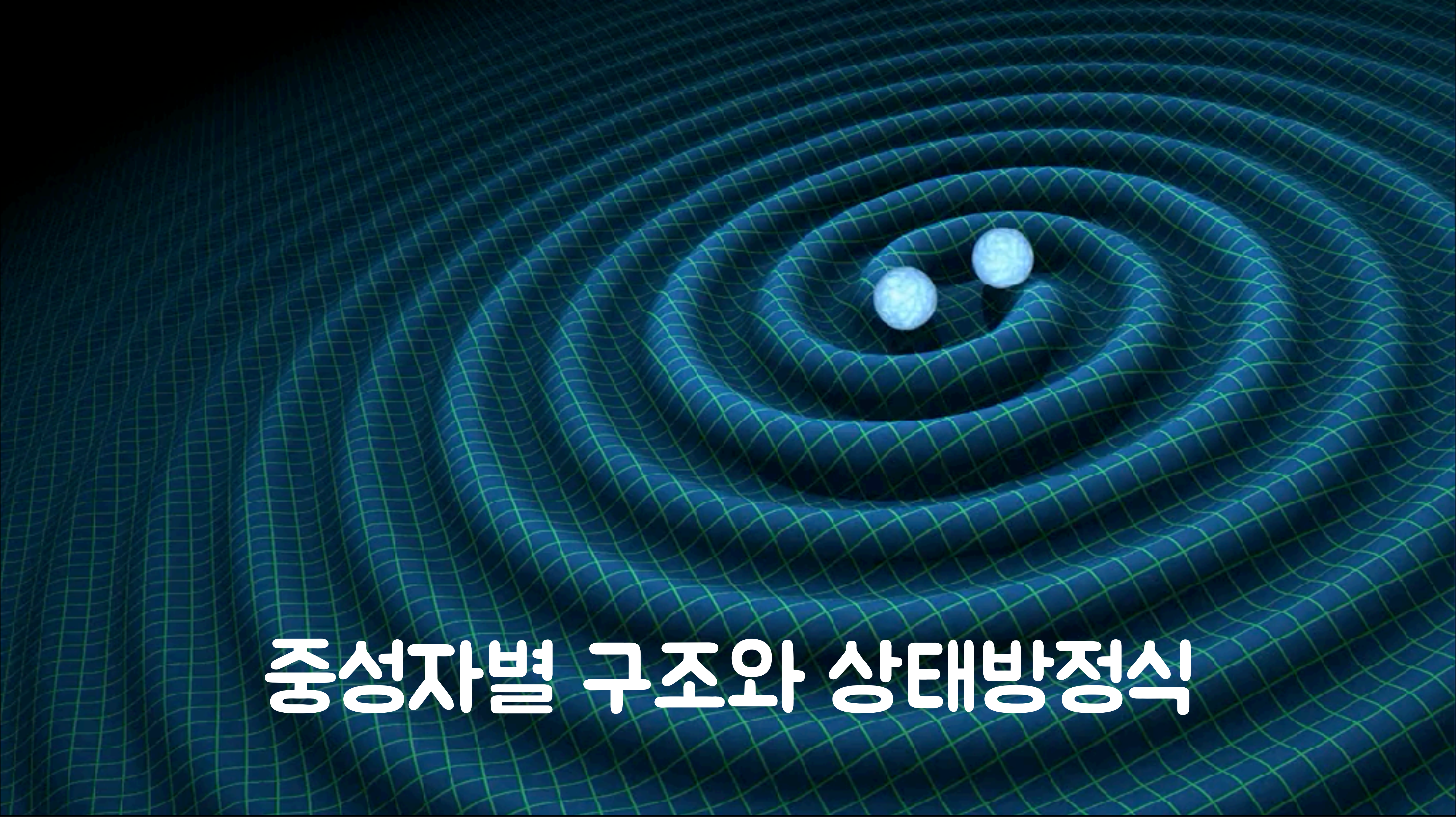
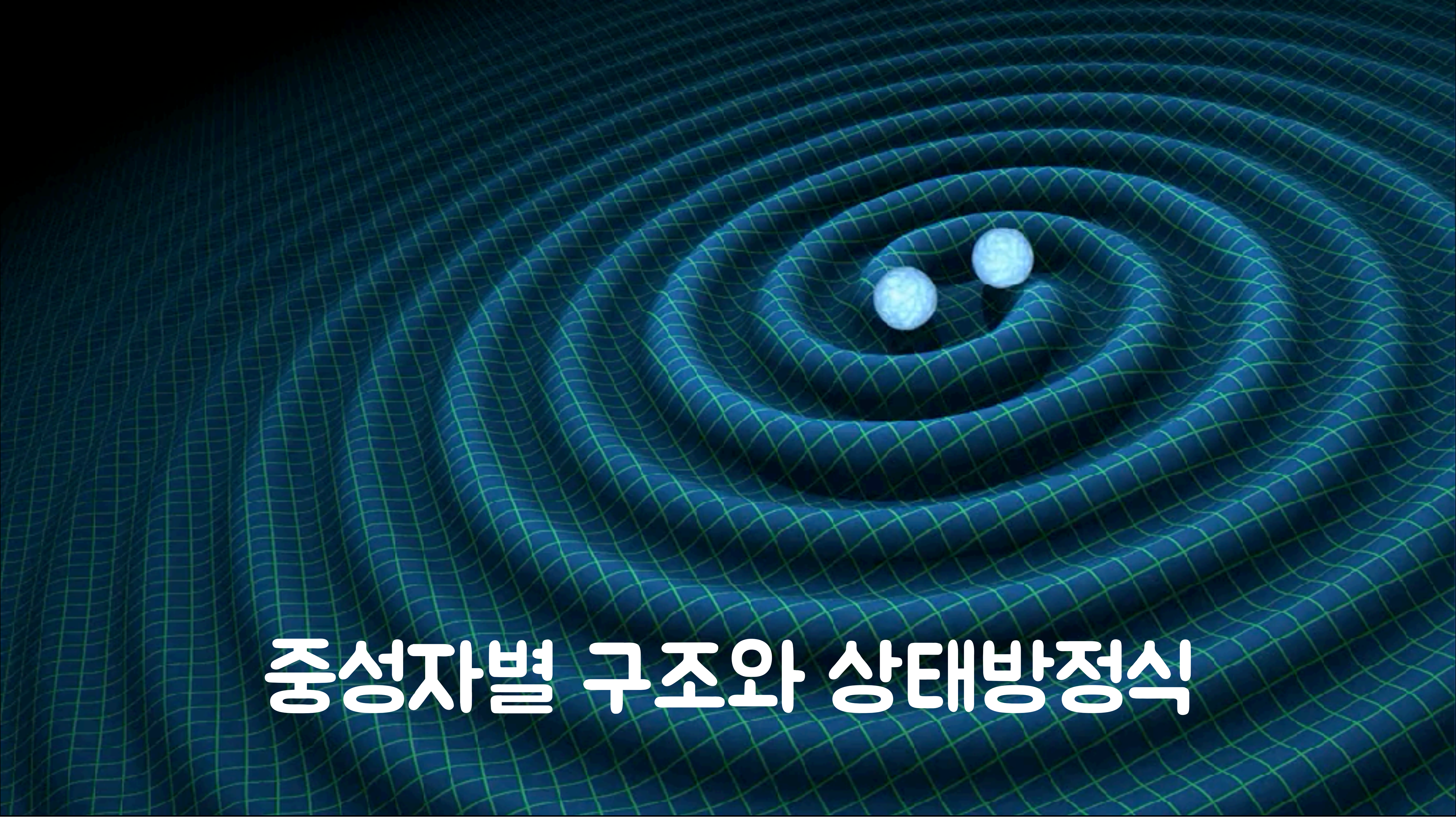


중성자별 구조와 상태방정식

김영민
한국천문연구원

연락처: ymkim@kasi.re.kr



중성자별 구조와 상태방정식

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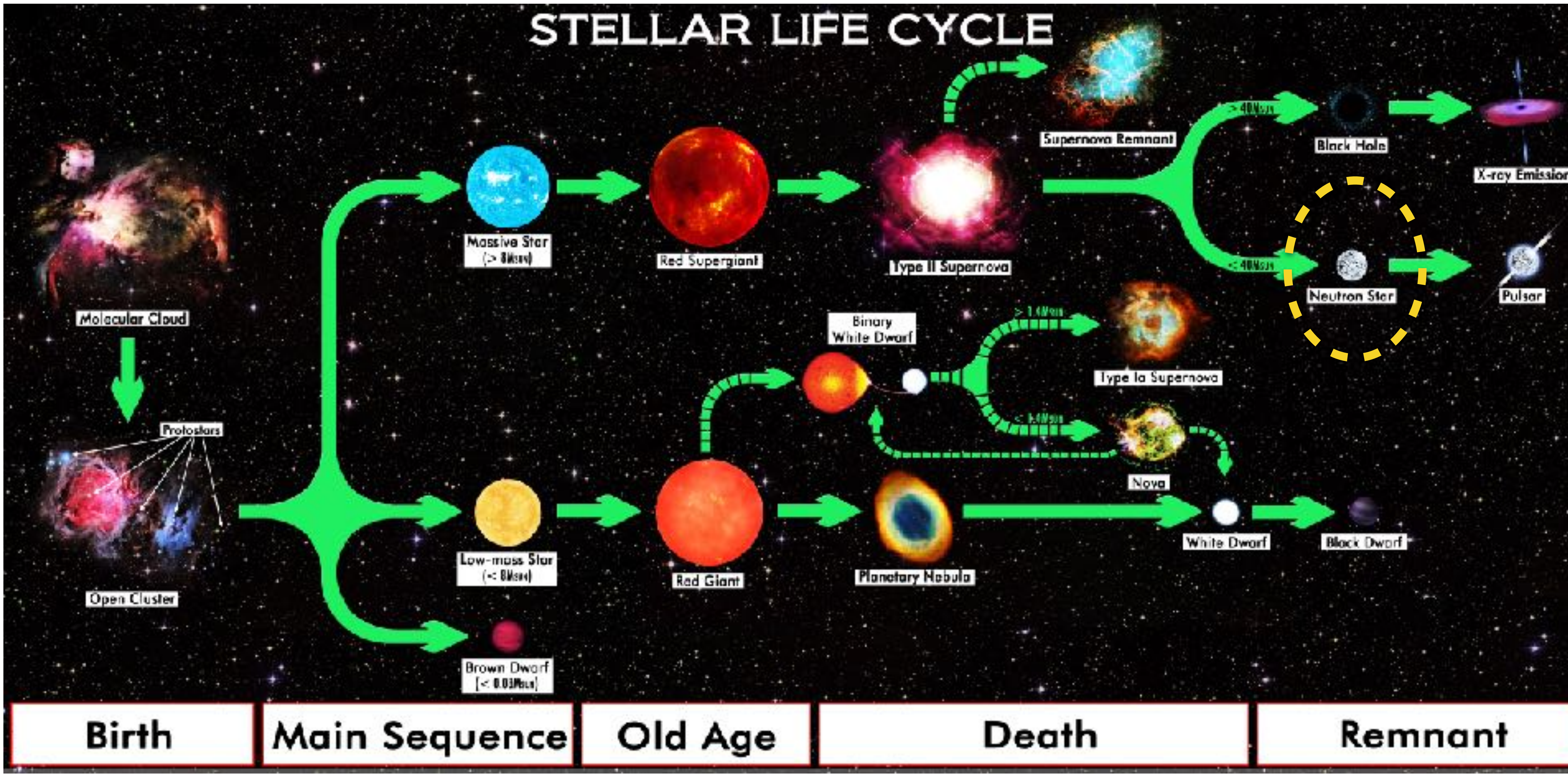
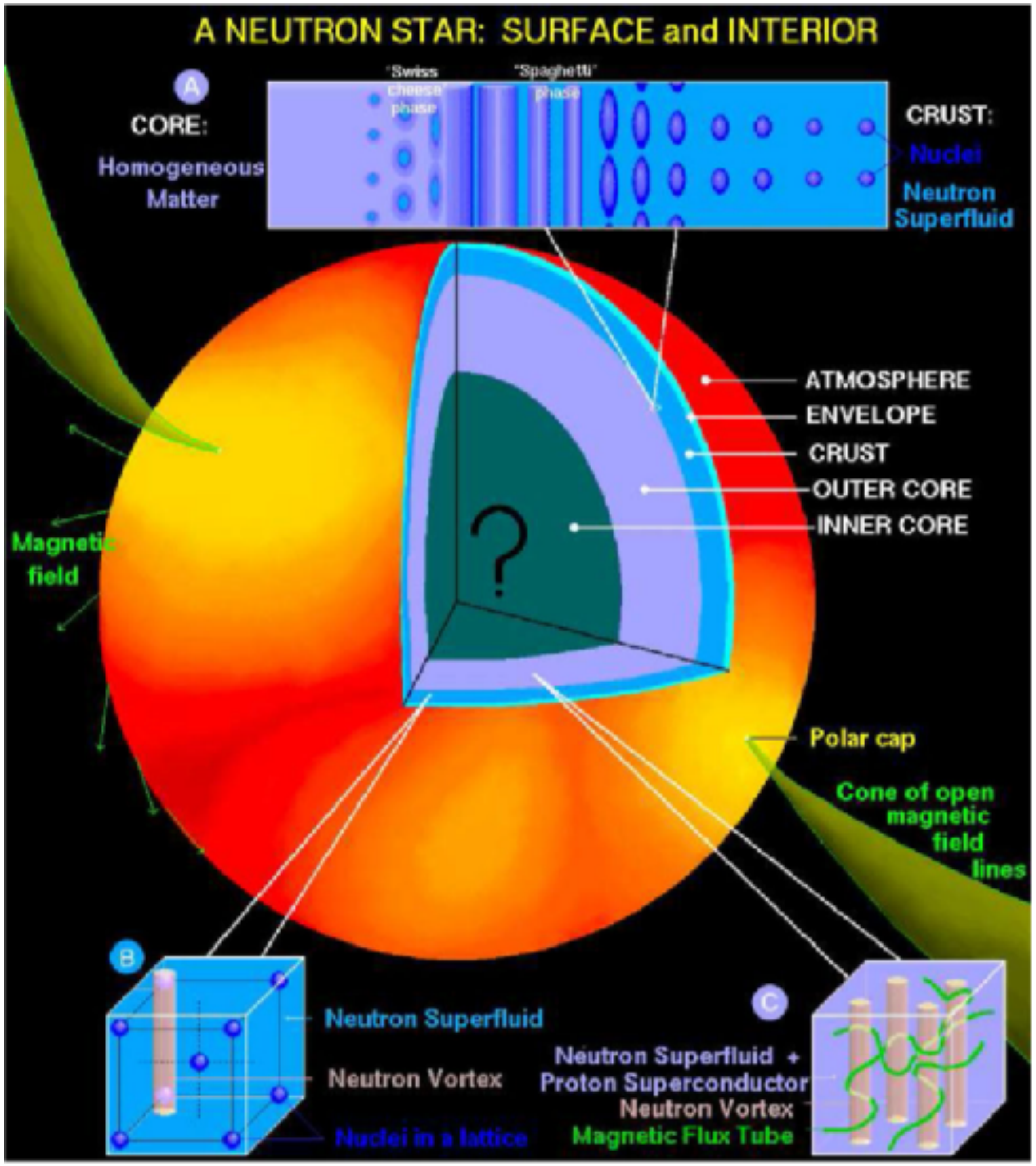
중성자별 (Neutron Star) 이란?

위키피디아/항성진화



Neutron Star



courtesy of D. Page

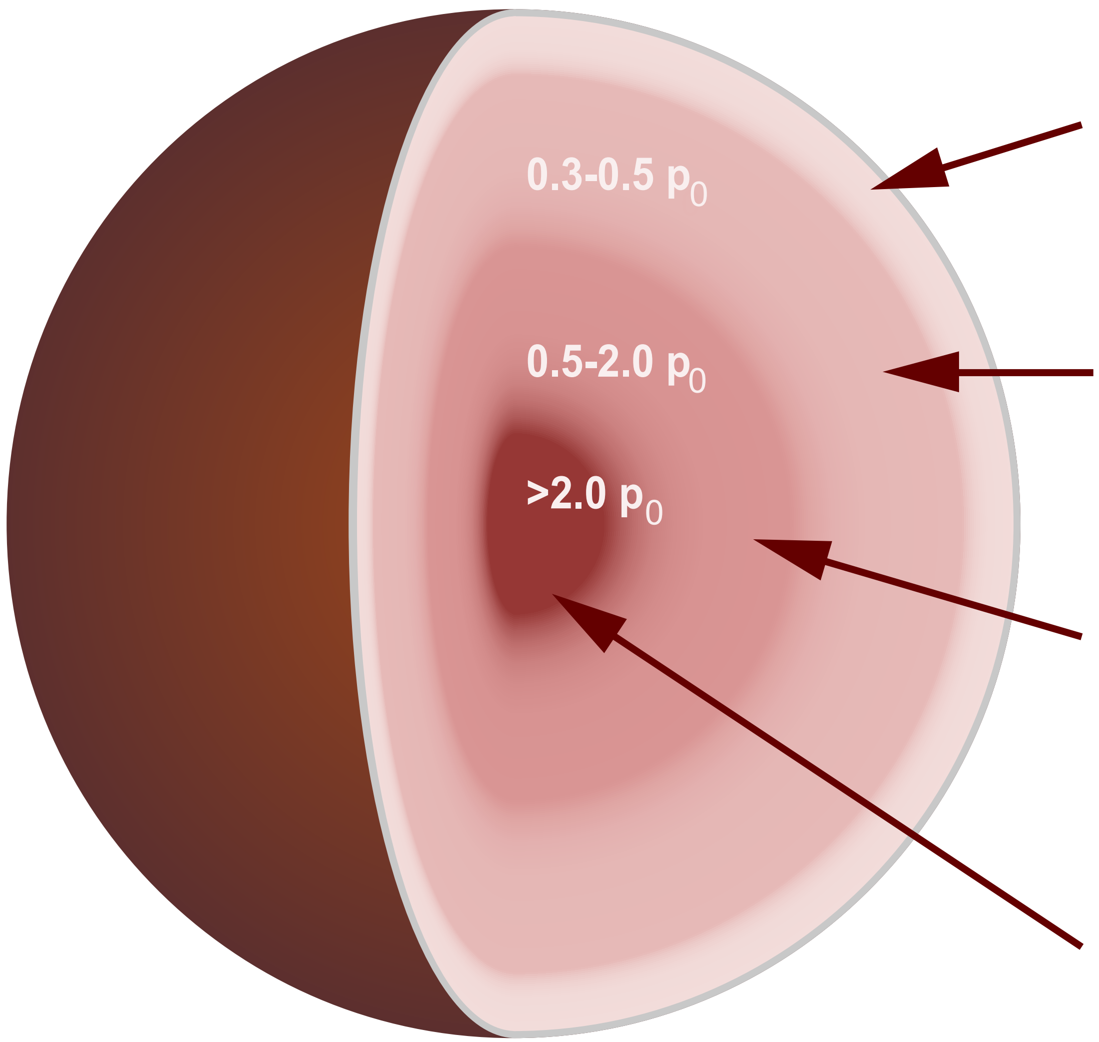
M= 1.4~2.0M$_\odot$
R=10~15km
A~ 10^{57} nucleons

$$\rho_{\text{center}} \approx \text{several} \times \rho_0$$

$$n_0 \approx 0.16\ \text{fm}^{-3} \approx 1.6 \times 10^{44}\text{m}^{-3}$$

$$\rho_0 \approx 2.04 \times 10^{17}\text{kg} \cdot \text{m}^{-3}$$

중성자별 (Neutron Star)



Wikipidea/Neutron_star

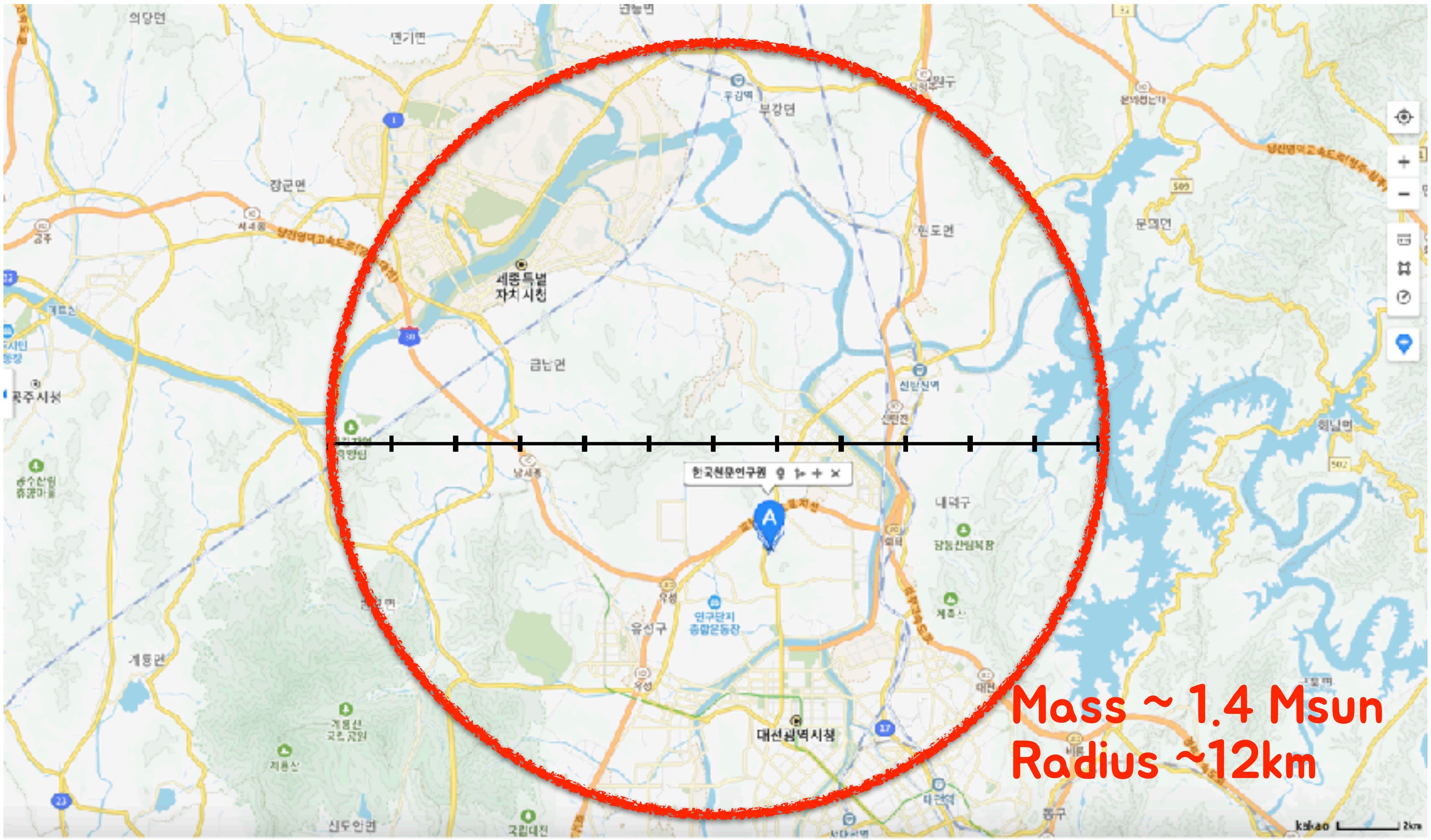
outer crust 0.3-0.5 km
ions, electrons

inner crust 1-2 km
electrons, neutrons, nuclei

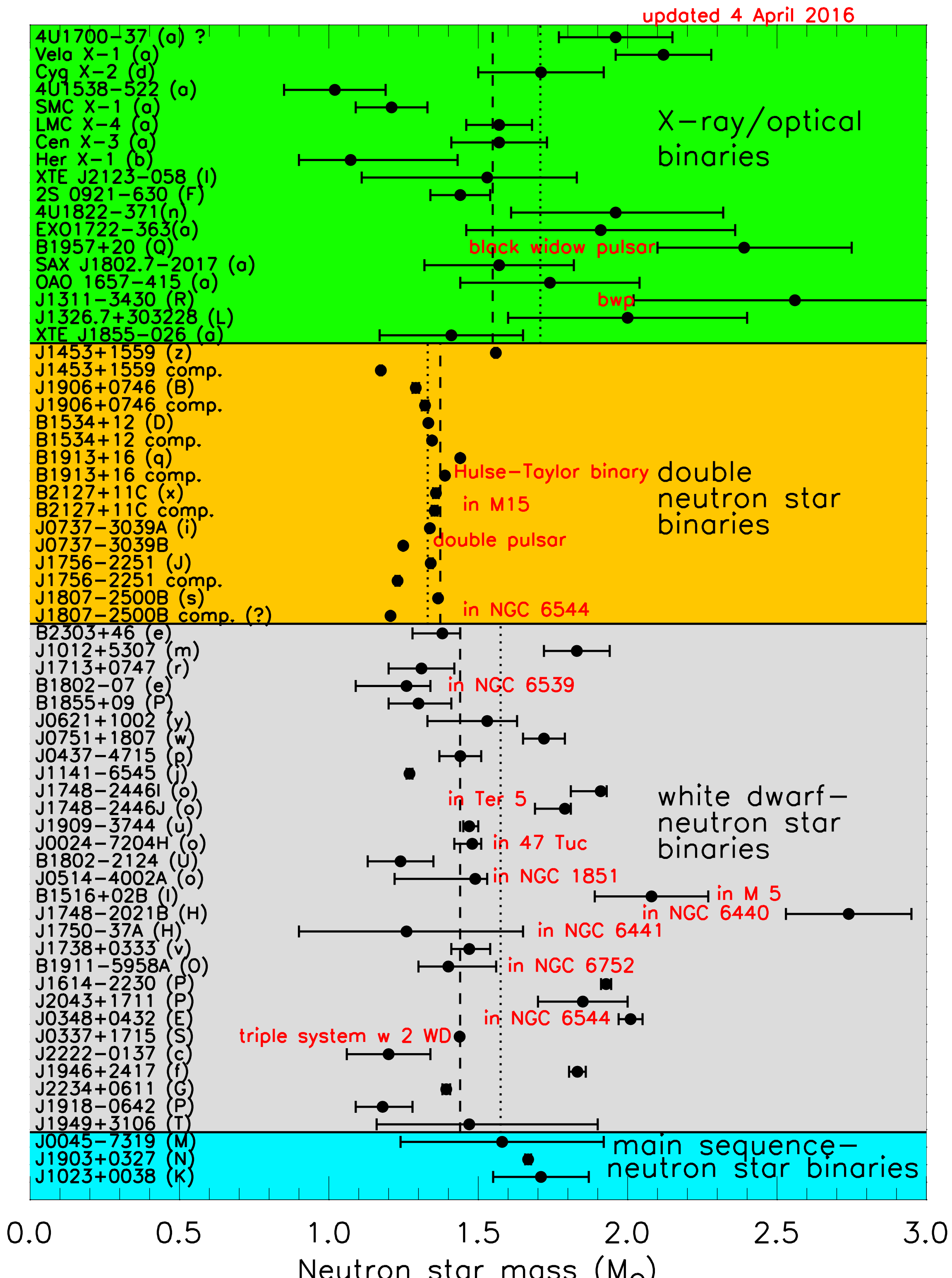
outer core ~ 9 km
neutron-proton Fermi liquid
few % electron Fermi gas

inner core 0-3 km
quark gluon plasma?

중성자별 (Neutron Star)

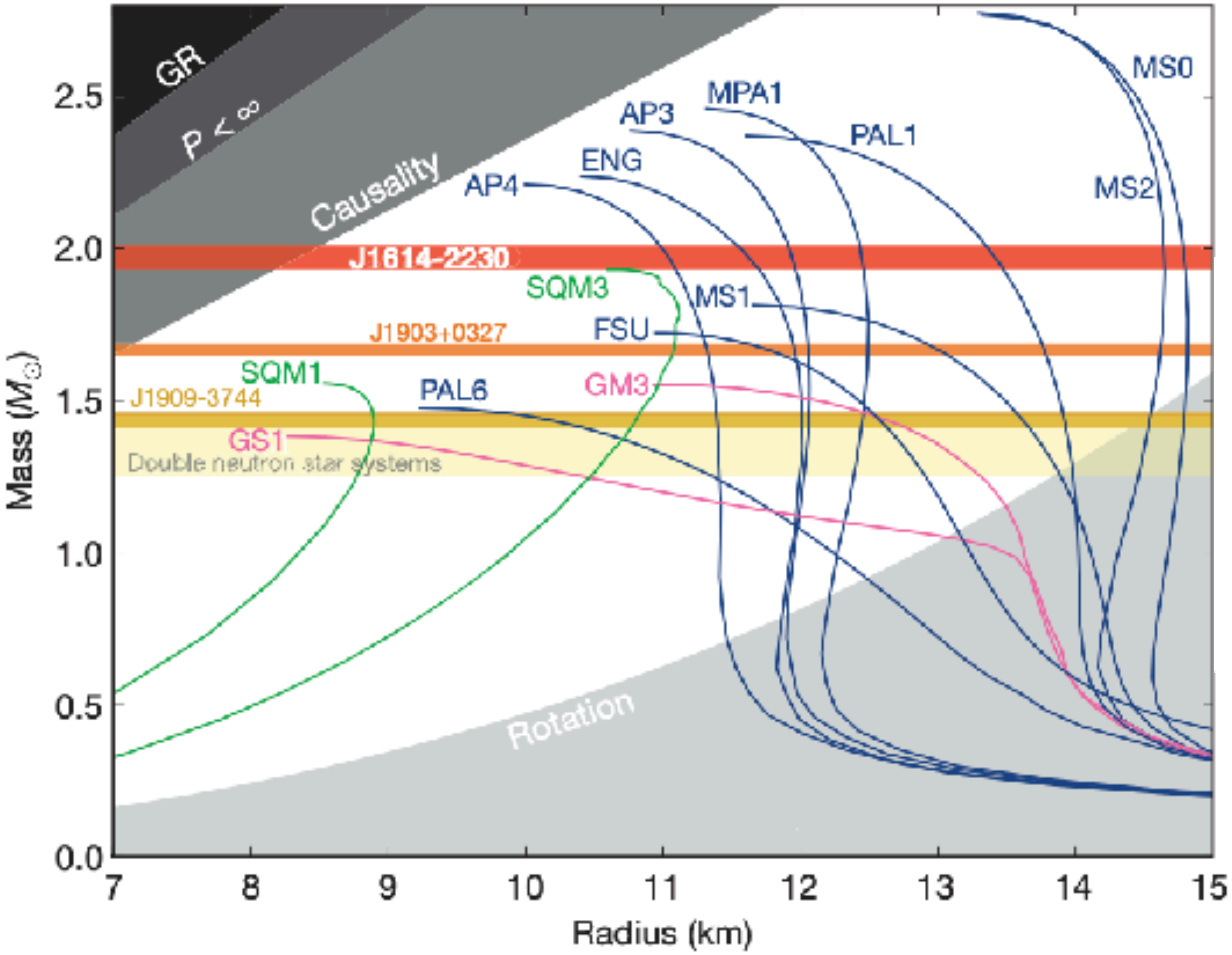


Neutron Star of Known Mass

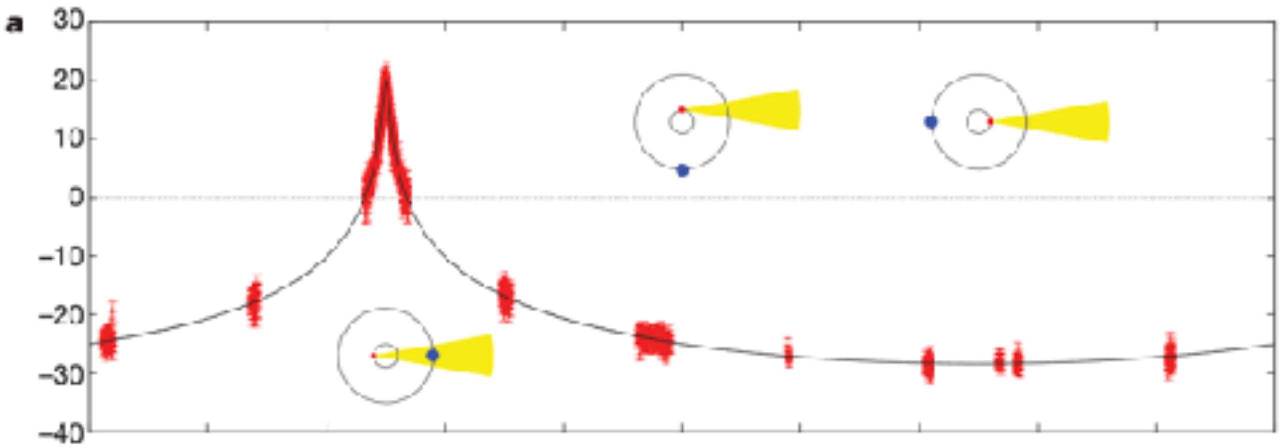


J. Lattimer, Annu.Rev.Nucl.Part.Sci.62,485(2012) and https://stellarcollapse.org by C. Ott

Maximum mass of Neutron Stars

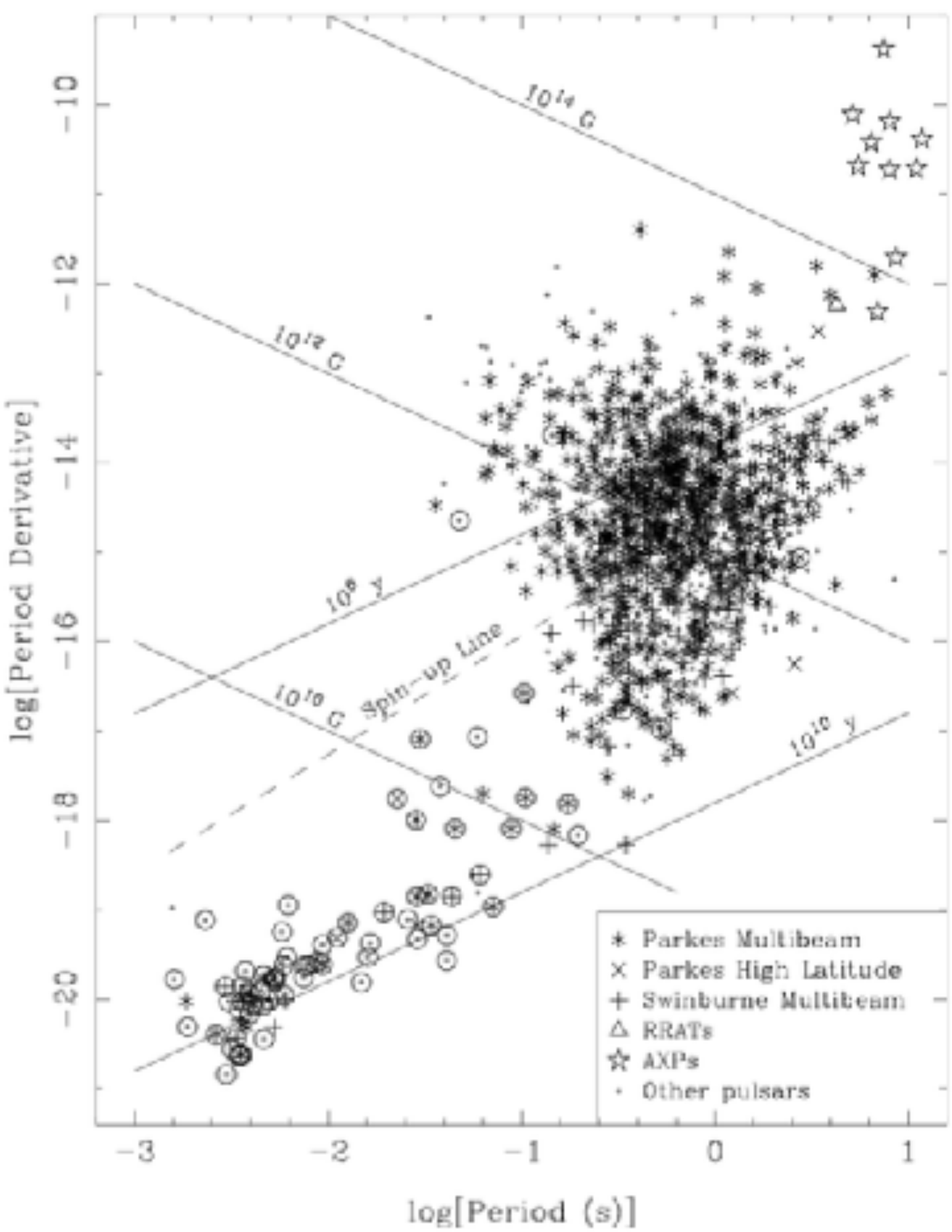


Shapiro delay



Demorest et al. Nature (2010)

Millisecond Pulsars / glitches



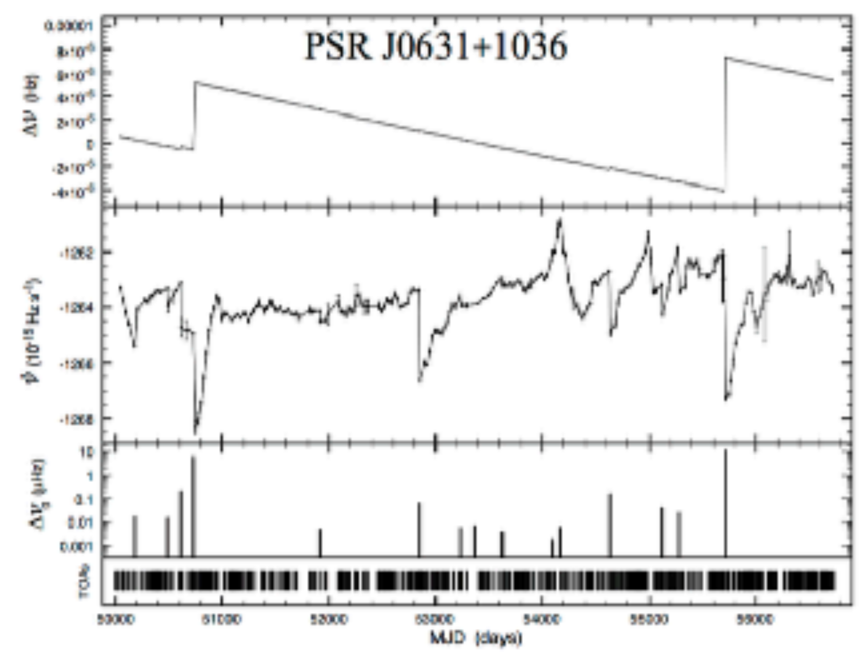
$$\dot{E}_{\rm rot} = I\Omega\dot{\Omega}$$

$$\dot{E}_{\rm dipole} = -\frac{B_\perp^2 R^6 \Omega^4}{6c^3}$$

$$\dot{\Omega} = -\frac{B_\perp^2 R^6 \Omega^3}{6Ic^3}$$

$$\dot{\Omega} \propto -\Omega^n$$

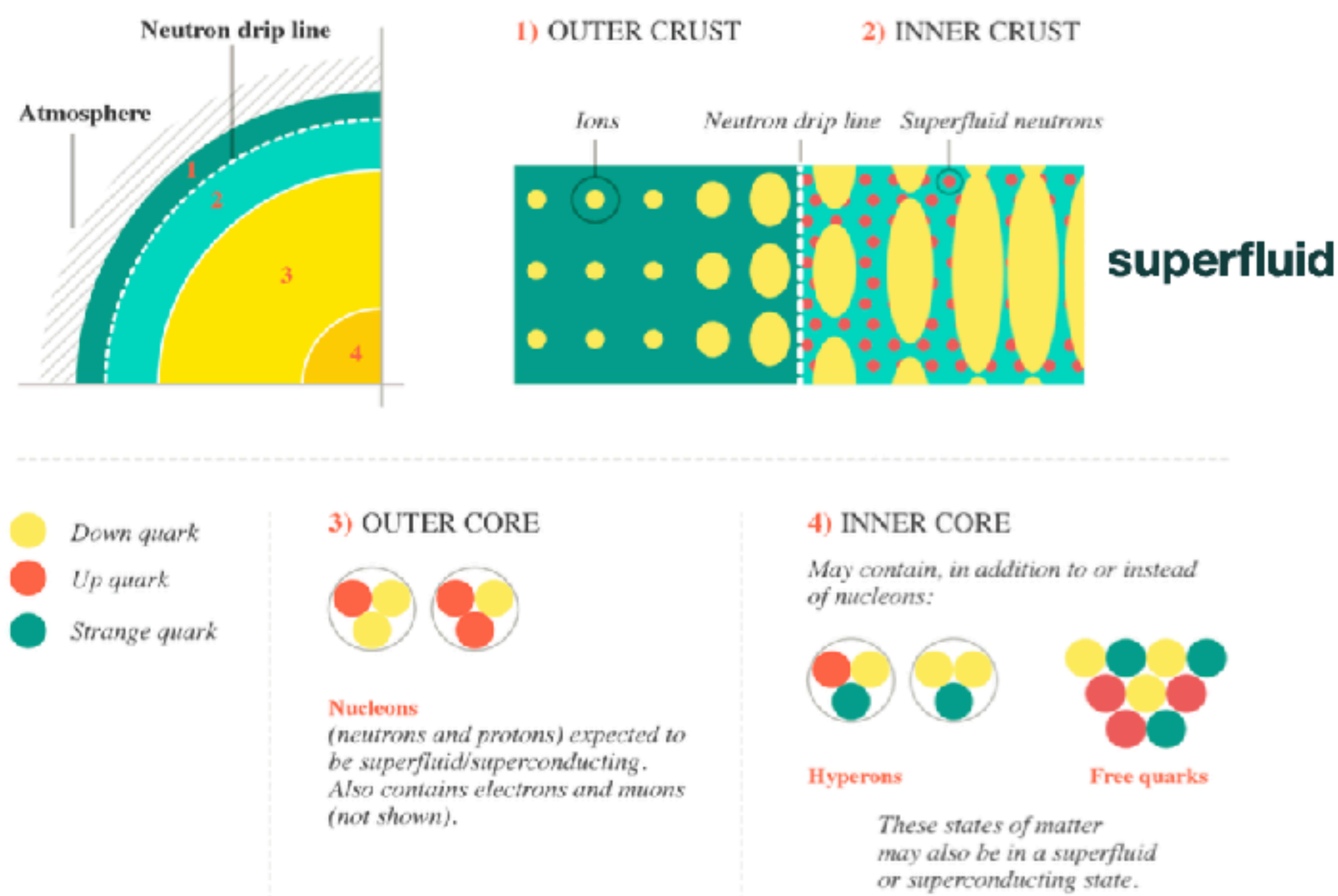
$$\tau_{\rm pulsar} = \frac{\Omega}{(1-n)\dot{\Omega}}$$



D. Antonopoulou (U. Amsterdam, 2015)

Superfluidity of neutrons

D. Antonopoulou (2015)



Low-Mass X-ray Binary

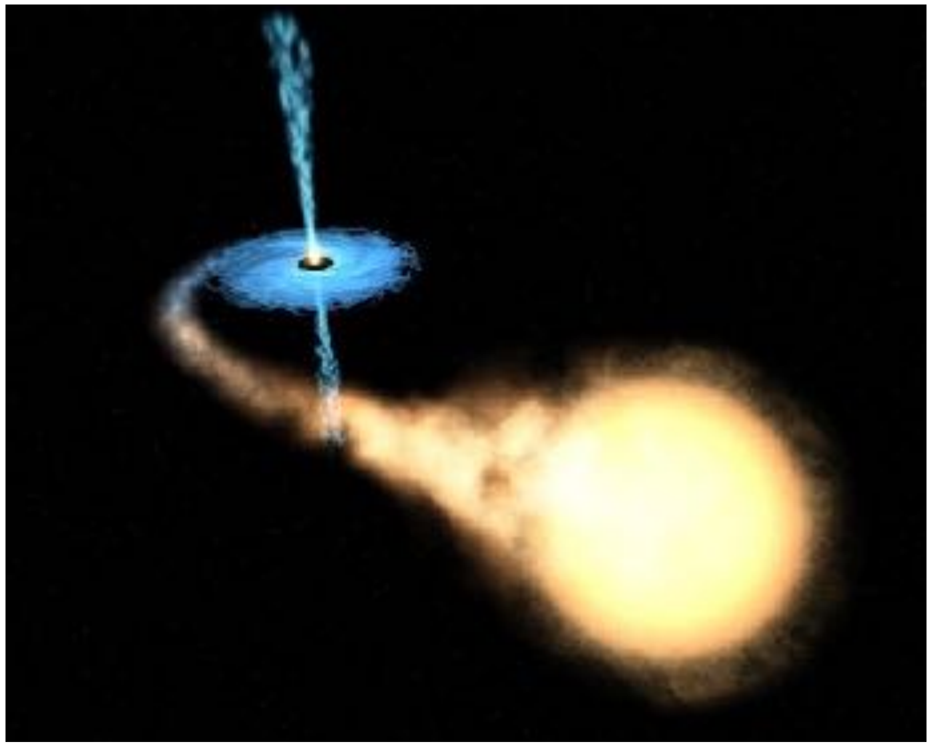
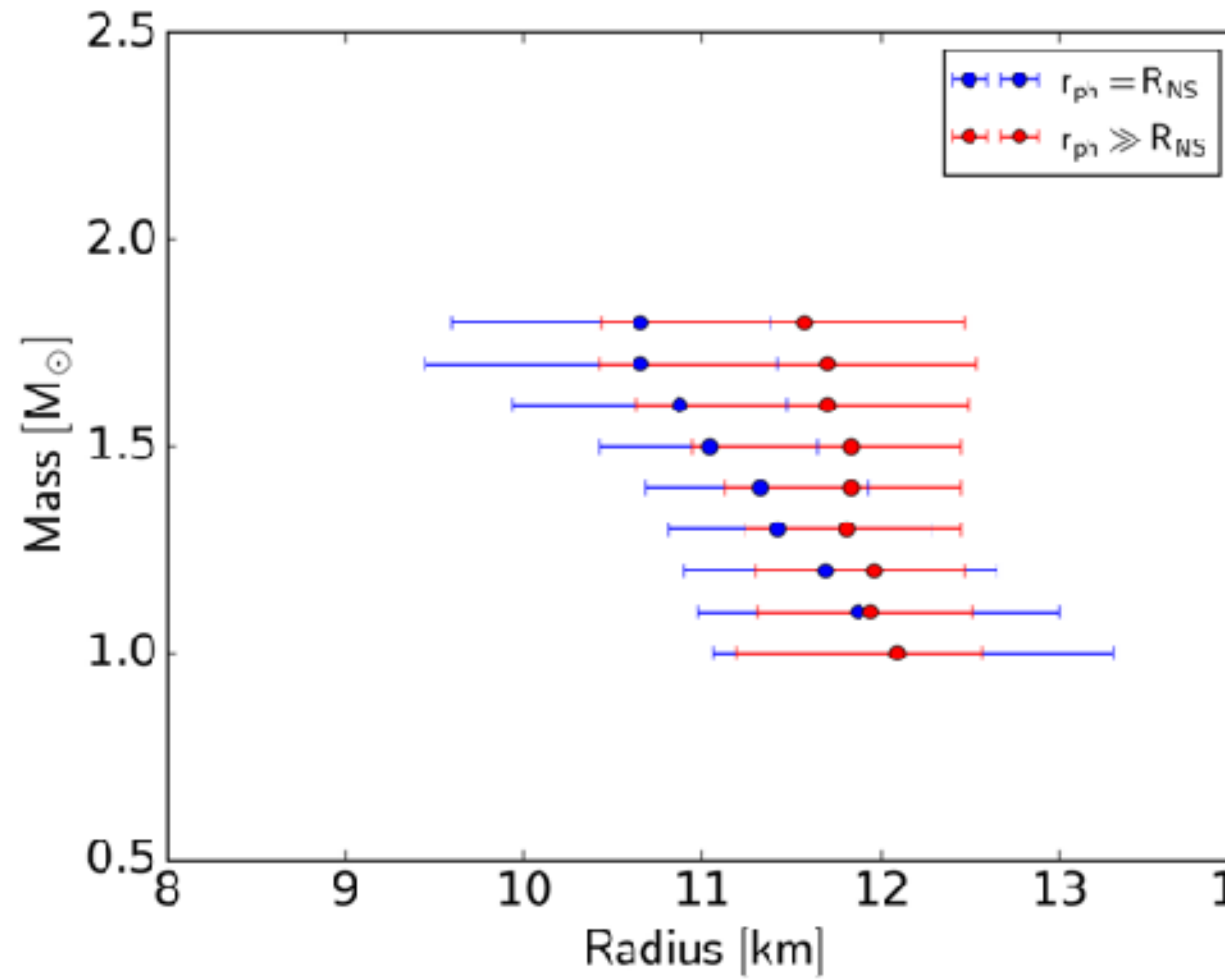


Table 9
Most Probable Values for Masses and Radii for Neutron Stars Constrained to Lie on One Mass Versus Radius Curve

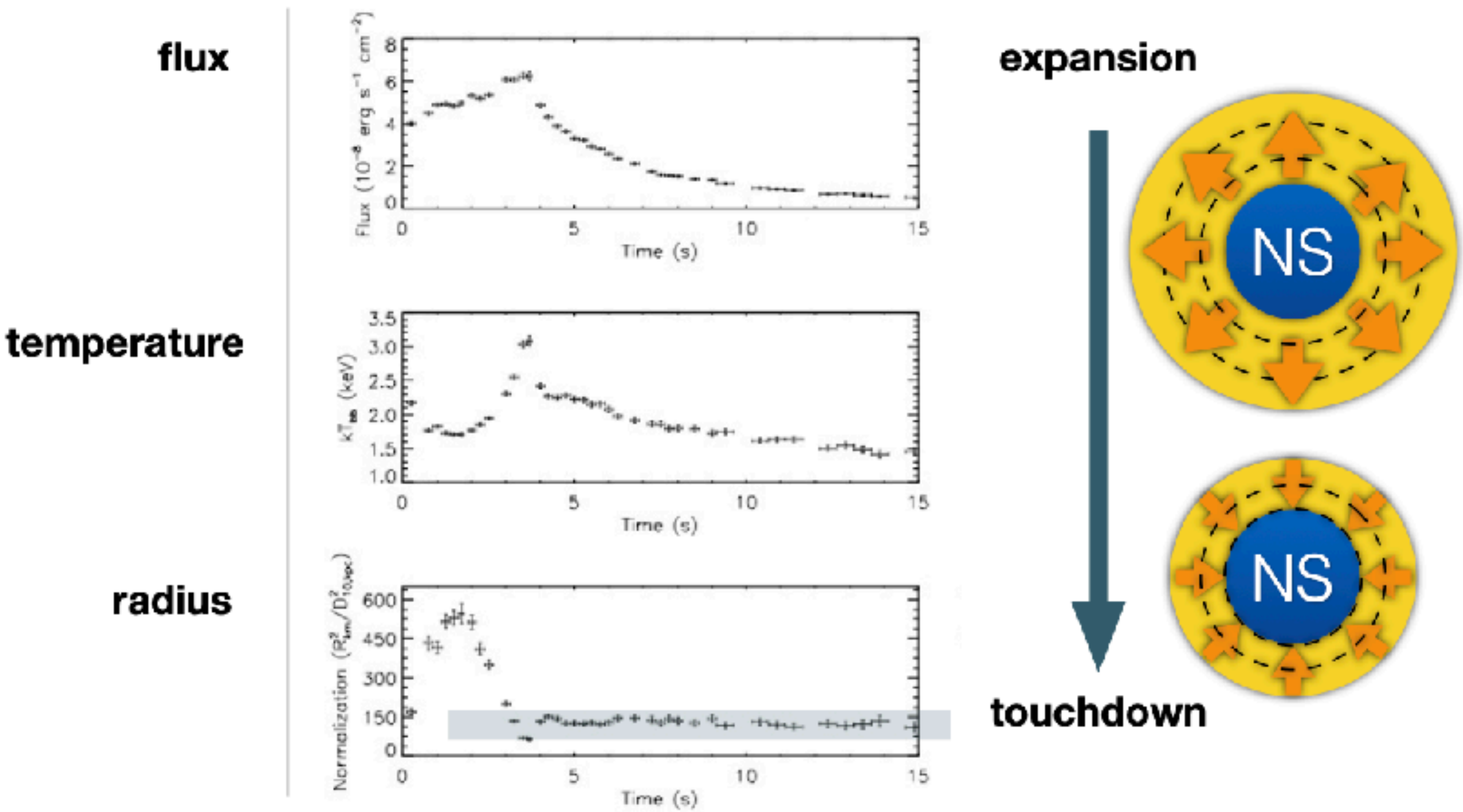
Object	M ($M_\odot$)	R (km)	M ($M_\odot$)	R (km)
	$r_{ph} = R$		$r_{ph} \gg R$	
4U 1608–522	$1.52^{+0.22}_{-0.18}$	$11.04^{+0.53}_{-1.50}$	$1.64^{+0.34}_{-0.41}$	$11.82^{+0.42}_{-0.89}$
EXO 1745–248	$1.55^{+0.12}_{-0.36}$	$10.91^{+0.86}_{-0.65}$	$1.34^{+0.450}_{-0.28}$	$11.82^{+0.47}_{-0.72}$
4U 1820–30	$1.57^{+0.13}_{-0.15}$	$10.91^{+0.39}_{-0.92}$	$1.57^{+0.37}_{-0.31}$	$11.82^{+0.42}_{-0.82}$
M13	$1.48^{+0.21}_{-0.64}$	$11.04^{+1.00}_{-1.28}$	$0.901^{+0.28}_{-0.12}$	$12.21^{+0.18}_{-0.62}$
ω Cen	$1.43^{+0.26}_{-0.61}$	$11.18^{+1.14}_{-1.27}$	$0.994^{+0.51}_{-0.21}$	$12.09^{+0.27}_{-0.66}$
X7	$0.832^{+1.19}_{-0.051}$	$13.25^{+1.37}_{-3.50}$	$1.98^{+0.10}_{-0.36}$	$11.3^{+0.95}_{-1.03}$

95% confidence limits by using MC sampling



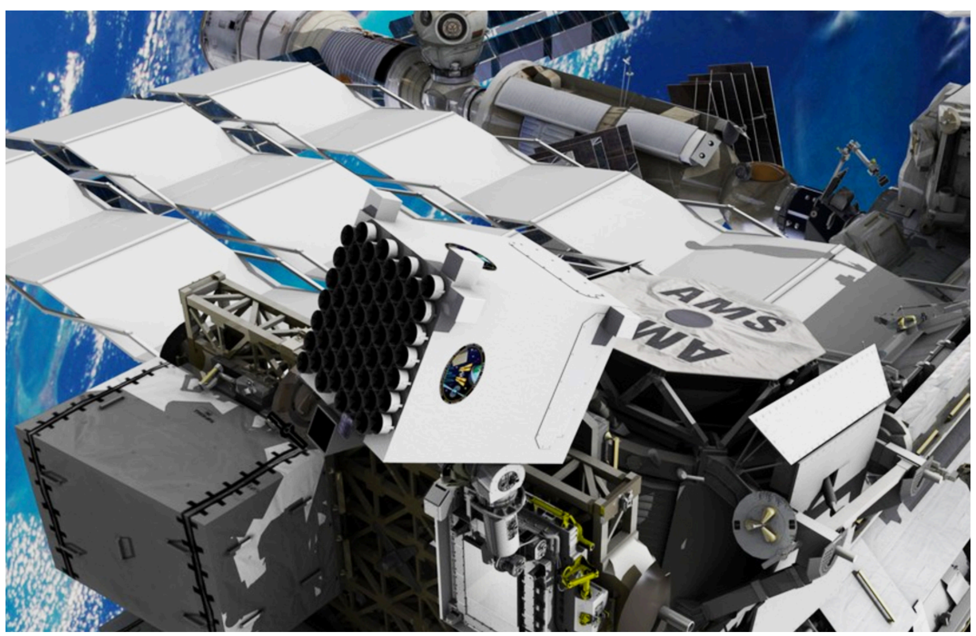
Steiner, Lattimer, Brown, ApJ 2010

Mass and Radius from LMXBs



Ozel et al. 2009

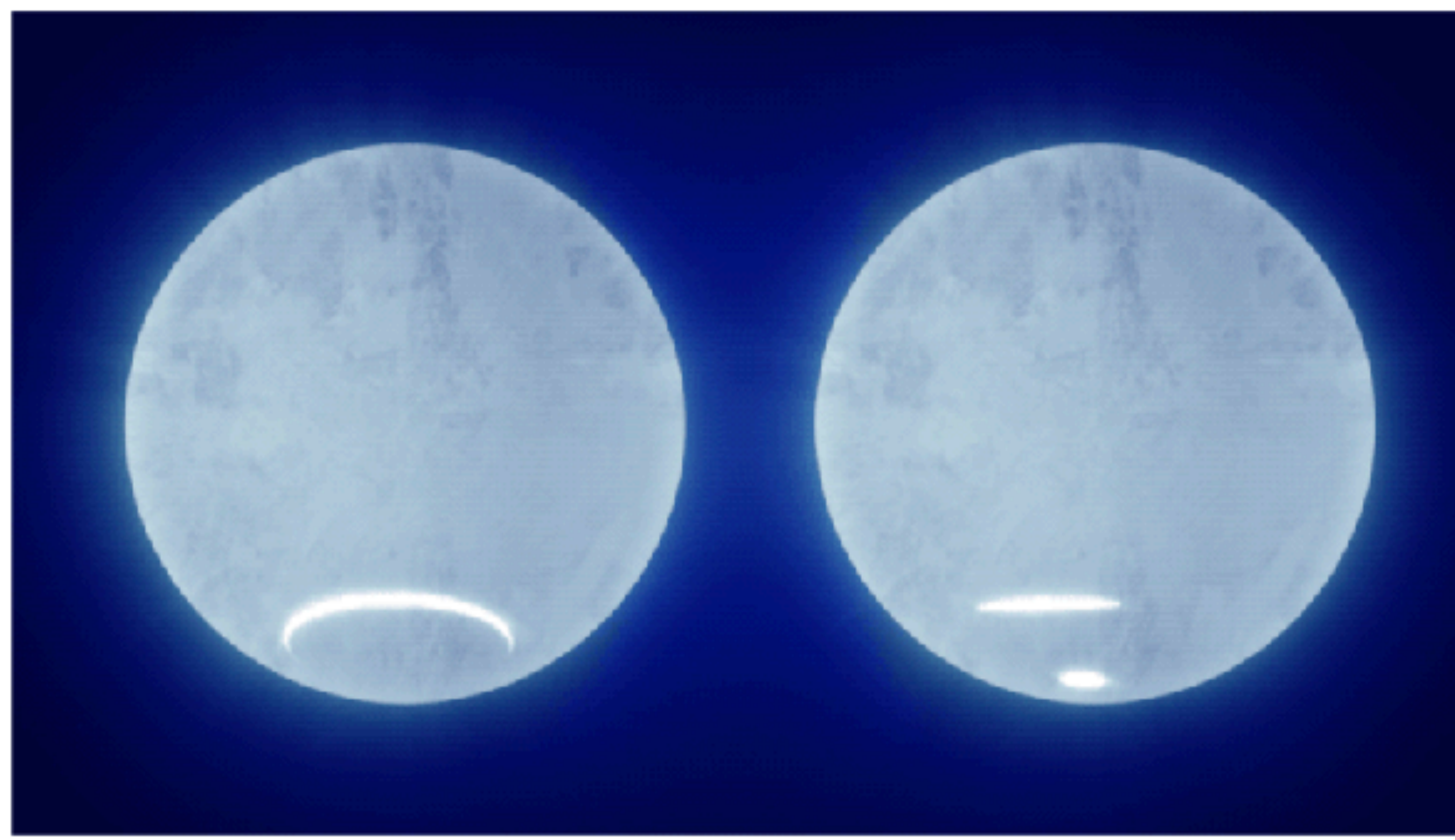
Observation by NICER



Focus on *NICER* Constraints on the Dense Matter Equation of State

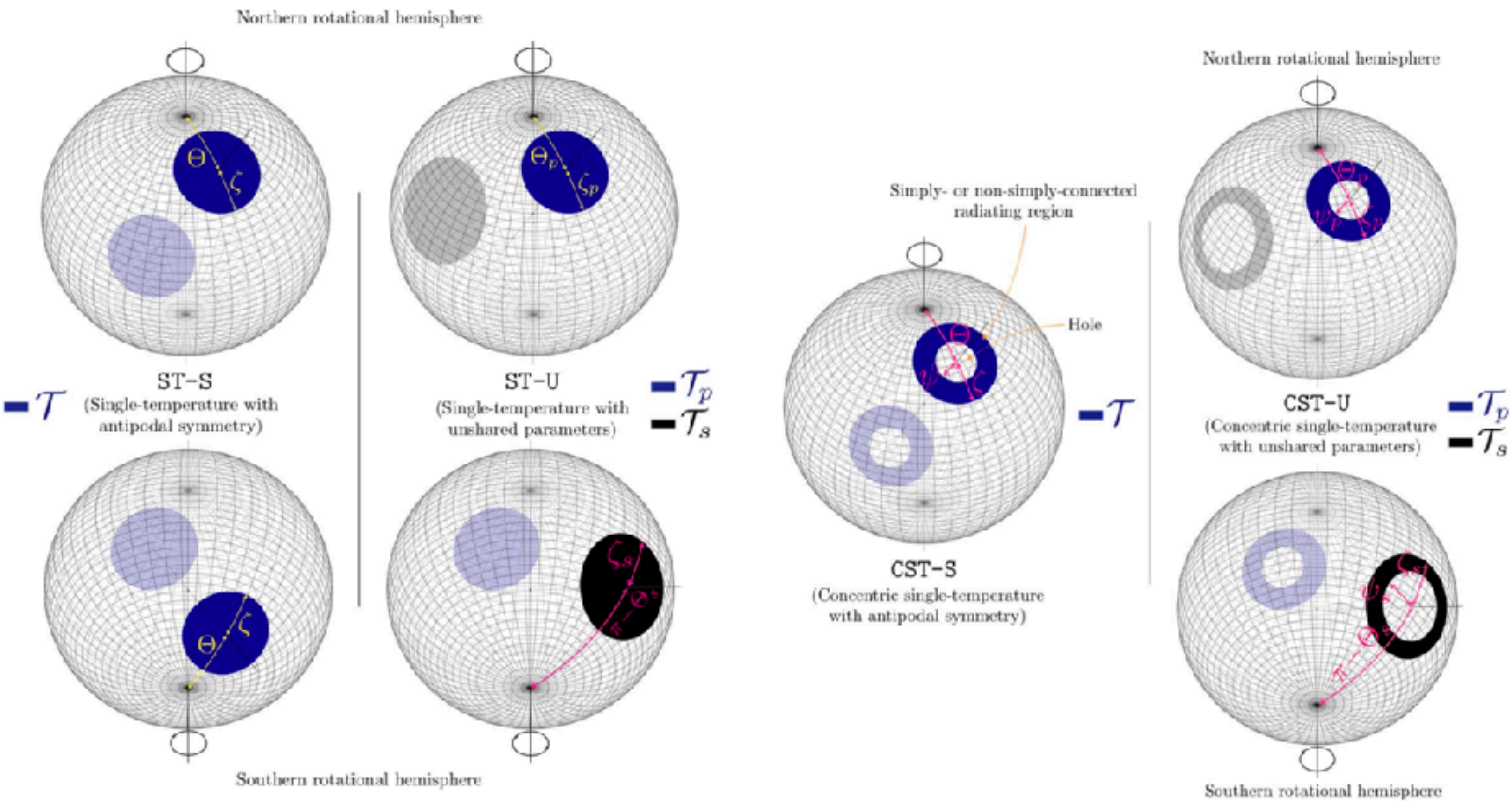
Zaven Arzoumanian & Keith C. Gendreau (NASA Goddard Space Flight Center)

December 2019



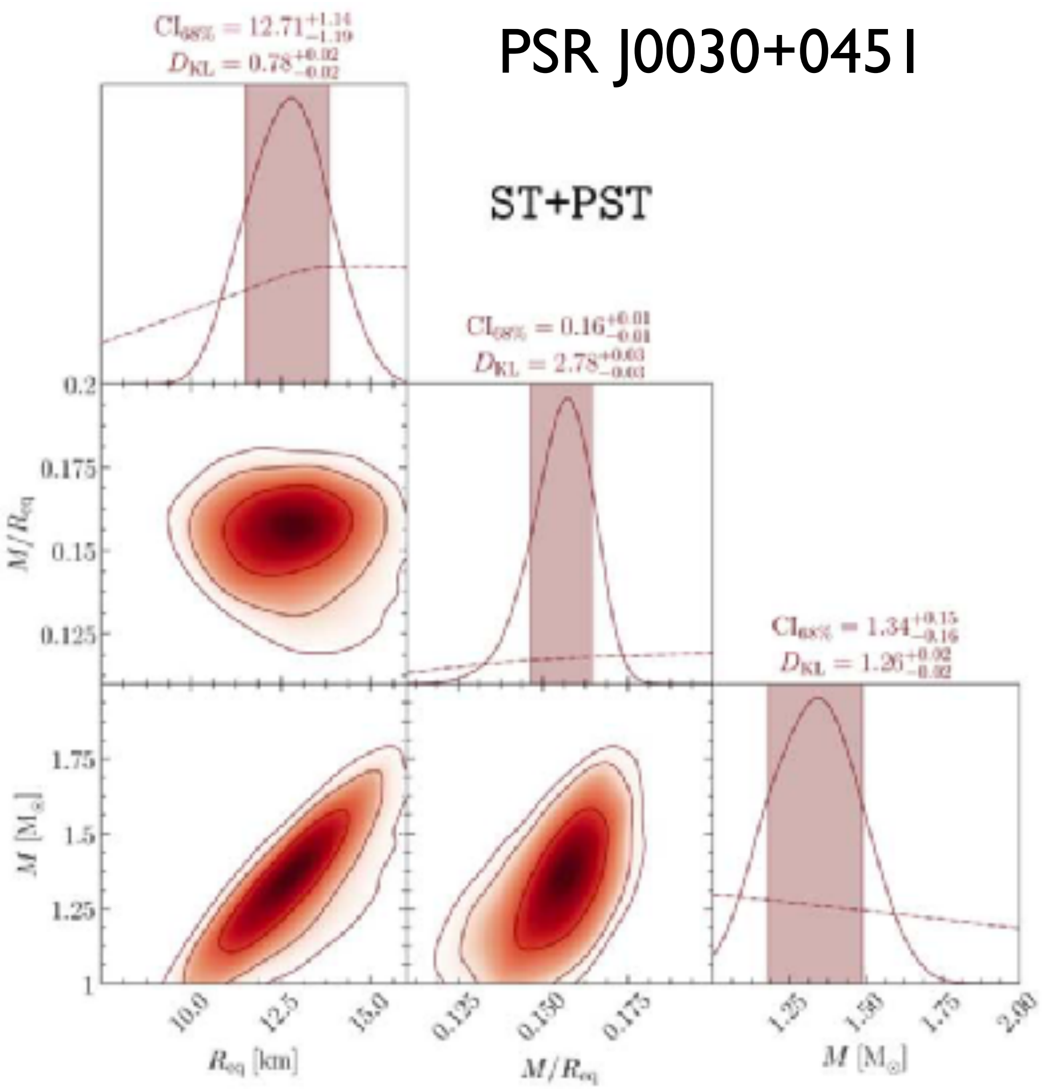
Hot spot region model in Riley 2019

Riley_2019_ApJL_887_L21



NICER observations

PSR J0030+0451

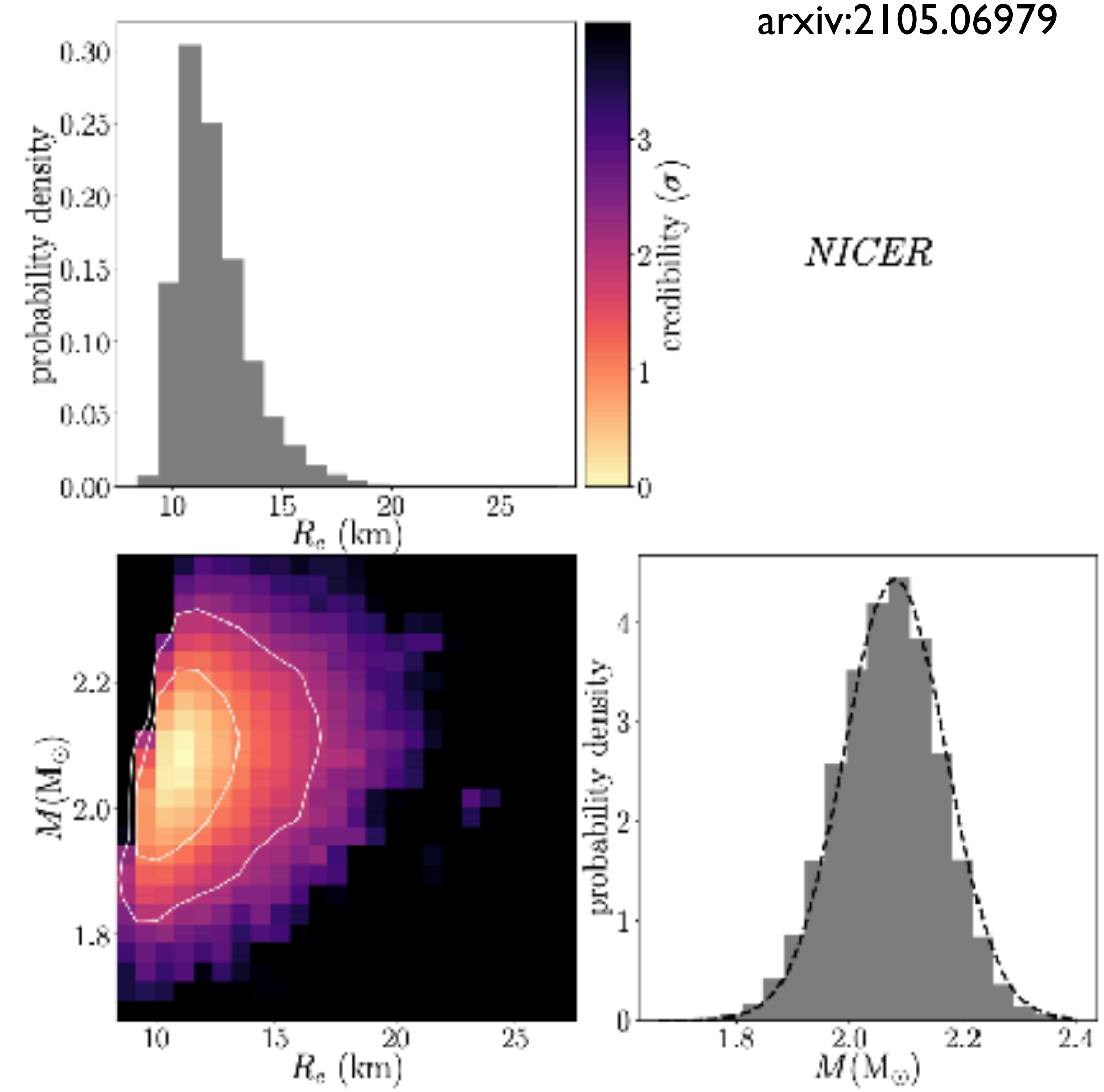


Riley_2019_ApJL_887_L21

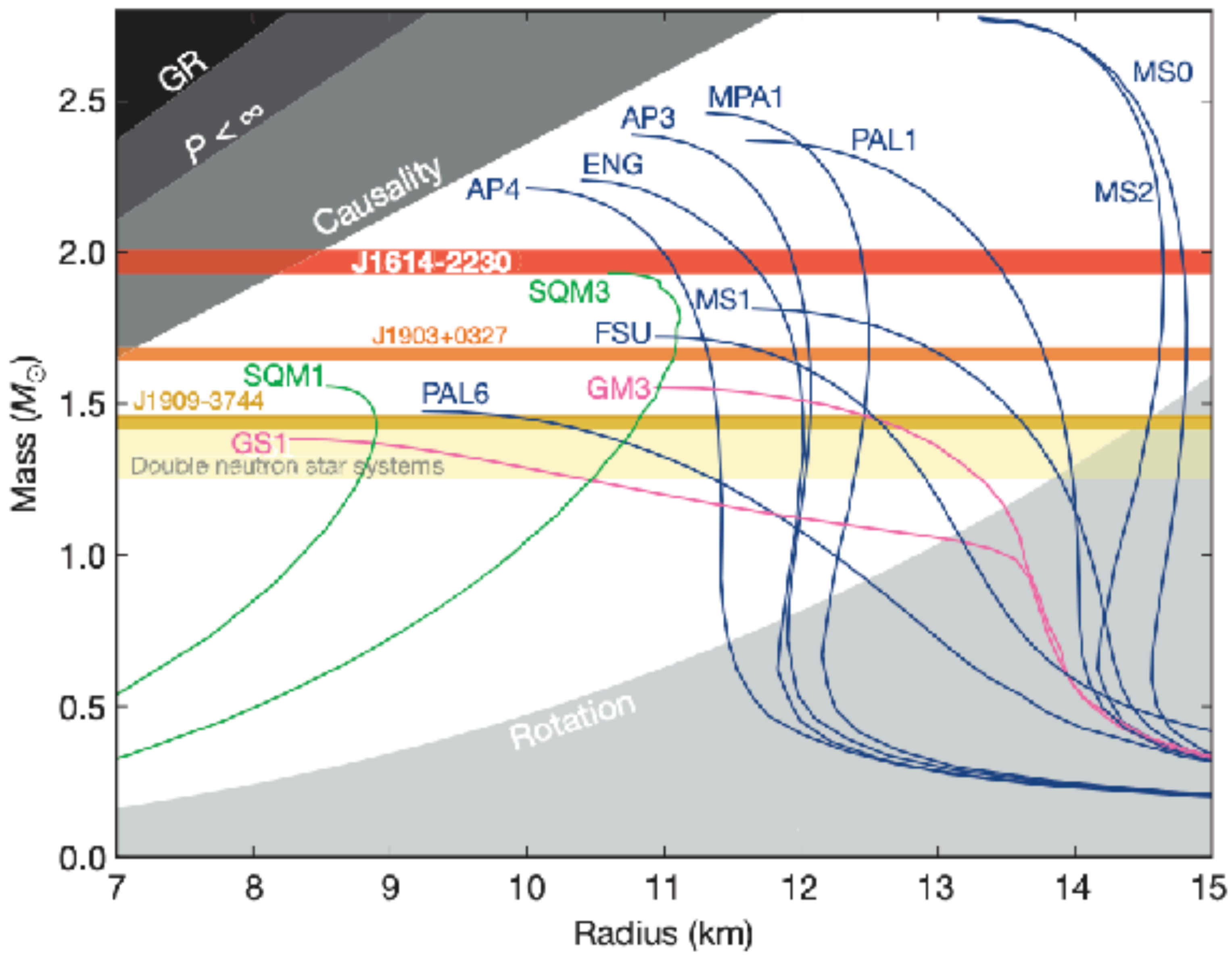
	Mass (M_{sun})	Radius (km)
Riley *et al.*[25]	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$
Miller *et al.*[26]	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$

PSR J0740+6620

arxiv:2105.06979



Mass and Radius of Neutron Stars



Demorest et al. Nature (2010)

TOV eq.

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi P r^3}{Mc^2}\right)\left(1-\frac{2GM}{rc^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2\left(\frac{\epsilon}{c^2}\right)$$

Deriving TOV eq.

Derving TOV eq. (1)

MTW ch. 23

$$ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + r^2 d\Omega^2$$

$\rho(r)$, $p(r)$, $n(r)$

$u^\mu(r)$: 4-velocity of fluid

$G = c = 1$

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p g^{\mu\nu}$$

Derving TOV eq. (2)

$$\nabla_\mu T^{\mu\nu} = 0$$

Energy-Momentum conservation in Hydrostatic equilibrium

$\Downarrow$

$$\frac{dP}{dr} + (p+\rho)\frac{d}{dr}\Phi(r) = 0.$$

Derving TOV eq. (3)

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$G_{00} \rightarrow \frac{1}{r^2}\frac{d}{dr}\left[r(1-e^{-2\Lambda})\right] = 8\pi\rho$$

$$2m \equiv r(1-e^{-2\Lambda}), \quad e^{2\Lambda} = (1-2m/r)^{-1}$$

Derving TOV eq. (4)

$$G_{00} \rightarrow \frac{2}{r^2}\frac{d\,m(r)}{dr} = 8\pi\rho$$

Integration $\longrightarrow$

$$m(r) = \int_0^r 4\pi\bar{r}^2\rho\, d\bar{r} + m(0)$$

$m(0) \rightarrow 0$

Derving TOV eq. (5)

$$G_{rr} \to -\frac{1}{r^2} + \frac{1}{r^2} e^{-2\Lambda} + \frac{2}{r} e^{-2\Lambda} \frac{d\Phi}{dr}$$

$$= 8\pi p$$

$$e^{-2\Lambda} = (1 - 2m/r)$$

$$\Rightarrow \frac{d\Phi(r)}{dr} = \frac{m + 4\pi r^3 p}{r(r-2m)}$$

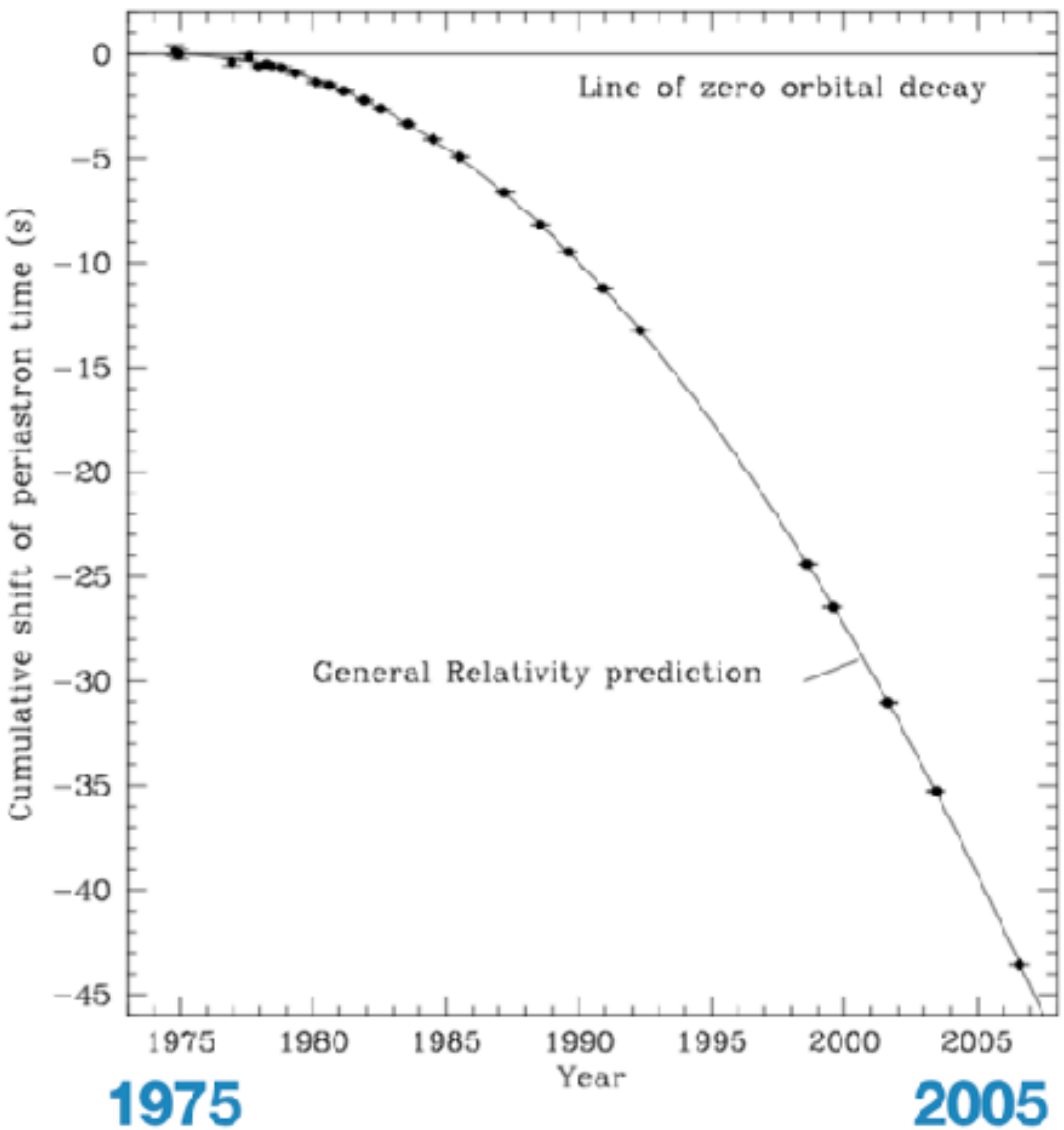
Derving TOV eq. (6)

$$\frac{d\,p(r)}{dr} = -(p+\rho)\frac{d}{dr}\Phi(r)$$

$$= -(p+\rho)\frac{m+4\pi r^3 p}{r(r-2m)}$$

$$\frac{d}{dr}m(r) = 4\pi r^2 \rho$$

NS Observation via Gravitational Waves



Weisberg, Nice, Taylor, ApJ (2010)

B1913+16
Hulse & Taylor (1975)

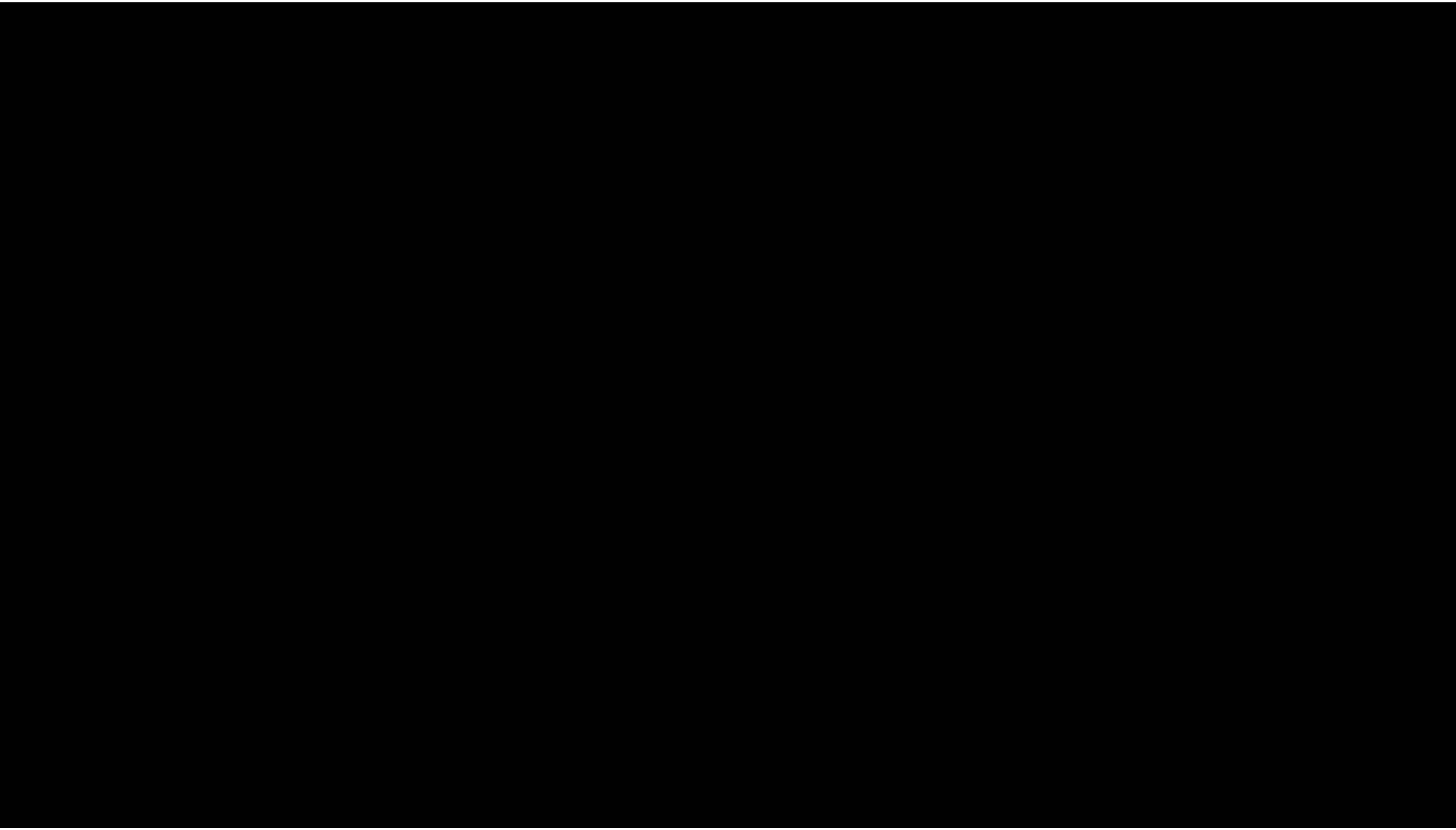
→ 1993 Nobel Prize

* change in the orbital period due to GW radiation

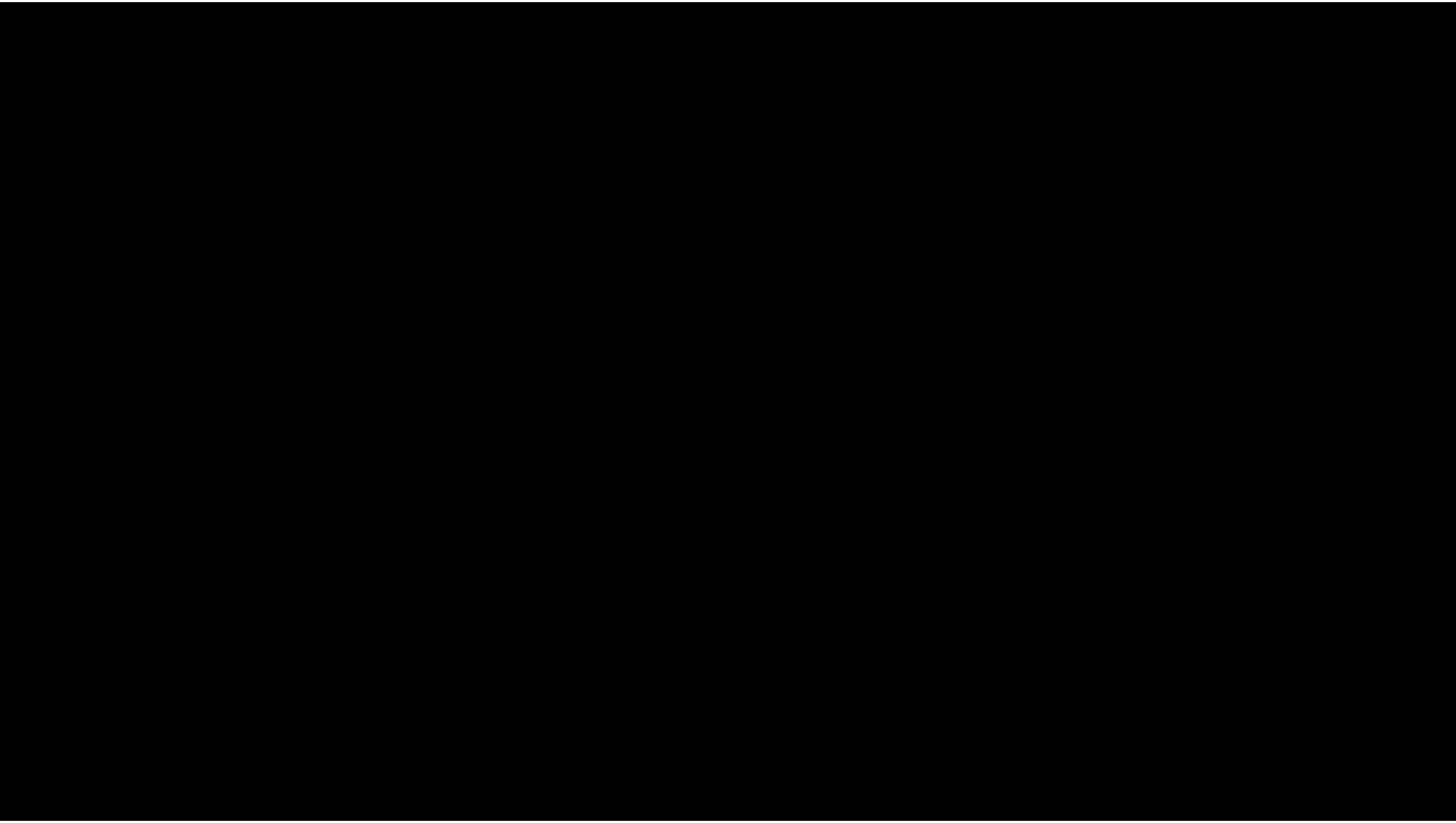
$$\dot{E}_{GW} = -\frac{32}{5}\frac{G^4 M^3 \mu^2}{c^5 a^5}[\text{GR correction}]$$

$$\dot{J}_{GW} = -\frac{32}{5}\frac{G^{7/2} M^{5/2} \mu^2}{c^5 a^{7/2}}[\text{GR correction}]$$

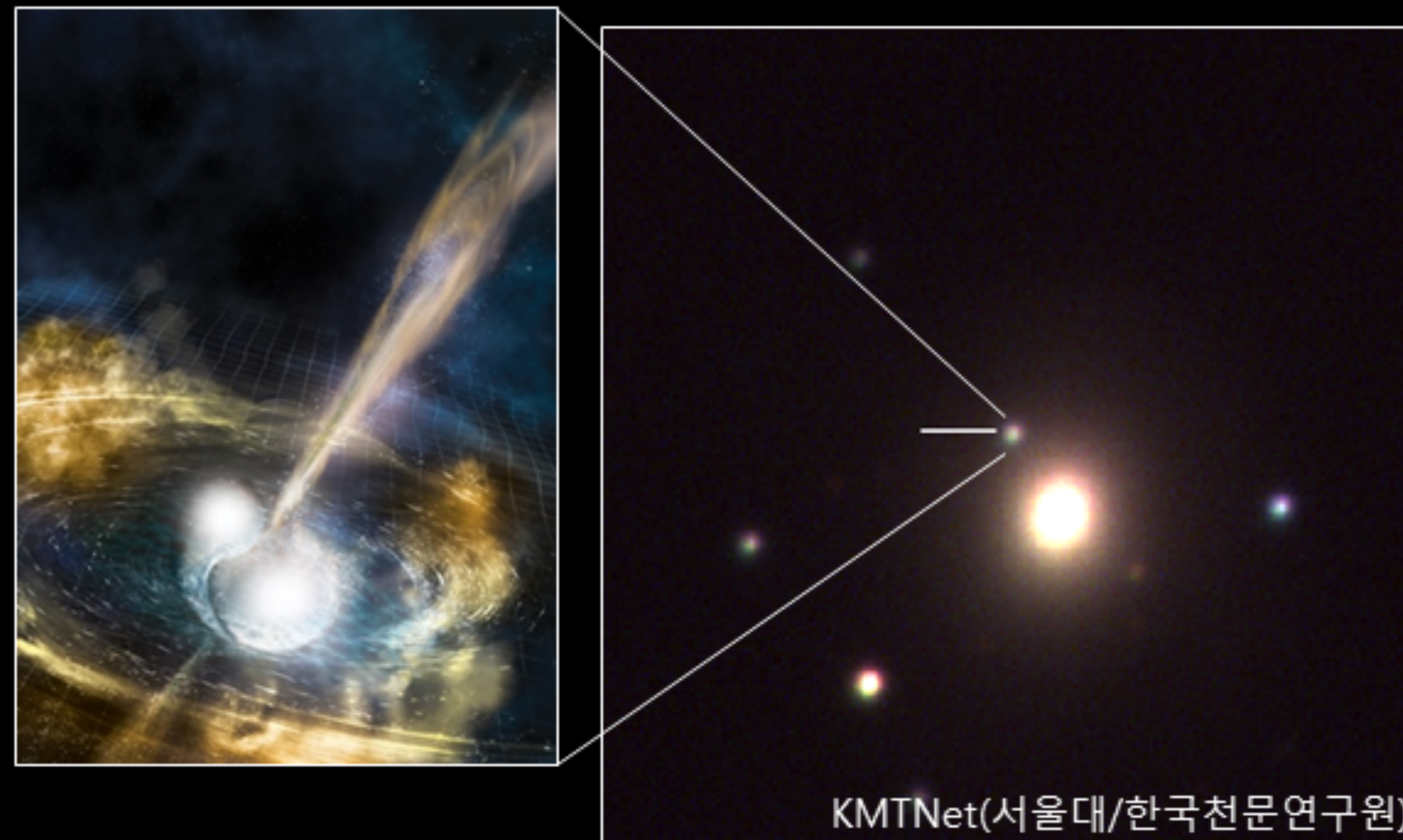
GW170817 : The Golden Binary

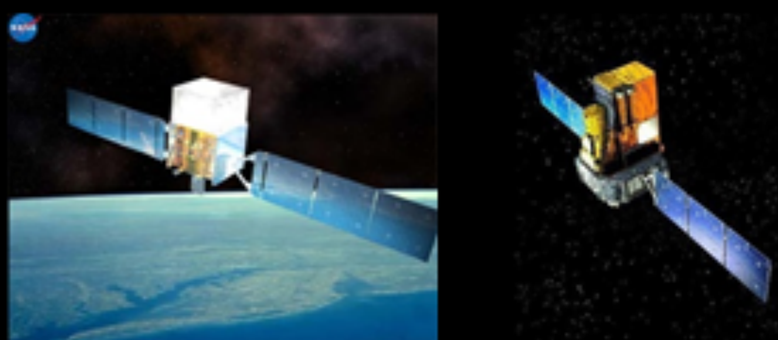


GW170817 : The Golden Binary



2017-08-17 12:41:04 UTC
라이고 및 비르고 중력파 신호 포착





+2초 후
페르미 및 인티그랄 감마선 신호 포착

+약 11시간 후
칠레 천문대 망원경들이
가시광선 신호 포착

+약 21시간 후
국내연구진 호주 이상각망원경으로
추적관측시작. 이후 약 4주간 추적관측
(KMTNet, BOOTES-5망원경 등)

+약 9일 후
찬드라 우주망원경
X-선 신호 포착

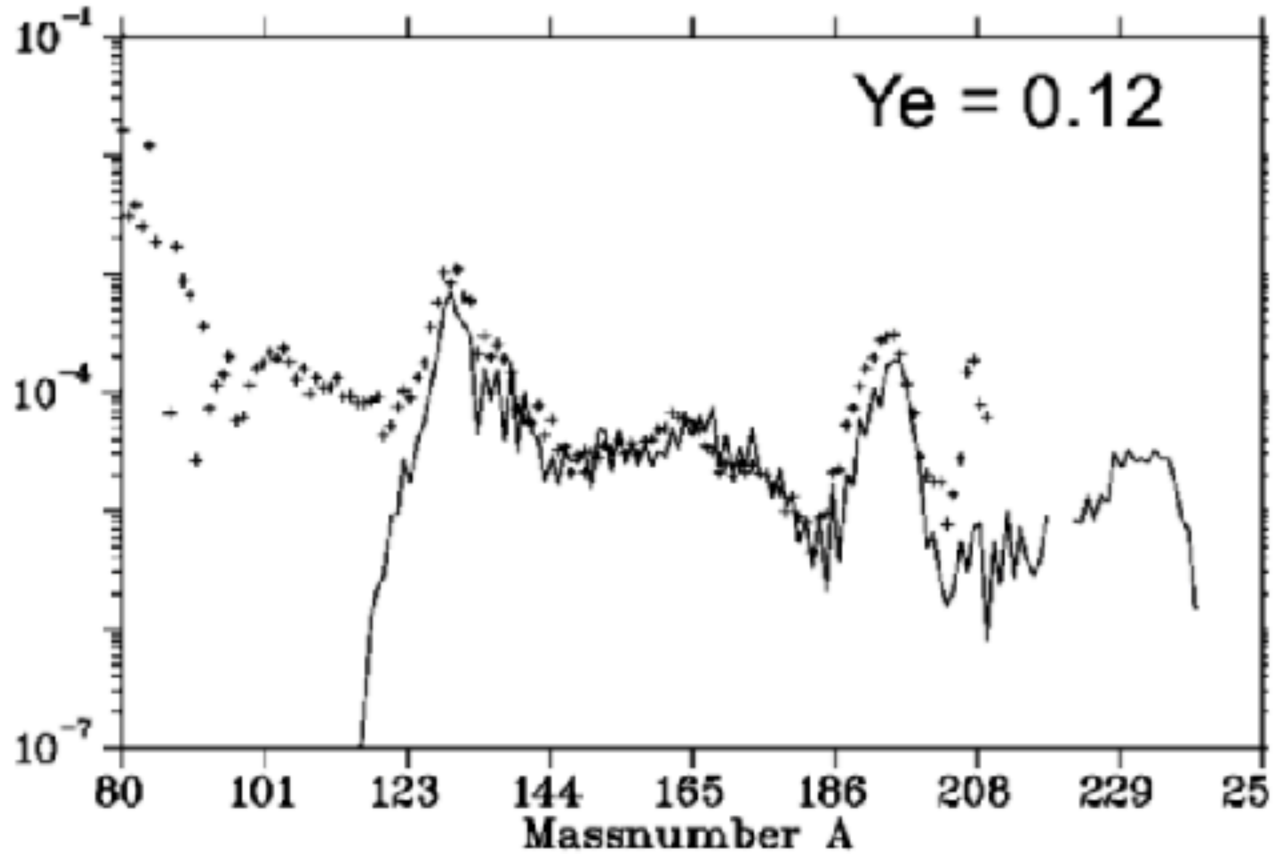
+약 16일 후
지상 전파망원경
전파 신호 포착

그림출처: LIGO, NASA, 1M2H/UC Santa Cruz and Carnegie Observatories/Ryan Foley,
서울대/한국천문연구원/임명신, NSF/Sonoma 주립대/A. Simonnet, NRAO

서울대학교 초기우주천체연구단
(임명신)

Heavy Elements from BNS mergers

S Rosswog 2015

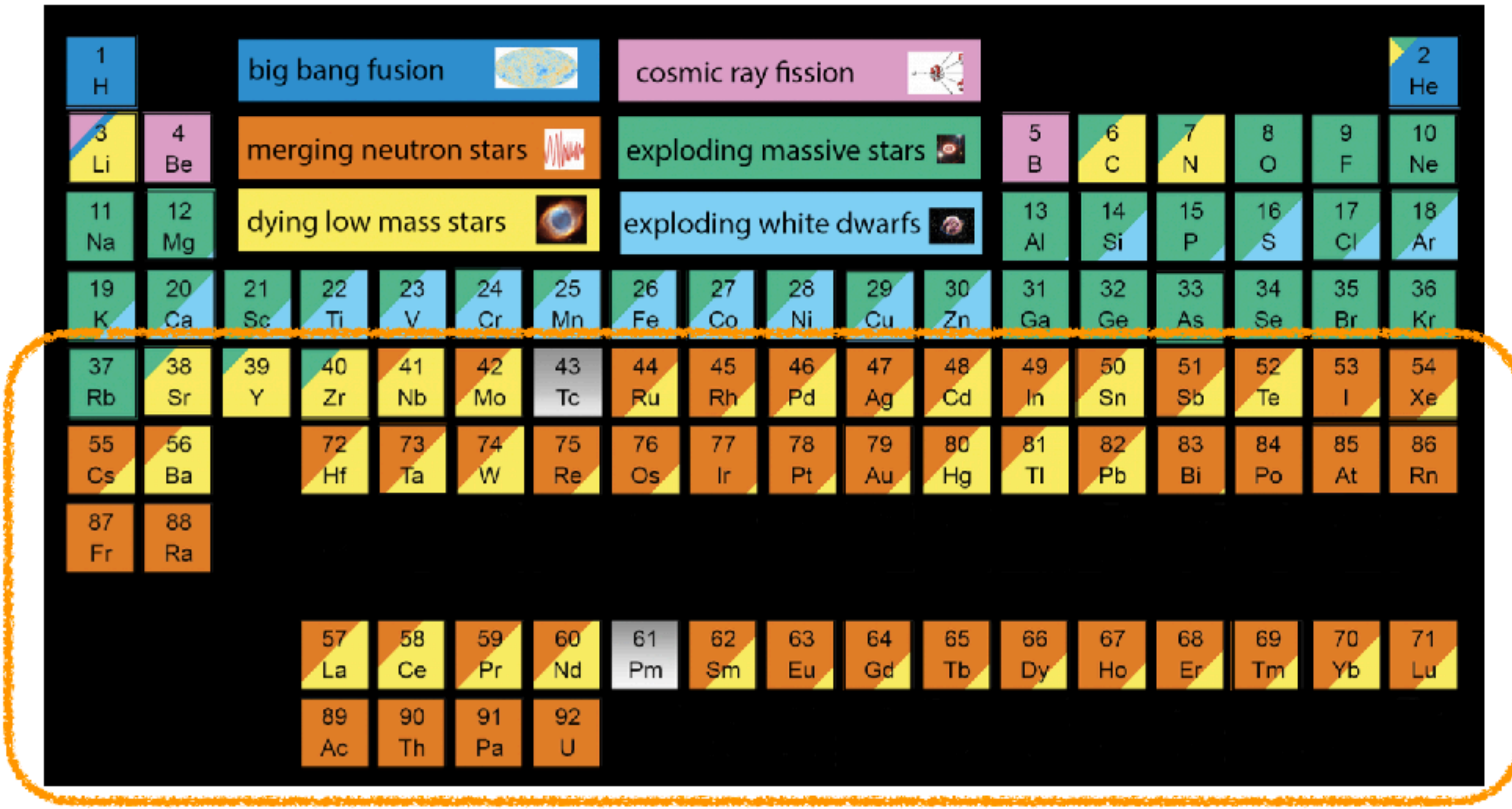


solar pattern vs NS-merger

- **Supernovae:**
  neutrino-driven wind
  r-process **peak at A~130**

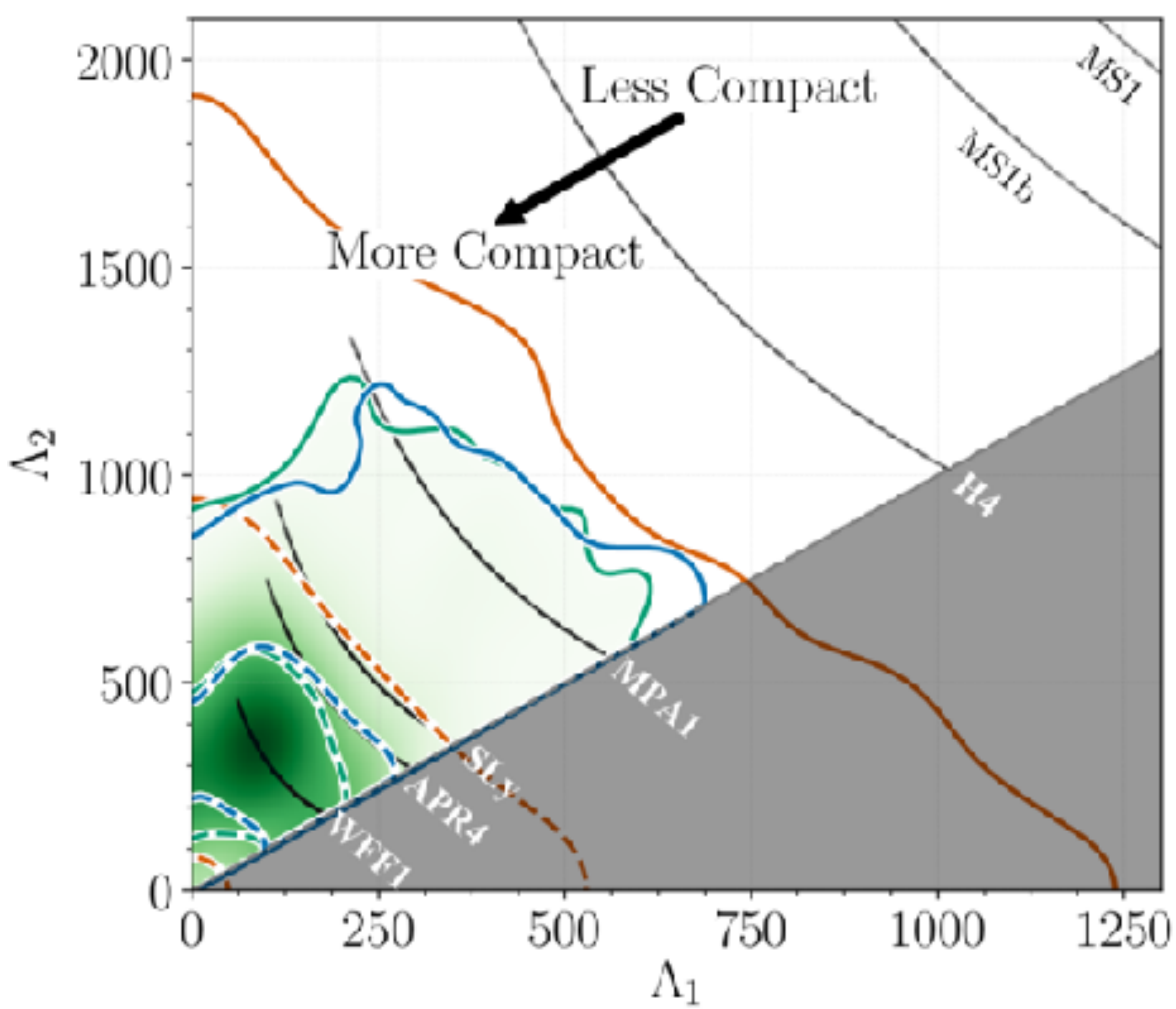
- **NS mergers:**
  r-process **peak at A~195**

Heavy Elements from BNS mergers



A new constraint by GW Observation

$\Lambda(1.4M_\odot)=190^{+390}_{-120}$



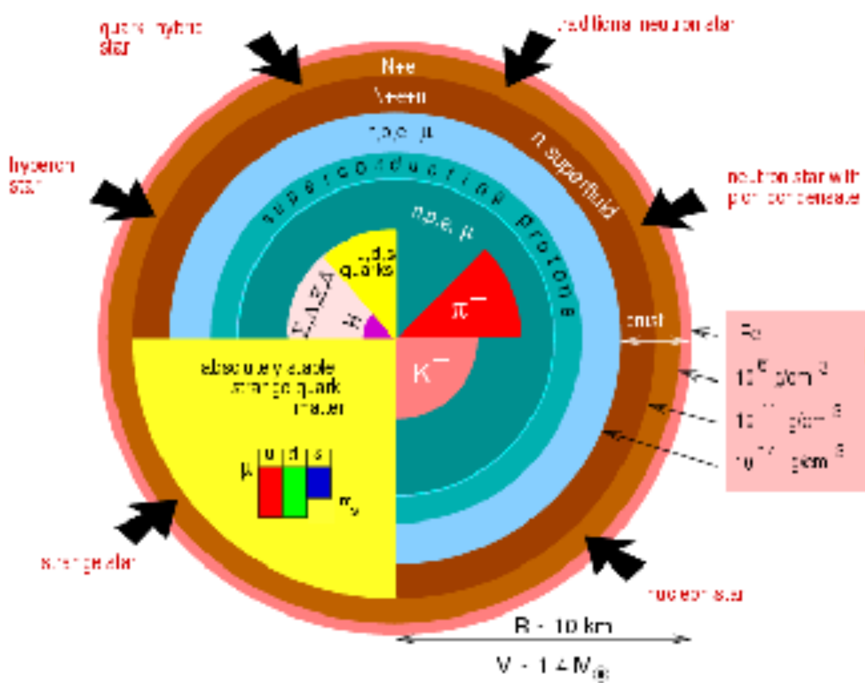
$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T}(f)$$

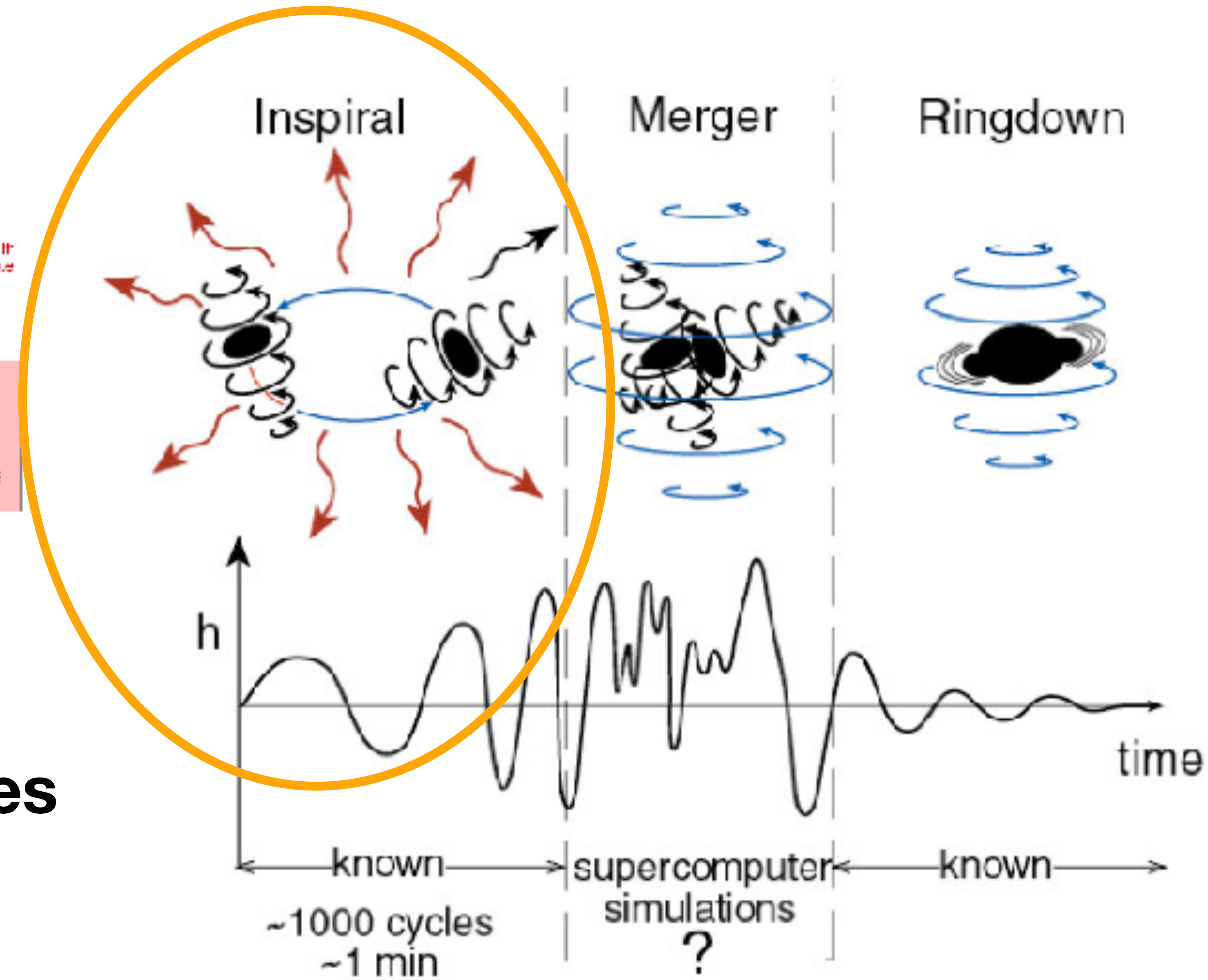
$$\Psi_T(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{pp}_{3.5PN} + \Delta\Psi^{spin}_{3PN} + \Delta\Psi^{ecc.}_{2PN} + \Delta\Psi^{tidal}_{6PN} + \Delta\Psi^{tm}_{6PN})$$

$$\Delta\Psi^{tidal}_{6PN} = -\frac{39}{2}\tilde{\Lambda} v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right)$$

Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PhysRevLett.121.161101)

Response of NS to GW during inspiral





perturbative approaches

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ :Tidal deformability

k2 :Tidal Love number

Measurability of tidal deformability

Tidal term in GW waveform

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T(f)}$$

M. Favata, PRL.112.101101 (2014)

$$\Psi_T(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{\rm pp}_{\rm 3.5PN} + \Delta\Psi^{\rm spin}_{\rm 3PN} + \Delta\Psi^{\rm ecc.}_{\rm 2PN} + \Delta\Psi^{\rm tidal}_{\rm 6PN} + \Delta\Psi^{\rm tm}_{\rm 6PN}), \quad (1)$$

$$\Delta\Psi^{\rm tidal}_{\rm 6PN} = -\frac{39}{2}\tilde{\Lambda} v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right), \quad (4)$$

$$v = (\pi f M)^{1/3}$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

$$\Lambda = \lambda/M^5 \to G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Accumulated GW phase (1)

the number of wave cycles in frequency domain

$$\Delta N_{\text{cyc},\Psi} = \frac{1}{2\pi}\left[\Psi(f_2) - \Psi(f_1) + (f_1 - f_2)\frac{d\Psi}{df_1}\right], \qquad (7.8)$$

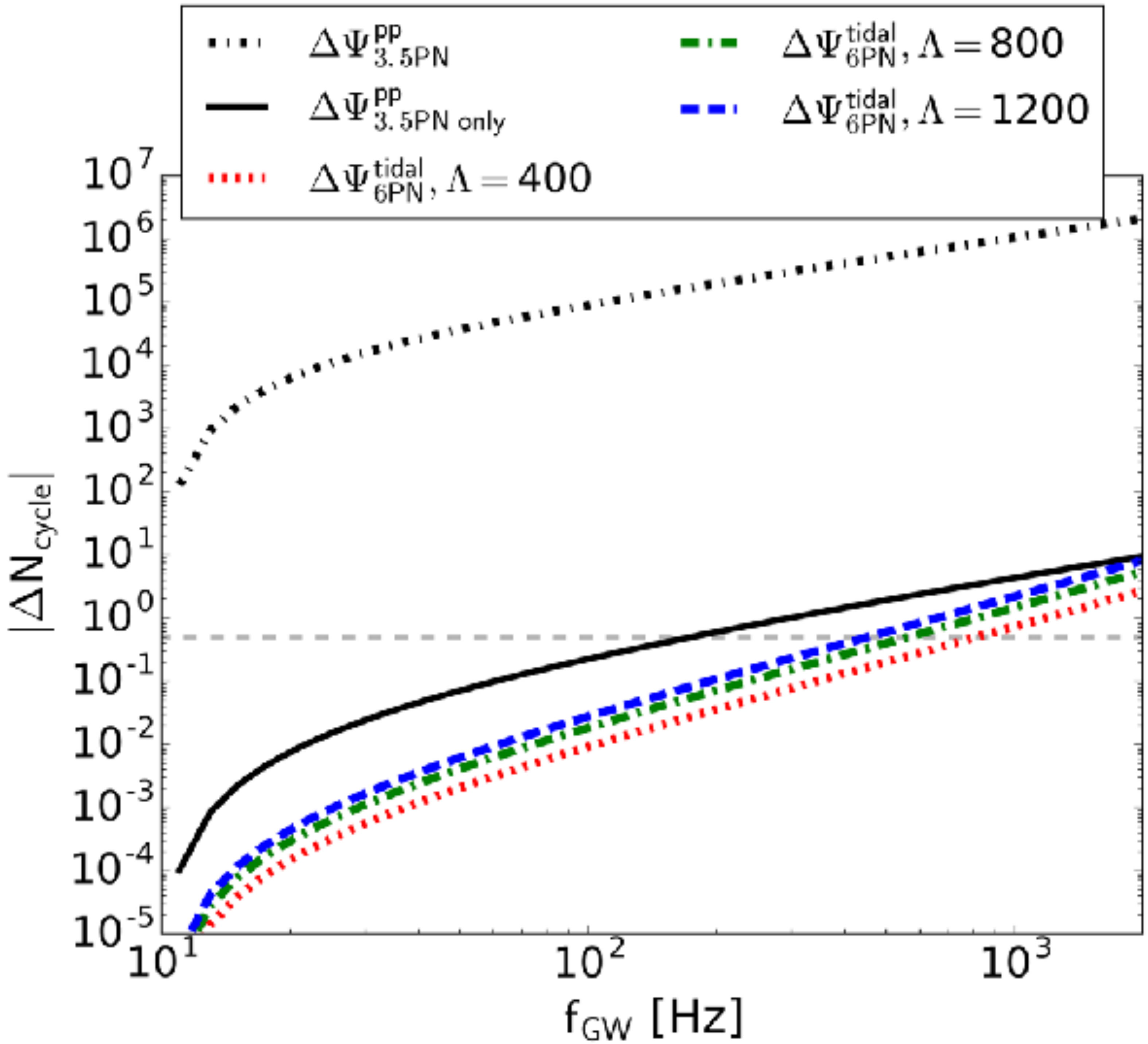
f_l = 10 Hz,
the low frequency cutoff for Advanced LIGO
due to seismic noises

Waveform models:
TaylorT2 for ΔN_{cyc}
TaylorF2(SPA) $\Delta N_{cyc,\Psi}$

Moore et al., PRD.93.124061(2016)

	$1.4M_\odot + 1.4M_\odot$, $f_2 = 1000$ Hz		
PN order	ΔN_{cyc}	$\Delta N_{\text{cyc},\Psi}$	$\Delta N_{\text{useful}}^{\text{norm}}$
0PN(circ)	16 031	986 372	1821
0PN(ecc)	−463	−36 137	−6.37
1PN(circ)	439	21 743	125
1PN(ecc)	−15.8	−1193	−0.332
1.5PN(circ)	−208	−8520	−94.8
1.5PN(ecc)	1.67	103	0.113
2PN(circ)	9.54	294	6.70
2PN(ecc)	−0.215	−15.4	−0.008 17
2.5PN(circ)	−10.6	−218	−10.6
2.5PN(ecc)	0.0443	2.61	0.004 73
3PN(circ)	2.02	18.2	2.80
3PN(ecc)	0.002 00	0.119	−0.000 238
3.5PN(circ)	−0.662	−4.39	−0.977
Total	15 785	962 445	1843

Accumulated GW phase (2)

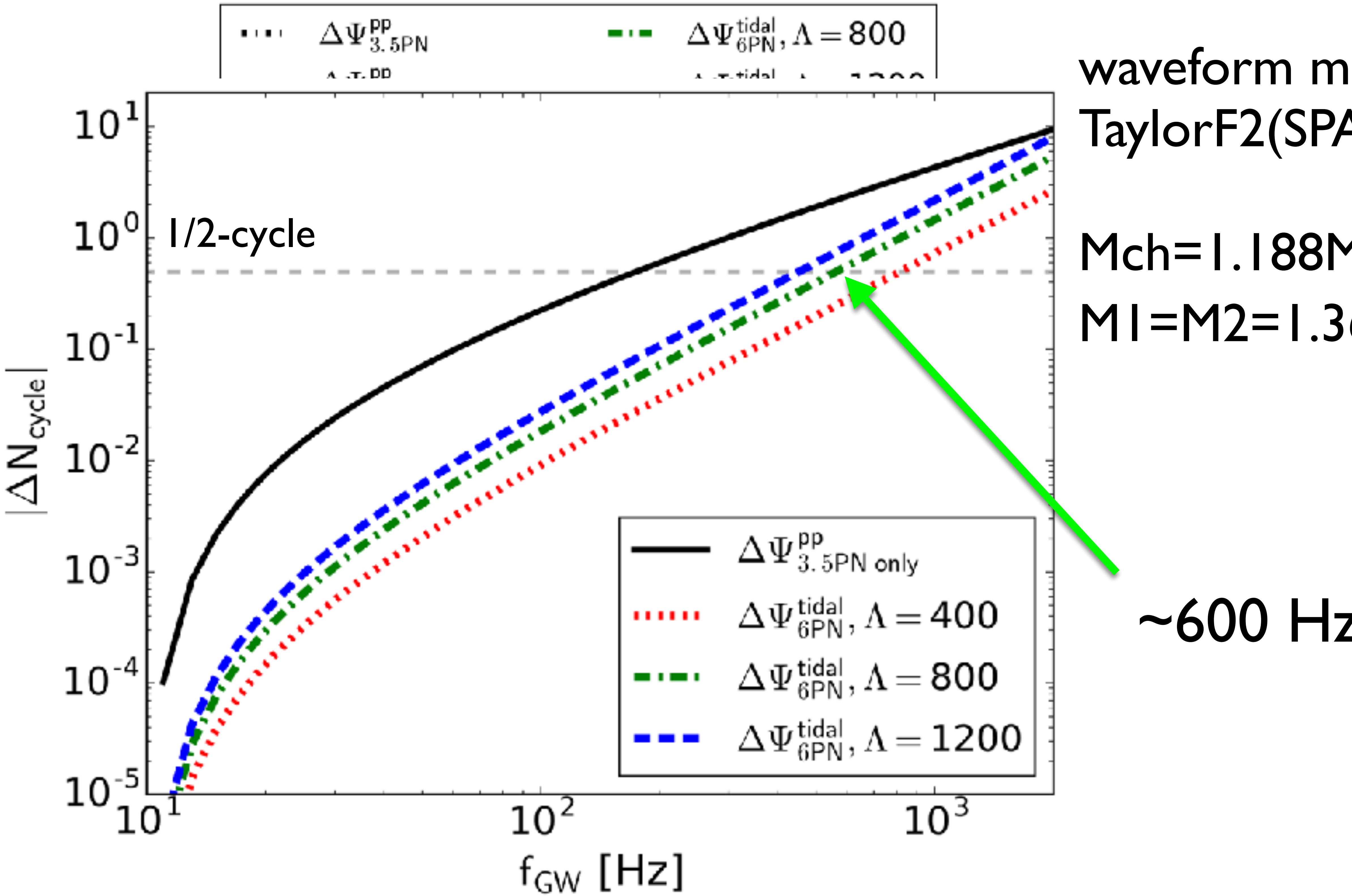


waveform model:
TaylorF2(SPA)

$M_{ch}=1.188M_\odot$

$M1=M2=1.365M_\odot$

Accumulated GW phase (2)



waveform model:
TaylorF2(SPA)

Mch=1.188M$_\odot$

M1=M2=1.365M$_\odot$

Measurement Errors via Fisher Matrix

Fisher matrix

$$\Gamma_{ij} = \mathrm{Re} < \frac{\partial h}{\partial \lambda_i} | \frac{\partial h}{\partial \lambda_j} >$$

$$\lambda_i = M_{\mathrm{chirp}}, \eta, \phi_c, t_c, s_{1,2}, e_0, \tilde{\Lambda}, ...$$

covariance matrix

$$\Sigma_{ij} = \Gamma_{ij}^{-1}$$

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T}(f)$$

$$\Psi_{\mathrm{T}}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}\left(\Delta\Psi^{\mathrm{pp}}_{3.5\mathrm{PN}} + \Delta\Psi^{\mathrm{spin}}_{3\mathrm{PN}} + \Delta\Psi^{\mathrm{ecc.}}_{2\mathrm{PN}} + \Delta\Psi^{\mathrm{tidal}}_{6\mathrm{PN}} + \Delta\Psi^{\mathrm{tm}}_{6\mathrm{PN}}\right)$$

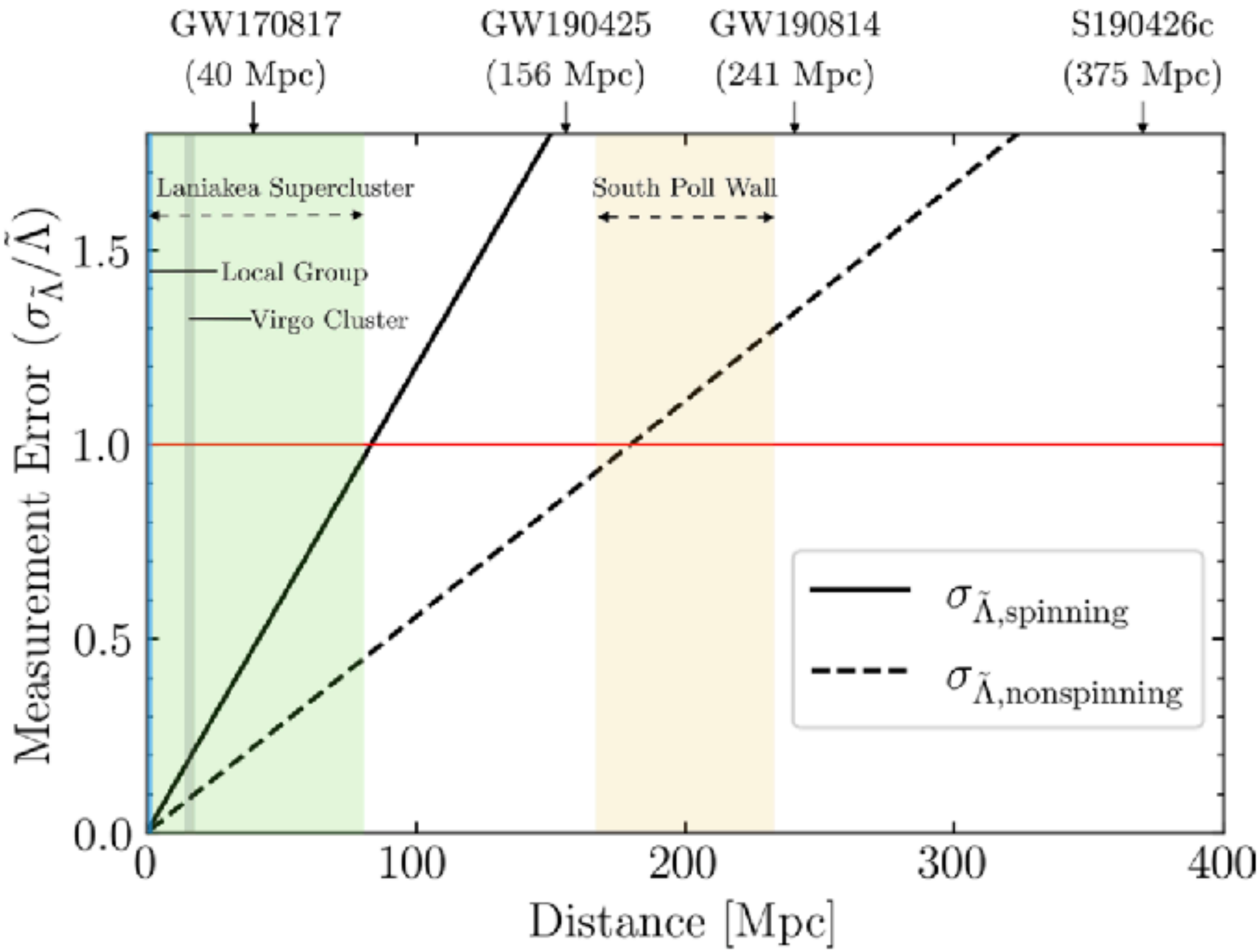
correlation matrix

$$c_{ij} = \Sigma_{ij} / \sqrt{\Sigma_{ii}\Sigma_{jj}}$$

Measurement Errors

$$\sigma_i = \sqrt{\Sigma_{ii}}$$

Measurement Errors of Tidal Deformability



Kim, Kim, Kwak, Choi, Cho and Lee, JKPS (2021)

Derive 2nd order Diff. eq of H for tidal deformability

Selected references

- **A.E.H. Love** (1909) - The Yielding of the Earth to Disturbing Forces
- **K.S. Thorne** & A. Campolattaro (1967) - non-radial pulsation of NS
- J.B. Hartle & **K.S. Thorne** (1969) - stability of rotating NS
- … …
- **K.S. Thorne** (1998) - Tidal stabilization of rigid rotating, fully relativistic neutron star
- … …
- **T. Hinderer** (2008) - Tidal Love numbers of neutron stars
- **T. Damour and A. Nagar** (2009) - Relativistic tidal properties of neutron stars

Love number

The Yielding of the Earth to Disturbing Forces.

By A. E. H. Love, F.R.S., Sedleian Professor of Natural Philosophy in the University of Oxford.

(Received November 28, 1908,—Read January 14, 1909.)

1. Any estimate of the rigidity of the Earth must be based partly on some observations from which a deformation of the Earth's surface can be inferred, and partly on some hypothesis as to the internal constitution of the Earth. The observations may be concerned with tides of long period, variations of the vertical, variations of latitude, and so on. The hypothesis must relate to the arrangement of the matter as regards density in different parts, and to the state of the parts in respect of solidity, compressibility, and so on.

Love number

terms which are the products of the spherical solid harmonic W_n and functions of r. In the most important cases $n = 2$, and we may write

$$U = H(r)\frac{W_2}{g}, \quad \Delta = f(r)\frac{W_2}{g}, \quad V = V_0 + K(r)W_2, \qquad (5)$$

where $H(r)$, $f(r)$, $K(r)$ are functions of r, and the constant $1/g$ has been inserted in the expression for U for the sake of convenience. If, in fact, W_2 is a periodic term of the tide-generating potential, W_2/g is the "true equilibrium height" of the corresponding tide, that is to say, it is the height of the harmonic inequality which the forces answering to W_2 would produce in an ocean covering a rigid spherical nucleus, of the same size and mass as the Earth, if the depth and density of the ocean were negligible. From the definition of $K(r)$ we easily find the formula

$$K(a) = \frac{3}{5\rho a^6}\int_0^a \rho_0 \left[\frac{d}{dr}\{r^6 H(r)\} - r^6 f(r)\right] dr. \qquad (6)$$

In what follows we shall write h for $H(a)$ and k for $K(a)$, so that the equation of the disturbed surface is

$$r = a + hW_2/g,$$

Tidal Love number and Tidal deformability

T. Hinderer, ApJ, 677, 1216 (2008)

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O(r^3), \quad (1)$$

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ : Tidal deformability

Q_{ij} : Quadrupole moment of NS

$\mathcal{E}_{ij}$: External quadrupole tidal field

k_2 : $l = 2$ Tidal Love number

$$\Lambda = \lambda / M^5 \rightarrow G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Tidal coefficients (deformability)

Multipole moments induced by a external tidal field

electric-type tidal coefficient (even parity)

Mass moments $M_L^A = \mu_\ell^A G_L^A,$ electric-type tidal field

Spin moments $S_L^A = \sigma_\ell^A H_L^A.$ magnetic-type tidal field

magnetic-type tidal coefficient (odd parity)

Metric in linearized perturbation

even parity

$$h_{\mu\nu}=\begin{vmatrix} (1-2m^*/r)H_0(T,r)Y_L{}^M & H_1(T,r)Y_L{}^M & h_0(T,r)(\partial/\partial\theta)Y_L{}^M & h_0(T,r)(\partial/\partial\varphi)Y_L{}^M \\ H_1(T,r)Y_L{}^M & (1-2m^*/r)^{-1}H_2(T,r)Y_L{}^M & h_1(T,r)(\partial/\partial\theta)Y_L{}^M & h_1(T,r)(\partial/\partial\varphi)Y_L{}^M \\ \text{Sym} & \text{Sym} & r^2[K(T,r)+G(T,r)(\partial^2/\partial\theta^2)]Y_L{}^M & \text{Sym} \\ \text{Sym} & \text{Sym} & r^2G(T,r)(\partial^2/\partial\theta\partial\varphi-\cos\theta\partial/\sin\theta\partial\varphi)Y_L{}^M & r^2[K(T,r)\sin^2\theta+G(T,r)(\partial^2/\partial\varphi\partial\varphi+\sin\theta\cos\theta\partial/\partial\theta)]Y_L{}^M \end{vmatrix}. \quad (13)$$

odd parity

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & -h_0(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_0(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ 0 & 0 & -h_1(T,r)(\partial/\sin\theta\partial\varphi)Y_L{}^M & h_1(T,r)(\sin\theta\partial/\partial\theta)Y_L{}^M \\ \text{Sym} & \text{Sym} & h_2(T,r)(\partial^2/\sin\theta\partial\theta\partial\varphi-\cos\theta\partial/\sin^2\theta\partial\varphi)Y_L{}^M & \text{Sym} \\ \text{Sym} & \text{Sym} & \frac{1}{2}h_2(T,r)(\partial^2/\sin\theta\partial\varphi\partial\varphi+\cos\theta\partial/\partial\theta-\sin\theta\partial^2/\partial\theta\partial\theta)Y_L{}^M & -h_2(T,r)(\sin\theta\partial^2/\partial\theta\partial\varphi-\cos\theta\partial/\partial\varphi)Y_L{}^M \end{vmatrix}. \quad (12)$$

Regge-Wheeler gauge and M=0

even parity

$$h_{\mu\nu}=\exp(-ikT)P_L(\cos\theta)$$

$$\times\begin{vmatrix} H_0(1-2m^*/r) & H_1 & 0 & 0 \\ H_1 & H_2(1-2m^*/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{vmatrix} \quad (20)$$

odd parity

$$h_{\mu\nu}=\begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & h_1(r) \\ 0 & 0 & 0 & 0 \\ \mathrm{Sym} & \mathrm{Sym} & 0 & 0 \end{vmatrix}$$

$$\times\exp(-ikT)(\sin\theta\partial/\partial\theta)P_L(\cos\theta). \quad (18)$$

Regge-Wheeler gauge and M=0

even parity

H=H0=H2,K

$$h_{\mu\nu} = \exp(-ikT) P_L(\cos\theta)$$

$$\times \begin{vmatrix} H_0(1-2m^*/r) & \cancelto{0}{H_1} & 0 & 0 \\ \cancelto{0}{H_1} & H_2(1-2m^*/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2K\sin^2\theta \end{vmatrix} \qquad (20)$$

k=0, stationary case

odd parity

Ψ=h0

$$h_{\mu\nu} = \begin{vmatrix} 0 & 0 & 0 & h_0(r) \\ 0 & 0 & 0 & \cancelto{0}{h_1}(r) \\ 0 & 0 & 0 & 0 \\ \mathrm{Sym} & \cancelto{0}{\mathrm{Sym}} & 0 & 0 \end{vmatrix}$$

$$\times \exp(-ikT)(\sin\theta\,\partial/\partial\theta) P_L(\cos\theta). \qquad (18)$$

Even parity (Electric-type) metric Fn. H

Regge & Wheeler (1957), Thorne & Campolattaro (1967)

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

$$h_{\alpha\beta} =$$
$$\mathrm{diag}\left[-e^{\nu(r)} H_0(r), e^{\lambda(r)} H_2(r), r^2 K(r), r^2 \sin^2\theta K(r)\right] Y_{2m}(\theta, \varphi).$$

$$\delta T_0^0 = -\delta\rho = -(dp/d\rho)^{-1}\delta p \qquad \delta T_i^i = \delta p$$

$$H'' + C_1 H' + C_0 H = 0.$$

$$C_1 = \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') = \frac{2}{r} + e^{\lambda}\left[\frac{2m}{r^2} + 4\pi r(p - e)\right], \tag{28}$$

$$C_0 = e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(e+p)\right] + \nu'' + (\nu')^2 + \frac{1}{2r}(2 - r\nu')(3\nu' + \lambda')$$
$$= e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(5e+9p)\right] - (\nu')^2, \tag{29}$$

Electric-type tidal coefficients (1)

General solution

$$H = a_P \hat{P}_{\ell 2}(x) + a_Q \hat{Q}_{\ell 2}(x),$$

$$x \equiv r/M - 1,$$

$$c \equiv M/R$$

$$y^{\rm ext} \equiv rH'/H$$

$$a_\ell \equiv a_Q/a_P$$

$$y_\ell^{\rm ext}(x) = (1+x)\frac{\hat{P}'_{\ell 2}(x) + a_\ell \hat{Q}'_{\ell 2}(x)}{\hat{P}_{\ell 2}(x) + a_\ell \hat{Q}_{\ell 2}(x)}.$$

$$a_\ell = -\left.\frac{\hat{P}'_{\ell 2}(x) - c y_\ell \hat{P}_{\ell 2}(x)}{\hat{Q}'_{\ell 2}(x) - c y_\ell \hat{Q}_{\ell 2}(x)}\right|_{x=1/c-1}.$$

Electric-type tidal coefficients (2)

Damour and Nagar , PRD 80, 084035 (2009)

$$-(\delta H_{00}e^{-\nu})^{\rm growing} = H^{\rm growing}(r) = a_P\hat{P}_{\ell 2}(x)Y_{\ell m} \simeq a_P\left(\frac{r}{M}\right)^{\ell}Y_{\ell m}, \quad (40)$$

$$-(\delta H_{00}e^{-\nu})^{\rm decreasing} = H^{\rm decreasing}(r) = a_Q\hat{Q}_{\ell 2}(x)Y_{\ell m} \simeq a_Q\left(\frac{r}{M}\right)^{-(\ell+1)}Y_{\ell m}, \quad (41)$$

$$\partial_L r^{-1} = (-)^{\ell}(2\ell-1)!!\hat{n}^L r^{-(\ell+1)},$$

$$M_L^A = \mu_\ell^A G_L^A,$$

$$(2\ell-1)!!G\mu_\ell = \frac{a_Q}{a_P}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = a_\ell\left(\frac{GM}{c_0^2}\right)^{2\ell+1}.$$

$$\bar{W} = \frac{1}{\ell!}\hat{X}^L G_L^A = \frac{1}{\ell!}r^{\ell}\hat{n}^L G_L^A,$$

$$W^+ = G\frac{(-)^{\ell}}{\ell!}\partial_L\left(\frac{M_L^A}{r}\right),$$

$$G_{00}^A(X) = -\exp(-2W^A/c^2),$$

$$G_{0a}^A(X) = -\frac{4}{c^3}W_a^A,$$

$$E_a^A(X) = \partial_a W^A + \frac{4}{c^2}\partial_T W_a^A,$$

$$G_L^A(T) \equiv [\partial_{\langle L-1}\bar{E}^A_{a_\ell\rangle}(T,\mathbf{X})]_{X^a\to(}$$

Electric-type tidal coefficients (3)

Damour and Nagar , PRD 80, 084035 (2009)

$$(2\ell - 1)!!G\mu_\ell = \frac{a_Q}{a_P}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = a_\ell\left(\frac{GM}{c_0^2}\right)^{2\ell+1}. \quad (45)$$

$$G\mu_\ell = \frac{a_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{2k_\ell}{(2\ell-1)!!}R^{2\ell+1}. \quad (48)$$

$$\begin{aligned} k_\ell &= \frac{1}{2}c^{2\ell+1}a_\ell \\ &= -\frac{1}{2}c^{2\ell+1}\frac{\hat{P}'_{\ell 2}(x) - cy_\ell \hat{P}_{\ell 2}(x)}{\hat{Q}'_{\ell 2}(x) - cy_\ell \hat{Q}_{\ell 2}(x)}\bigg|_{x=1/c-1}. \end{aligned} \quad (49)$$

Electric-type tidal coefficients (4)

Damour and Nagar , PRD 80, 084035 (2009)

Electric-type tidal coefficients

$$G\mu_\ell = \frac{a_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{2k_\ell}{(2\ell-1)!!}R^{2\ell+1}. \quad (48)$$

$$H'' + C_1 H' + C_0 H = 0.$$

$$C_1 = \frac{2}{r} + \frac{1}{2}(\nu' - \lambda') = \frac{2}{r} + e^{\lambda}\left[\frac{2m}{r^2} + 4\pi r(p-e)\right], \quad (28)$$

$$\begin{aligned} C_0 = {} & e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(e+p)\right] \\ & + \nu'' + (\nu')^2 + \frac{1}{2r}(2 - r\nu')(3\nu' + \lambda') \\ = {} & e^{\lambda}\left[-\frac{\ell(\ell+1)}{r^2} + 4\pi(e+p)\frac{de}{dp} + 4\pi(5e+9p)\right] \\ & - (\nu')^2, \end{aligned} \quad (29)$$

we calculated the case, l=2

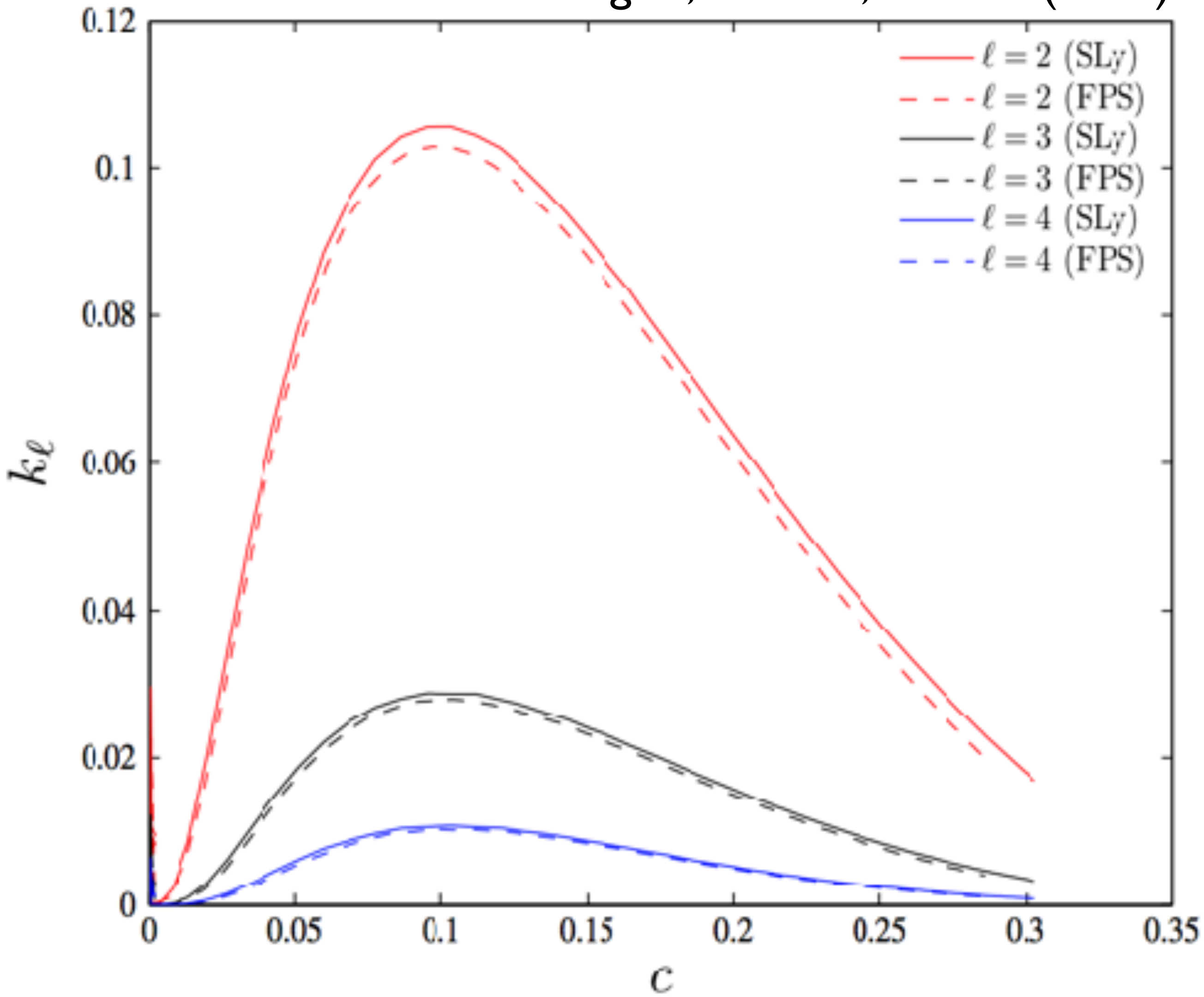
$$\begin{aligned} k_2 = {} & \frac{8}{5}(1-2c)^2 c^5 [2c(y-1) - y + 2]\Big\{2c(4(y+1)c^4 + (6y-4)c^3 + (26-22y)c^2 + 3(5y-8)c - 3y + 6) \\ & - 3(1-2c)^2(2c(y-1) - y + 2)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (50)$$

$$y_R = \frac{rH'(r)}{H(r)}\Big|_{r=R}$$

$$\begin{aligned} k_3 = {} & \frac{8}{7}(1-2c)^2 c^7 [2(y-1)c^2 - 3(y-2)c + y - 3]\Big\{2c[4(y+1)c^5 + 2(9y-2)c^4 - 20(7y-9)c^3 + 5(37y-72)c^2 \\ & - 45(2y-5)c + 15(y-3)] - 15(1-2c)^2(2(y-1)c^2 - 3(y-2)c + y - 3)\log\left(\frac{1}{1-2c}\right)\Big\}^{-1}, \end{aligned} \quad (51)$$

Higher Tidal Love Numbers

Damour and Nagar , PRD 80, 084035 (2009)



$$x \equiv (M\omega)^{2/3}$$

$$\Delta\Psi_2^{tidal} \sim \lambda_2 \, x^{5/2}$$

$$\Delta\Psi_3^{tidal} \sim \lambda_3 \, x^{9/2}$$

$$|\Delta\Psi_3^{tidal}/\Delta\Psi_2^{tidal}| \sim \mathcal{O}(10^{-3})$$

We hardly expect to observe higher terms of tidal deformability in the waveform

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$G\sigma_\ell = \frac{\ell-1}{4(\ell+2)} \frac{b_\ell}{(2\ell-1)!!} \left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{\ell-1}{4(\ell+2)} \frac{j_\ell}{(2\ell-1)!!} R^{2\ell+1},$$

$$G\sigma_2 = \frac{1}{48} j_2 R^5 = \frac{1}{48} b_2 \left(\frac{GM}{c_0^2}\right)^5,$$

$$\psi'' + \frac{e^\lambda}{r^2}[2m + 4\pi r^3(p-e)]\psi' - e^\lambda\left[\frac{\ell(\ell+1)}{r^2} - \frac{6m}{r^3} + 4\pi(e-p)\right]\psi = 0. \quad (31)$$

$$j_\ell \equiv c^{2\ell+1} b_\ell = -c^{2\ell+1} \frac{\psi'_P(\hat{r}) - c y_{\rm odd}\psi_P(\hat{r})}{\psi'_Q(\hat{r}) - c y_{\rm odd}\psi_Q(\hat{r})}\bigg|_{\hat{r}=1/c}.$$

$$j_2 = \frac{96c^5(2c-1)(y-3)}{5(2c(12(y+1)c^4 + 2(y-3)c^3 + 2(y-3)c^2 + 3(y-3)c - 3y + 9) + 3(2c-1)(y-3)\log(1-2c))}. \quad (73)$$

Magnetic-type tidal coefficients

Damour and Nagar , PRD 80, 084035 (2009)

Likewise,

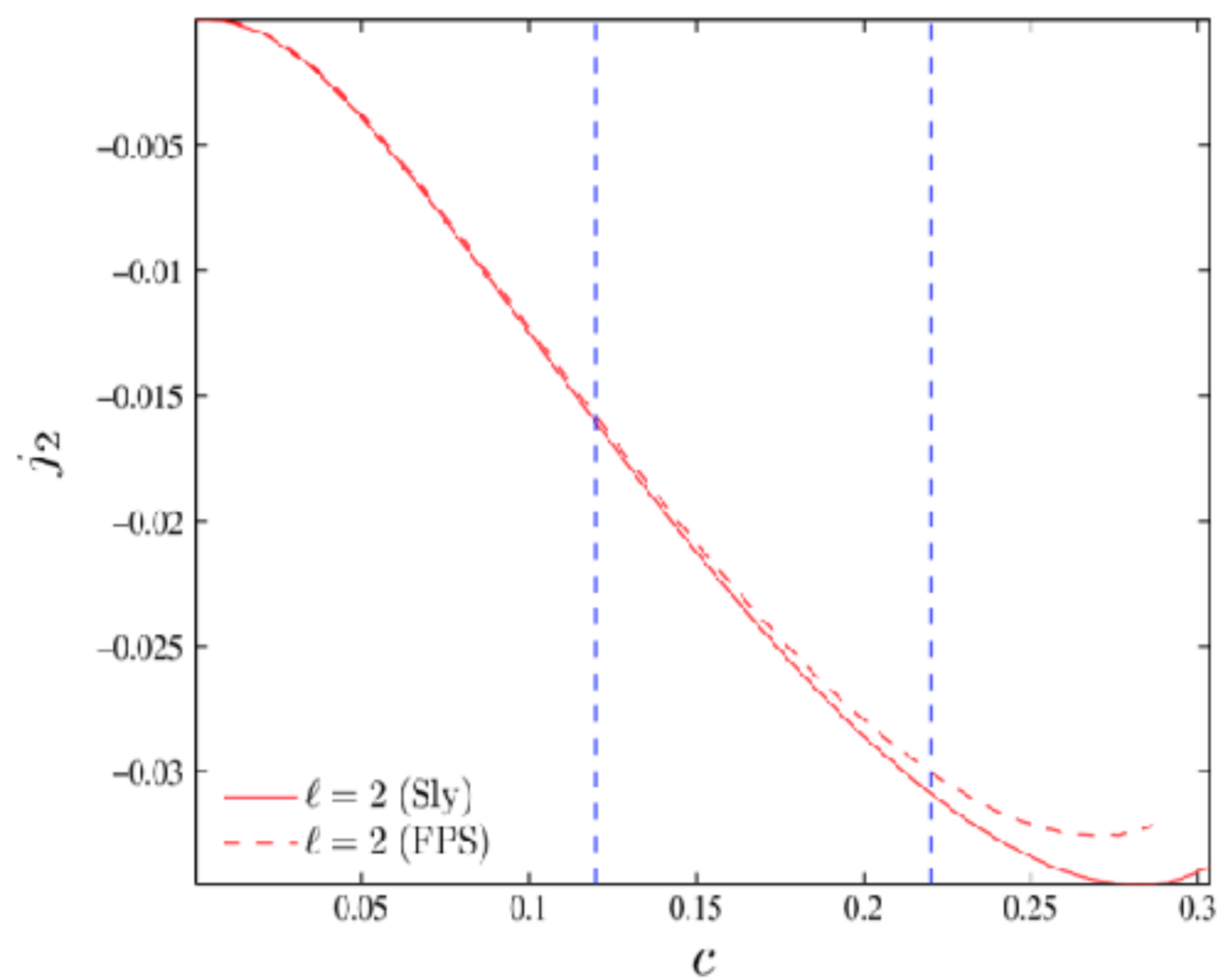
Magnetic-type tidal coefficients

$$S_L^A = \sigma_\ell^A H_L^A.$$

$$\psi'' +$$

$$G\sigma_\ell = \frac{\ell-1}{4(\ell+2)}\frac{b_\ell}{(2\ell-1)!!}\left(\frac{GM}{c_0^2}\right)^{2\ell+1} = \frac{\ell-1}{4(\ell+2)}\frac{j_\ell}{(2\ell-1)!!}R^{2\ell+1},$$

$$j_\ell \equiv$$



$$G\sigma_2 = \frac{1}{48}j_2R^5 = \frac{1}{48}b_2\left(\frac{GM}{c_0^2}\right)^5,$$

$$j_2 = \frac{96c^5(2c-1)(y-3)}{5(2c(12(y+1)c^4 + 2(y-3)c^3 + 2(y-3)c^2 + 3(y-3)c - 3y + 9) + 3(2c-1)(y-3)\log(1-2c))}. \quad (73)$$

Tidal Love number and Tidal deformability

$$\frac{(1-g_{tt})}{2} = -\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + O\left(\frac{1}{r^3}\right) + \frac{1}{2}\mathcal{E}_{ij}x^i x^j + O(r^3), \qquad (1)$$

T. Hinderer, ApJ, 677, 1216 (2008)

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$\lambda = \frac{2}{3}\frac{R^5}{G}k_2$$

λ : Tidal deformability

Q_{ij} : Quadrupole moment of NS

$\mathcal{E}_{ij}$: External quadrupole tidal field

k_2 : l = 2 Tidal Love number

$$y = \frac{R\beta(R)}{H(R)} \qquad C = \frac{GM}{Rc^2}$$

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y] \times \Big\{2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)] + 3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C)\Big\}^{-1}$$

Tidal term in GW waveform

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T(f)}$$

M. Favata, PRL.112.101101 (2014)

$$\Psi_{\rm T}(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}\left(\Delta\Psi^{\rm pp}_{\rm 3.5PN} + \Delta\Psi^{\rm spin}_{\rm 3PN} + \Delta\Psi^{\rm ecc.}_{\rm 2PN} + \Delta\Psi^{\rm tidal}_{\rm 6PN} + \Delta\Psi^{\rm tm}_{\rm 6PN}\right), \qquad (1)$$

$$\Delta\Psi^{\rm tidal}_{\rm 6PN} = -\frac{39}{2}\tilde{\Lambda} v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right), \qquad (4)$$

$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12 m_2) m_1^4 \Lambda_1 + (m_2 + 12 m_1) m_2^4 \Lambda_2}{(m_1 + m_2)^5}$$

$$\Lambda = \lambda / M^5 \rightarrow G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

Higher order terms in GW phase (1)

Post-Newtonian spin-tidal couplings for compact binaries - arxiv:1805.01487

PN order	λ_2	σ_2	$\lambda_{23,32},\ \sigma_{23,32}$	λ_3	σ_3
5	LO $\propto \Lambda$				
6	NLO $\propto \delta\Lambda$	LO $\propto \Sigma$			
6.5	**NNLO $\propto \tilde{\Lambda}$**	**NLO $\propto \tilde{\Sigma}$**	**LO $\propto \tilde{\Gamma}$**		
7	...	...	...	LO	
8	...	...	...	...	LO

$$\psi(x) = \frac{3}{128\nu x^{5/2}}\Bigg\{1+\left(\frac{3715}{756}+\frac{55}{9}\nu\right)x+\left(\frac{113}{3}\times \times(\eta_1\chi_1+\eta_2\chi_2)-\frac{38}{3}\nu(\chi_1+\chi_2)\right)x^{1.5}+O(x^2) + \Lambda x^5+(\delta\Lambda+\Sigma)x^6+(\tilde{\Lambda}+\tilde{\Sigma}+\tilde{\Gamma})x^{6.5}+O(x^7)\Bigg\}, \tag{15}$$

$$\Lambda = \left(264-\frac{288}{\eta_1}\right)\frac{c^{10}\lambda_2^{(1)}}{M^5}+(1\leftrightarrow 2), \tag{16}$$

$$\delta\Lambda = \left(\frac{4595}{28}-\frac{15895}{28\eta_1}+\frac{5715\eta_1}{14}-\frac{325\eta_1^2}{7}\right)\frac{c^{10}\lambda_2^{(1)}}{M^6} + (1\leftrightarrow 2), \tag{17}$$

$$\Sigma = \left(\frac{6920}{7}-\frac{20740}{21\eta_1}\right)\frac{c^8\sigma_2^{(1)}}{M^5}+(1\leftrightarrow 2).$$

$$\tilde{\Lambda} = \left[\left(\frac{593}{4}-\frac{1105}{8\eta_1}+\frac{567\eta_1}{8}-81\eta_1^2\right)\chi_2 + \left(-\frac{6607}{8}+\frac{6639\eta_1}{8}-81\eta_1^2\right)\chi_1\right]\frac{c^{10}\lambda_2^{(1)}}{M^5} + (1\leftrightarrow 2), \tag{19}$$

$$\tilde{\Sigma} = \left[\left(-\frac{9865}{3}+\frac{4933}{3\eta_1}+1644\eta_1\right)\chi_2-\chi_1\right]\frac{c^8\sigma_2^{(1)}}{M^5} + (1\leftrightarrow 2), \tag{20}$$

$$\tilde{\Gamma} = \frac{c^{10}\chi_1}{M^4}\left[\left(856\eta_1-816\eta_1^2\right)\lambda_{23}^{(1)} - \left(\frac{4993\eta_1}{18}-\frac{2497\eta_1^2}{9}\right)\sigma_{23}^{(1)} - \nu\left(272\lambda_{32}^{(1)}-204\sigma_{32}^{(1)}\right)\right]+(1\leftrightarrow 2). \tag{21}$$

Higher order terms in GW phase (2)

Rotational-tidal phasing of the binary neutron star waveform

- arxiv:1805.01882

$$\Psi = \frac{3M}{128\mu} x^{-2.5} \left[1 - \frac{39}{2} \tilde{\Lambda}\, x^5 + \tilde{\Sigma}\, x^6 - \tilde{X}\, x^{6.5} - \tilde{\Lambda}_3\, x^7 + \tilde{\Sigma}_3\, x^8 \right], \qquad (8)$$

$$\tilde{X} = \frac{1}{21 M^6} c^{12} \Big\{ \chi^{(1)} \left[36(35 + 614q)\hat{\lambda}_2^{(1)} - (7 - 4751q)\hat{\sigma}_2^{(1)} - 2316q\hat{\lambda}_3^{(1)} - 3474q\hat{\sigma}_3^{(1)} \right] + \chi^{(2)} \left[36(35 + 614/q)\hat{\lambda}_2^{(2)} - (7 - 4751/q)\hat{\sigma}_2^{(2)} - 2316\hat{\lambda}_3^{(2)}/q - 3474\hat{\sigma}_3^{(2)}/q \right] \Big\},$$

$$\tilde{\Lambda}_3 = \frac{4000}{9M^7} c^{14} (q\lambda_3^{(1)} + \lambda_3^{(2)}/q),$$

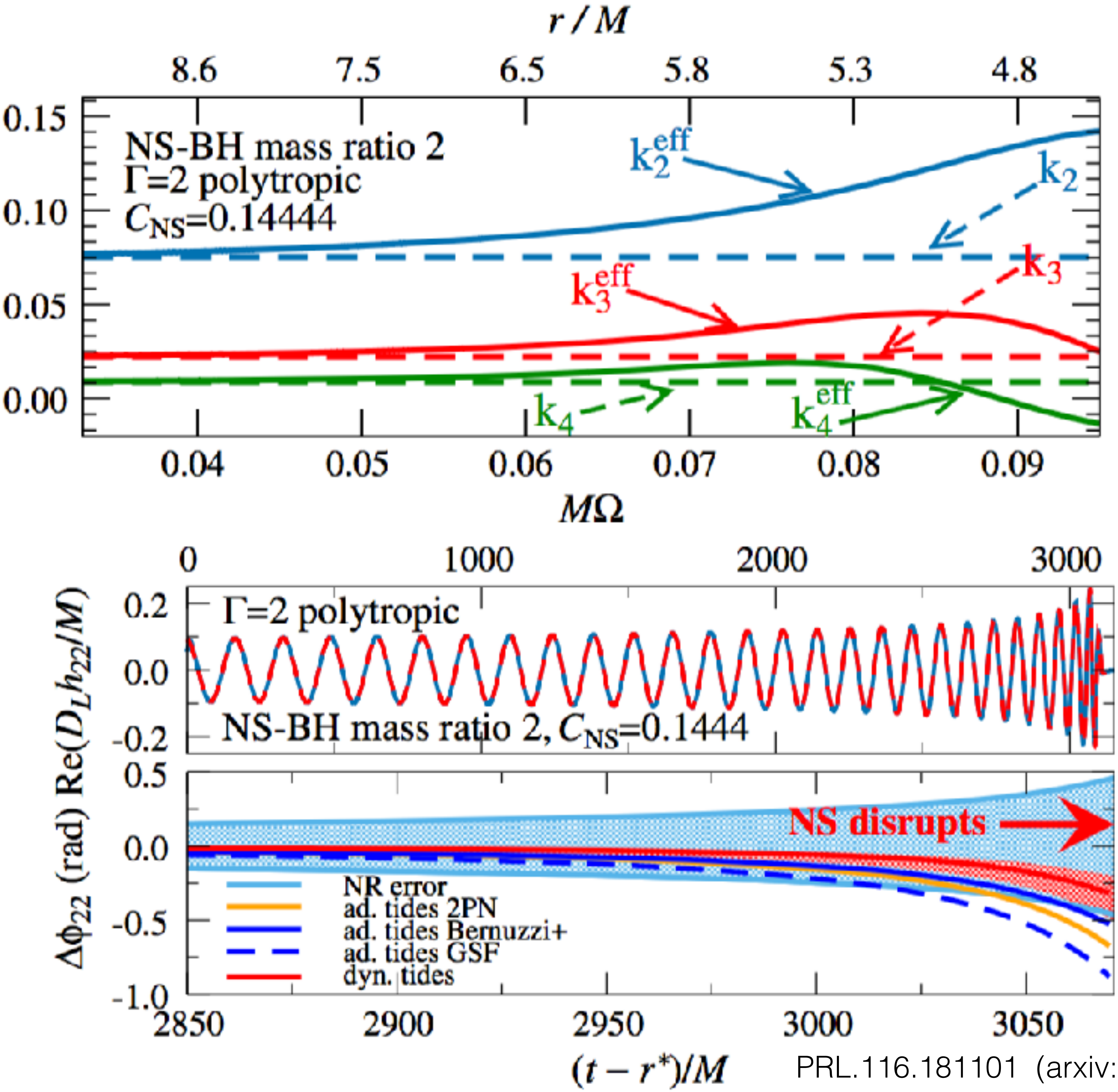
$$\tilde{\Sigma}_3 = \frac{29925}{11M^7} c^{14} (q\sigma_3^{(1)} + \sigma_3^{(2)}/q).$$

Dynamic tide

$$k_\ell^{\rm eff} = k_\ell \left[a_\ell + \frac{b_\ell}{2} \left(\frac{Q^{\rm DT}_{m=\ell}}{Q^{\rm AT}_{m=\ell}} + \frac{Q^{\rm DT}_{m=-\ell}}{Q^{\rm AT}_{m=-\ell}} \right) \right],$$

$$\frac{Q^{\rm DT}_m}{Q^{\rm AT}_m} \approx \frac{\omega_f^2}{\omega_f^2 - (m\Omega)^2} + \frac{\omega_f^2}{2(m\Omega)^2 \epsilon_f \Omega_f' (\phi - \phi_f)} \pm \frac{i\omega_f^2}{(m\Omega)^2 \sqrt{\epsilon_f}} e^{\pm i\Omega_f' \epsilon_f (\phi - \phi_f)^2} \int_{-\infty}^{\sqrt{\epsilon_f}(\phi - \phi_f)} e^{\mp i \Omega_f' s^2} ds, \quad (2)$$

PRL.116.181101 (arxiv:1602.00599)



PRL.116.181101 (arxiv:1602.00599)

Calculation of Tidal deformability

TOV Eq. vs. Diff. Eq. for Tidal deformability

a spherical symmetric star in hydrostatic equilibrium

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$\begin{aligned} ds_0^2 &= g^{(0)}_{\alpha\beta}\, dx^\alpha\, dx^\beta \\ &= -e^{\nu(r)}\, dt^2 + e^{\lambda(r)}\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \end{aligned}$$

$$T_{\alpha\beta} = (\rho + p) u_\alpha u_\beta + p g^{(0)}_{\alpha\beta},$$

TOV eq.

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{Mc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

Mass & Radius

TOV Eq. vs. Diff. Eq. for Tidal deformability

static linearized perturbations due to an external tidal field

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu}$$

$$g_{\alpha\beta} = g^{(0)}_{\alpha\beta} + h_{\alpha\beta},$$

T. Hinderer (2008), K. Thorne and A. Campolattaro (1967)

$$h_{\alpha\beta} = \mathrm{diag}\left[-e^{\nu(r)}H_0(r), e^{\lambda(r)}H_2(r), r^2K(r), r^2\sin^2\theta K(r)\right]Y_{2m}(\theta,\varphi).$$

$$\delta T^0_0 = -\delta\rho = -(dp/d\rho)^{-1}\delta p \qquad \delta T^i_i = \delta p$$

$$H'' + H'\left\{\frac{2}{r} + e^{\lambda}\left[\frac{2m(r)}{r^2} + 4\pi r(p-\rho)\right]\right\} + H\left[-\frac{6e^{\lambda}}{r^2} + 4\pi e^{\lambda}\left(5\rho + 9p + \frac{\rho+p}{dp/d\rho}\right) - \nu'^2\right] = 0,$$

k2 or λ

To obtain Tidal Love number, k2 (1)

T. Hinderer, ApJ, 677, 1216 (2008)

$$k_2 = \frac{8C^5}{5}(1-2C)^2[2+2C(y-1)-y]$$
$$\times \Big\{ 2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)]$$
$$+ 3(1-2C)^2[2-y+2C(y-1)]\ln(1-2C) \Big\}^{-1}$$

$$y = \frac{R\beta(R)}{H(R)} \quad C = \frac{GM}{Rc^2}$$

$$\frac{dH}{dr} = \beta$$

$$\frac{d\beta}{dr} = 2\left(1 - 2\frac{M}{r}\right)^{-1} \times H \left\{ -2\pi\left[5\epsilon + 9P + (d\epsilon/dP)(\epsilon+P)\right] \right.$$
$$\left. + \frac{3}{r^2} + 2\left(1-2\frac{M}{r}\right)^{-1}\left(\frac{M}{r^2} + 4\pi r P\right)^2 \right\}$$
$$+ \frac{2\beta}{r}\left(1 - 2\frac{M}{r}\right)^{-1}\left\{ -1 + \frac{M}{r} + 2\pi r^2(\epsilon - P) \right\}$$

To obtain Tidal Love number, k2 (2)

$$\frac{dH}{dr} = \beta \qquad \frac{d\beta}{dr} = 2\left(1 - 2\frac{M}{r}\right)^{-1} \times H \left\{ -2\pi \left[5\epsilon + 9P + (d\epsilon/dP)(\epsilon + P)\right] + \frac{3}{r^2} + 2\left(1 - 2\frac{M}{r}\right)^{-1} \left(\frac{M}{r^2} + 4\pi r P\right)^2 \right\} + \frac{2\beta}{r}\left(1 - 2\frac{M}{r}\right)^{-1} \left\{ -1 + \frac{M}{r} + 2\pi r^2 (\epsilon - P) \right\}$$

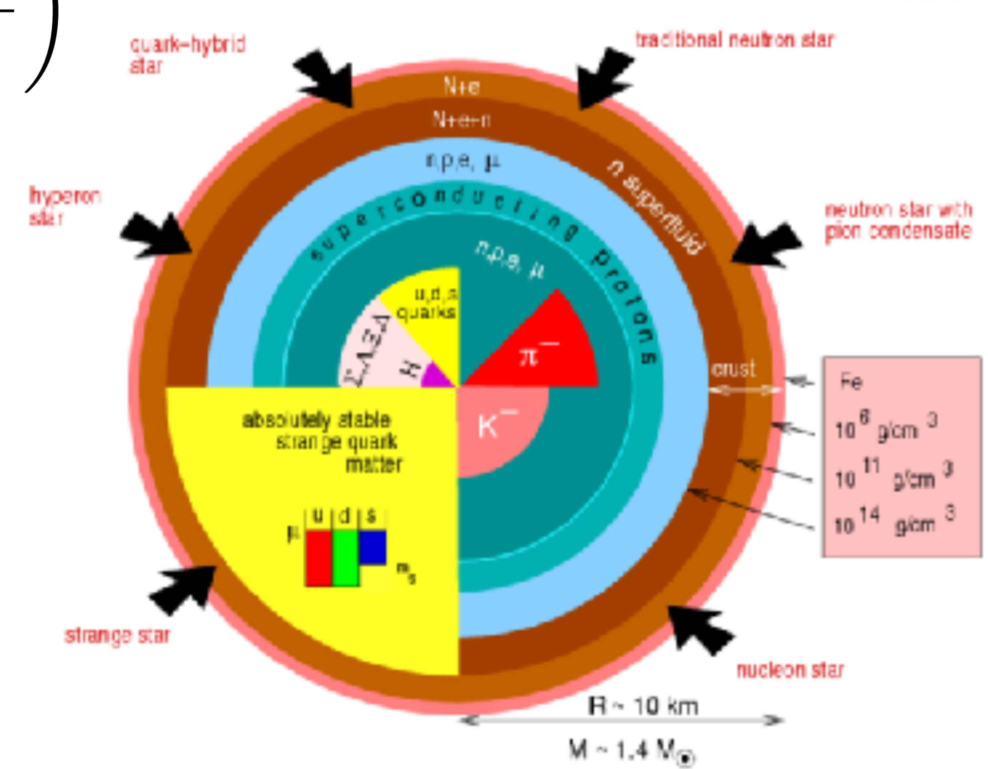
TOV

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi P r^3}{Mc^2}\right)\left(1 - \frac{2GM}{rc^2}\right)$$

$$\frac{dM}{dr} = 4\pi r^2 \left(\frac{\epsilon}{c^2}\right)$$

Compositions of a NS

F. Weber 2005

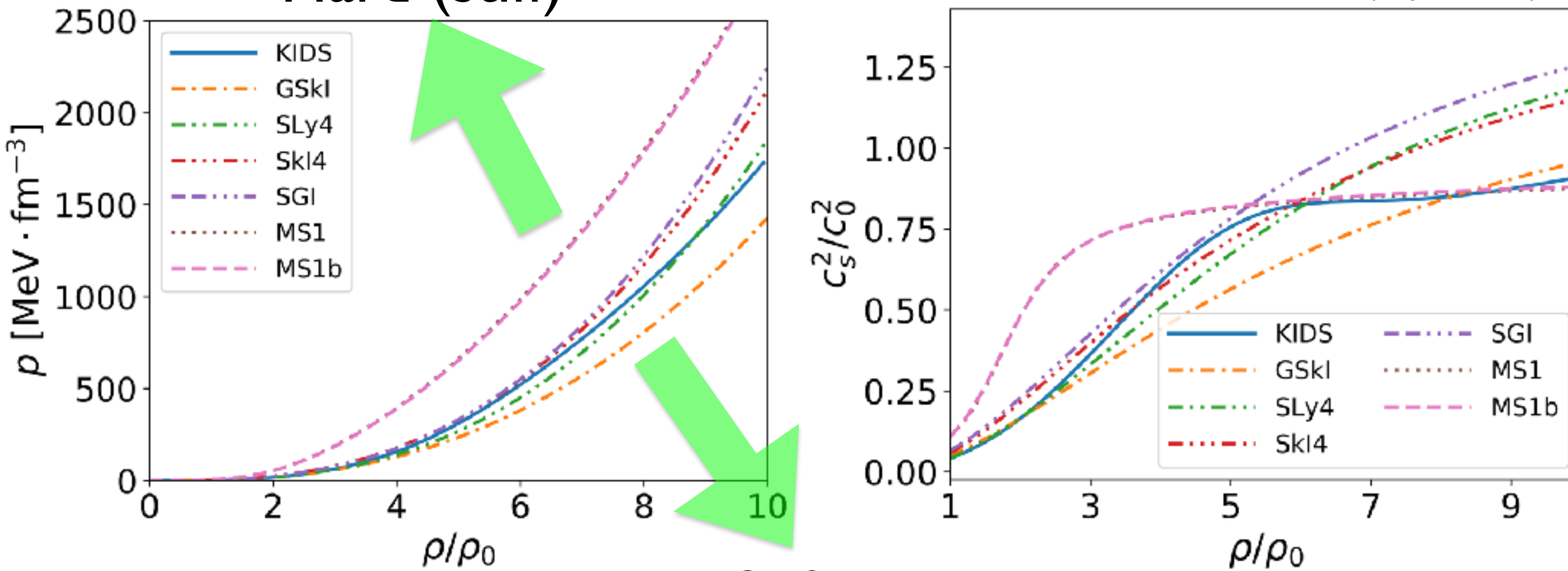


k2 (λ,Λ) depends on NS EoS !!

Equation of State

Hard (stiff)

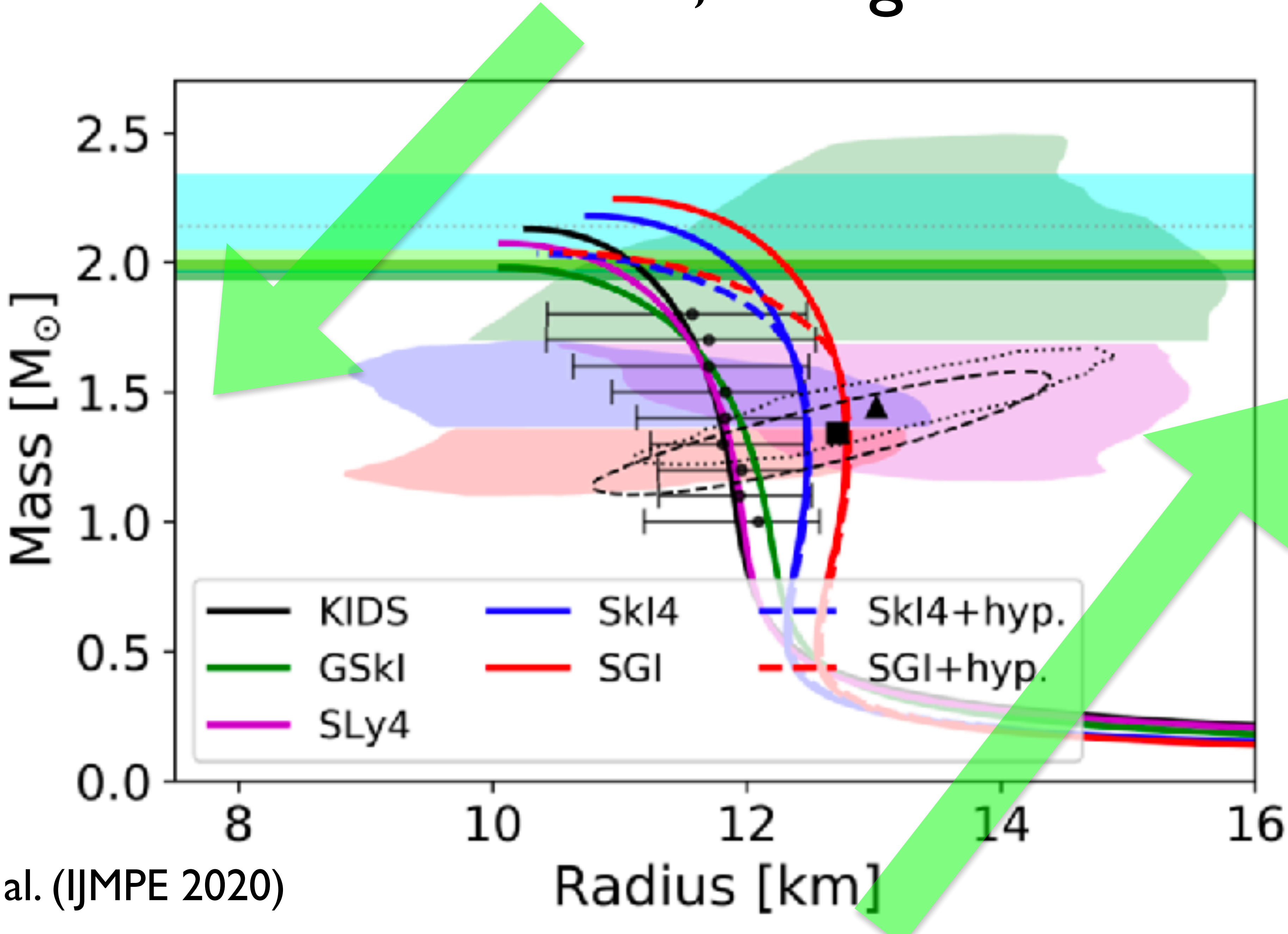
Y.-M. Kim et al. (EPJA 2020)



Soft

Constraints on Equation of State

prefer soft EoS: GW170817,strangeness



M. Kim et al. (IJMPE 2020)

prefer harder EoS: M_max, NICER

Polytropic equation of state

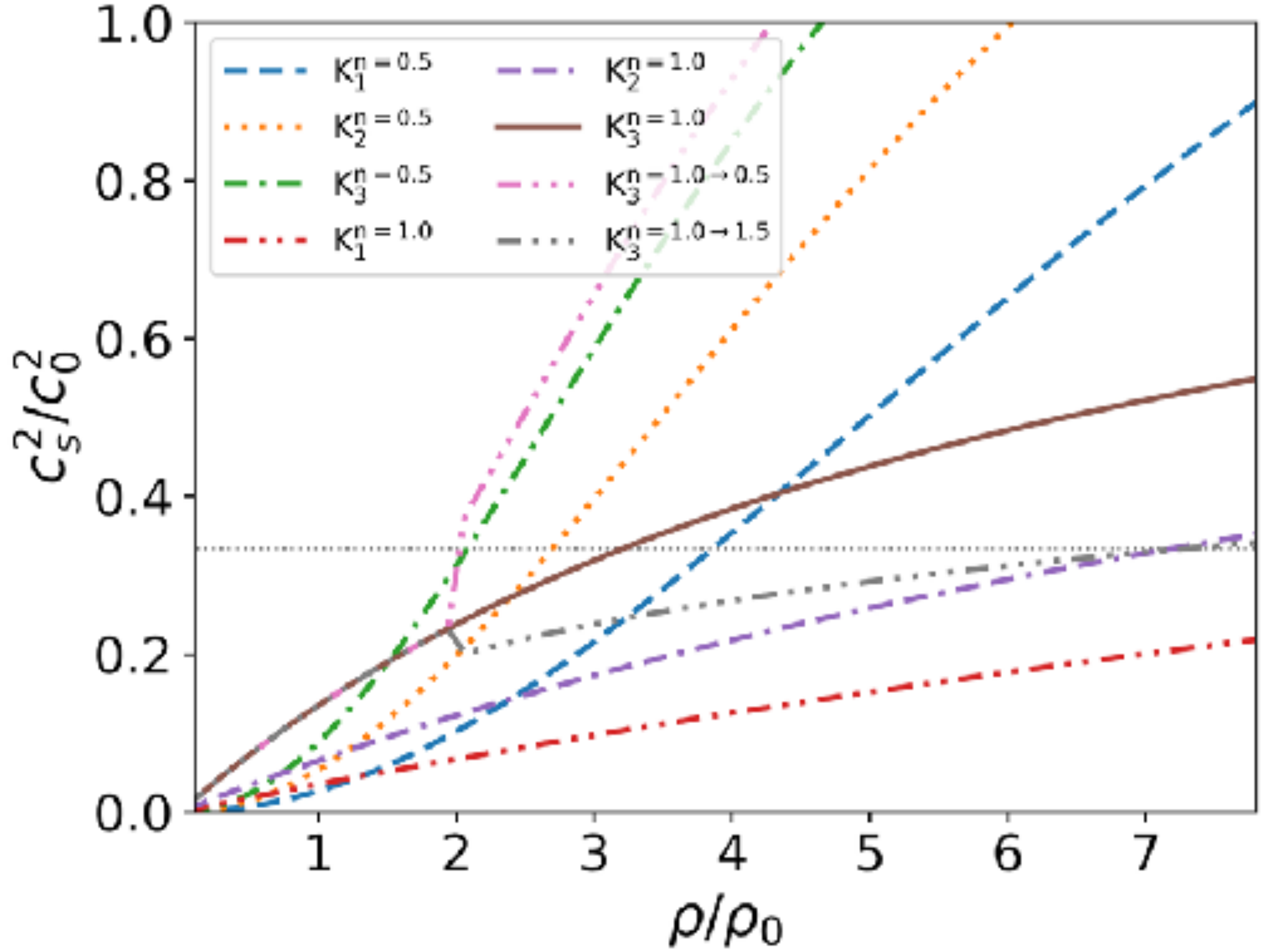
$$p = K\rho^{\Gamma} = K\rho^{1+1/n}$$

$$\begin{aligned}\epsilon &= (1+a)\rho c_0^2 + \frac{p}{\Gamma - 1} \\ &= (1+a)\rho c_0^2 + \frac{K}{\Gamma - 1}\rho^{\Gamma},\end{aligned}$$

(a=0)

$$\frac{c_s^2}{c_0^2} = \frac{\mathrm{d}p}{\mathrm{d}\epsilon} = \Gamma\frac{p}{\epsilon + p}$$

$K_1 < K_2 < K_3$

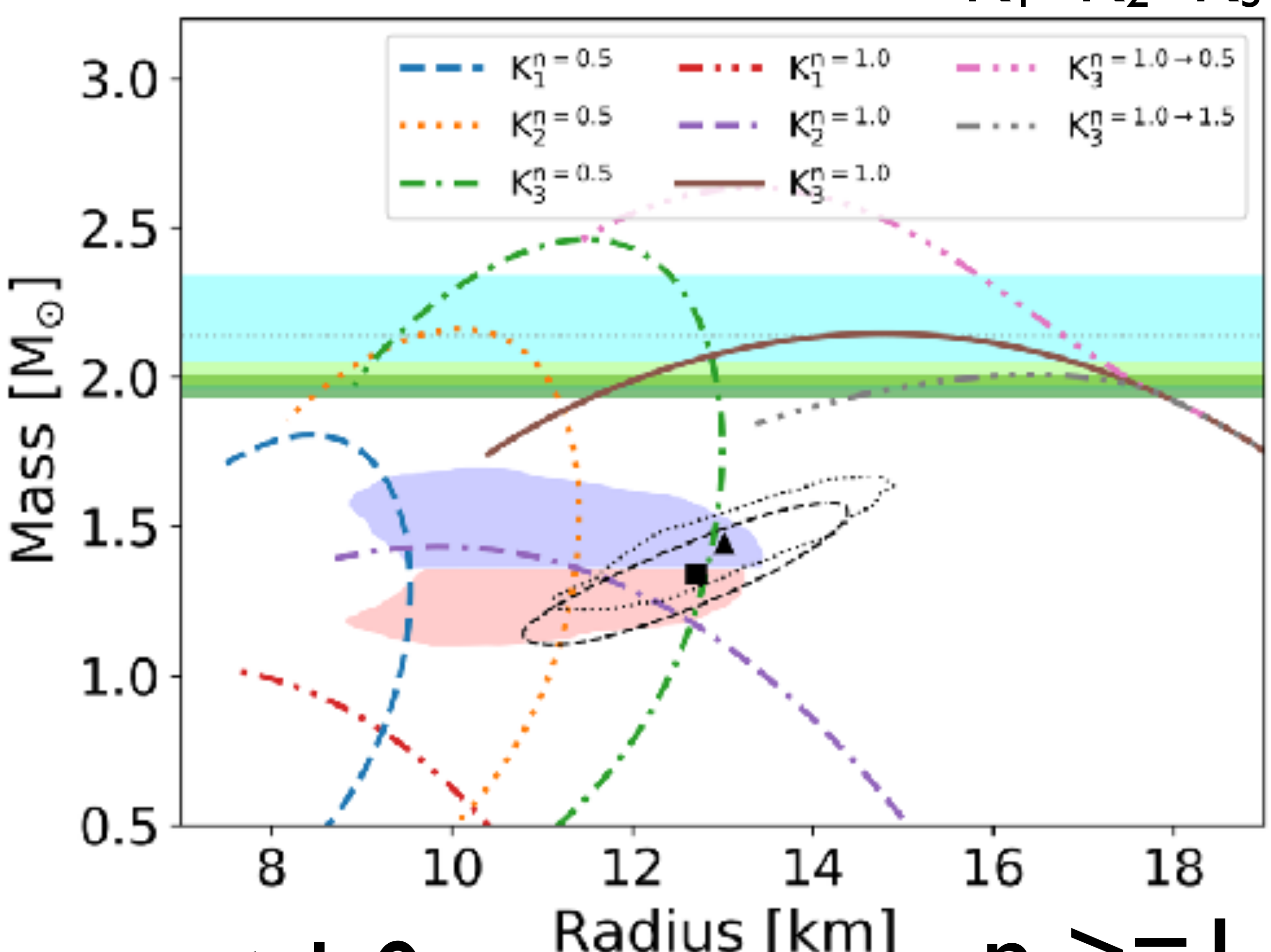


M. Kim et al., JKPS, 78, 932-941(2021)
https://link.springer.com/article/10.1007/s40042-021-00084-4

Polytropic EoSs vs. Realistic EoSs (1)

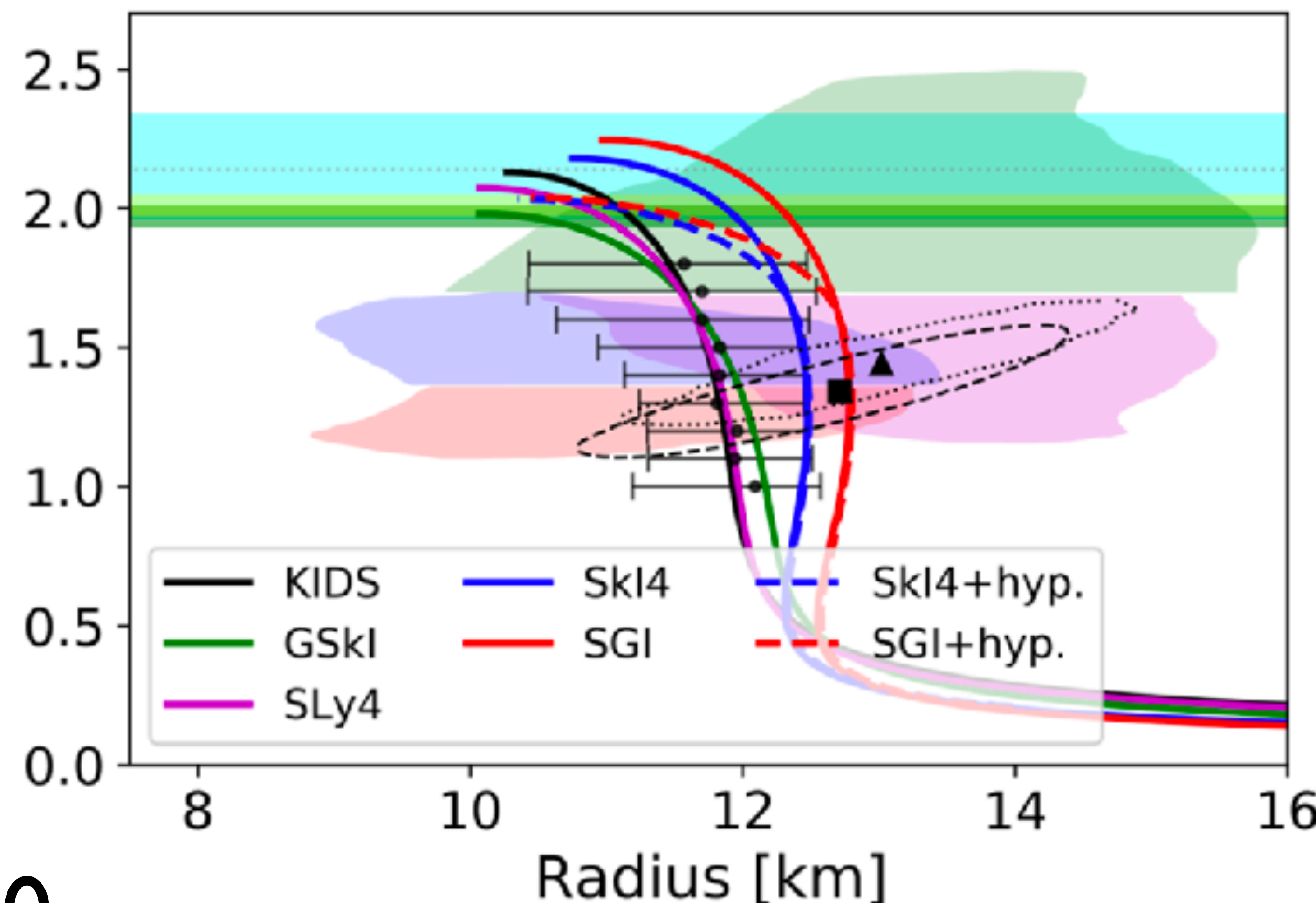
M. Kim et al. (JKPS 2021)

$K_1<K_2<K_3$



n < 1.0

M. Kim et al. (IJMPE 2020)

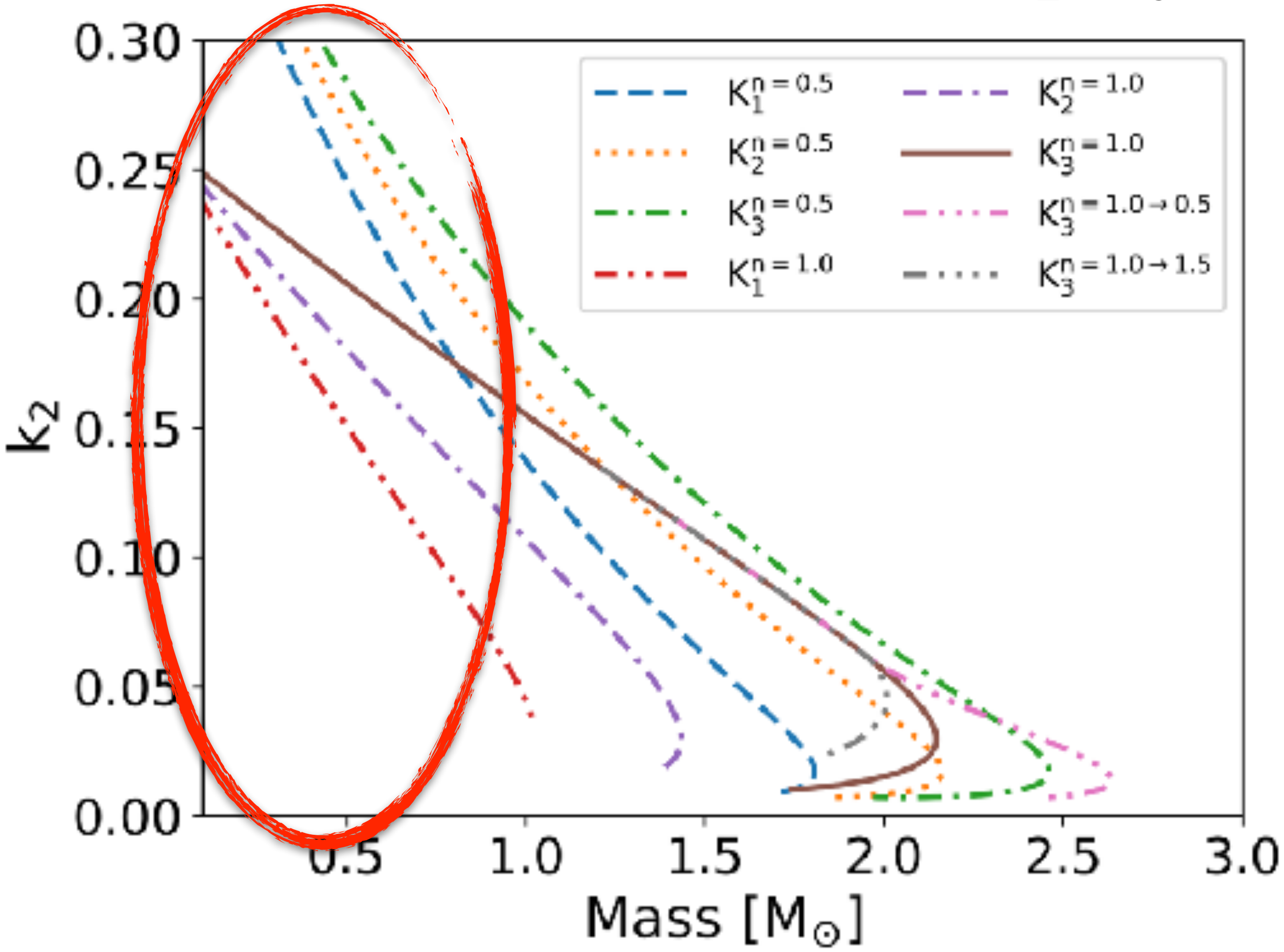


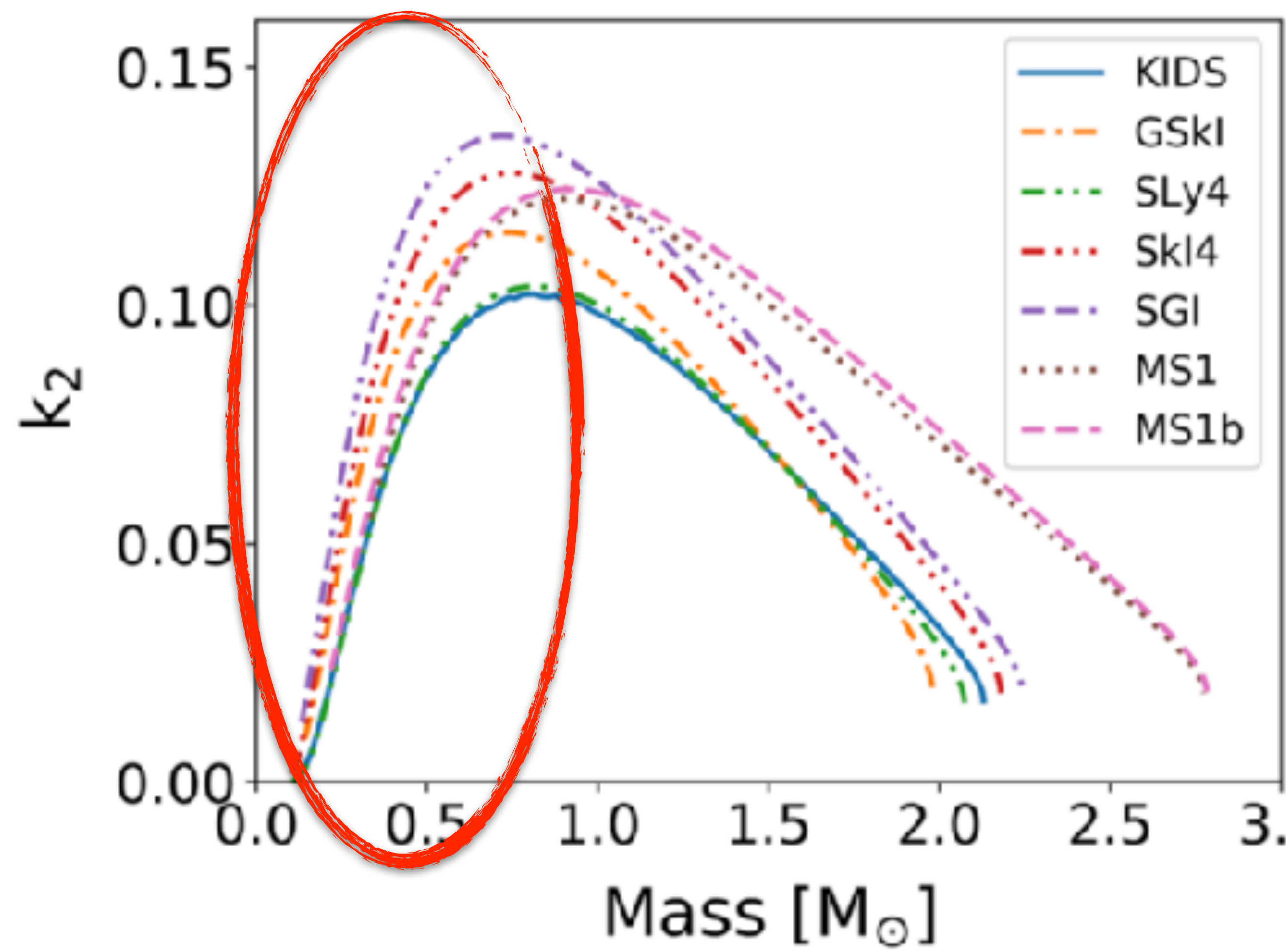
n >=1.0

Polytropic EoSs vs. Realistic EoSs (2)

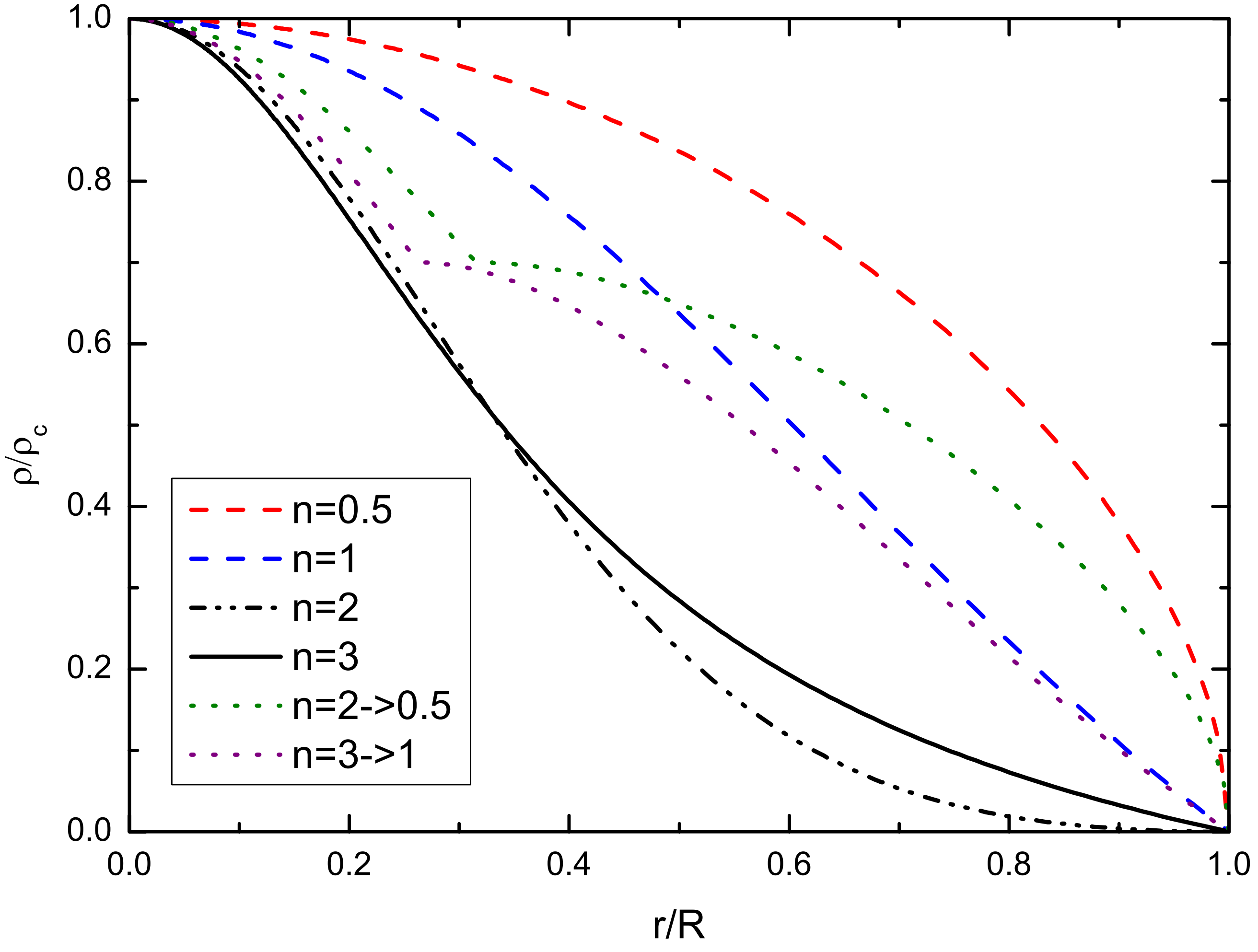
$$\Lambda = \lambda/M^5 \rightarrow G\left(\frac{c^2}{GM}\right)^5 \lambda = \frac{2}{3}\left(\frac{Rc^2}{GM}\right)^5 k_2$$

$K_1 < K_2 < K_3$

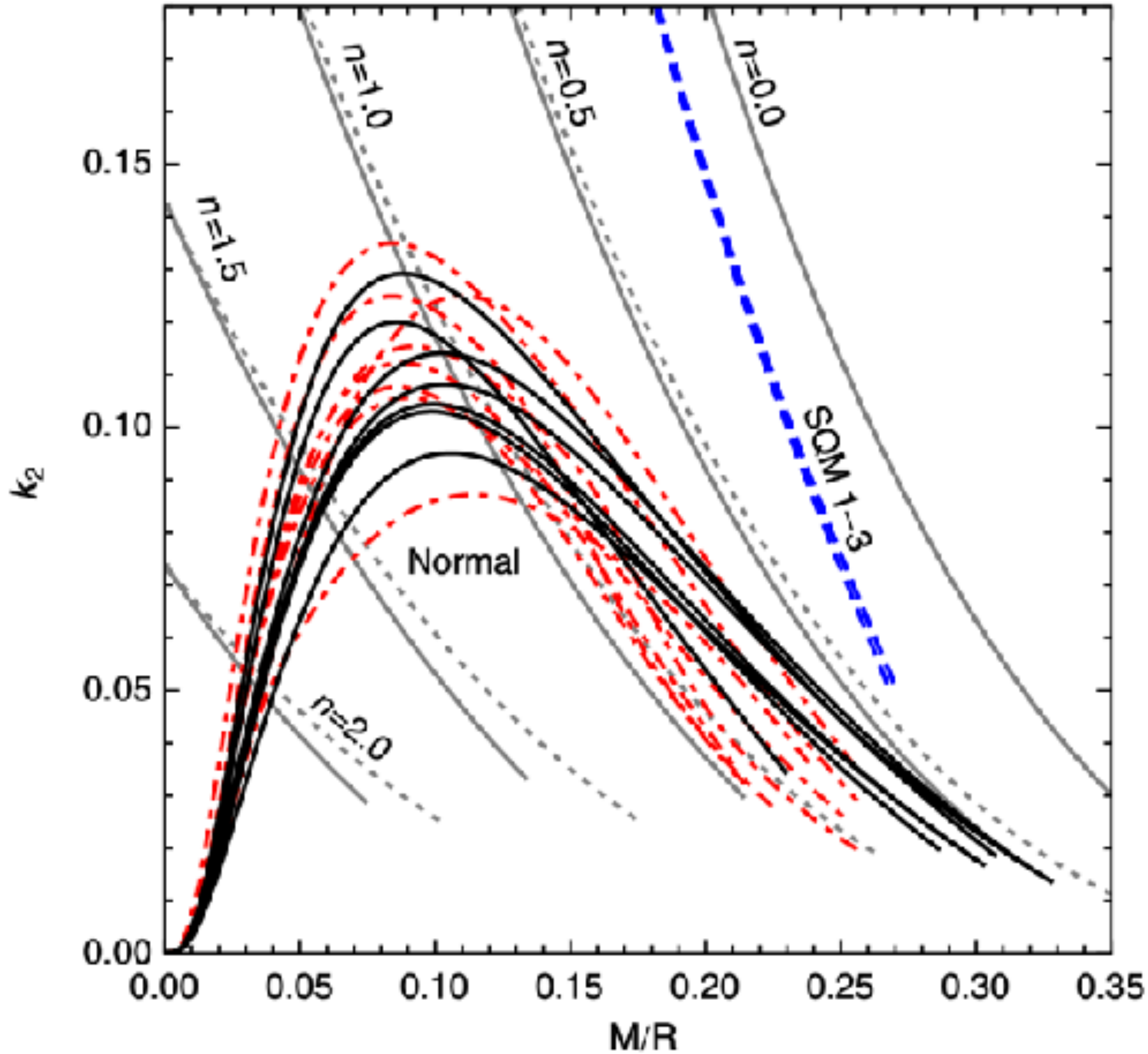




Density profiles for polytropic eos

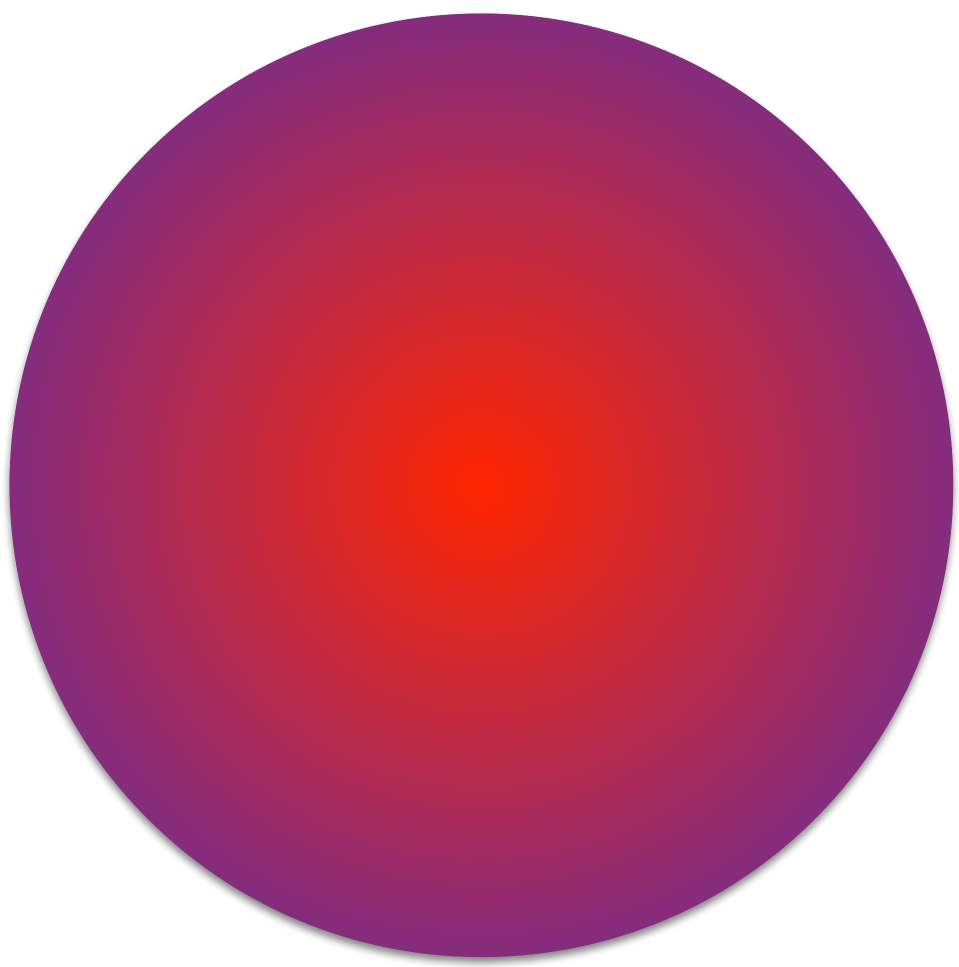


PHYSICAL REVIEW D **81**, 123016 (2010)

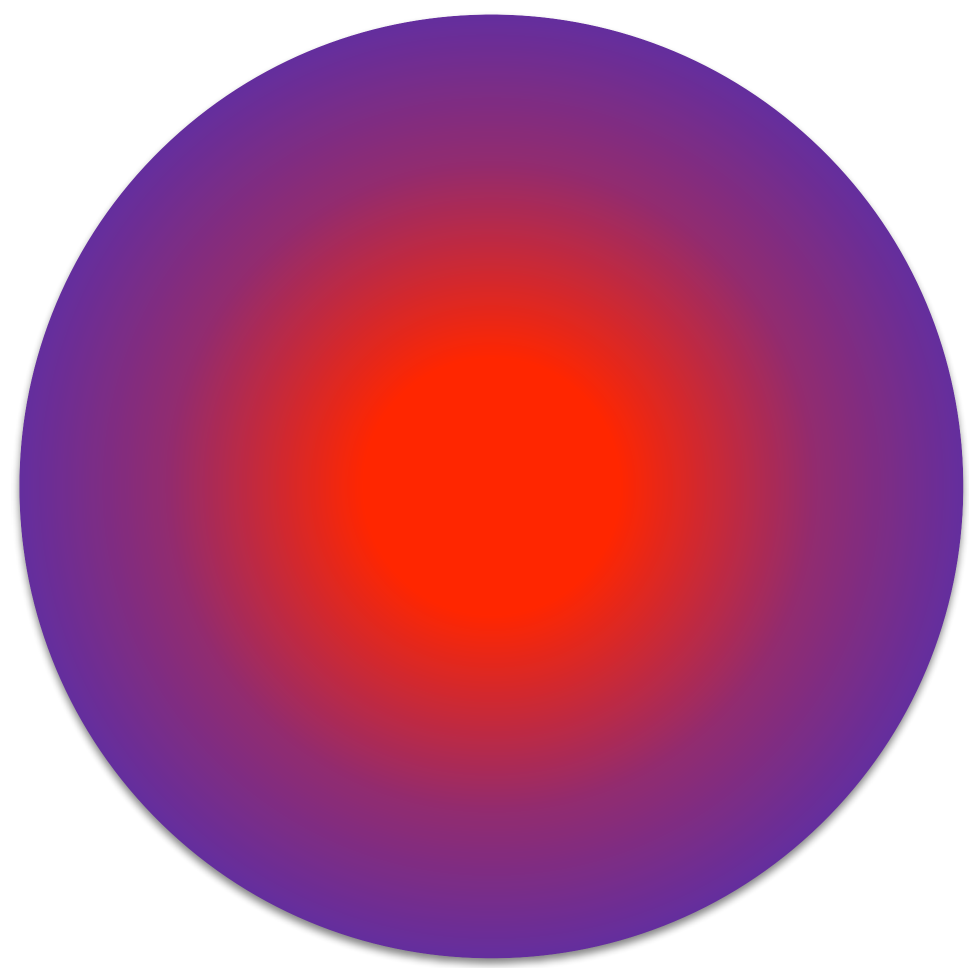


Tidal Love number, k2

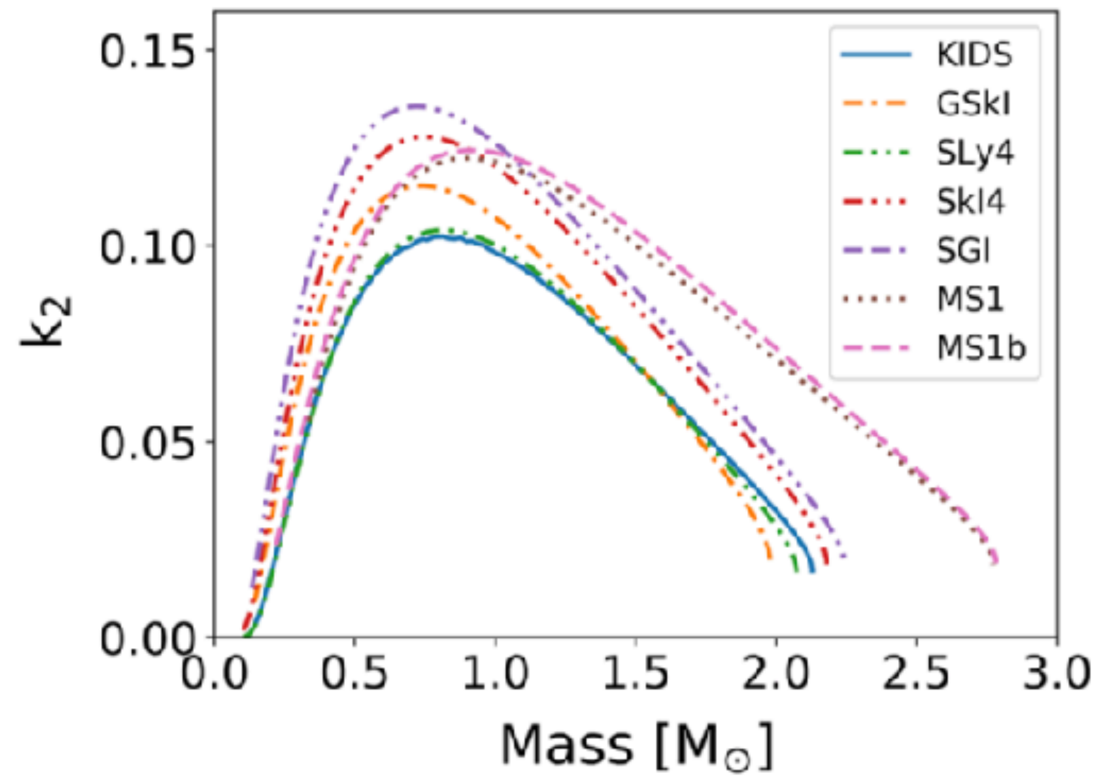
High k2

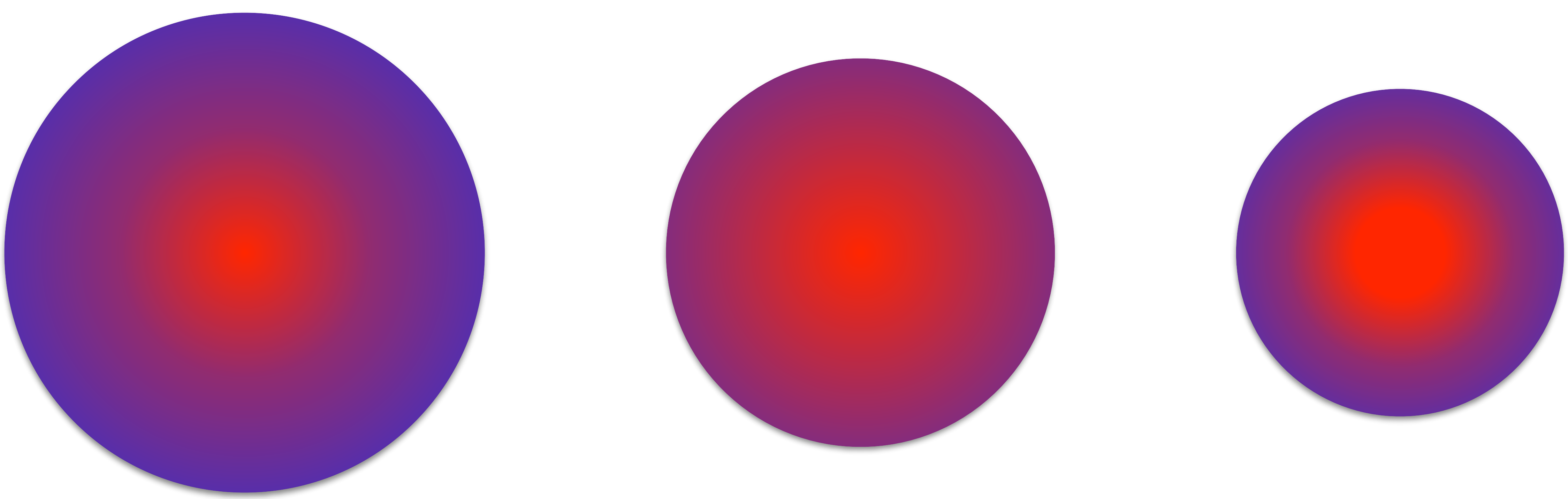


low k2



Tidal Love number, k2



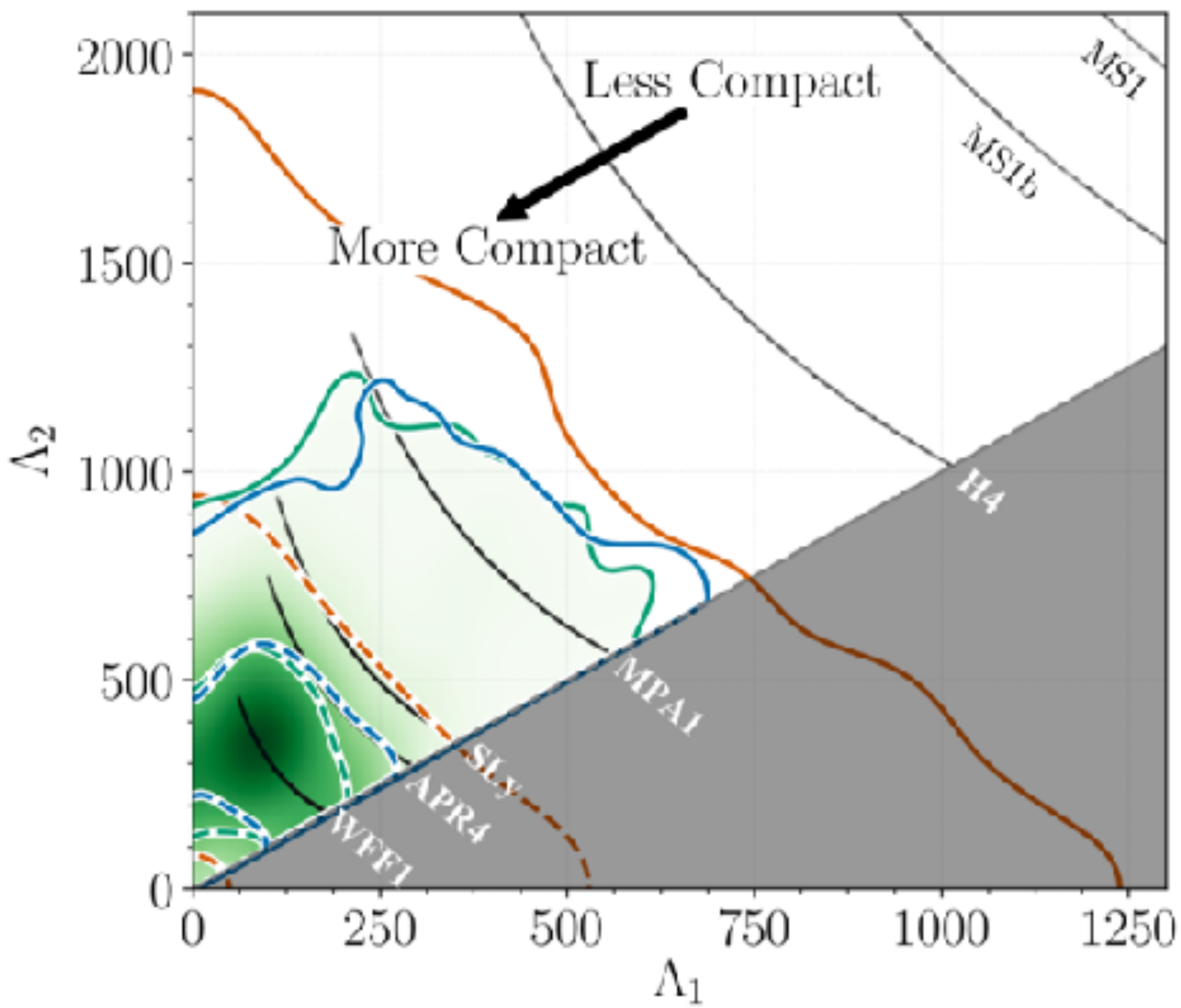


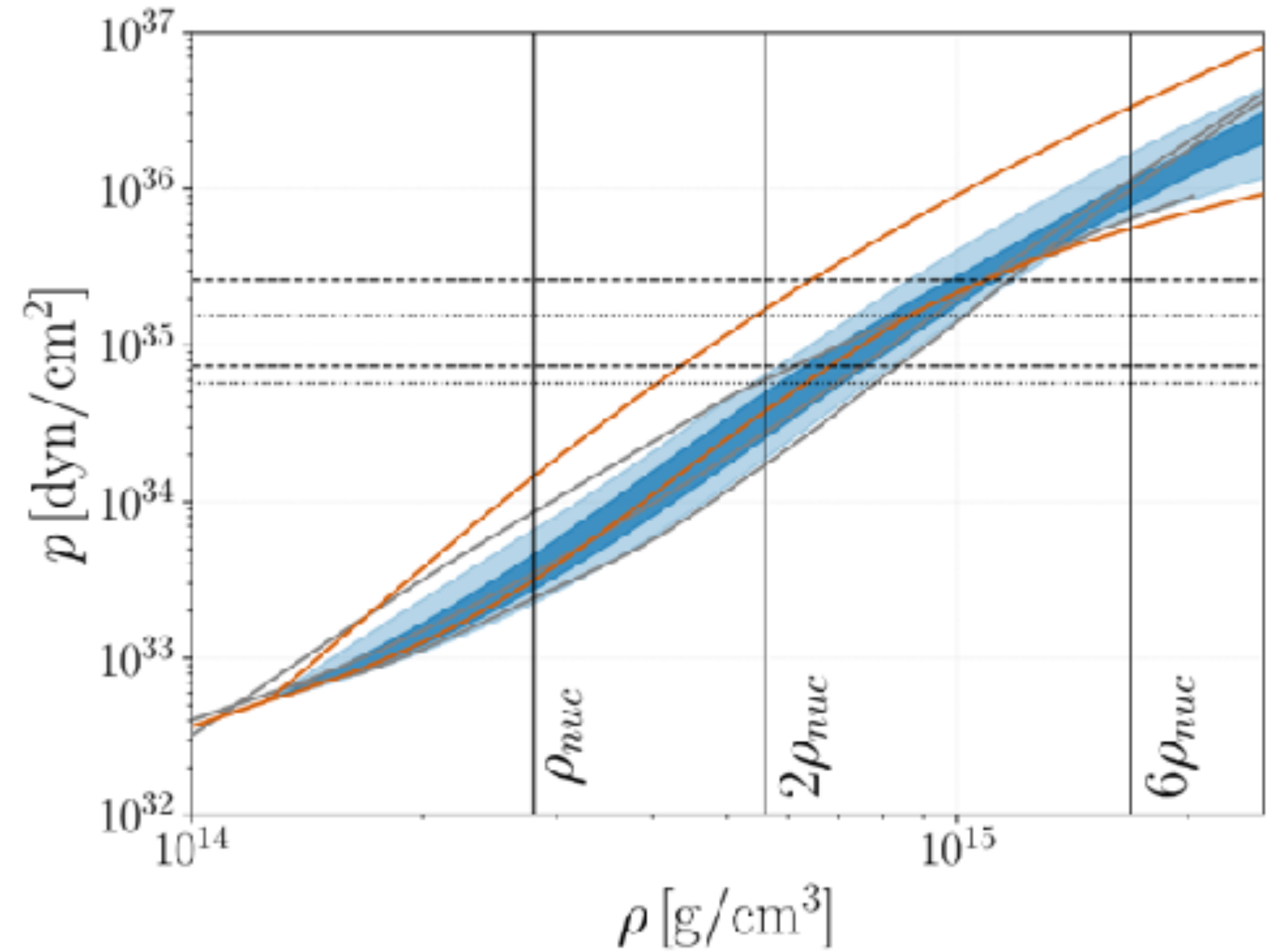
Measurements of GW170817

$\Lambda(1.4M_\odot)=190^{+390}_{-120}$

$P(2\,\rho_{nuc}) = 3.5^{+2.7}_{-1.7} \times 10^{34}$ dyne/cm^2

$P(6\,\rho_{nuc}) = 9.0^{+7.9}_{-2.6} \times 10^{35}$ dyne/cm^2

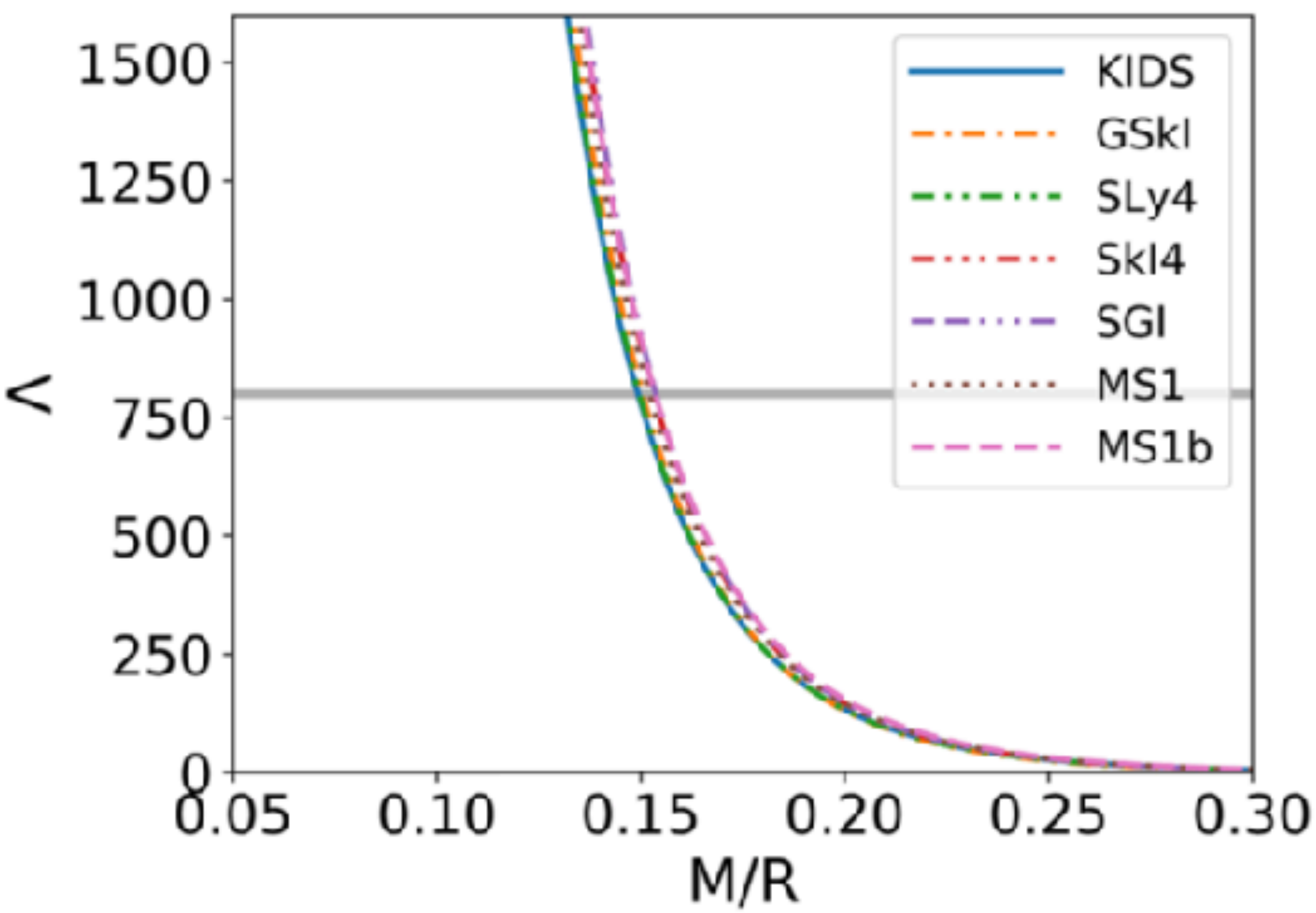




Abbott et al. (LSC and Virgo), arxiv:1805.11581 (PhysRevLett.121.161101)

$\rho_{nuc} = 2.8 \times 10^{14}$ g/cm^3

Eos-insensitive relation



Insensitive to EoS

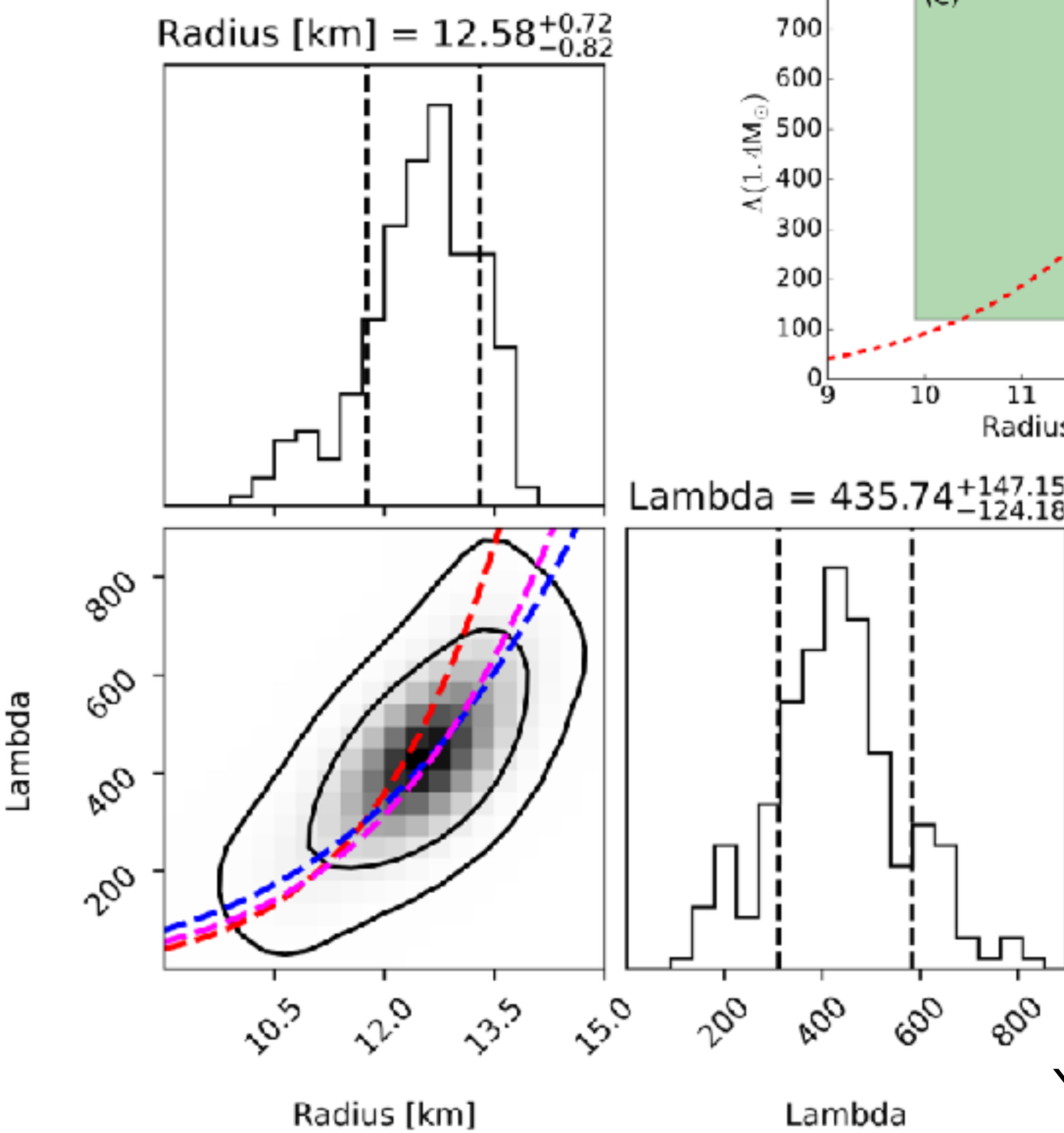
K. Yagi and N. Yunes, Phys. Rep. 681 (2017) 1

$$C = a_0 + a_1(\ln\Lambda) + a_2(\ln\Lambda)^2$$

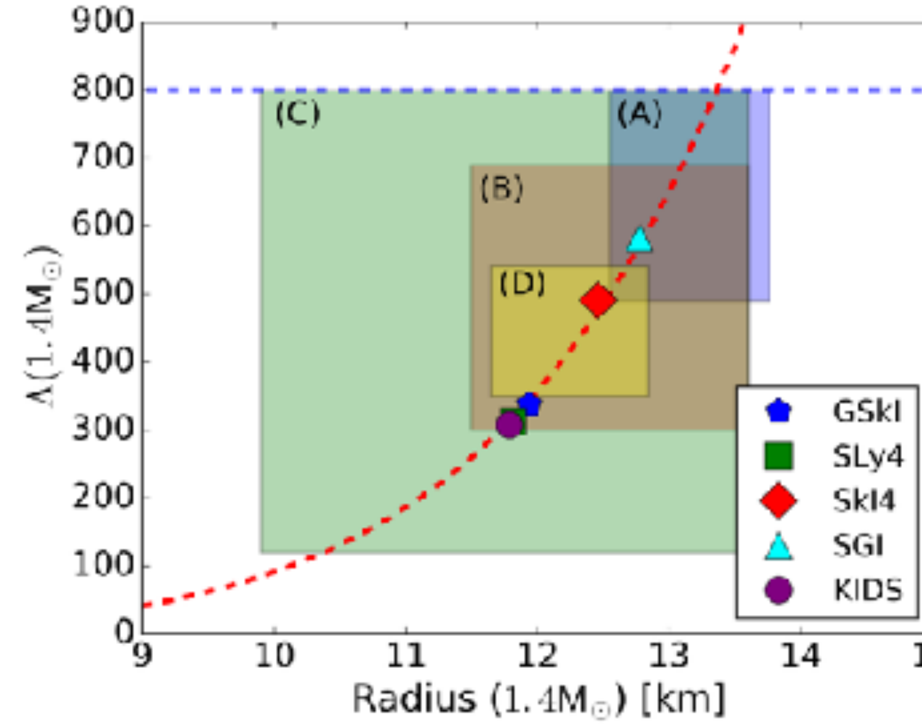
a_0=0.360, a_1= - 0.0355, a_2= 0.000705

C=GM/Rc^2

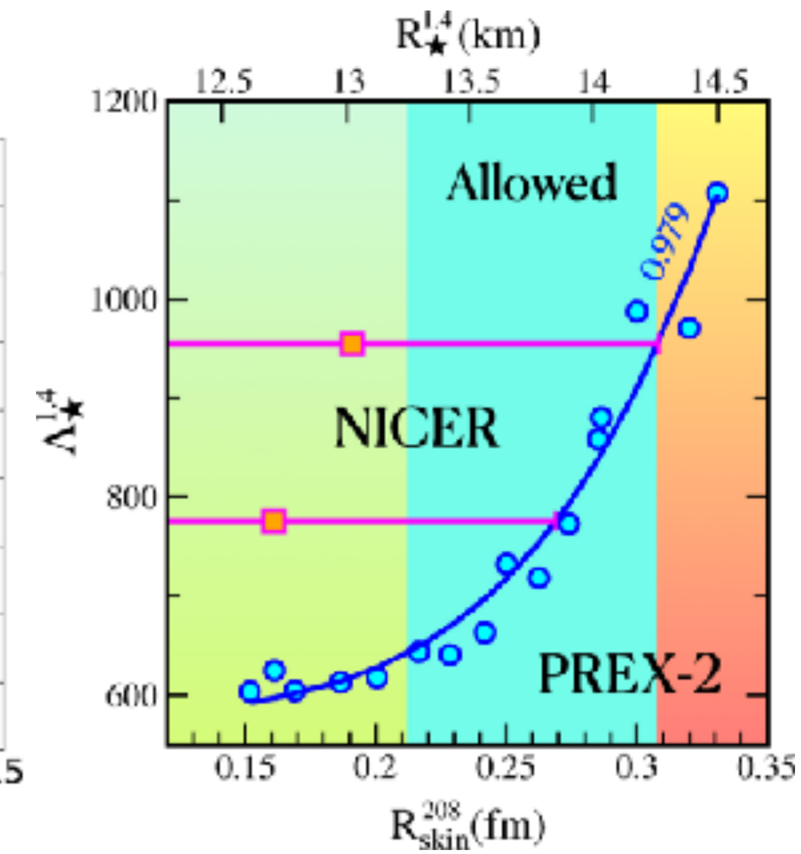
Lambda-Radius relation



Kim et al., PhysRevC.98.065805 (2018)



Implication of PREX 2 - PhysRevLett.126.172503 (2021)



Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6}\ (R/km)^{7.5}$

[C] PhysRevLett.120.172703.pdf

Blue line: $\Lambda(1.4M_\odot) = 1.35 * 10^{-3}\ (R/km)^{5.0}$

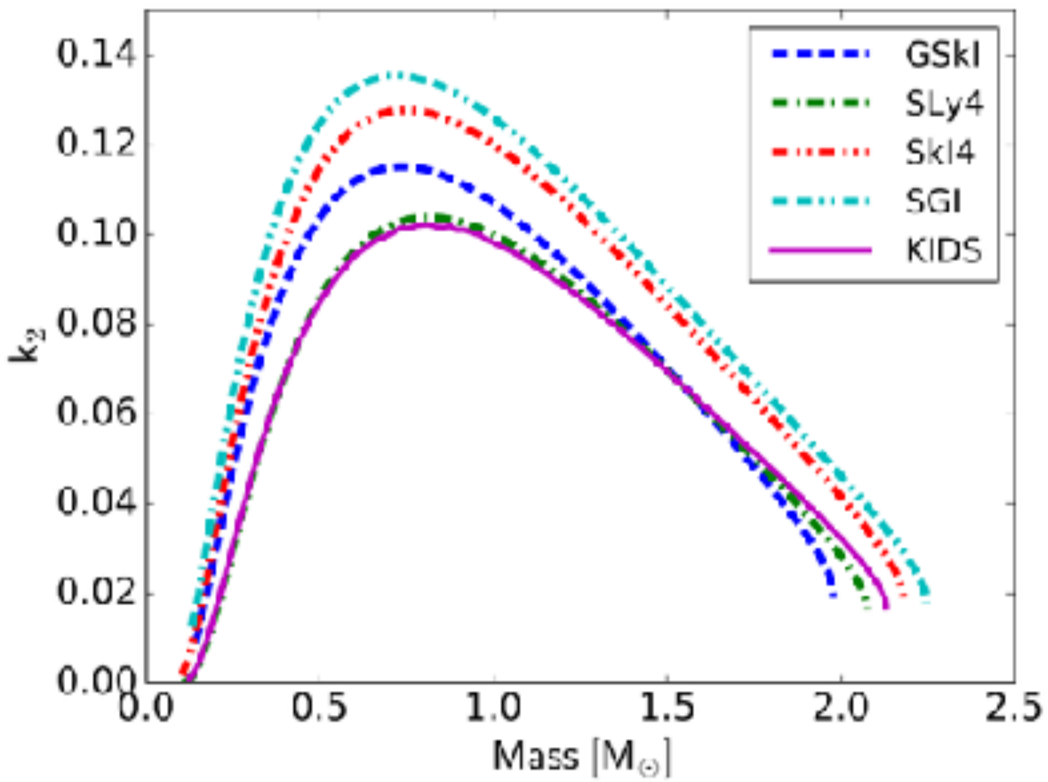
Implication of PREX 2 - PhysRevLett.126.172503 (2021)

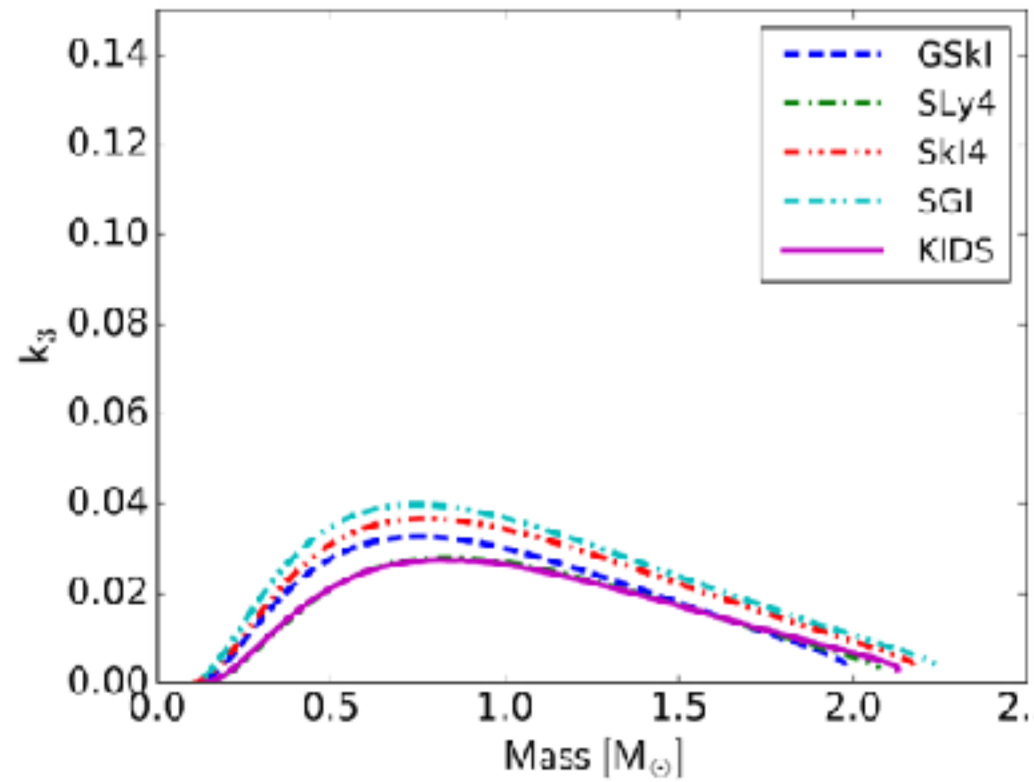
=> $\Lambda \sim R^{4.8}$

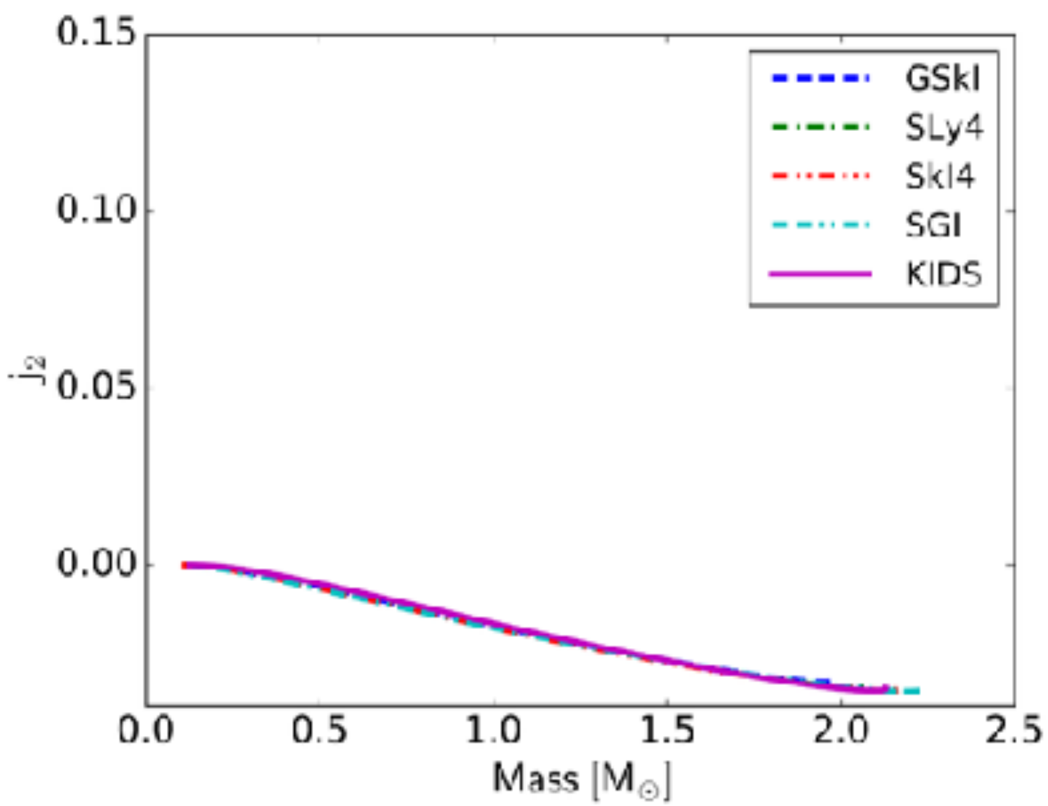
Magenta line: $\Lambda(1.4M_\odot) = 1.05 * 10^{-4}\ (R/km)^{6.0}$

Y.-M. Kim, in progress

Beyond k2







$$\psi(x) = \frac{3}{128\nu x^{5/2}}\Bigg\{1 + \left(\frac{3715}{756} + \frac{55}{9}\nu\right)x + \Bigg(\frac{113}{3} \times \times (\eta_1\chi_1 + \eta_2\chi_2) - \frac{38}{3}\nu(\chi_1 + \chi_2)\Bigg)x^{1.5} + O(x^2) + \Lambda x^5 + (\delta\Lambda + \Sigma)x^6 + (\tilde{\Lambda} + \tilde{\Sigma} + \tilde{\Gamma})x^{6.5} + O(x^7)\Bigg\}, \tag{15}$$

TOV solver in lalsimulation

```
import lalsimulation as lalsim
```

EoS generation

Polytrope EoS 내장함수 사용

```
eos = lalsim.SimNeutronStarEOSPolytrope(Gamma,
                                         reference_pressure_SI,
                                          reference_density_SI)
```

Polytrope EoS table 직접 계산해서 eosfile 생성 (첫번째 컬럼 pressure, 두번째 컬럼 density)

```
eos = lalsim.SimNeutronStarEOSFromFile(eosfile)
```

Solving TOV

```
eosfam = lalsim.CreateSimNeutronStarFamily(eos)

mass = 1.4 * lal.MSUN_SI
radius = lalsim.SimNeutronStarRadius(mass,eosfam)
k2 = lalsim.SimNeutronStarLoveNumberK2(mass,eosfam)
```

참고 자료: https://lscsoft.docs.ligo.org/lalsuite/lalsimulation/group___l_a_l_sim_neutron_star__h.html


To be continued ...

궁금한 것은 이메일로 물어보세요~ : ymkim@kasi.re.kr