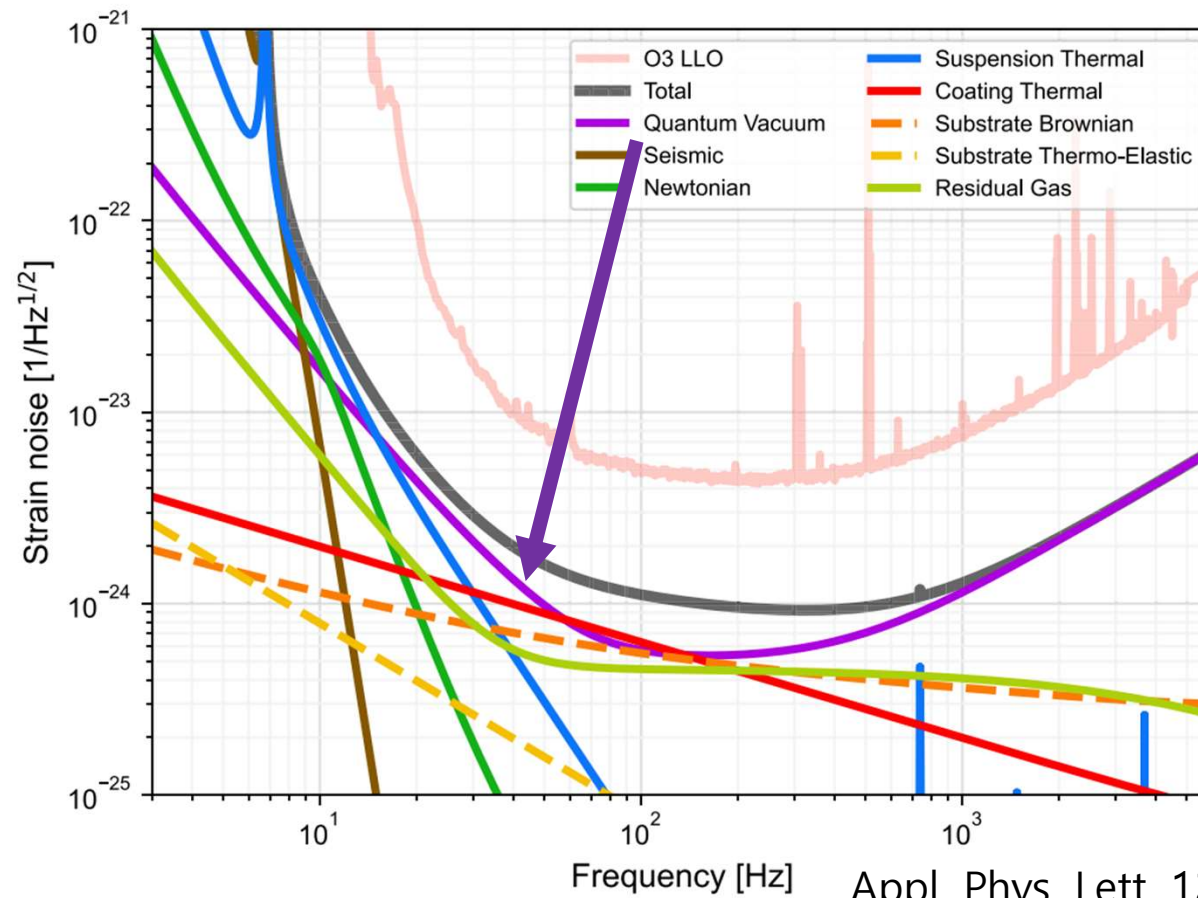


3. Quantum noise of gravitational wave detector

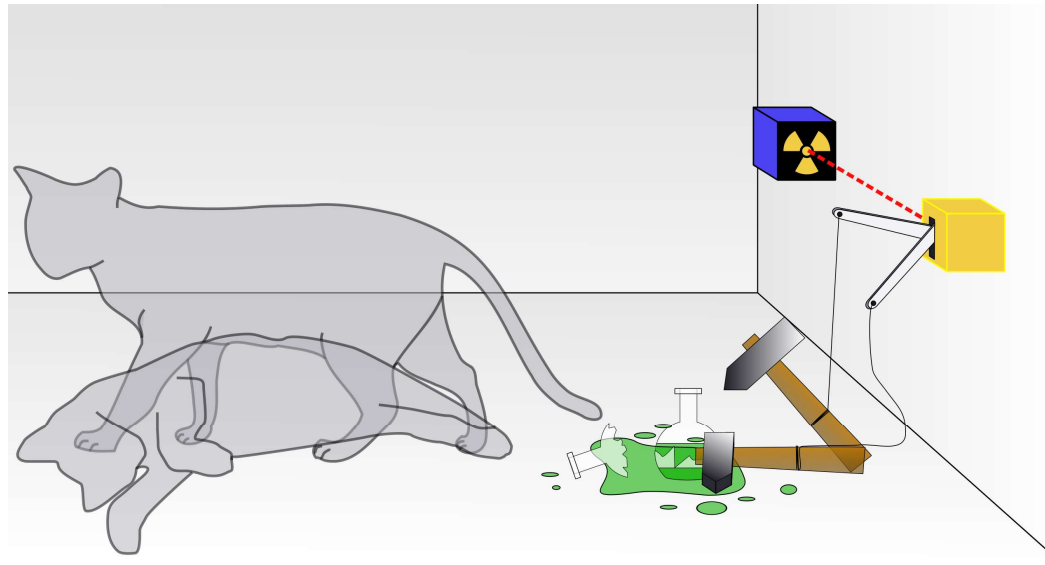


LIGO sensitivity

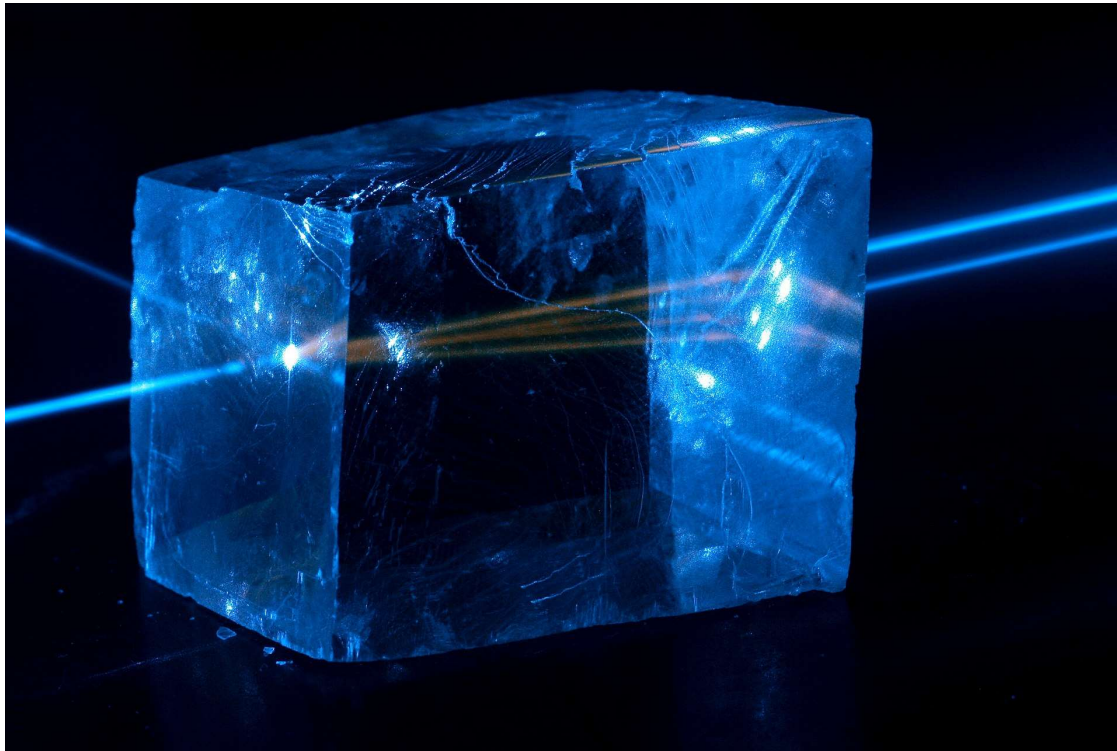


Appl. Phys. Lett. 122, 110502 (2023)

Quantum noise



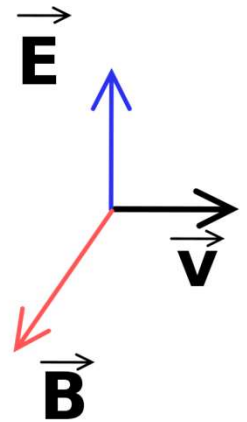
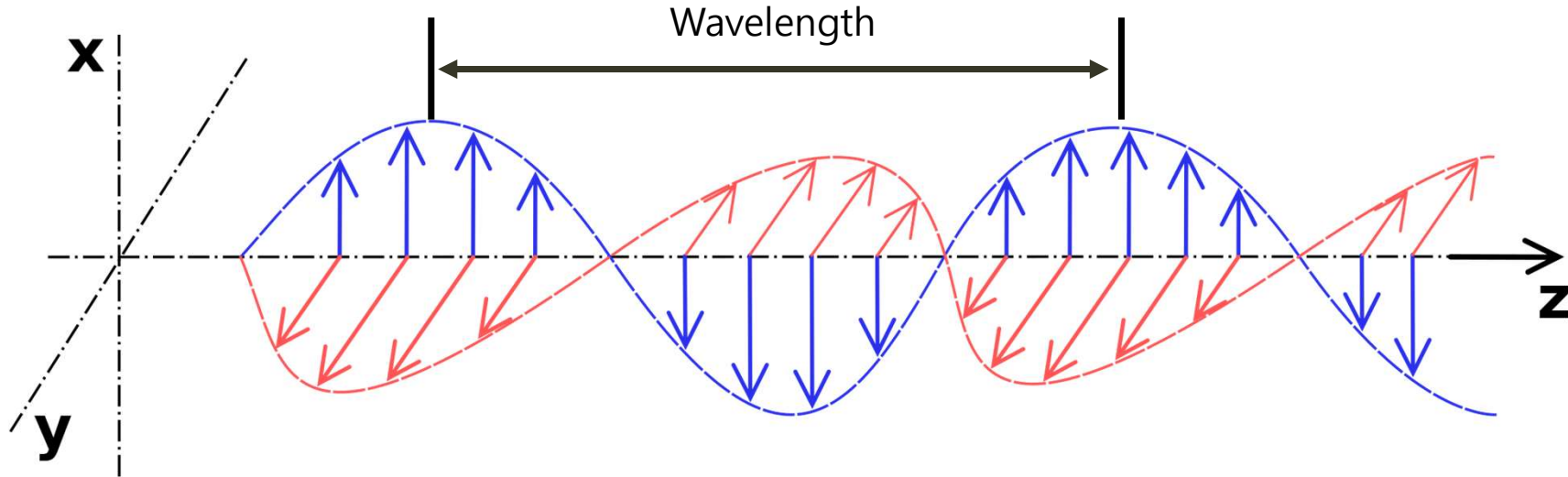
Wave-particle duality



Wave-particle duality

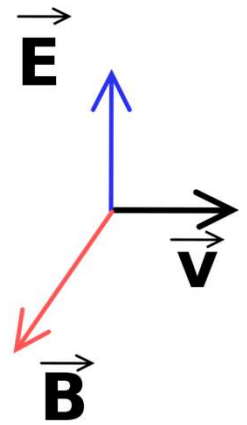
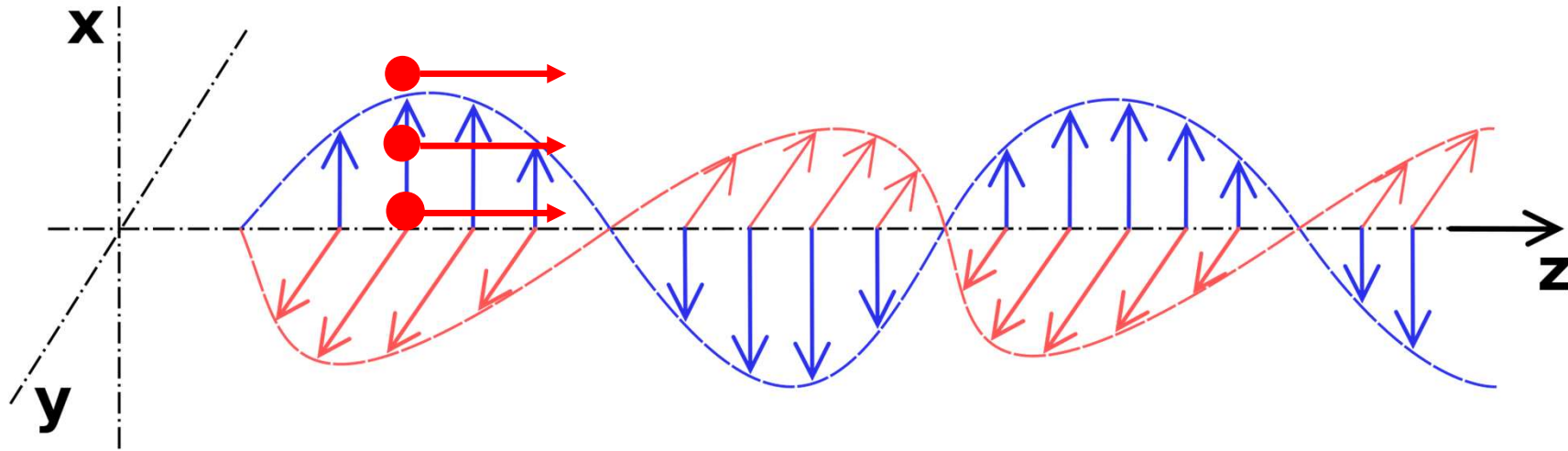


Light(Electromagnetic wave)

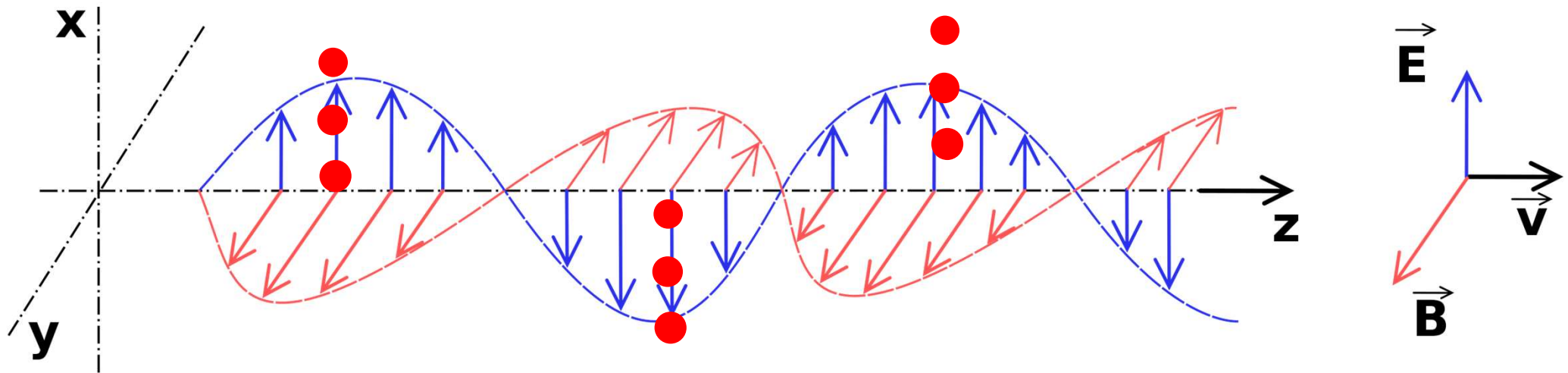


$$|\vec{E}| \propto \text{Intensity of Light}$$

Wave-particle duality



Wave-particle duality



Observed Photon

Photoelectric effect

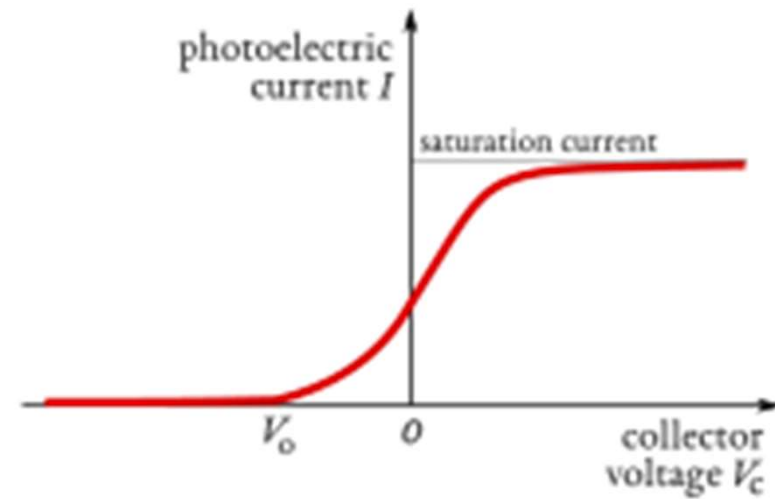
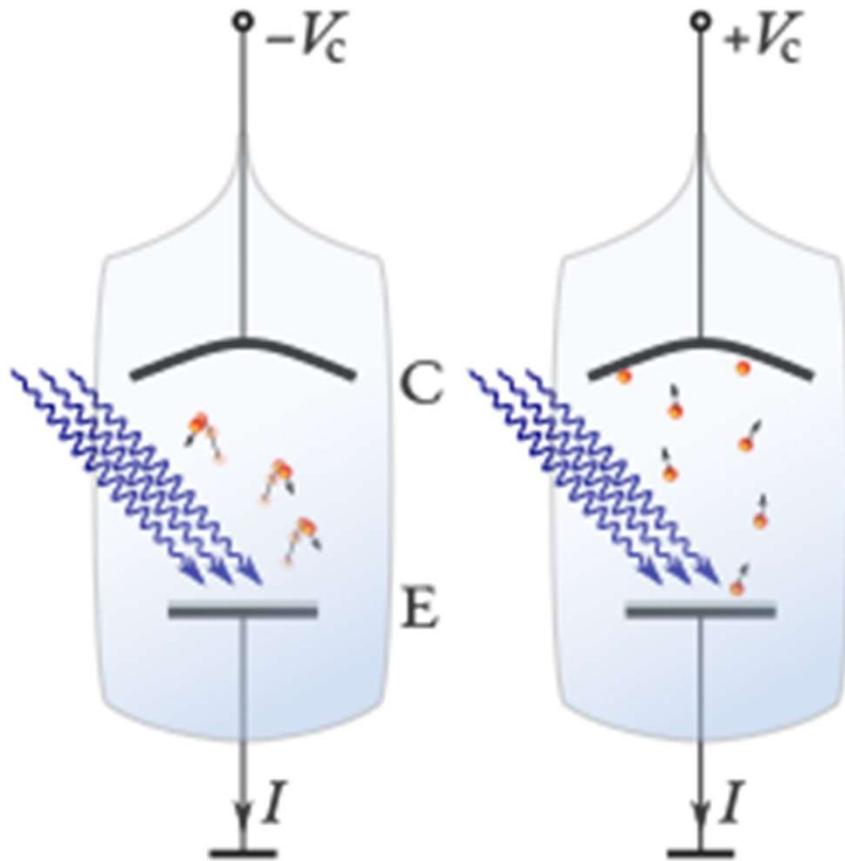


Photo detector

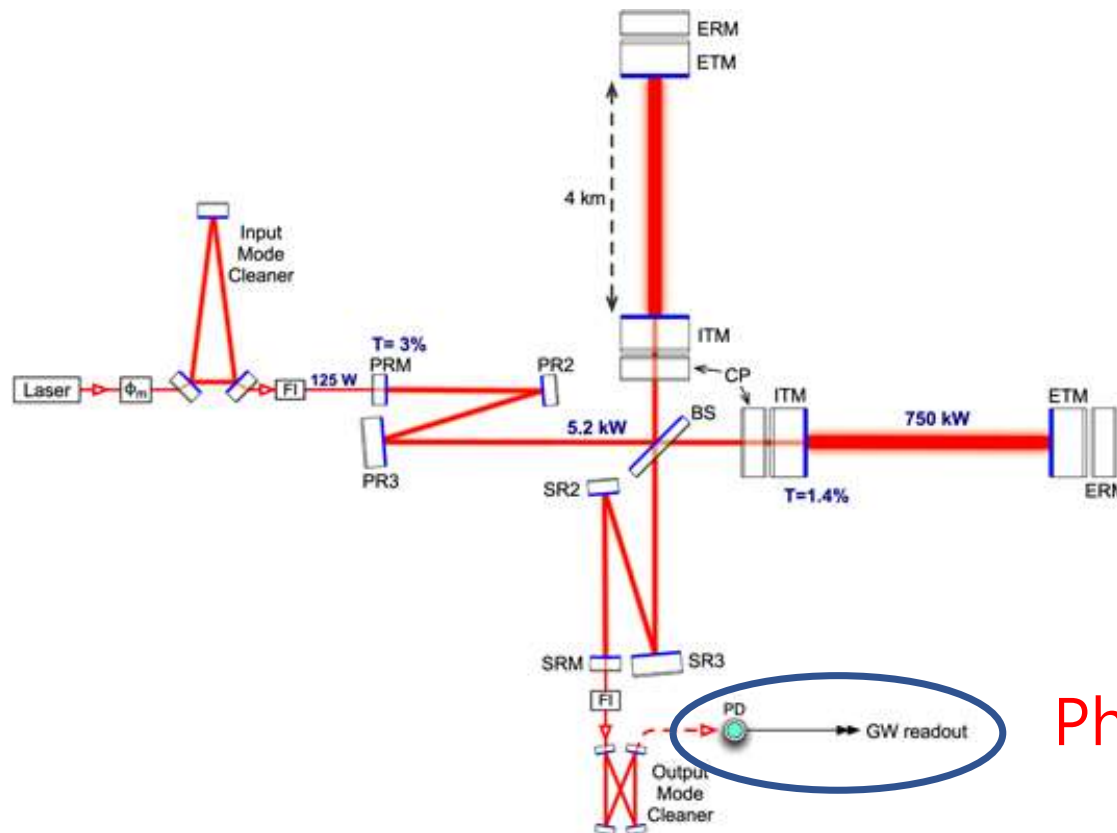


Amplitude of electricfield
 $\propto$ *Number of photon*

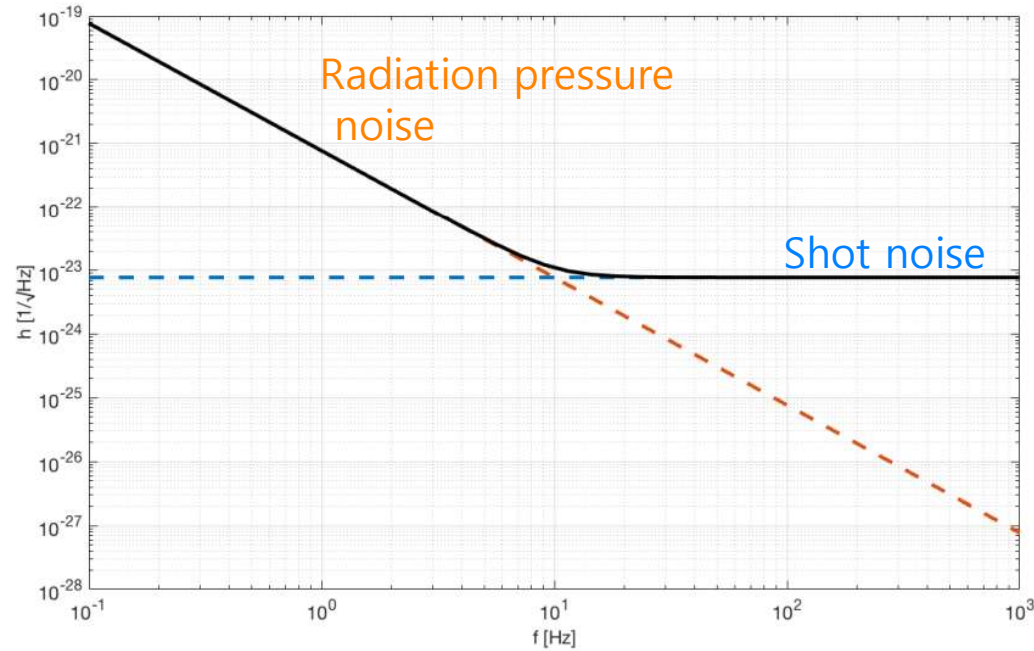
Measuring the intensity of light
= Counting the number of photon



Gravitational wave detector



Standard quantum limit of GW detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

Shot noise



Shot noise



Shot noise of interferometer

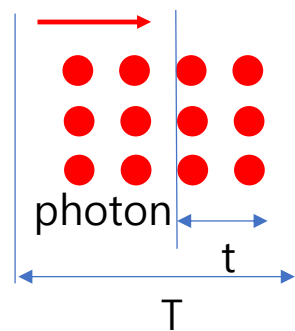


$$E = h\nu$$

Photon energy

Laser power(W) = the number of photon(n) / time(s)

$$nh\nu = \quad / s$$



Shot noise

Shot noise of interferometer

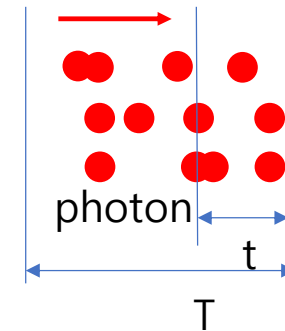
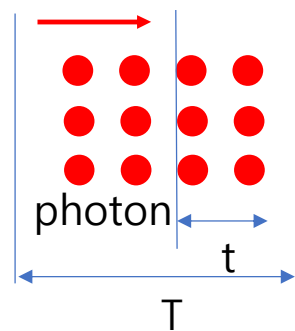


$$E = h\nu$$

Photon energy

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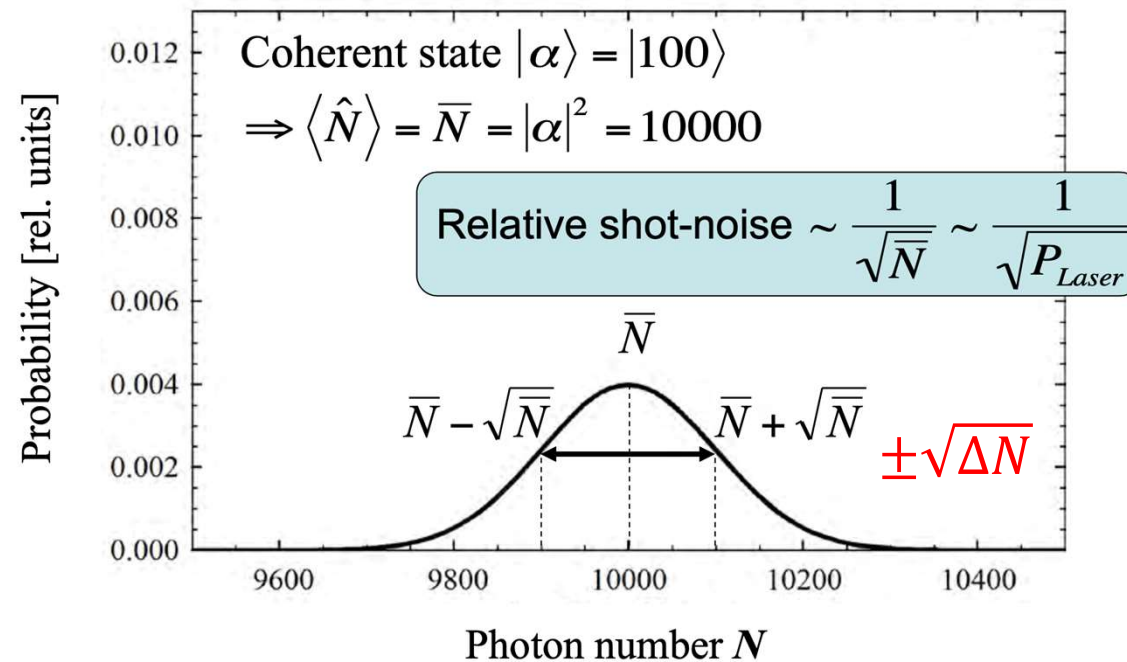
Shot noise

Shot noise of interferometer



Photon Counting Statistics

$$h = 6.62607015 \times 10^{-34} \text{ J}\cdot\text{Hz}^{-1}$$



Albert-Einstein-Institut



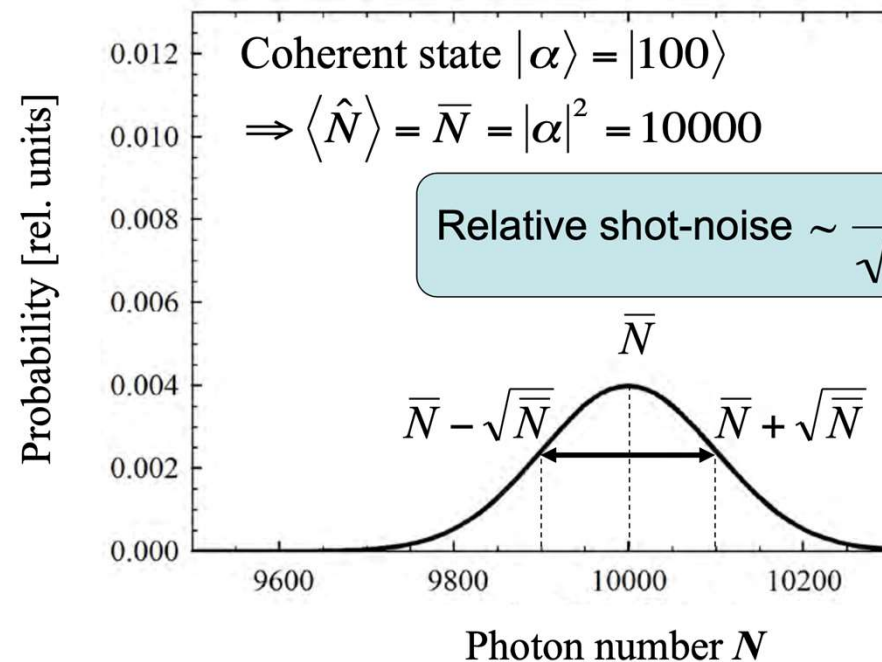
Roman Schnabel, 17 / June/ 2013

4

Shot noise of interferometer



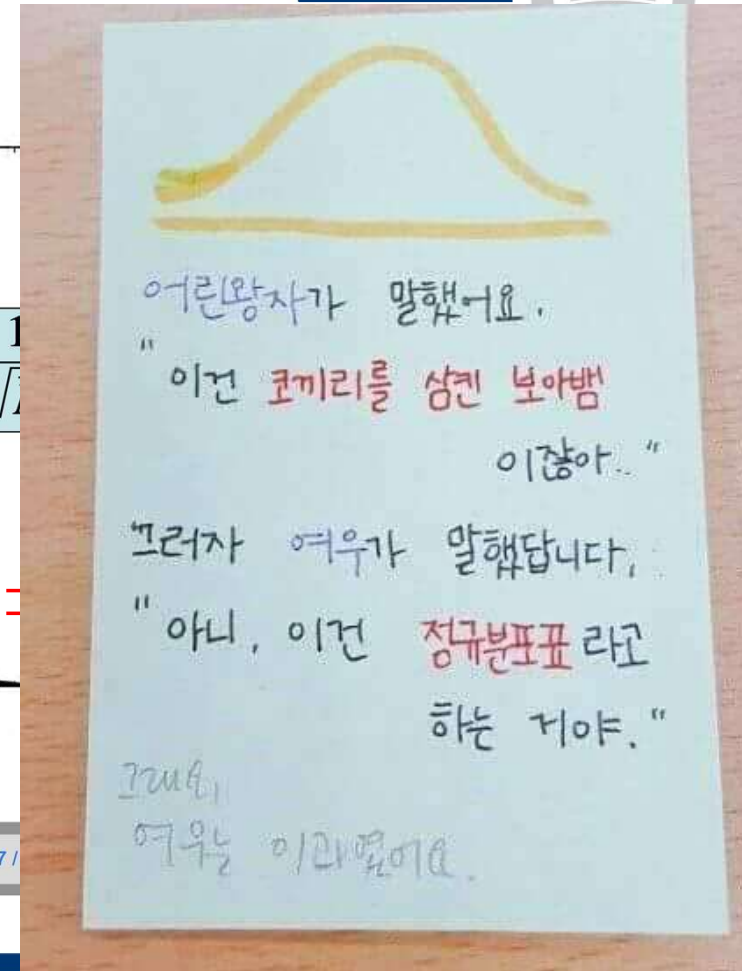
Photon Counting Statistics



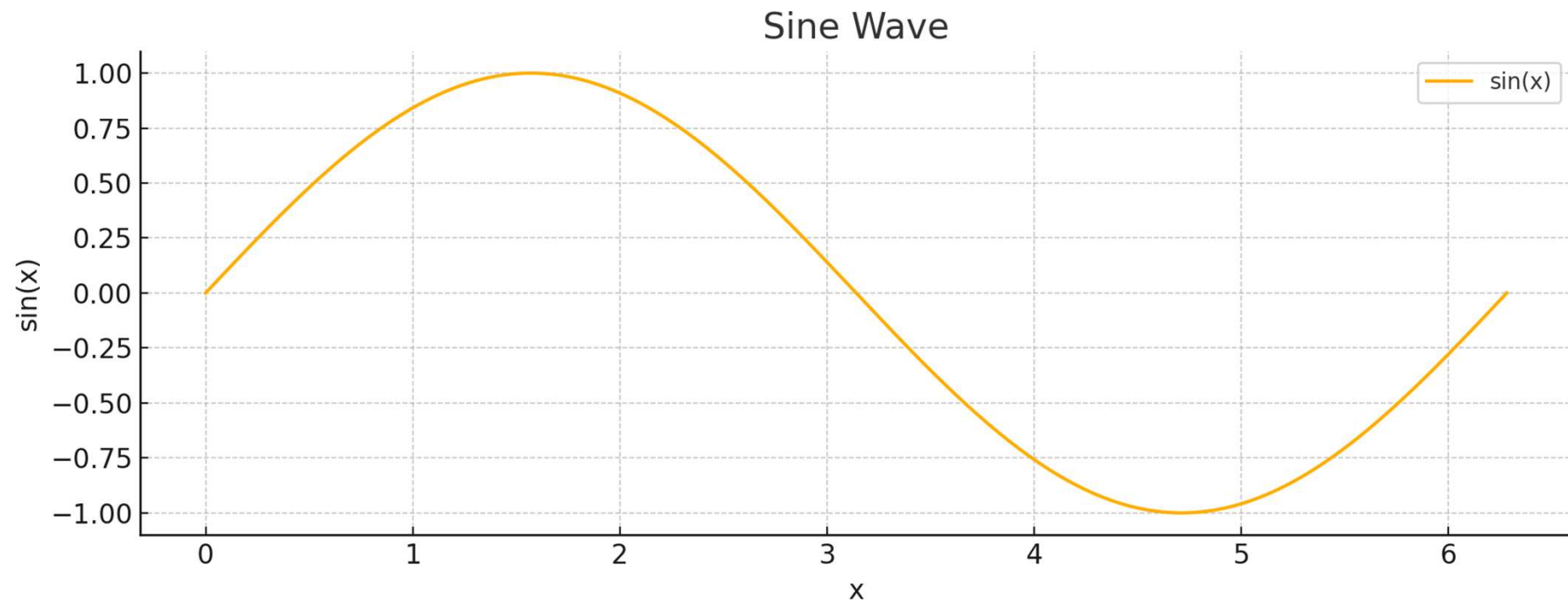
Albert-Einstein-Institut



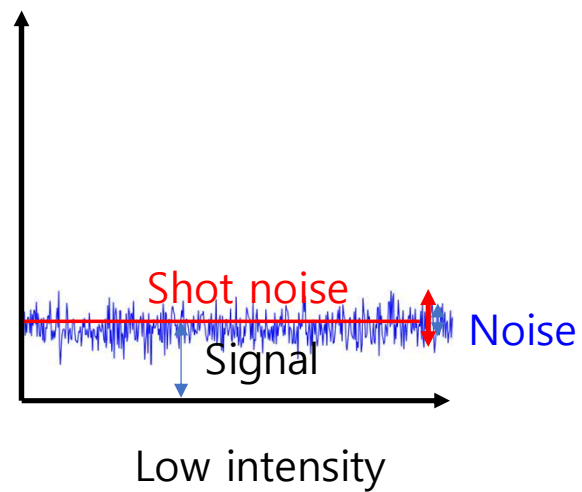
Roman Schnabel, 17 /



Sine wave

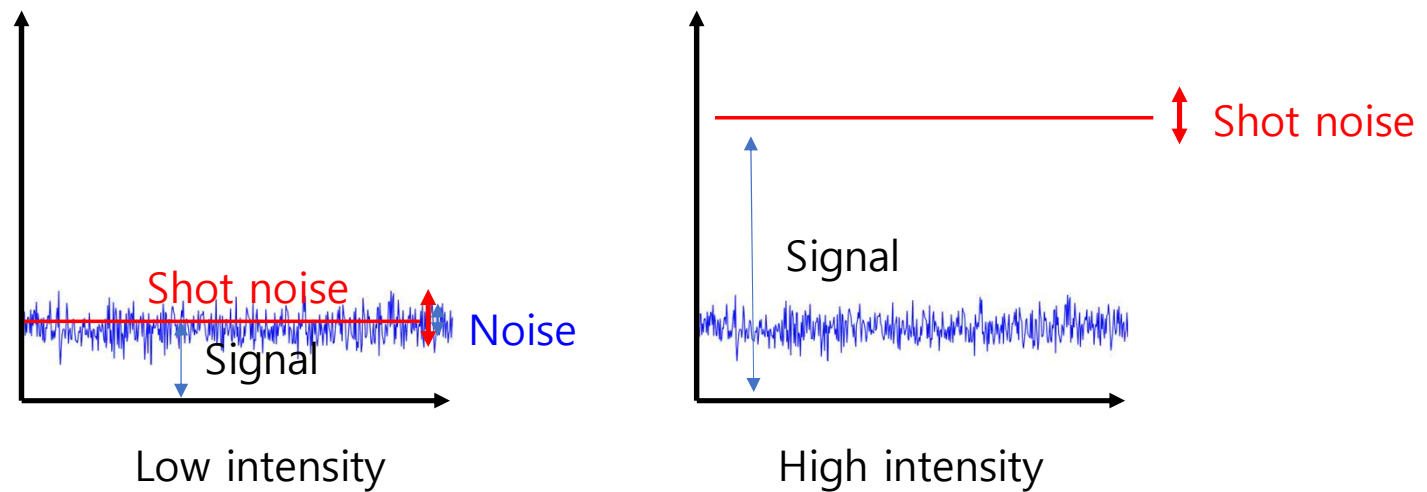


Shot noise of interferometer



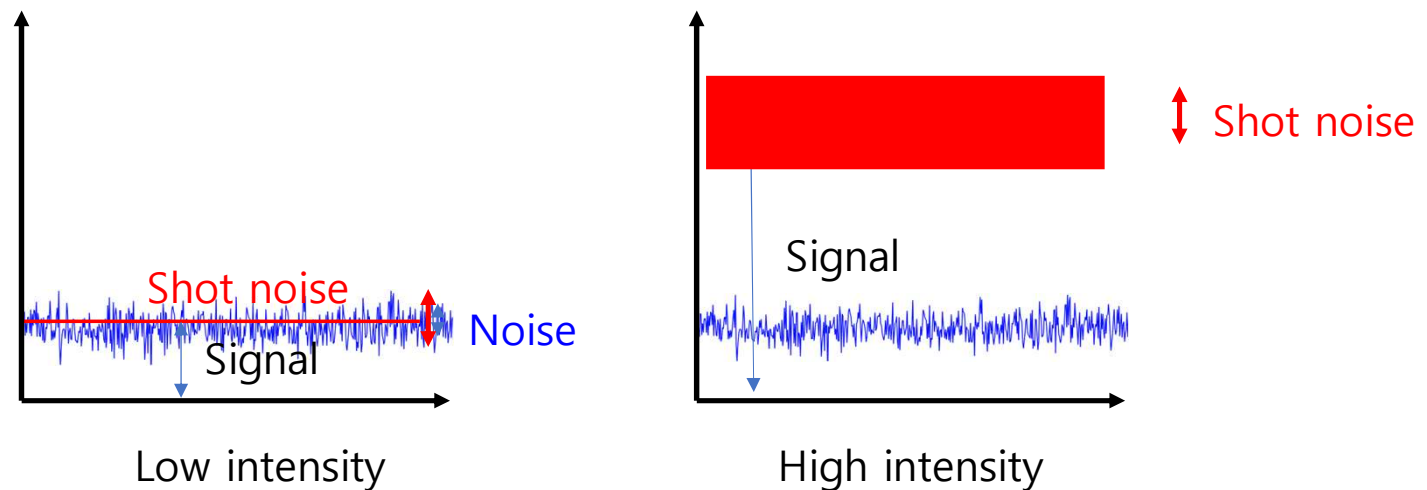
$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

Shot noise of interferometer



$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

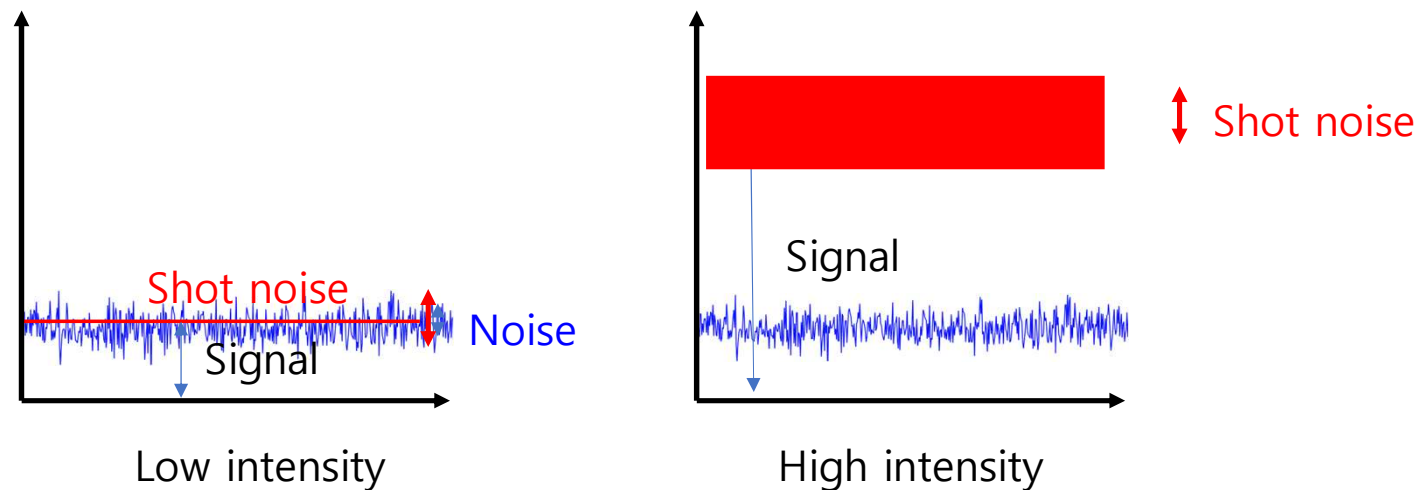
Shot noise of interferometer



$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
We say that it has **shot noise limit sensitivity**

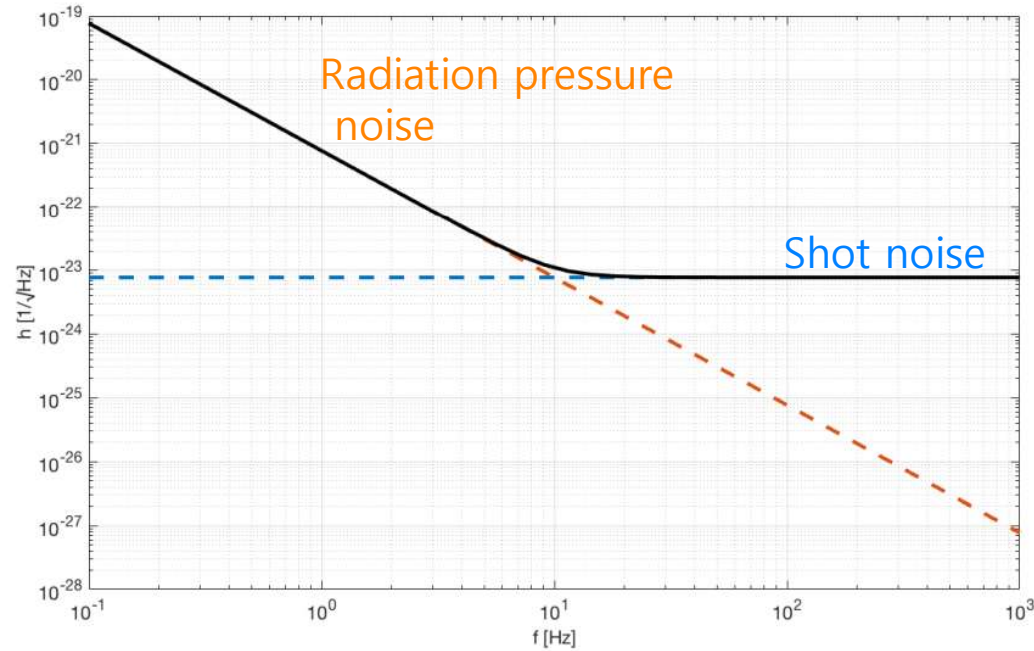
Shot noise of interferometer



$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}} \quad \begin{array}{l} N=100, \text{ SNR} = 10 \\ N=10000, \text{ SNR} = 100 \end{array}$$

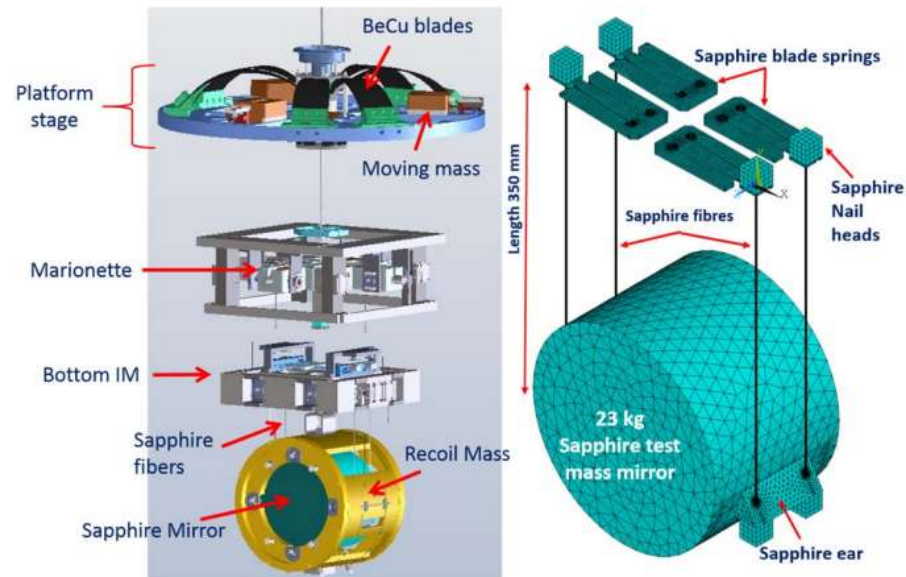
If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
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Standard quantum limit of GW detector

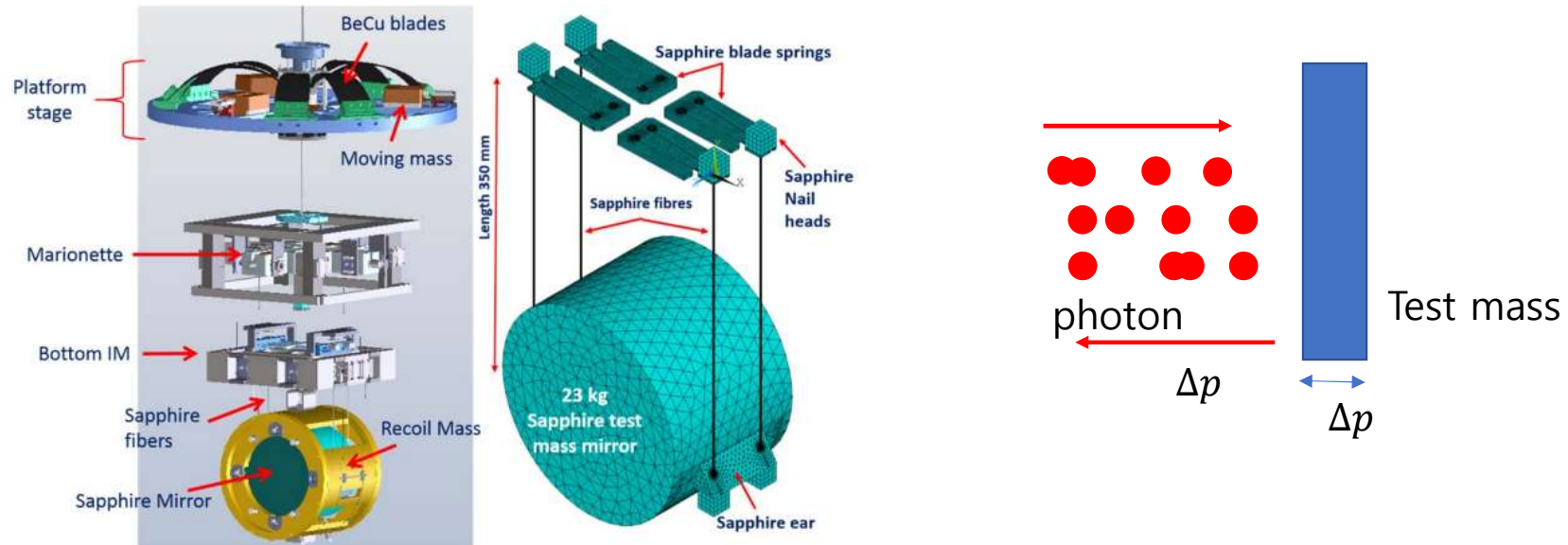


Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

Radiation pressure noise

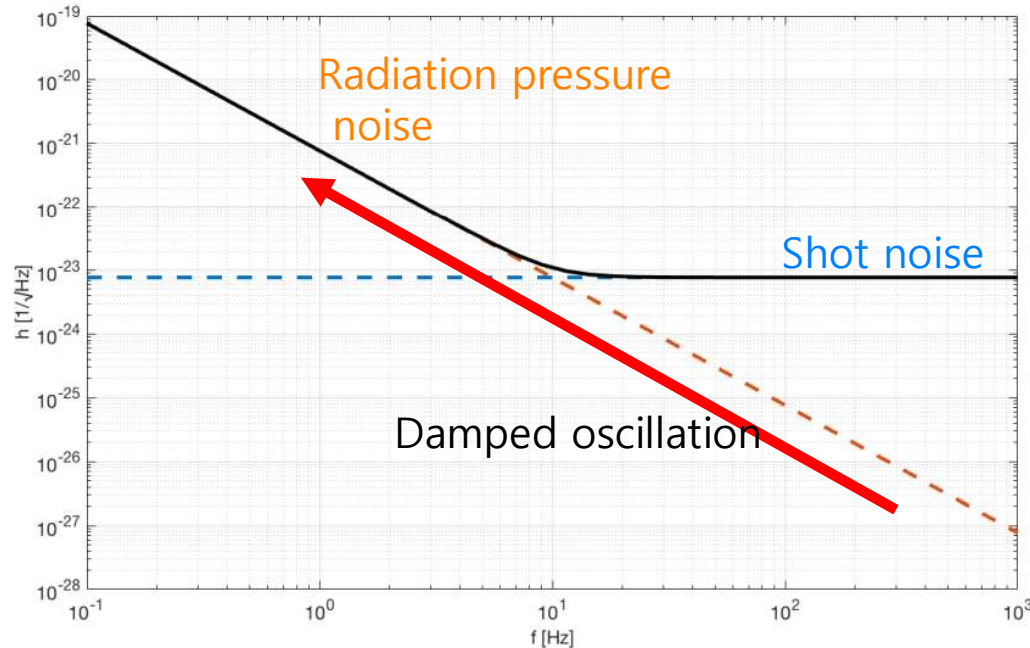


Radiation pressure noise



- Stored energy is very high (750 kW)
- Desired sensitivity is very high ($10^{-21} \sim 10^{-24}$)

Standard quantum limit of GW detector



Radiation pressure noise

- Photon pressure fluctuation at **mirror**

-> **Intensity noise**

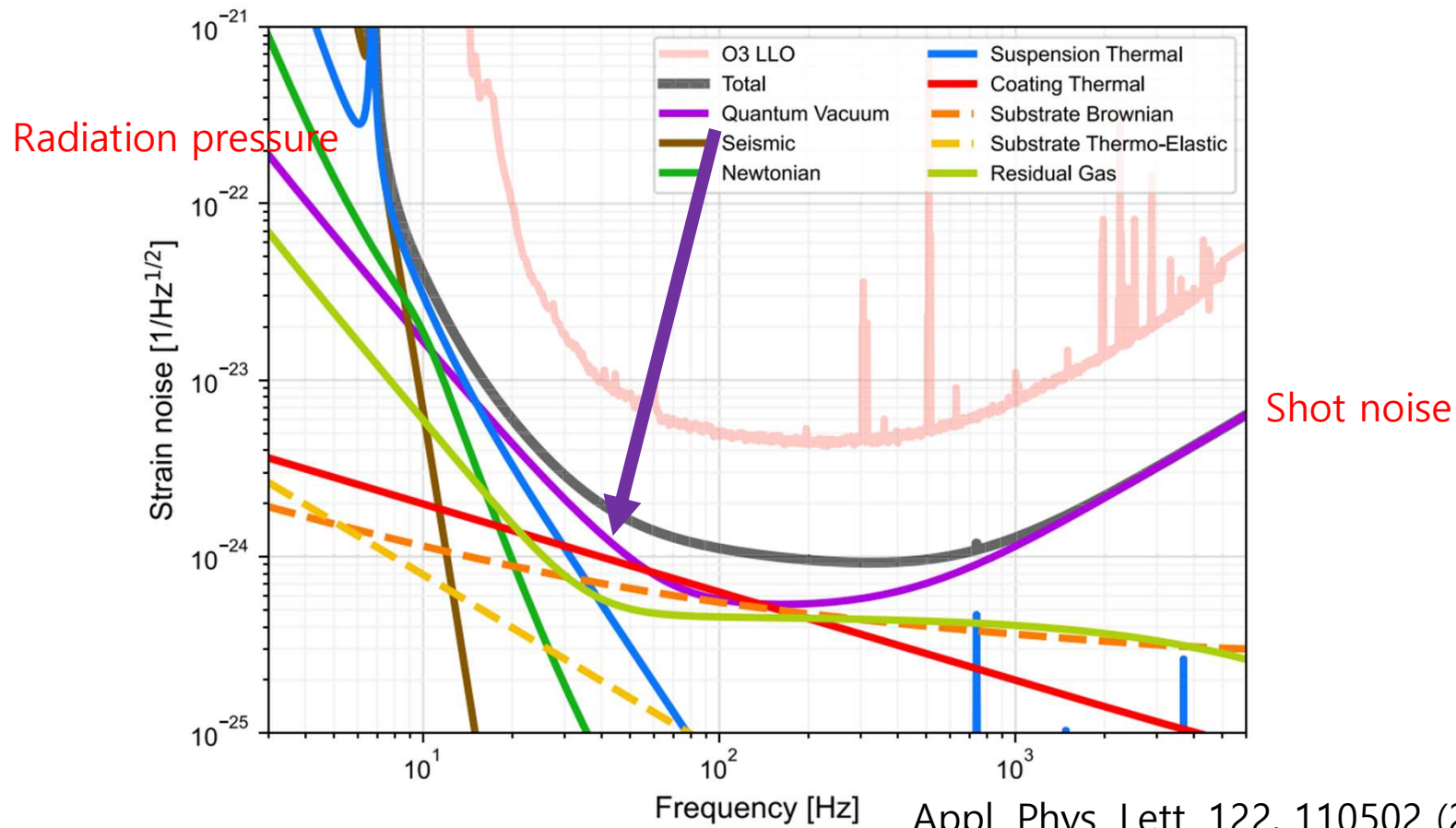
Shot noise

- Photon number fluctuation at **PD**

-> **Phase noise**

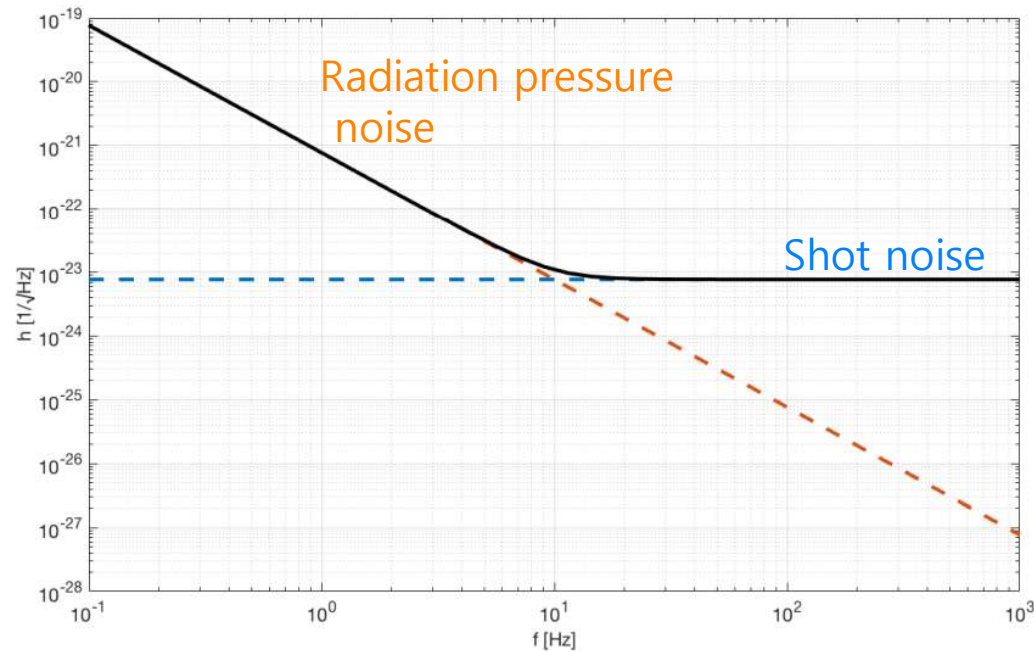
Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

LIGO sensitivity



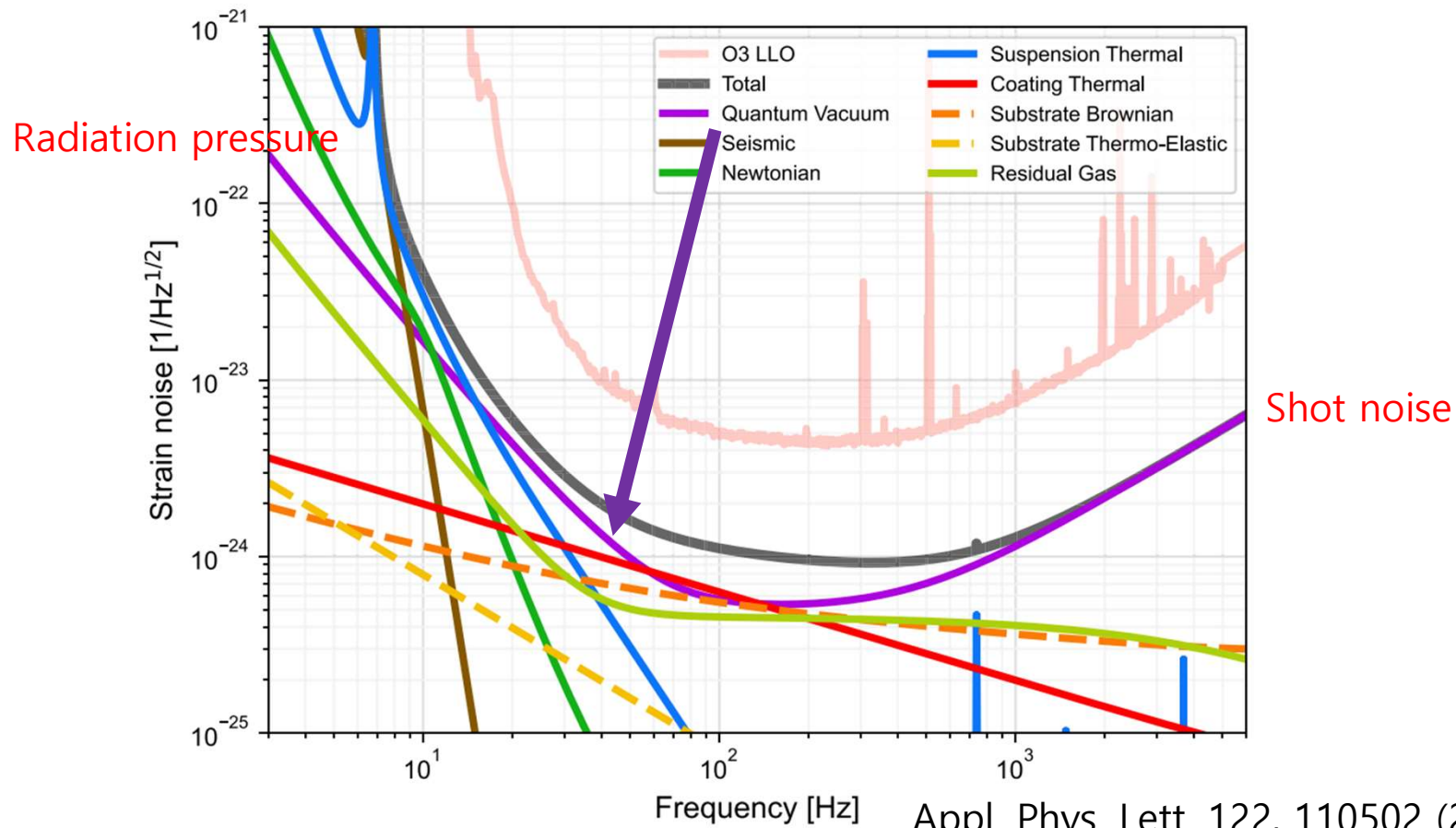
Appl. Phys. Lett. 122, 110502 (2023)

Standard quantum limit of GW detector



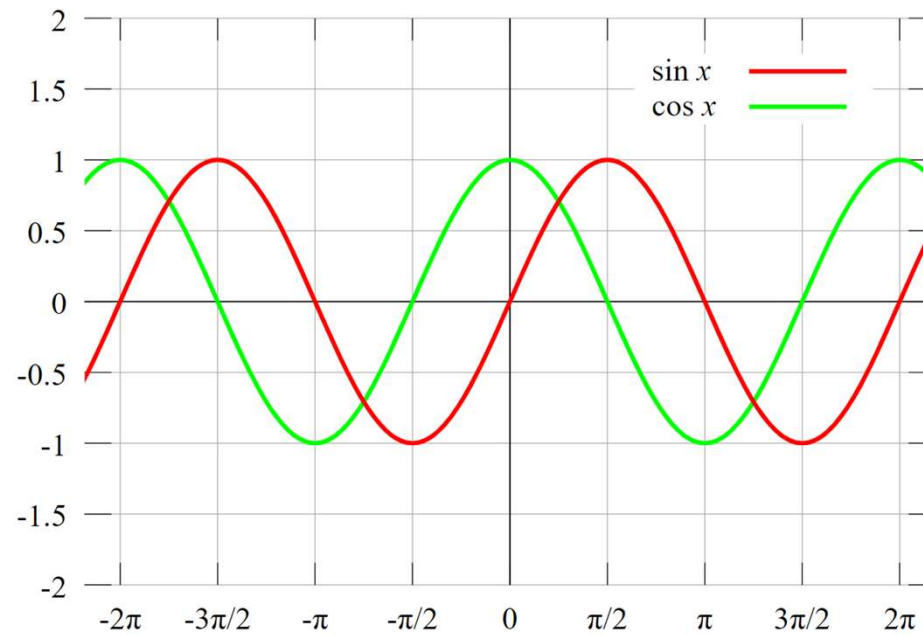
Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

LIGO sensitivity

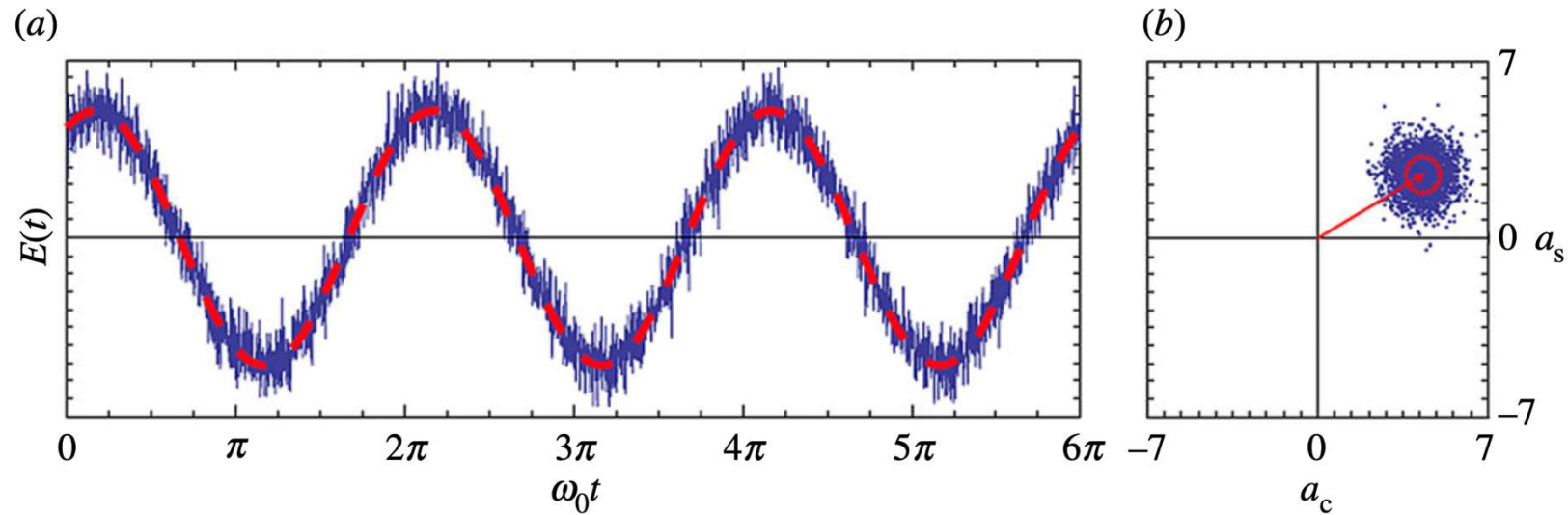


Appl. Phys. Lett. 122, 110502 (2023)

Classical electromagnetic wave

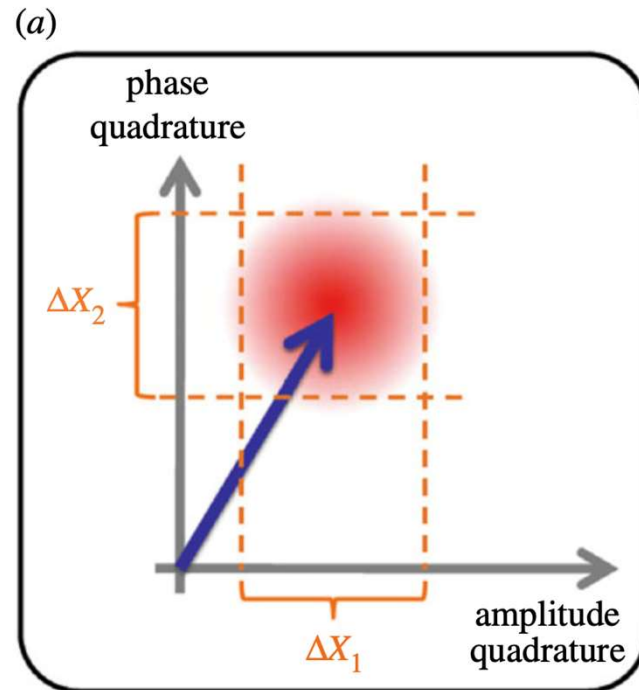


Quantum noise of coherent light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Phase and amplitude quadrature



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

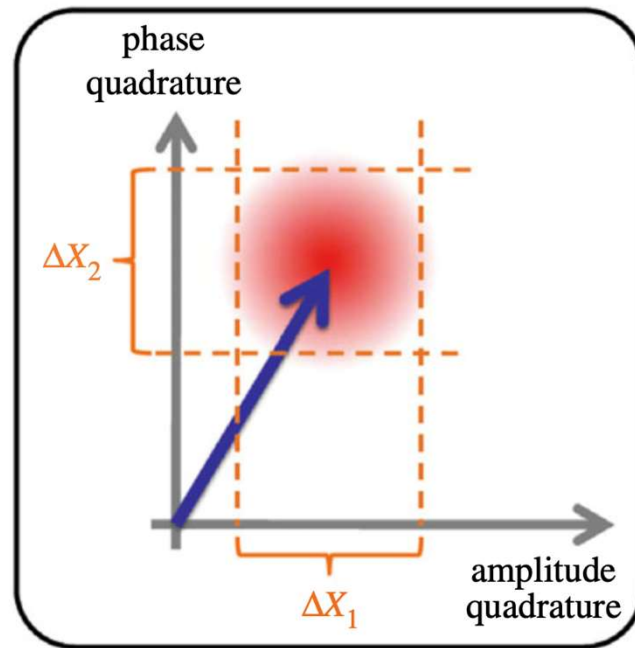
Uncertainty principle

Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Phase and amplitude quadrature



(a) Shot noise



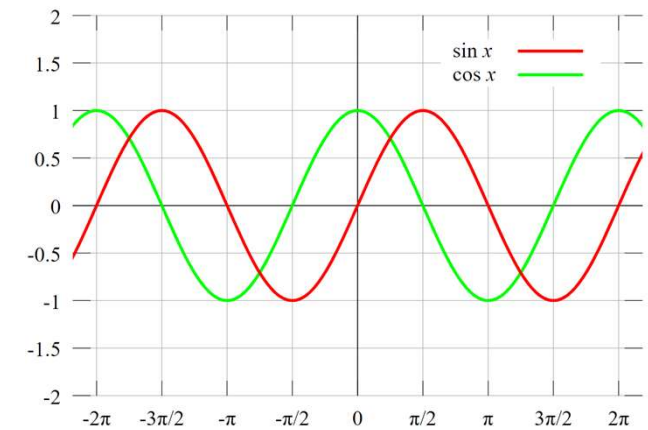
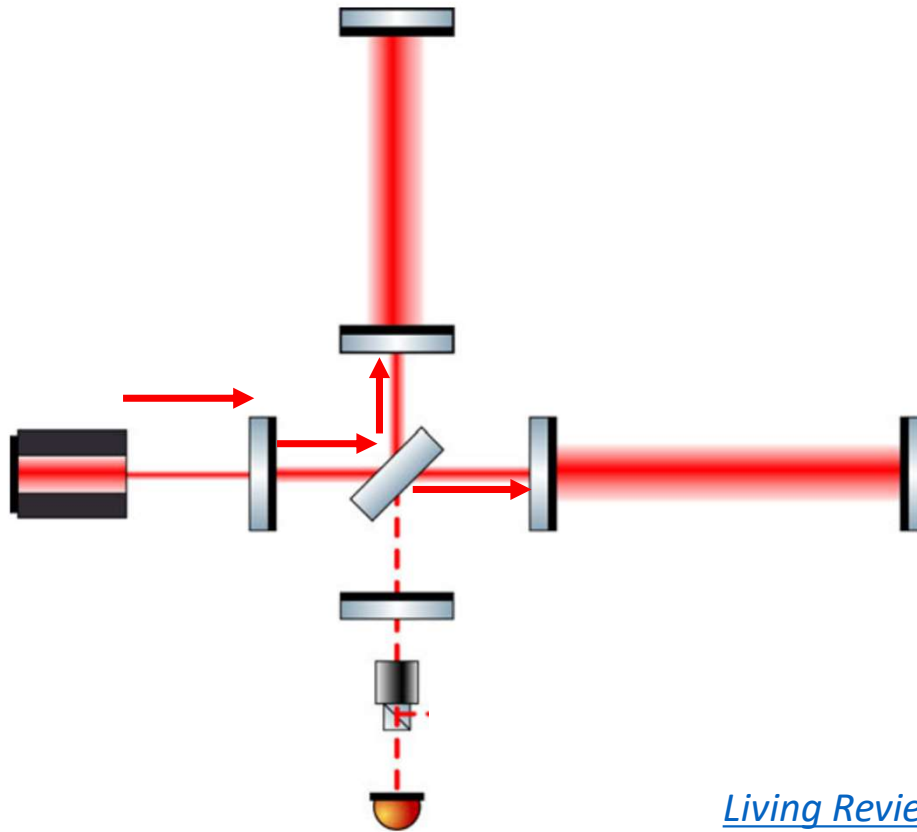
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Uncertainty principle

Radiation pressure noise

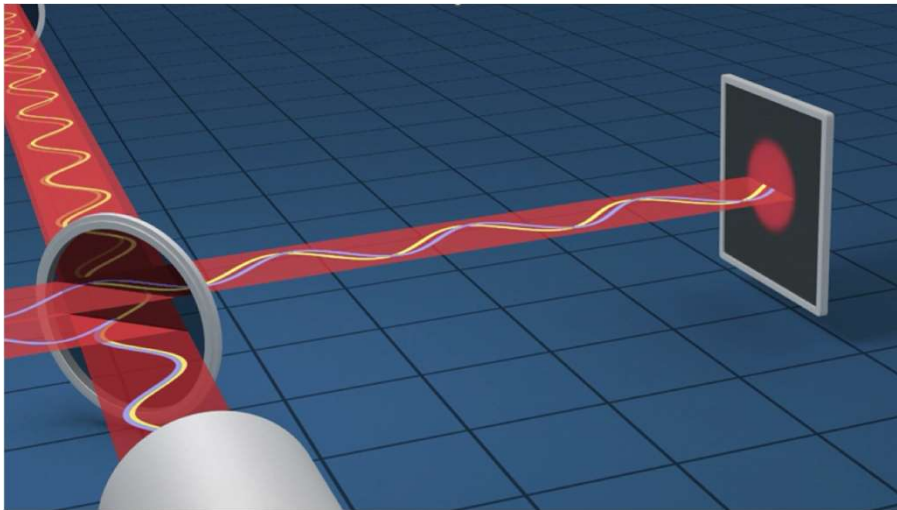
Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Gravitational wave detector

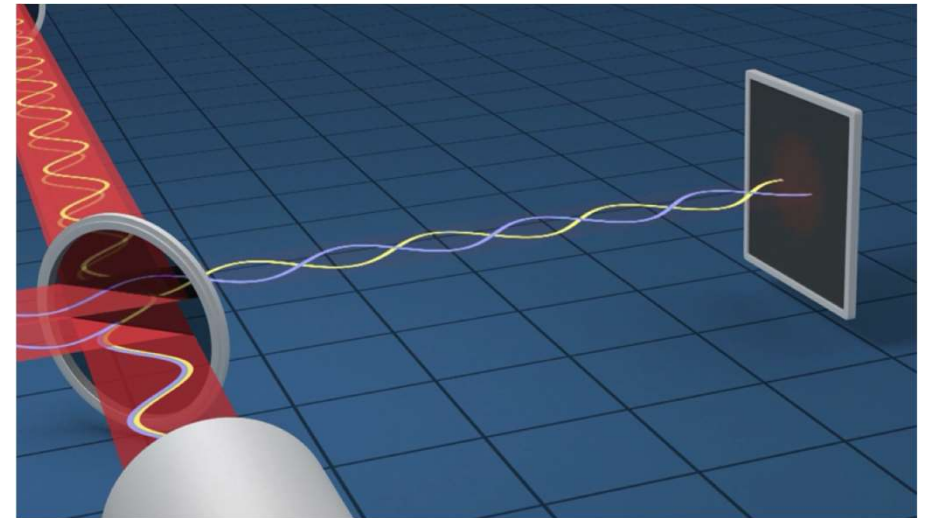


[*Living Reviews in Relativity*](#) volume 22, Article number: 2 (2019)

Bright port and dark port

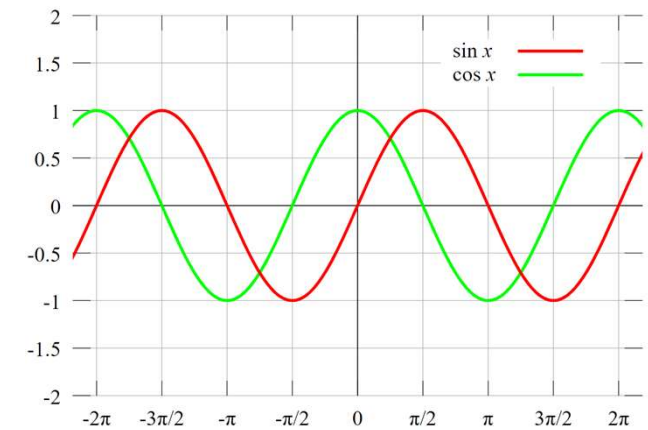
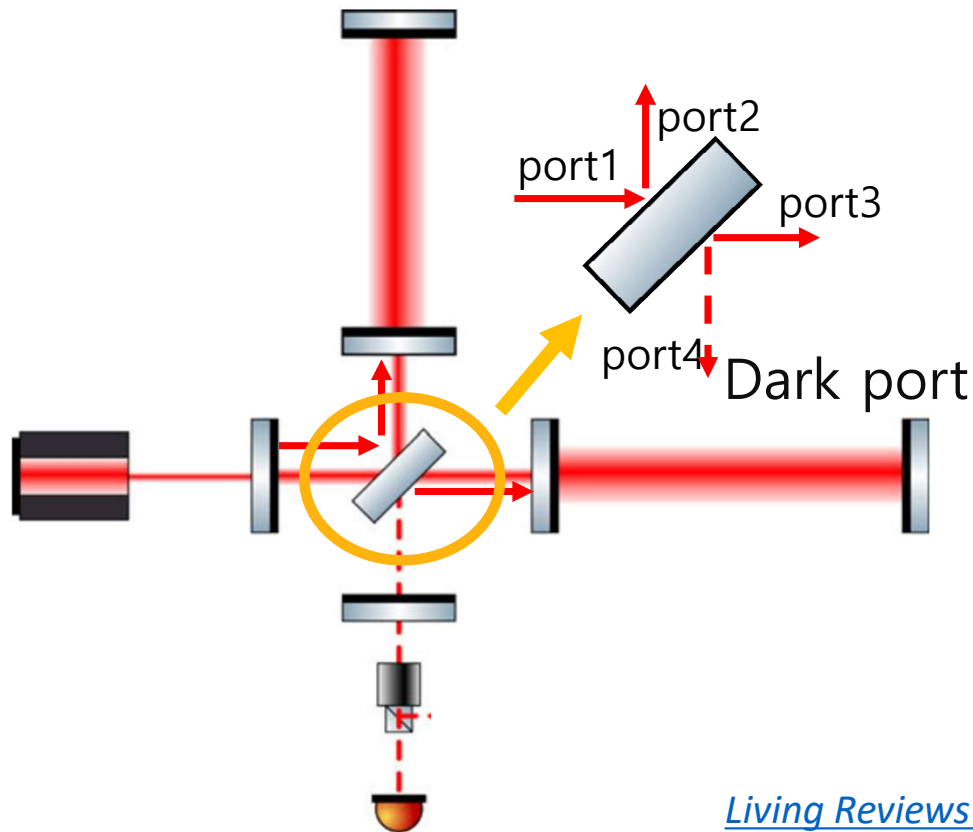


Bright



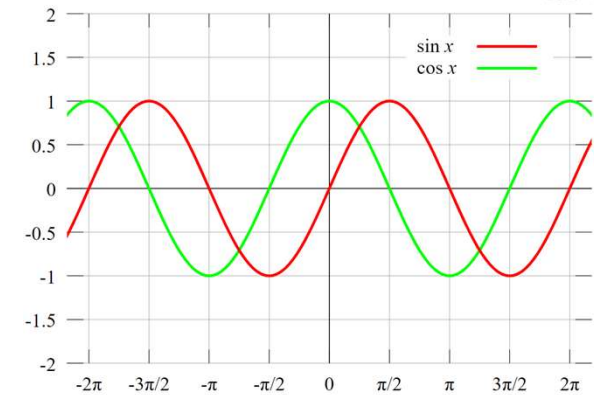
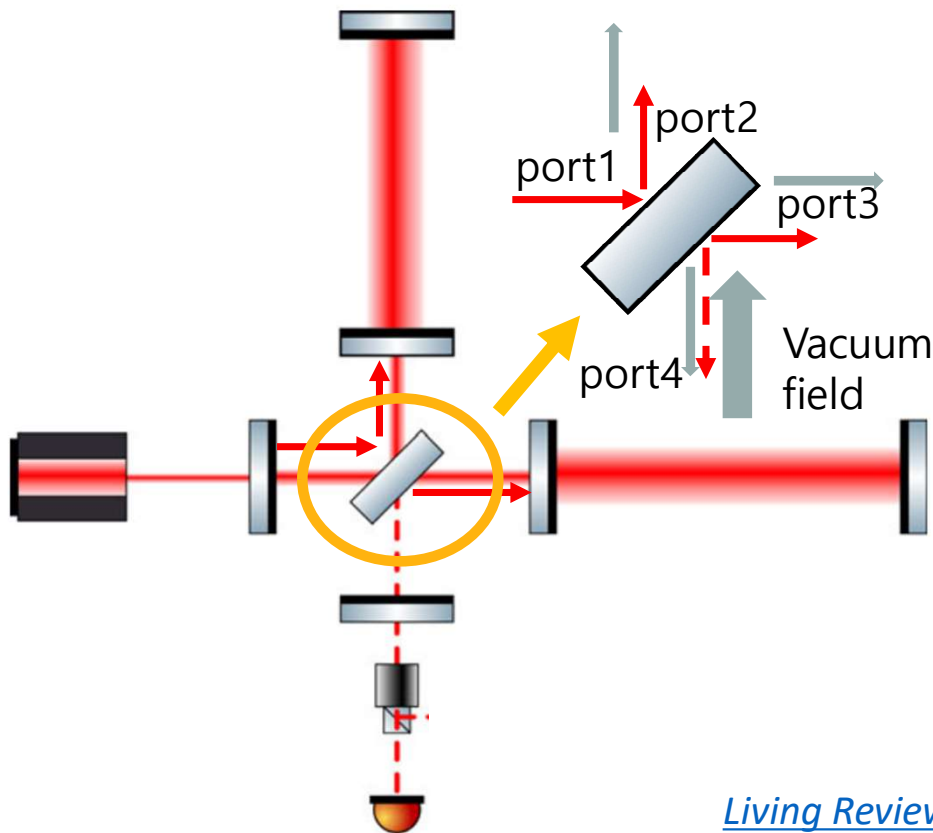
Dark

Gravitational wave detector



[Living Reviews in Relativity](#) volume 22, Article number: 2 (2019)

Quantum noise of gravitational wave detector



[Living Reviews in Relativity](#) volume 22, Article number: 2 (2019)

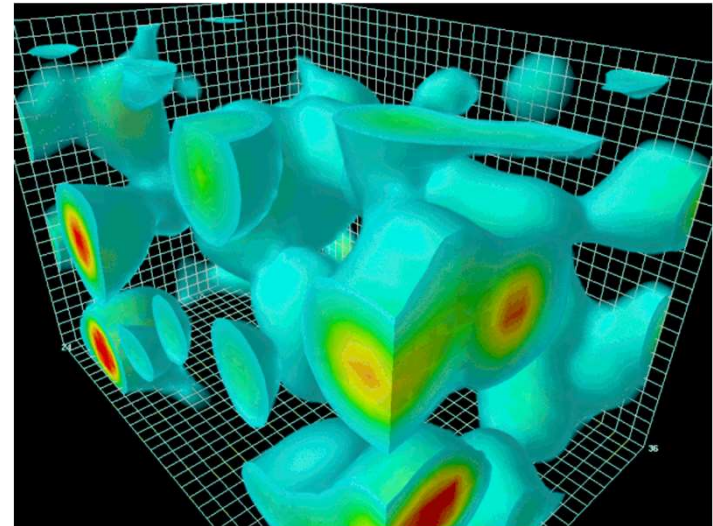
Quantum vacuum fluctuation



Vacuum $\neq$
empty

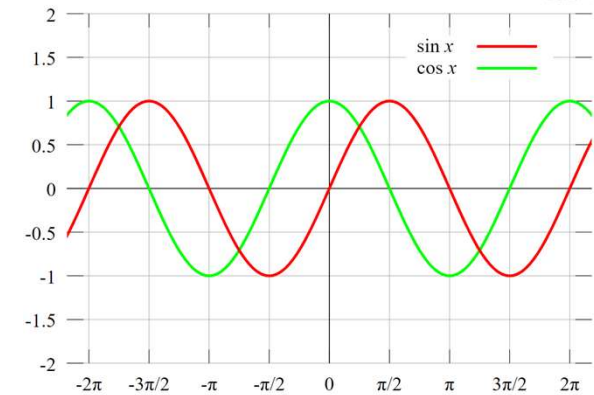
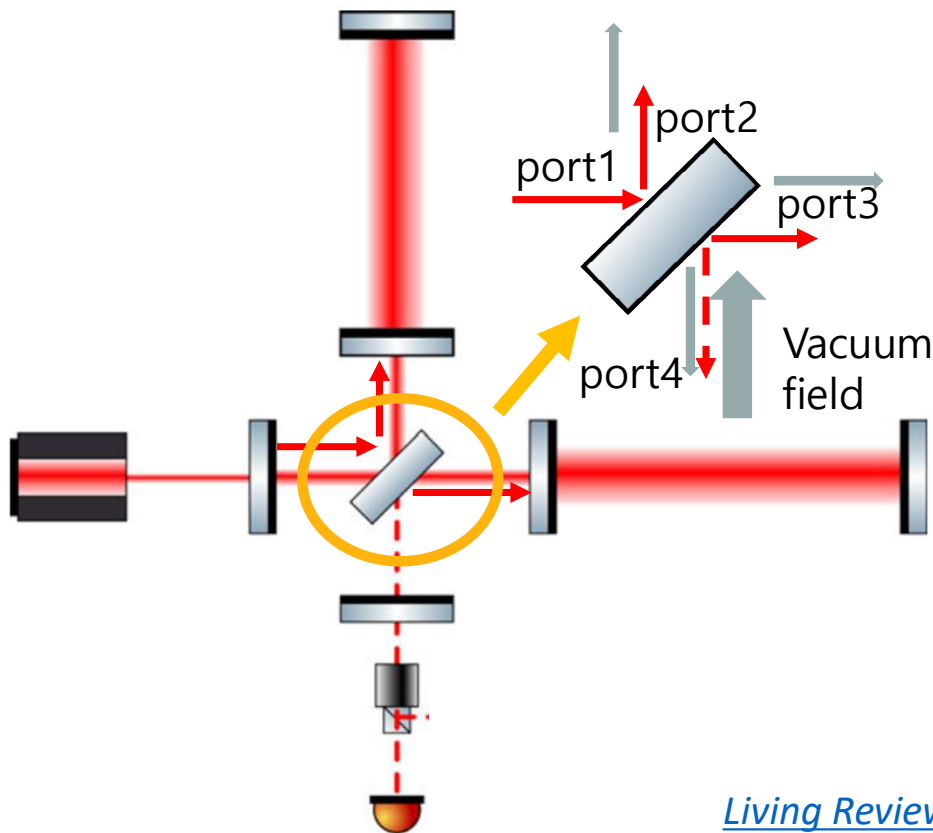
Vacuum

Transmission medium



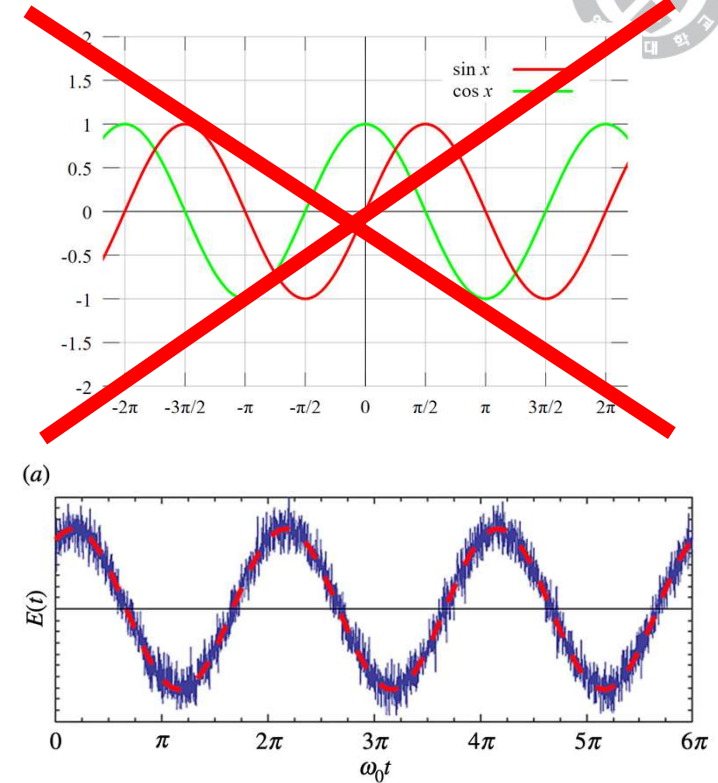
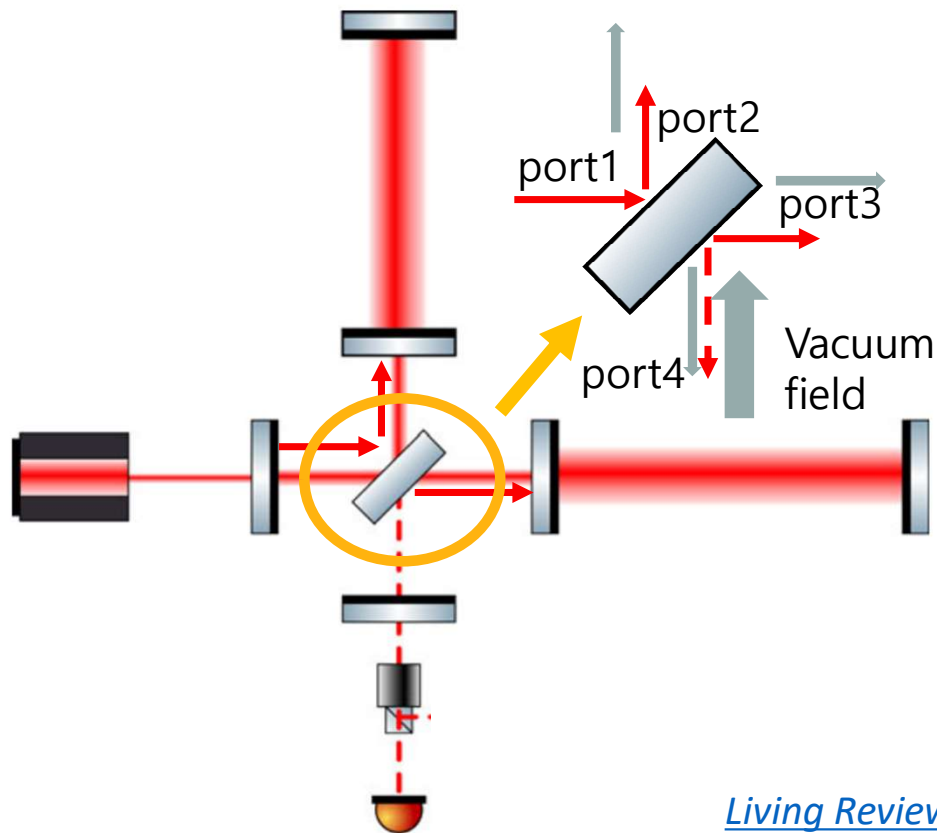
Average energy = zeropoint energy

Quantum noise of gravitational wave detector



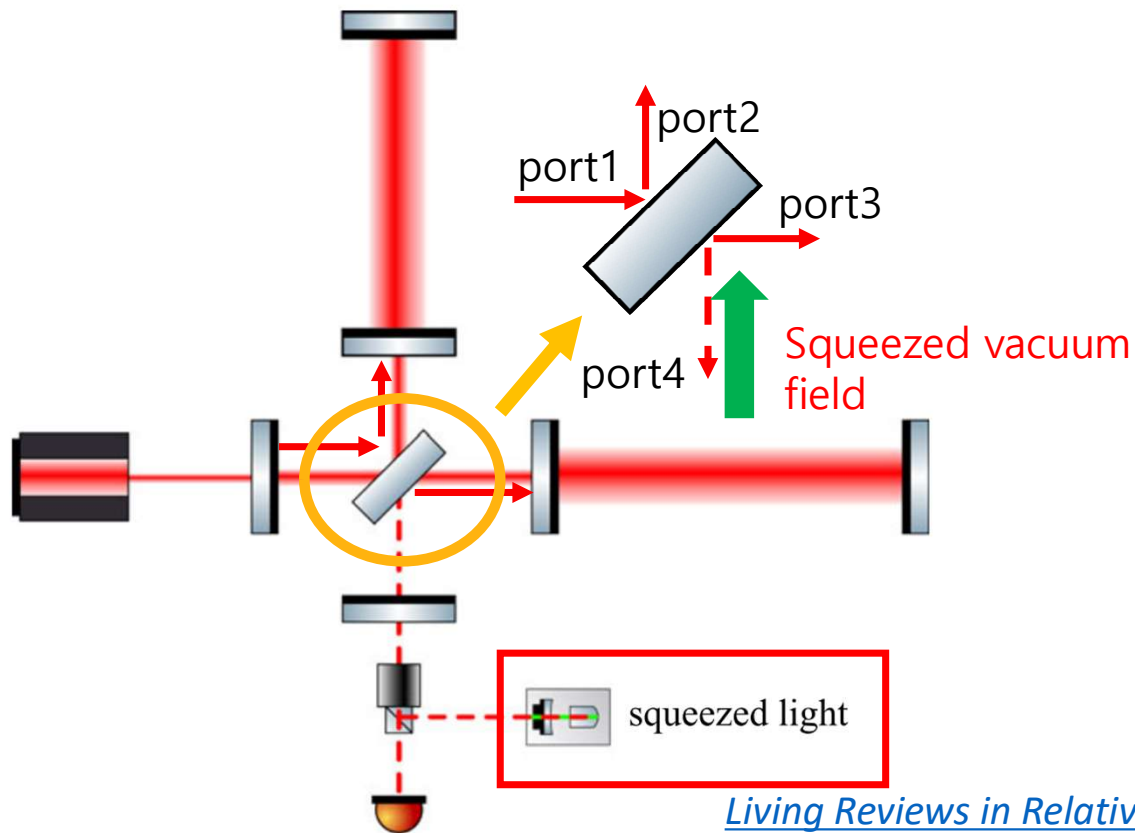
[Living Reviews in Relativity](#) volume 22, Article number: 2 (2019)

Quantum noise of gravitational wave detector



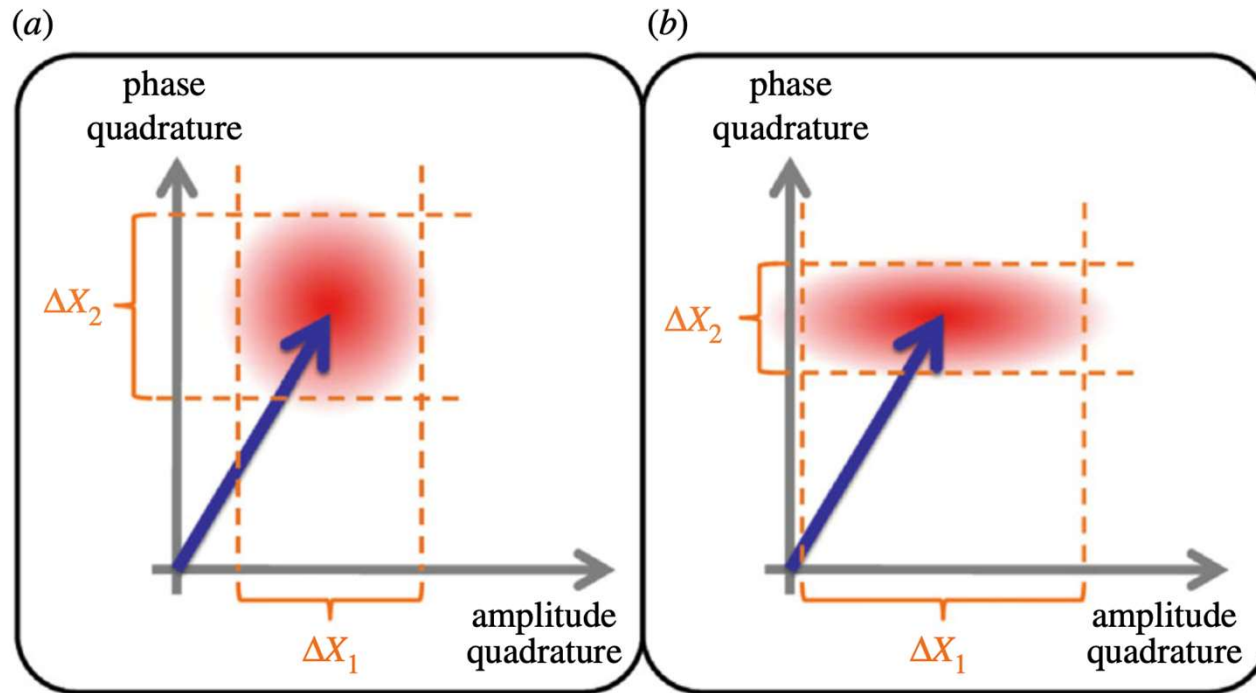
[Living Reviews in Relativity](#) volume 22, Article number: 2 (2019)

Gravitational wave detector



[Living Reviews in Relativity](#) volume 22, Article number: 2 (2019)

Squeezed light

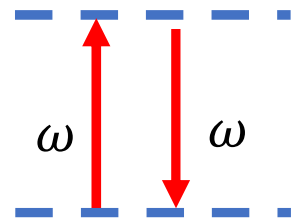


Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

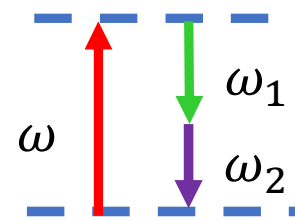
Non linear crystal



Non-linear crystal

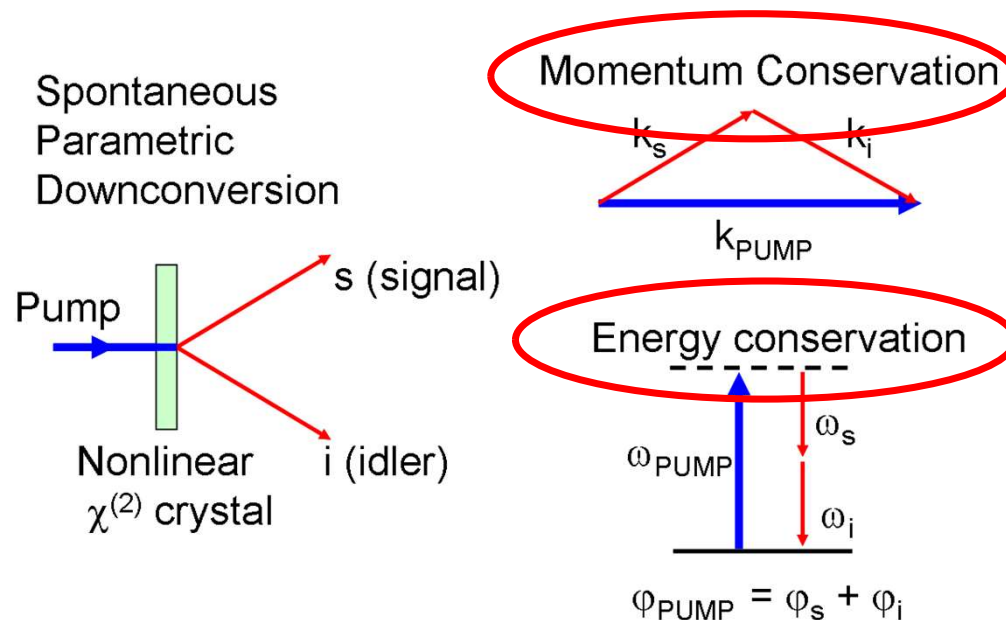


linear optics



Non linear optics

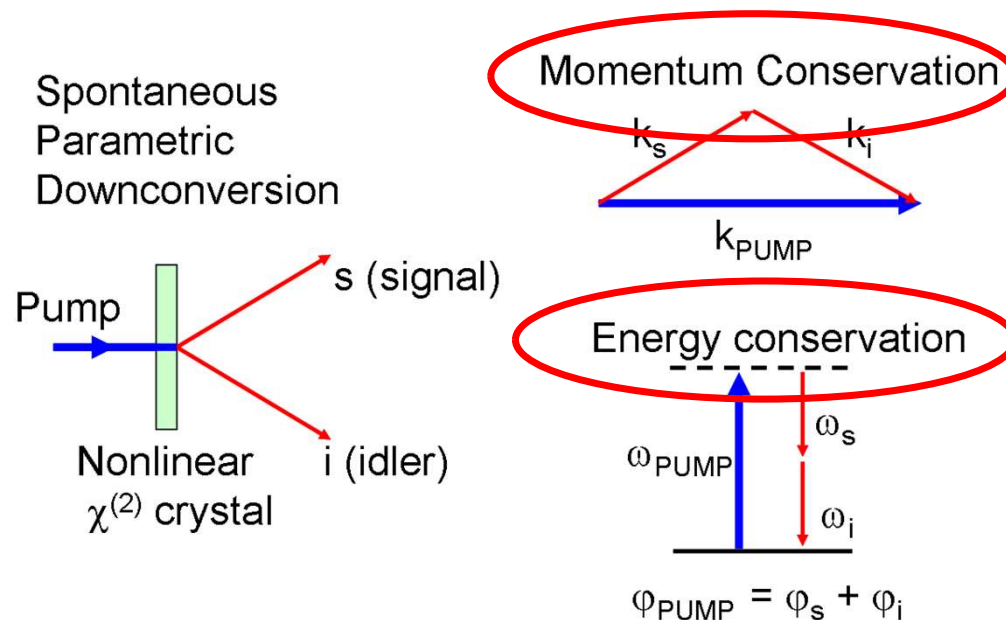
Parametric down conversion



Every field which is participated in Non-linear conversion process, Must obey energy, momentum conservation.

[J S Lundeen](#)

Parametric down conversion

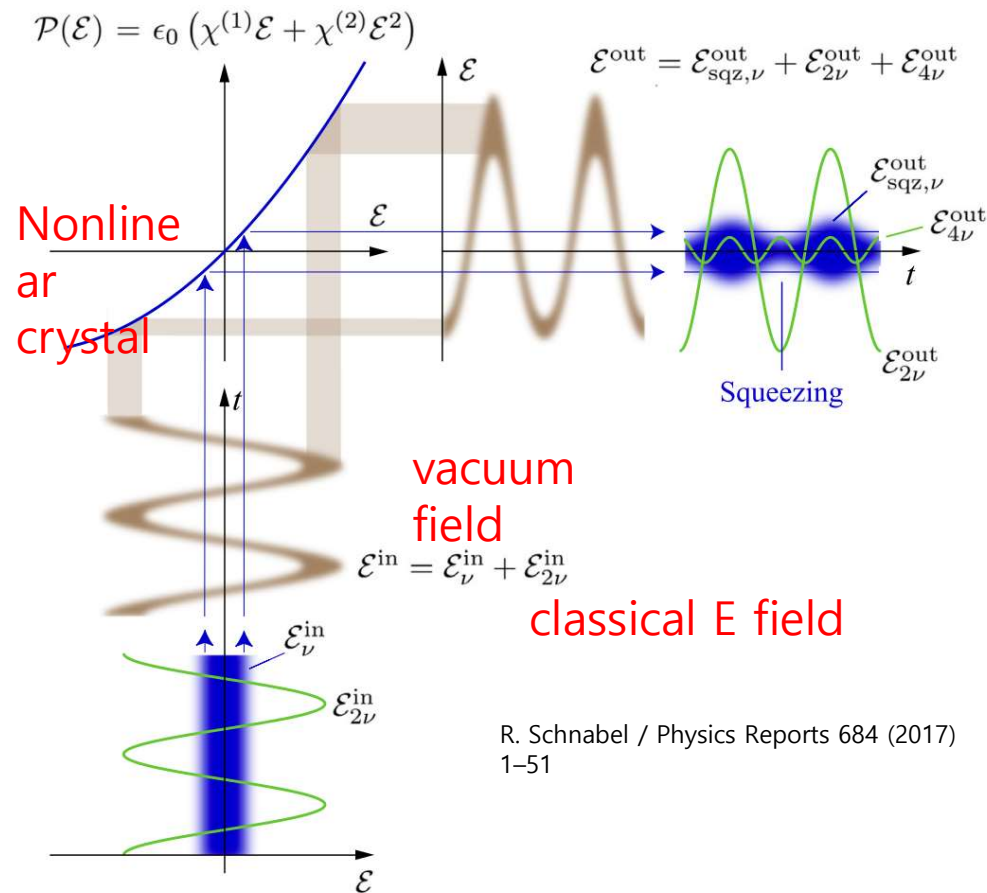


Every field which is participated in Non-linear conversion process, Must obey energy, momentum conservation.

Even for vacuum field!!

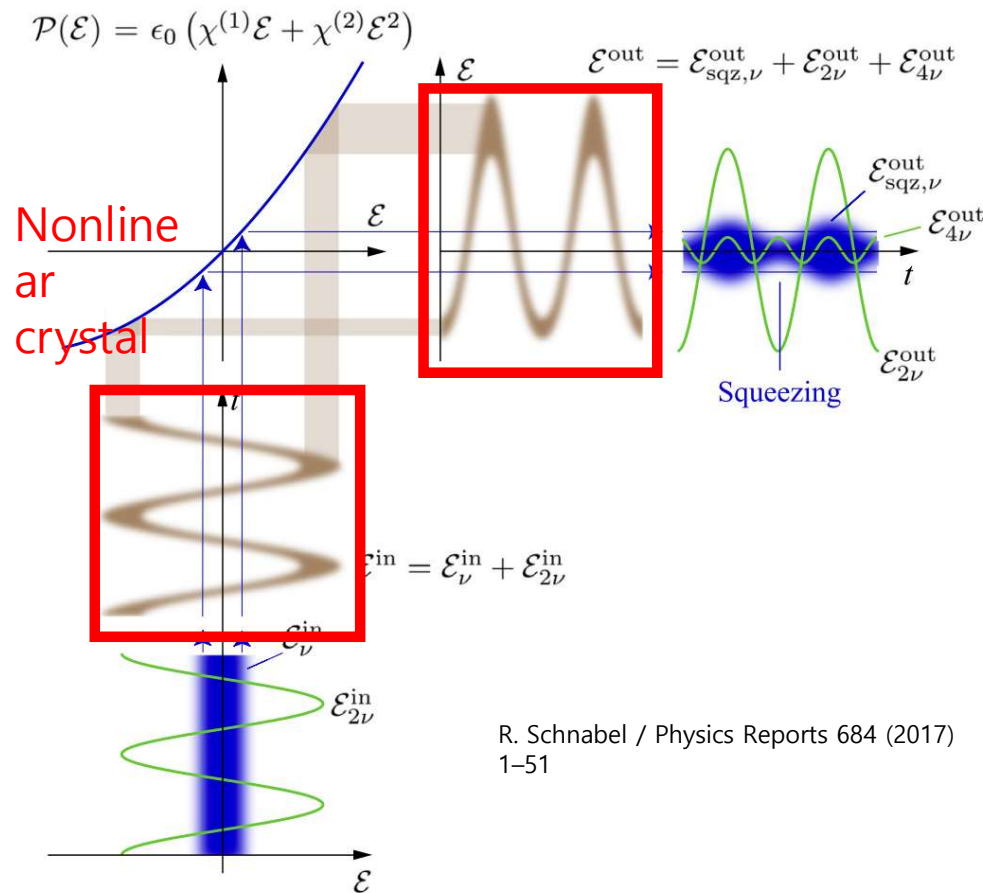
[J S Lundeen](#)

Parametric down conversion

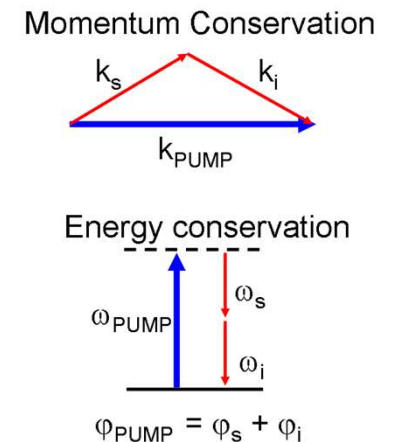
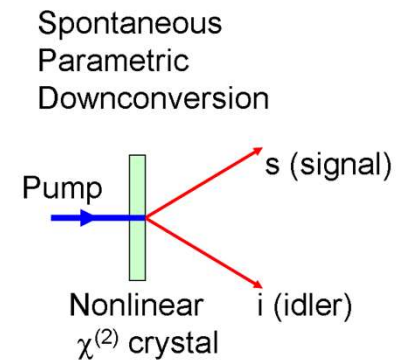


R. Schnabel / Physics Reports 684 (2017)
1-51

Parametric down conversion

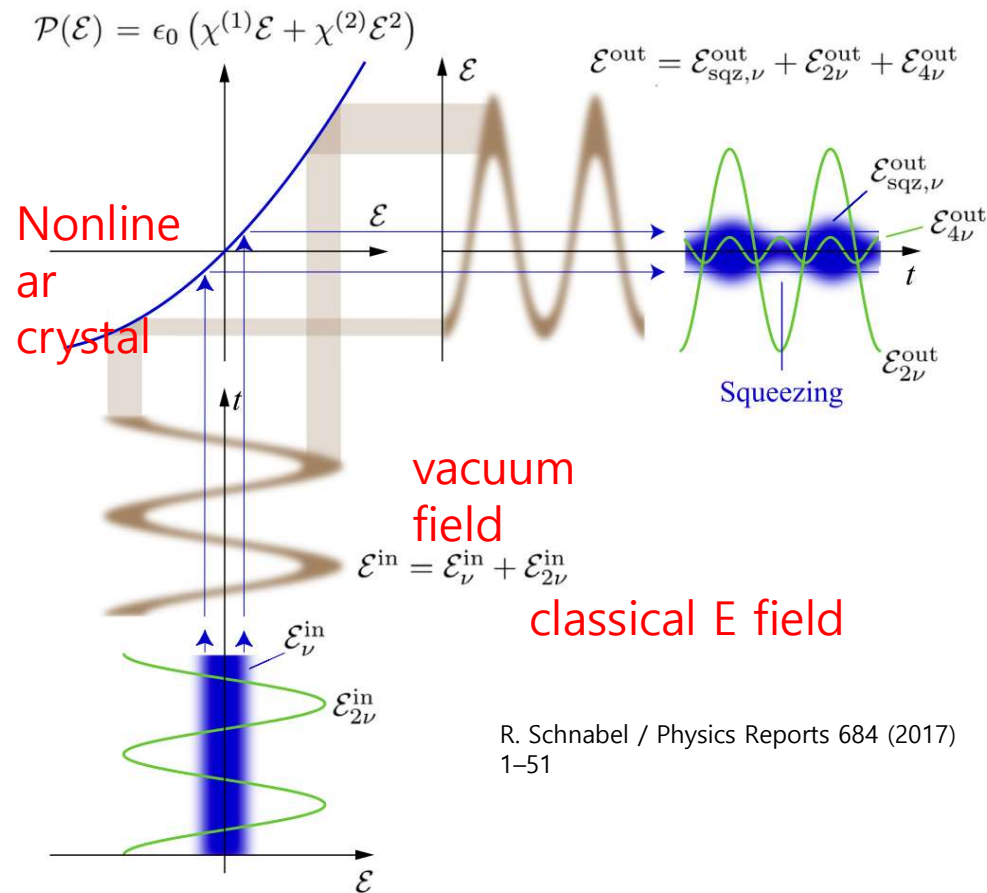


R. Schnabel / Physics Reports 684 (2017) 1–51



Boundary condition

Parametric down conversion



State of vacuum is changed during
conversion process
due to momentum and energy conservation

R. Schnabel / Physics Reports 684 (2017)
1-51

Boundary condition



Squeezed vacuum

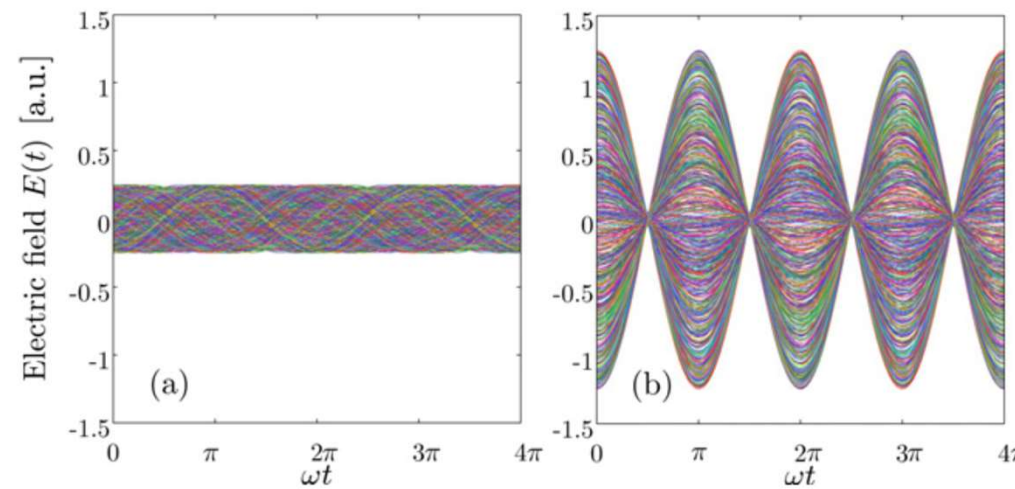
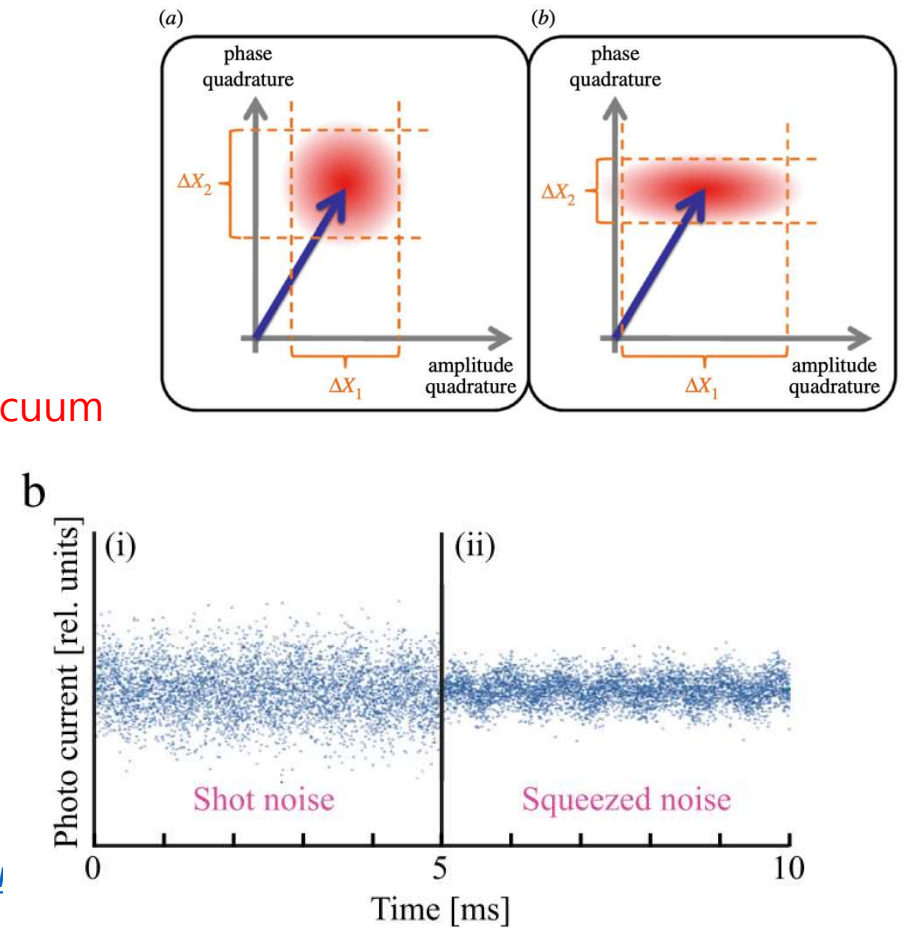
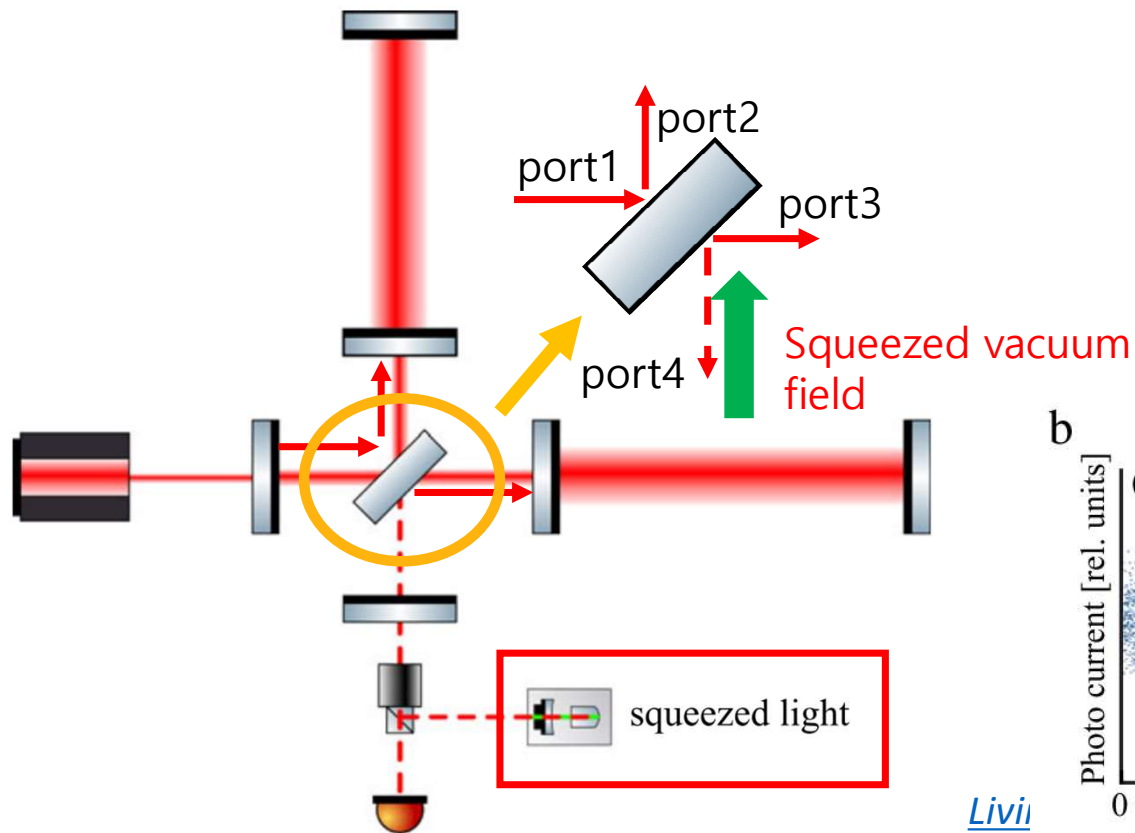


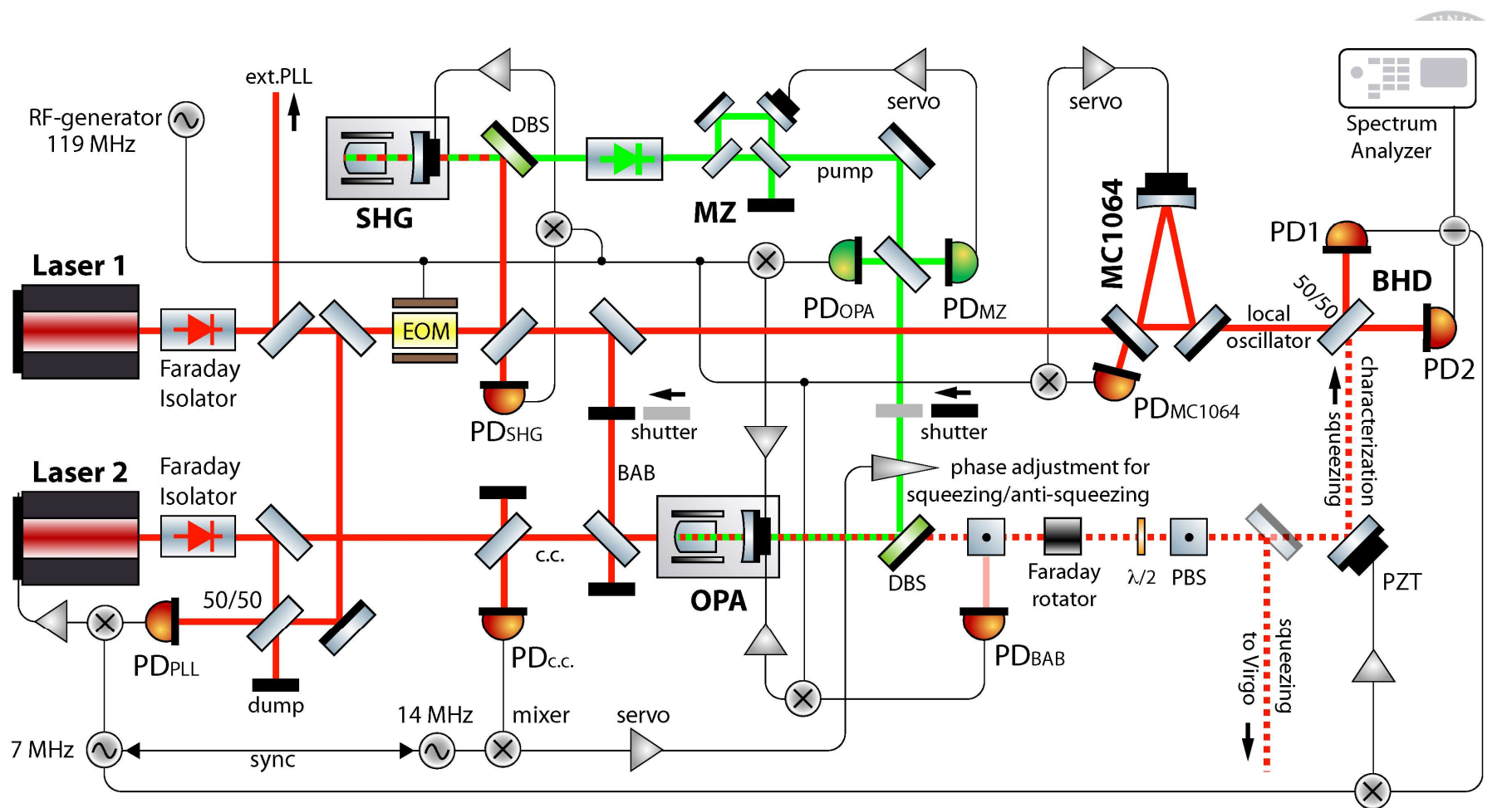
Figure 1.6: Simulation of electric field in time for (a) vacuum state and for (b) squeezed vacuum.

Gravitational wave detector

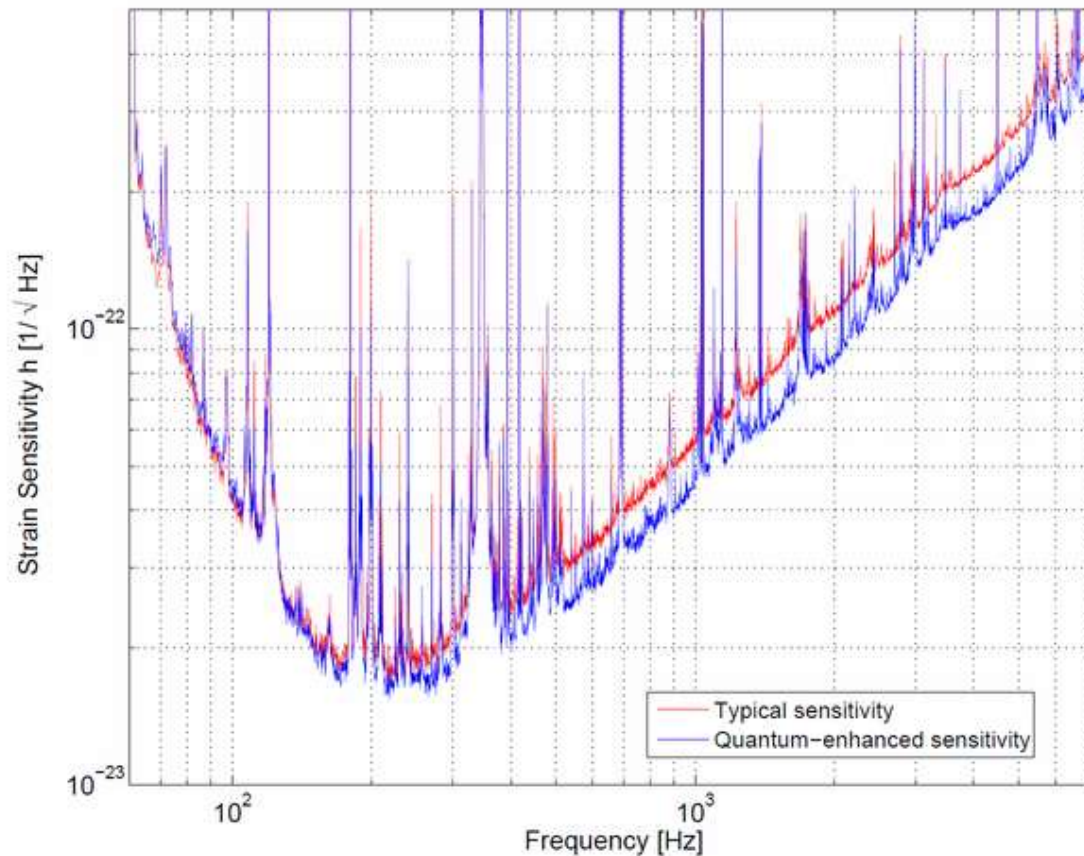


[Livi](#)

2019)



Quantum noise enhancement of LIGO





지금 지쳤나요?



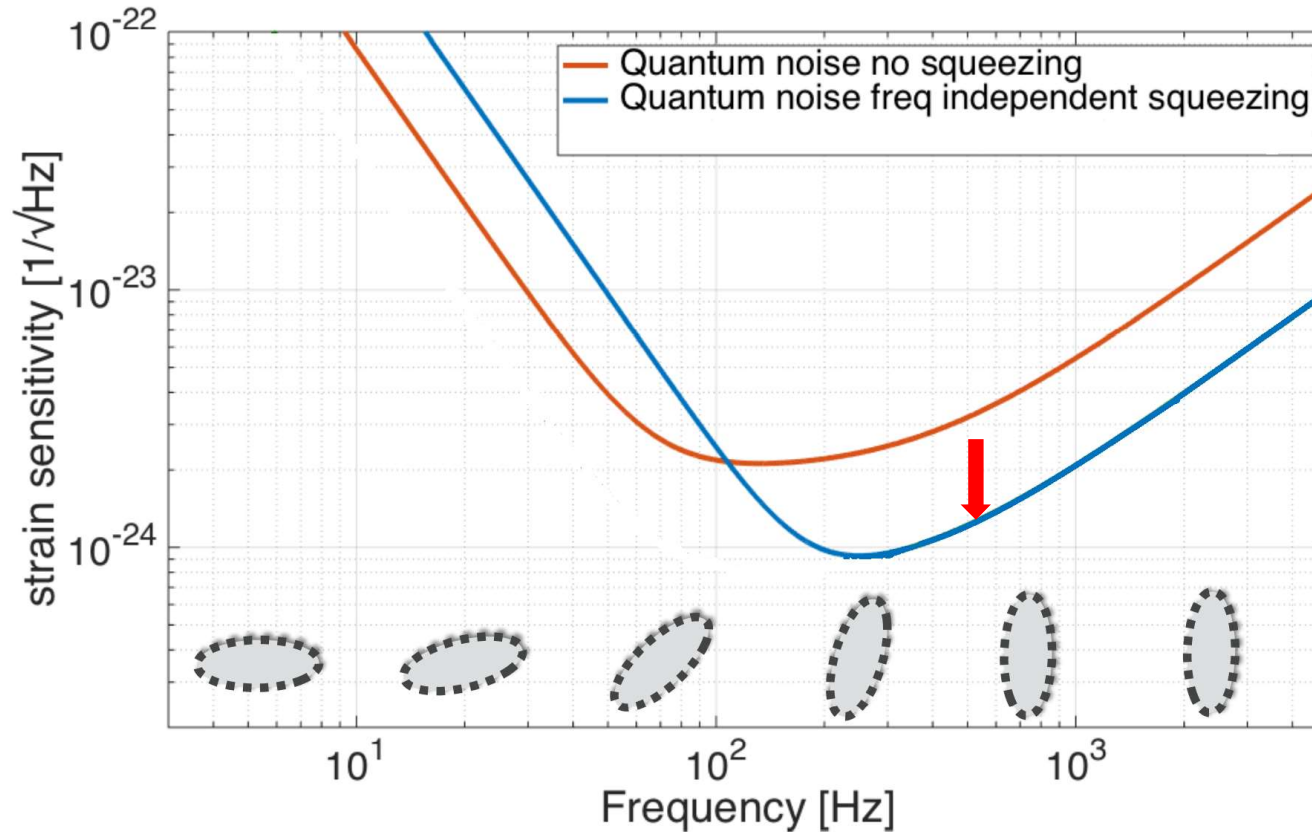
아니요



지금 지쳤나요?



Frequency independent squeezing

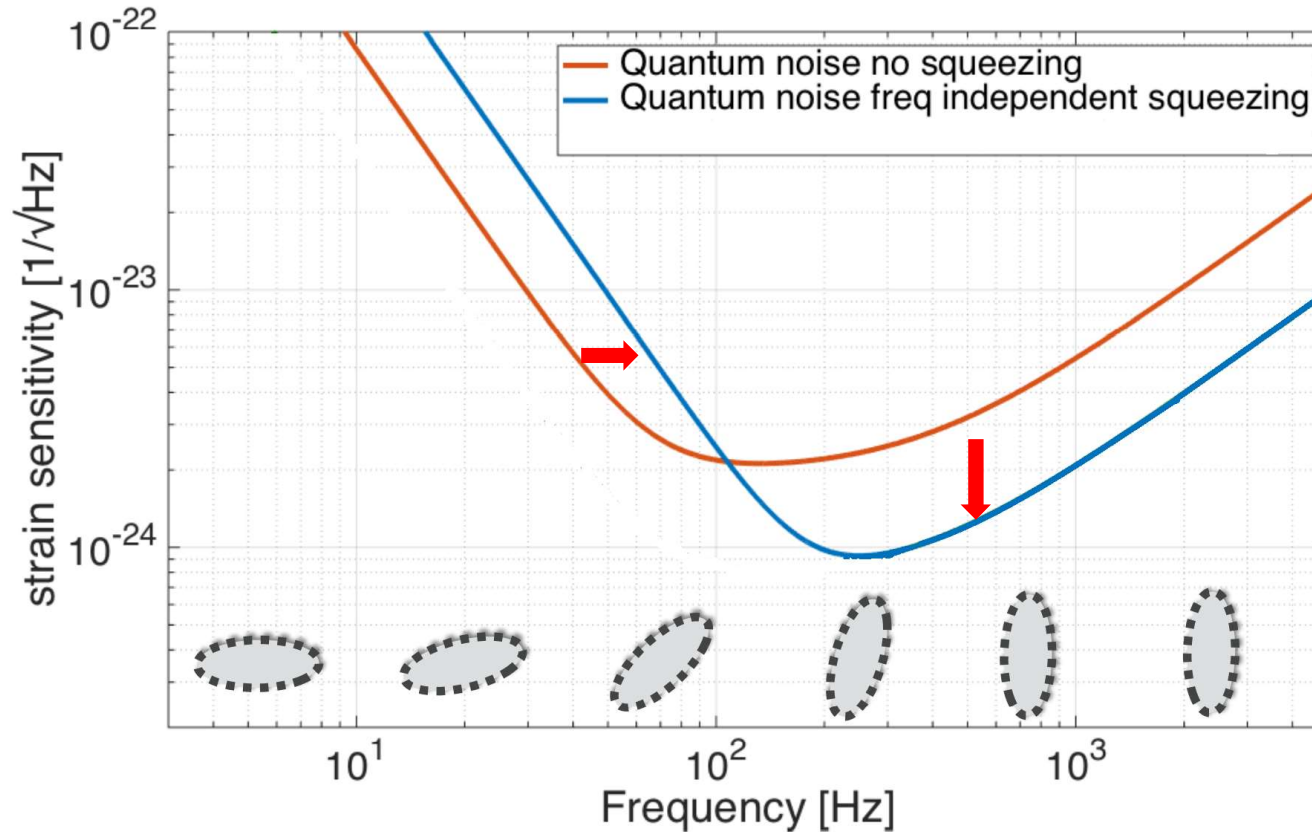


Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

중력파 여름학교 2025

YONSEI UNIVERSITY

Frequency independent squeezing

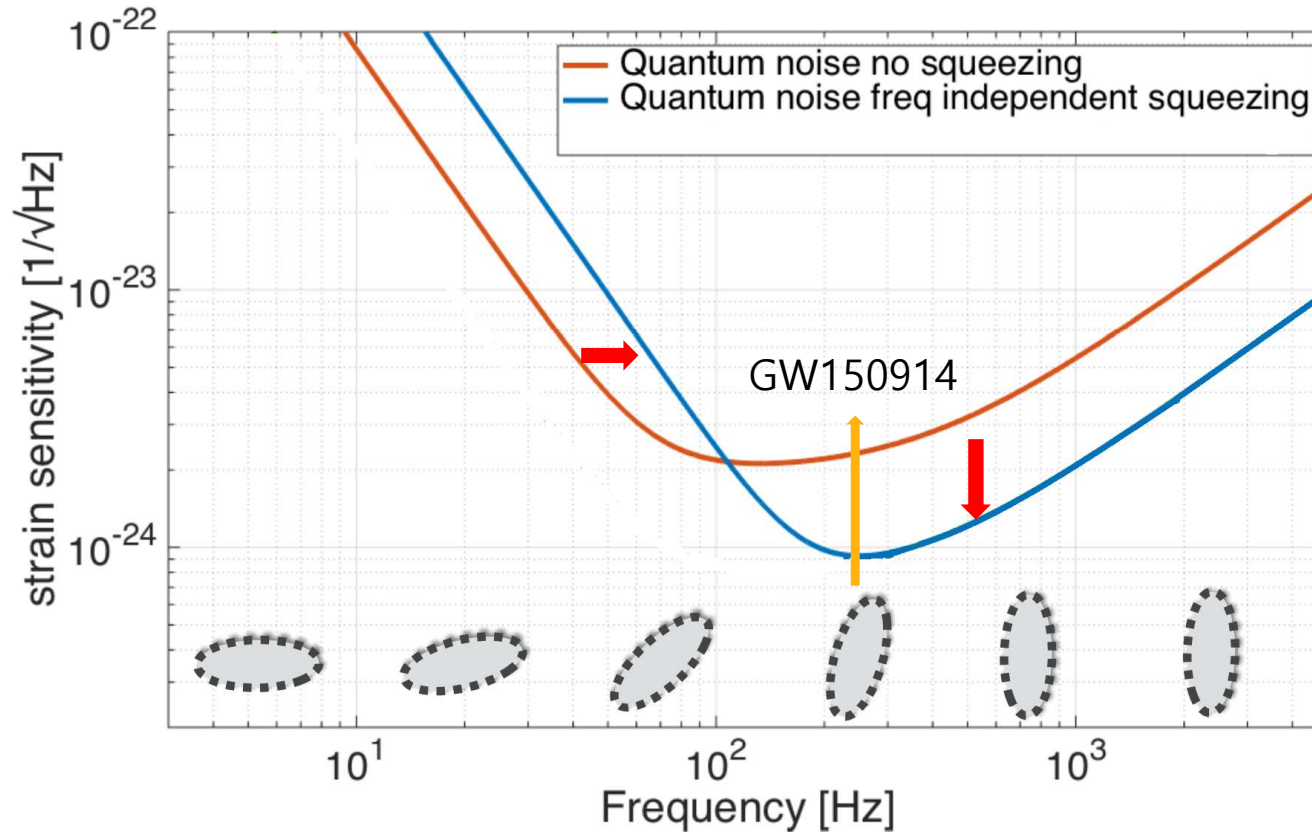


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Frequency independent squeezing

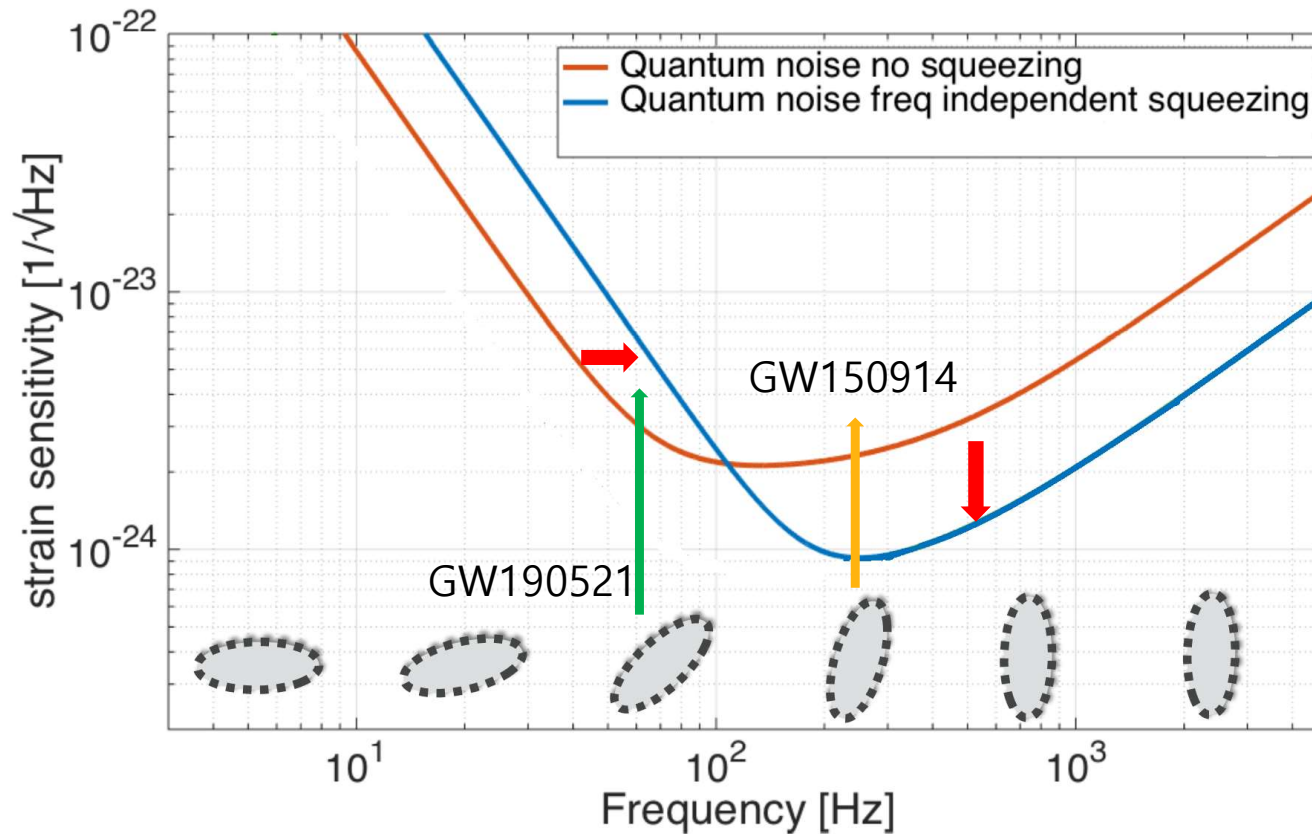


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중력파 여름학교 2025

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Frequency independent squeezing

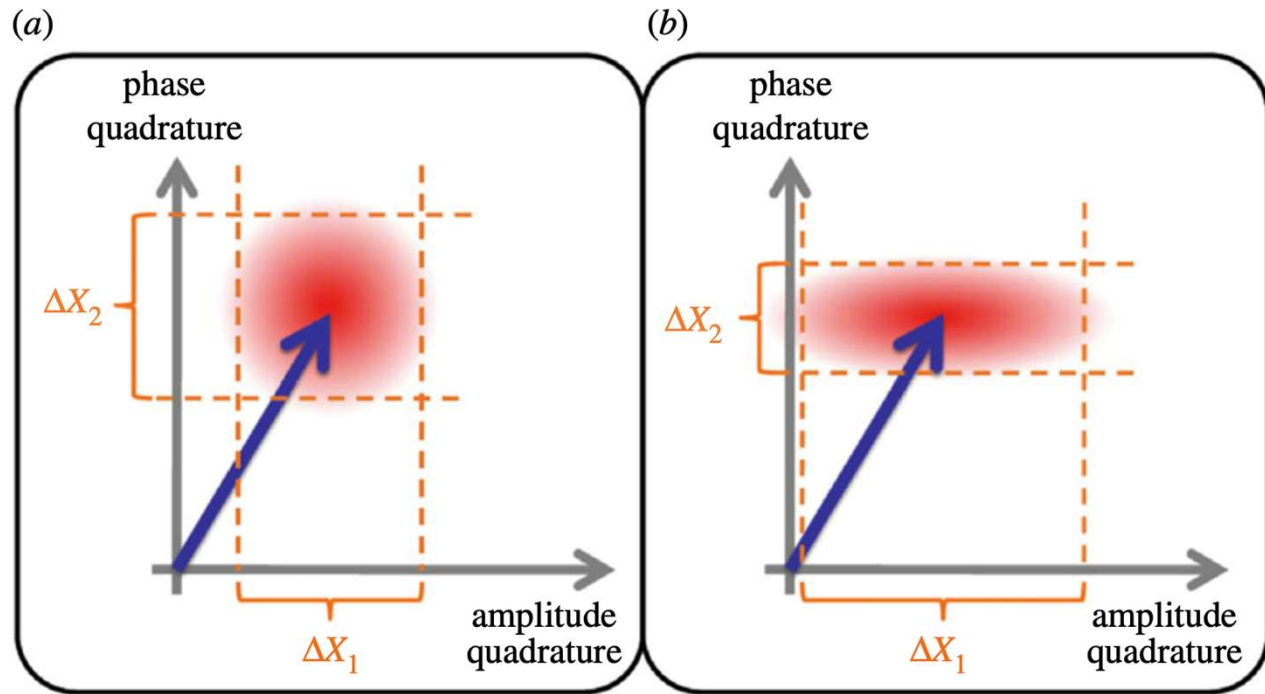


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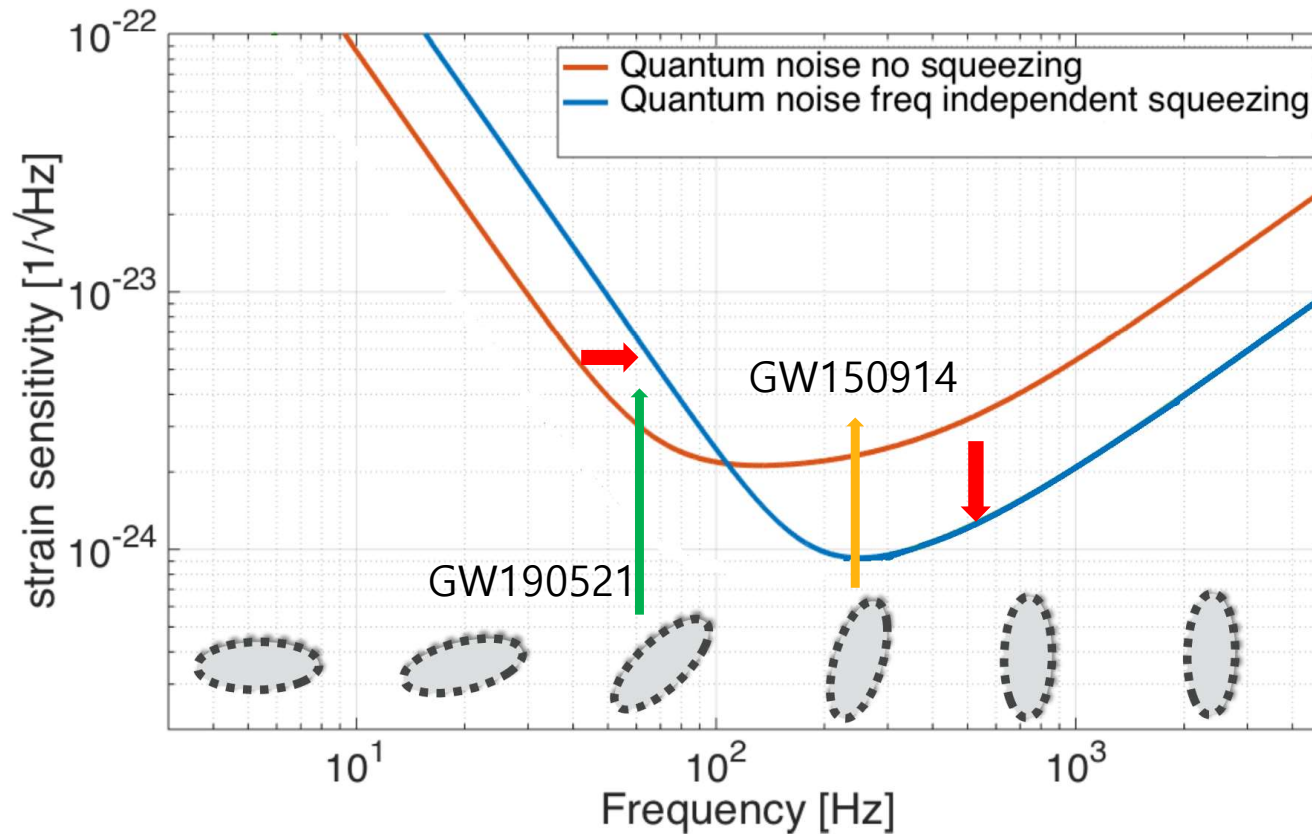
YONSEI UNIVERSITY

Squeezed light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Frequency independent squeezing

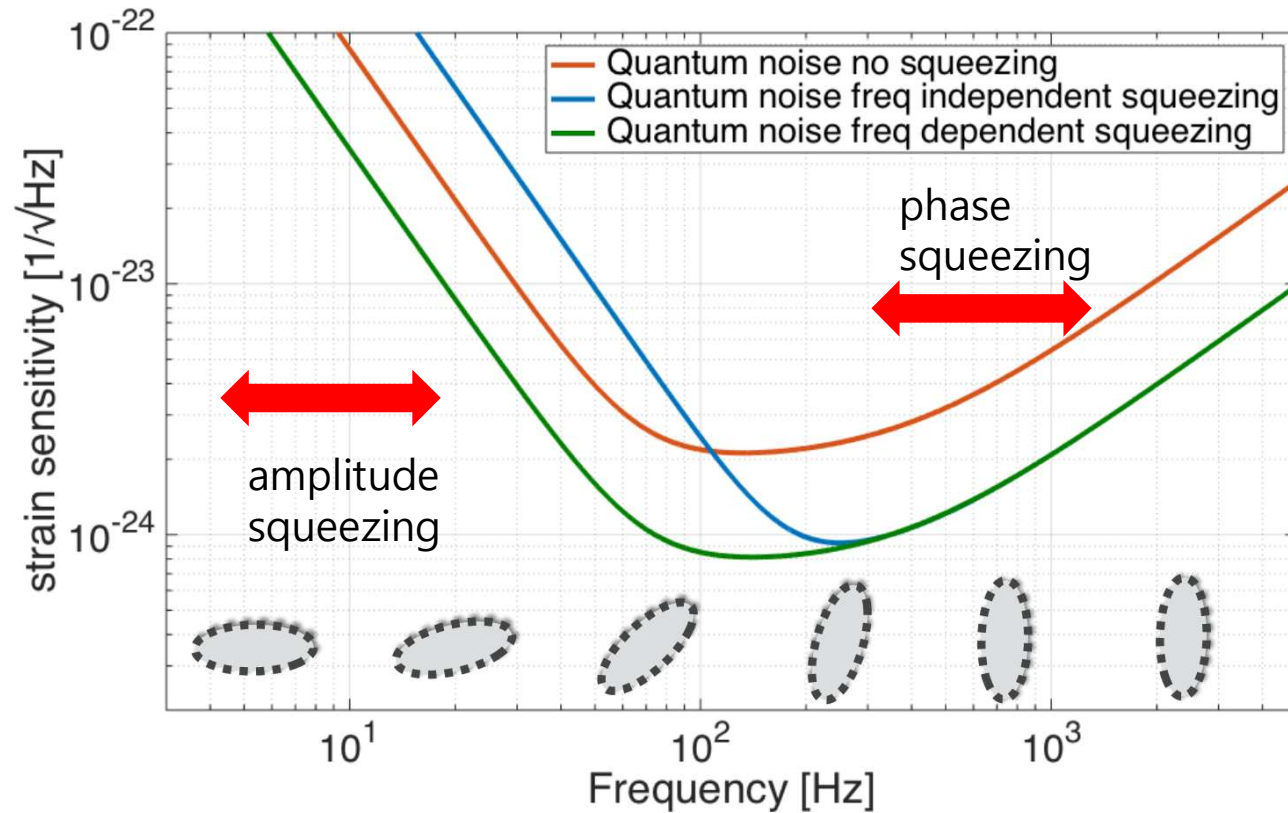


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Frequency dependent squeezing(FDS)

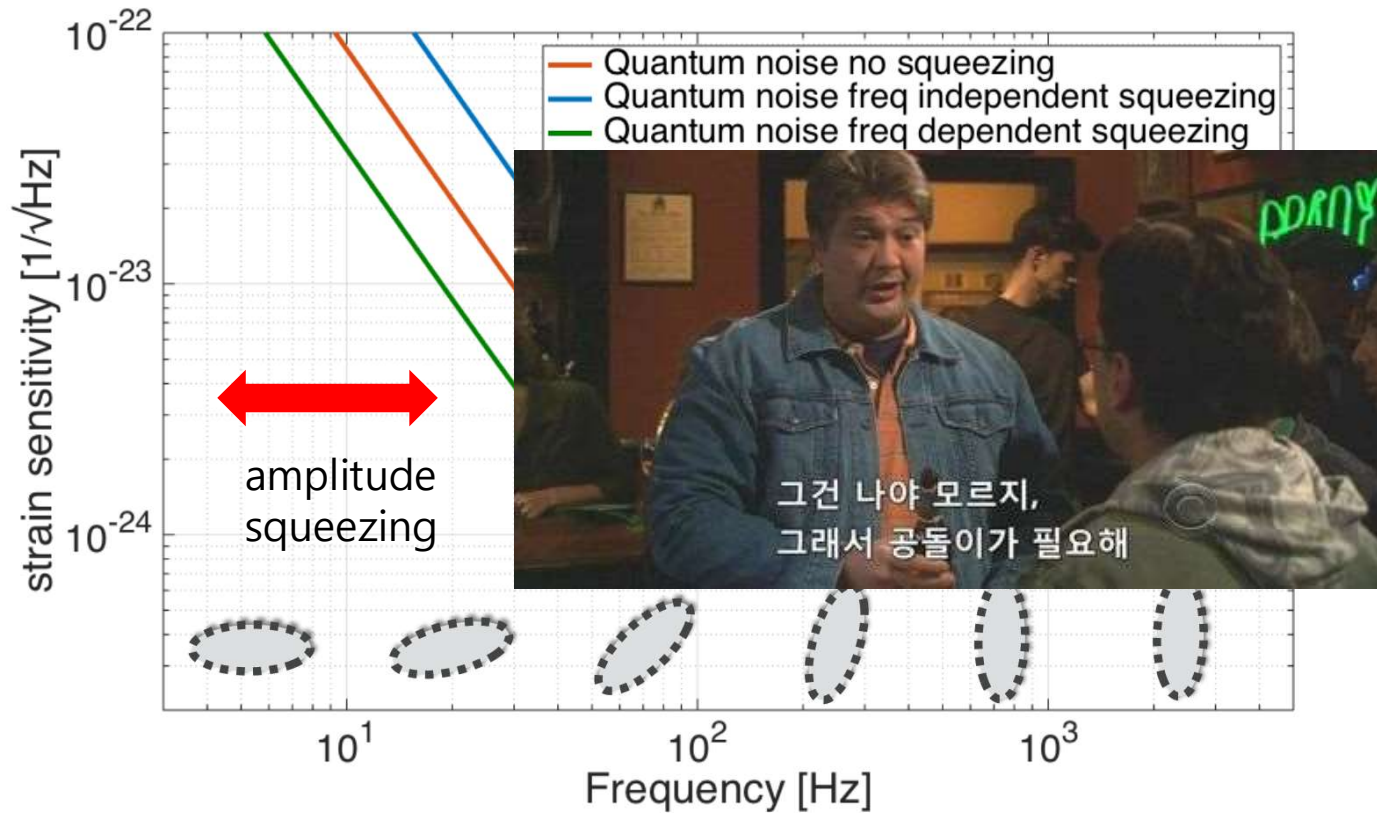


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in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

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Frequency dependent squeezing(FDS)

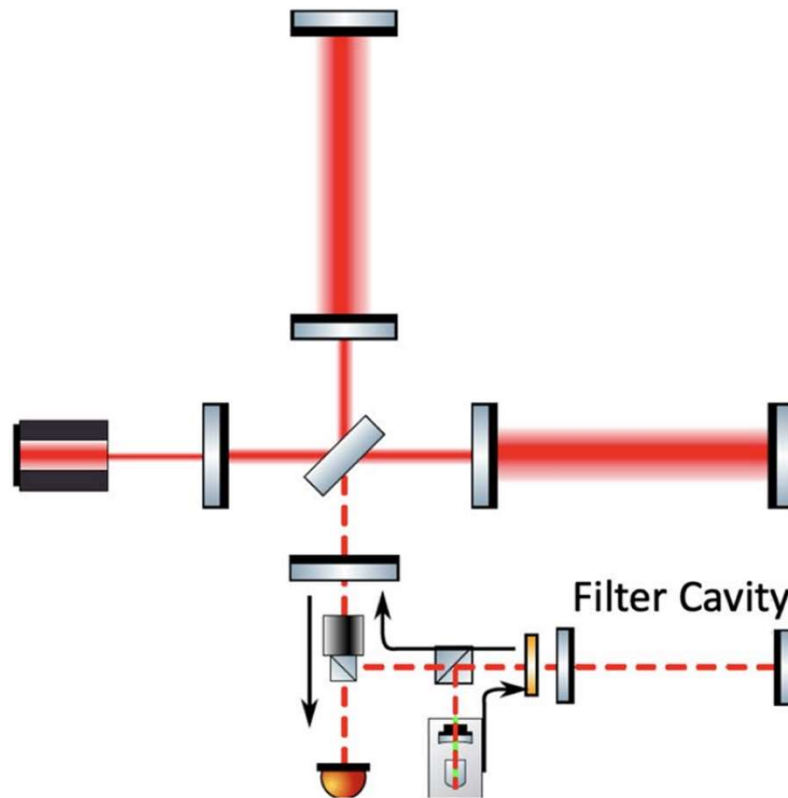


Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

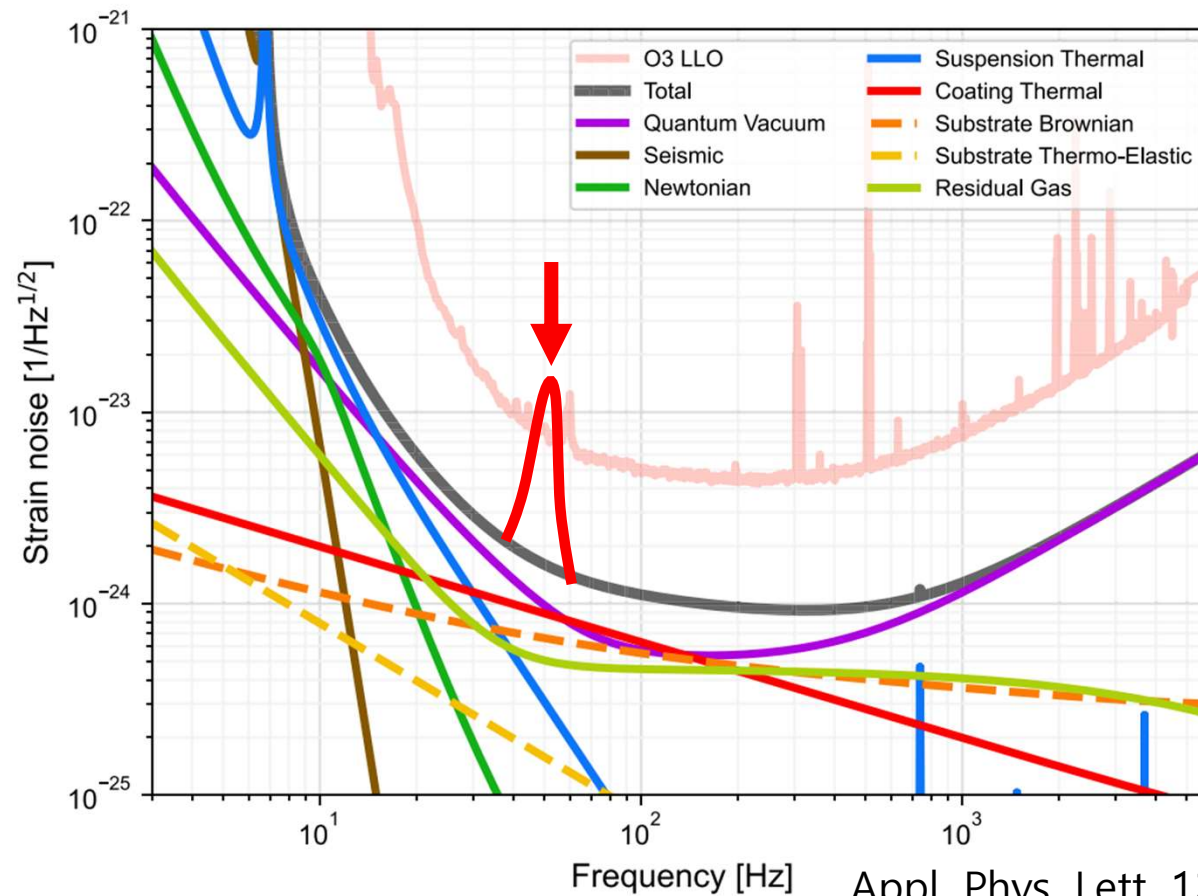
중력파 여름학교 2025

YONSEI UNIVERSITY

Frequency dependent squeezing using filter cavity

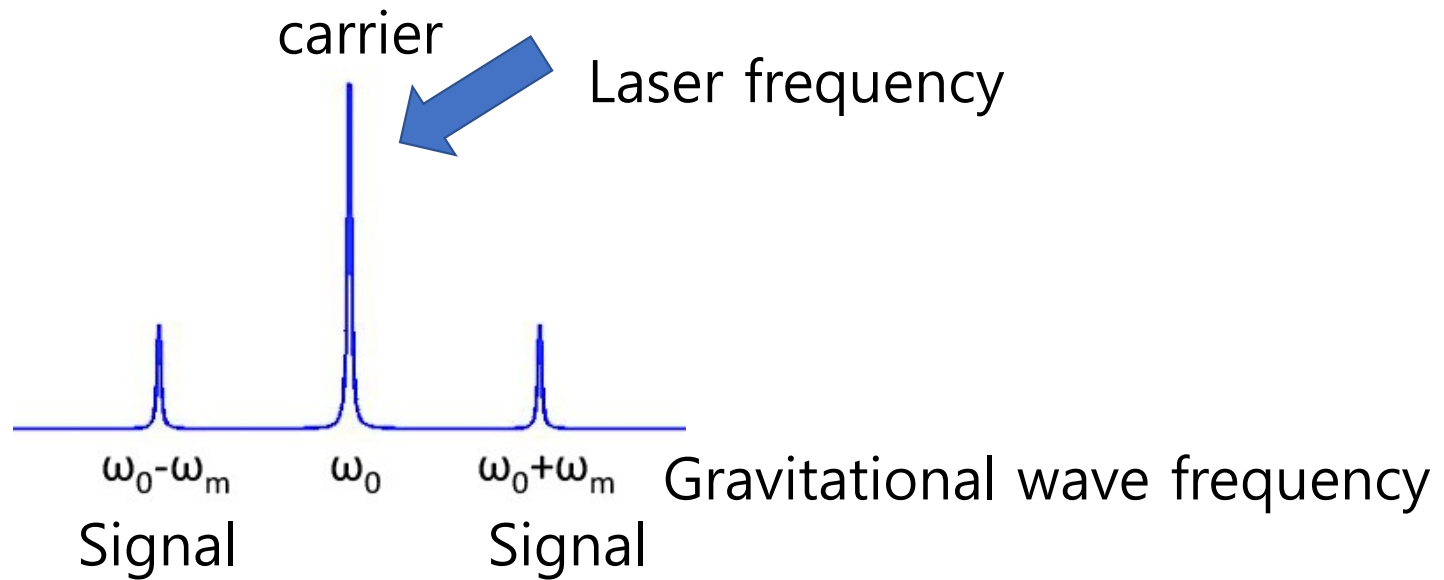


LIGO sensitivity

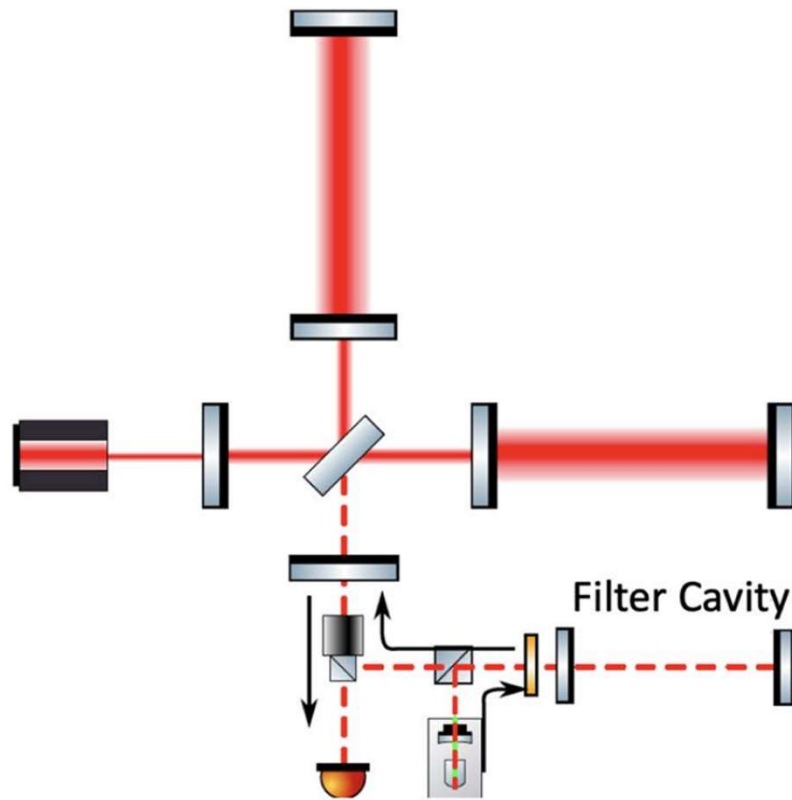


Appl. Phys. Lett. 122, 110502 (2023)

Side band figure

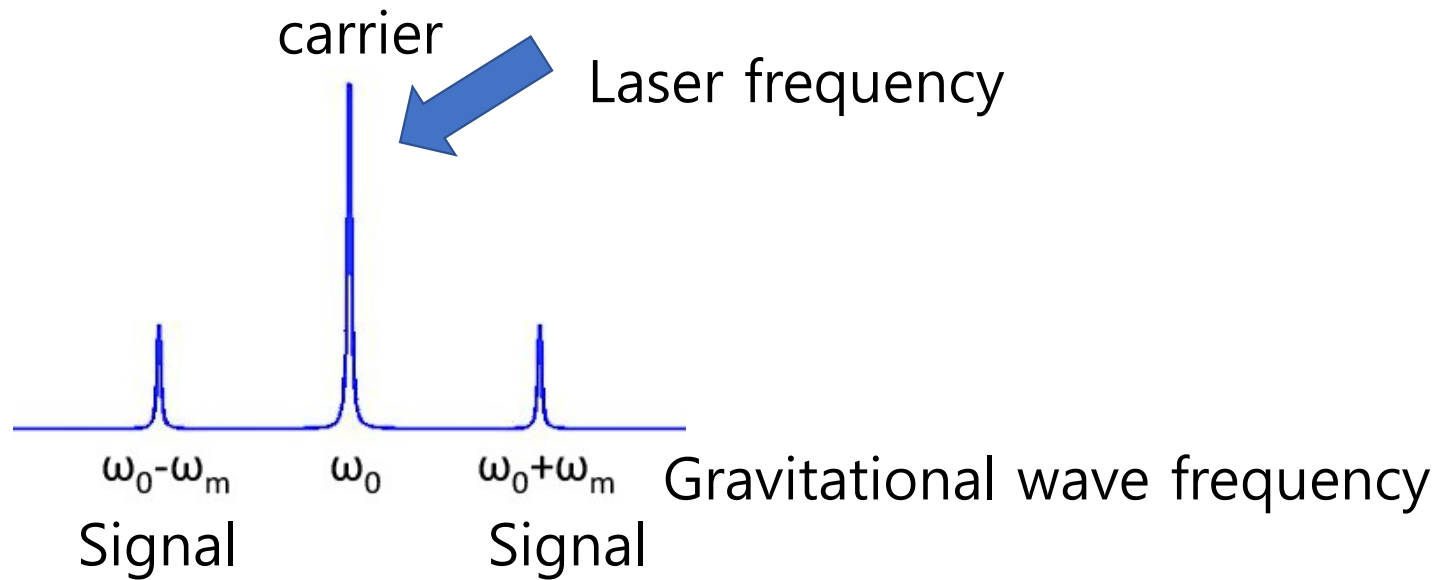


Frequency dependent squeezing using filter cavity

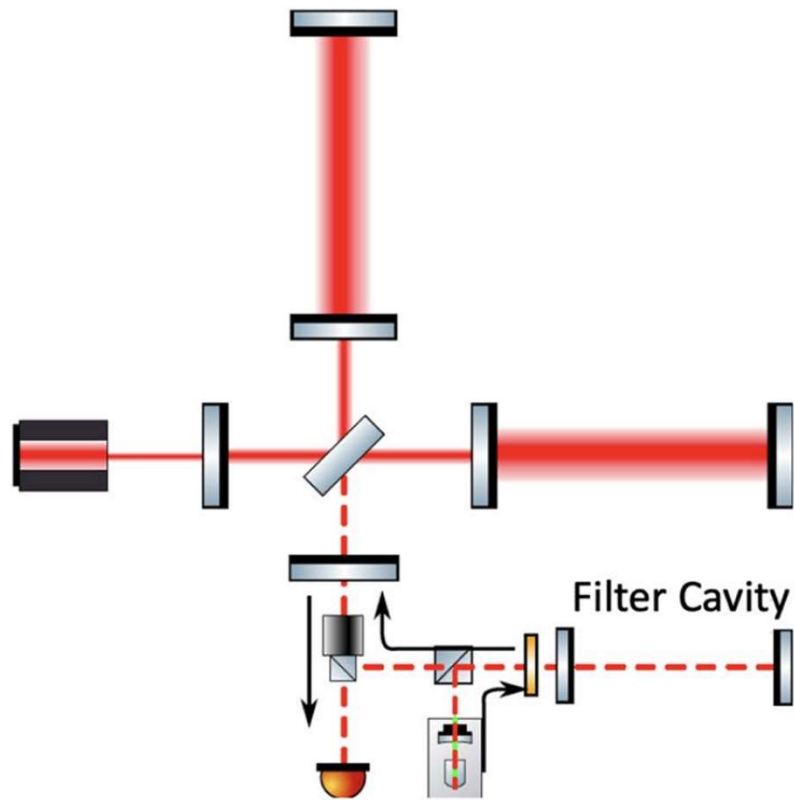


$$E^{(+)}(t) = \int_0^{\infty} E(\omega) e^{-i\omega t} \frac{d\omega}{2\pi}$$

Side band figure



Frequency dependent squeezing using filter cavity



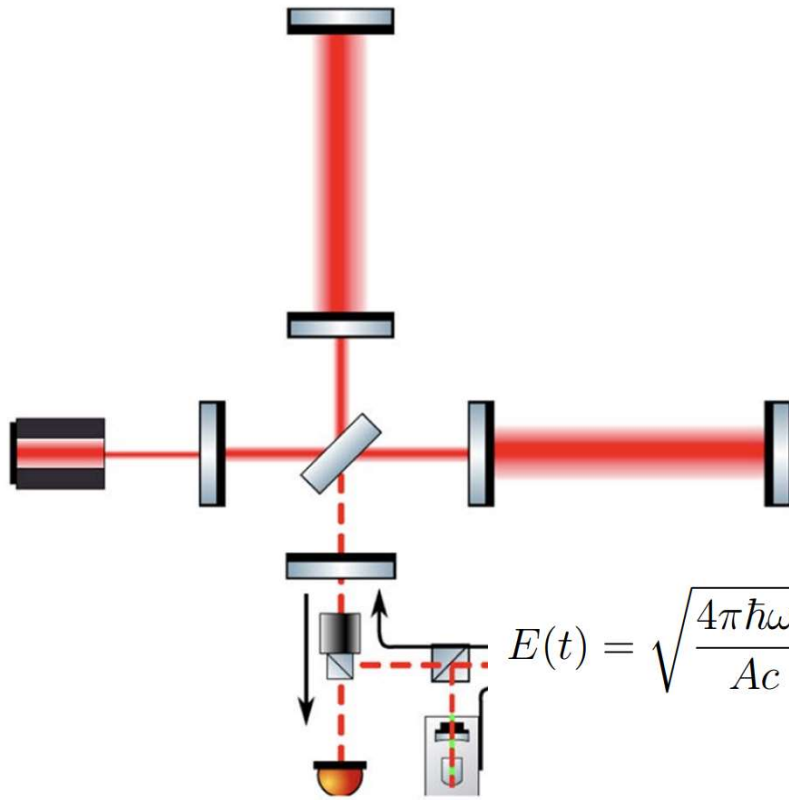
$$E^{(+)}(t) = \int_0^{\infty} E(\omega) e^{-i\omega t} \frac{d\omega}{2\pi}$$

$$E^{(+)}(t) = e^{-i\omega_0 t} \int_0^{\infty} (E(\Omega) e^{-i\Omega t} + E(-\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi}$$

$$\omega = \omega_0 + \Omega$$

Laser frequency GW signal frequency

Frequency dependent squeezing using filter cavity



$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} [a_1(t) \cos(\omega_0 t) + a_2(t) \sin(\omega_0 t)]$$

$$\text{Amplitude Quadrature : } a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

$$\text{Phase Quadrature : } a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$

$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} \left[\cos(\omega_0 t) \int_0^\infty (a_1(\Omega) e^{-i\Omega t} + a_1^\dagger(\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right. \\ \left. + \sin(\omega_0 t) \int_0^\infty (a_2(\Omega) e^{-i\Omega t} + a_2^\dagger(\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right]$$

도전선사

TIME MARKET TIME

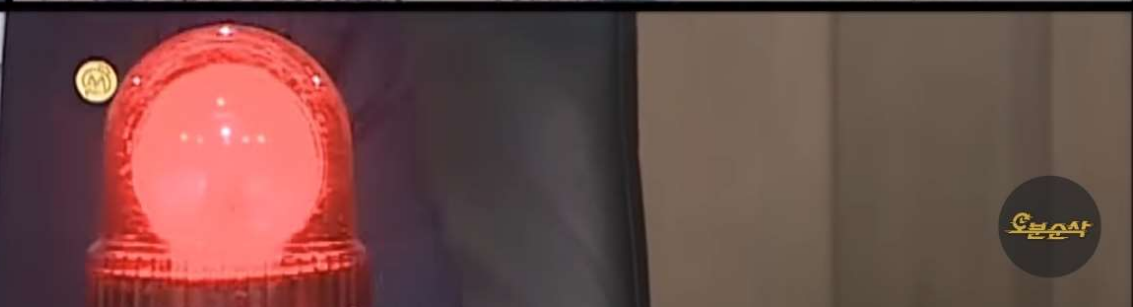
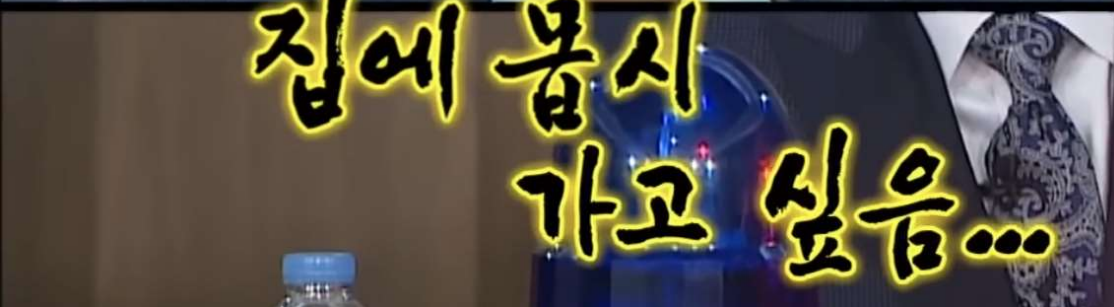
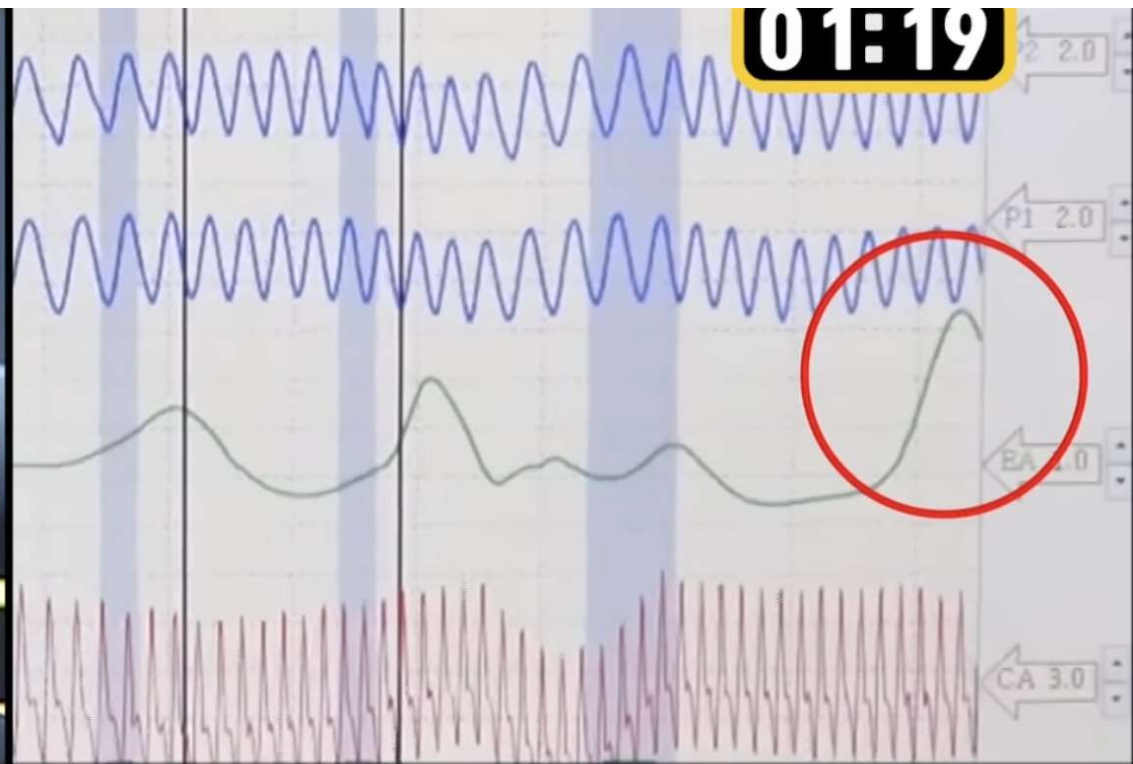
01:21

아니요

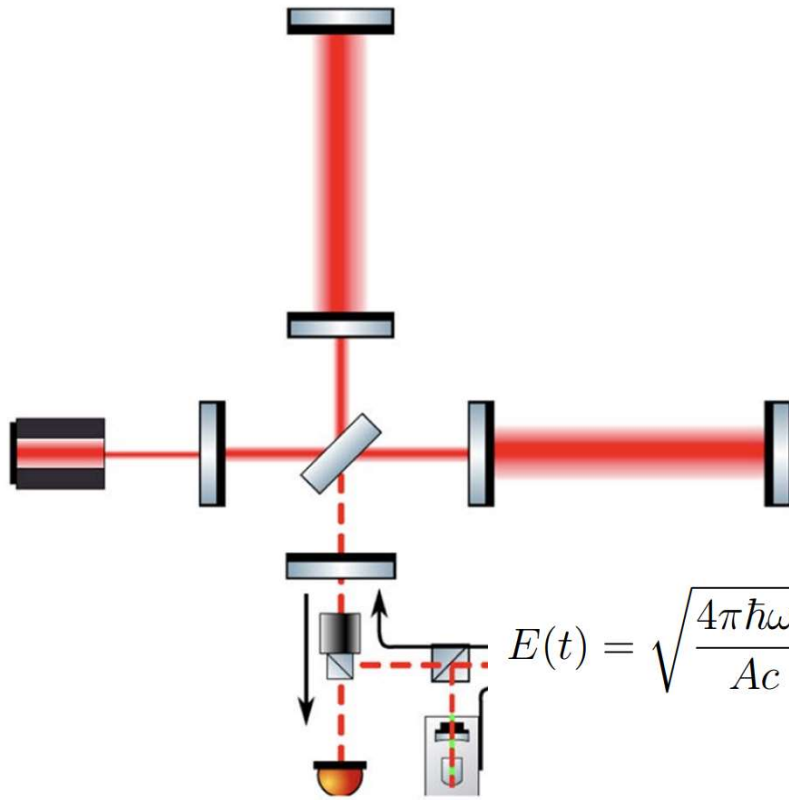
Q

집에 가고 싶으세요?

도전선사



Frequency dependent squeezing using filter cavity



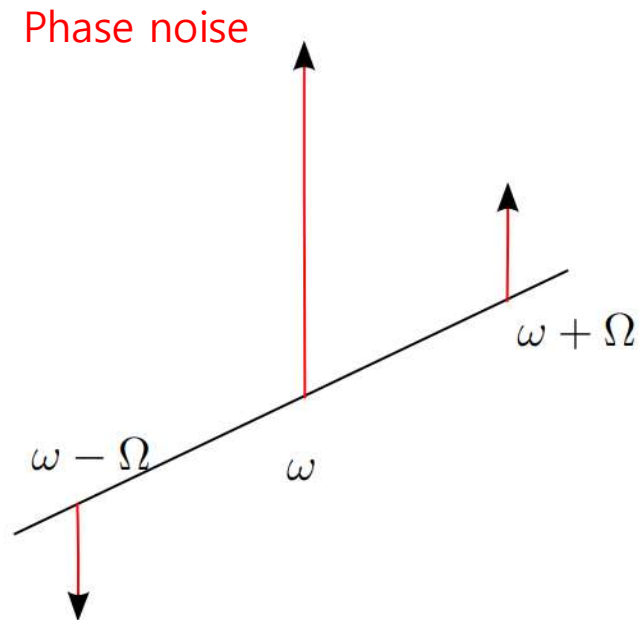
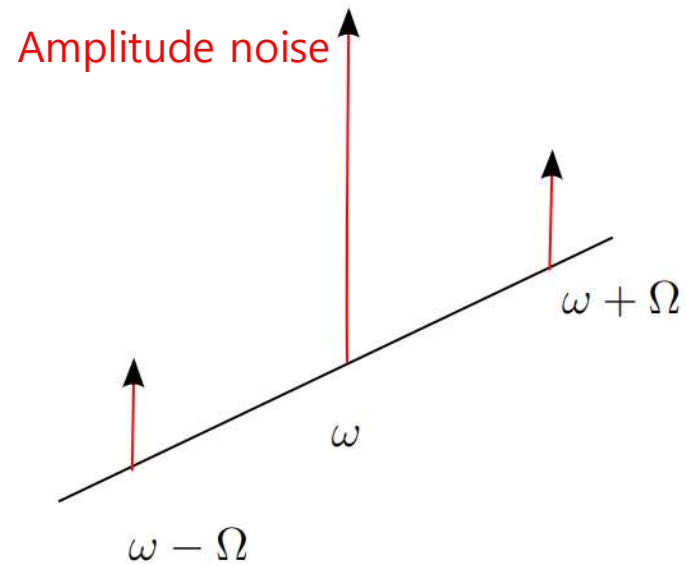
$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} [a_1(t) \cos(\omega_0 t) + a_2(t) \sin(\omega_0 t)]$$

$$\text{Amplitude Quadrature : } a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

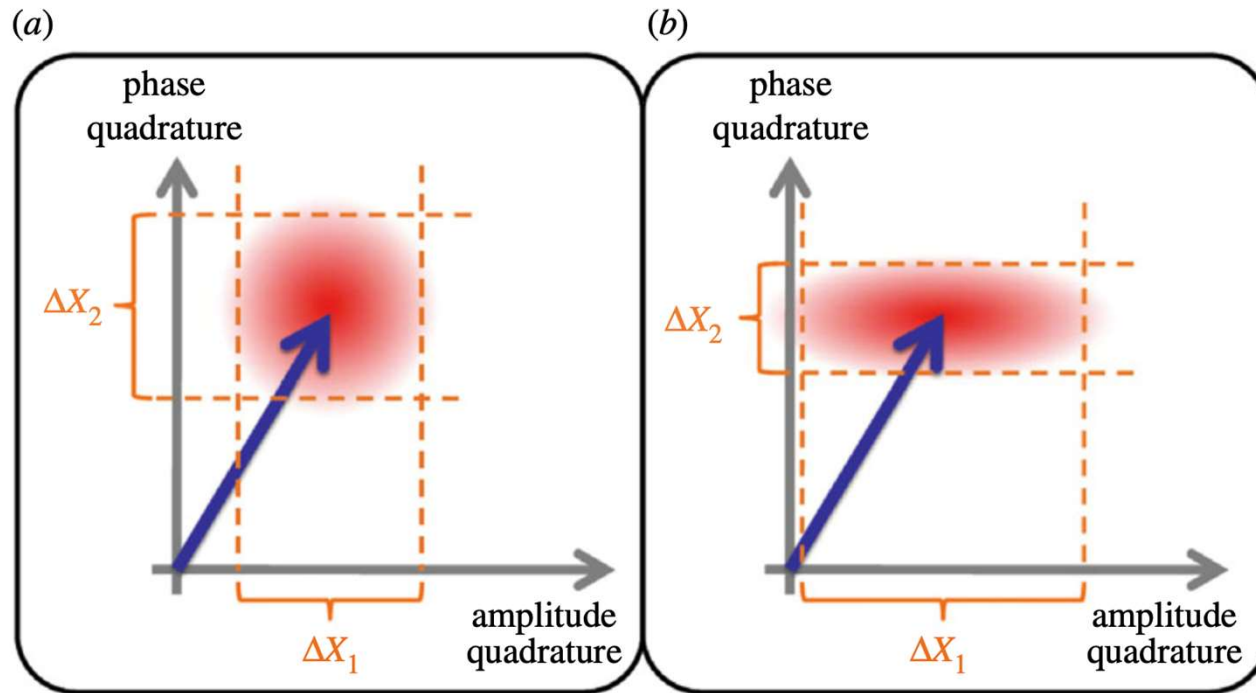
$$\text{Phase Quadrature : } a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$

$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} \left[\cos(\omega_0 t) \int_0^\infty (a_1(\Omega) e^{-i\Omega t} + a_1^\dagger(\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right. \\ \left. + \sin(\omega_0 t) \int_0^\infty (a_2(\Omega) e^{-i\Omega t} + a_2^\dagger(\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right]$$

Quantum noise side band figure

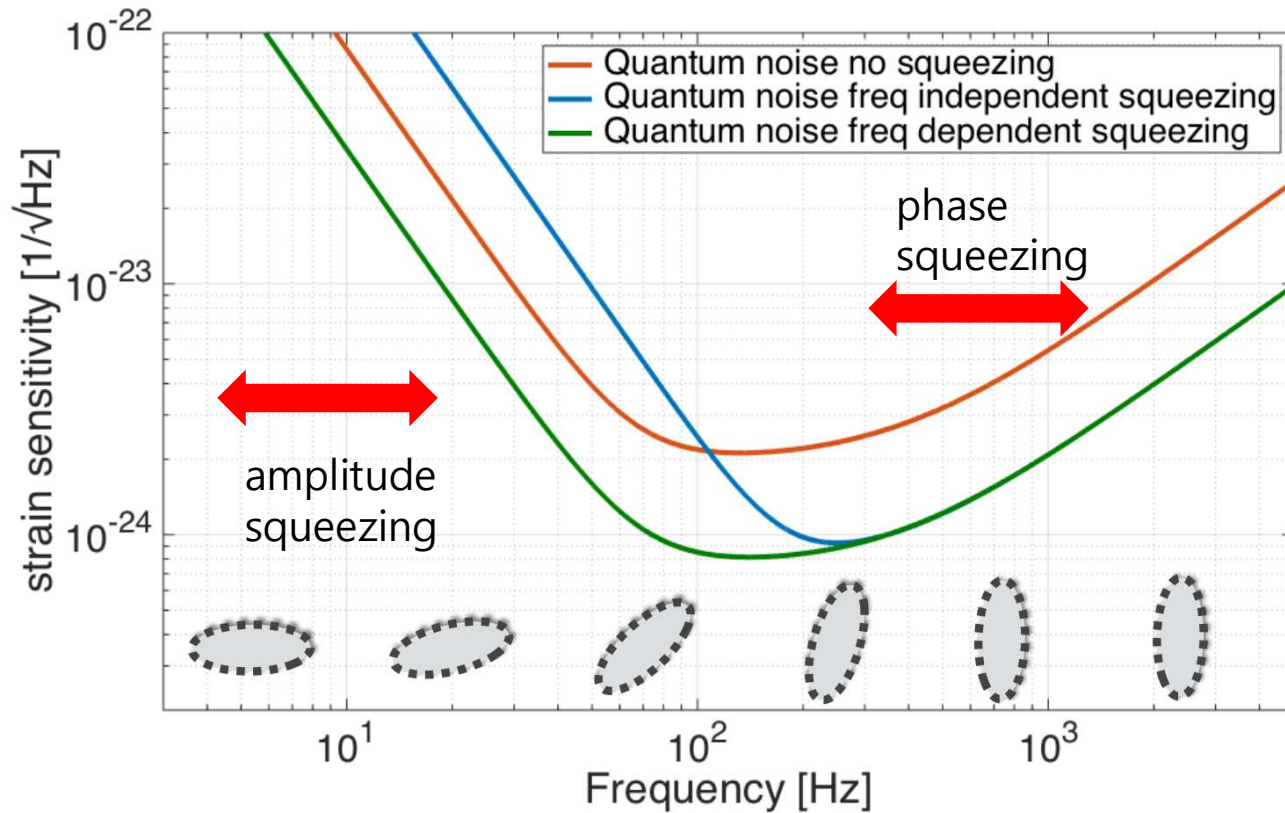


Squeezed light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Frequency dependent squeezing(FDS)

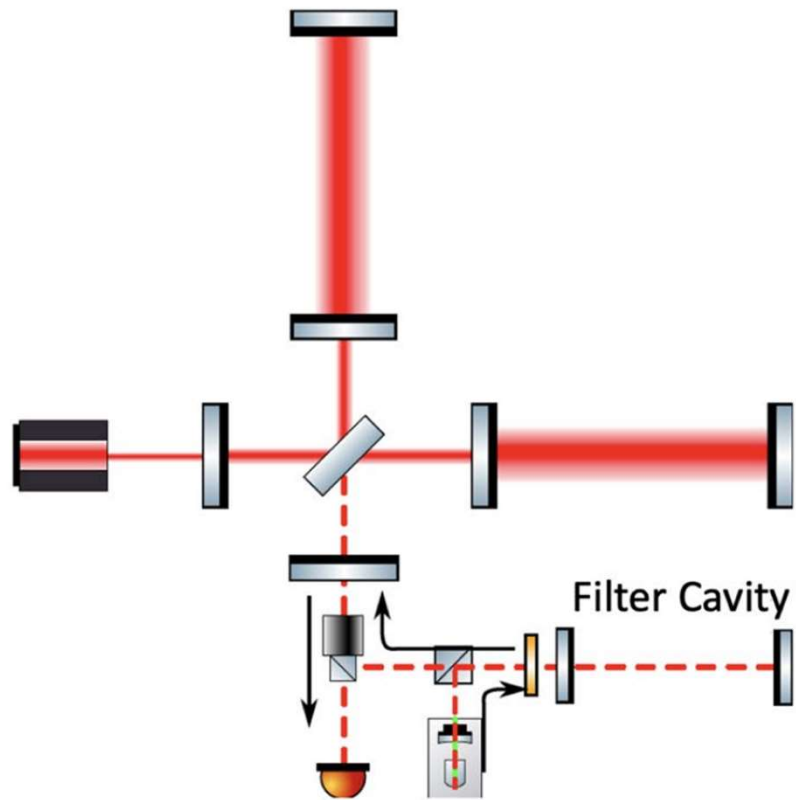


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중력파 여름학교 2025

YONSEI UNIVERSITY

Frequency dependent squeezing using filter cavity

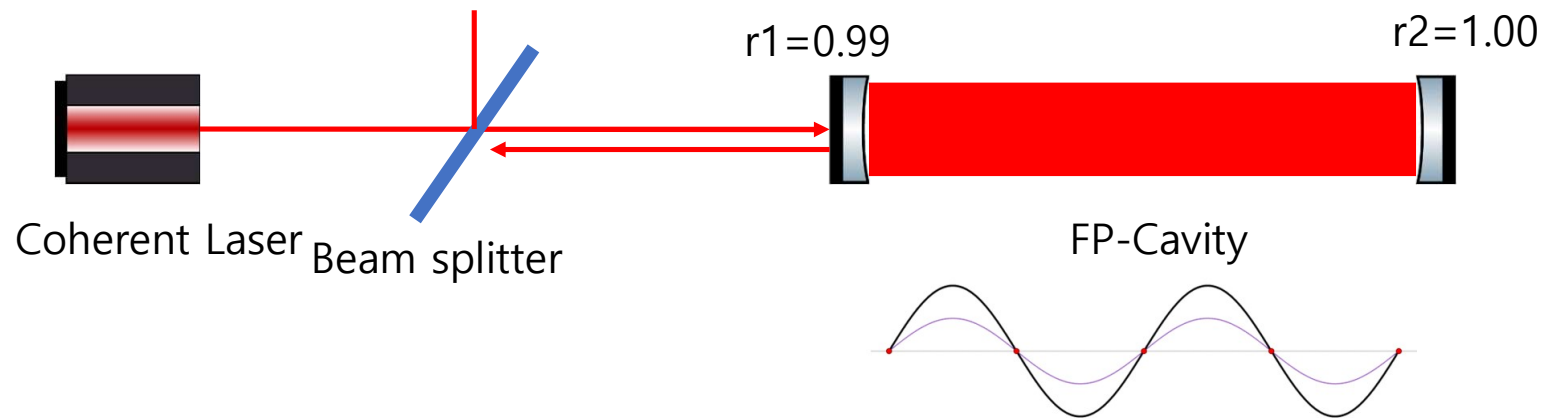


Fabry-perot cavity

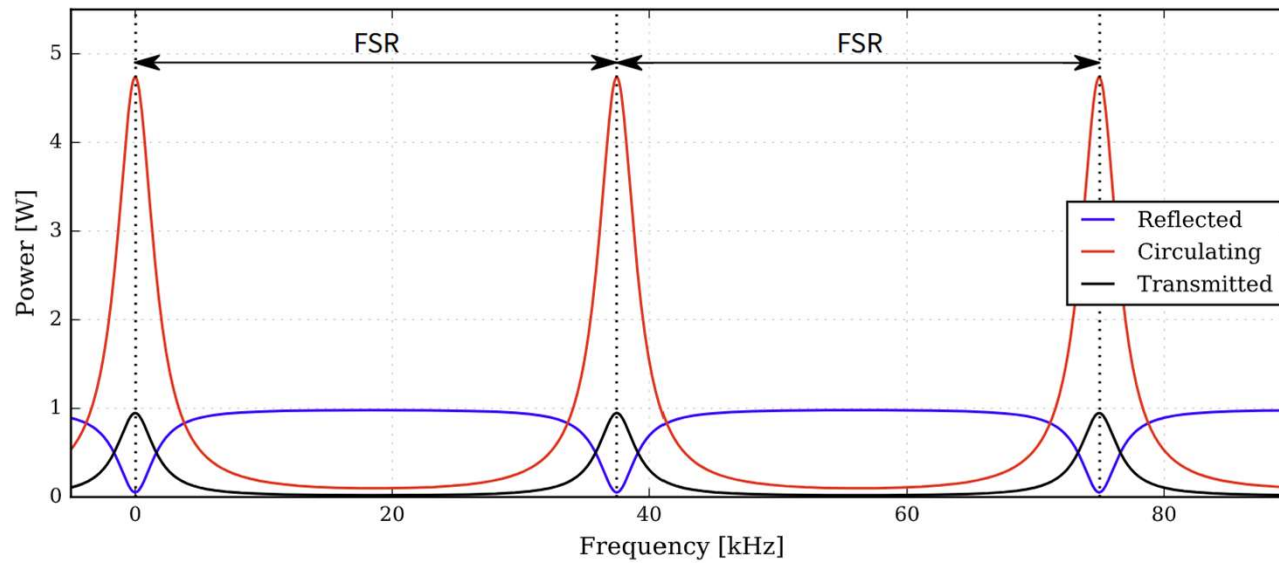


Number of round trip > 250

4 km x 250 ~ 1000 km



Free spectral range of cavity



Detuned cavity



Simple picture

Stefan Hild et al, "Detuned arm cavities", 3rd GEO simulation workshop, Hannover, June 2007

B:

- less carrier light in cavity => less GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced.

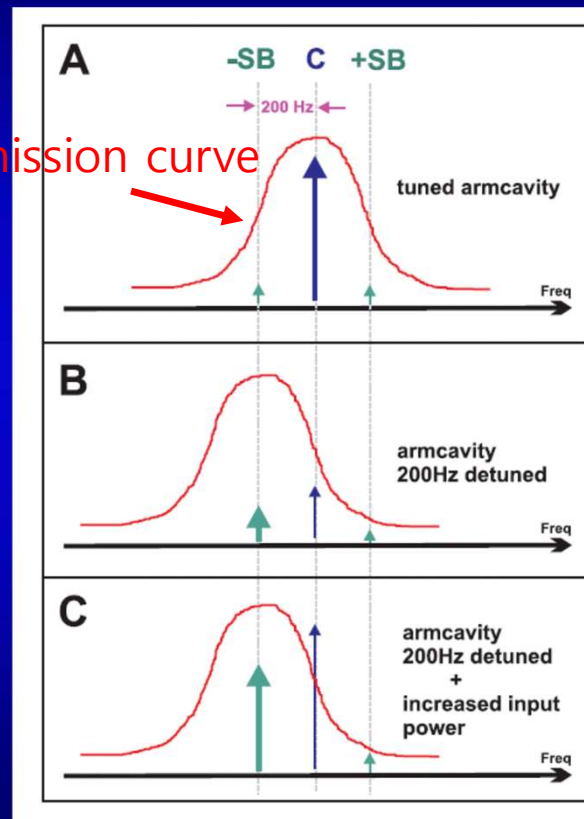
=> **Smaller GW signal**

C:

- optical power is restored in the cavity by larger PR-gain.
 - Same amount of GW sidebands are produced.
 - Since one GW sideband is resonant, it gets enhanced.
- Overall we win GW signal.

=> **Larger GW signal**

Transmission curve



Quantum noise side band figure

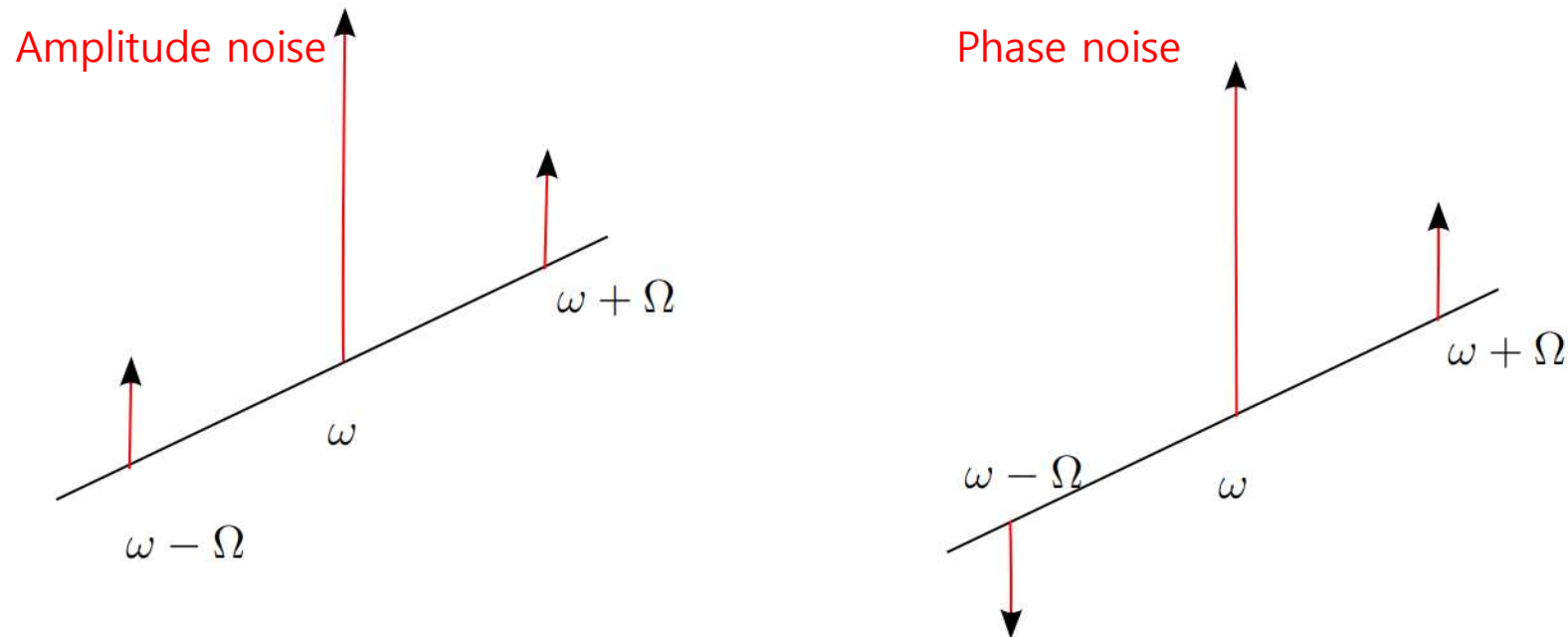
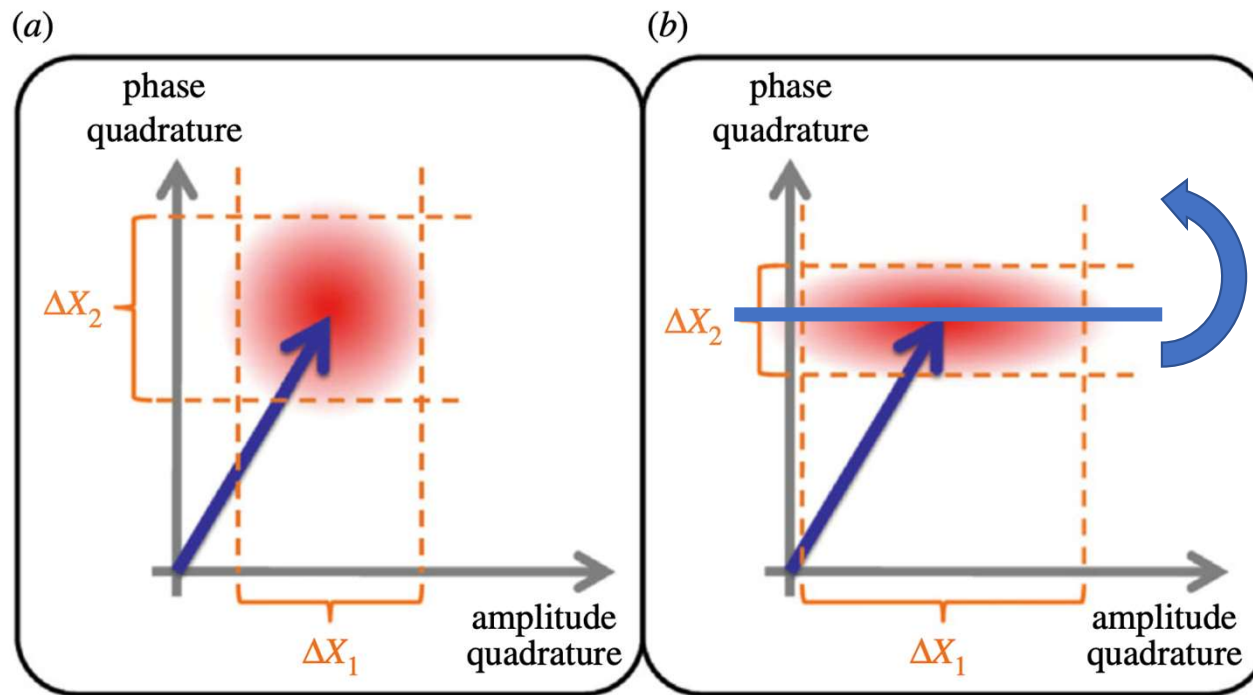


Figure 1-2: Phasors of amplitude noise (left) and phase noise (right) in the sideband picture. In the frame rotating at the carrier frequency ω the carrier is still in these diagrams while the sidebands rotate at Ω , the signal at $\omega + \Omega$ rotating clockwise while the idler at $\omega - \Omega$ rotates counter clockwise. (Sidebands have equal amplitudes)

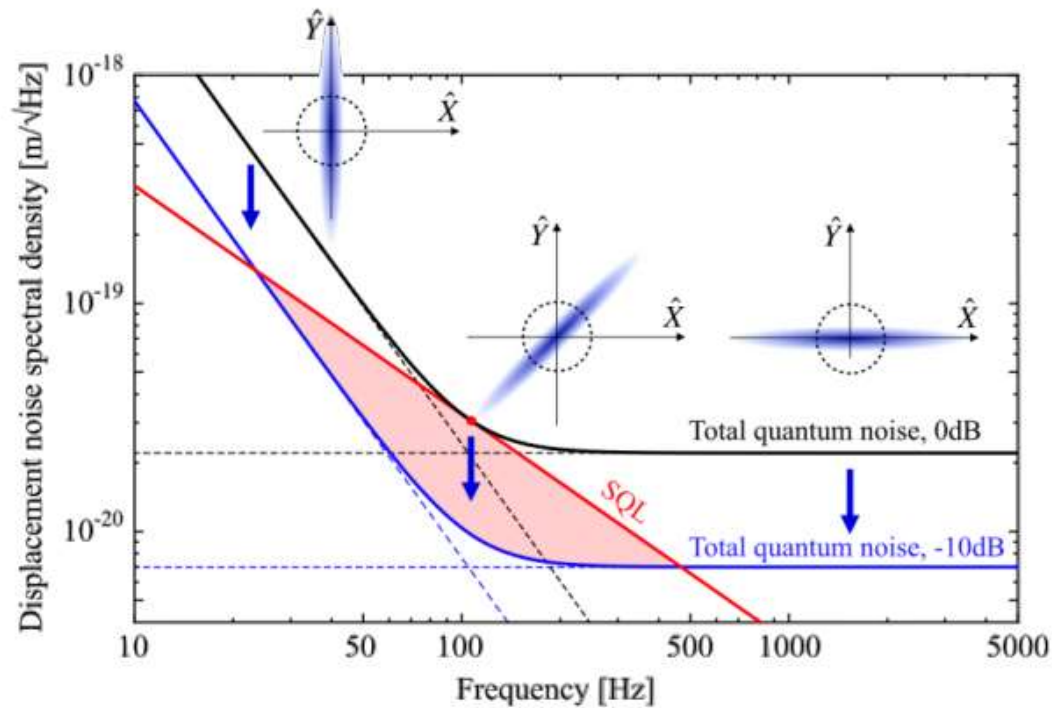
Squeezed light



Squeeze angle rotation

Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

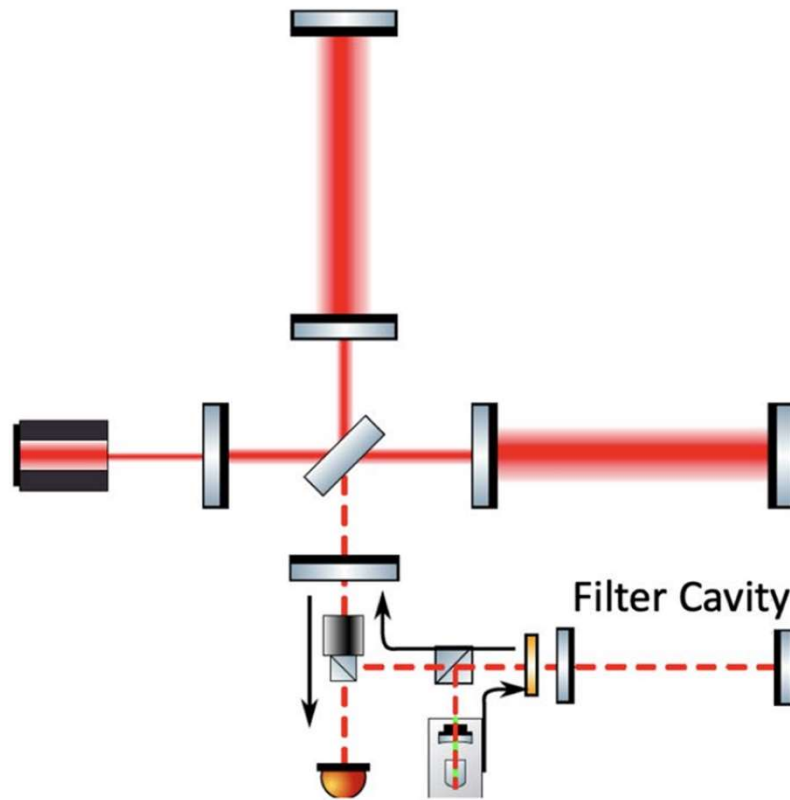
Squeeze angle rotation



$$\alpha_p = \arctan \left(\frac{2\gamma_{fc}\Delta\omega_{fc}}{\gamma_{fc}^2 - \Delta\omega_{fc}^2 + \Omega^2} \right)$$

$$t_{st} = \frac{1}{\gamma_{fc}} = \frac{\sqrt{2}}{\Omega_{SQL}} \simeq 3 \text{ ms}$$

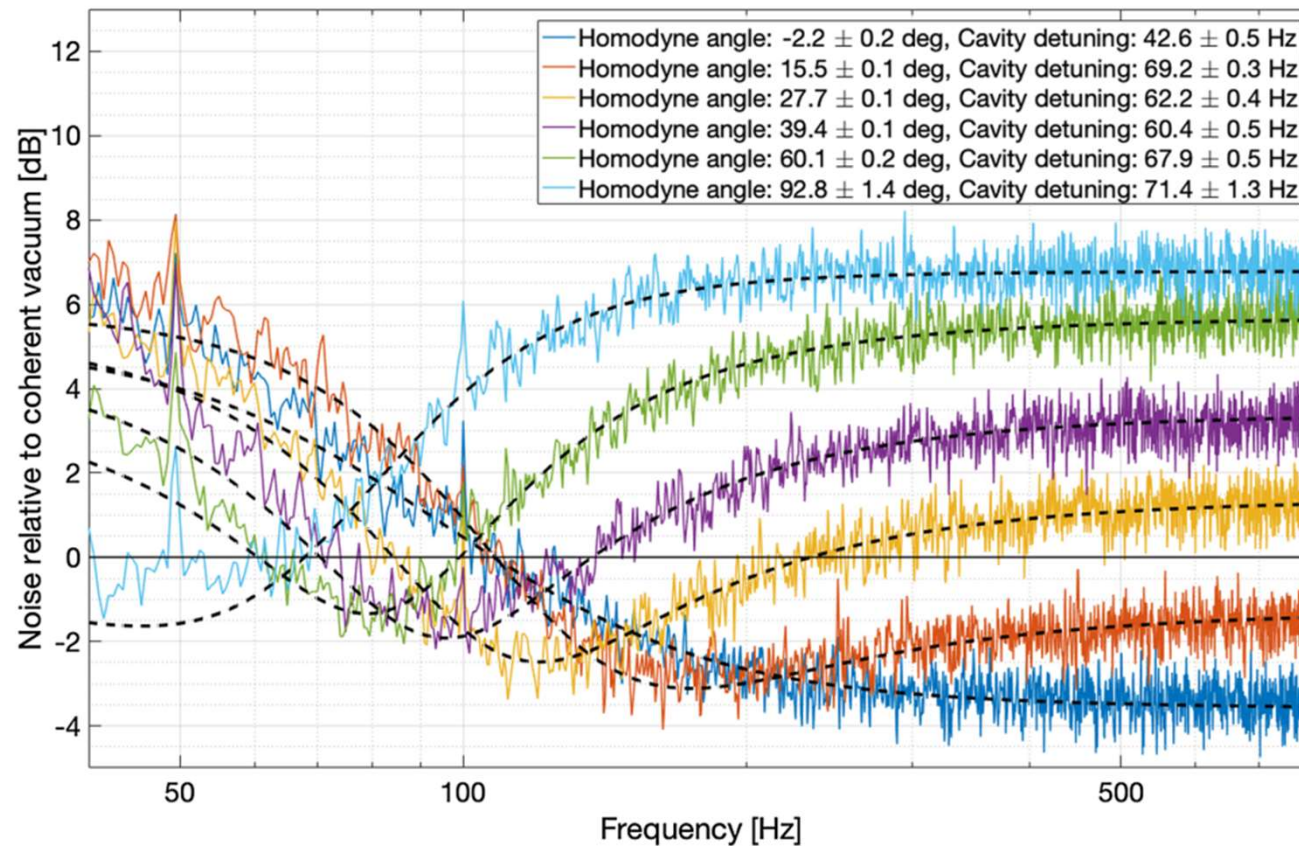
Frequency dependent squeezing using filter cavity



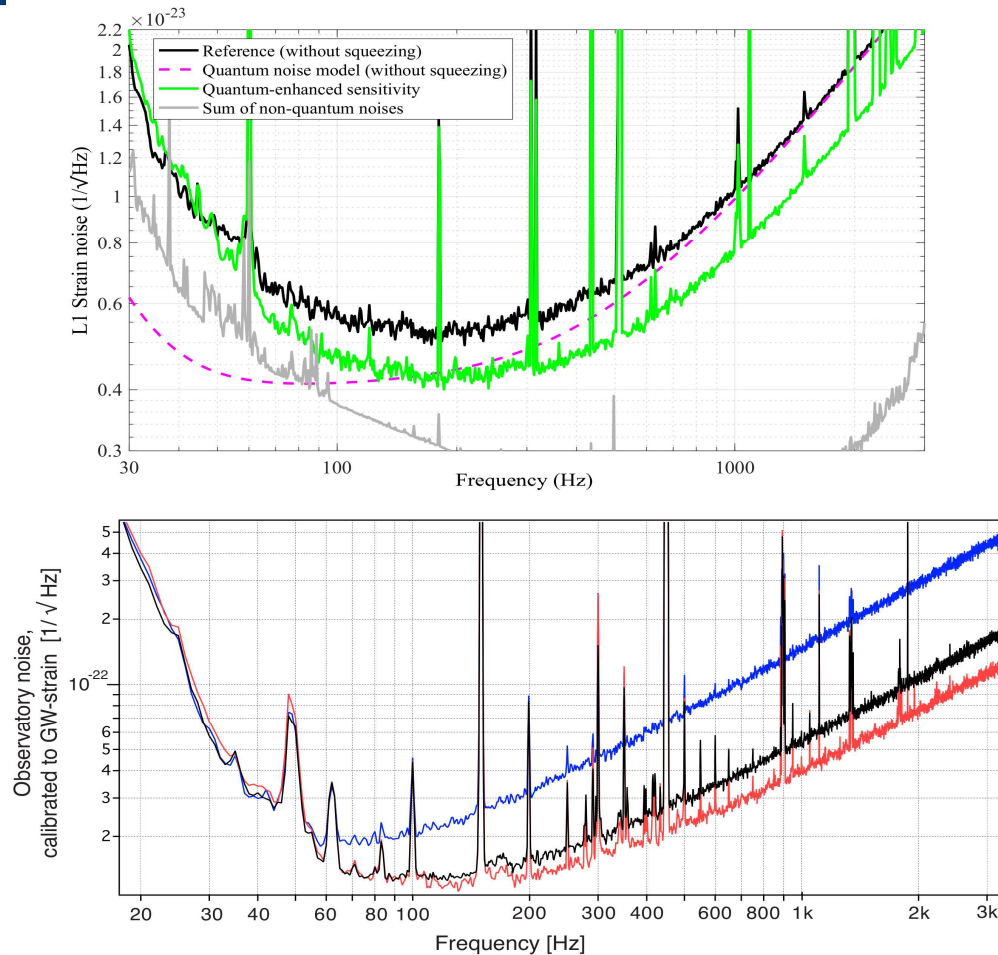
Frequency dependent squeezing - KAGRA



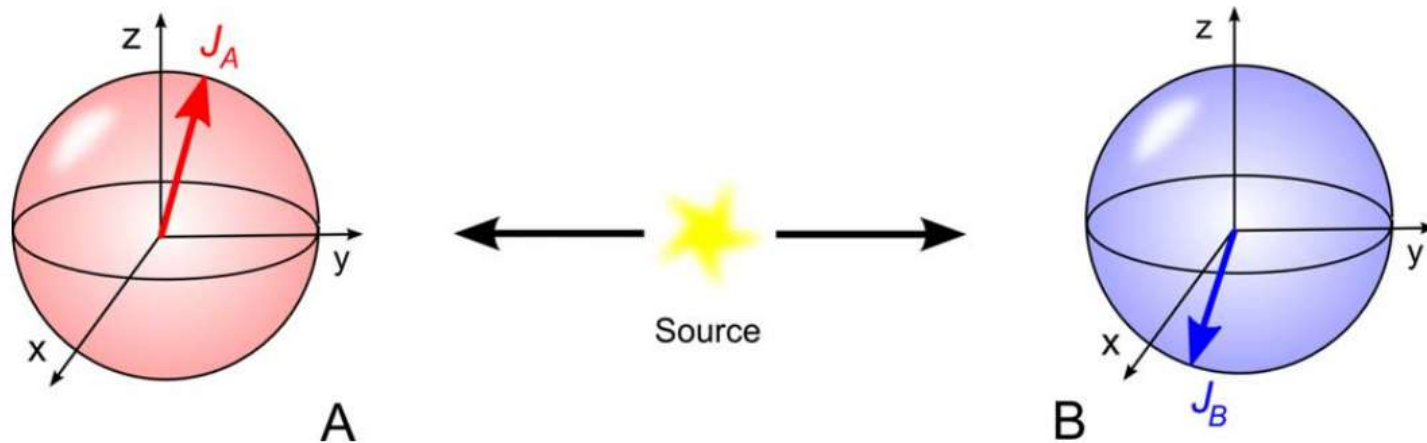
PHYSICAL REVIEW LETTERS **124**, 171101 (2020)



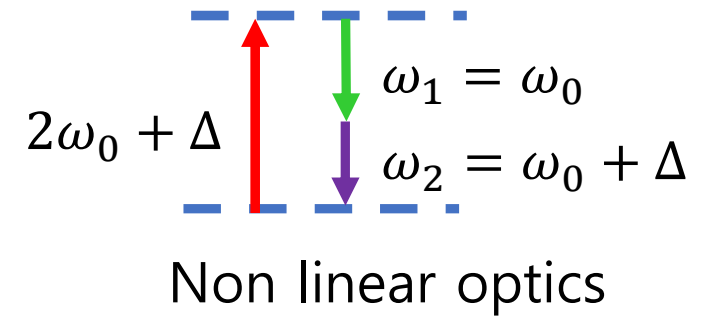
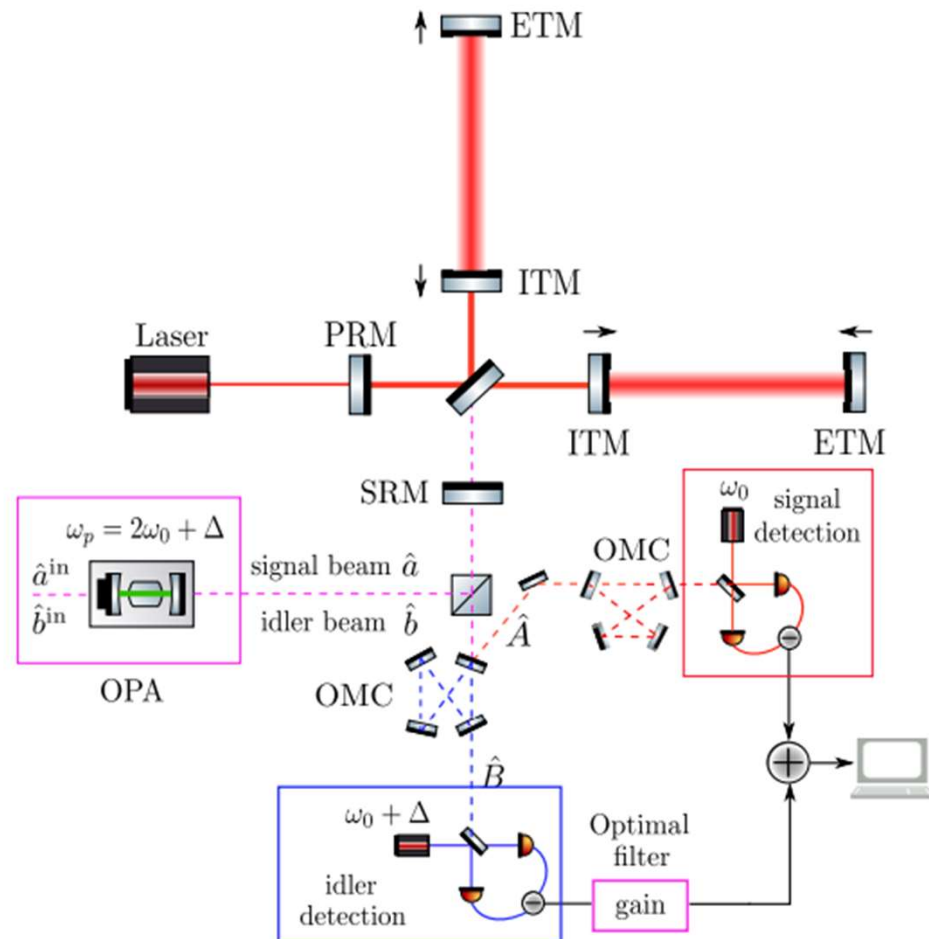
Quantum noise of gravitational wave detector



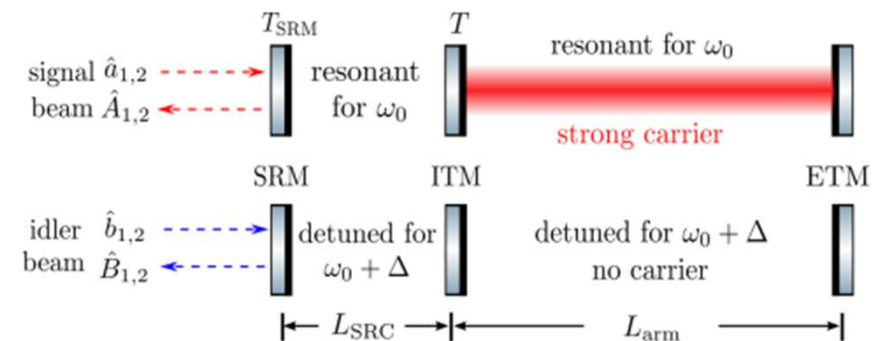
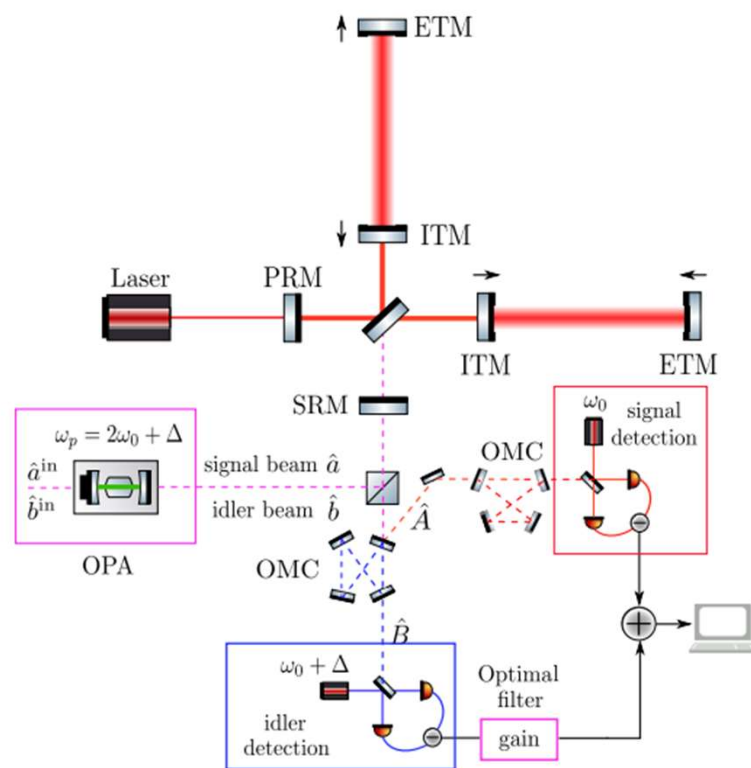
Einstein-Podolsky-Rosen paradox

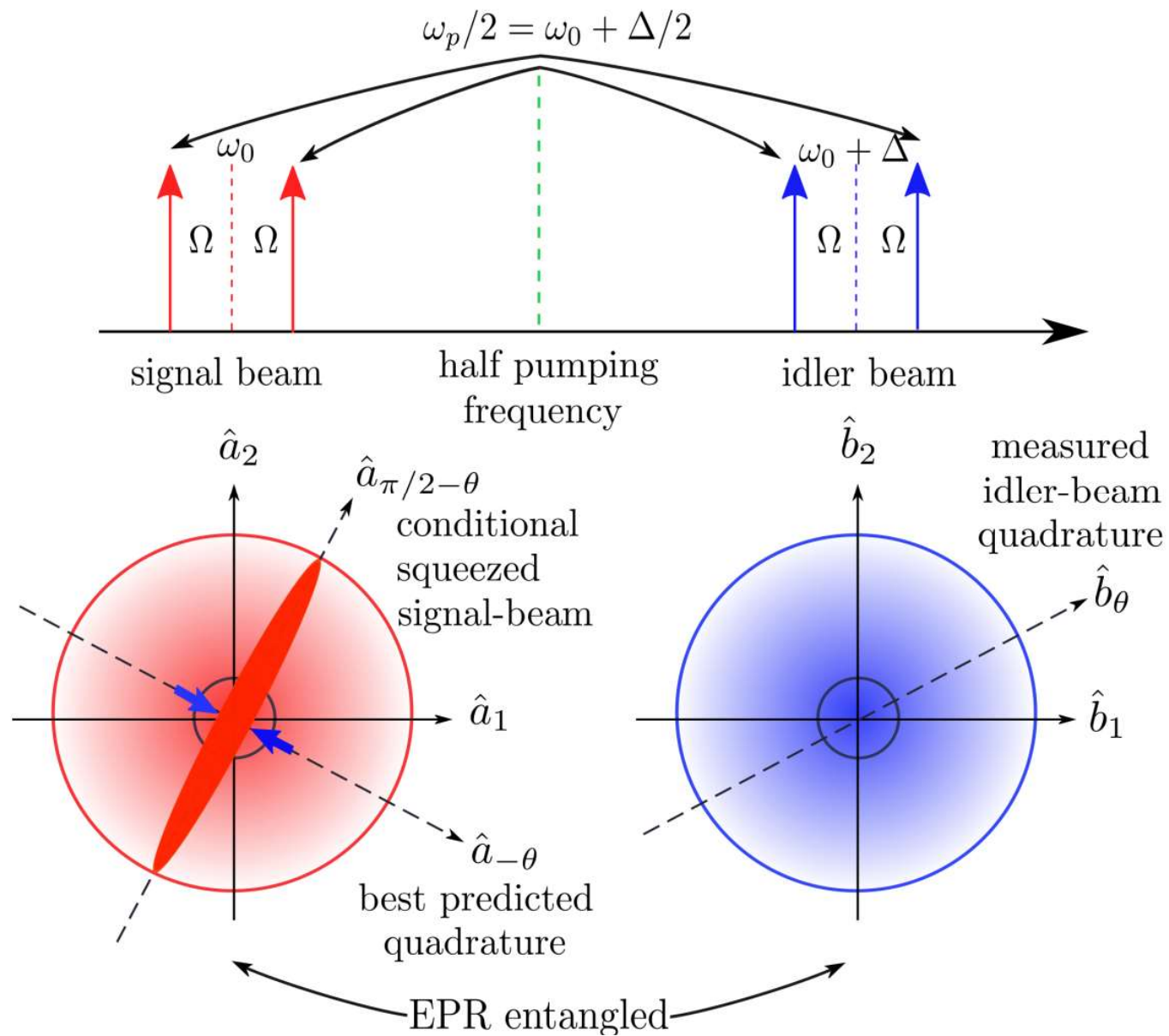


EPR squeezing for gravitational wave detector

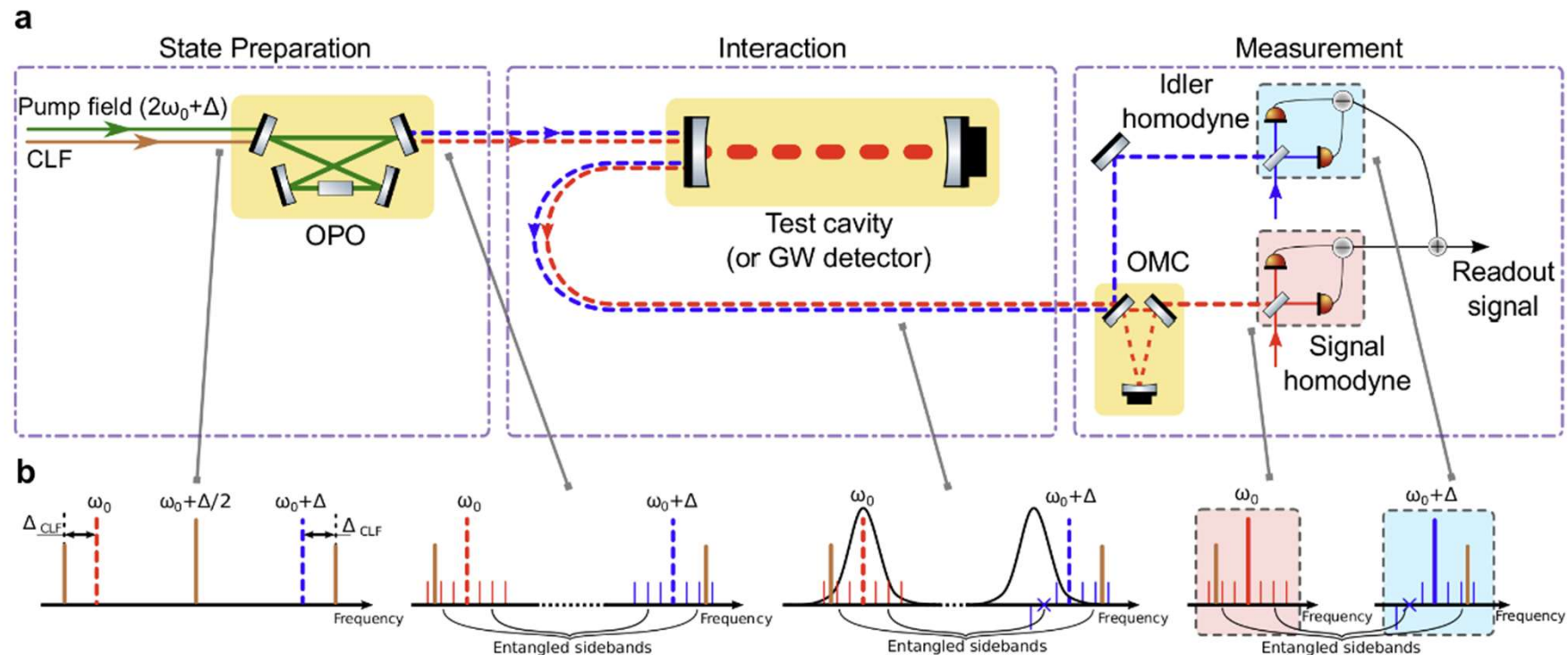


EPR squeezing for gravitational wave detector





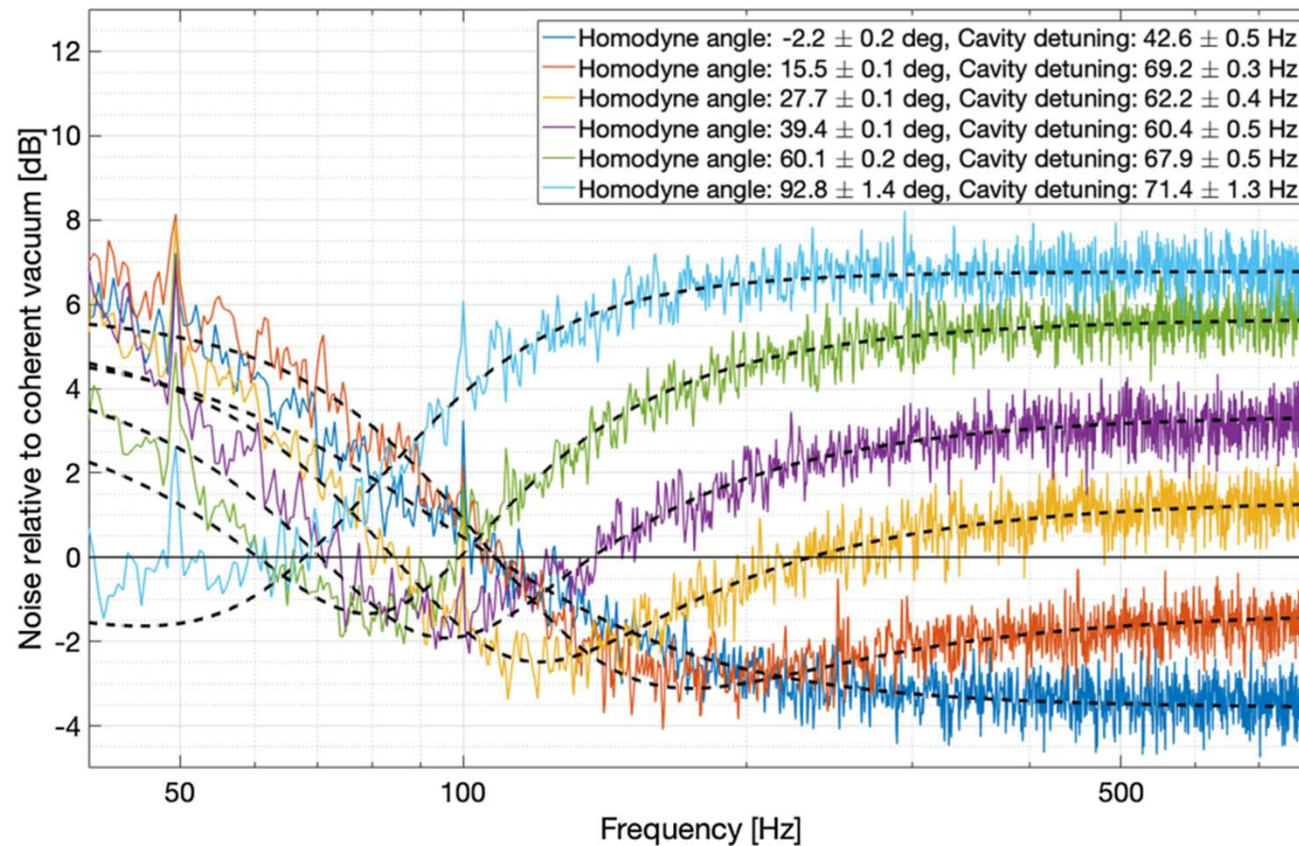
Generation and control of frequency dependent squeezing via EPR entanglement



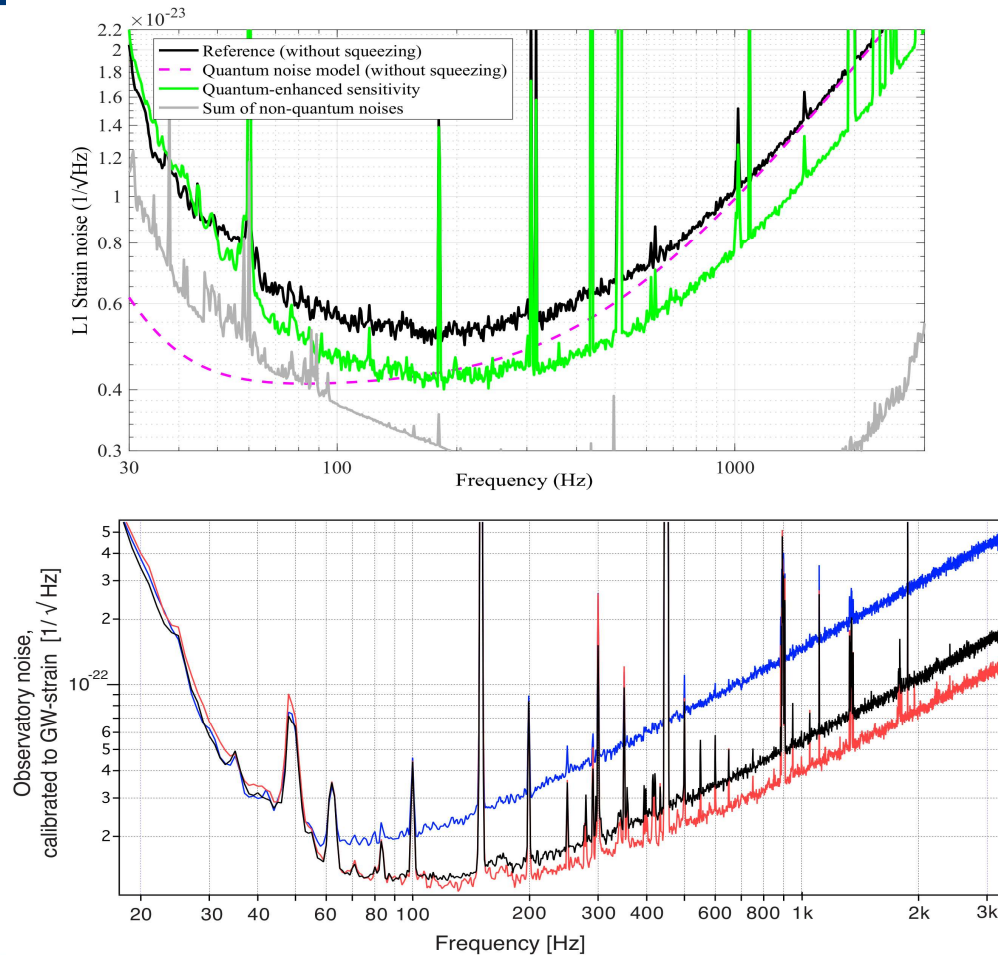
Frequency dependent squeezing - KAGRA



PHYSICAL REVIEW LETTERS **124**, 171101 (2020)



Quantum noise of gravitational wave detector



오분순작

IX THMA MAKETH N

01:06

꾸욱~





복잡한심경대변(?)

잔뜩 뒤텀킨 그래프💧

