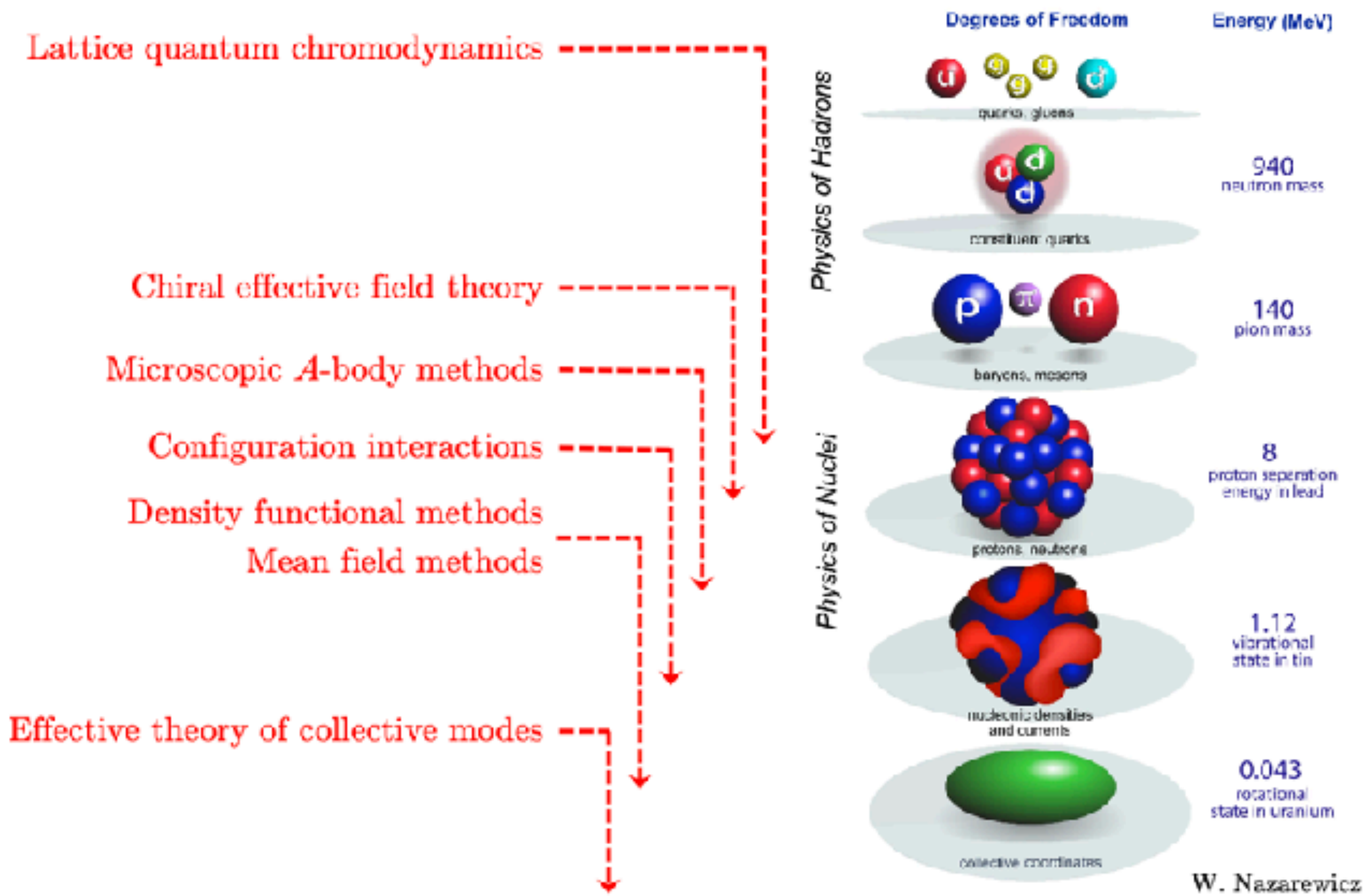


Image: NASA

Effective field theories and energy scales



Slide from Dean Lee' lecture in Nuclear Physics School 2020

EoS for a Neutron Star

Energy per nucleon

$$\frac{E}{A}(\rho, \delta) = \frac{V}{A}\epsilon(\rho, \delta) = \epsilon(\rho, \delta)/\rho$$

ρ : baryon density $[\mathrm{fm}^{-3}]$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

Pressure

$$P = -\frac{\partial E}{\partial V}(\rho, \delta)\big|_A = \frac{A}{V^2}\frac{\partial E}{\partial \rho}(\rho, \delta)\big|_A = \rho^2 \frac{\partial E/A(\rho, \delta)}{\partial \rho}\big|_A$$

Beta Equilibrium

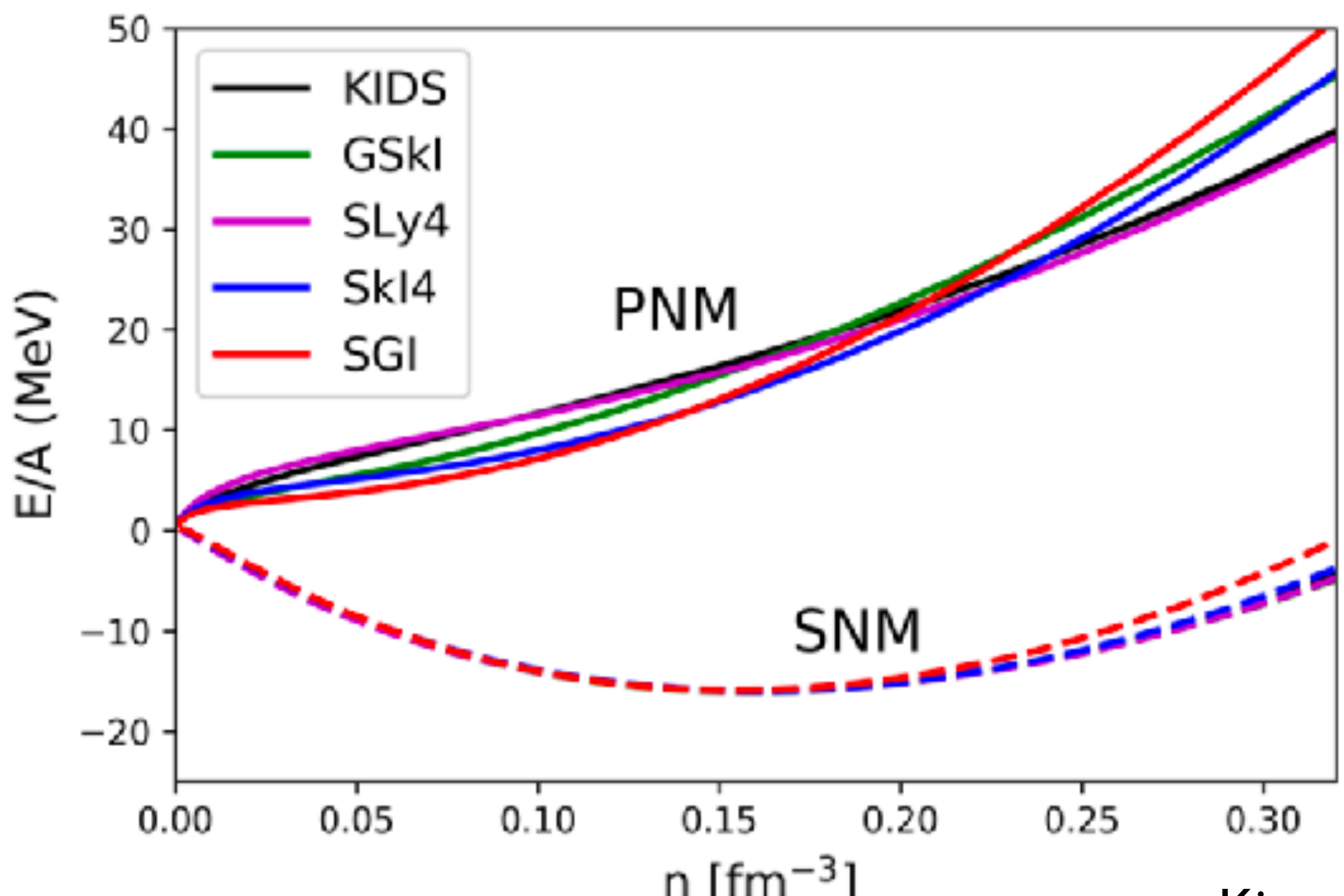
$$\rho_p = \rho_e + \rho_\mu \ (+\rho_K)$$

$$\mu_n = \mu_p + \mu_e \qquad \mu_e = \mu_\mu (= \mu_K)$$

Nuclear Equation of States

$$\frac{E}{A}(\rho,\delta) = \frac{V}{A}\epsilon(\rho,\delta) = \epsilon(\rho,\delta)/\rho \equiv \mathscr{E}(\rho,\delta)$$

$$= E(\rho,\delta=0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) + \ldots,$$



$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

$\rho_0 \sim 0.16$: saturation density

Kim et al. (IJMPE 2020)

Nuclear Equation of States

$$\mathscr{E}(\rho, 0) = E_0 + \tfrac{1}{2}K_0x^2 + \tfrac{1}{6}Q_0x^3 + O(x^4),$$

$$\mathcal{S}(\rho) = J + Lx + \tfrac{1}{2}K_{\text{sym}}x^2 + \tfrac{1}{6}Q_{\text{sym}}x^3 + \tfrac{1}{24}R_{\text{sym}}x^4 + O(x^5),$$

$$x = \frac{(\rho - \rho_0)}{3\rho_0}$$

$$K_0 \equiv 9\rho_0^2 \frac{d^2}{d\rho^2} \frac{\mathscr{E}(\rho, 0)}{\rho}\bigg|_{\rho=\rho_0},$$

$$Q_0 \equiv 27\rho_0^3 \frac{d^3}{d\rho^3} \mathscr{E}(\rho, 0)\bigg|_{\rho=\rho_0}.$$

Incompressibility	$K_0 \simeq 230$ MeV
Skewness	$K_0' \sim -2000$ MeV

Nuclear Symmetry Energy

$$\mathscr{E}(\rho, 0) = E_0 + \tfrac{1}{2}K_0 x^2 + \tfrac{1}{6}Q_0 x^3 + O(x^4),$$

$$\mathcal{S}(\rho) = J + Lx + \tfrac{1}{2}K_{\rm sym}x^2 + \tfrac{1}{6}Q_{\rm sym}x^3 + \tfrac{1}{24}R_{\rm sym}x^4 + O(x^5),$$

$$S \equiv E_{\rm sym}(\rho_0)$$

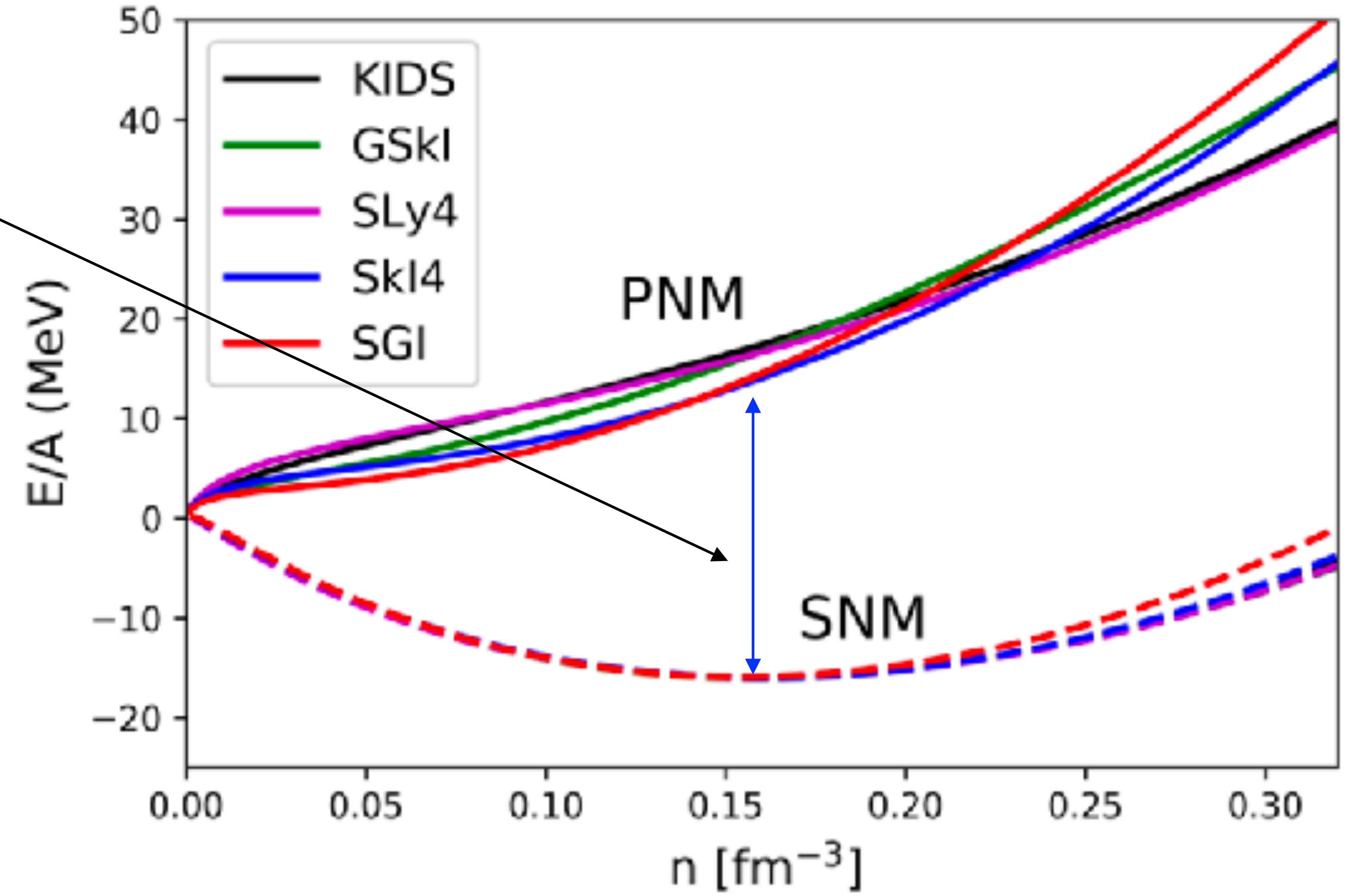
$$L \equiv 3\rho_0 \frac{d}{d\rho}\mathcal{S}(\rho)\Big|_{\rho=\rho_0}, \qquad Q_{\rm sym} \equiv 27\rho_0^3 \frac{d^3}{d\rho^3}\mathcal{S}(\rho)\Big|_{\rho=\rho_0},$$

$$K_{\rm sym} \equiv 9\rho_0^2 \frac{d^2}{d\rho^2}\frac{\mathcal{S}(\rho)}{\rho}\Big|_{\rho=\rho_0}. \qquad R_{\rm sym} \equiv 81\rho_0^4 \frac{d^4}{d\rho^4}\mathcal{S}(\rho)\Big|_{\rho=\rho_0}.$$

Nuclear Symmetry Energy

$$\frac{E}{A} = E(\rho, \delta = 0) + E_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4) + \ldots,$$

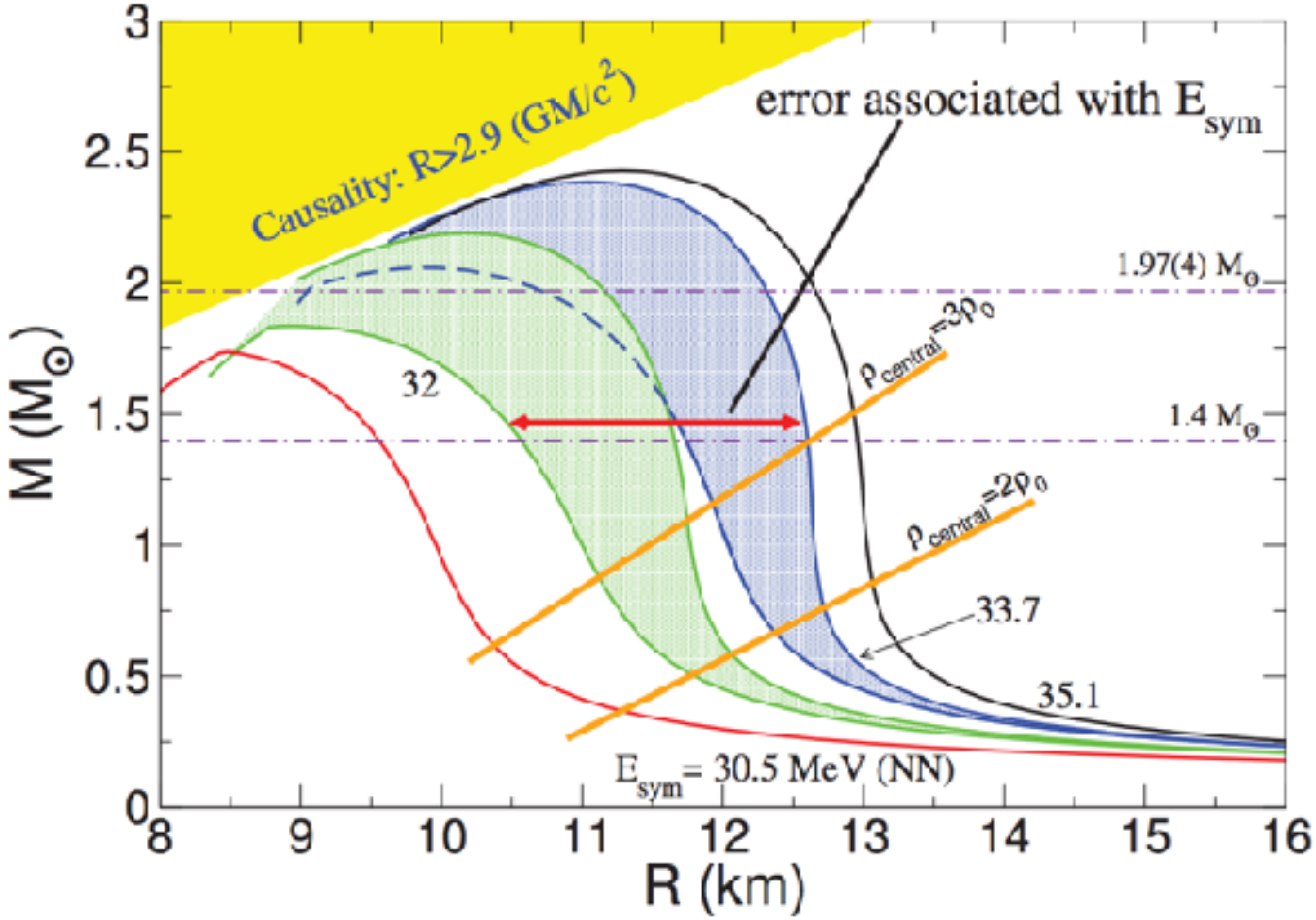
$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L}{3}\left(\frac{\rho - \rho_0}{\rho_0}\right) + \frac{K_{sym}}{18}\left(\frac{\rho - \rho_0}{\rho_0}\right)^2$$



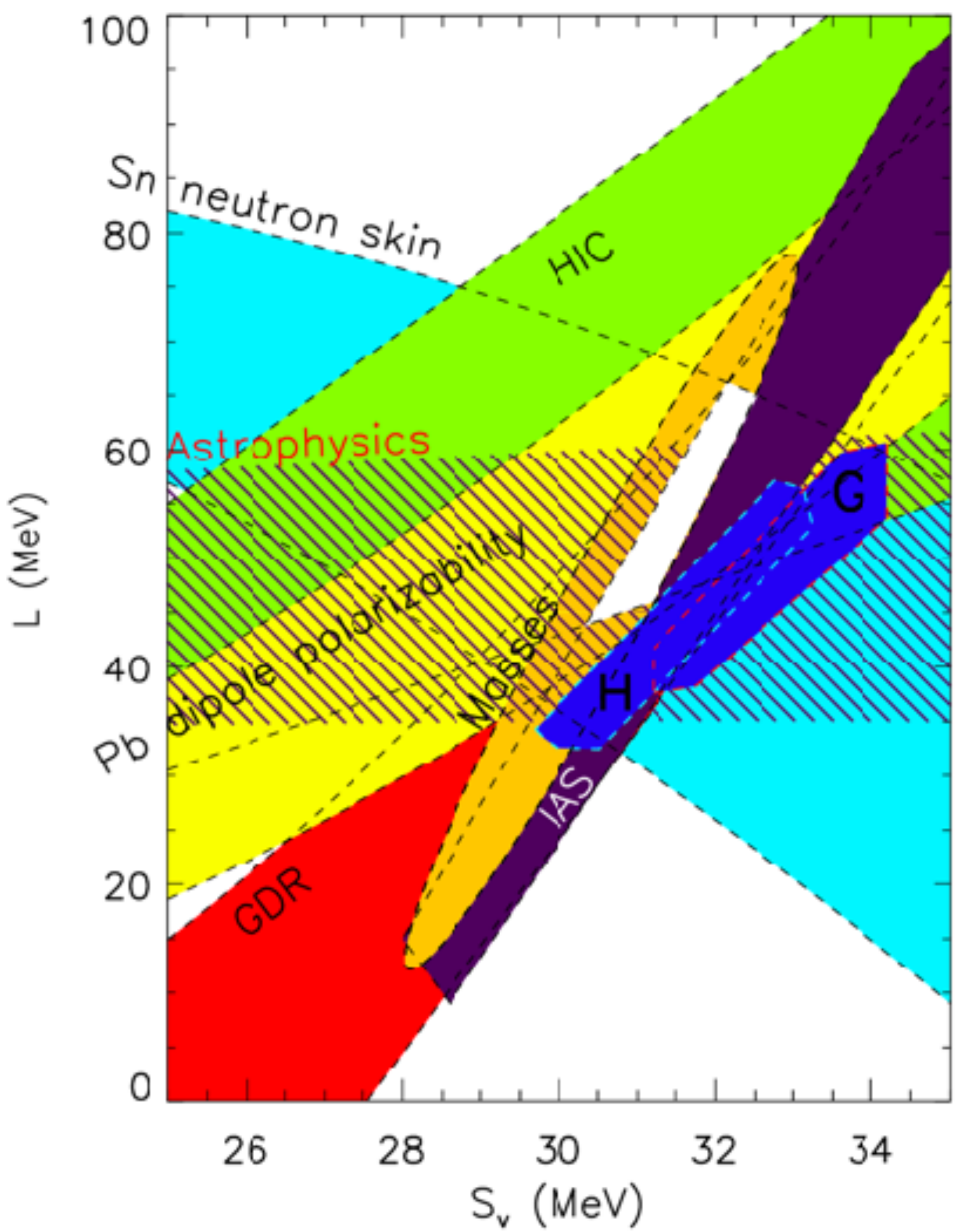
Kim et al. (IJMPE 2020)

Nuclear Symmetry Energy

Gandolfi and Steiner (2016)



J.M. Lattimer / Nuclear Physics A 928 (2014) 276–295



Constraints on Symmetry Energy

Model	ρ_0	E_0	K_0	$-Q_0$	J	L	$-K_\tau$	$M_{\max}$
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	$200 \sim 260$	$200 \sim 1200$	$30 \sim 35$	$40 \sim 76$	$372 \sim 760$	$\geq 1.93 \sim 2.05$
CSkP	-	-	$202.0 \sim 240.3$	$362.5 \sim 425.6$	$30.0 \sim 35.5$	$48.6 \sim 67.1$	$360.1 \sim 407.1$	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

Kim et al., PhysRevC.98.065805

Constraints on EoS

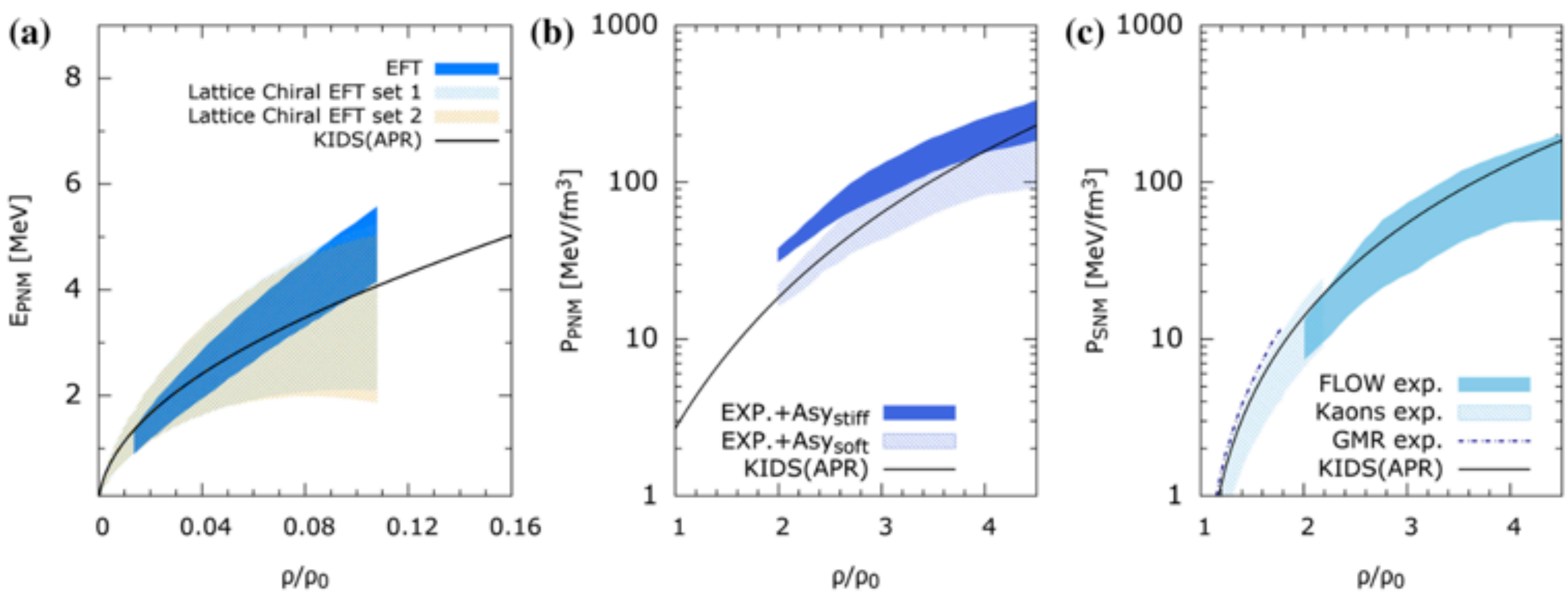
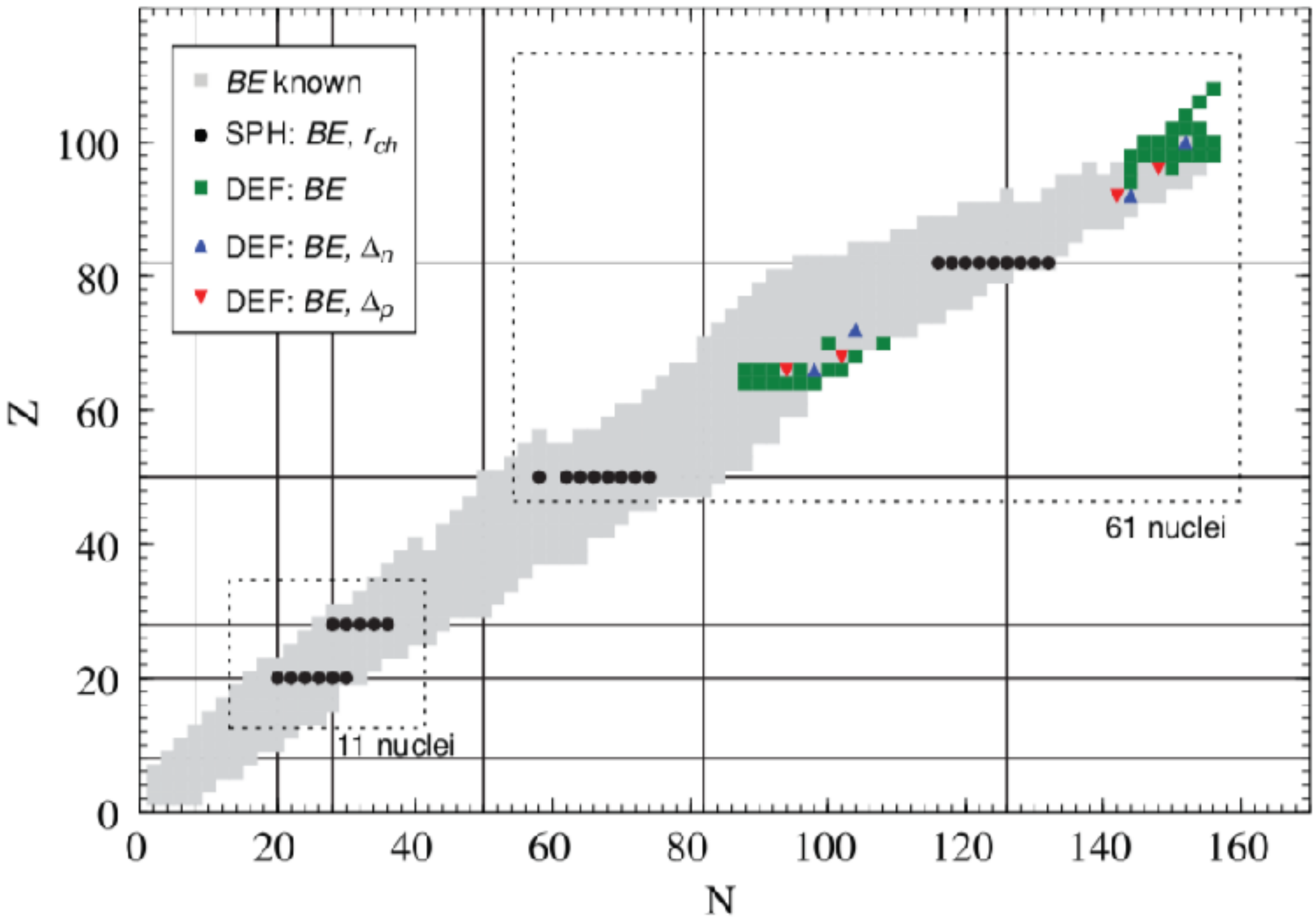


Fig. 1 Energy below (**a**) and pressure above (**b**, **c**) the saturation density. The result of an EFT and lattice chiral EFT in **a** is taken from [16] and [17], respectively. The results from experiment in **b** are from [21,22], and those in **c** are from [22–25]

Fitting to Observables

PHYSICAL REVIEW C **82**, 024313 (2010)



Standard Skyrme effective interaction

N1LO Skyrme pseudopotential

E. Chabanat et al. / Nuclear Physics A 627 (1997) 710–746 713

$$V(\boldsymbol{r}_1,\boldsymbol{r}_2) = t_0\,(1+x_0P_\sigma)\,\delta(\boldsymbol{r}) \qquad \text{central term}$$

$$+\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] \qquad \text{2nd order derivative}$$

$$+t_2\,(1+x_2P_\sigma)\,\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} \qquad \text{non-local terms} \qquad (2.1)$$

$$+\frac{1}{6}t_3\,(1+x_3P_\sigma)\left[\rho(\boldsymbol{R})\right]^\sigma\delta(\boldsymbol{r}) \qquad \text{density-dependent term}$$

$$+iW_0\boldsymbol{\sigma}\cdot\left[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}\right] \qquad \text{spin–orbit term}.$$

with:

$$\boldsymbol{r}=\boldsymbol{r}_1-\boldsymbol{r}_2\,, \qquad \boldsymbol{R}=\frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2)\,,$$

$$\boldsymbol{P}=\frac{1}{2i}(\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)\,, \qquad \boldsymbol{P}' \text{ cc of } \boldsymbol{P} \text{ acting on the left}$$

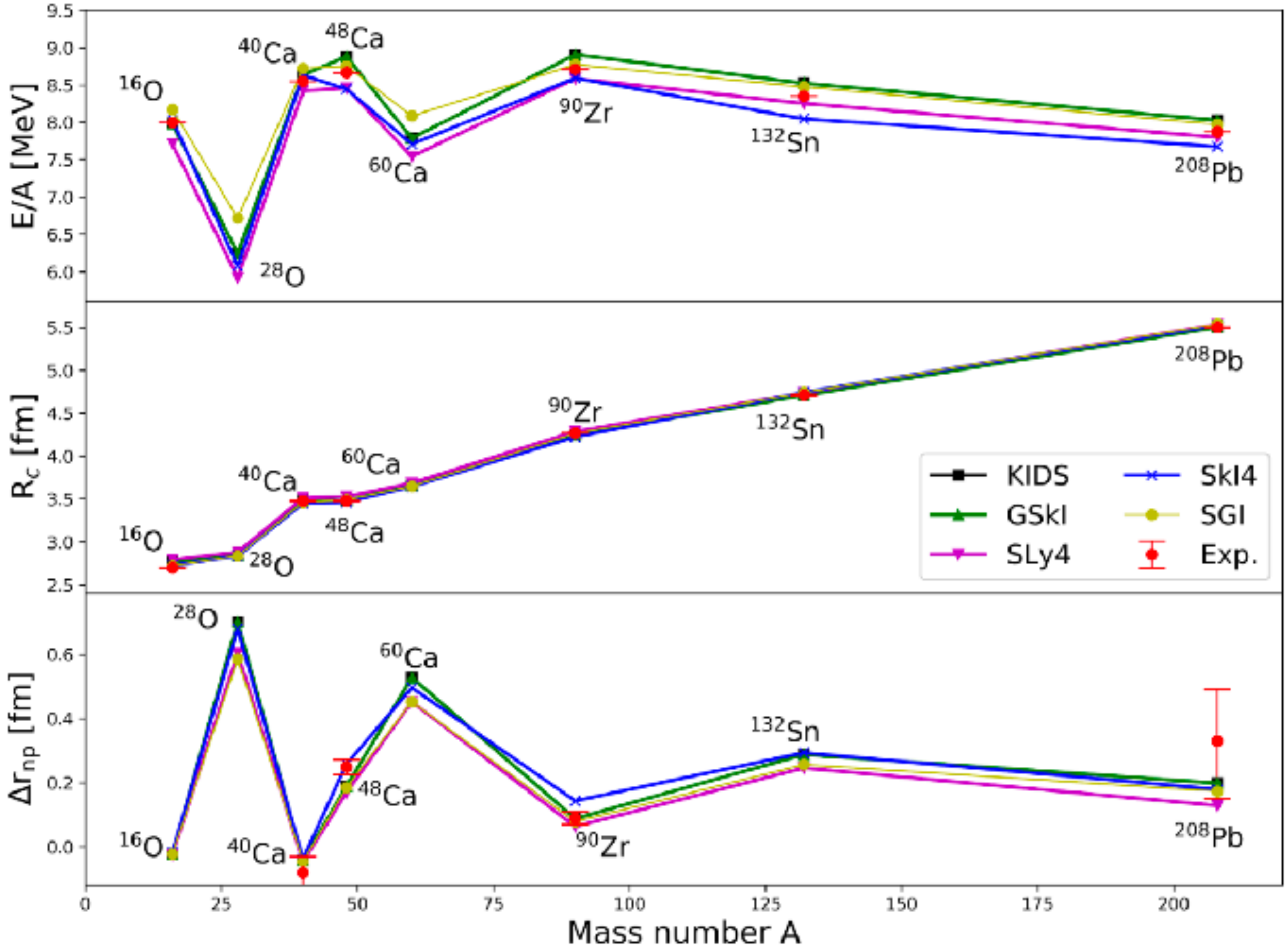
and

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2\,, \qquad P_\sigma=(1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)/2\,.$$

Skyrme-type Energy Density Functional

$$
\begin{aligned}
\mathscr{E} = {} & \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho - \frac{1}{8}(t_0 + 2y_0)\rho\delta^2 + \frac{1}{16}\sum_{n=1}^{N-1} t_{3n}\rho^{1+n/3} \\
& - \frac{1}{48}\sum_{n=1}^{N-1}(t_{3n} + 2y_{3n})\rho^{1+n/3}\delta^2 + \frac{1}{64}(9t_1 - 5t_2 - 4y_2) \\
& \times \frac{(\nabla\rho)^2}{\rho} - \frac{1}{64}(3t_1 + 6y_1 + t_2 + 2y_2)\frac{(\nabla\rho\delta)^2}{\rho} \\
& + \frac{1}{8}(2t_1 + y_1 + 2t_2 + y_2)\tau - \frac{1}{8}(t_1 + 2y_1 - t_2 - 2y_2) \\
& \times \sum_q \frac{\rho_q\tau_q}{\rho} + \frac{1}{2}W_0\left(\frac{\mathbf{J}\cdot\nabla\rho}{\rho} + \sum_q \frac{\mathbf{J}_q\cdot\nabla\rho_q}{\rho}\right), \qquad (15)
\end{aligned}
$$

Properties of Nuclei



Kim et al. (IJMPE 2020)

Constraints on Nuclear EoS

- Nuclear data: hundreds of models (Skyrme force, RMF, …)
- Neutron star maximum mass

  1.97 ± 0.04 M⊙ [Nature 467, 1081 (2010)]

  2.01 ± 0.04 M⊙ [Science 340, 448 (2013)]

- 11 experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

16/240 Skyrme force models satisfy 11 constraints.

Relativistic Mean-Field (RMF) theory

$$\mathcal{L} = \sum_B \overline{\psi}_B(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho B}\gamma_\mu\tau\cdot\rho^\mu)\psi_B$$
$$+ \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu$$
$$- \tfrac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \tfrac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu - \tfrac{1}{3}bm_n(g_\sigma\sigma)^3 - \tfrac{1}{4}c(g_\sigma\sigma)^4$$
$$+ \sum_\lambda \overline{\psi}_\lambda(i\gamma_\mu\partial^\mu - m_\lambda)\psi_\lambda \,. \qquad (5.38)$$

$$U = \frac{m_\sigma^2}{2}\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4.$$

σ-ω-ρ model

$$\partial^2\sigma + \frac{\partial U}{\partial\sigma} = -g_\sigma\rho_S$$
$$(\partial^2 + m_\omega^2)\omega^\nu = g_\omega j_b^\nu$$
$$(\partial^2 + m_\rho^2)\boldsymbol{\rho}^\nu = g_\rho \boldsymbol{j}_I^\nu$$

$$\rho_S = g\sum_{a=p,n,\bar{p},\bar{n}}\int\frac{d^3p}{(2\pi)^3E_a^*}m_a^*f_a(x,\mathbf{p}_a)$$
$$j_b^\mu = g\sum_{a=p,n,\bar{p},\bar{n}}(-1)^a\int\frac{d^3p}{(2\pi)^3E_a^*}p_a^\mu f_a(x,\mathbf{p}_a)$$
$$j_I^{3\mu} = g\sum_{a=p,n,\bar{p},\bar{n}}\tau_a^3\int\frac{d^3p}{(2\pi)^3E_a^*}p_a^\mu f_a(x,\mathbf{p}_a)$$

Relativistic Mean-Field (RMF) theory

$$\mathcal{T}^{\mu\nu} = -g^{\mu\nu}\mathcal{L} + \sum_{\phi} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial^\nu \phi \,,$$

TABLE I. Parameter sets.

Parameter	Set I	Set II	NL3
f_σ (fm^2)	10.33	same	15.73
f_ω (fm^2)	5.42	same	10.53
f_ρ (fm^2)	0.95	3.15	1.34
f_δ (fm^2)	0.00	2.50	0.00
A (fm^{-1})	0.033	same	-0.01
B	-0.0048	same	-0.003

and compare with (3.26) to read the energy dens
equations to rewrite the energy density as

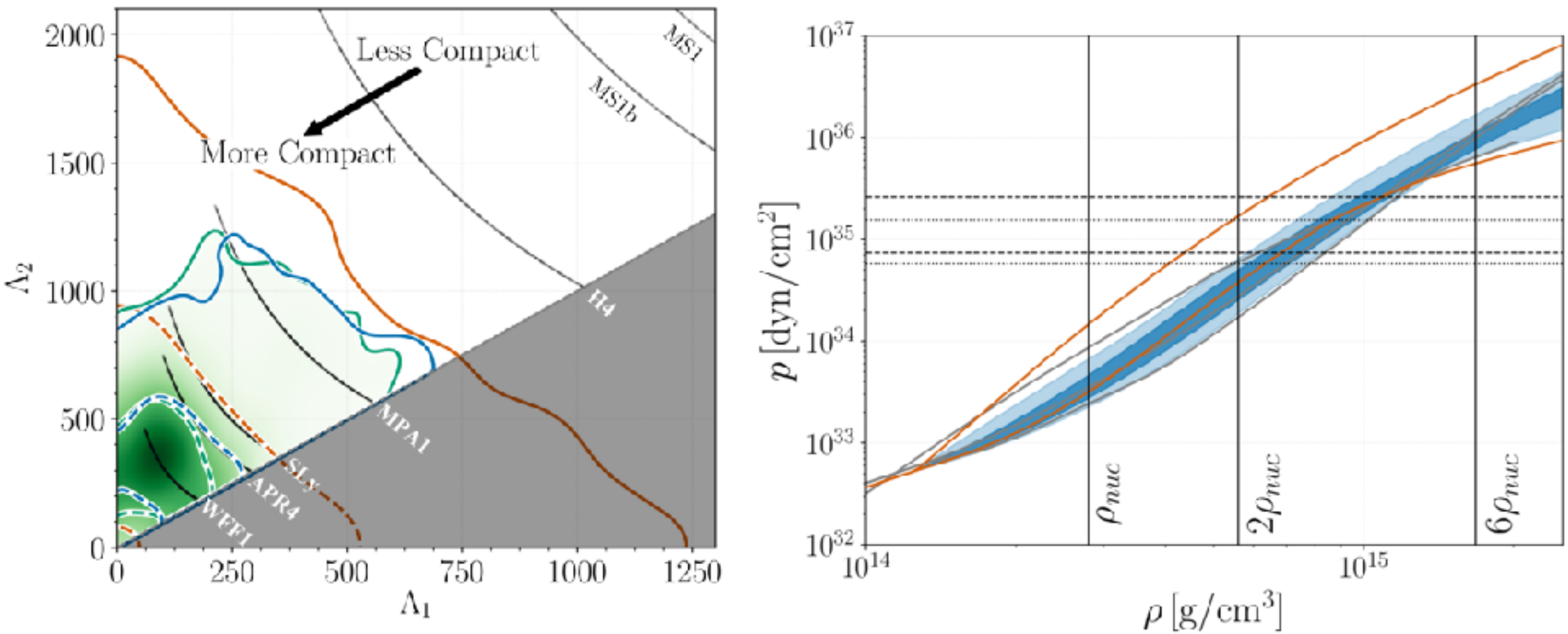
$$\begin{aligned}\epsilon = {} & \tfrac{1}{3}bm_n(g_\sigma\sigma)^3 + \tfrac{1}{4}c(g_\sigma\sigma)^4 + \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B}\sqrt{k^2+(m_B-g_{\sigma B}\sigma)^2}\,k^2\,dk \\ & + \sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda}\sqrt{k^2+m_\lambda^2}\,k^2\,dk\,. \end{aligned} \qquad (5.50)$$

The pressure is given by

$$\begin{aligned} p = {} & -\tfrac{1}{3}bm_n(g_\sigma\sigma)^3 - \tfrac{1}{4}c(g_\sigma\sigma)^4 - \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \tfrac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B} k^4\,dk/\sqrt{k^2+(m_B-g_{\sigma B}\sigma)^2} \\ & + \frac{1}{3}\sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda} k^4\,dk/\sqrt{k^2+m_\lambda^2}\,. \end{aligned} \qquad (5.51)$$

EoS from GW observation

GW170817



Abbott et al. (LSC and Virgo), PhysRevLett.121.161101

Bayesian Inference

Bayes' theorem

The likelihood could be the function of errors

Posterior $$p(\theta|d) = \frac{p(d|\theta)}{p(d)} \cdot p(\theta)$$

Prior choices can influence results

$$= \frac{p(d|\theta)}{\int p(d|\theta)p(\theta)d\theta} \cdot p(\theta)$$

The evidence is unimportant for parameter estimation (but not model selection !)

$$p(\theta|d) \sim p(d|\theta)p(\theta)$$

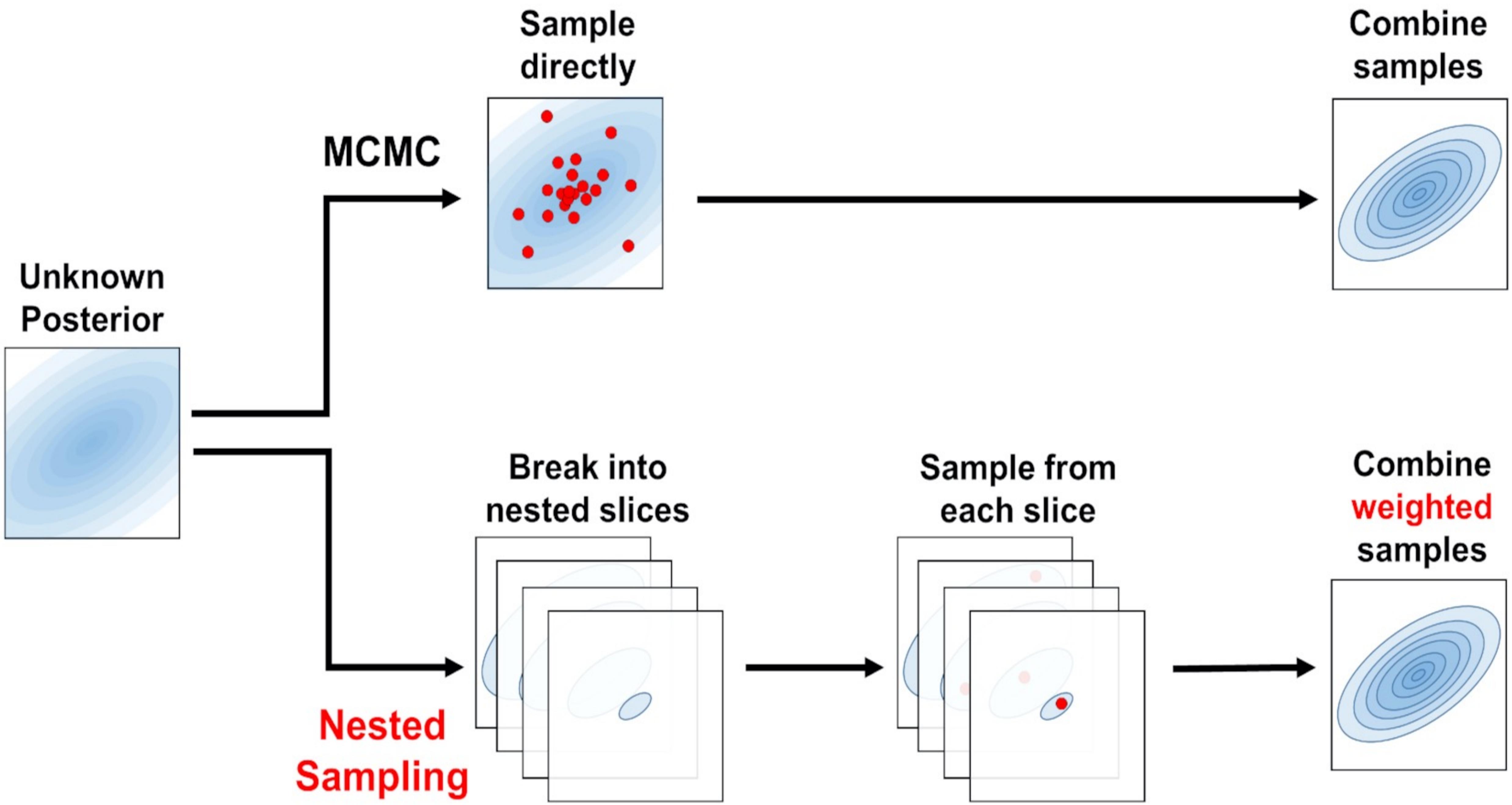
Prior, p(θ) : the distribution of the parameter(s) before any data is observed

Likelihood, p(d|θ) : the distribution of the observed data conditional on its parameters

Posterior, p(θ|d): the distribution of the parameter(s) after taking into account the observed data

Model evidence, p(d):the distribution of the observed data marginalized over the parameter(s)

Figure 1. A schematic representation of the different approaches MCMC methods and nested sampling methods take to ...



Mon Not R Astron Soc, Volume 493, Issue 3, April 2020, Pages 3132–3158, https://doi.org/10.1093/mnras/staa278

Polytropic equation of state

$$p = K\rho^{\Gamma} = K\rho^{1+1/n}$$

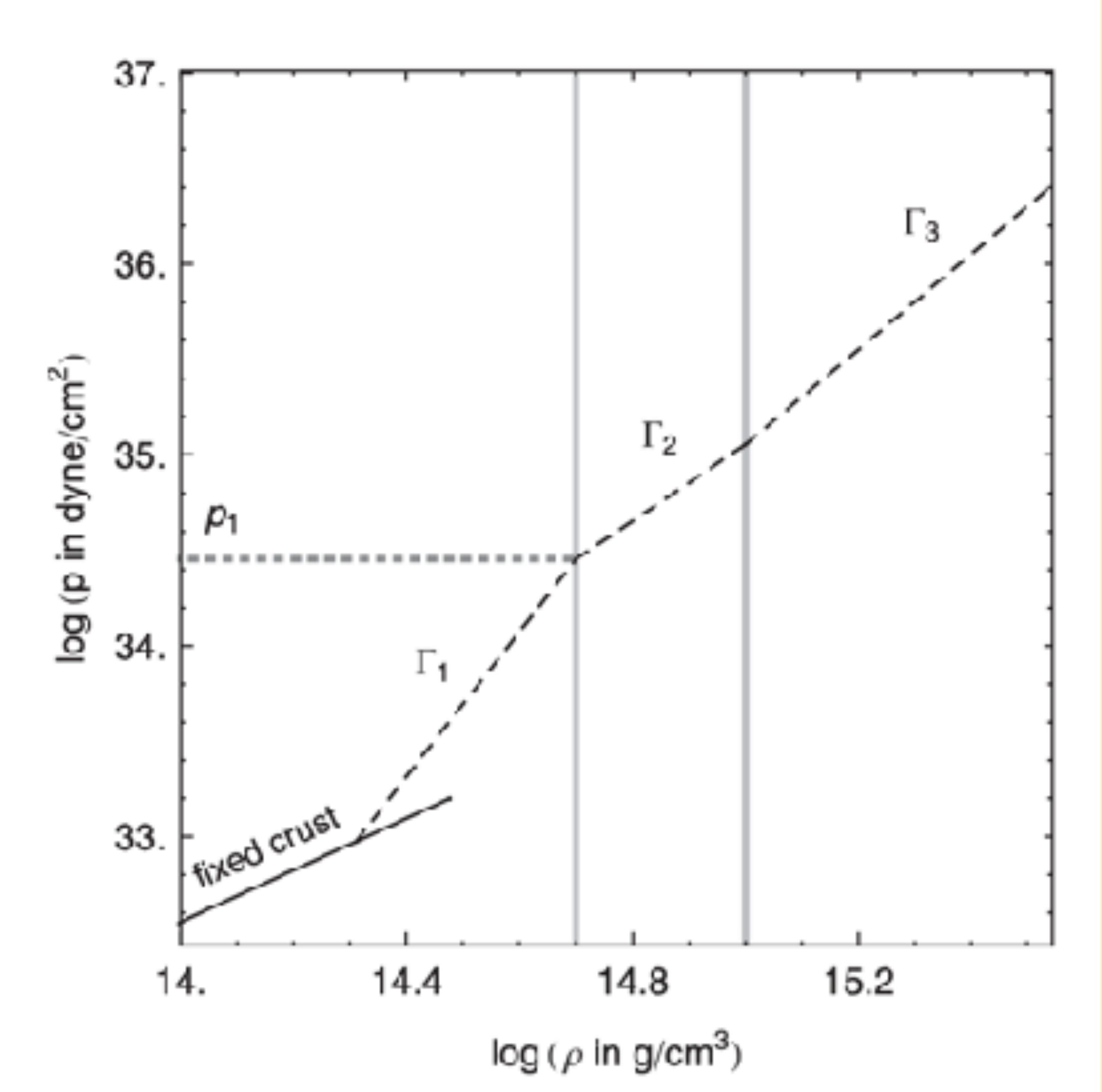
$$\begin{aligned}\epsilon =& (1+a)\rho c_0^2 + \frac{p}{\Gamma - 1} \\ =& (1+a)\rho c_0^2 + \frac{K}{\Gamma - 1}\rho^{\Gamma}.\end{aligned}$$

(a=0)

$$\frac{c_s^2}{c_0^2} = \frac{dp}{d\epsilon} = \Gamma\frac{p}{\epsilon + p}$$

Piece-wise polytope: 4 parameters

$$\epsilon(\rho) = (1 + a_i)\rho + \frac{K_i}{\Gamma_i - 1}\rho^{\Gamma_i},$$



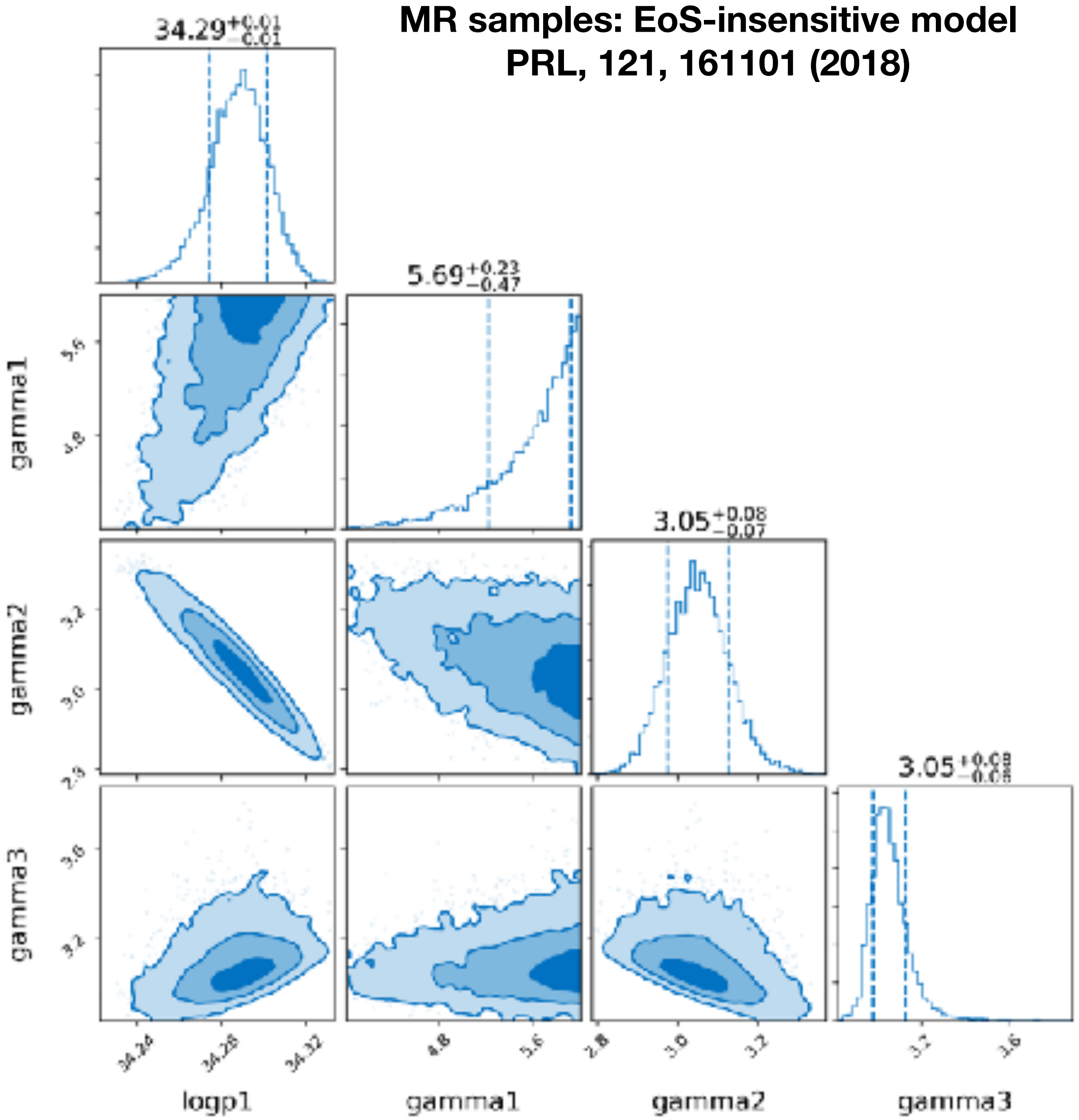
Read et al. PRD 79, 124032 (2009)

Posteriors w/ Piece-wise Polytropic EoSs

Reference for Piece-wise Polytropic EoSs : Read et al. PRD 79, 124032 (2009)

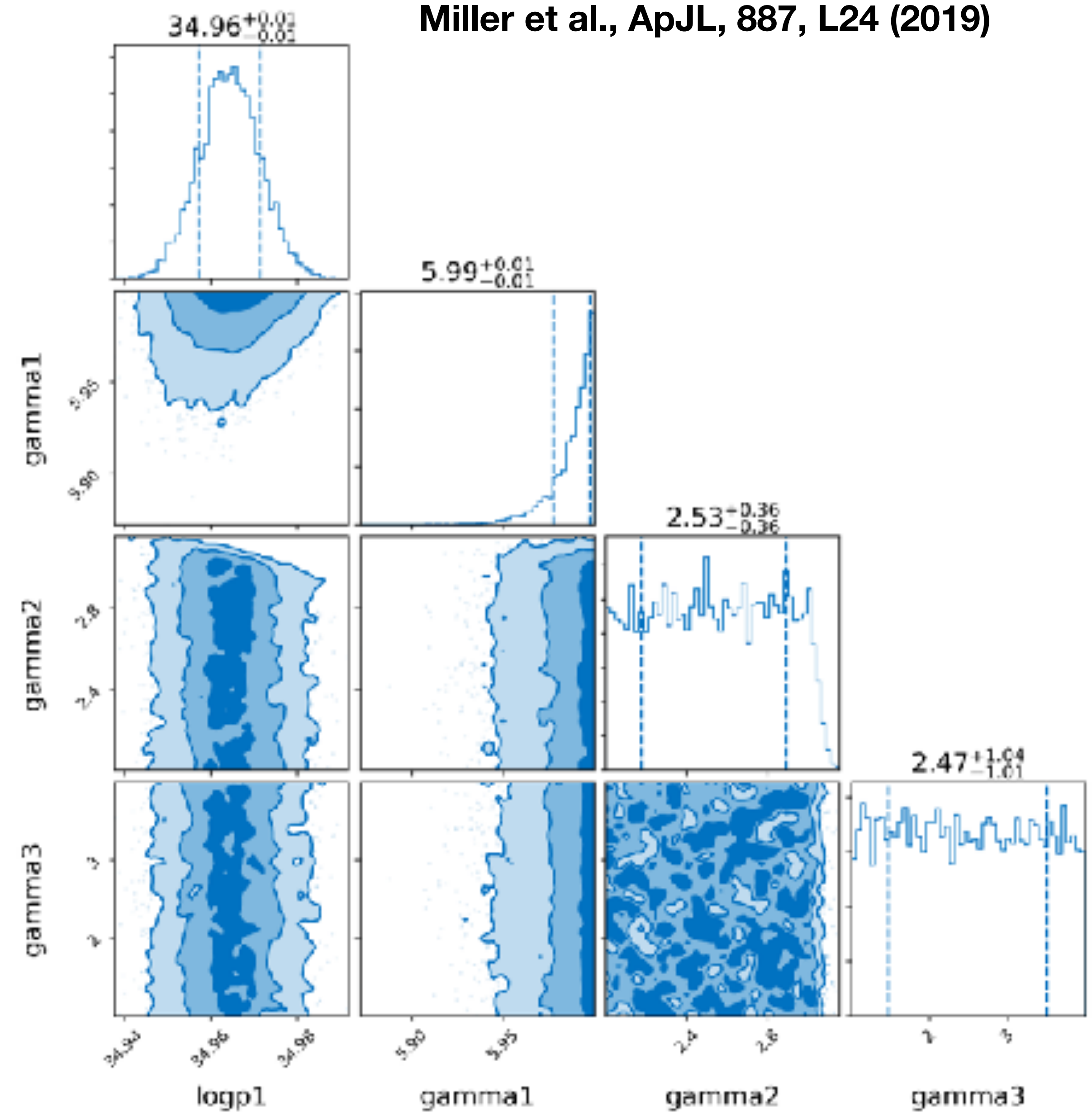
GW170817

MR samples: EoS-insensitive model
PRL, 121, 161101 (2018)

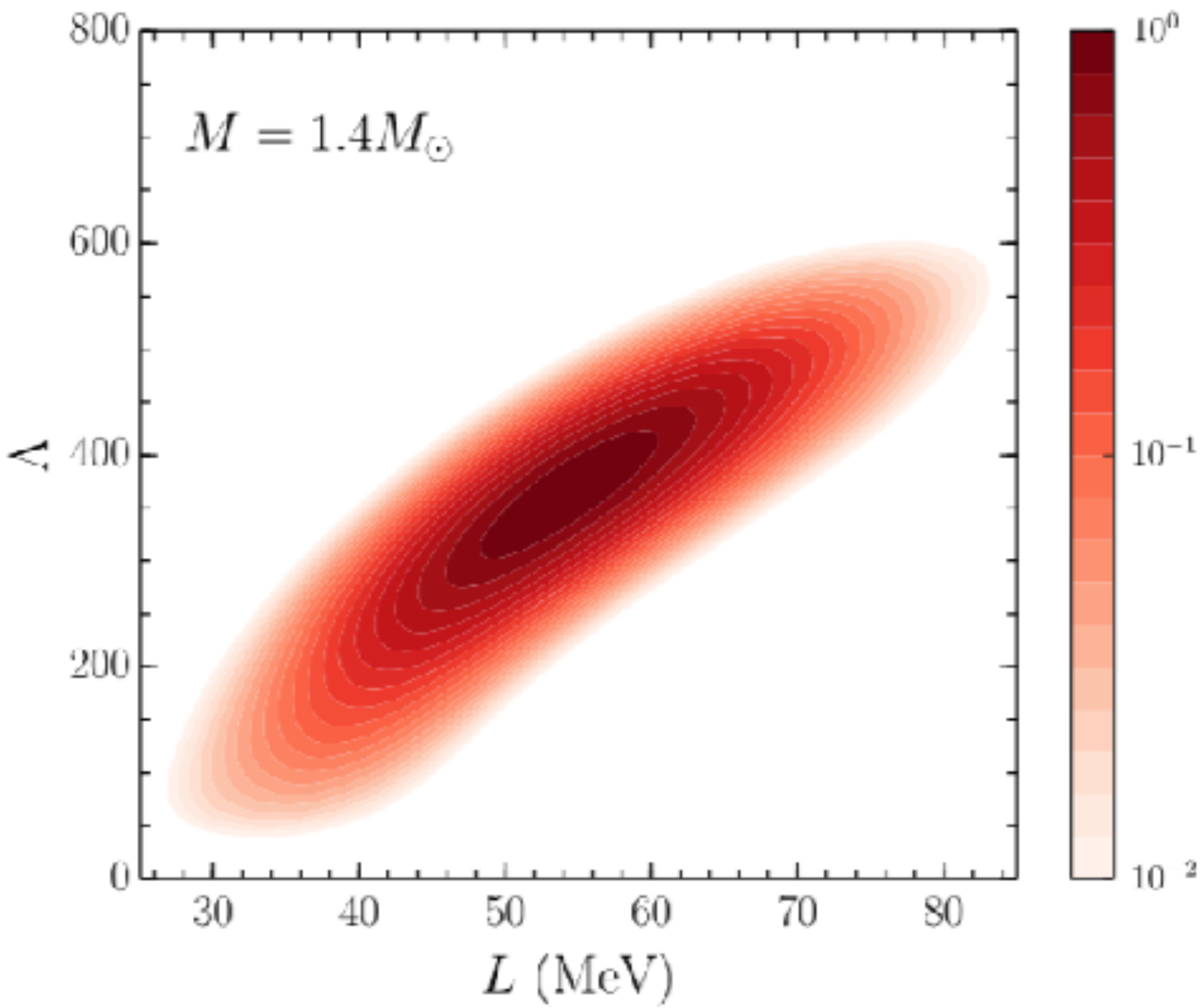


PSR J0030+0451

MR samples: 2 spot model
Miller et al., ApJL, 887, L24 (2019)

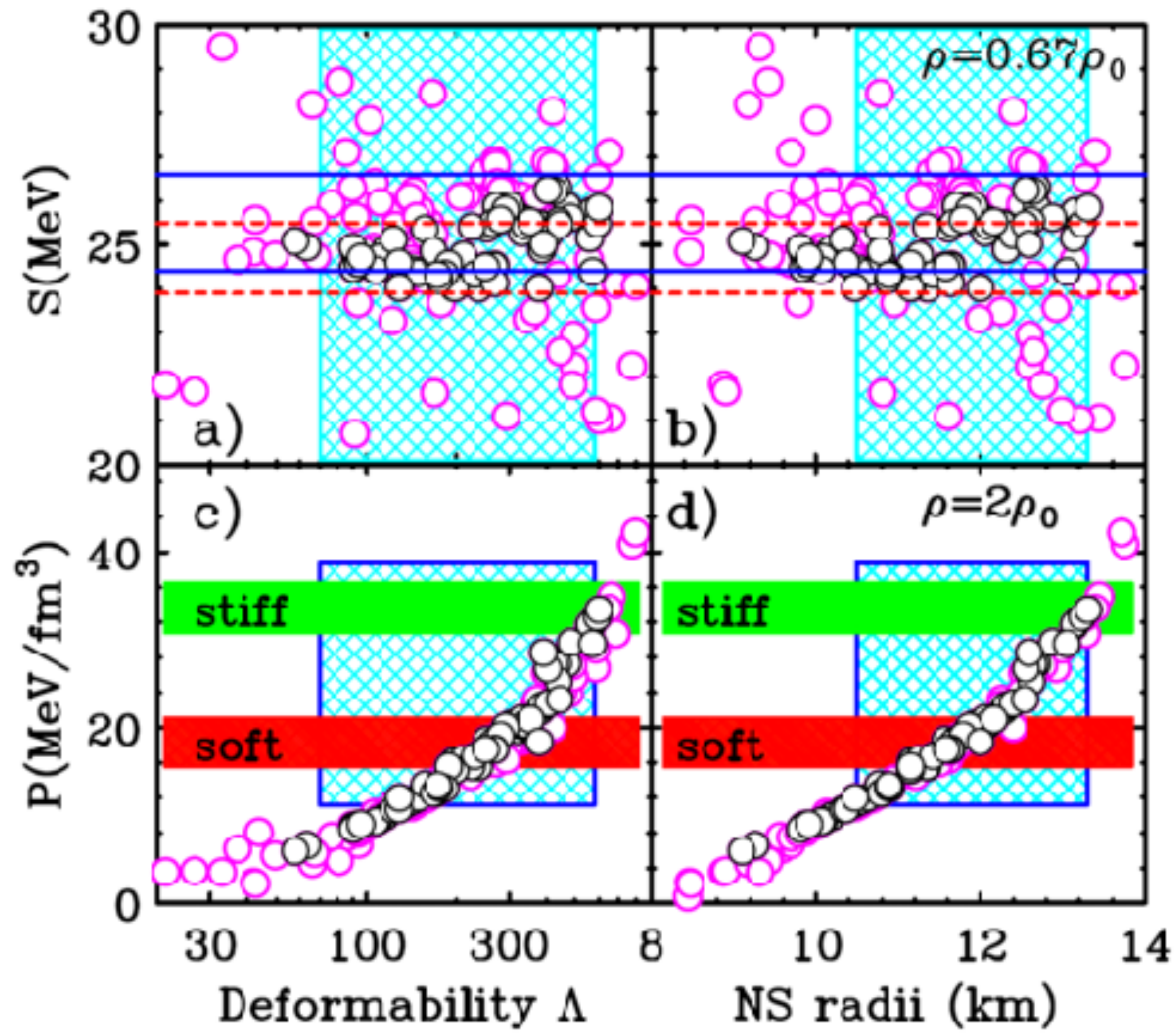


Symmetry Energy from GW observation



isospin-asymmetry energy slope parameter L

[D] Y.Lim and J. Holt, PRL.121.062701 (arXiv:1803.02803v2)



C.Y. Tsang et al. arxiv:1807.06571

KIDS Energy Density Functional

Papakonstantinou et al.,
Phys. Rev. C 97, 014312 (2018)

KIDS Energy density functional form

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho,\delta) + \Sigma_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$

$$\delta = \frac{\rho_n - \rho_p}{\rho}$$

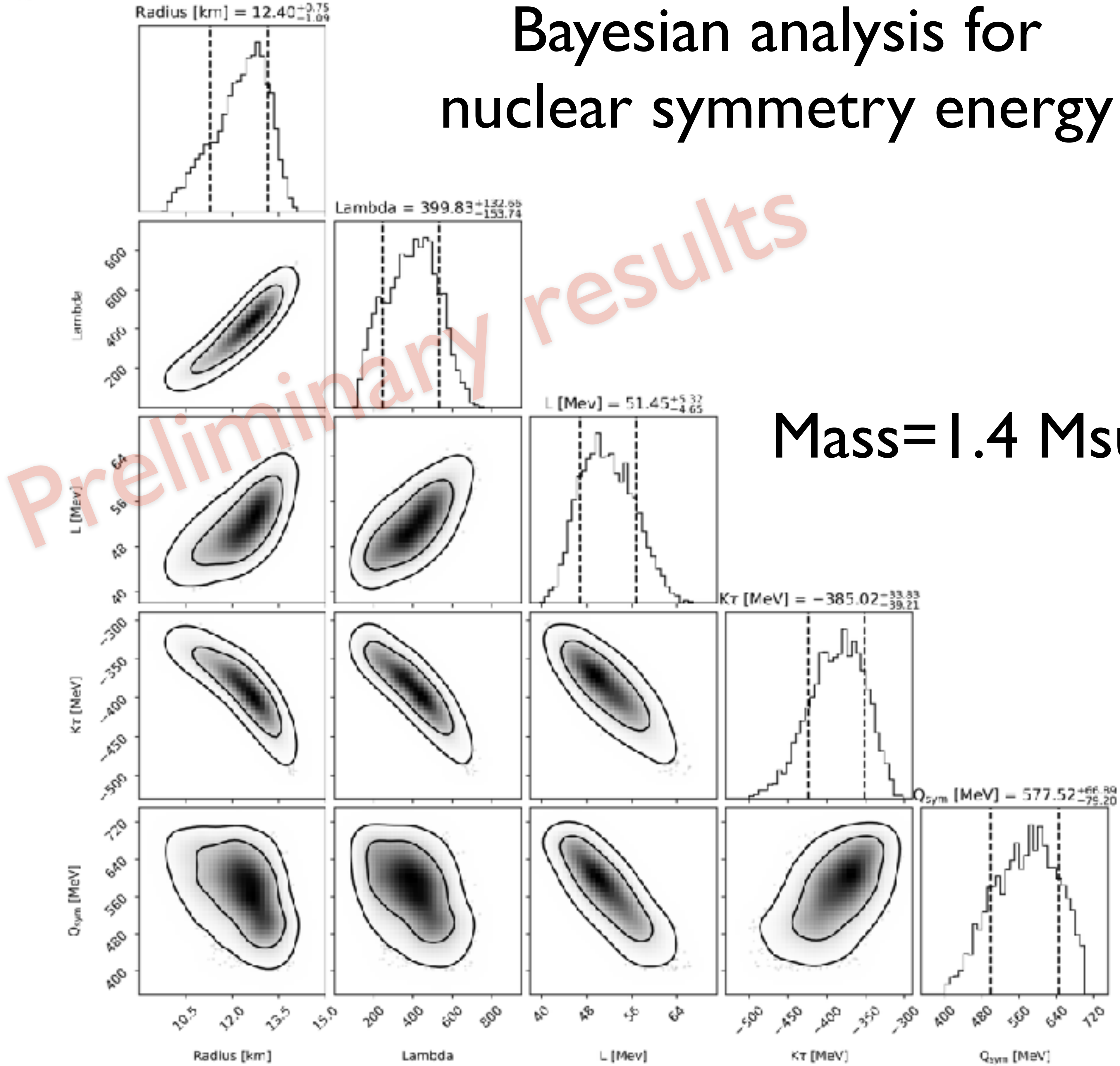
$$\mathcal{T}(\rho,\delta) = \frac{3}{5}\left[\frac{\hbar^2}{2m_p}\left(\frac{1-\delta}{2}\right)^{5/3} + \frac{\hbar^2}{2m_n}\left(\frac{1+\delta}{2}\right)^{5/3}\right](3\pi^2\rho)^{2/3}$$

$$c_i(\delta) = \alpha_i + \beta_i\delta^2$$ to be determined by fitting to the observables

at zero temperature $k_F = (3\pi^2\rho/2)^{1/3}$ $k_{F_\tau} = k_F(1+\tau\delta)^{1/3}$

Bayesian analysis for nuclear symmetry energy

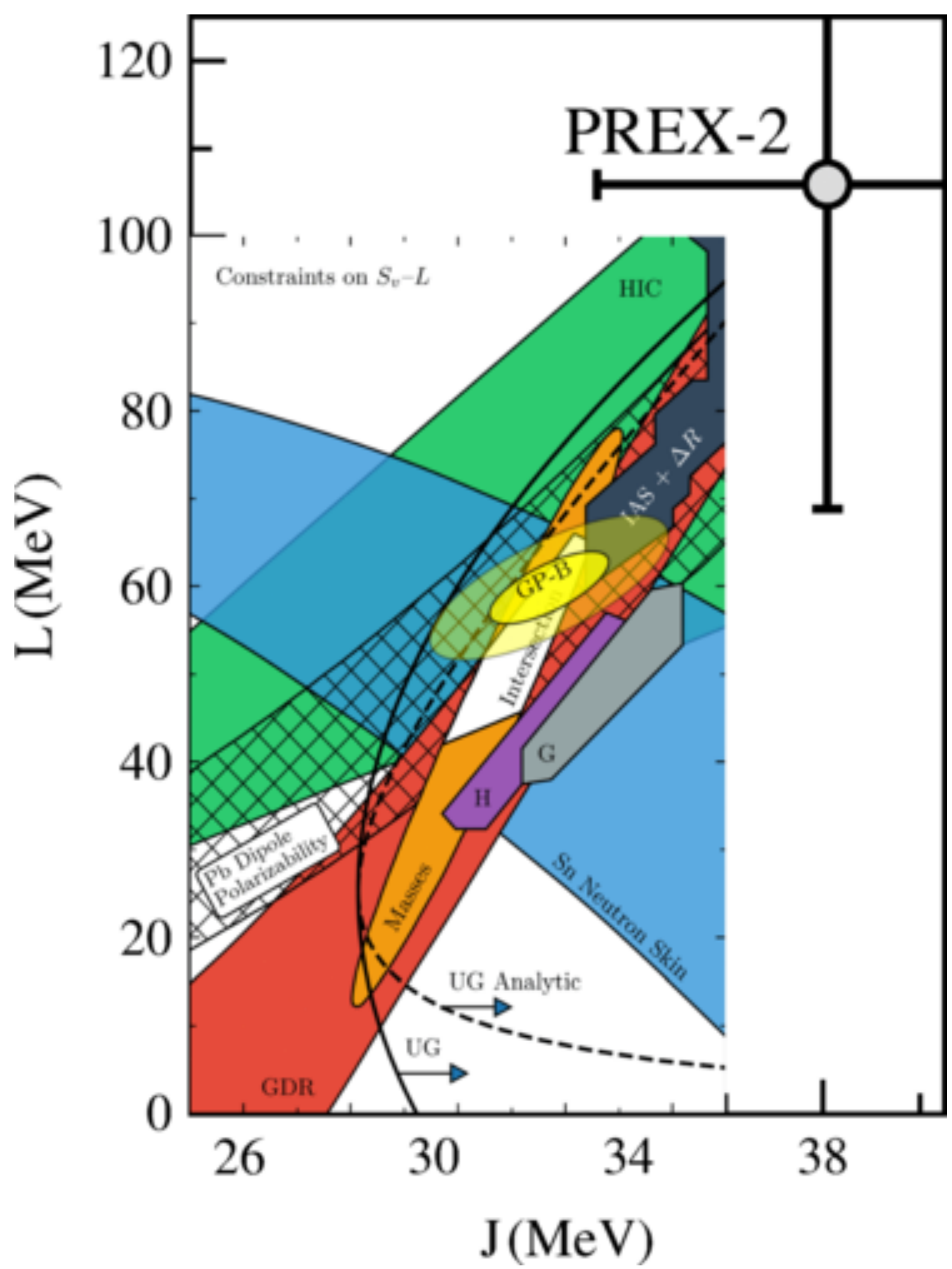
Mass=1.4 Msun

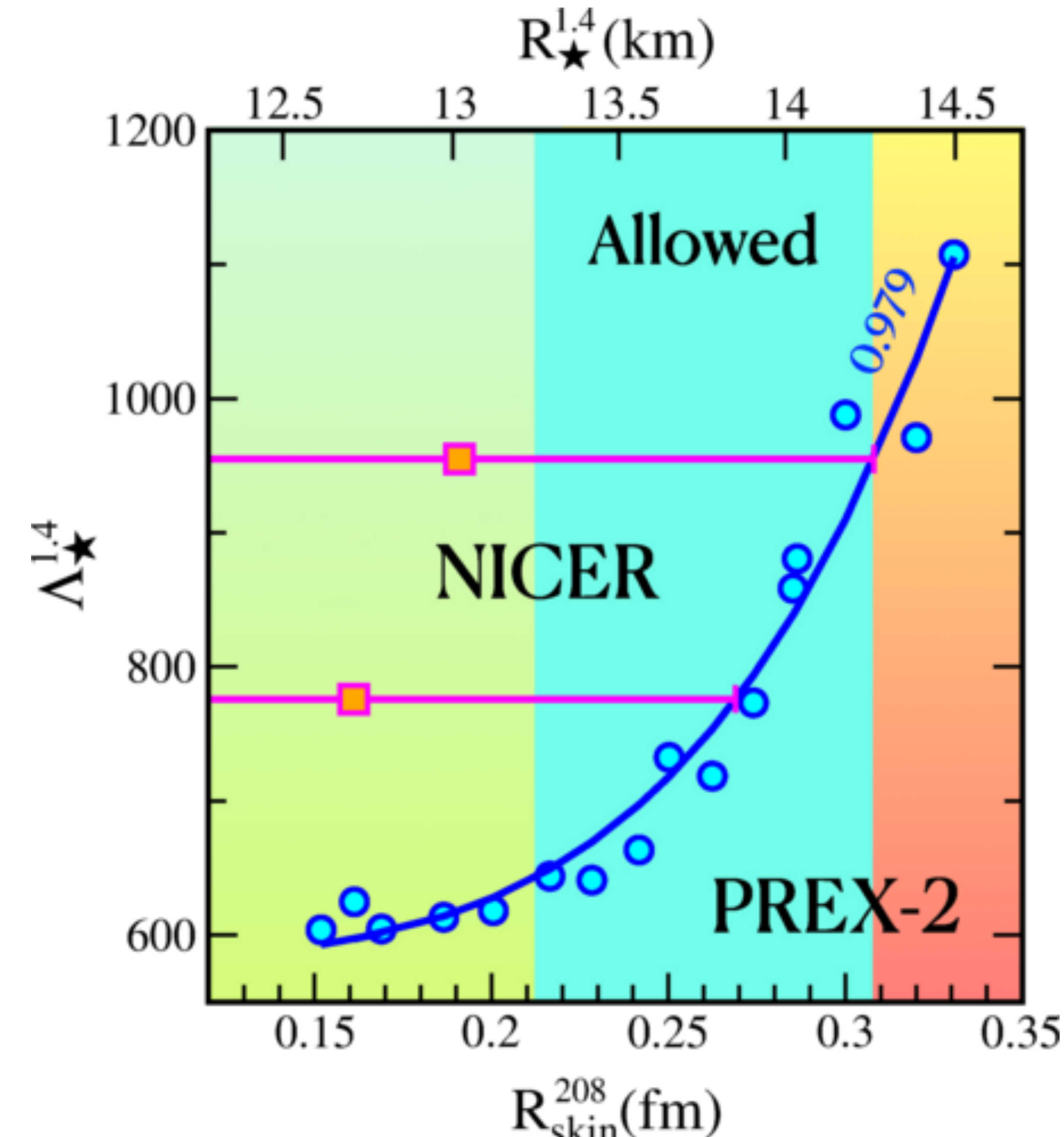


Implication of PREX 2 and NICER Obs.

PREX 2 - PhysRevLett.126.172502 (2021) Neutron skin thickness, Rn-Rp = 0.278 +/- 0.078 (exp.) +/- 0.012 (theo.) fm

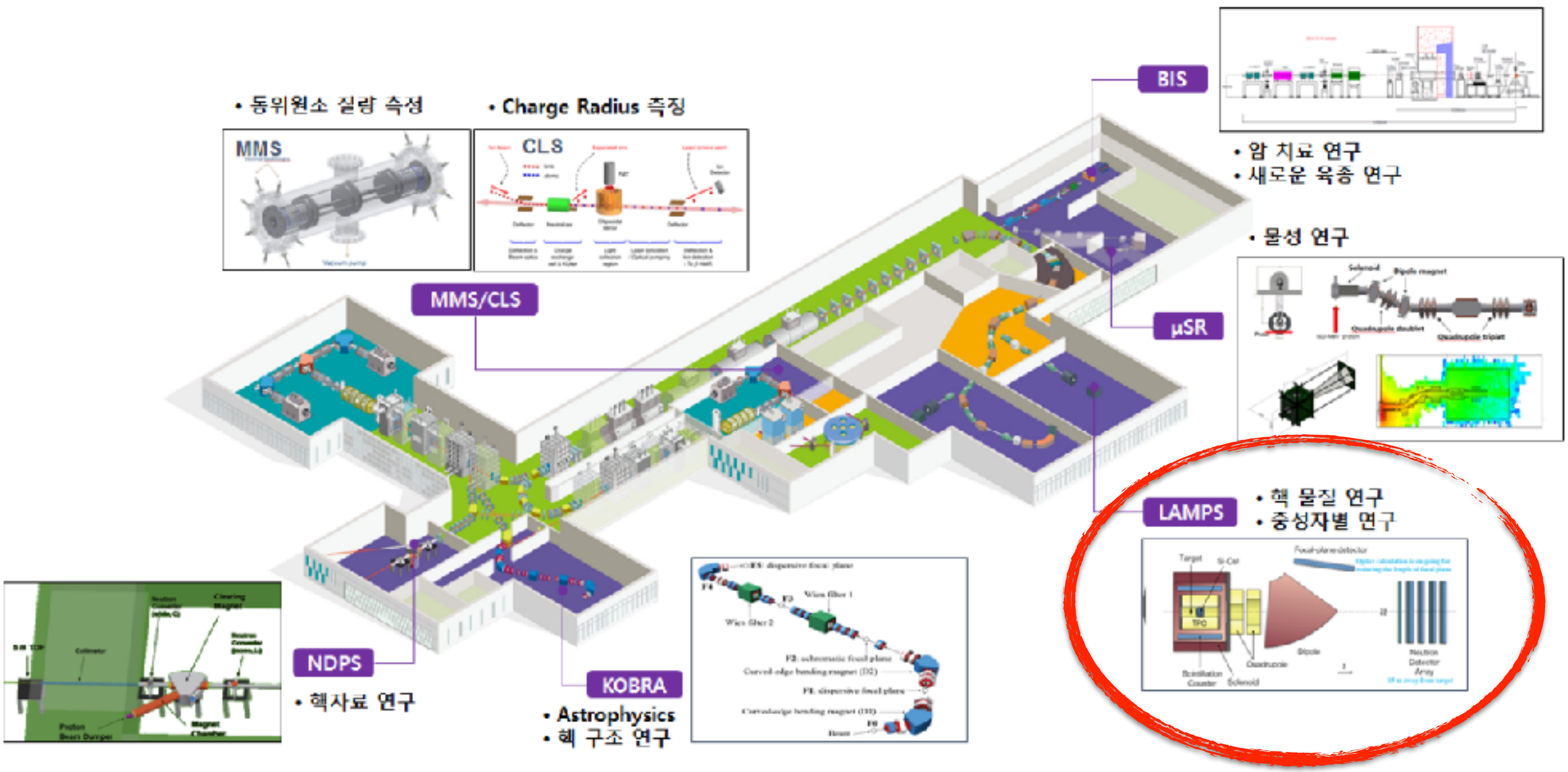
Implication of PREX 2 - PhysRevLett.126.172503 (2021)





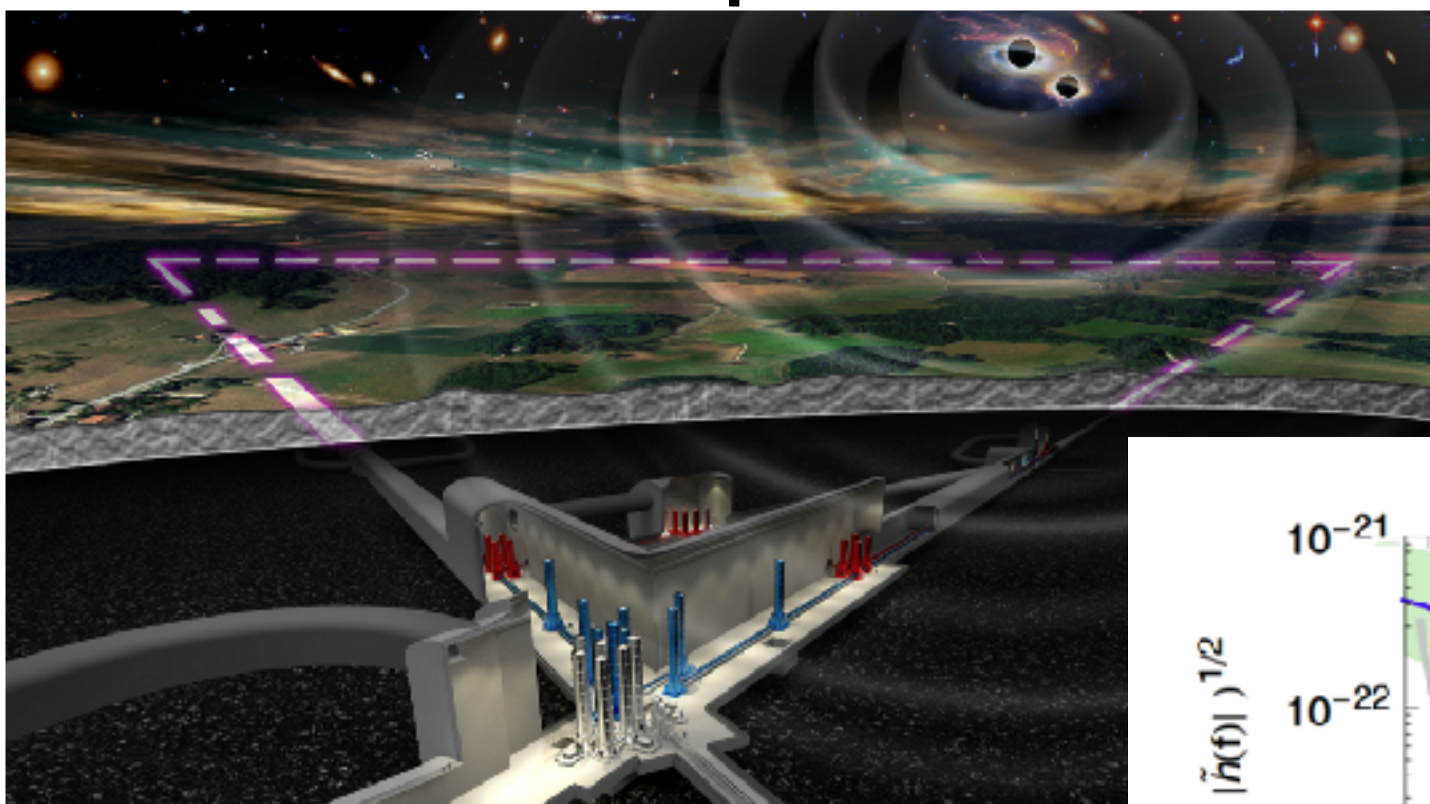
Astrophysics w/ nuclear physics

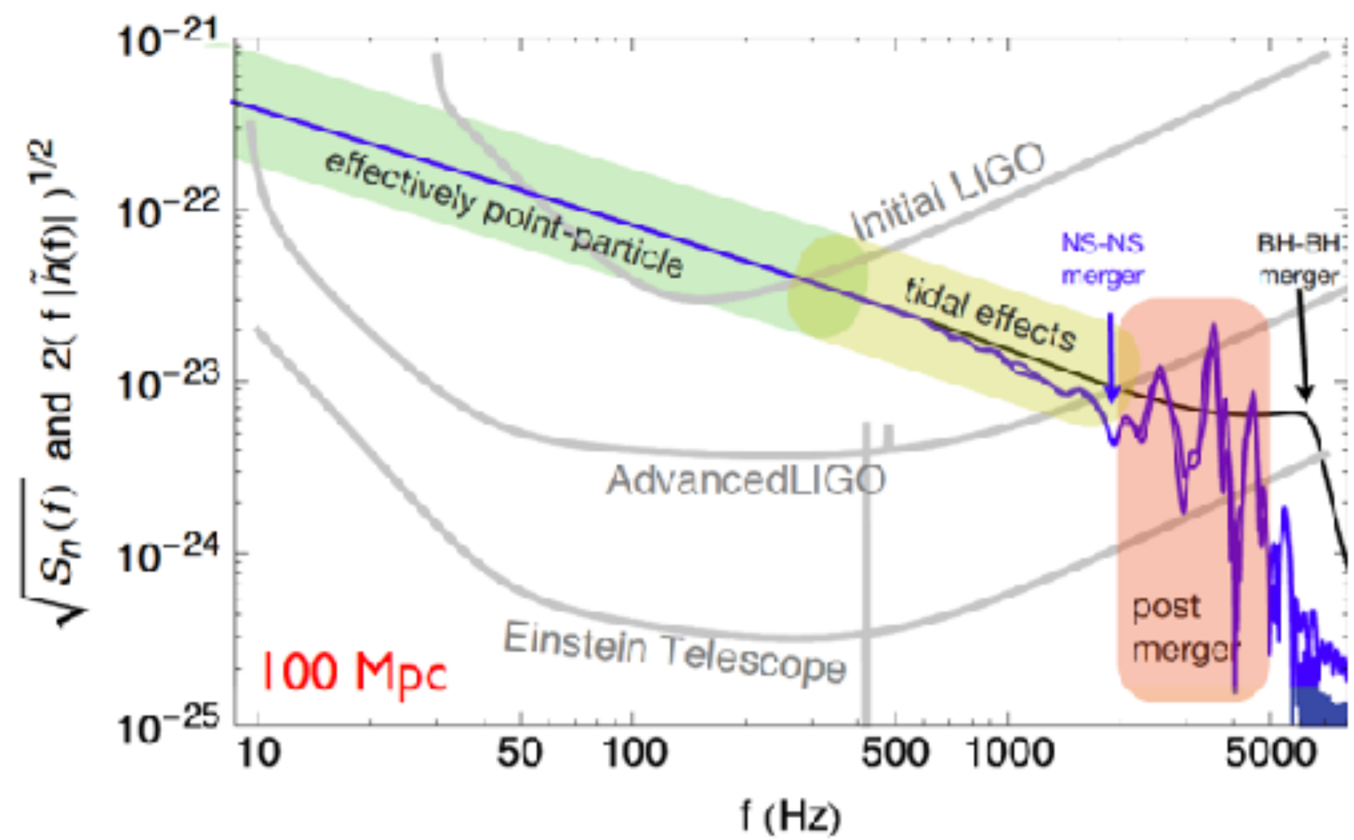
▪ **Experimental Systems**



Next Generation GW detectors

Einstein Telescope





ET-bluebook

중성자별에 대해서 알아야할게
더 없을까 …

Equilibrium Structure of Star

Fluid equation (Euler equation).

1. Continuity Equation

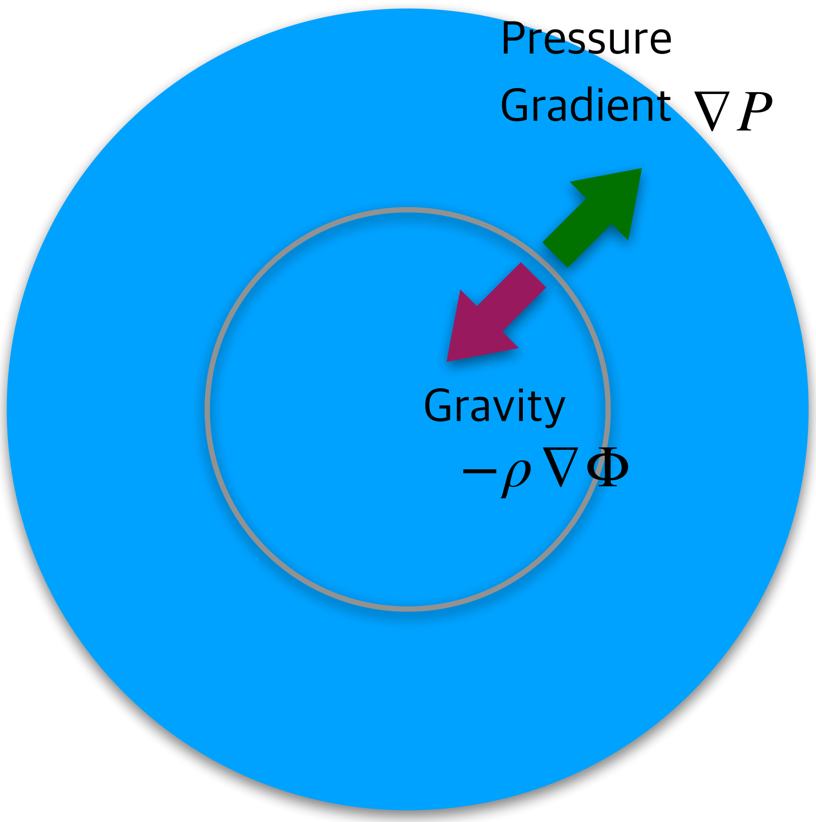
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

2. Momentum Equation

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} + \nabla P = -\rho \nabla \Phi$$

3. Energy Equation

$$\rho \frac{\partial e}{\partial t} + \vec{v} \cdot \nabla e + \frac{P}{\rho} \nabla \cdot \vec{v} = 0$$



For spherical star (non-rotating, non-magnetized), LHS of 1. Continuity and 3. Energy equation is 0. The remaining equation is 2. Momentum equation and can be rewritten in spherical coordinates as

$$\frac{dP}{dr} = -\rho \frac{GM_r}{r^2} \quad \text{where } M_r = 4\pi \int_0^r \rho(r) r^2 dr \quad \Rightarrow \quad \frac{1}{r^2} \frac{d}{dr} \left(\frac{r^2}{\rho} \frac{dP}{dr} \right) = -4\pi G \rho$$

김진호 박사님 강의자료중에서

Linear Stability Analysis

Let us revisit hydrodynamic (Euler) equation in spherical coordinates

1. Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho v\right) = 0,$$

2. Momentum Equation

$$\rho \frac{\partial v}{\partial t} + \rho v \frac{\partial v}{\partial r} + \frac{\partial P}{\partial r} = -\rho \frac{\partial \Phi}{\partial r},$$

3. Equation of State

$$P = K\rho^{\Gamma},$$

4. Poisson Equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) = 4\pi G \rho.$$

Equilibrium (background) solution

$$\frac{\partial P_0}{\partial r} = -\rho_0 \frac{\partial \Phi_0}{\partial r}.$$

Assume,

$$\rho = \rho_0 + \delta\rho,$$
$$P = P_0 + \delta P,$$
$$v = 0 + \delta v,$$
$$\Phi = \Phi_0 + \delta \Phi,$$

where subscript 0 denotes background (equilibrium) solution while δ-ed variables represents Eulerian perturbations.

We can define radial displacement, ζ, then the perturbation of velocity can be written as

$$\delta v = \frac{\partial \zeta}{\partial t}.$$

김진호 박사님 강의자료중에서

Linear Stability Analysis

Then the linearize equation becomes

1. Continuity Equation

$$\delta\rho + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho_0\zeta\right) = 0,$$

2. Momentum Equation

$$\rho_0\frac{\partial^2\zeta}{\partial t^2} + \frac{\partial\delta P}{\partial r} = -\rho_0\frac{\partial\delta\Phi}{\partial r} - \delta\rho\frac{\partial\Phi_0}{\partial r},$$

3. Equation of State

$$\frac{\delta P}{P_0} = \Gamma\frac{\delta\rho}{\rho_0},$$

4. Poisson Equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\delta\Phi}{\partial r}\right) = 4\pi G\delta\rho.$$

We assume that the perturbation has a form as follows,

$$\zeta(t,r) = \xi(r)e^{i\omega t}.$$

All other δ-ed variables also follow the above form. Then the continuity and momentum equation become

$$\delta\rho + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho_0\xi\right) = 0,$$

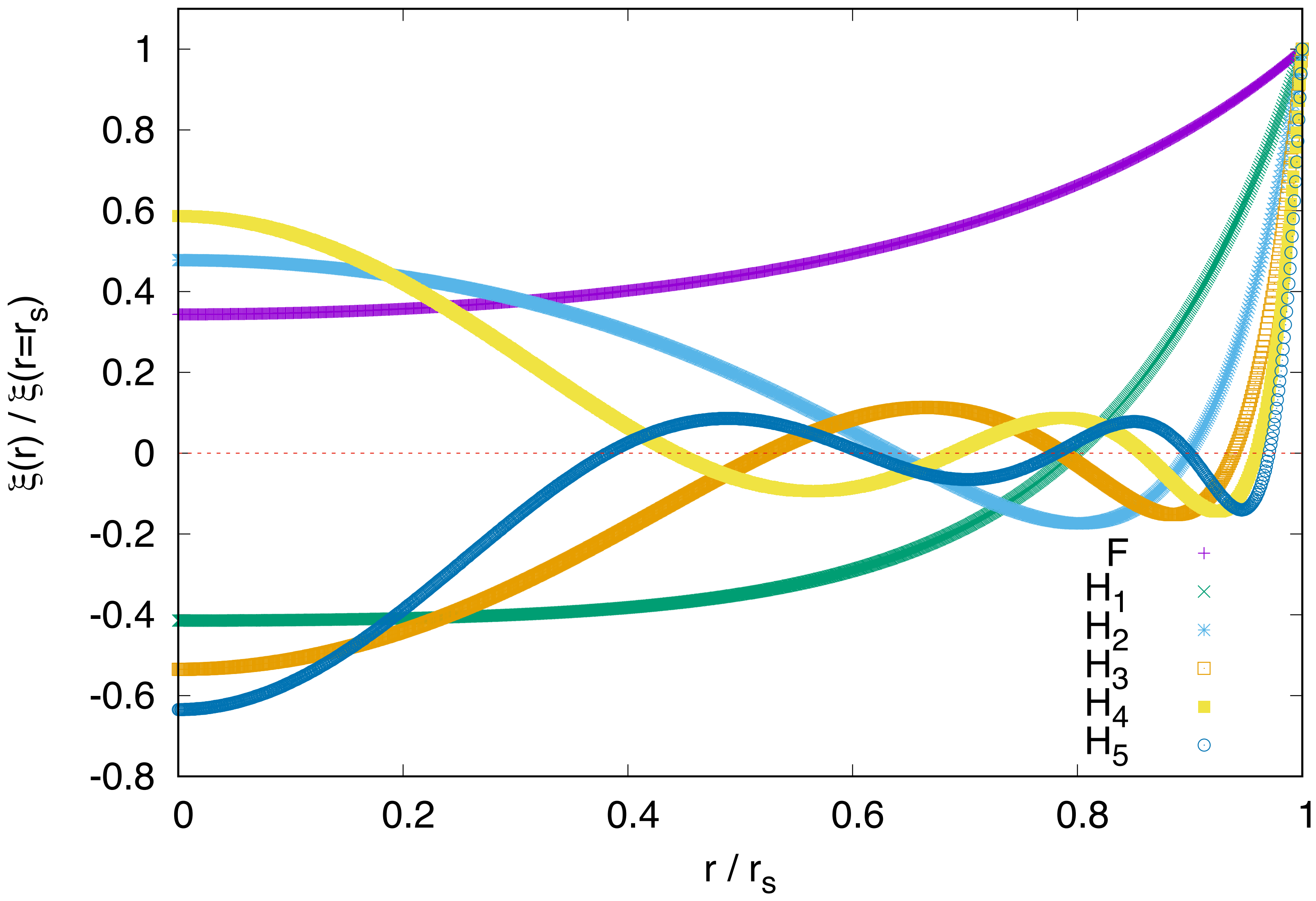
$$-\rho_0\omega^2\xi + \frac{\partial\delta P}{\partial r} = -\rho_0\frac{\partial\delta\Phi}{\partial r} - \delta\rho\frac{\partial\Phi_0}{\partial r}.$$

Boundary condition:

1. $r = 0$: $\xi \sim r$
2. $r = r_s$: $\Delta P = 0$

김진호 박사님 강의자료중에서

Radial Pulsation Modes



F: Fundamental mode. No node inside star.

H_n: n^{th} overtone modes. n nodes interior of the star.

김진호 박사님 강의자료중에서

Models of Rapidly Rotating NS

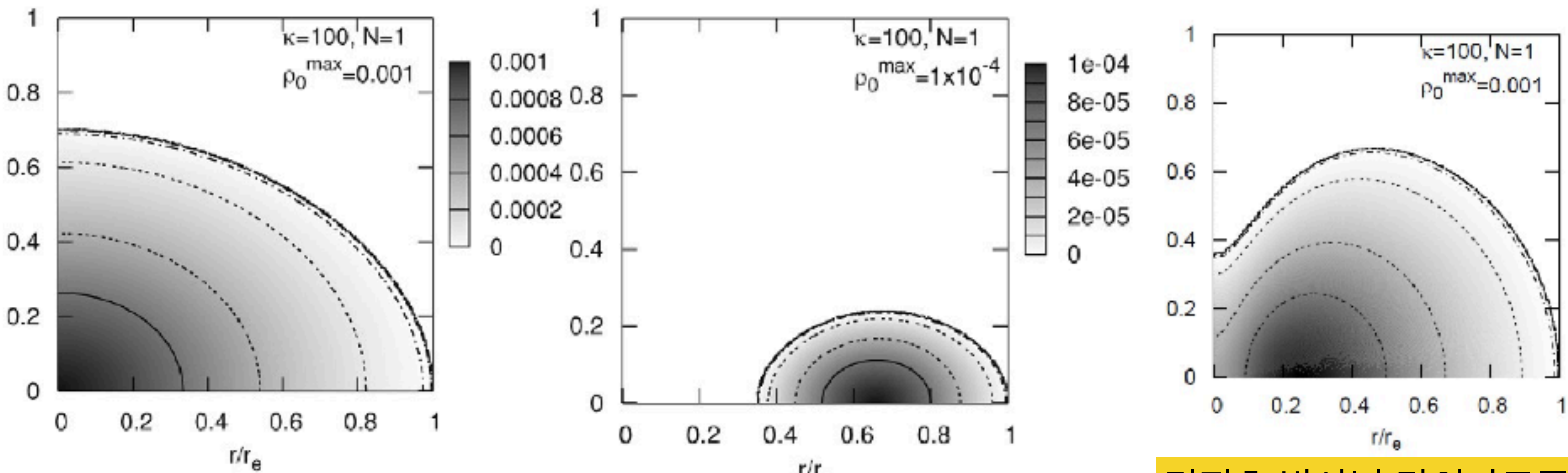
- Rotating star (2D structure) depends on r, z (or theta) coordinates.
  - Integral representation of equilibrium equation (see previous lecture and homework, it is also known as Self-Consistent Field Method).
  - Hachisu (1986): Newtonian

$$H + \Phi - \int \Omega^2 R dR = C.$$

  - Komatsu, Eriguchi & Hachisu (KEH, 1989), Cook, Shapiro & Teukolsky (CST, 1992): GR

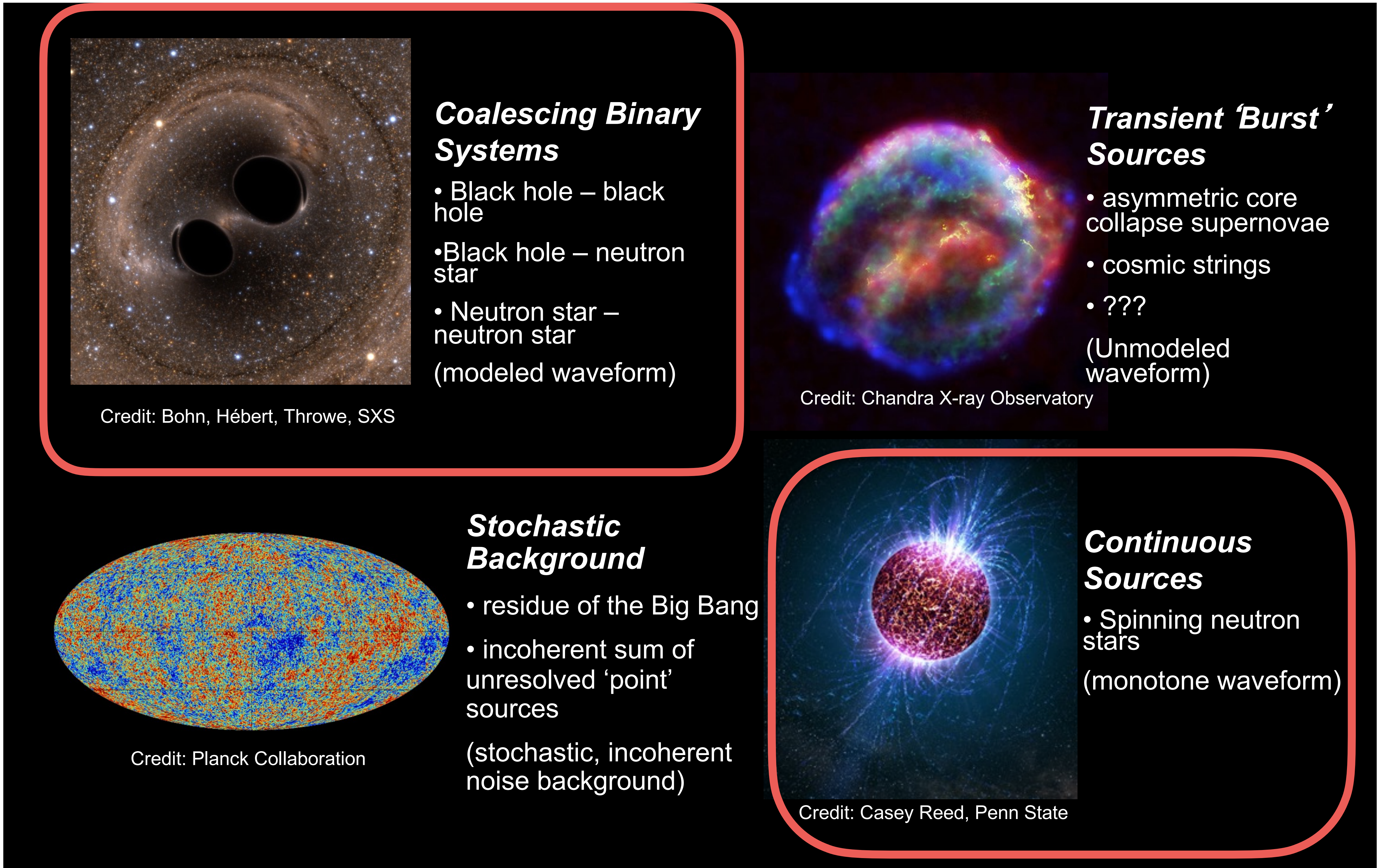
Metric: $ds^2 = -e^{2\nu} dt^2 + e^{2\alpha}\left(dr^2 + r^2 d\theta^2\right) + e^{2\beta} r^2 \sin^2\theta \left(d\phi - \omega dt\right)^2,$

Integral equation: $\ln H + \nu + \ln\left(1 - v^2\right) + \int v^2 \frac{d\Omega}{\Omega - \omega}.$



김진호 박사님 강의자료중에서

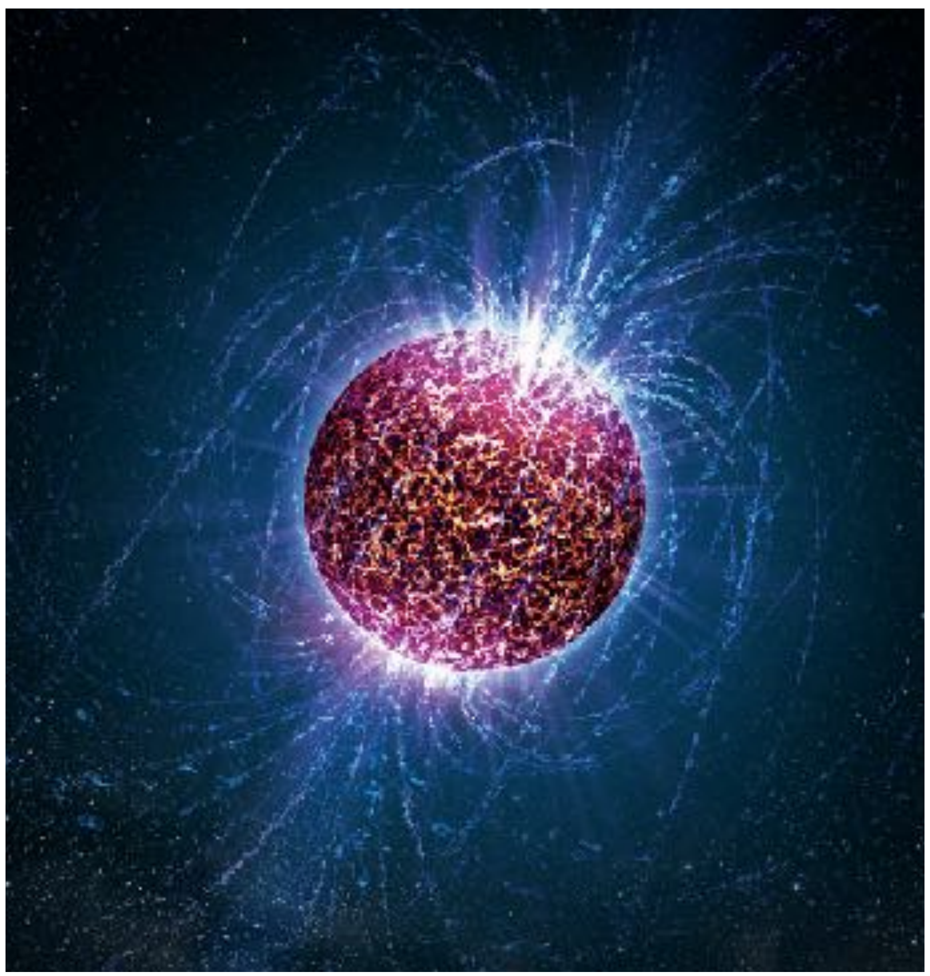
Astrophysical GW Sources - NS



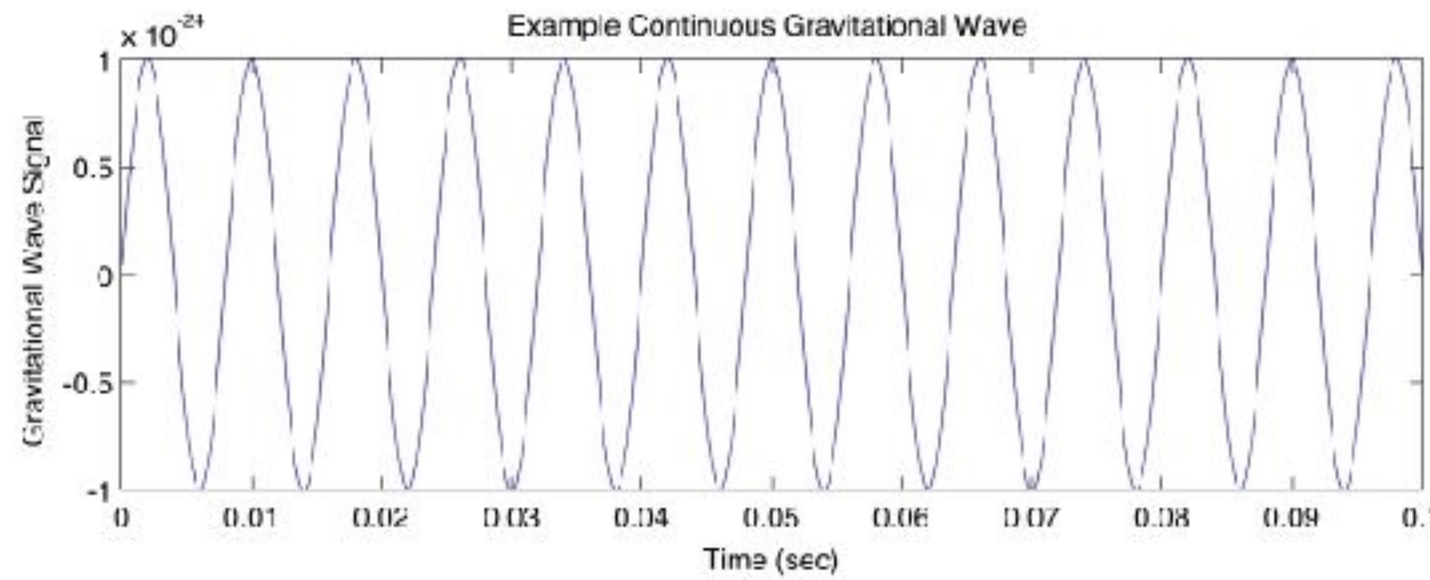
In D. Reitze's presentation in LIGO ODW #1, 2018

Continuous Gravitational Waves (CW)

non-Transient sources



Credit: Casey Reed, Penn State



credit: A. Stuver/ LIGO

Continuous Sources

CW from Neutron Star Spin-down

Keith Riles, Living. Rev. Rel. 26, 3(2023)

Phenomenological model

$$\dot{f} = Kf^n$$

Breaking index

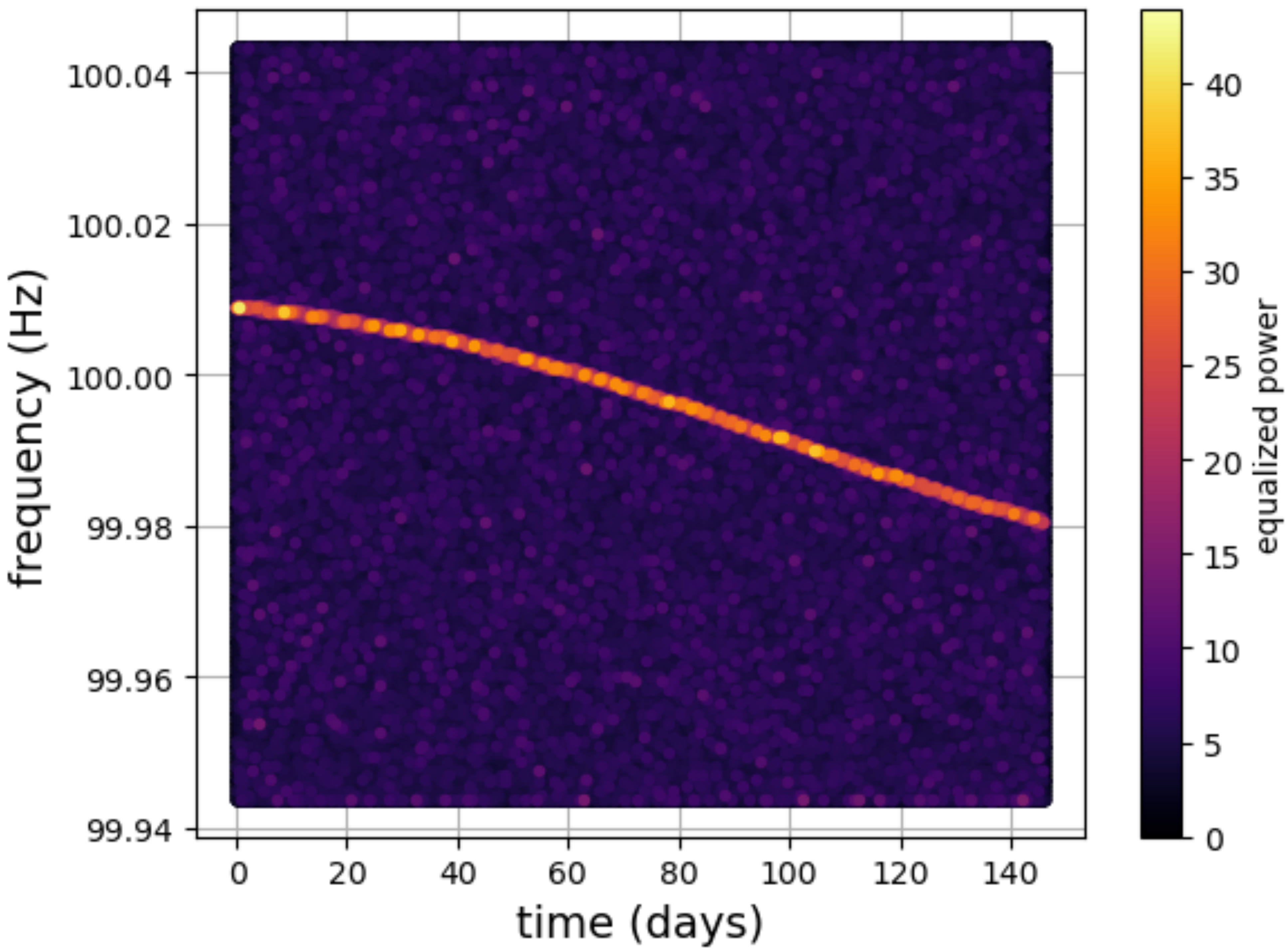
- $n = 1$—"Pulsar wind" (extreme model)
- $n = 3$—Magnetic dipole radiation
- $n = 5$—Gravitational mass quadrupole radiation ("mountain")
- $n = 7$—Gravitational mass current quadrupole radiation (r-modes).

$$h_0 = \sqrt{\frac{512\,\pi^7}{5}\frac{G}{c^5}\frac{1}{d}} f_{\rm GW}^3 \alpha M R^3 \tilde{J}$$

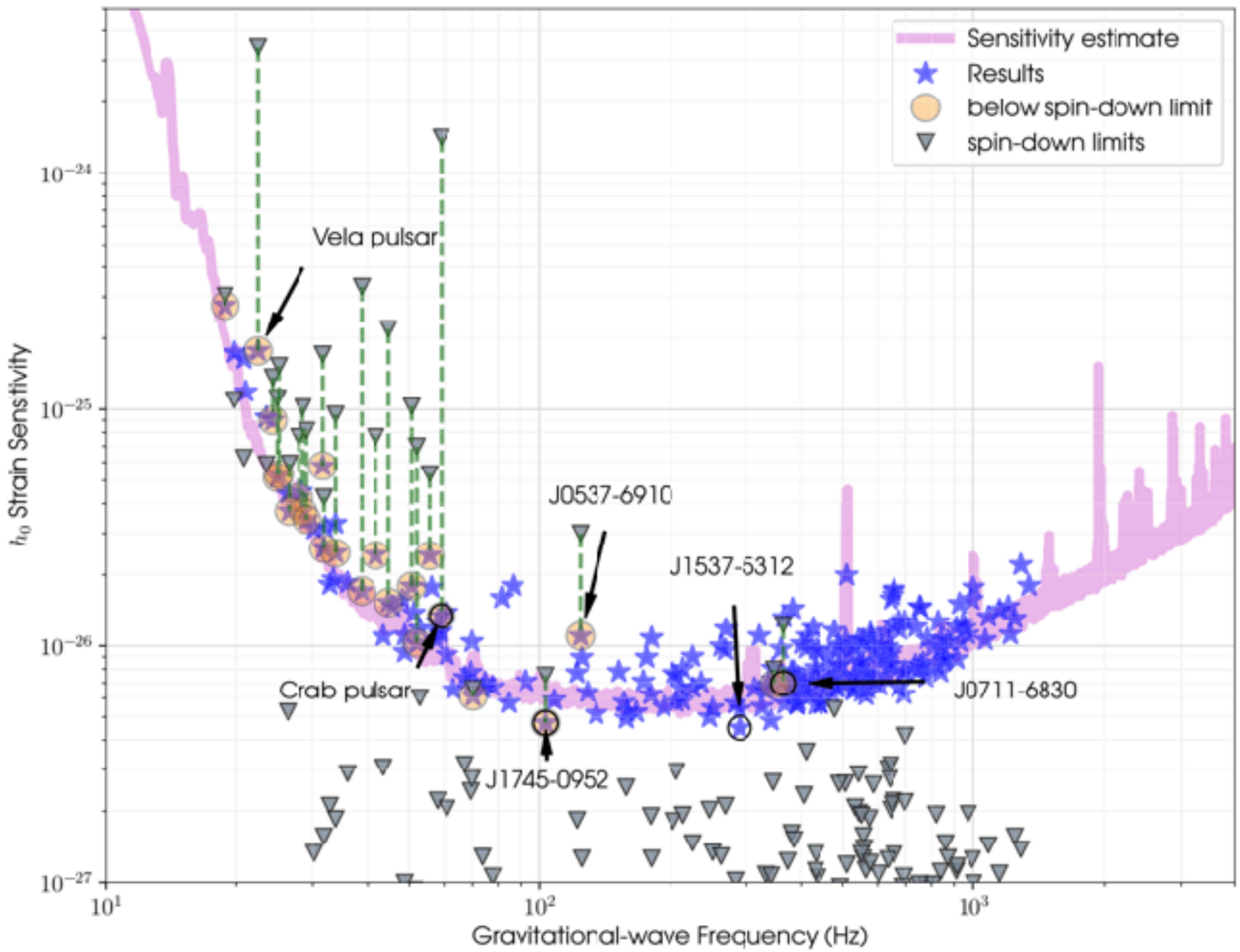
$$= 3.6\times 10^{-26}\left(\frac{1\,\mathrm{kpc}}{d}\right)\left(\frac{f_{\rm GW}}{100\,\mathrm{Hz}}\right)^3\left(\frac{\alpha}{10^{-3}}\right)\left(\frac{R}{11.7\,\mathrm{km}}\right)^3$$

$$h_{\text{spin-down}} = \frac{1}{r}\sqrt{-\frac{45}{8}\frac{G}{c^3}I_{zz}\frac{\dot{f}_{\rm GW}}{f_{\rm GW}}},$$

CW from Neutron Star Spin-down



CW targeted searches for known pulsars



$$h_0 = 2C_{22} = \frac{16\pi^2 G}{c^4}\frac{I_{zz}\varepsilon f_{\rm rot}^2}{d},$$

$$h_0^{\rm sd} = \frac{1}{d}\left(\frac{5GI_{zz}}{2c^3}\frac{|\dot{f}_{\rm rot}|}{f_{\rm rot}}\right)^{1/2}$$

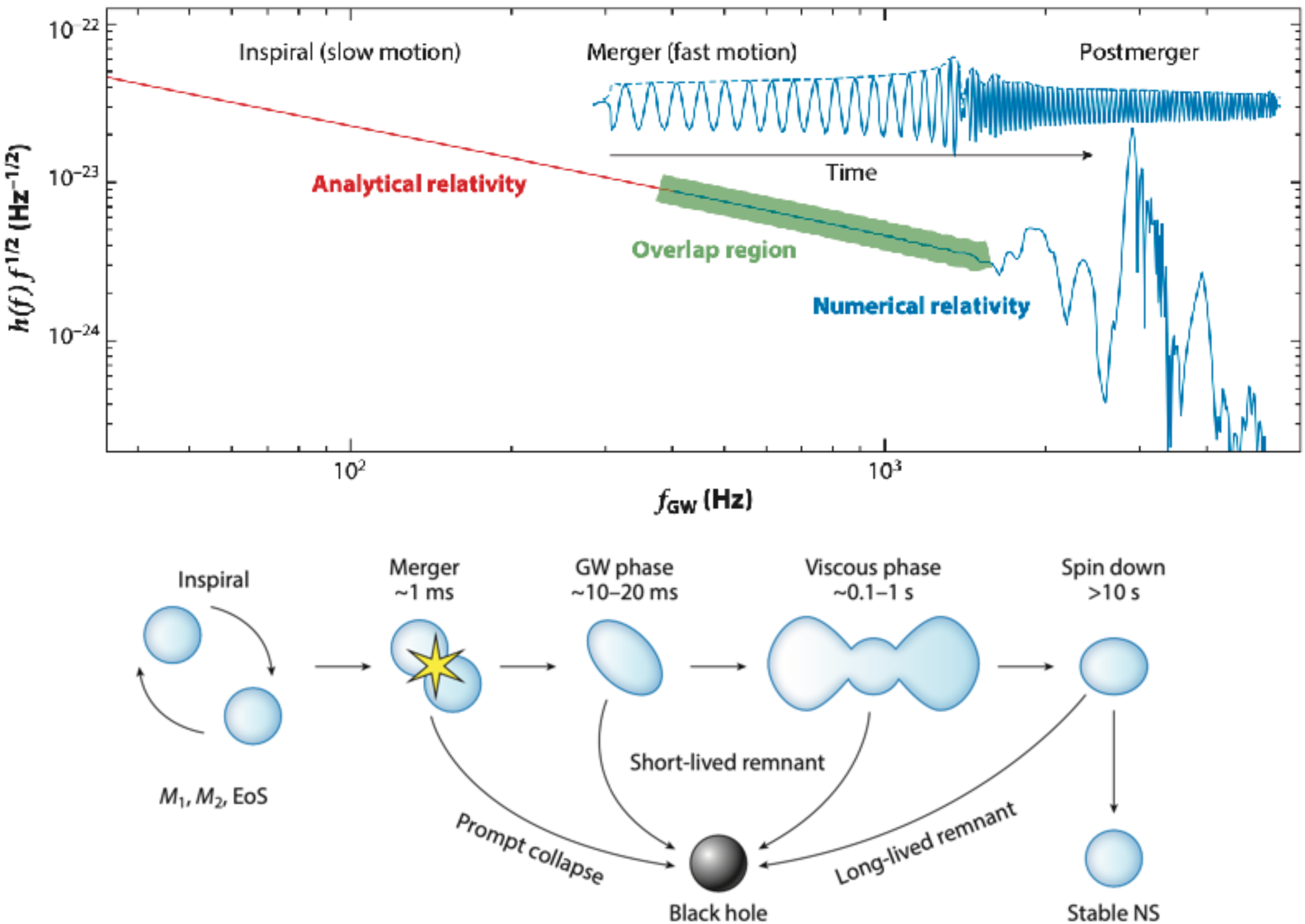
$$\epsilon \equiv \frac{|I_{xx} - I_{yy}|}{I_{zz}}$$

Searches for GW from Known Pulsars at Two Harmonics in O2 and O3

LVK, ApJ, 935, 1 (2022)

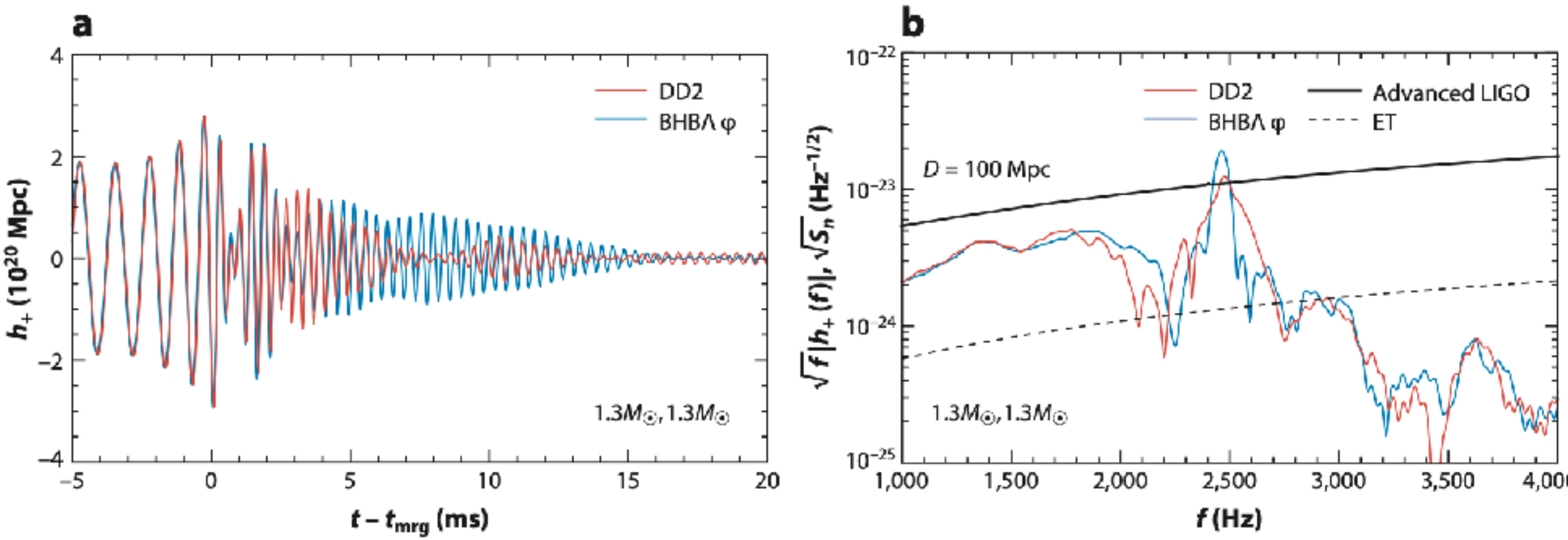
Radice et al. ,Annu. Rev. Nucl. Part. Sci. 70, 95–119 (2020)

"The Dynamics of Binary Neutron Star Mergers and GW170817"



Radice et al. , Annu. Rev. Nucl. Part. Sci. 70, 95–119 (2020)

"The Dynamics of Binary Neutron Star Mergers and GW170817"



Stergioulas, Bauswein, Zagkouris , and Janka, MNRAS. **418,** 427–436 (2011)

“Gravitational waves and non-axisymmetric oscillation modes in mergers of compact object binaries ”

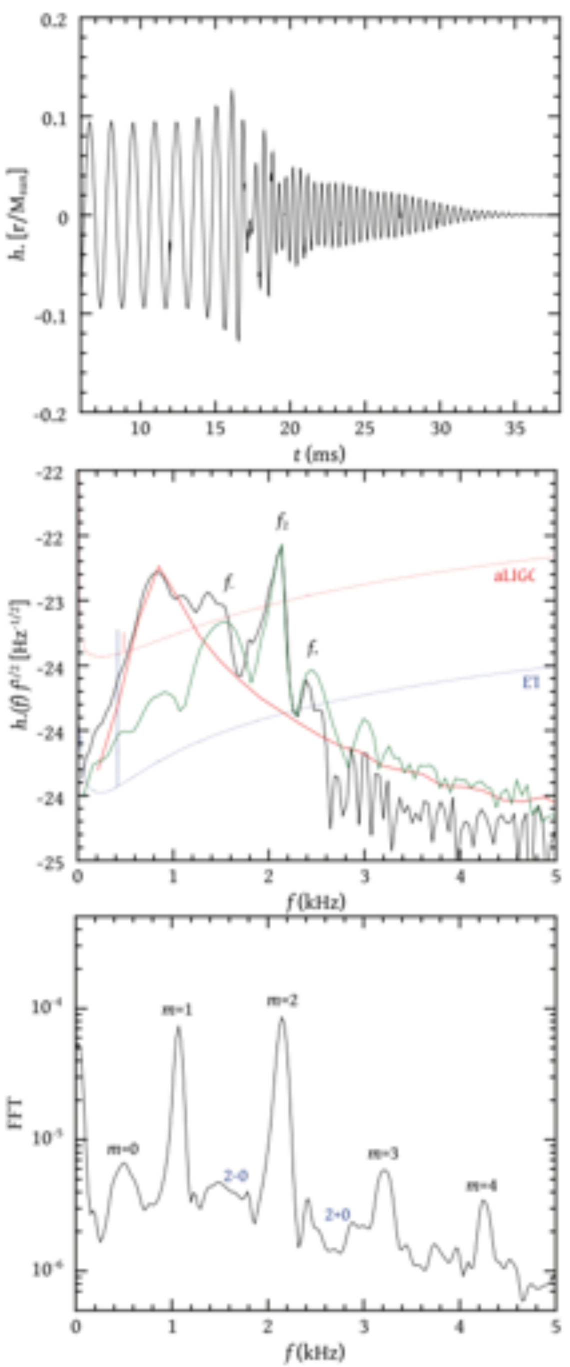


Figure 3. Model Shen 12135. Top panel: time evolution of the GW amplitude h_+. Middle panel: total (black), pre-merger (red) and post-merger (green) scaled power spectral density, compared to the Advanced LIGO and

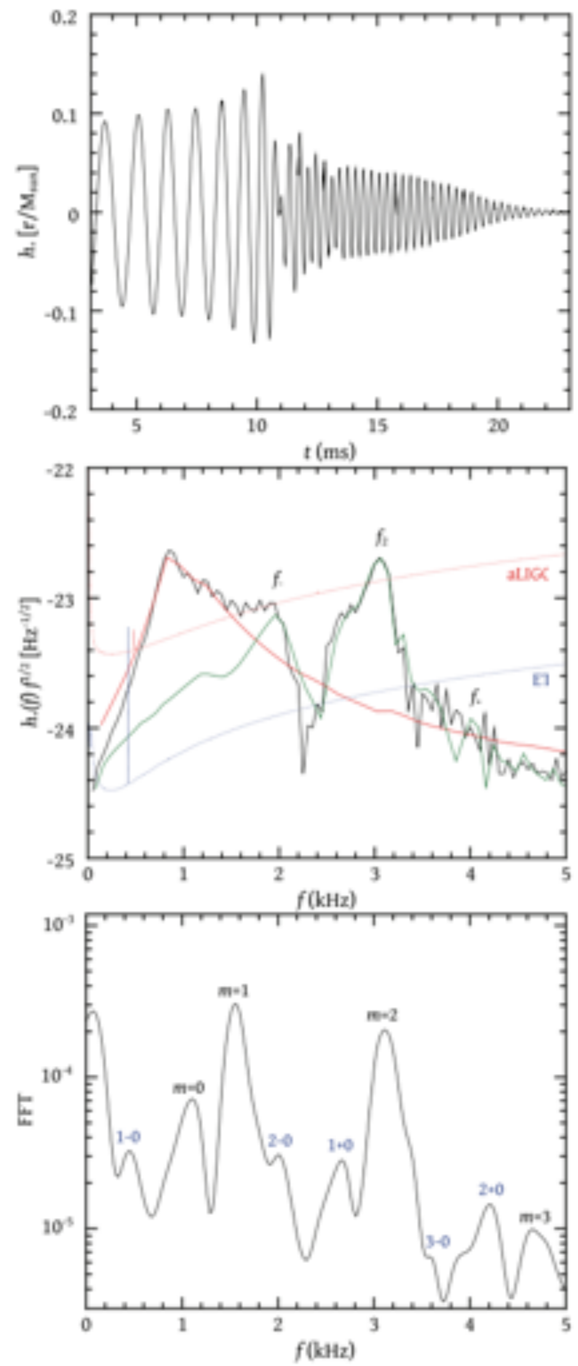


Figure 5. Same as Fig. 3, but for model LS 12135.

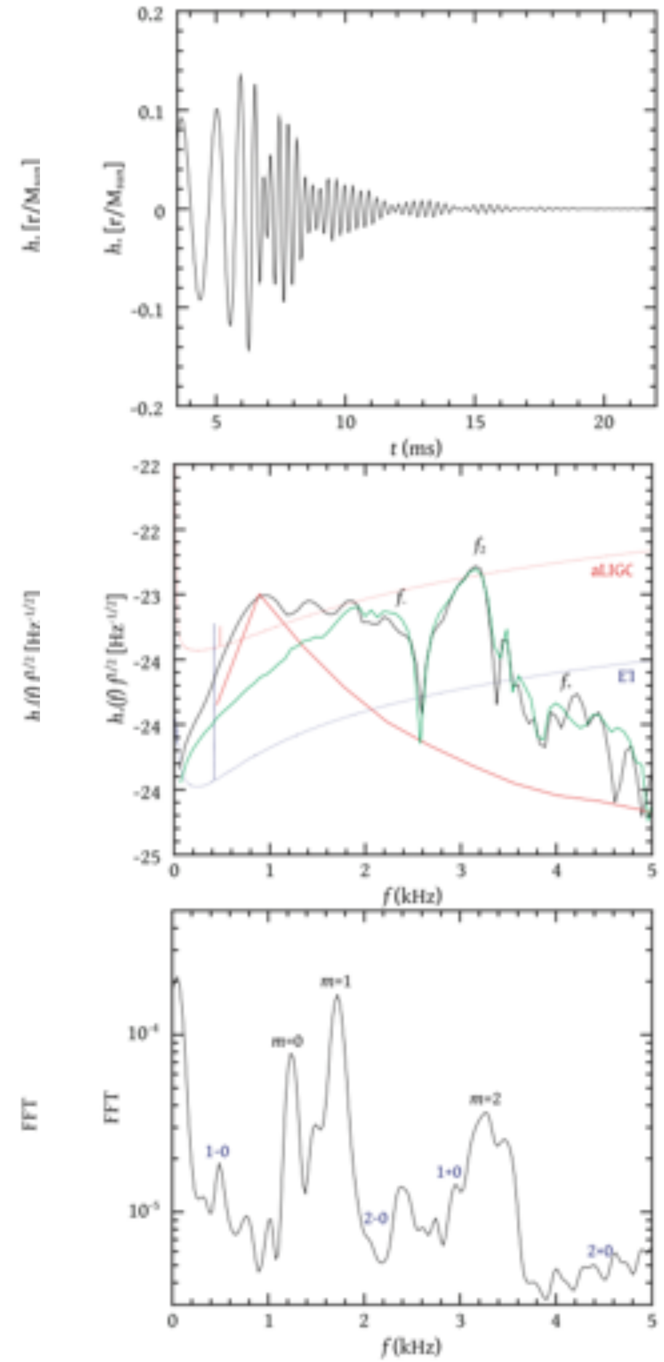


Figure 8. Same as Fig. 3, but for model MIT60 12135.

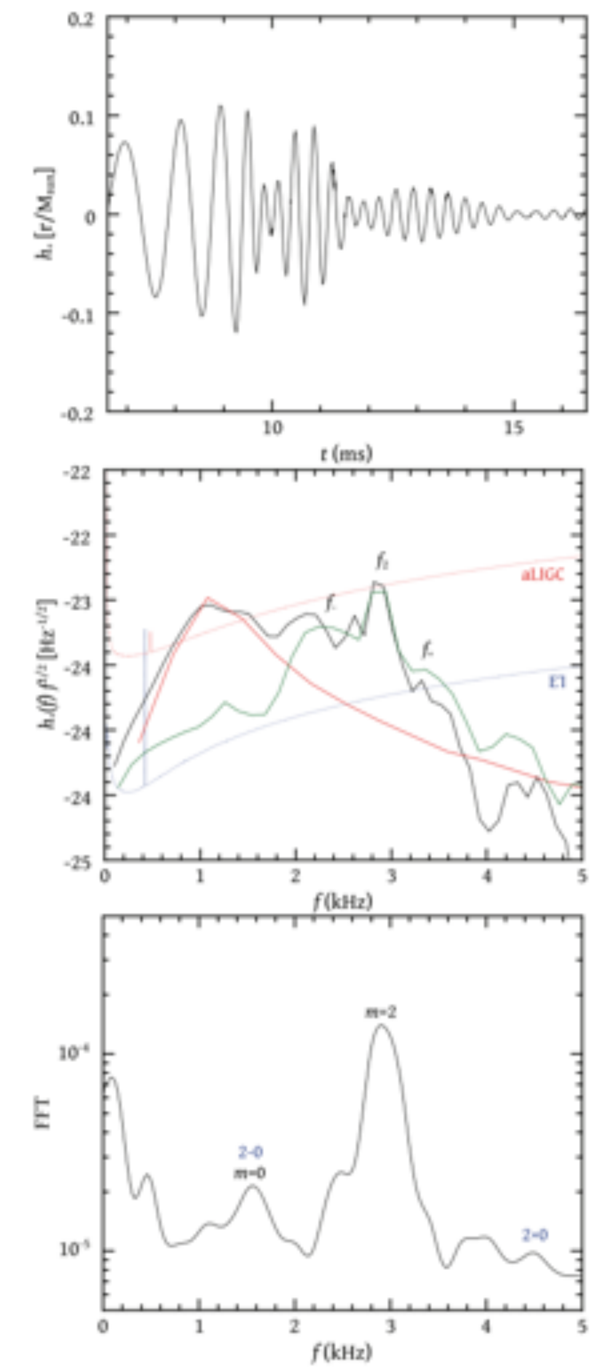
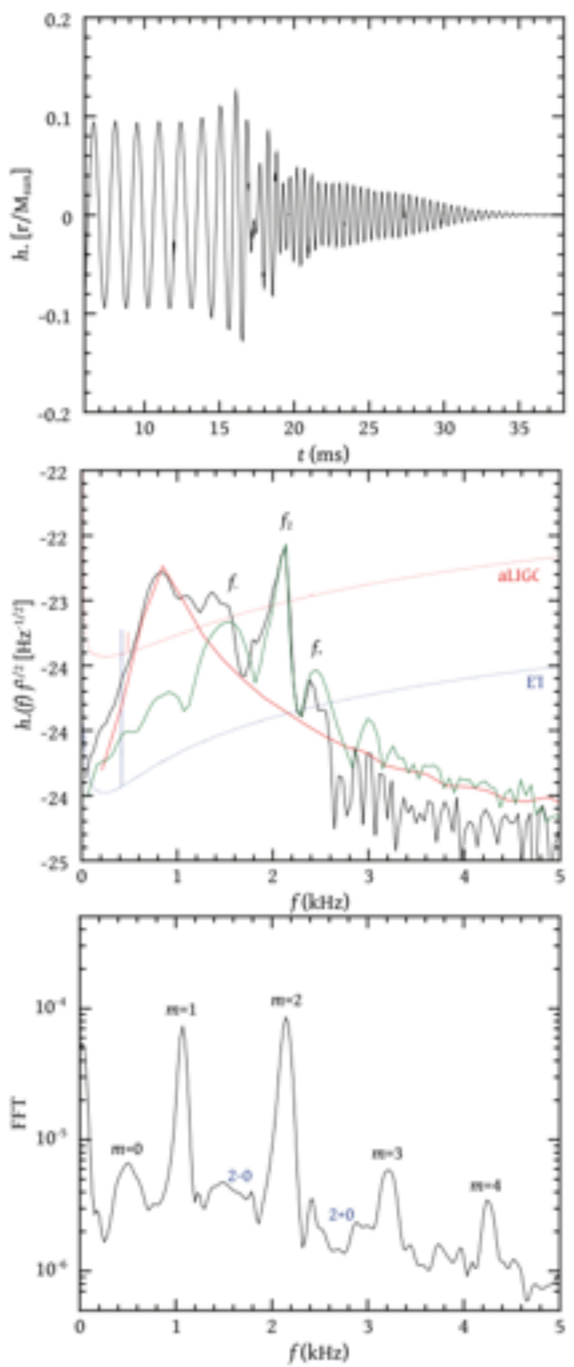


Figure 7. Same as Fig. 3, but for model MIT60 1111.

Stergioulas, Bauswein, Zagkouris , and Janka, MNRAS. **418,** 427–436 (2011)

"Gravitational waves and non-axisymmetric oscillation modes in mergers of compact object binaries "



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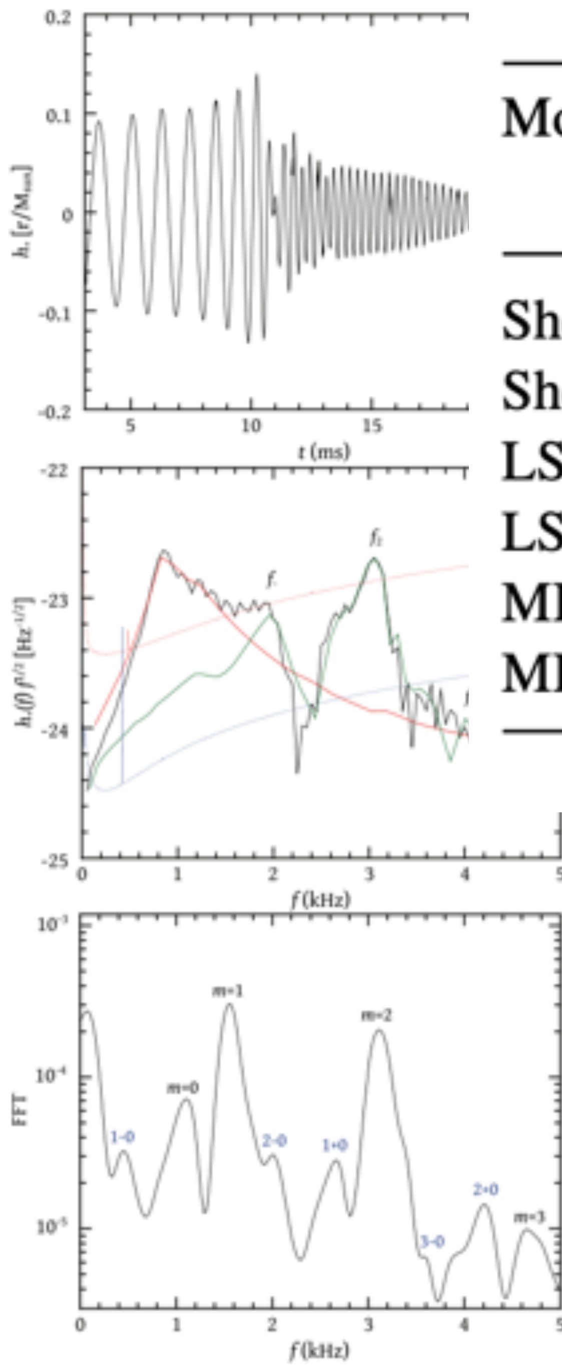


Figure 5. Same as Fig. 3, but for model LS 12135.

Table 1. Extracted mode frequencies.

Model	$f_{m=0}$ (kHz)	$f_{m=1}$ (kHz)	$f_{m=2}$ (kHz)	$f_{m=3}$ (kHz)	$f_{m=4}$ (kHz)
Shen 12135	0.50	1.07	2.14	3.22	4.26
Shen 135135	0.46	–	2.24	–	4.11
LS 12135	1.10	1.55	3.12	4.66	–
LS 135135	0.98	–	3.30	–	–
MIT60 1111	1.56	–	2.92	–	5.94
MIT60 12135	1.24	1.72	3.26	–	–

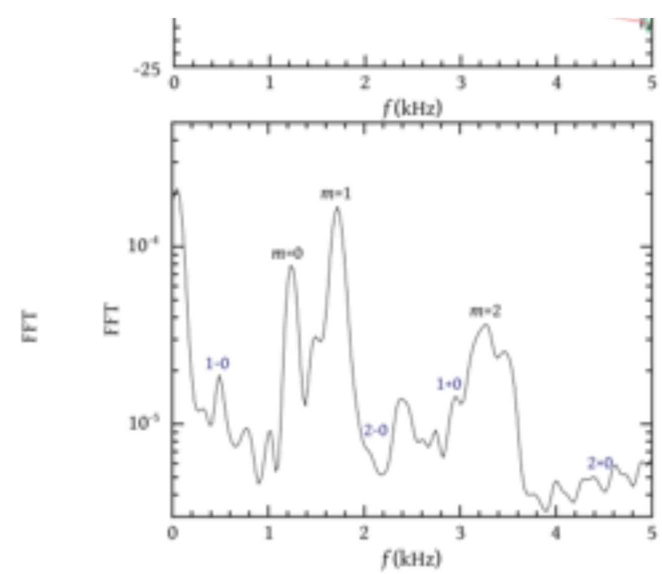
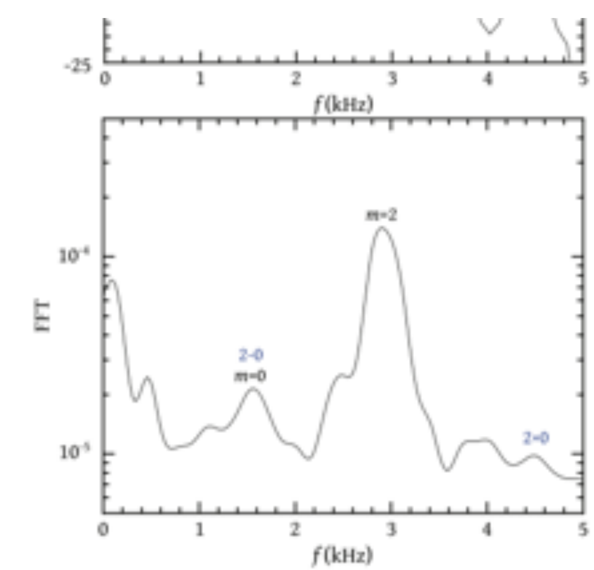


Figure 8. Same as Fig. 3, but for model MIT60 12135.



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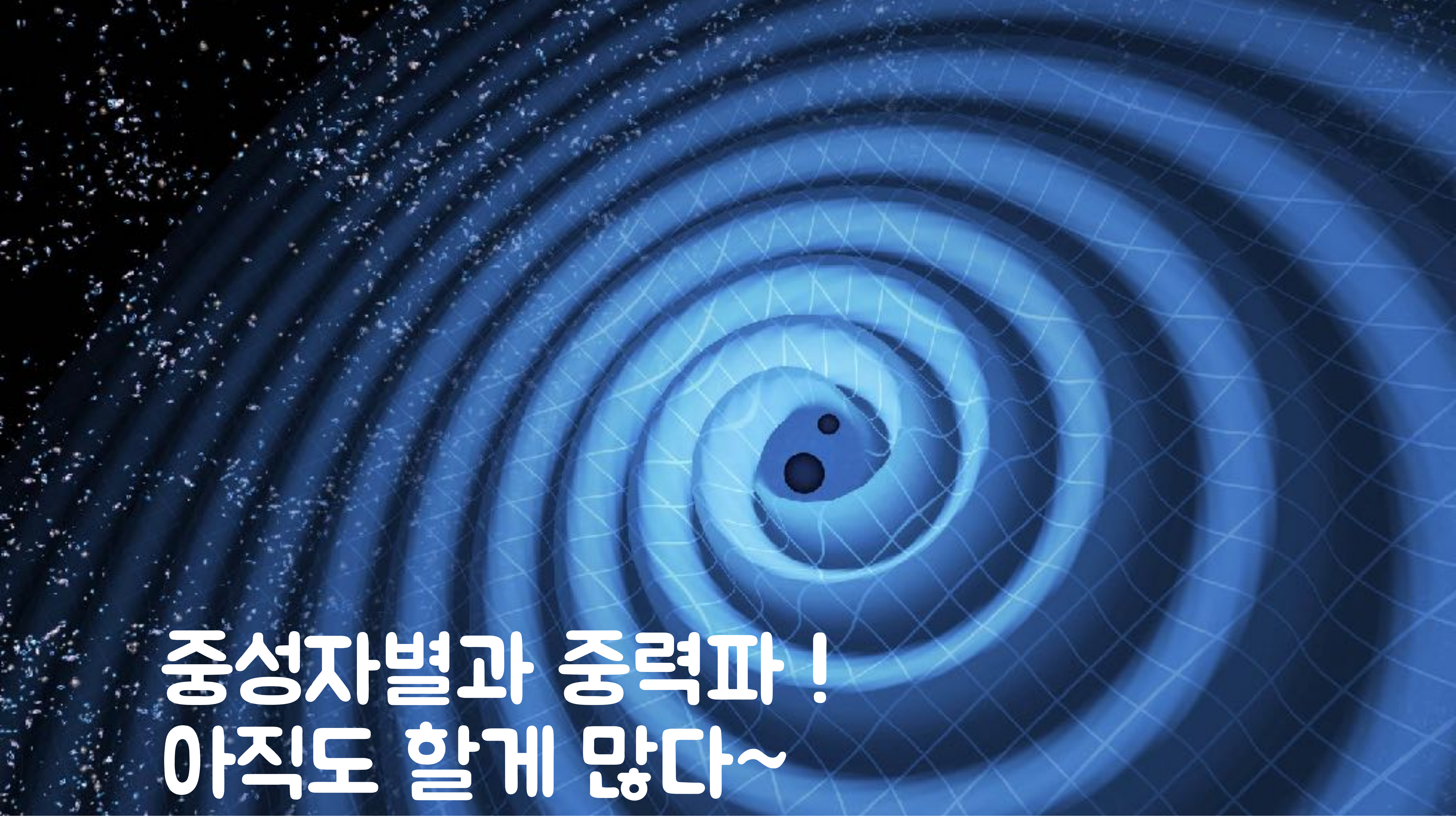
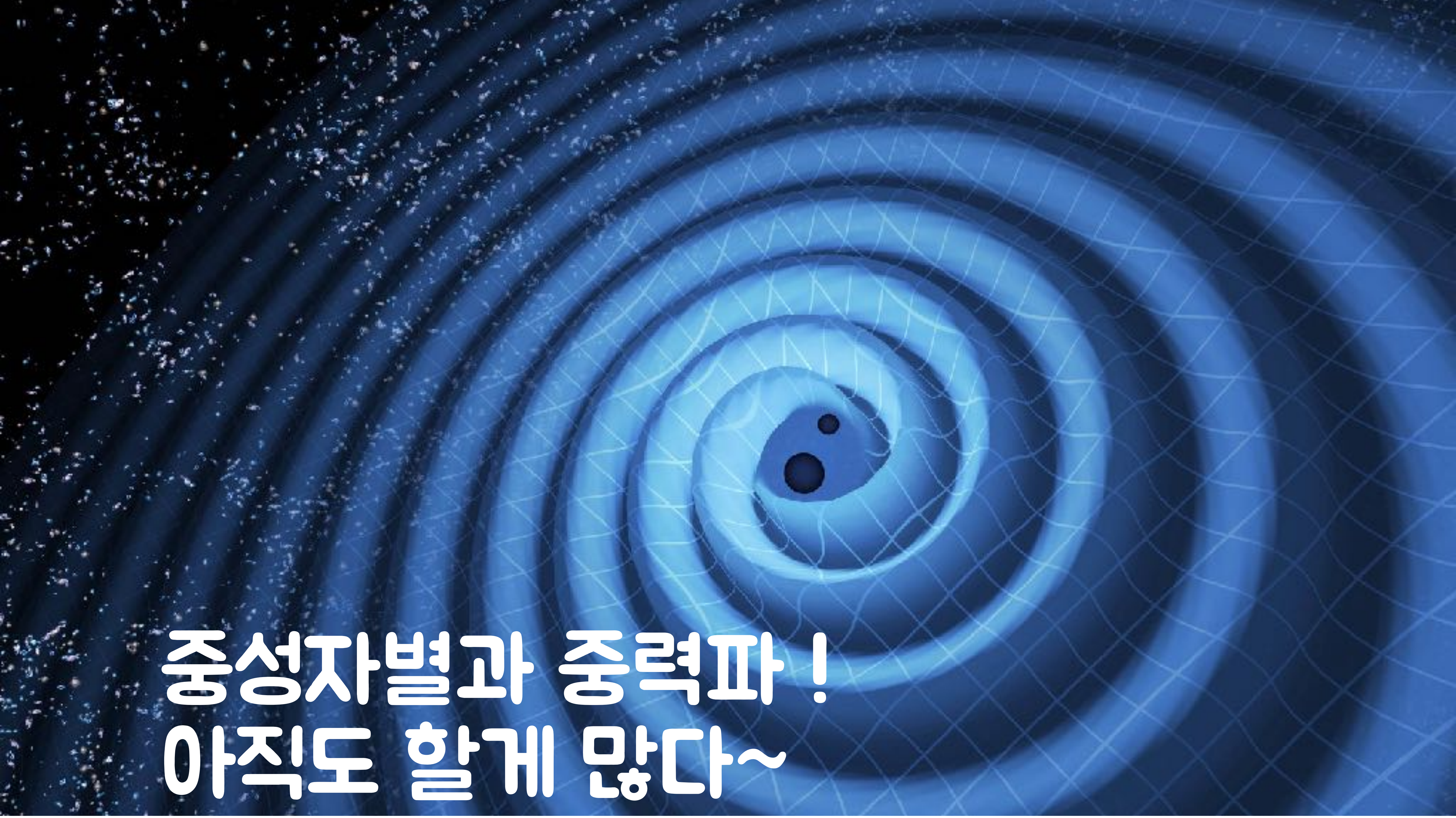

중성자별과 중력파 !
아직도 할게 많다~



중성자별과 중력파 !
아직도 할게 많다~

궁금한 것은 이메일로 물어보세요~ : ymkim@kasi.re.kr