

Gravitational Wave Cosmology

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Topics

- Cosmological models and parameters
- Hubble constant and Hubble Tension
- Measurements of distances to GW sources
- Multi-messenger astronomy for Hubble constant
- Future prospects of dark siren measurements

Big Bang Cosmology

- Cosmological Principle
 - Universe is homogenous and isotropic
- Einstein's equations :

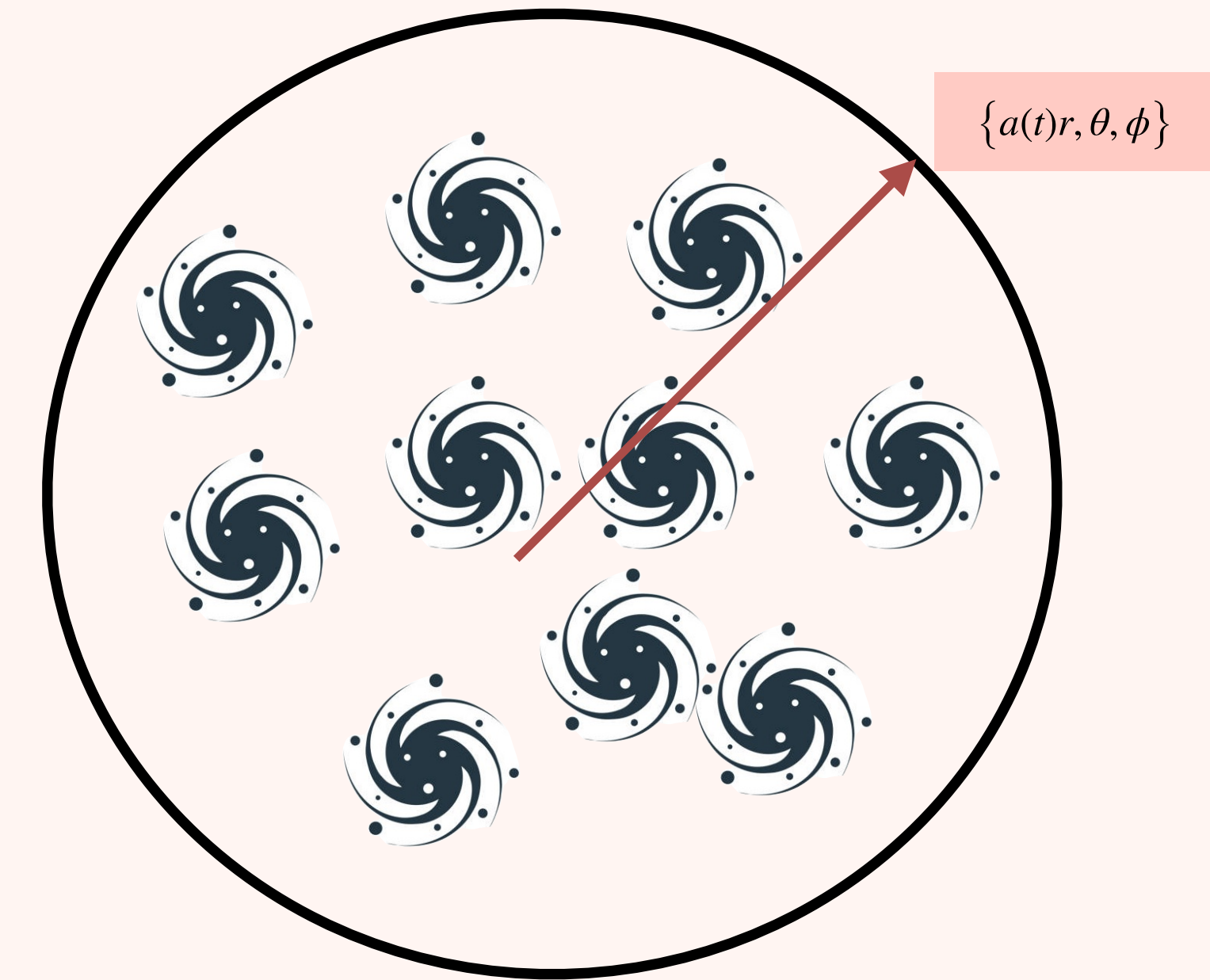
$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}$$

where Λ is the cosmological constant

- Friedman-Robertson-Walker metric for homogenous and isotropic universe

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right\}$$

where $(r, \vec{\Omega})$ is the coming coordinates and $a(t)$ is the scale factor.



Expansion of the universe

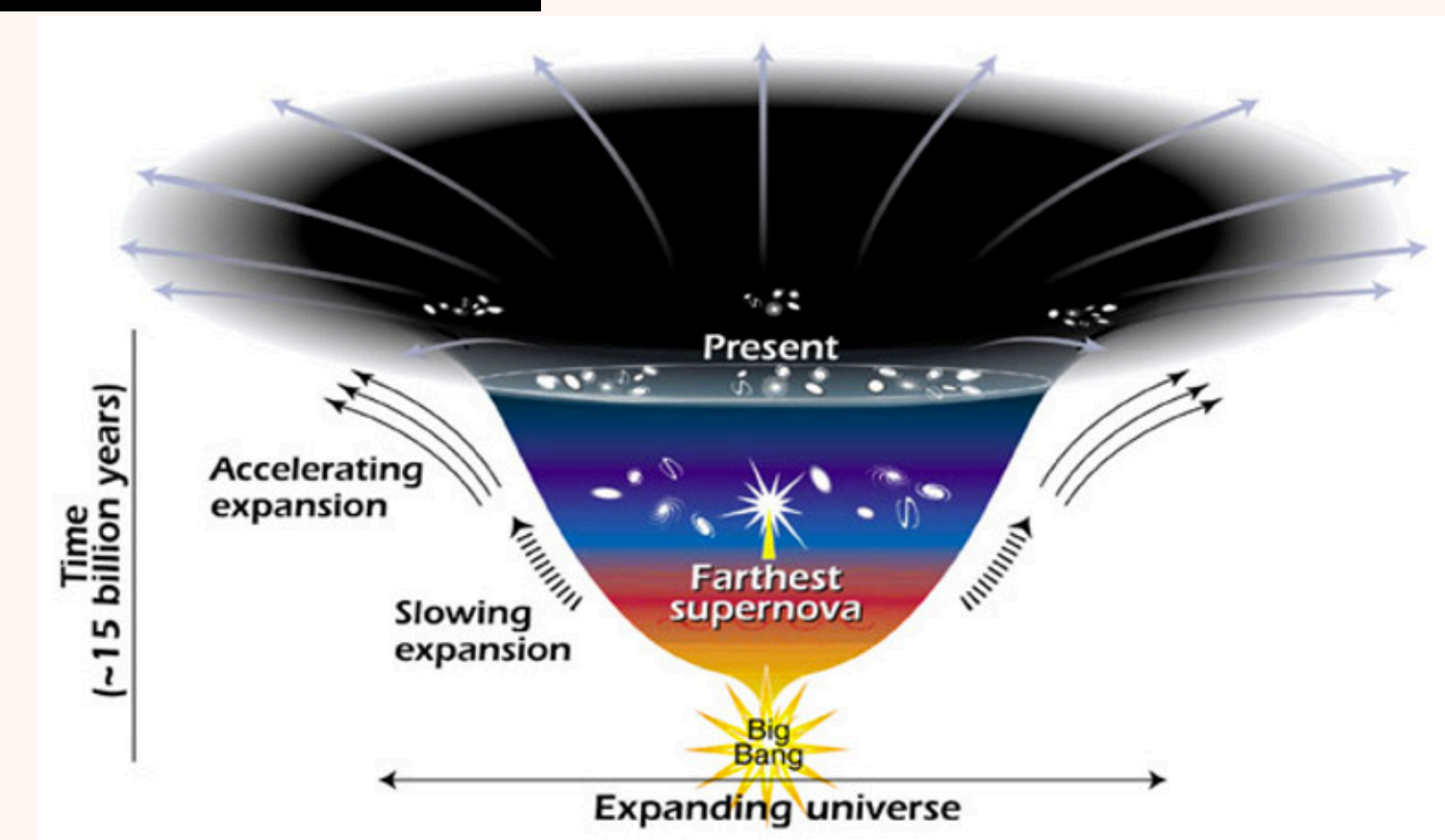
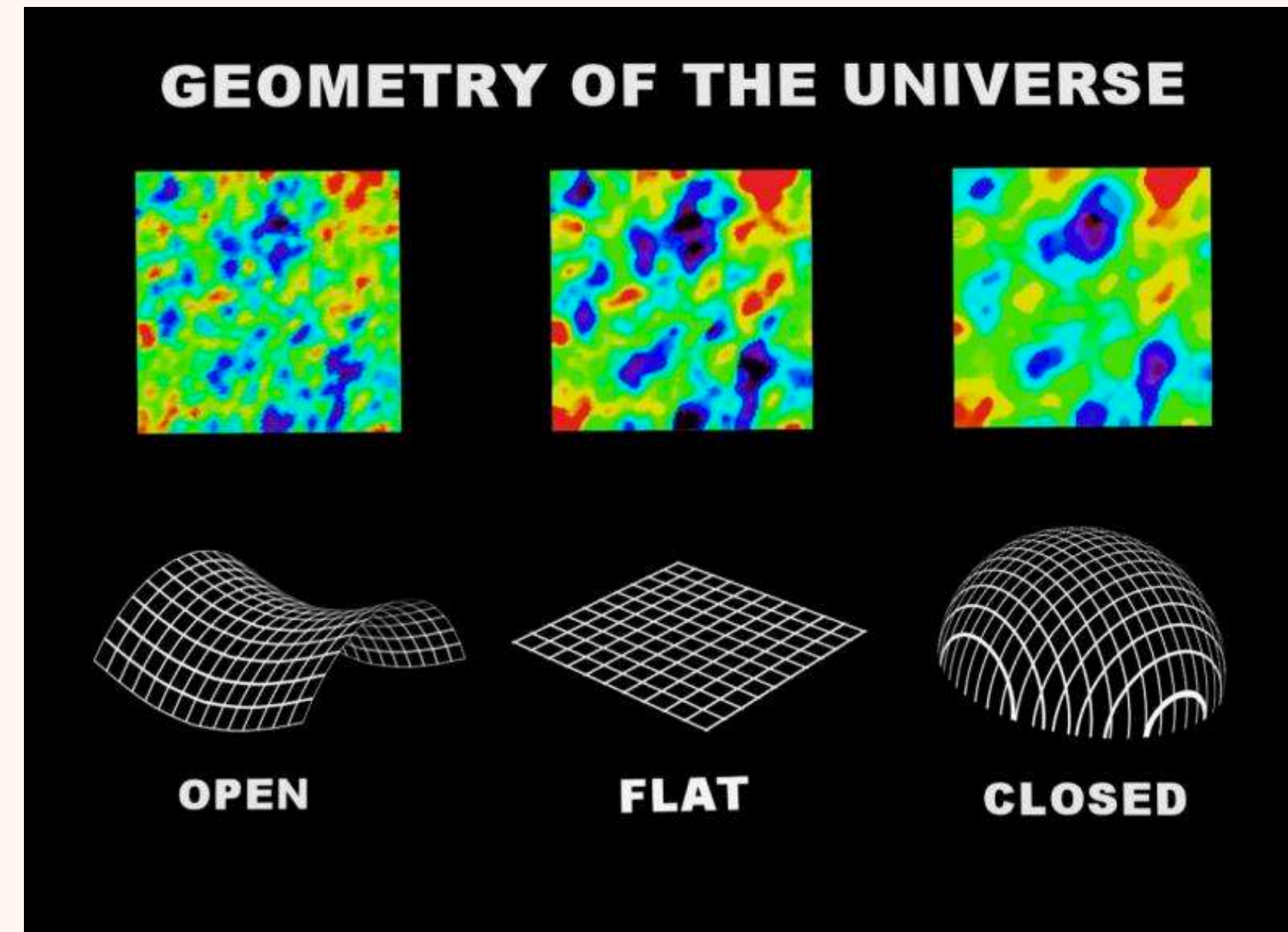
- The energy-momentum tensor $T_{\mu\nu}$
 - $T_{\mu\nu} = pg_{\mu\nu} + (p + \rho)U_\mu U_\nu$, U : velocity four vector of the Hubble flow
- The Einstein's equations become

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad \frac{d}{da}(\rho a^3) = -3pa^2$$

p : Pressure, ρ : energy density
- Constituents of the Universe
 - Matter: $p = 0, (p \ll \rho)$. $\rightarrow \rho \propto a^{-3}, \ddot{a} < 0$ (deceleration)
 - Radiation (or relativistic matter): $p = \frac{1}{3}\rho$, $\rightarrow \rho \propto a^{-4}, \ddot{a} < 0$ (deceleration)
 - Dark Energy: $p = -w\rho, w > \frac{1}{3}, \ddot{a} > 0$ (deceleration if $w > 1/3$)

Cosmological Parameters

- Hubble parameter
 - $H(t) = \frac{\dot{a}}{a}$, $H_0 = H(t_0)$: Hubble constant
- Density parameters: $\Omega_0 = \frac{\rho_0}{\rho_{crit}}$
 - $\Omega < 1$: Open,
 - $\Omega > 1$: Closed
 - $\Omega = 1$: Flat
- $\Omega_m = \Omega_b + \Omega_{DM}$ (Baryon, Dark matter)
- If the density of the universe is dominated by matter+radiation, deceleration
- If the density of the universe is dominated by dark energy, acceleration



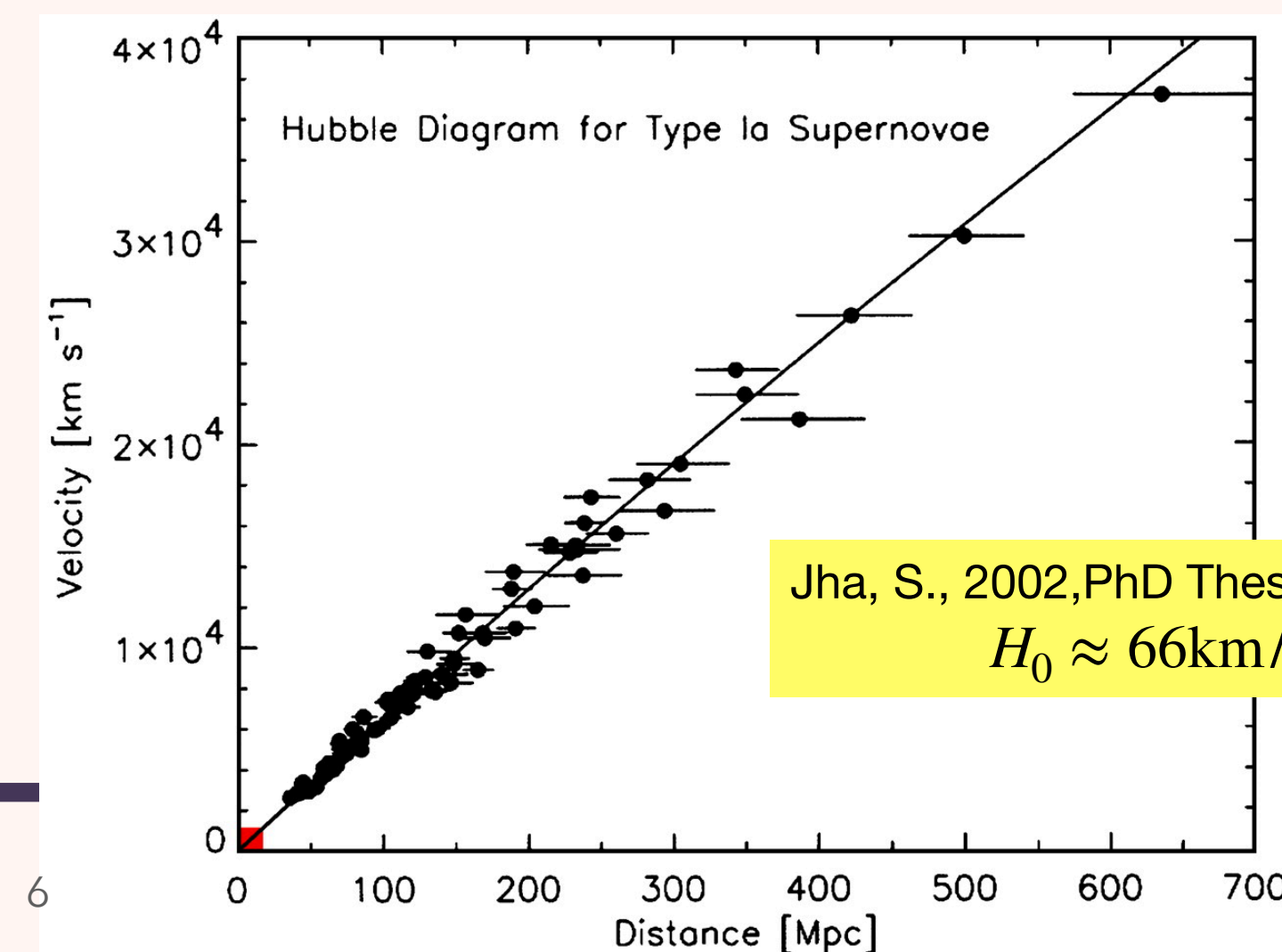
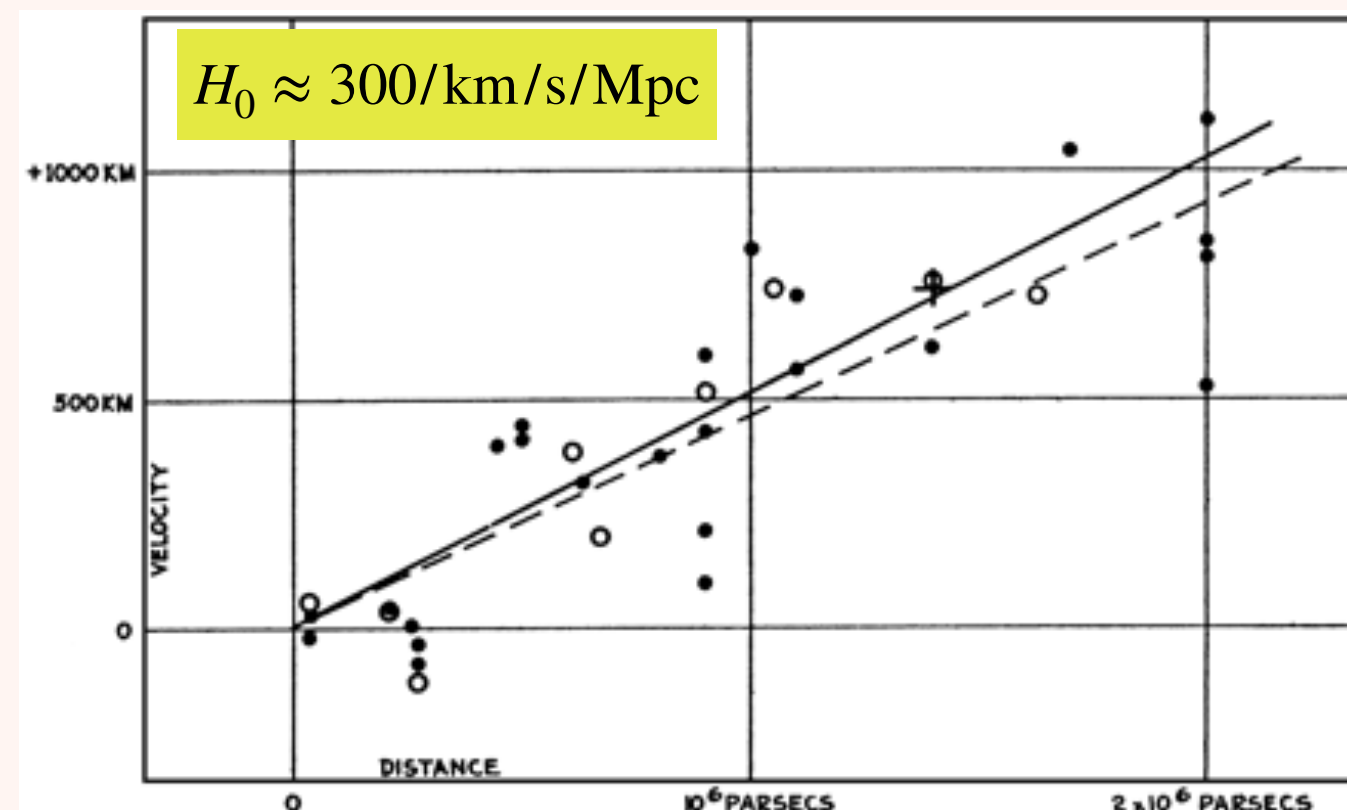
Accelerating universe: $\Omega_\Lambda > \Omega_m$

How the Hubble parameter is measured?

- Hubble's law;

$$v_r = H_0 d$$

- We need to measure radial velocity (v_r , or redshift z) and distance (d) simultaneously for large number of galaxies.
- Galaxies should be not too close (because of the peculiar motion) and not too far (because we are looking at the past).
- Distance measurements are difficult, especially for distant galaxies



Summary of local measurement: Hubble Key Project

(Freedman et al., 2001, ApJ, 553, 47)

- Cepheid Hubble Diagram (up to 25 Mc): $H_0 = 75 \pm 10(r)\text{km/sec/Mpc}$
- Relative distance methods (60 - 400 Mpc)
 - Type Ia SN: $H_0 = 71 \pm 2(r) \pm 6(s)\text{km/sec/Mpc}$
 - Tully-Fisher Relation $H_0 = 71 \pm 3(r) \pm 7(s)\text{km/sec/Mpc}$
 - Fundamental Plane: $H_0 = 70 \pm 5(r) \pm 6(s)\text{km/sec/Mpc}$
 - Surface Brightness Fluctuation $H_0 = 82 \pm 6(r) \pm 9(s)\text{km/sec/Mpc}$
 - Type II SN $H_0 = 72 \pm 9(r) \pm 7(s)\text{km/sec/Mpc}$
- Combination:

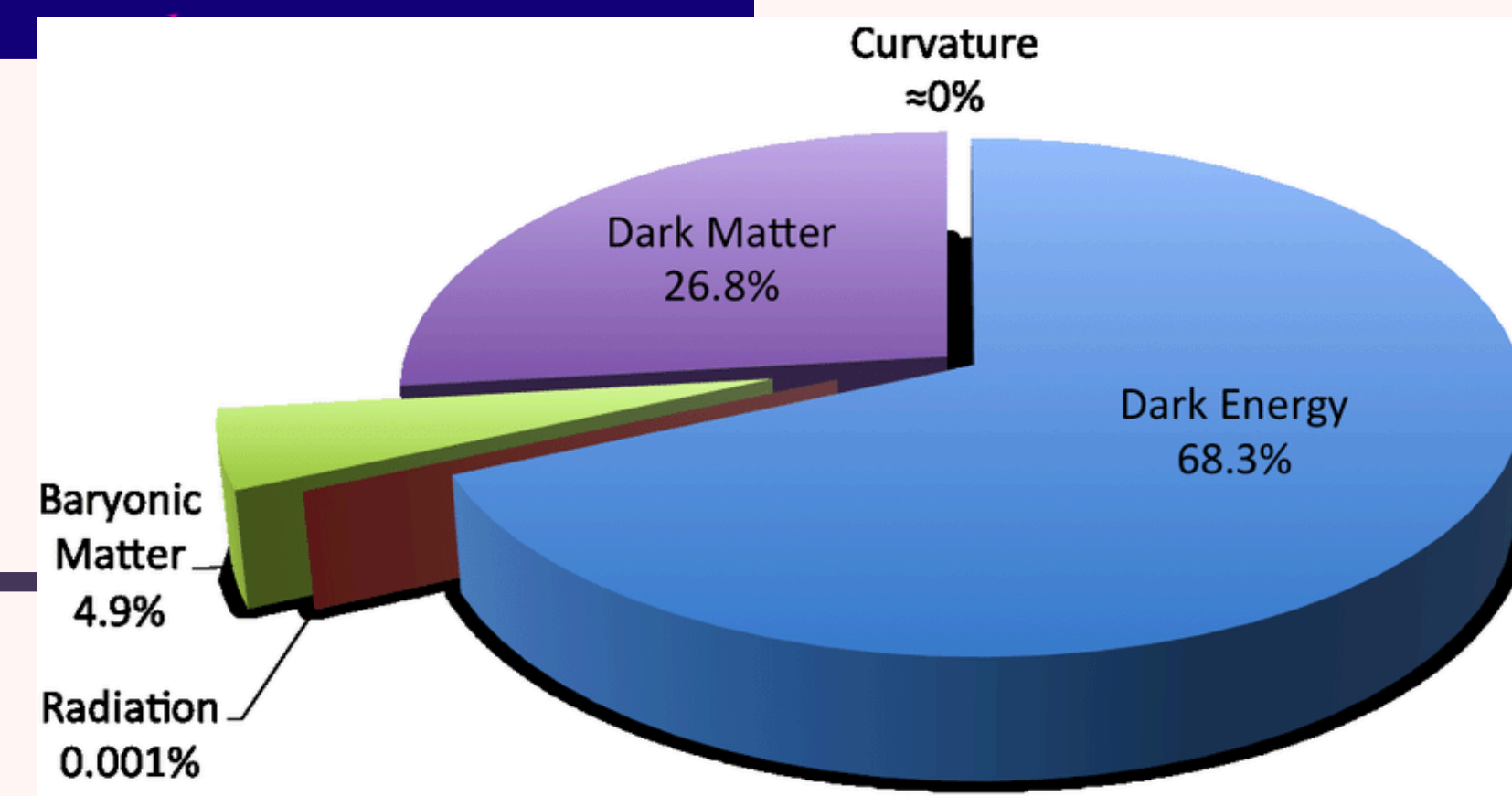
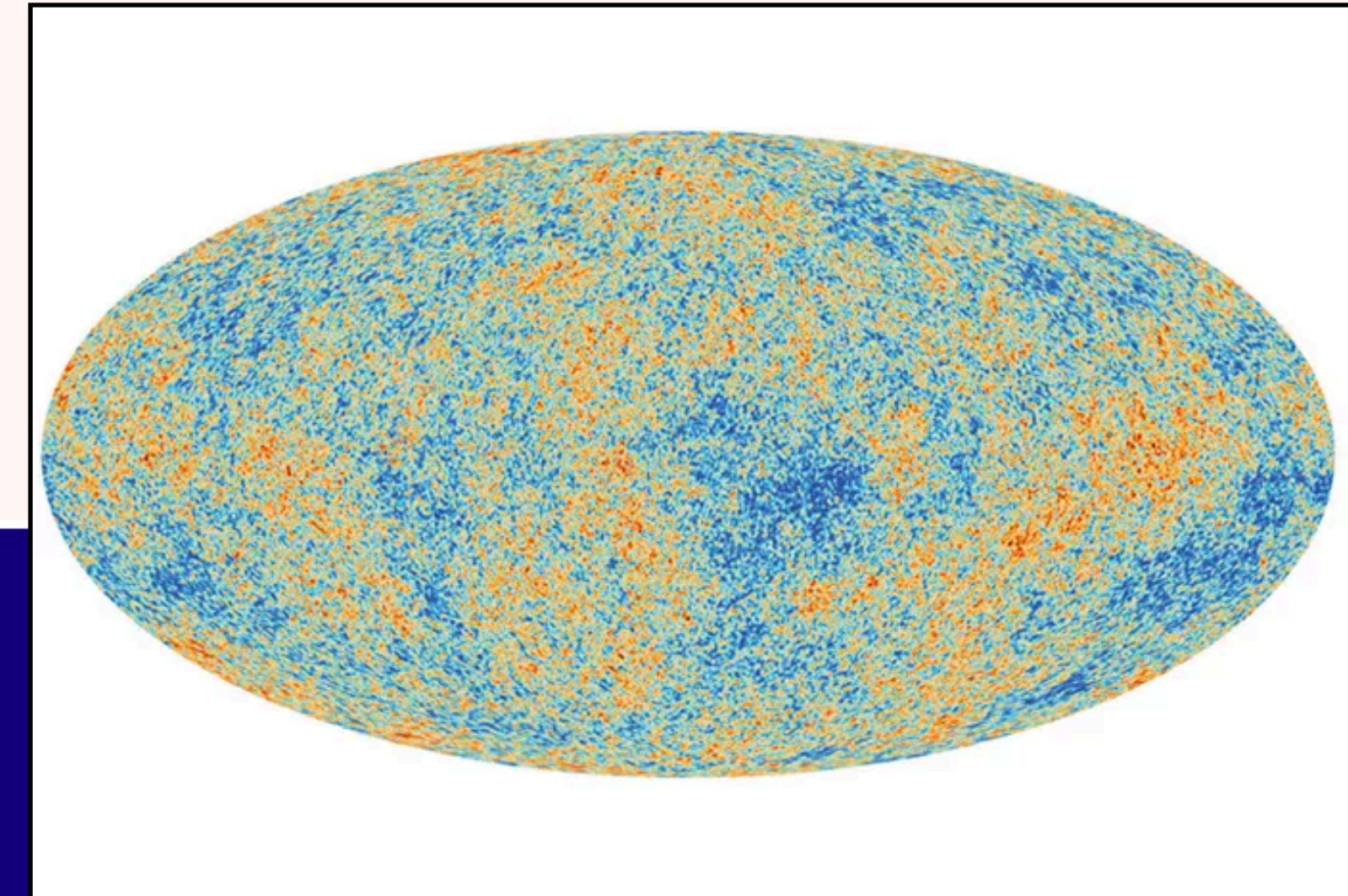
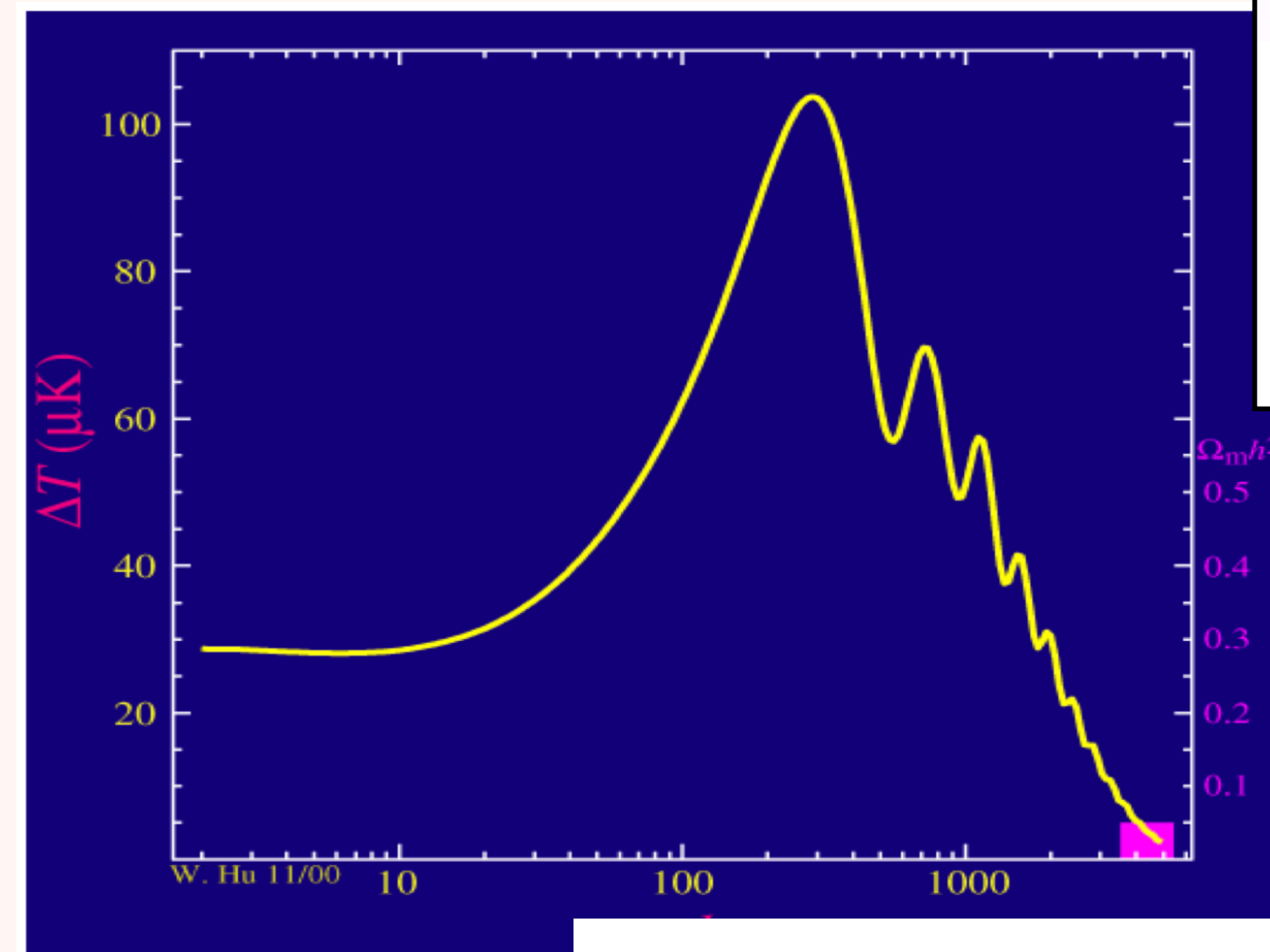
$$H_0 = 72 \pm 8\text{km/sec/Mpc}$$

Cosmic Microwave Background contains Rich Information on the Universe

- CMB power spectrum depends on various parameters: Ω_0 , Ω_b , Ω_m and H_0 .
- For flat universe,
 - Peak positions: $\sim \Omega_m h^3$
 - Height of the second peak $\sim \Omega_b h^2$
 - Peak Ratios $\sim \Omega_m h^2$

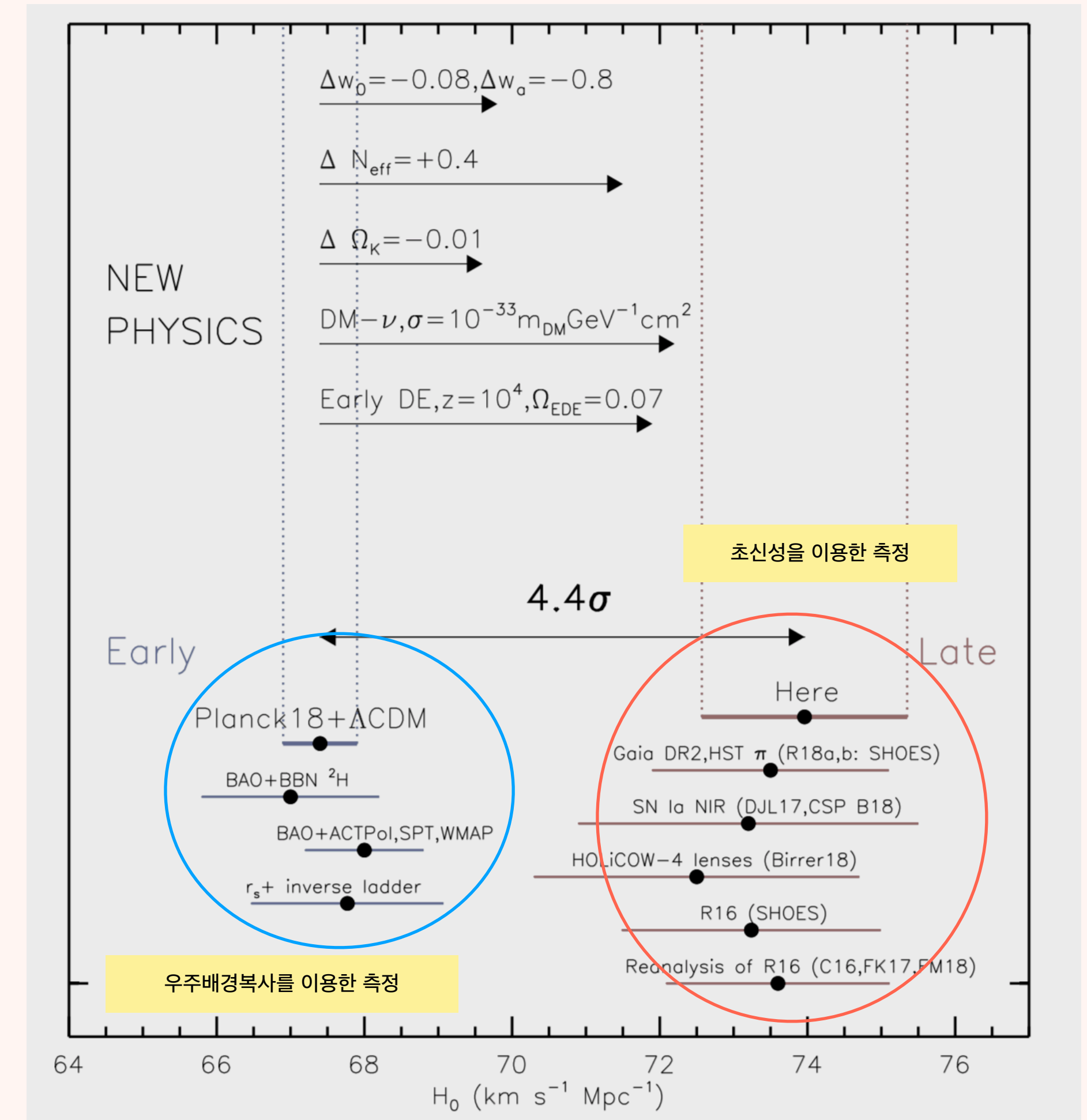
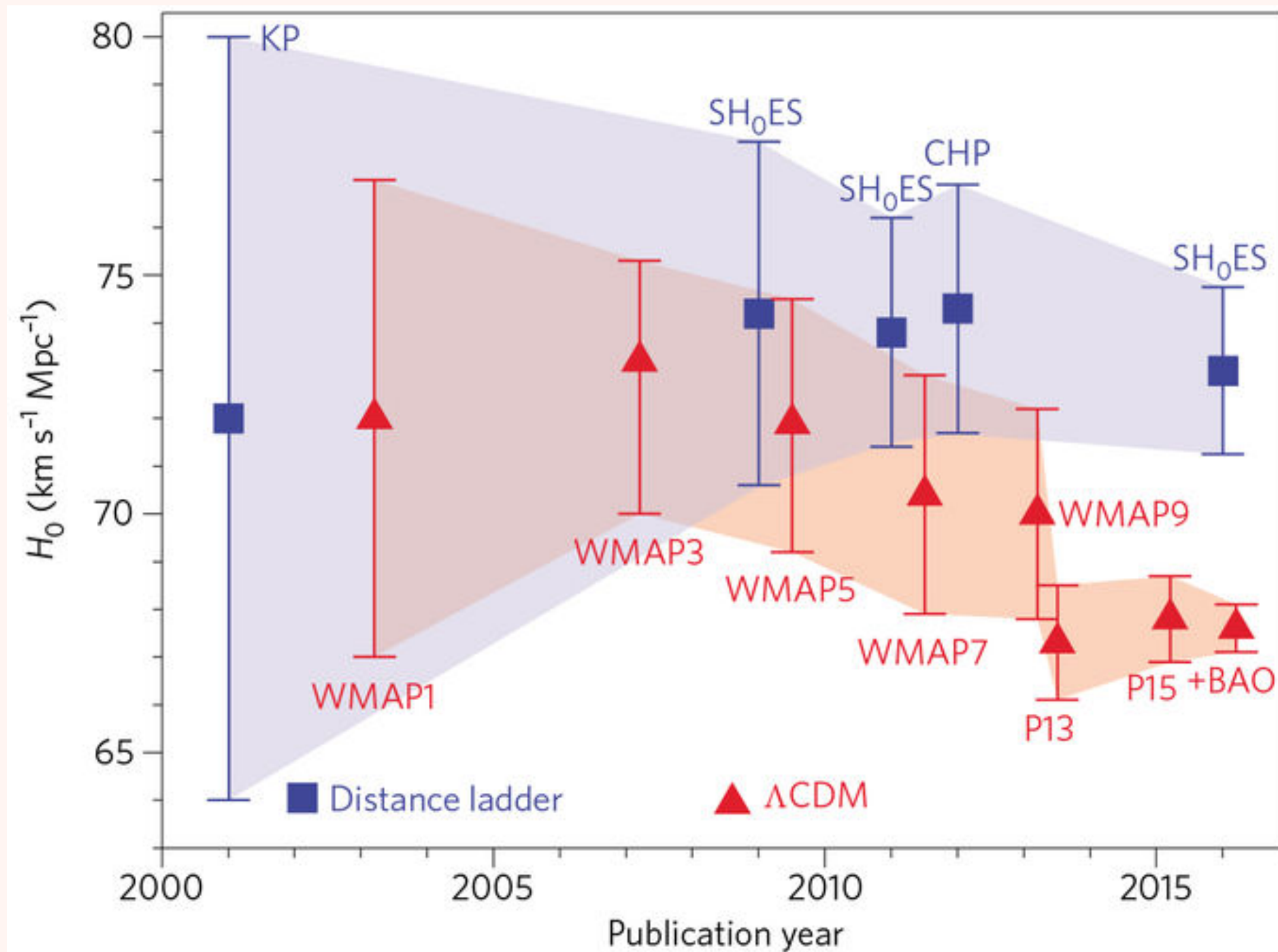
$$h = H_0 / 100 \text{ km/s}$$

Results from Planck (2018)
 $H_0 = 67.4 \pm 0.5 \text{ km/sec/Mpc}$



Hubble tension

- SH₀ES: SNe, H₀
- WMAP: Wilkinson Microwave Probe
- P: Planck
- BAO: Baryonic Acoustic Oscillation



Riess et al. (2019) ApJ, 876, 85

Cosmological Research with GWs

- Reconstruction of expansion history of the universe through direct measurements of distances
- GWs from compact binaries are '**standard sirens**' as the distances can be estimated from the detected waveforms and amplitudes
 - **Bright sirens** [optical counterparts] vs. Dark sirens [no optical counterparts]
- Need to identify the host galaxies in order to measure the redshifts in order to construct Hubble diagram

Estimation of Parameters from GW Observations

- What we measure is the waveforms form for certain duration

$$h(t) = F_+(\alpha, \delta, \psi, t_c)h_+(t) + F_\times(\alpha, \delta, \psi, t_c)h_\times(t)$$

- For circular binaries, 15 parameters are imprinted on the waveforms

Polarization angle

Coleasence Phase

- Intrinsic: m_1, m_2, s_1, s_2 (8)
- Extrinsic: $\Omega = (\alpha, \delta), l, d, \psi, t_c, \phi_c$ (7)
- If we ignore spins, the number of parameters reduces to 9.
- Duration of the observations from frequency f until merger

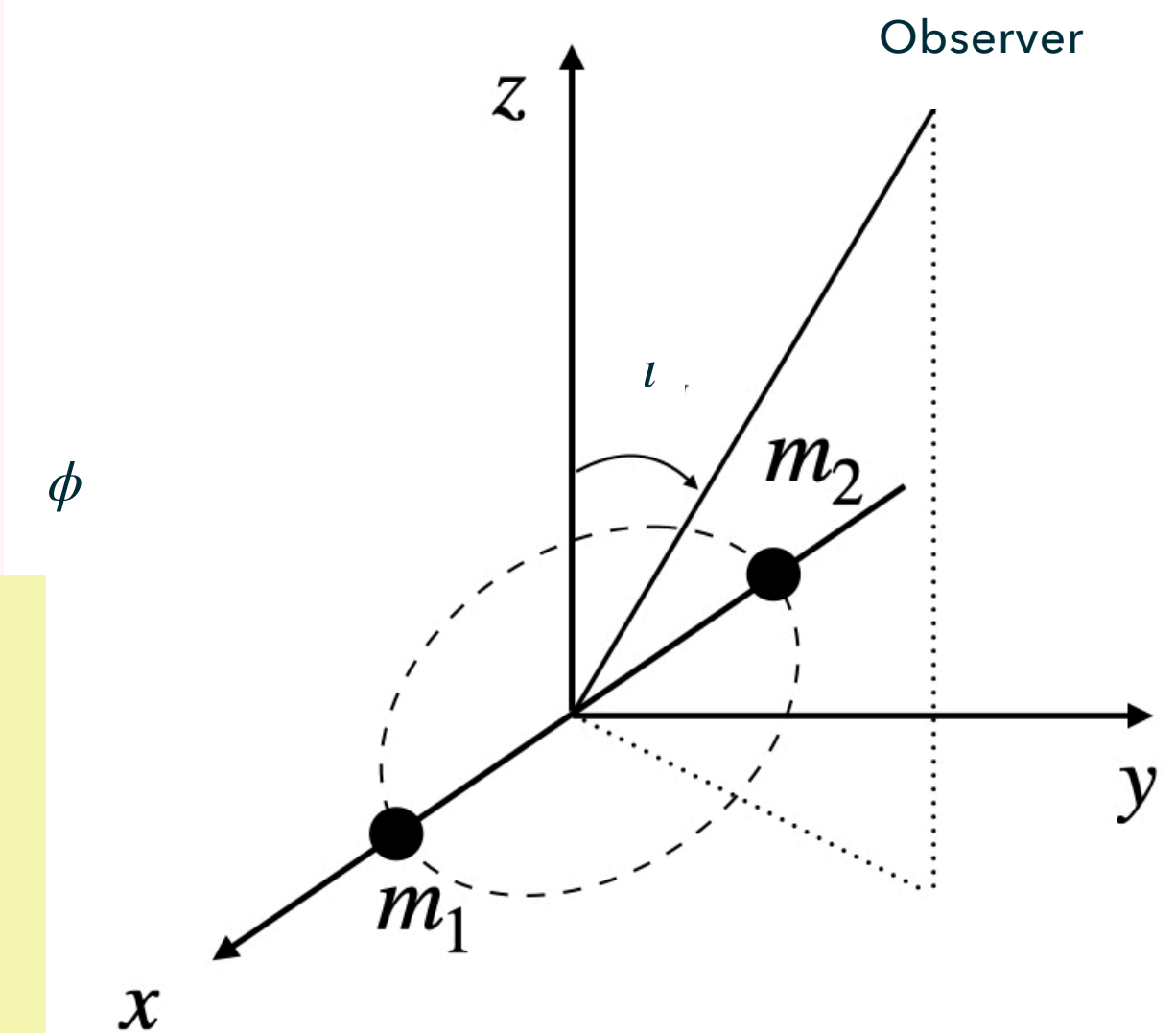
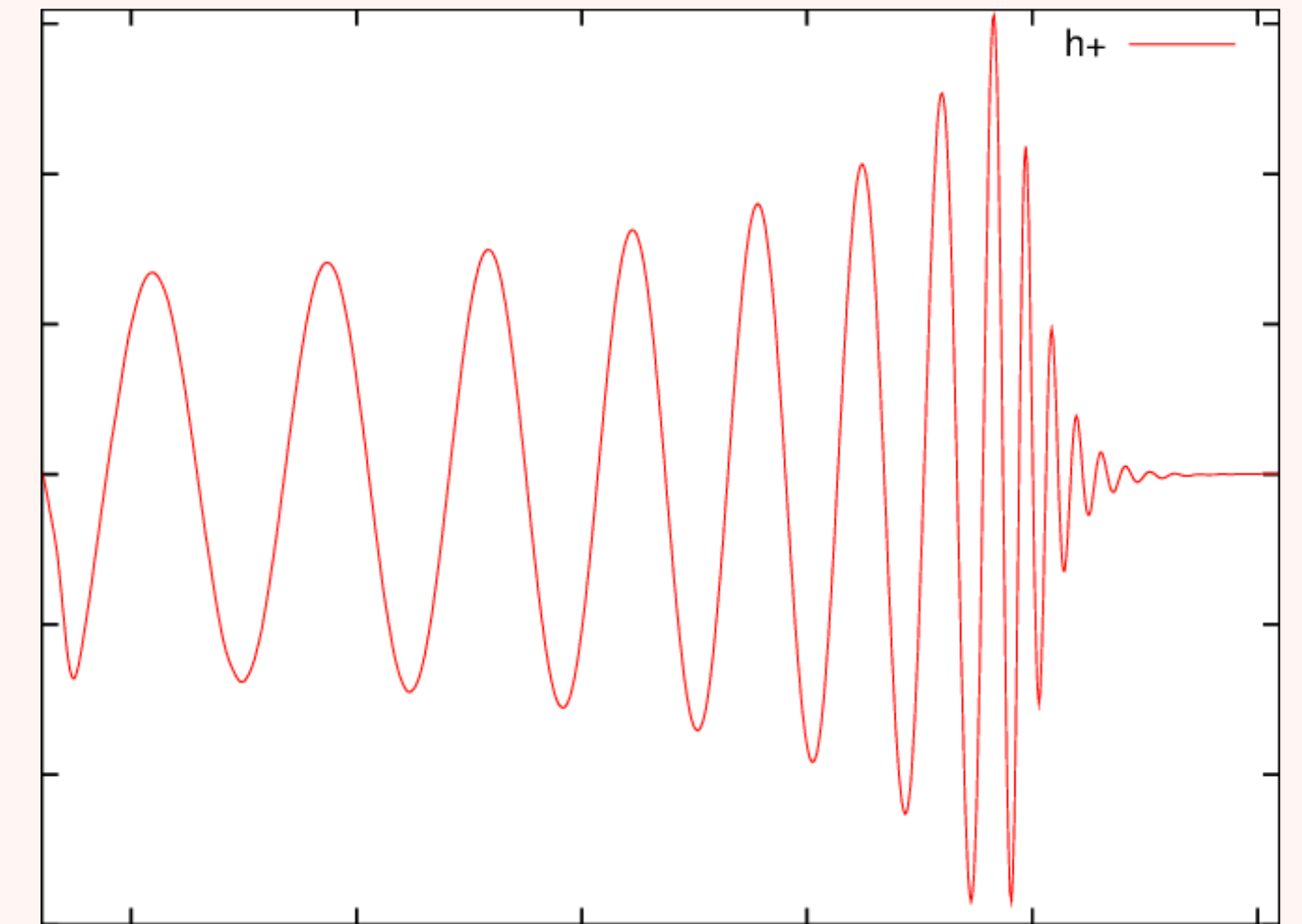
$$(f_{\text{merge}} \gg f)$$

$$T = \frac{f}{\dot{f}} = \frac{5}{96} \pi^{-8/3} \frac{c^5}{\eta(GM)^{5/3}} f^{-8/3}$$

For ground based detectors, $f_0 \sim 30$ Hz

- $T < 1$ second for BBH
- $T \sim$ minute for BNS

For lower frequency detectors, T could become very long (weeks to years)



Estimation of Distances

- The shape of the waveforms does not depend on distances → masses & spins
- The amplitude of the GW signal is **inversely proportional** to the distance.

$$d_L = \frac{5}{96\pi^2} \frac{c}{h} \frac{f_{GW}}{f_{GW}^3}$$

$$= 512 \frac{1}{h_{21}} \left(\frac{0.01s}{\tau} \right) \left(\frac{100\text{Hz}}{f_{GW}} \right)^2 \text{ Mpc}$$

Schutz 1982

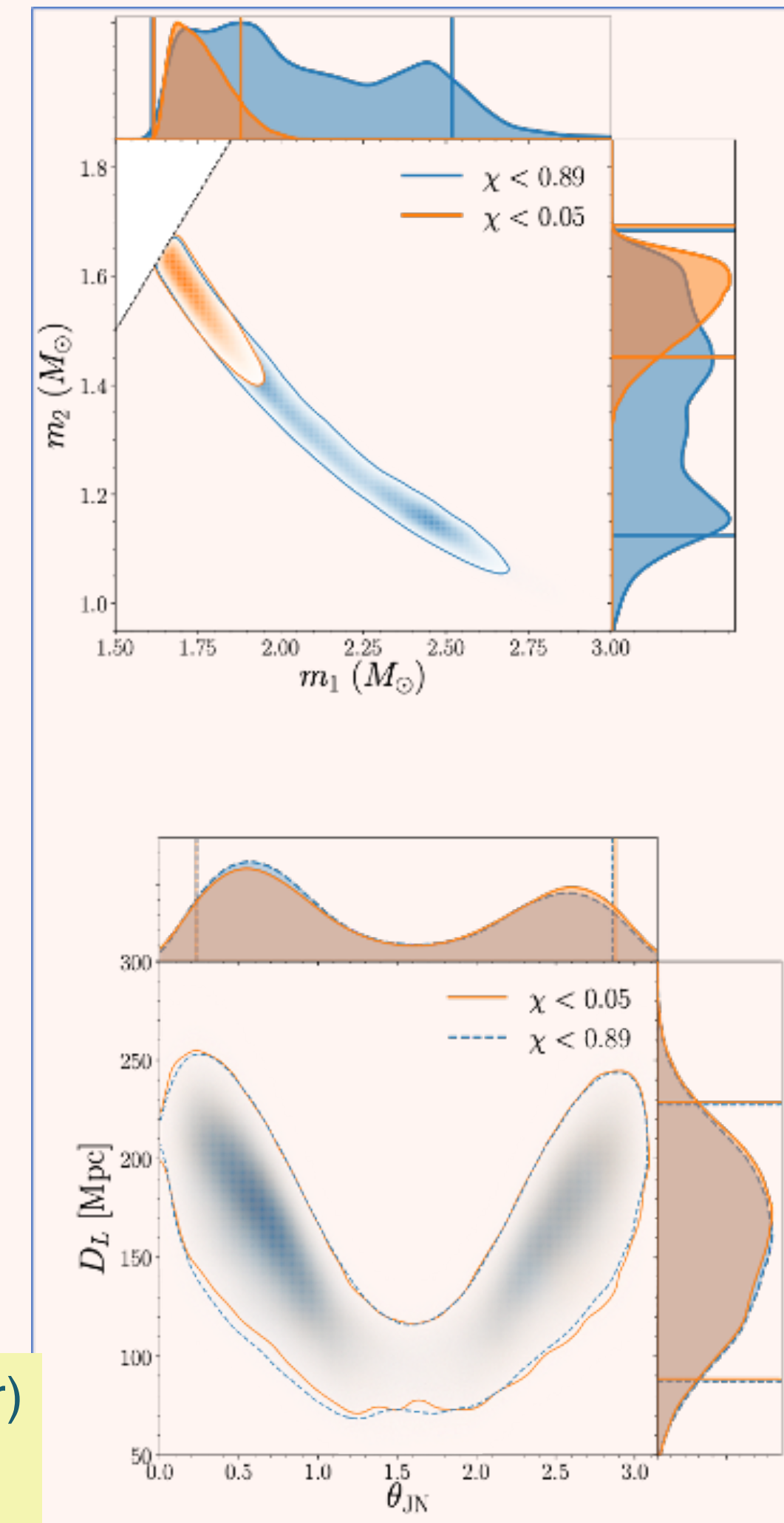
- The distance estimation from GWs does not suffer from **systematic uncertainty**,
- However, individual GW distance estimation is subject to large **statistical uncertainty**

$$h_+ = \frac{h_c}{d} (1 + \cos^2 \iota), \quad h_c \equiv 2\mu(M\Omega^{2/3})\cos(2[\Omega t - \phi_0])$$

$$h_\times = \frac{h_s}{d} \cos \iota, \quad h_s \equiv 2\mu(M\Omega^{2/3})\sin(2[\Omega t - \phi_0])$$

Ω : Orbital frequency

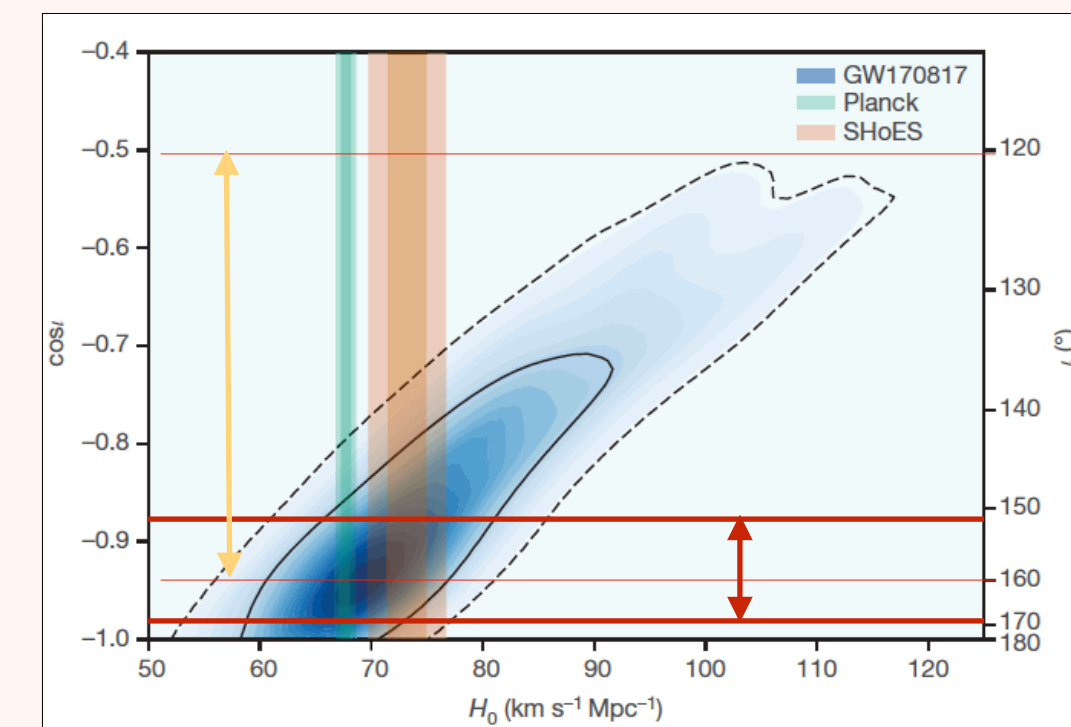
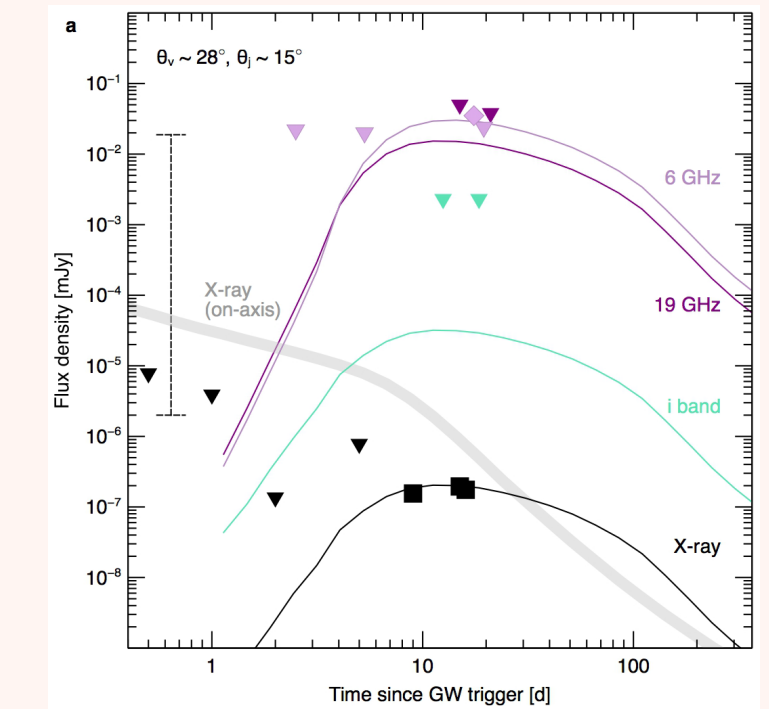
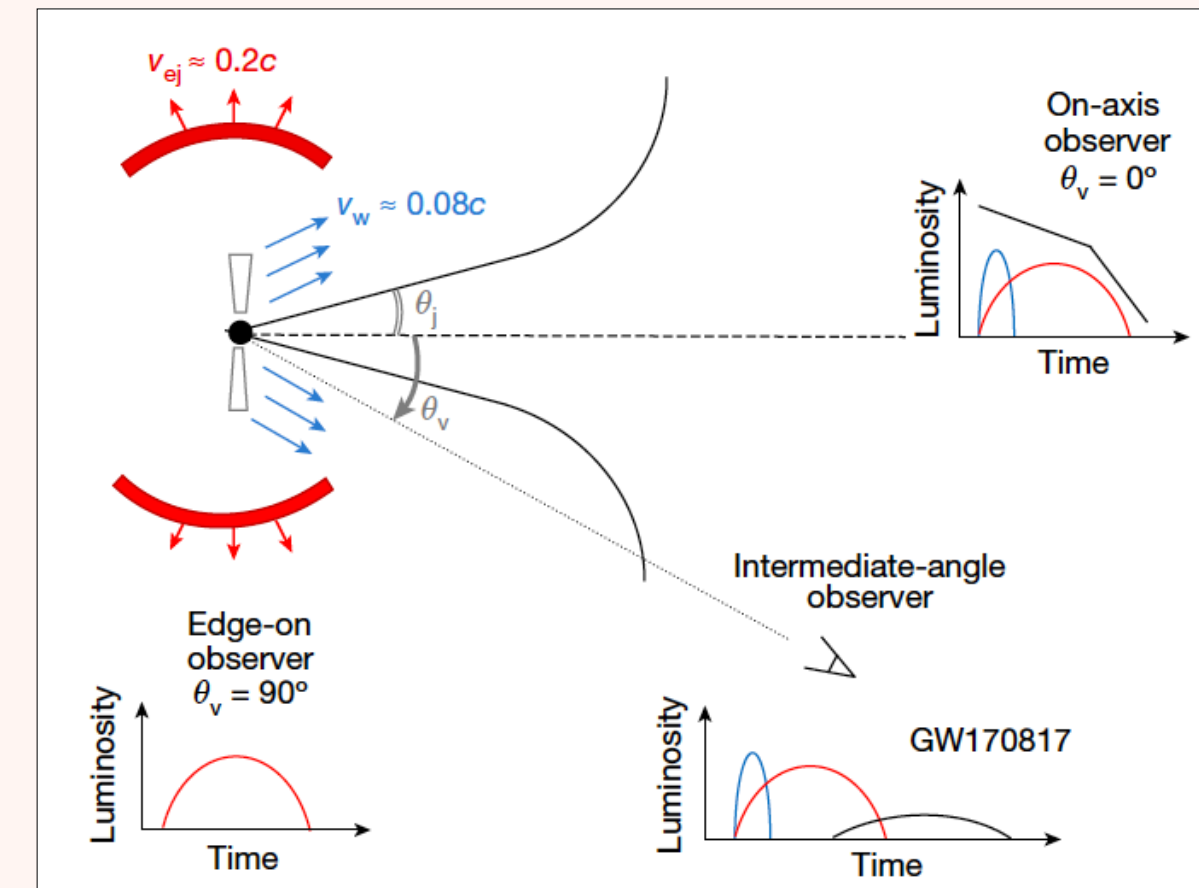
Estimated masses (upper) and distance (lower) to GW190425, a binary neutron star merger event. (Figure from Abbott et al. 2020, ApJL, 892, L3)



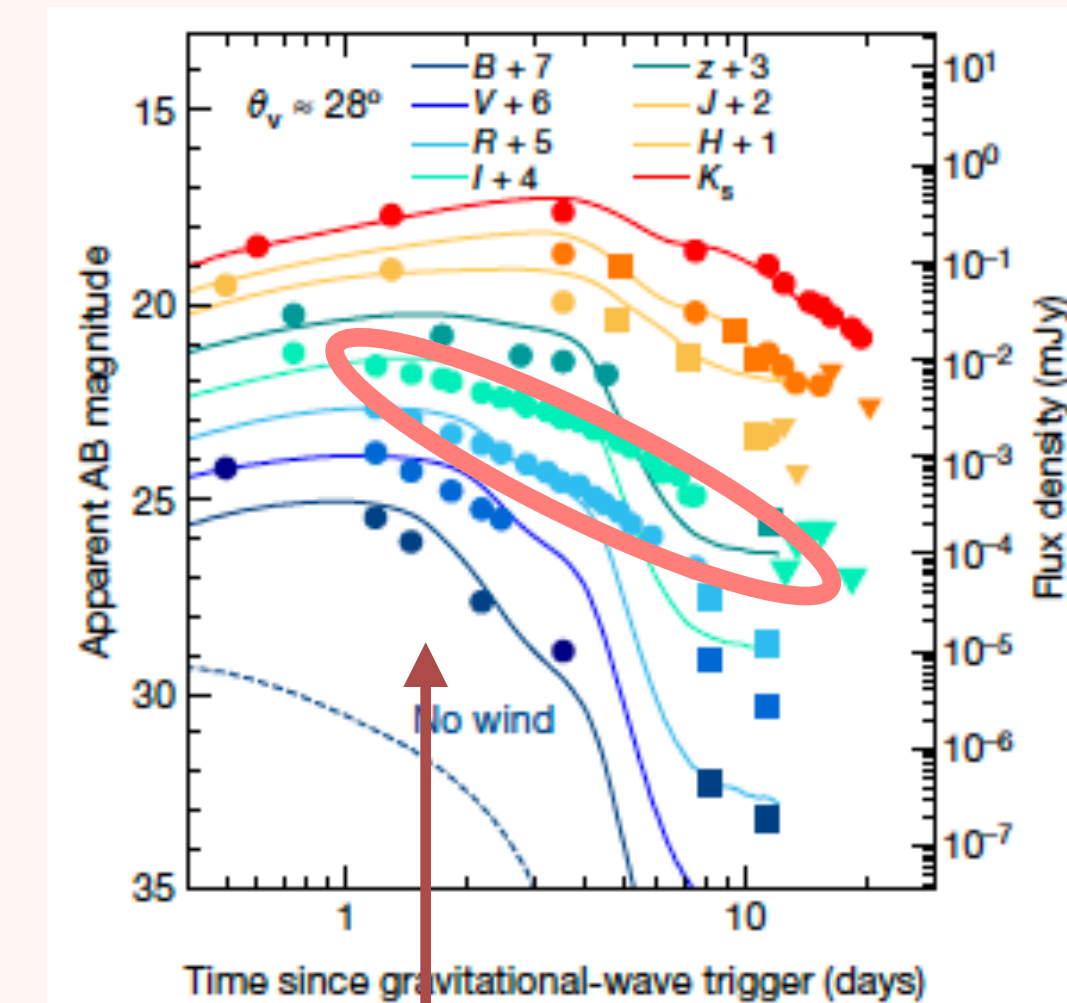
Can we constrain the viewing angle?

Troja et al. 2017, Nature

- If the GRB is observed, together with the GWs, the l-o-s should lie within the opening angle (θ_j) of the jet $\iota < \theta_j$
- Radio synchrotron emission contains the information on ι
- However, kilonova lightcurve in optical/IR is not sensitive to ι .
- GW170817:
 - Early constraint: $20^\circ \lesssim \iota \lesssim 60^\circ$ mostly from radio data (Troja et al. 2017)
 - Discovery of the superluminal radio jet (Mooley et al. 2019) is claimed to give the range of the viewing angle more tightly ($14^\circ \lesssim \iota \lesssim 28^\circ$).
 - Optical superluminal jet (Mooley et al. 2022) gave tighter constraints the viewing angle ($19^\circ \lesssim \iota \lesssim 25^\circ$)



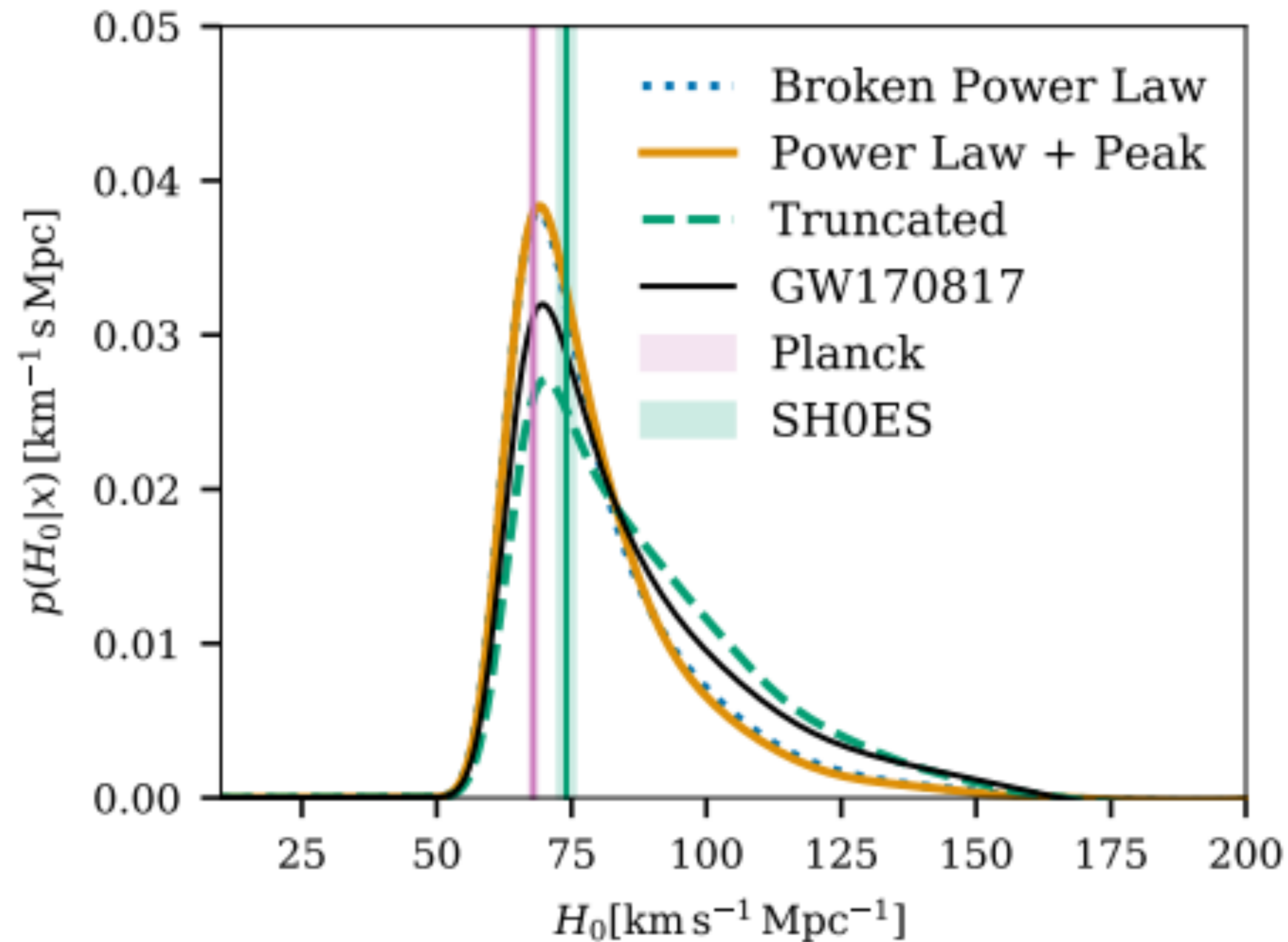
LSC (2017) Nature



KMTNet

Measurements of H_0 with GW170817

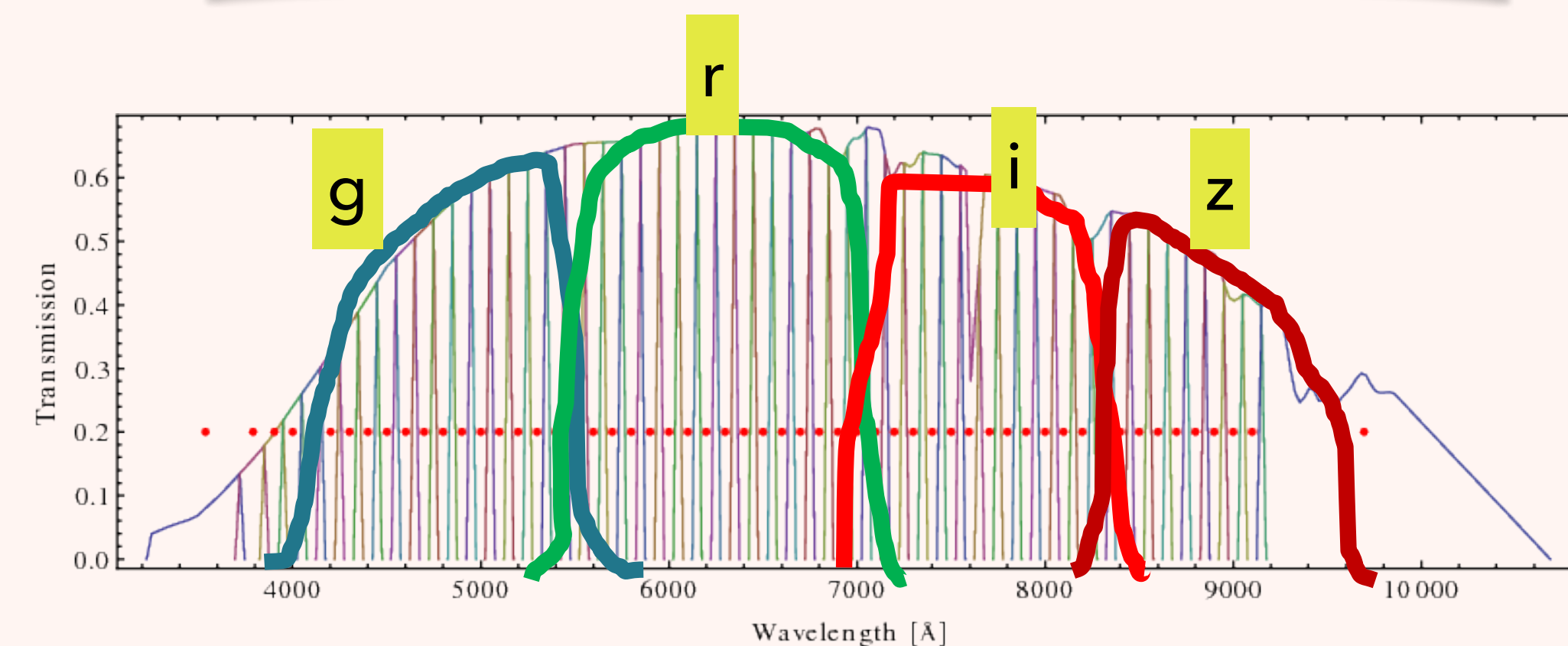
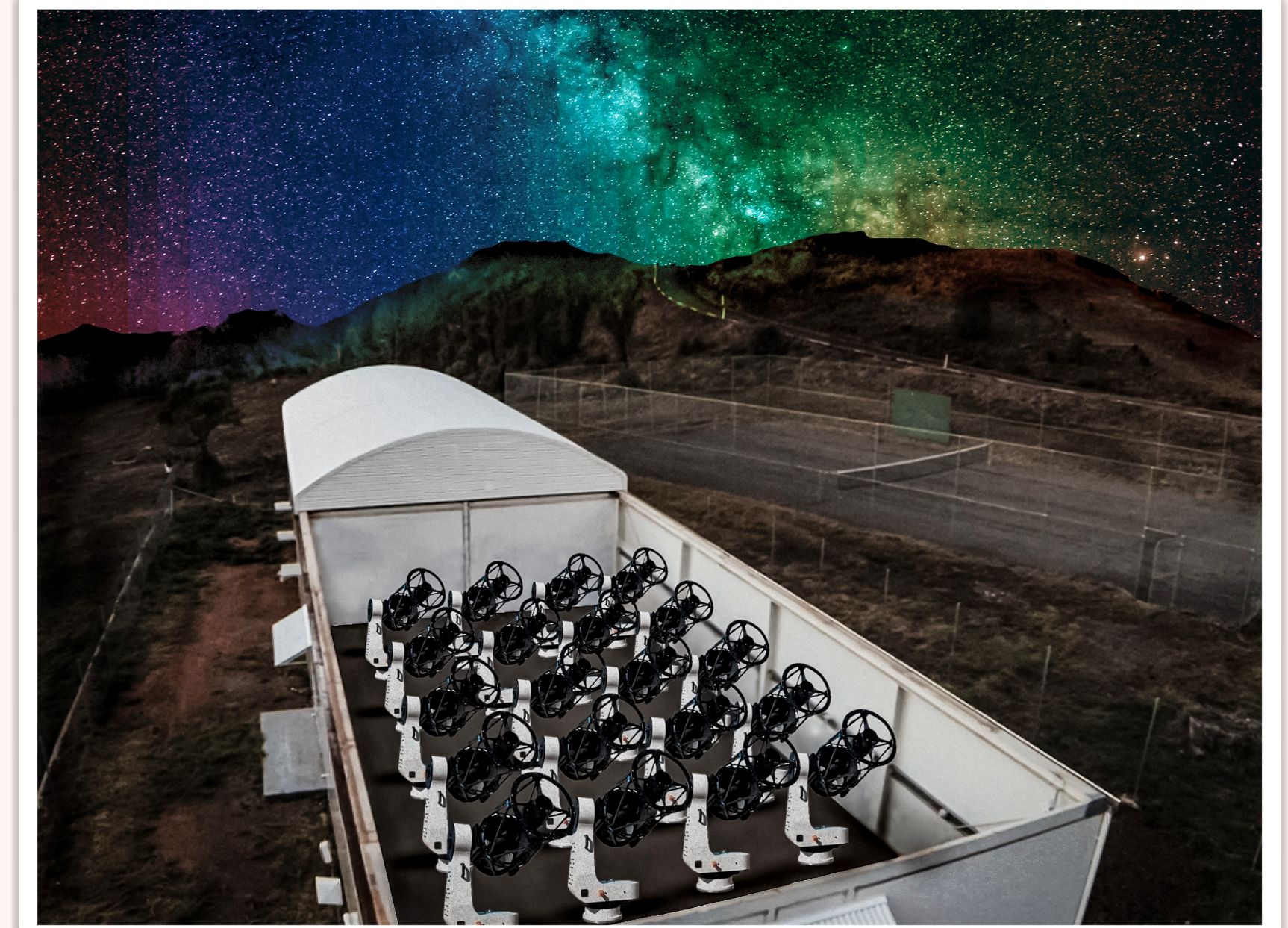
arXiv:2111.03604V1



- Accuracy of Hubble constant can be improved if we have many BNS events with EM counterparts.
- There will be a few more such events in during O4.
- Since the EM emission fades away very quickly, we need to identify the EM counterpart rather quickly.

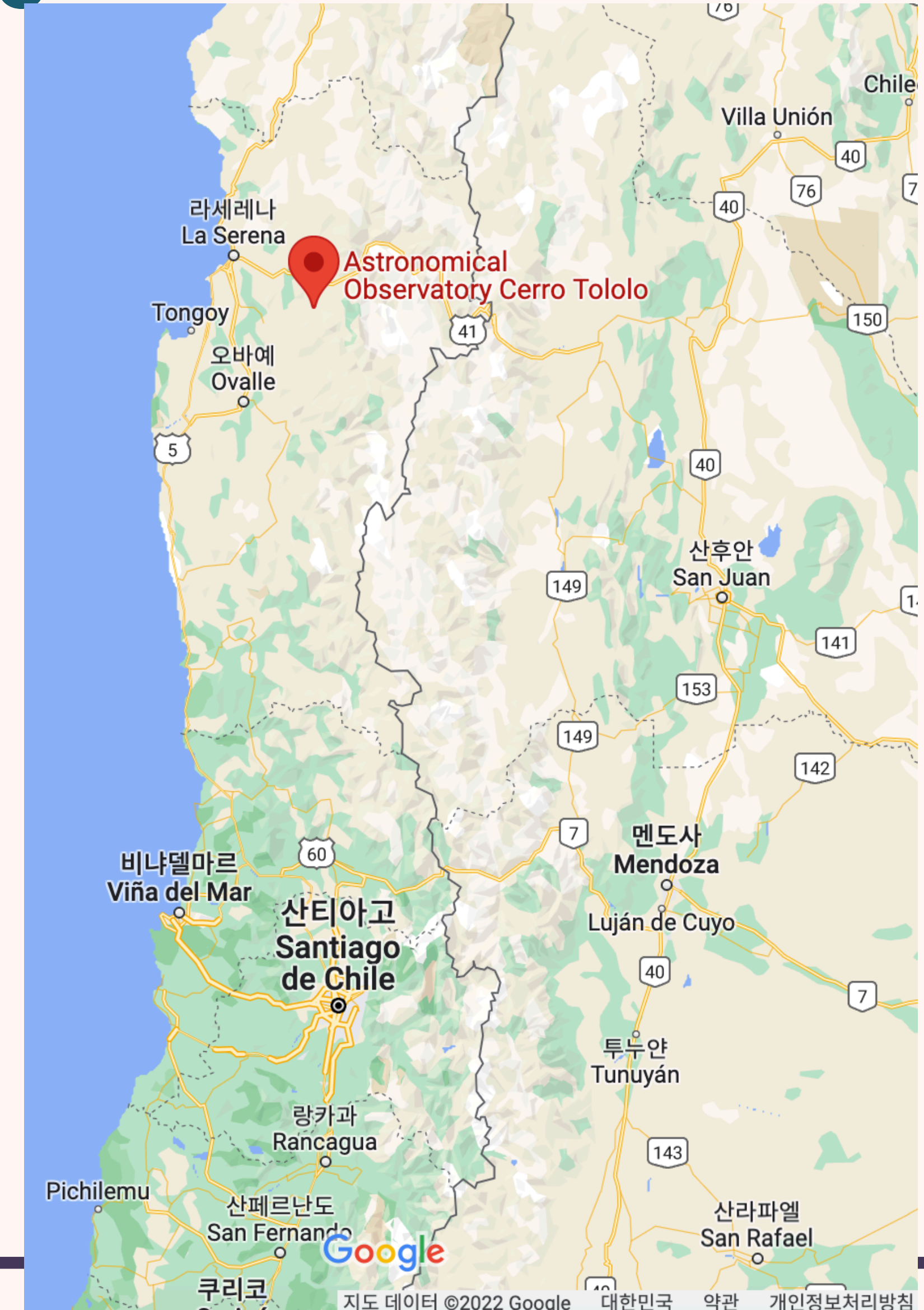
A new facility (7DT) for rapid followup of transients

- We are building a system of telescope composed of 20 telescopes of 50 cm aperture ($\sim$ Effective $D \sim 2.2$ m if all telescopes are combined)
- Imaging wide field with 40 medium band filters (each telescope has 2) $\rightarrow$ **low resolution spectroscopy** for every pixel in the field of view
- It can cover **large area** of sky repeatedly:
 - Wide-field, time domain, spectroscopic telescope
 - Suitable for the survey of transients such as GRB and Kilonovae



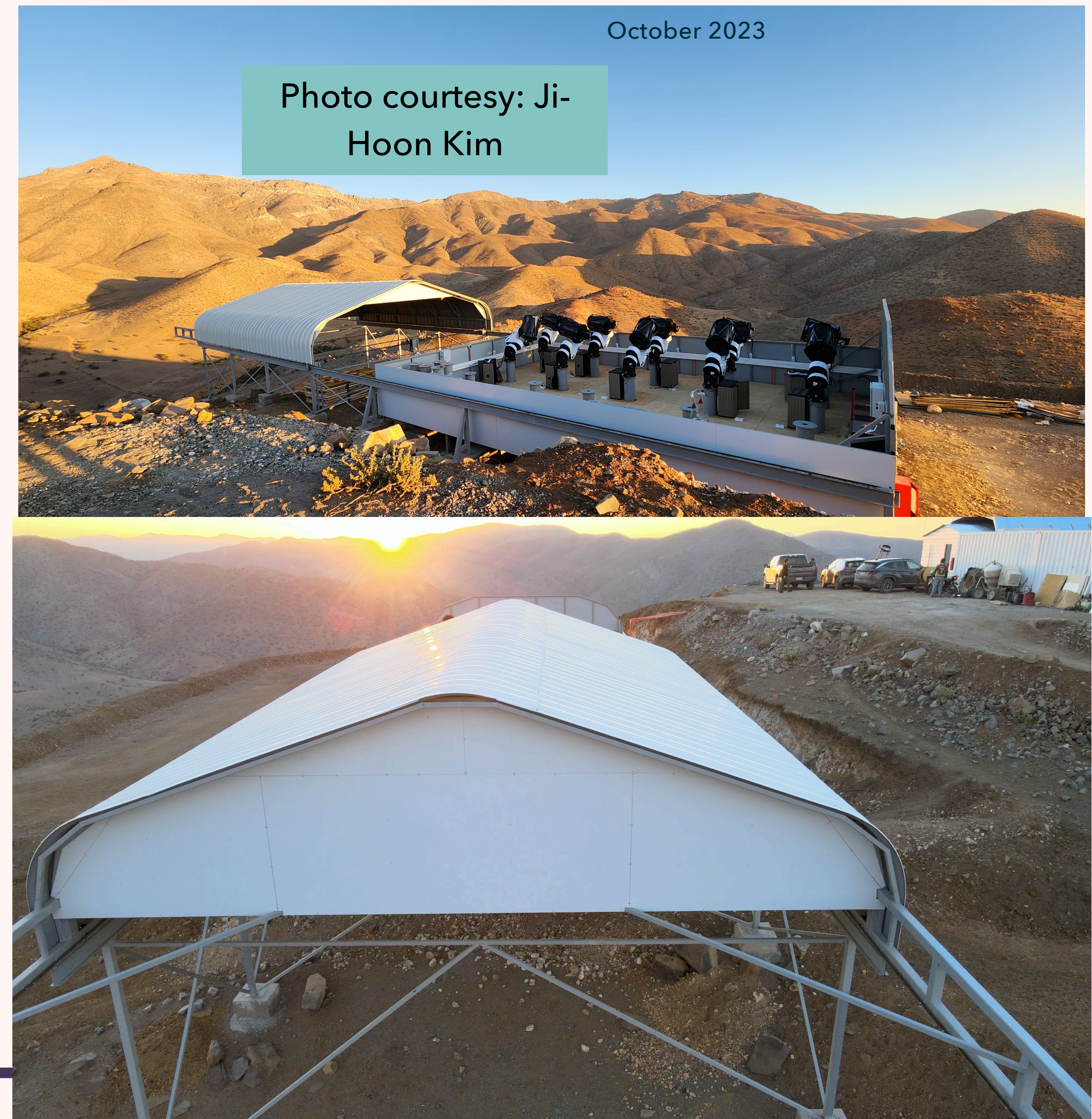
Telescope site

- Chile, Rio Hurtado (near CTIO/Cerro Pachon)
- Altitude: 1700 m
- 320 clear nights
- $<1''$ Seeing



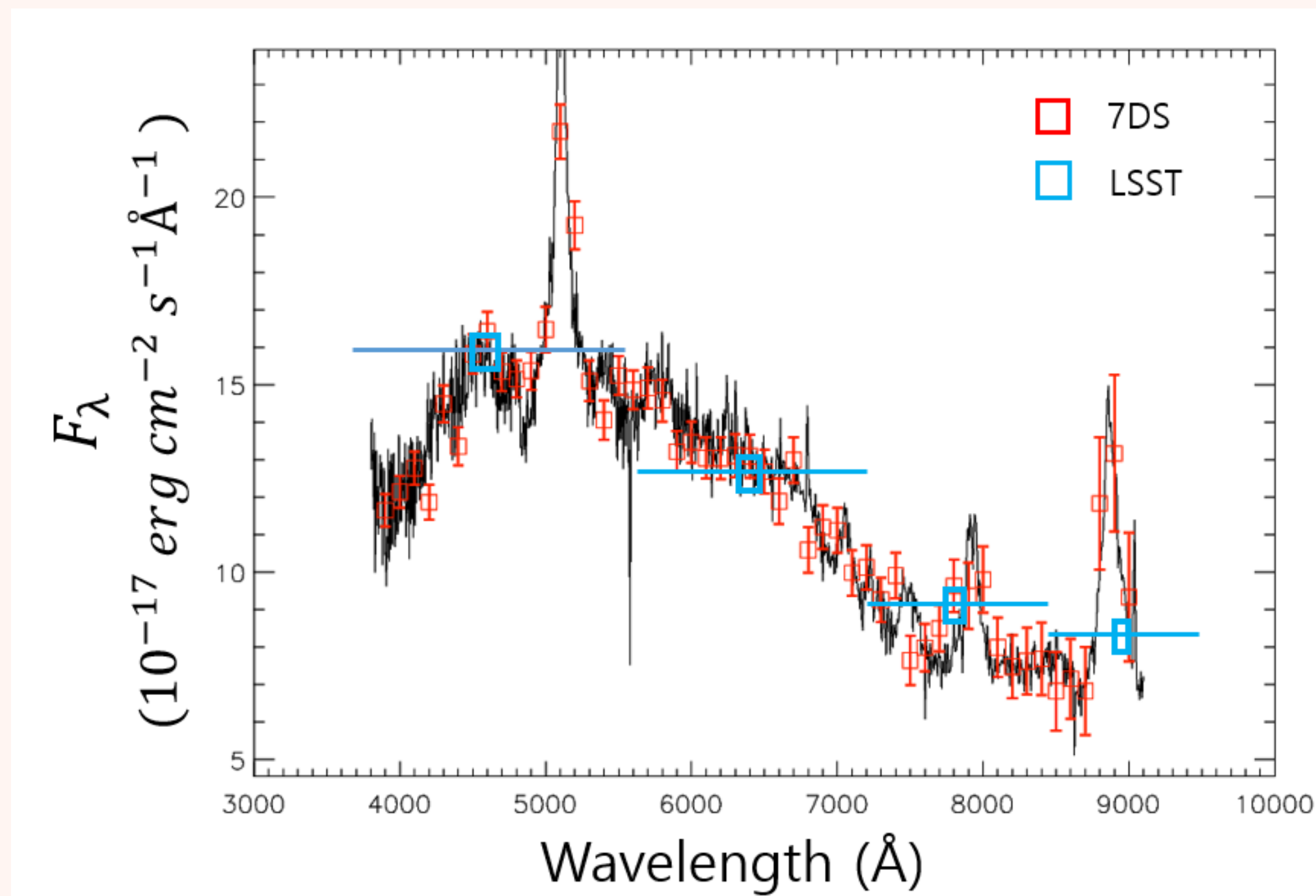
Current Status of 7DT

- 12 telescopes (out of planned 20) telescopes are installed and are being commissioned.
- Current Field of View 1.25 sq. deg. (1.4 deg x 0.9 deg)
- Twenty 25nm medium band filters + Sloan Broad-band filters
- Computers for data storage and processing (GPU) are ready
- A GPU-based data process server with 2 NVIDIA A100 cards, is ready.
- Network synchronization with KREONET has been tested.
 - Data rate = 400 Mbps
 - Data per day ~ 1 PBytes



Advantage of Medium Band Spectrum

- Best suited for broad continuum features and broad emission lines
- Photometric redshift: $< 0.3\% - 1\%$ accuracy
- Emission line/continuum can be separated well, and thus characterize the nature of the transients.



$z = 0.822$,
 $i = 18.3$ quasar spectrum
 (black: SDSS
 red: 7DS
 blue: LSST)

Separation between Kilonova and Supernova

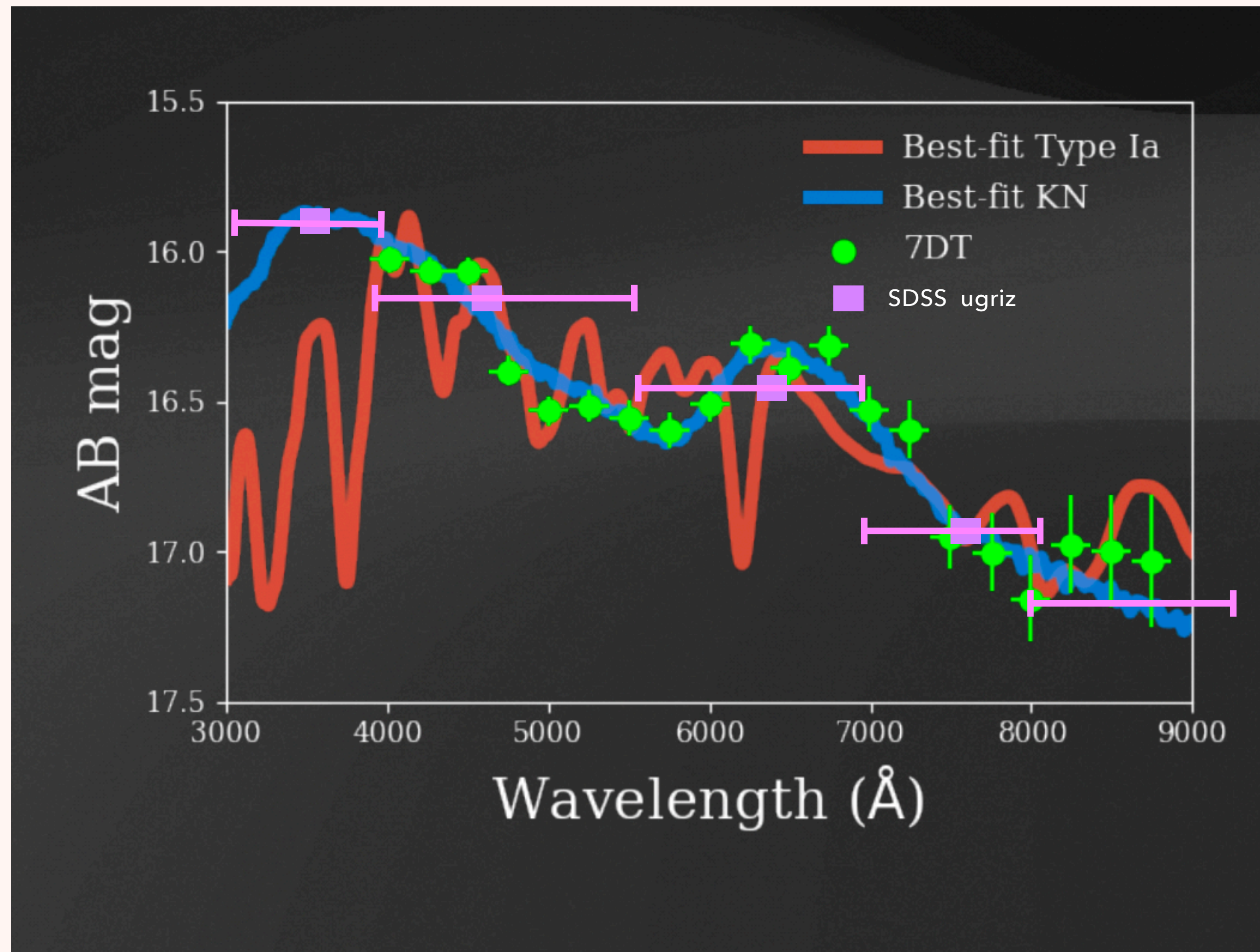


Figure Credit: Gregory Paek (SNU)

Dark sirens

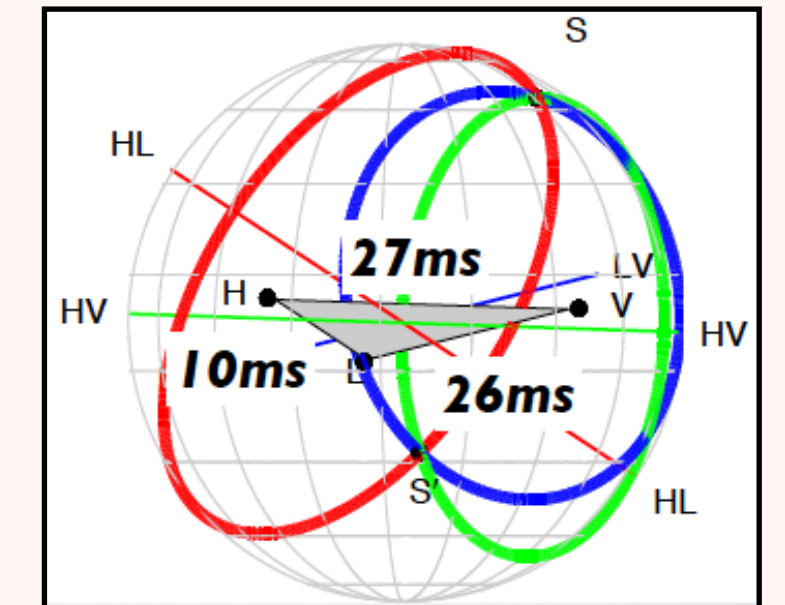
- BBH does not emit EM radiation, and distant BNS may emit EM radiation that is too faint to observe. Such objects are called **dark sirens**
- Unless the angular resolution of the dark sirens becomes very good, it would be very difficult to uniquely identify their host galaxies with ground-based detectors.
- One can still use dark sirens to constrain cosmological parameters (Hubble constant, DE equation of state, etc.) using dark sirens statistically.
- Photometric redshifts of galaxies within Ω and d_L range (e.g., Soares-Santos et al. 2021)
- Cross-correlation with galaxies (Mukherjee et al. 2018, 2021)
- Identification of host galaxies with future detectors

Why host identification is so difficult?

- Most of the GW sources do not emit electromagnetic (EM) radiation (e.g., BBH)
- BNS or NS-BH can emit EM radiation, but they are rare and generally rather faint.
- Sky localization is done by triangulation using differences in the arrival times to different detectors.
- Accuracy depends on the signal-to-noise ratio

$$\sin \theta d\theta = \frac{\sqrt{\sigma_1^2 + \sigma_2^2}}{\Delta t} \quad \sigma_t = \frac{1}{2\pi\rho\sigma_f}$$

$d\theta$: width of the ring, Δt : baseline, ρ : signal-to-noise ratio, σ_f : effective

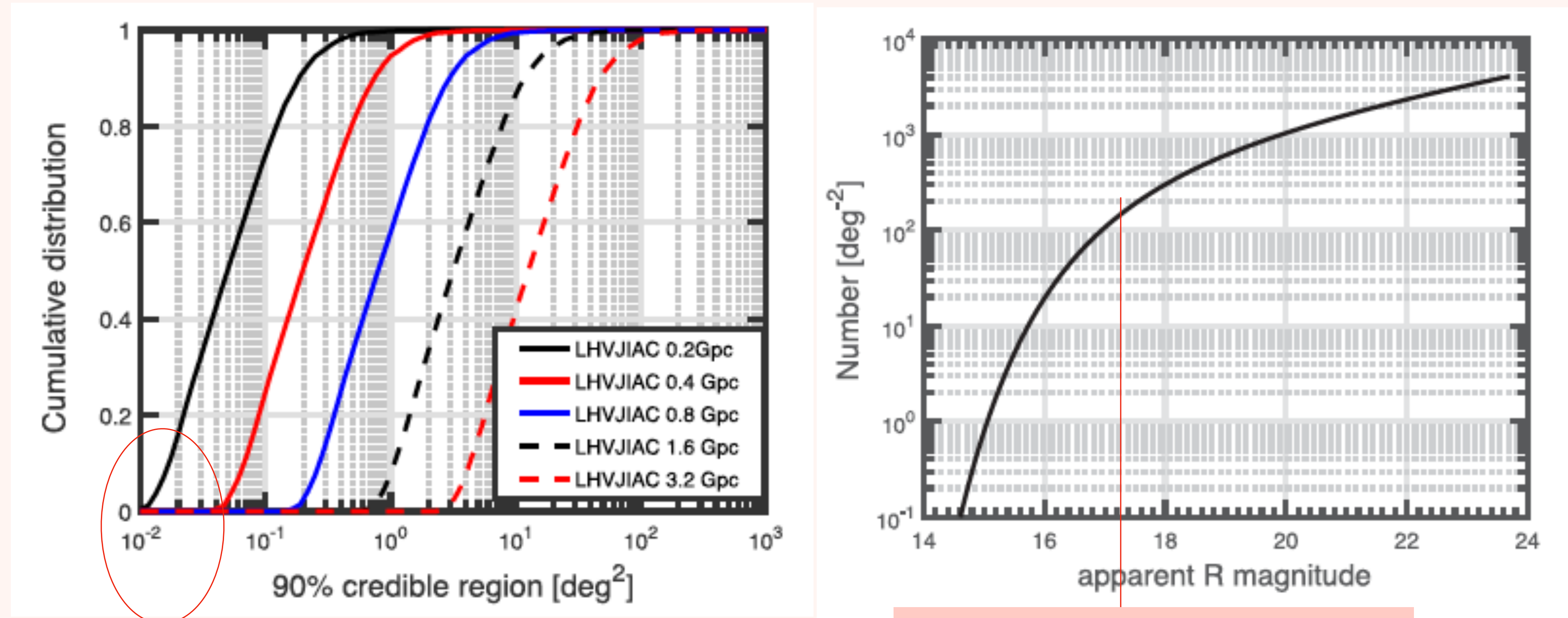


bandwidth of the source, (~100 Hz for NS binaries, smaller for BH binaries)

- $\Delta\Omega$ is very large (100 - 1000 sq. deg.)
- Localization accuracy improves with number and sensitivity of detectors

Can BBH hosts be identified with ground based detectors?

Howell, ... Lee, ... et al. 2018

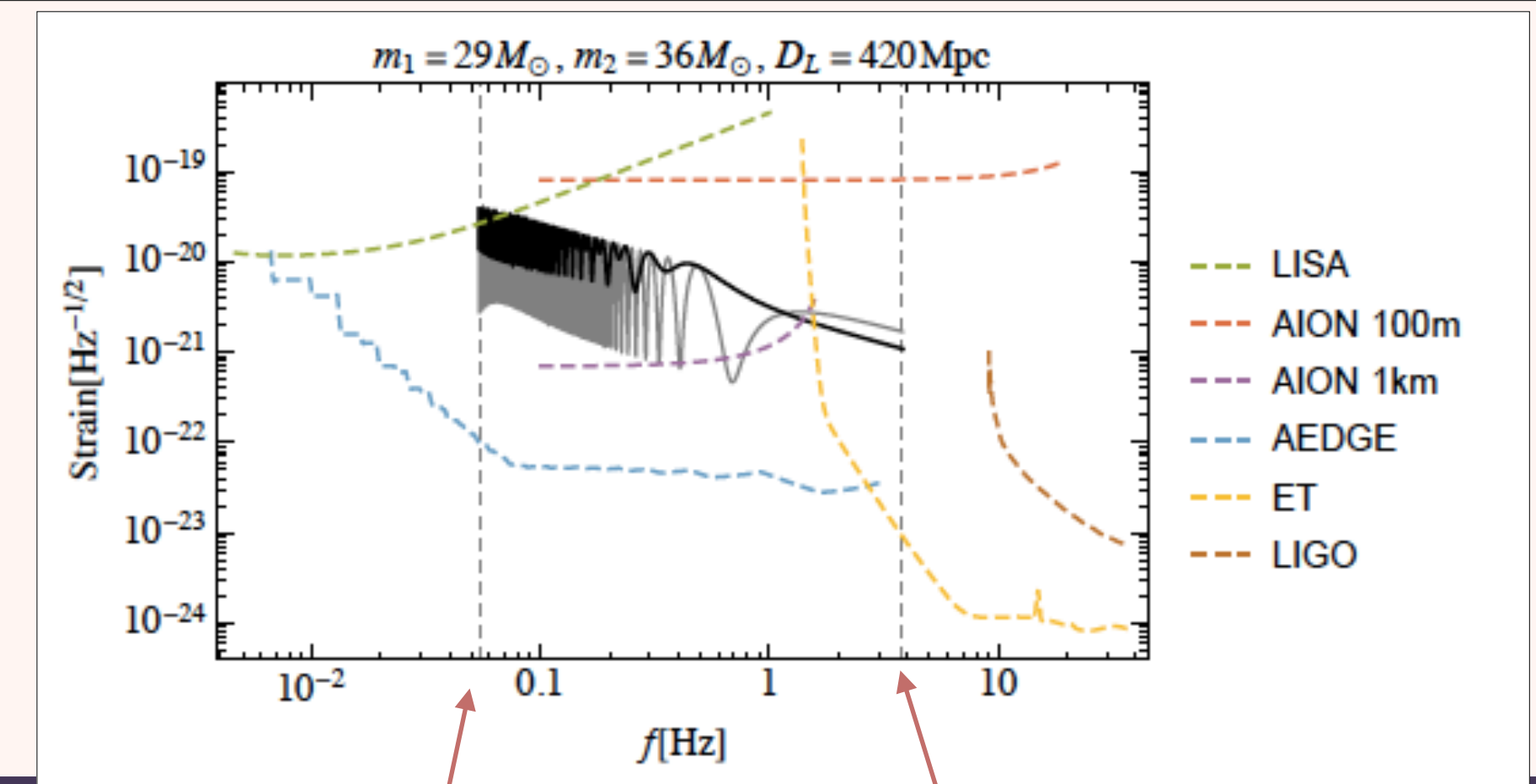
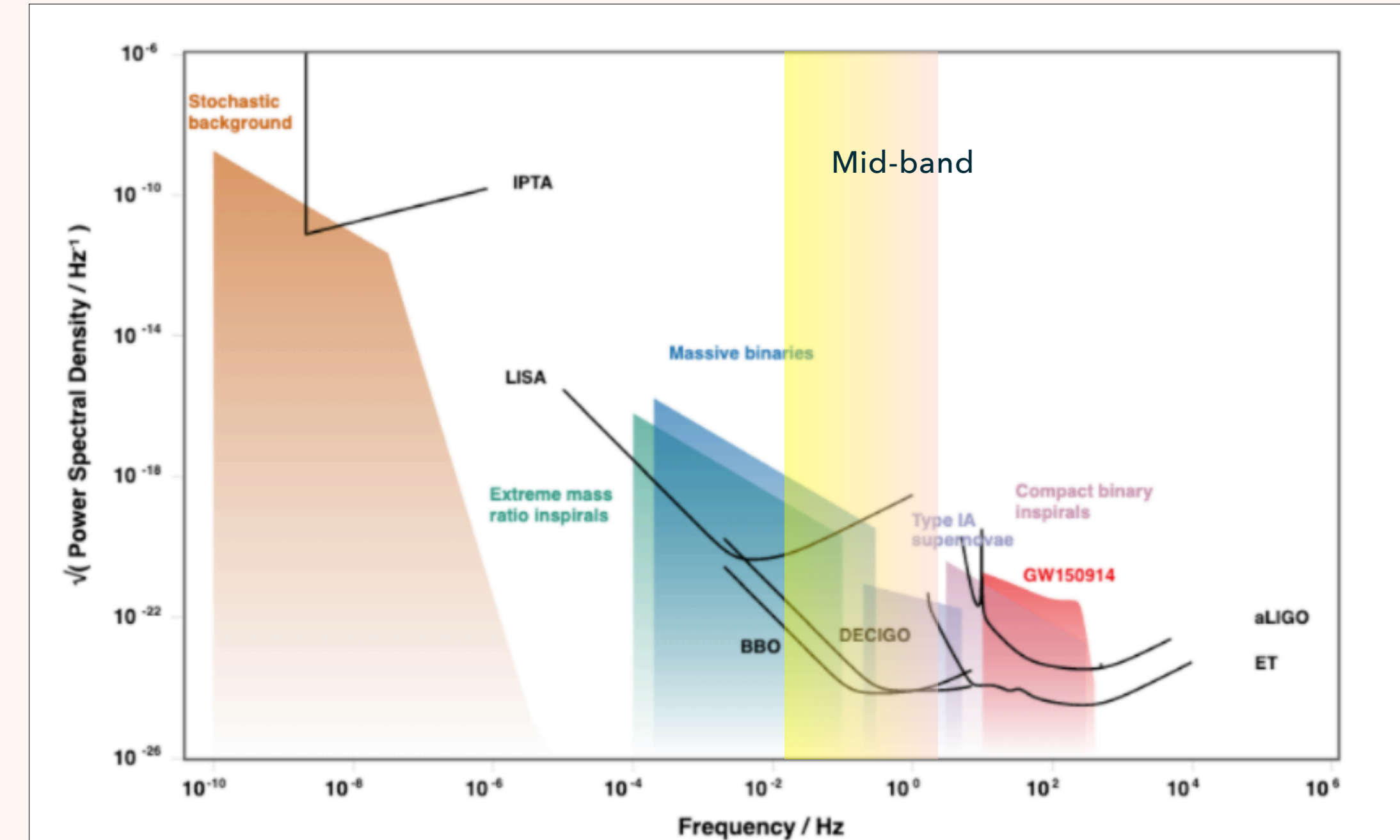


Milky Way Galaxy at 0.4 Gpc

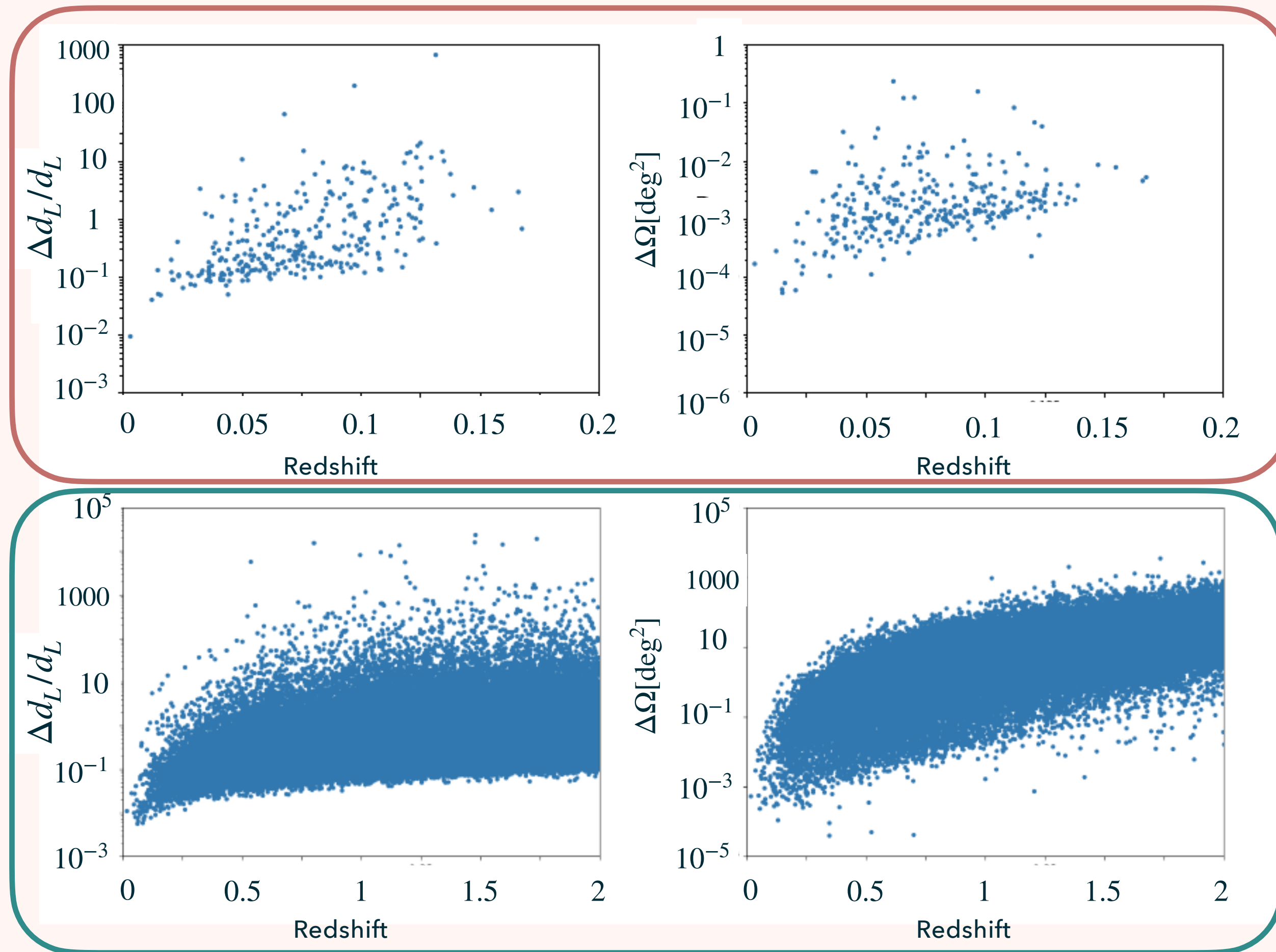
- With network of 5 advanced detectors and two additional detectors with better sensitivity, $\sim 20\%$ of the BBHs at 400 Mpc can be localized within 0.1 sq. deg.
- There could be 10-20 galaxies within the 0.1 sq. deg.
- **BBH host identification by ground based detectors is very challenging!**

Improvement of localization in 3D volume with mid-band detectors

- Detectors operating at lower frequencies can observe the merging binaries for a long time (**days to years**)
- The source position and inclination angle are encoded in the measured signal through
 - Relative amplitudes and phases of the two polarization components,
 - **Periodic Doppler shift** imposed on the signal by the detector's motion around the Sun,
 - Further modulation of the signal caused by the detector's **time-varying orientation**
- Accuracies of Ω and d_L can be significantly improved



A case study: Simulation of BBH and BNS observations with AEDGE (Yang, Lee+, 2022, JCAP [arXiv:2110.9967v1])



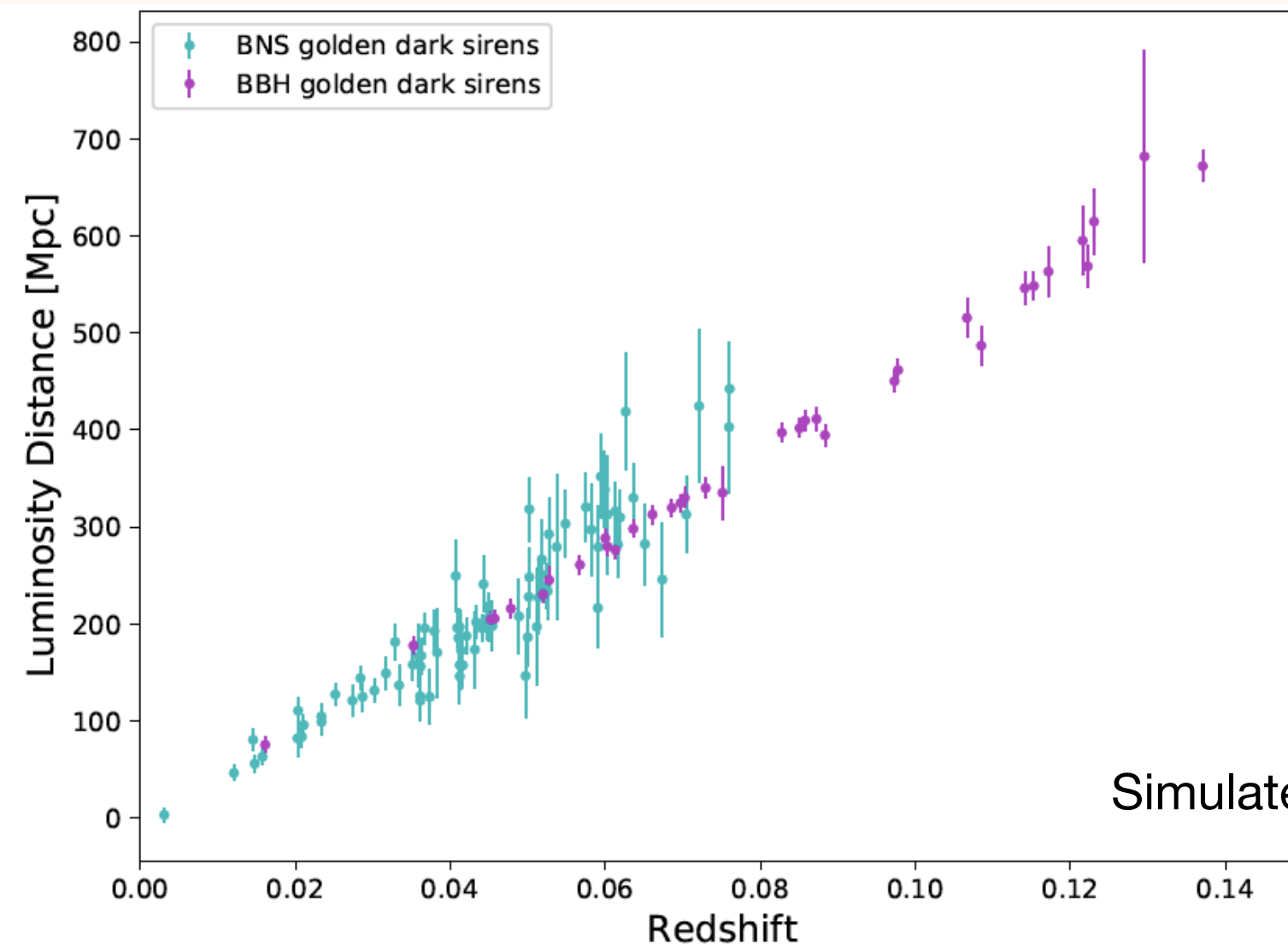
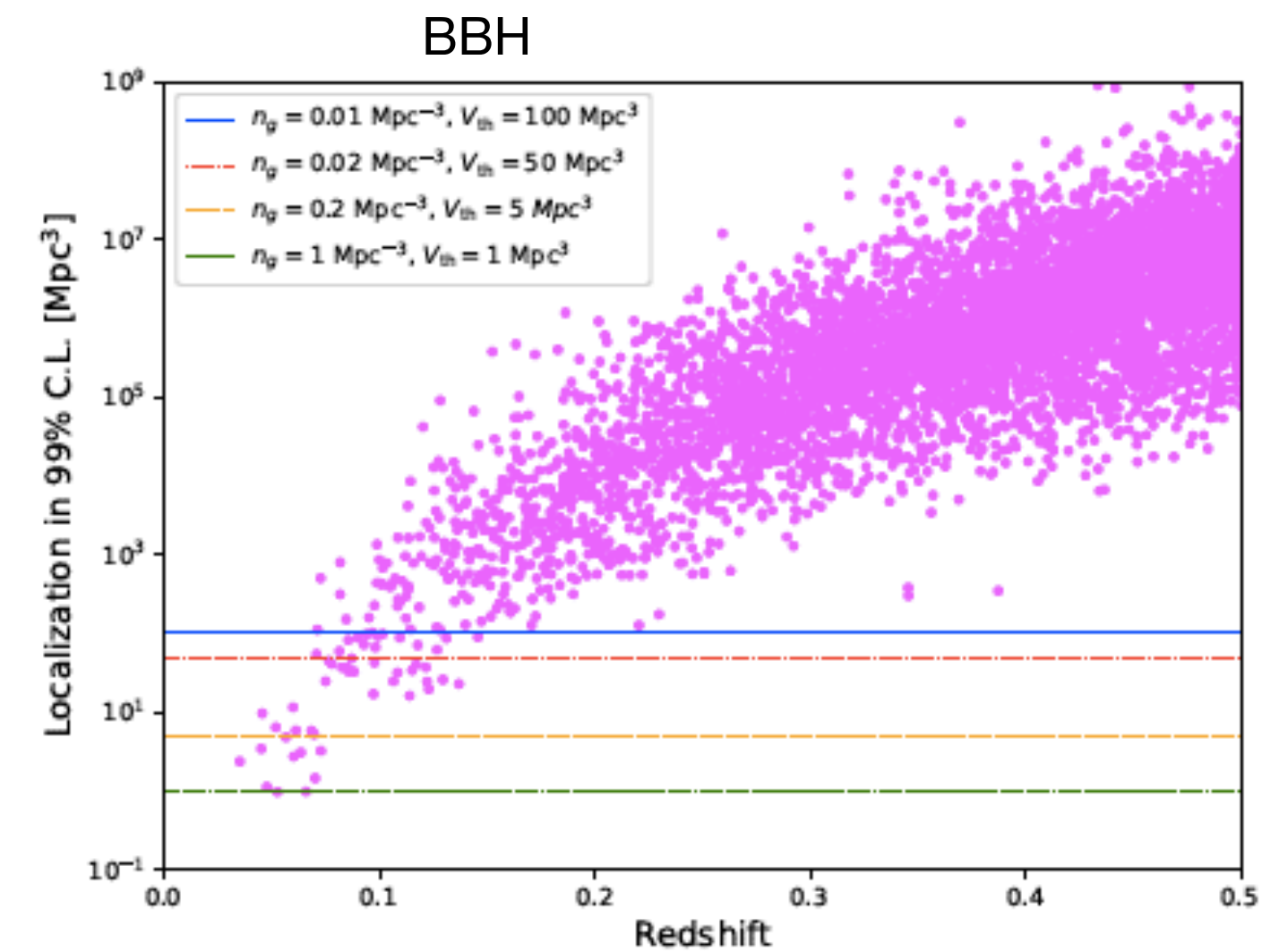
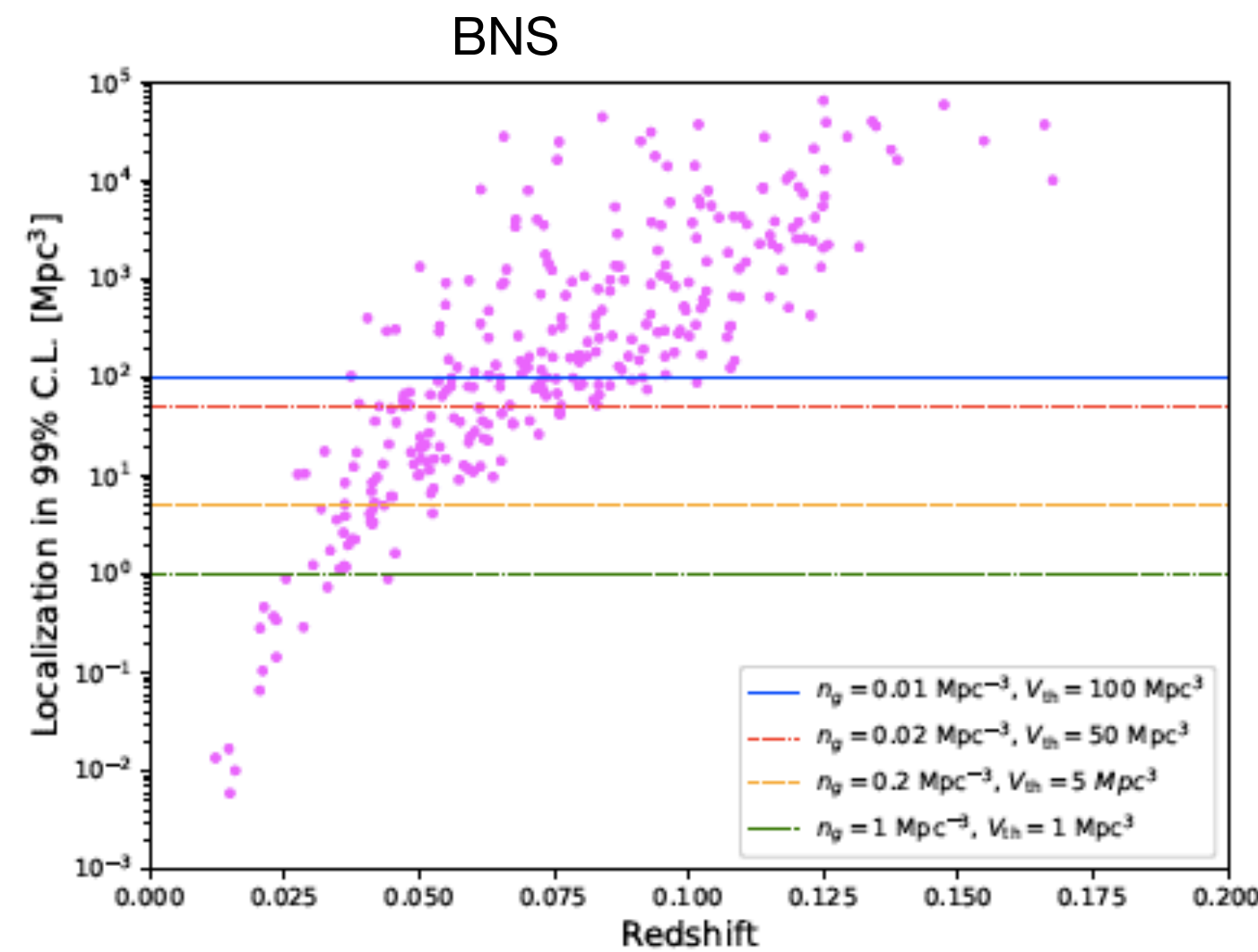
$$\Delta\Omega = 2\pi |\sin\theta| \sqrt{\Gamma_{\theta\theta}^{-1}\Gamma_{\phi\phi}^{-1} - (\Gamma_{\theta\phi}^{-1})^2}$$

where $\Gamma_{ij} = \left(\frac{\partial h}{\partial \lambda_i}, \frac{\partial h}{\partial \lambda_j} \right)$ is Fisher matrix.

Simulated results for 5 year run of AEDGE assuming GWTC-3 population

AEDGE is a proposed mid-band detector in space,

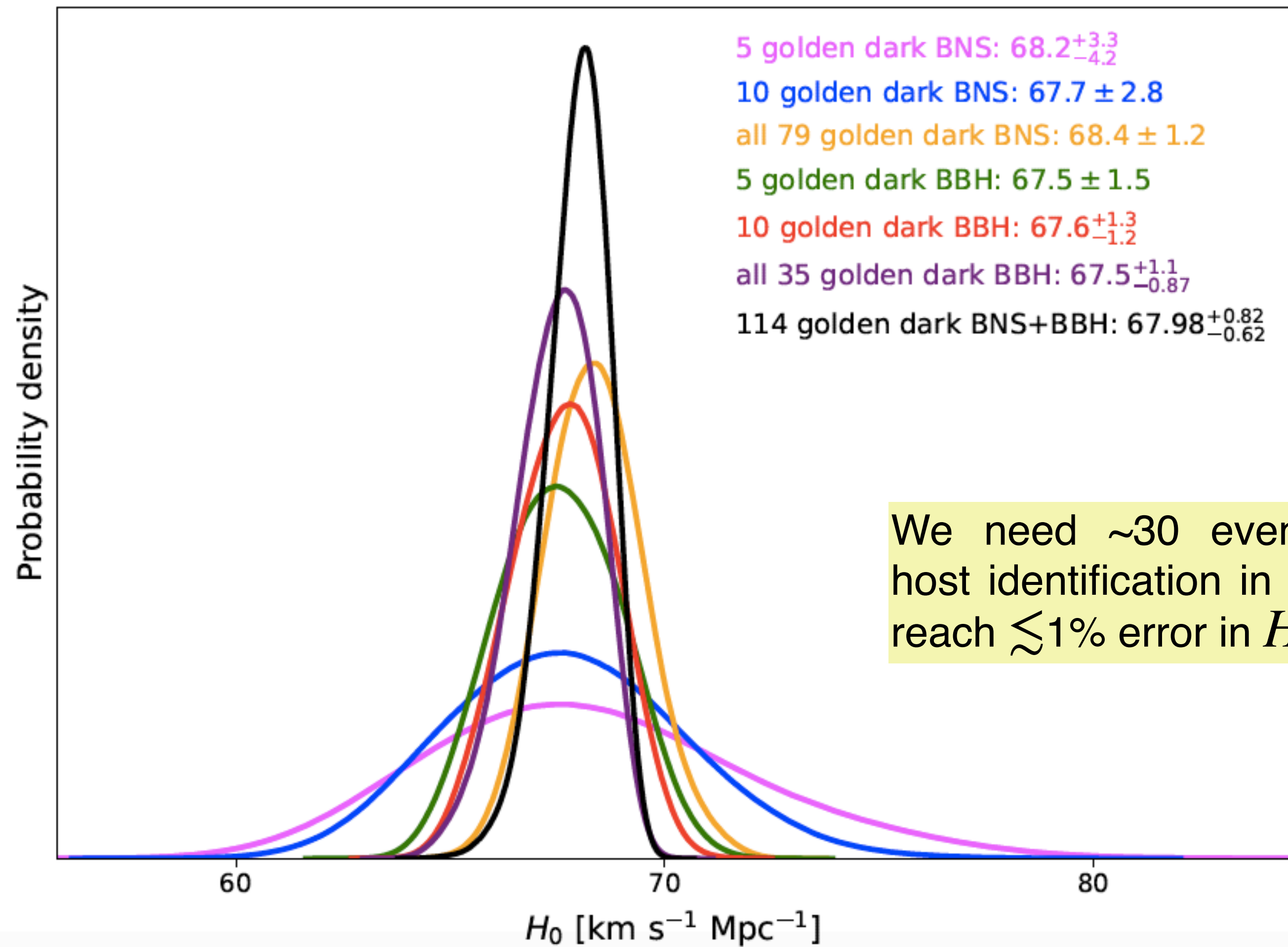
Localization volume and Hubble Diagram



Simulated Hubble Diagram

Various cuts are assumed galaxy number densities: below these lines, we can uniquely identify host galaxies within 5 year observation

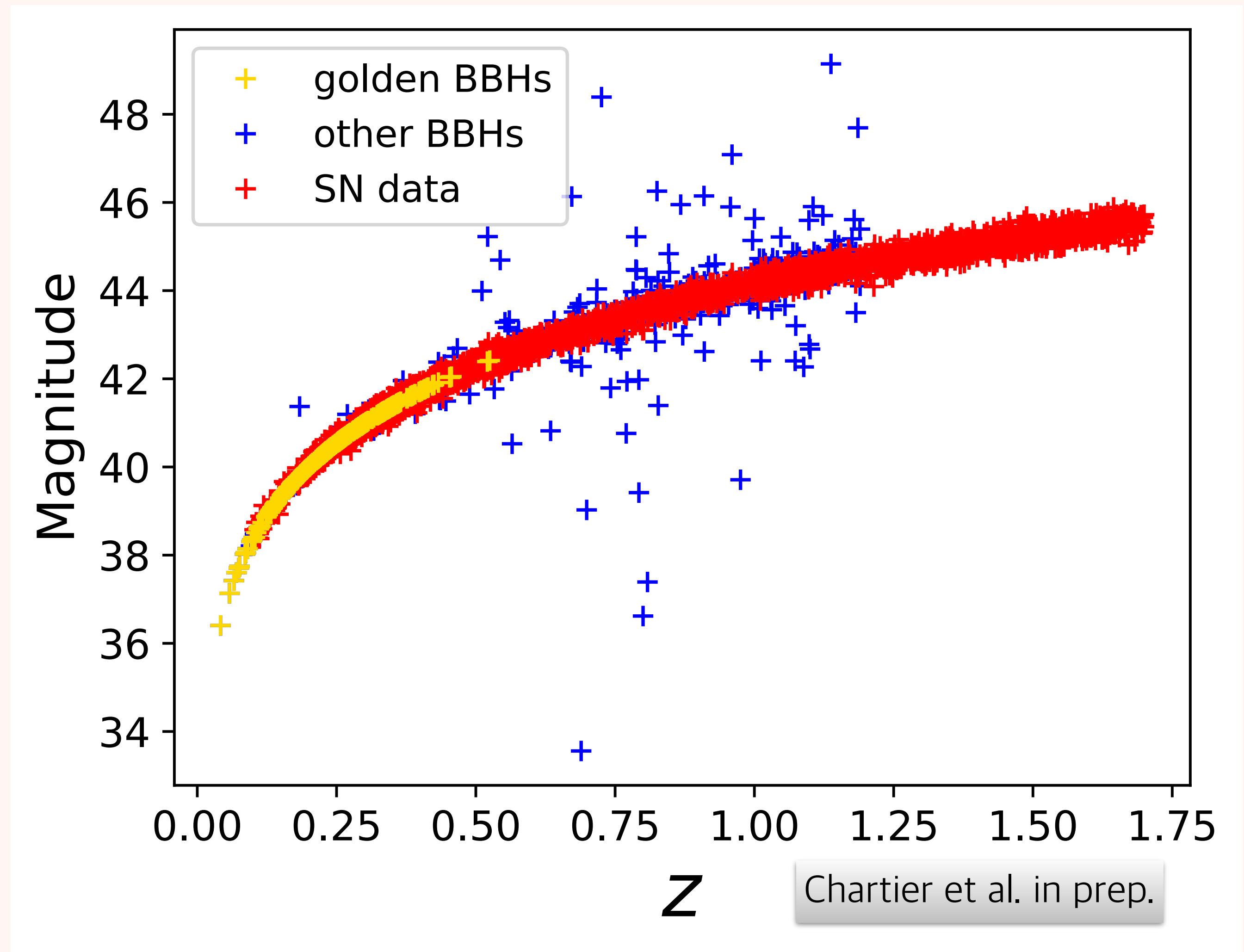
Simulation of Hubble Constant Estimation with Dark Sirens



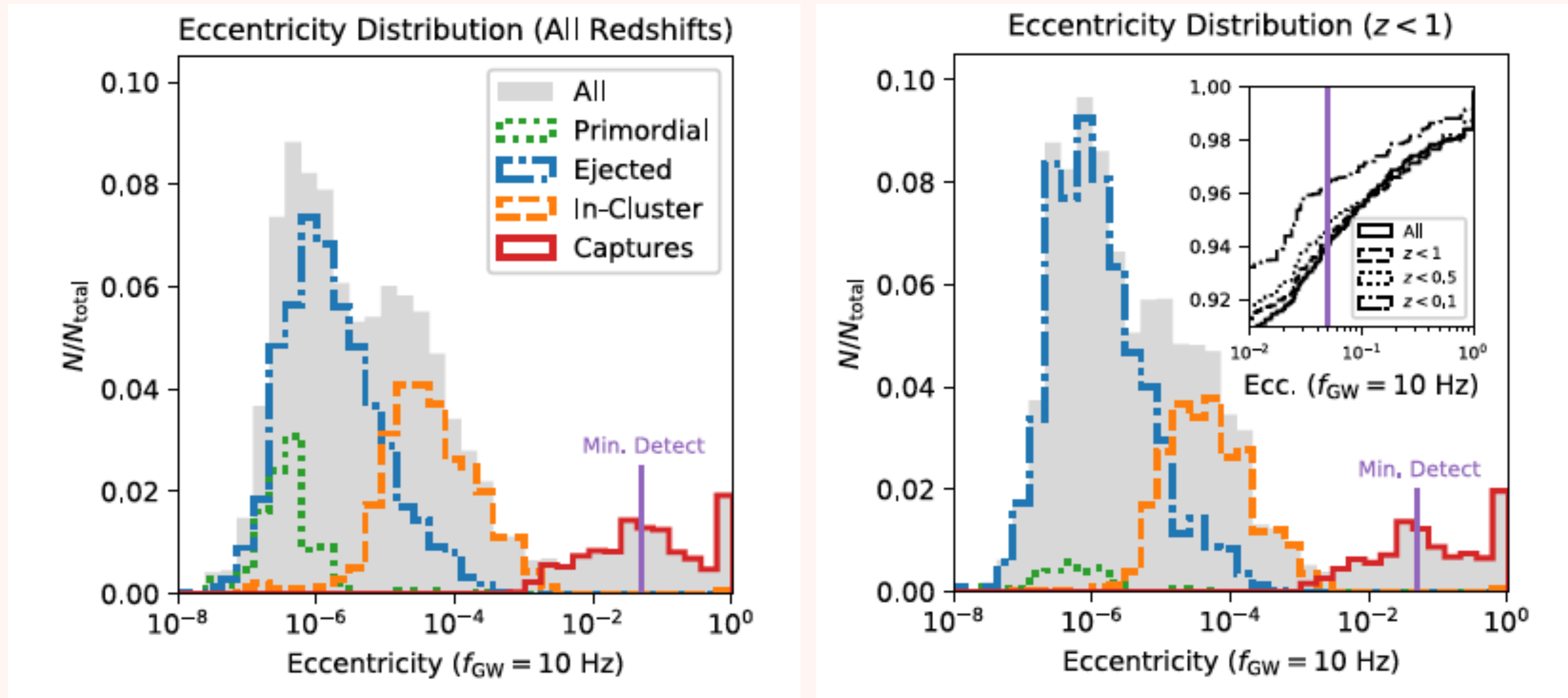
We need ~ 30 events with host identification in order to reach $\lesssim 1\%$ error in H_0

Hubble Diagram with GW Observations

- Sensitive GW detectors allow us to accurately constrain $z - d_L$ relation within $z < 0.5$



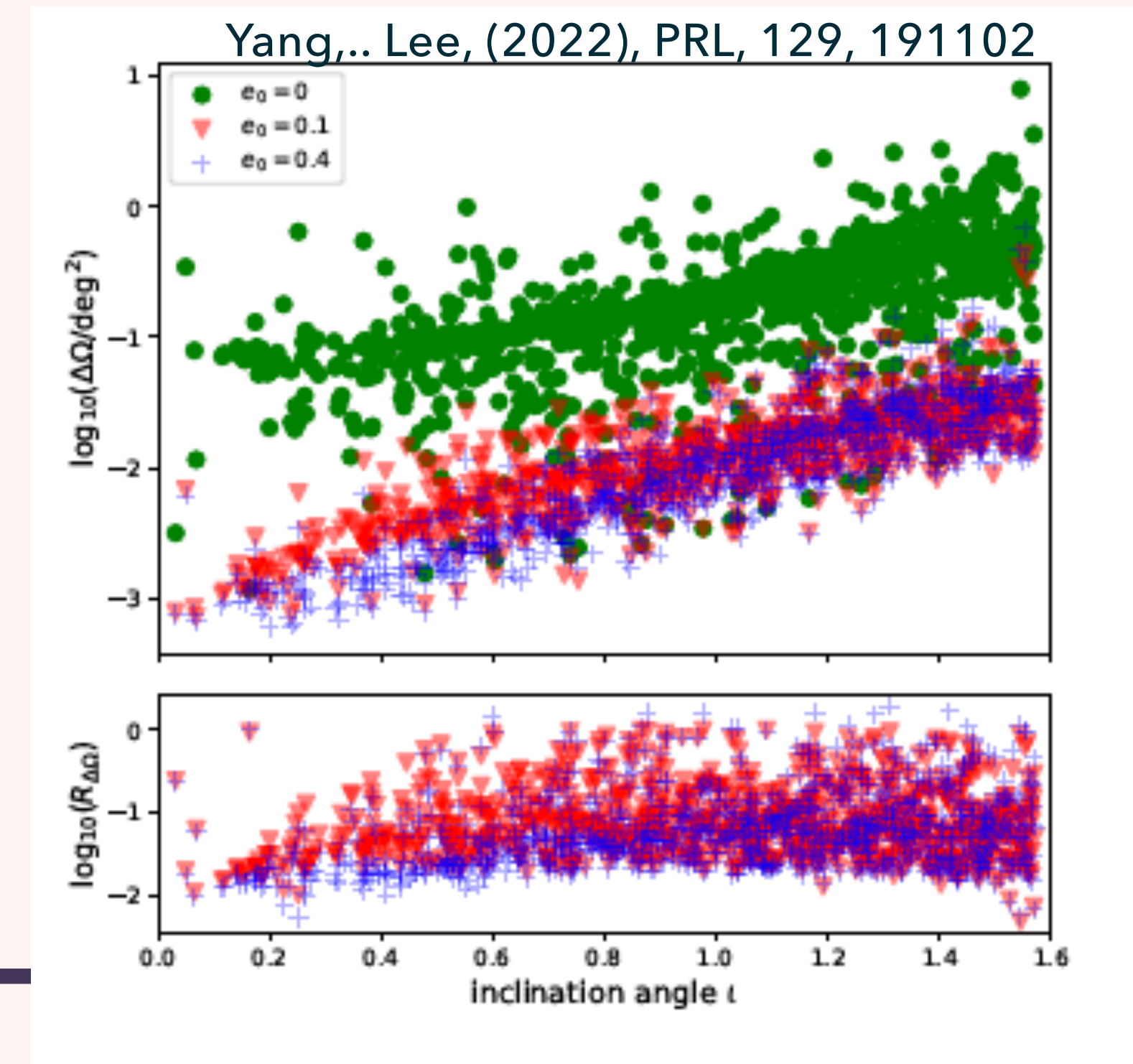
So far we assumed circular binaries, but dynamical processes produce eccentric binaries



Rodriguez et al., PRD 98. 123005 (2018)

Further improvements of estimated parameters for eccentric binaries

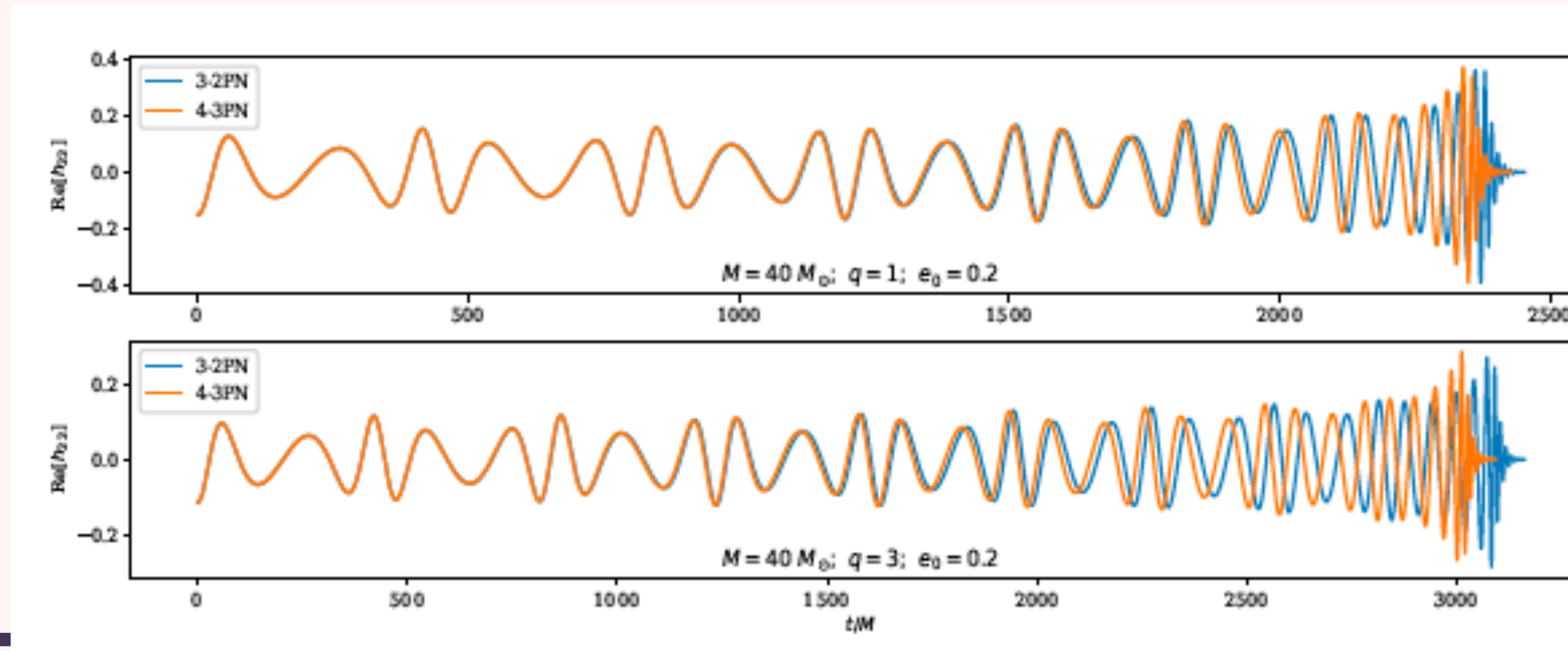
- In mid-frequency band, some binaries may have significant eccentricity (i.e., $e > 0.1$)
- The eccentric waveforms have more features than circular ones, and thus enable us to break some of the degeneracies during the inspiral phase → more accurate parameters can be inferred
- A case study with B-DECIGO:
 - $\Delta d_L / d_L$ can be improved near $\iota = 0$.
 - $(\Delta\Omega)_{e=0.1} \lesssim (\Delta\Omega)_{e=0}$
 - More improvement for larger e .



Accurate Waveforms for longer duration

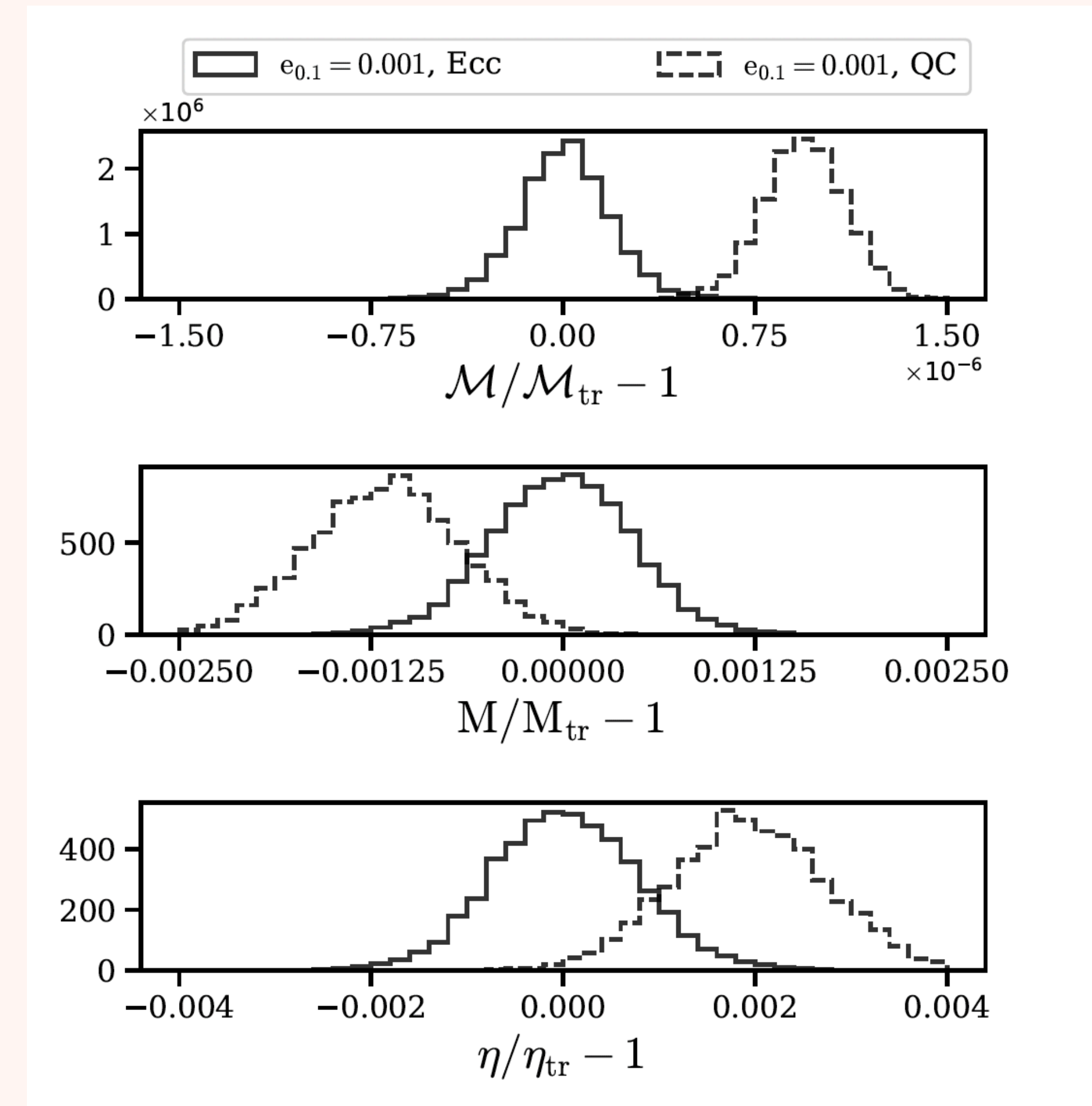
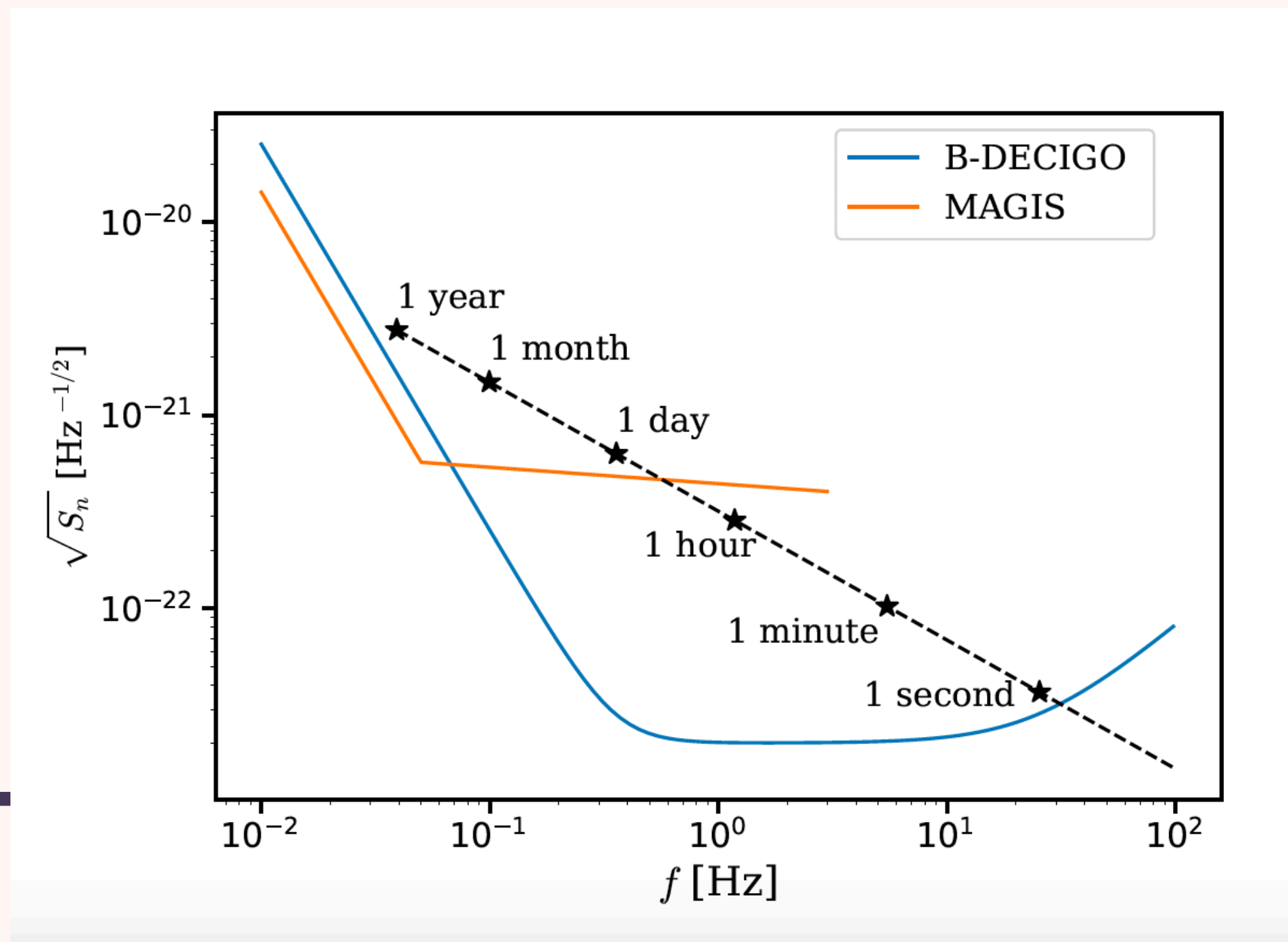
- In order to fully utilize the long duration observation data with mid-frequency, we need accurate waveforms from binaries with eccentricity and spins.
- Current status:
 - Time domain waveforms can be computed up to 4 PN. (Cho et al. 2022) for binaries with arbitrary eccentricity.
 - We need to transform the TD waveform into freq. domain: issue of higher modes.
- Spin:
 - Waveforms for spinning black hole binaries are not well understood yet.

Cho et al. 2022, PRD, 105, 064010



Systematic Errors due to uncertain waveforms (Choi, Yang & Lee, PRD, 2024)

- We made detailed parameter estimation for assumed B-DECIGO and MAGIS Detectors for GW150914-like BBH
- The injected waveform is eccentric model (TaylorF2ECC) but analyzed with Quasi-circular model
- Systematic Errors in mass estimations (thus eventually distance) greater than statistical errors arise.



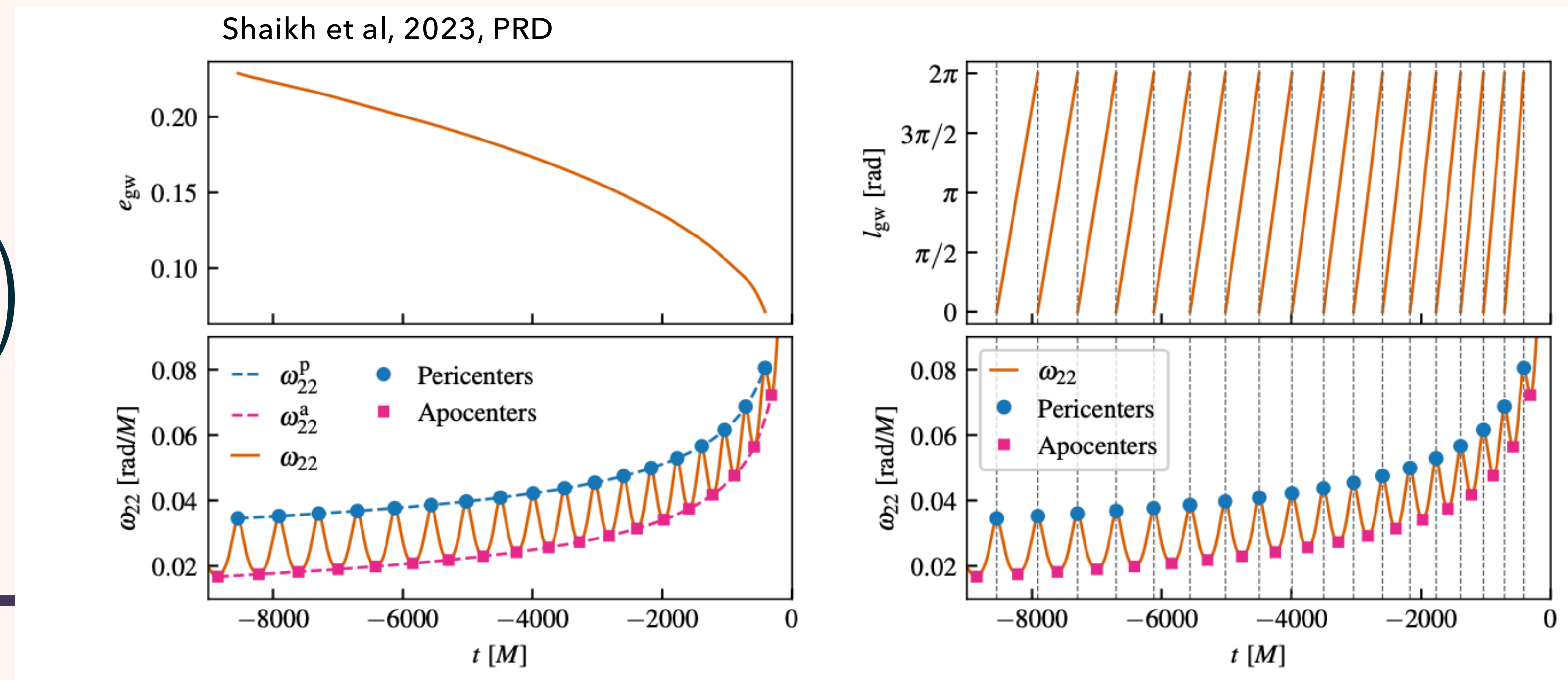
Eccentricities

- What is the eccentricity?
 - It is uniquely defined in Keplerian orbits only.
 - In PN theory, there are three eccentricities, e_r , e_ϕ and e_t with e_t being dependent on gauge.
 - Different models may be using eccentricity that is defined differently.
- Recently, Shaikh et al. (2023) proposed definition of gauge independent eccentricity from the GW frequency evolution.

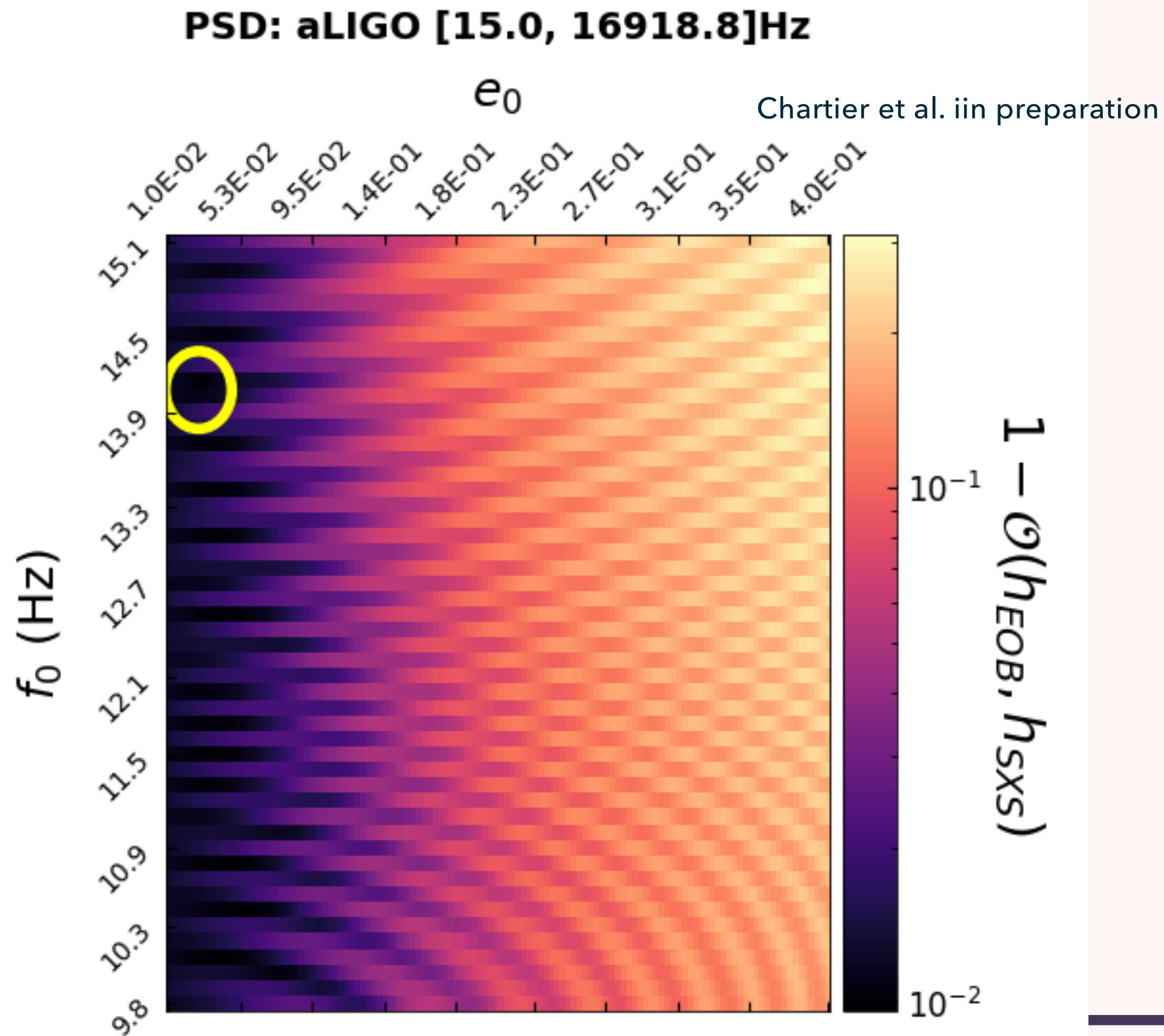
$$h_{22}(t) = A_{22}(t)e^{-i\phi_{22}(t)}, \quad \omega_{22}(t) = \frac{d\phi_{22}(t)}{dt}$$

$$e_{\omega_{22}} = \frac{\omega_{22}^p - \omega_{22}^a}{\omega_{22}^p + \omega_{22}^a} \quad \Psi = \arctan\left(\frac{1 - e_{\omega_{22}}^2}{2e_{\omega_{22}}}\right)$$

$$e_{gw} = \cos(\Psi/3) - \sqrt{3} \sin(\Psi/3)$$



Are eccentric models consistent each other (Chartier et al. in prep)

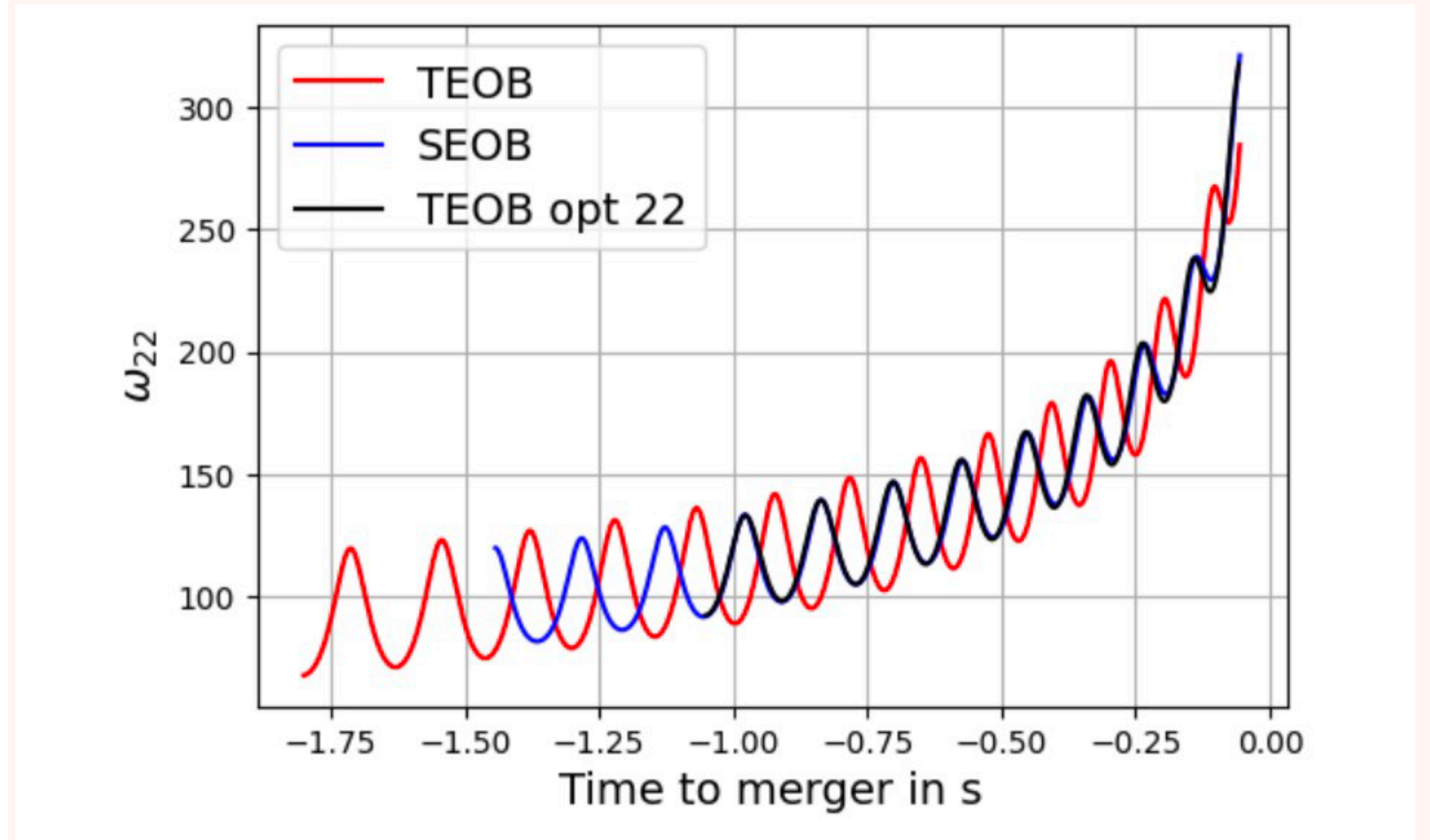
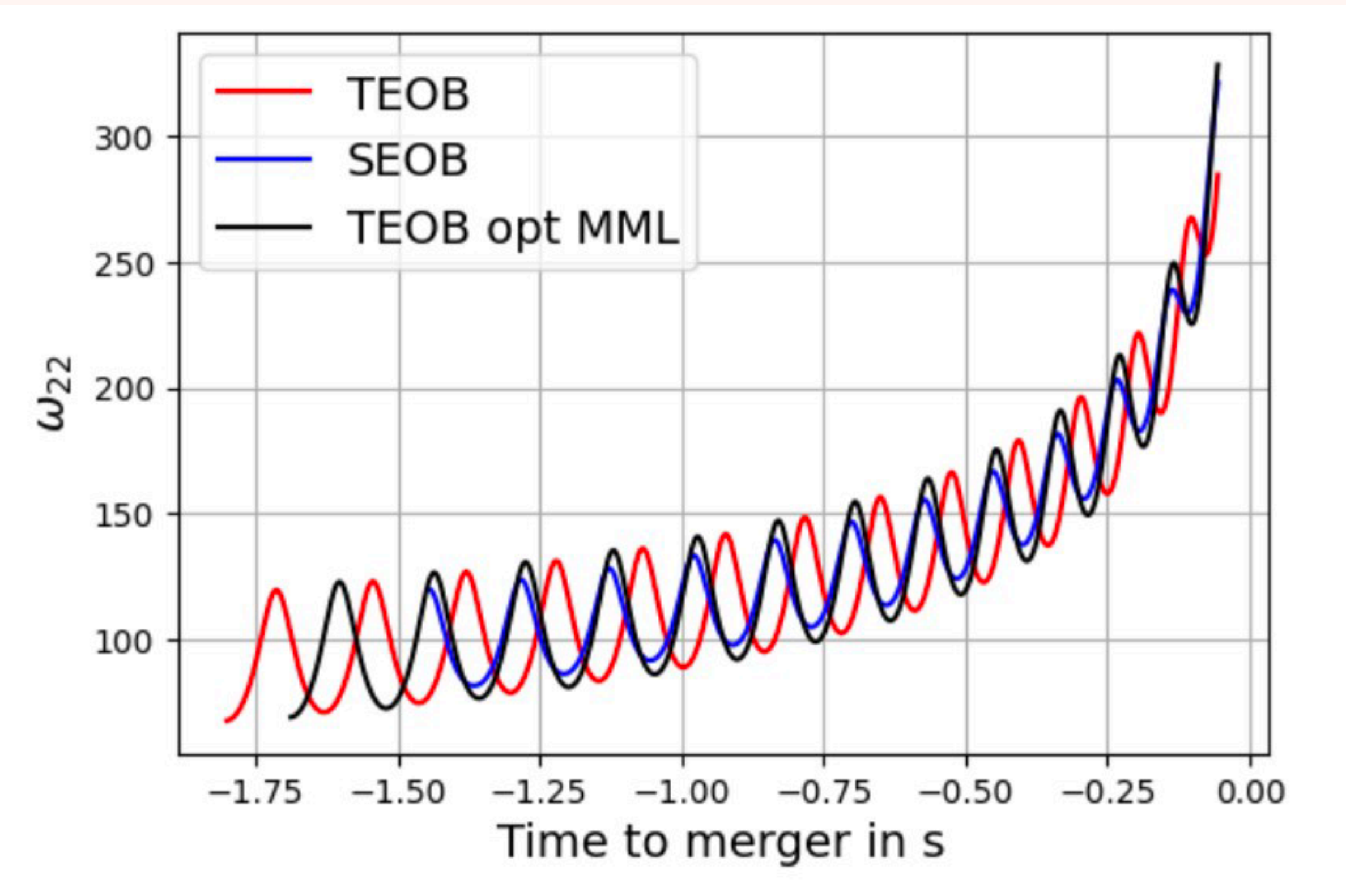


$$\mathcal{O}(h_1, h_2) = \frac{\langle h_1 h_2 \rangle}{|h_1| |h_2|}$$

- Comparison between EOB model and NR shows that there is a significant mismatch for $e > 0.05$ at high $f_0 \sim 10$ Hz.
- Further, the mismatch seems to have oscillating behavior depending on the f_0 where e_0 is defined.
- This indicates that the definition of eccentricity depends on the phase of the GW waves.

Optimize with the same e_{gw}

Chartier et al. in preparation



- If we require the same $e_{gw}(t)$ behavior for different models, we sometimes end up with different phase evolution that means significant mismatch.

Summary

- Hubble tension could be due to our lack of understanding of the universe or systematic error in local measurements
- GW sources can be used to make completely independent measurements of the Hubble constant.
- However, individual measurement of distances with GW sources have large random error due to the lack of information on inclination angle of the coalescing binaries.
- BBHs do not emit EM radiation. The pointing accuracy of the ground-based detectors (including the future ones) is very poor for host identification.
- Localization can be significantly improved with future detectors in mid-band (0.1 ~ 1 Hz).
 - If some binaries are eccentric, accuracies of directions and distances can be further improved.
- Cosmological parameters could be precisely constrained with dark sirens alone with mid-band detectors.