

# Multi messenger astronomy with a laser interferometer gravitational wave observatory

Department of astronomy  
June Gyu Park

중력파 여름학교 2025



# Introduction



## June Gyu Park

Laser optics

Laser interferometer

Science hall 623

Sogang University PhD (Physics)

Korea basics science institute (KBSI)

Korea research institute of standards and science (KRISS)

Korea astronomy and space science institute (KASI)

### Gravitational wave detector

LVK collaboration

KAGRA

VIRGO

Einstein telescope

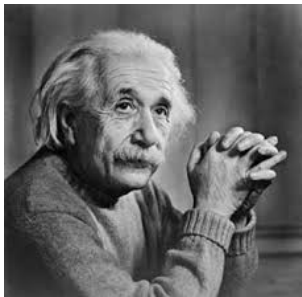
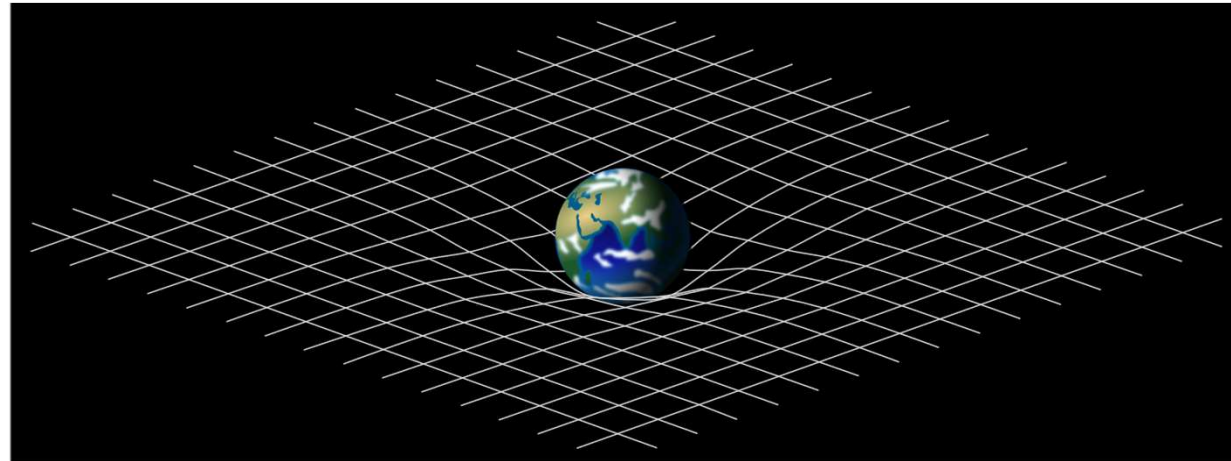
Local Sensor and detector design

### Space optics metrology

KRISS

KASI

# Gravity and general relativity

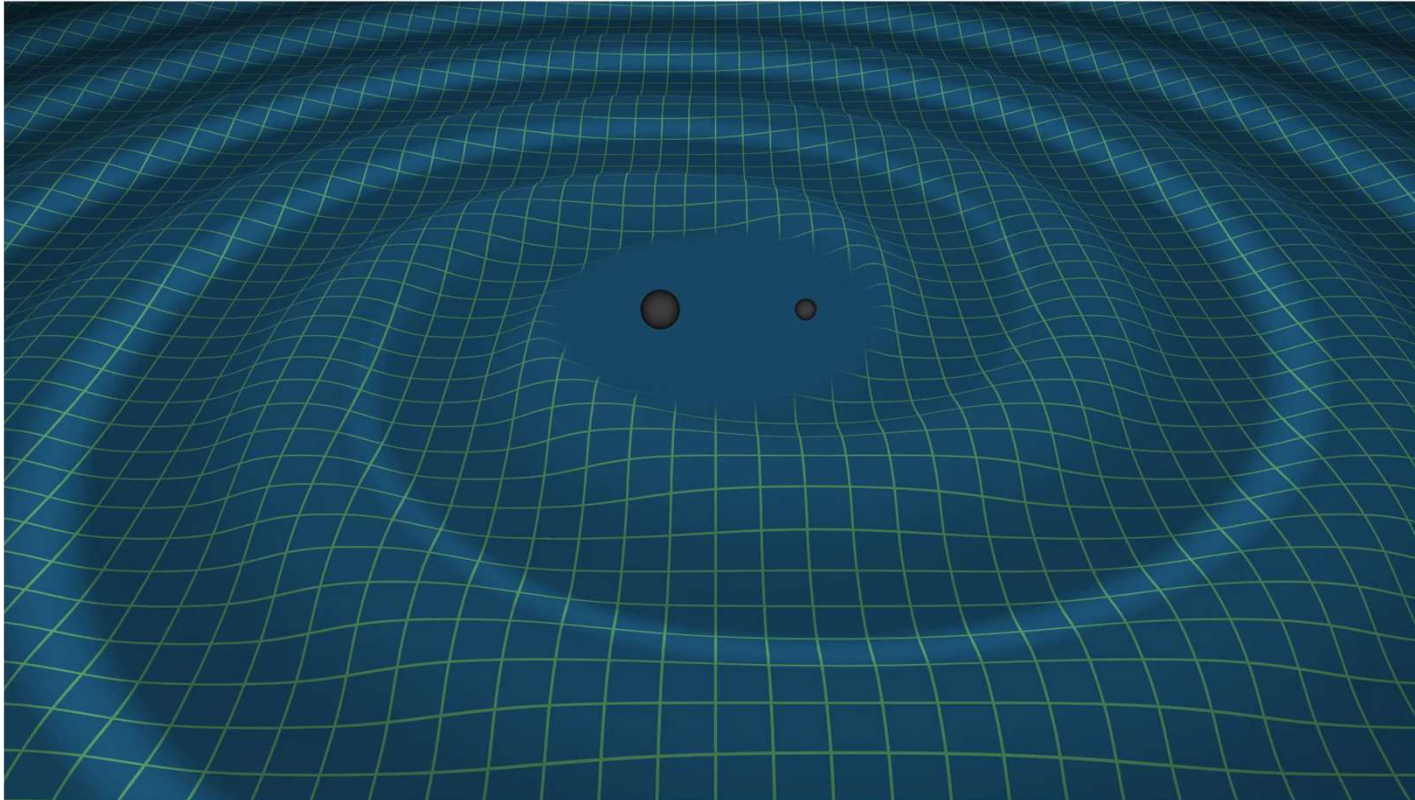


$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

Local space time curvature    Local energy, momentum stress

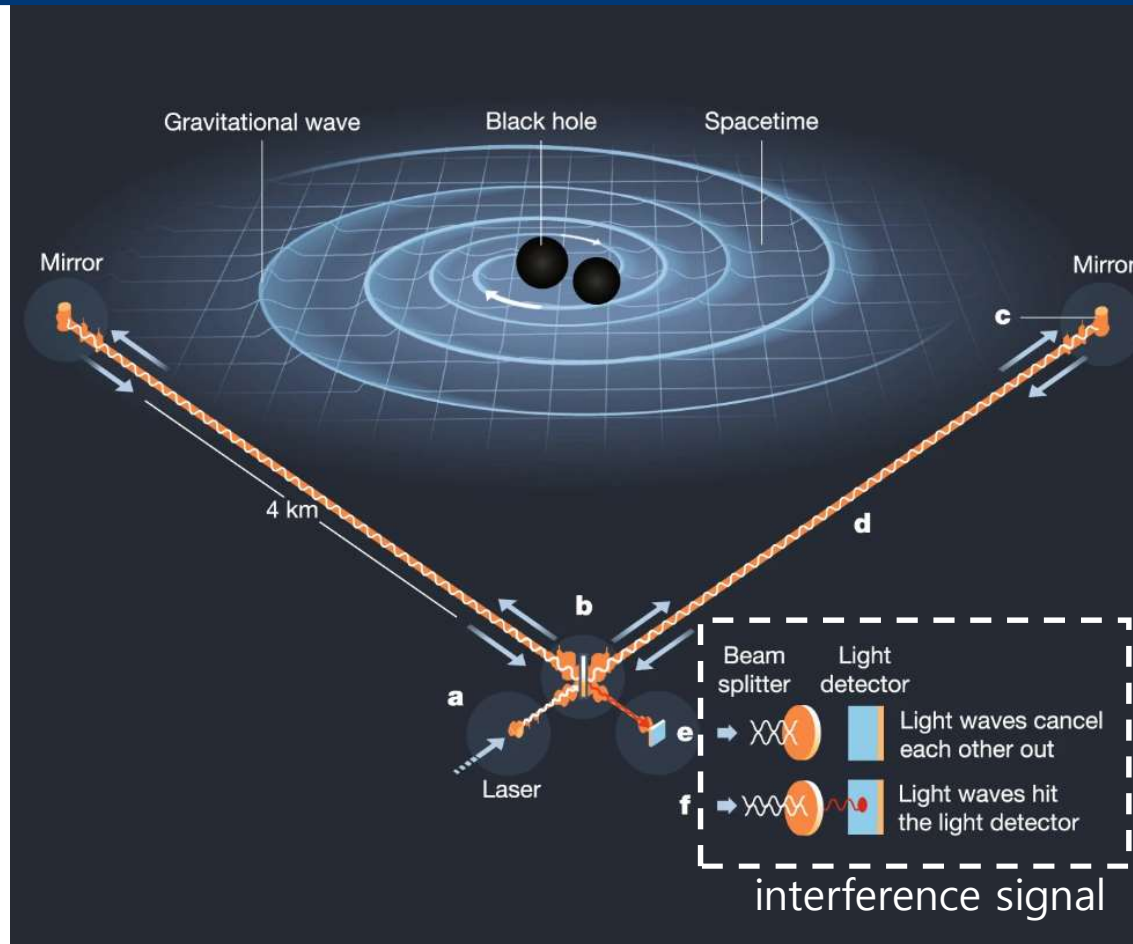
Mass of object

# Gravitational wave





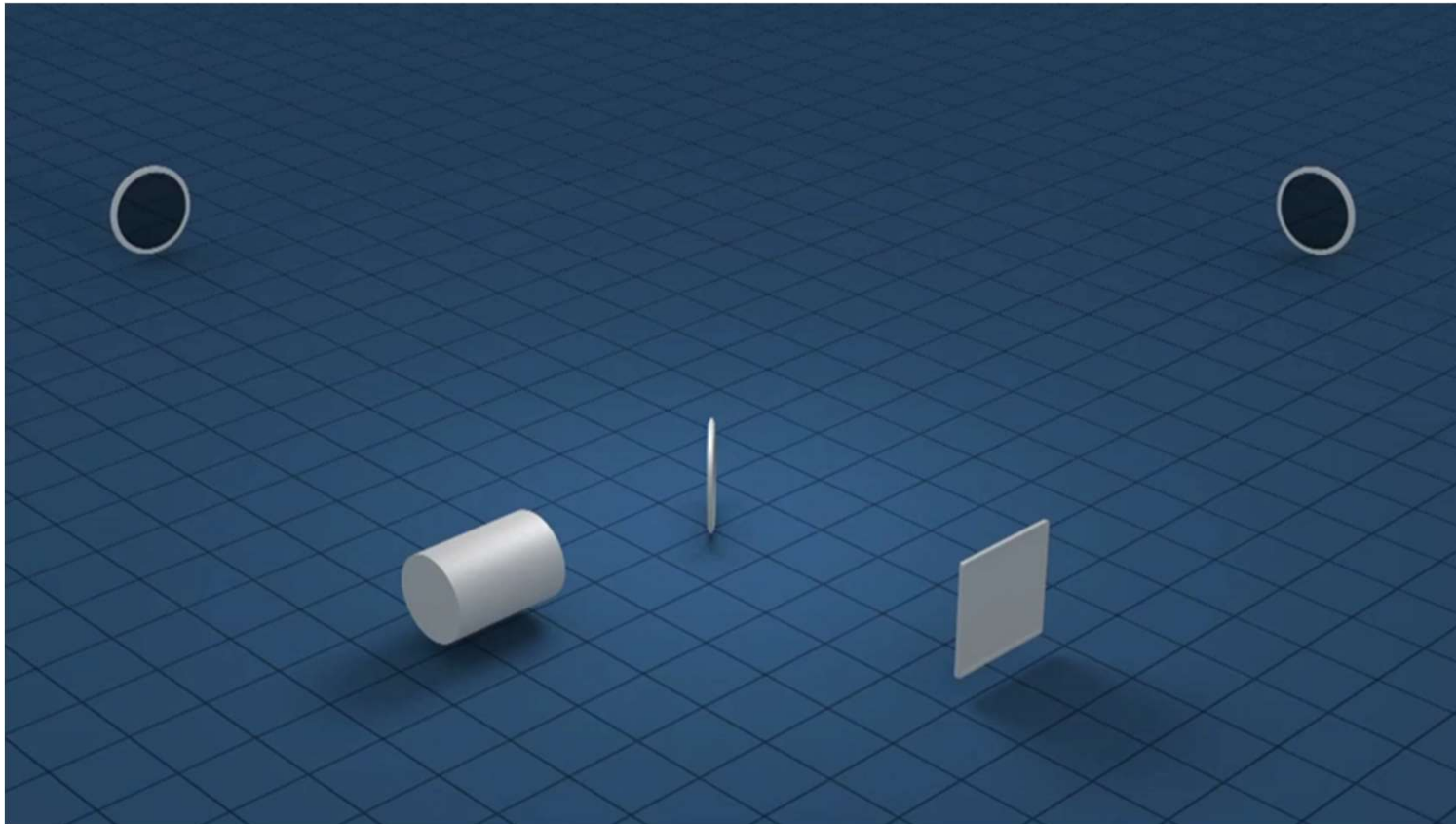
# Gravitational wave detector



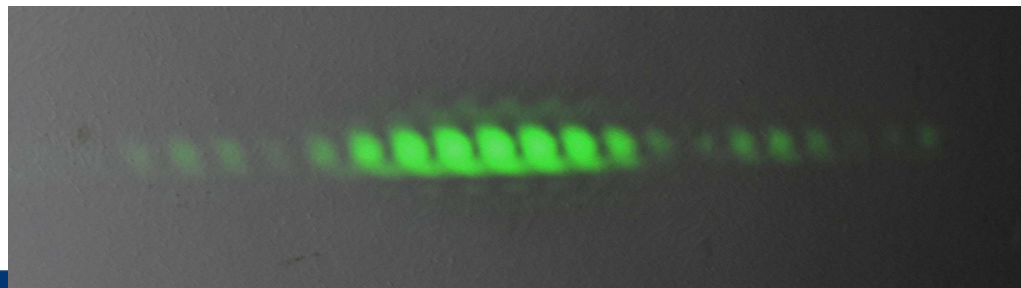
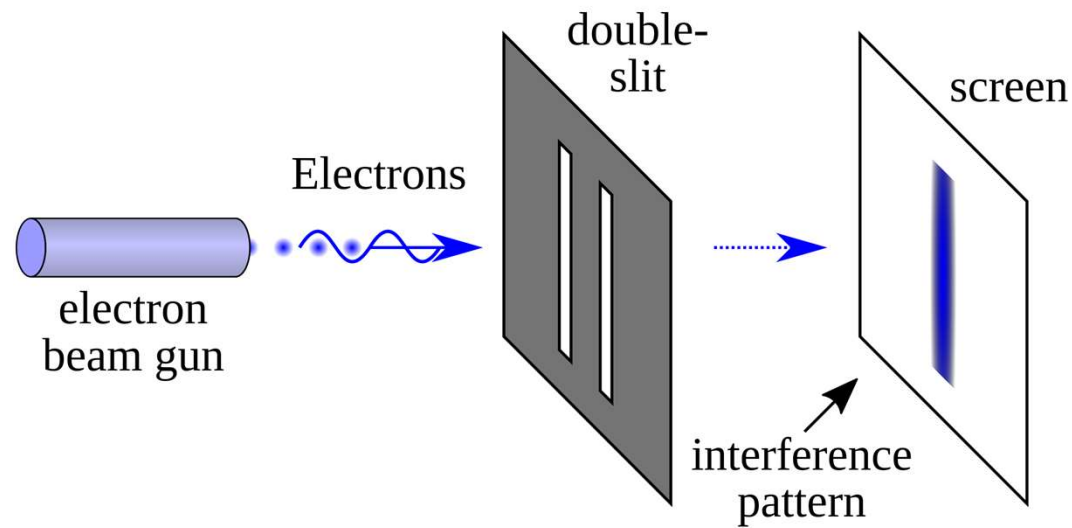
- Ground scale Michelson interferometer
- 4 km vacuum tunnel arm
- Over 1000 km interaction length

Miller, M.C., Yunes, N. The new frontier of gravitational waves. *Nature* **568**, 469–476 (2019)

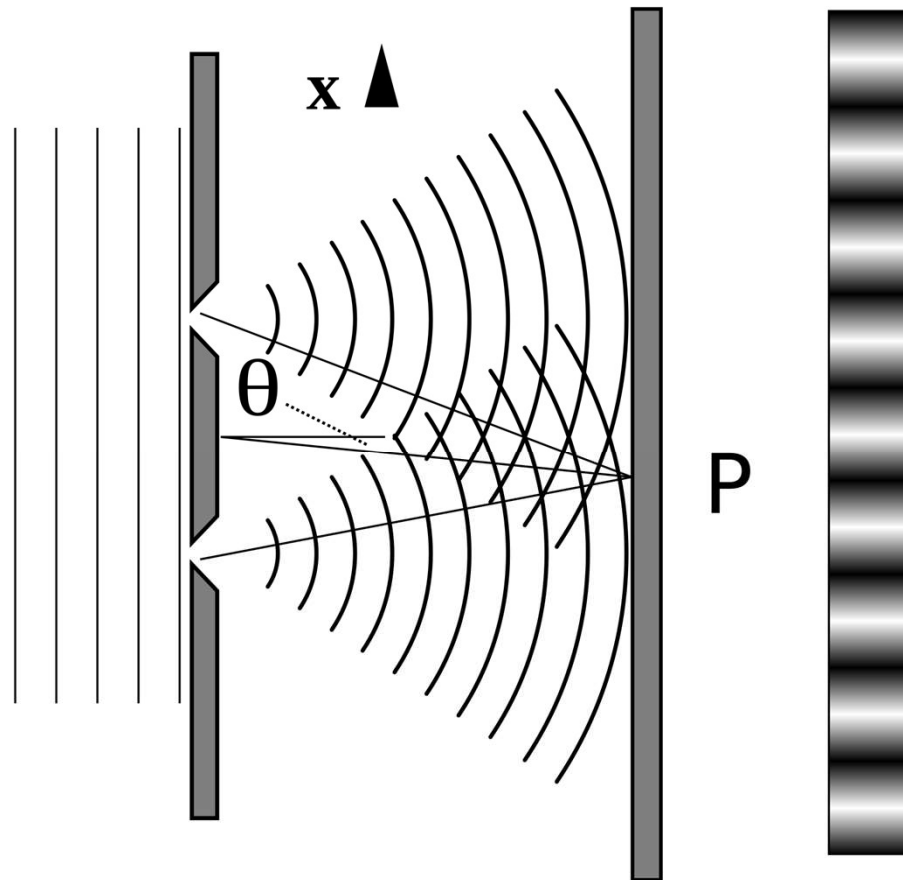
# Michelson interferometer



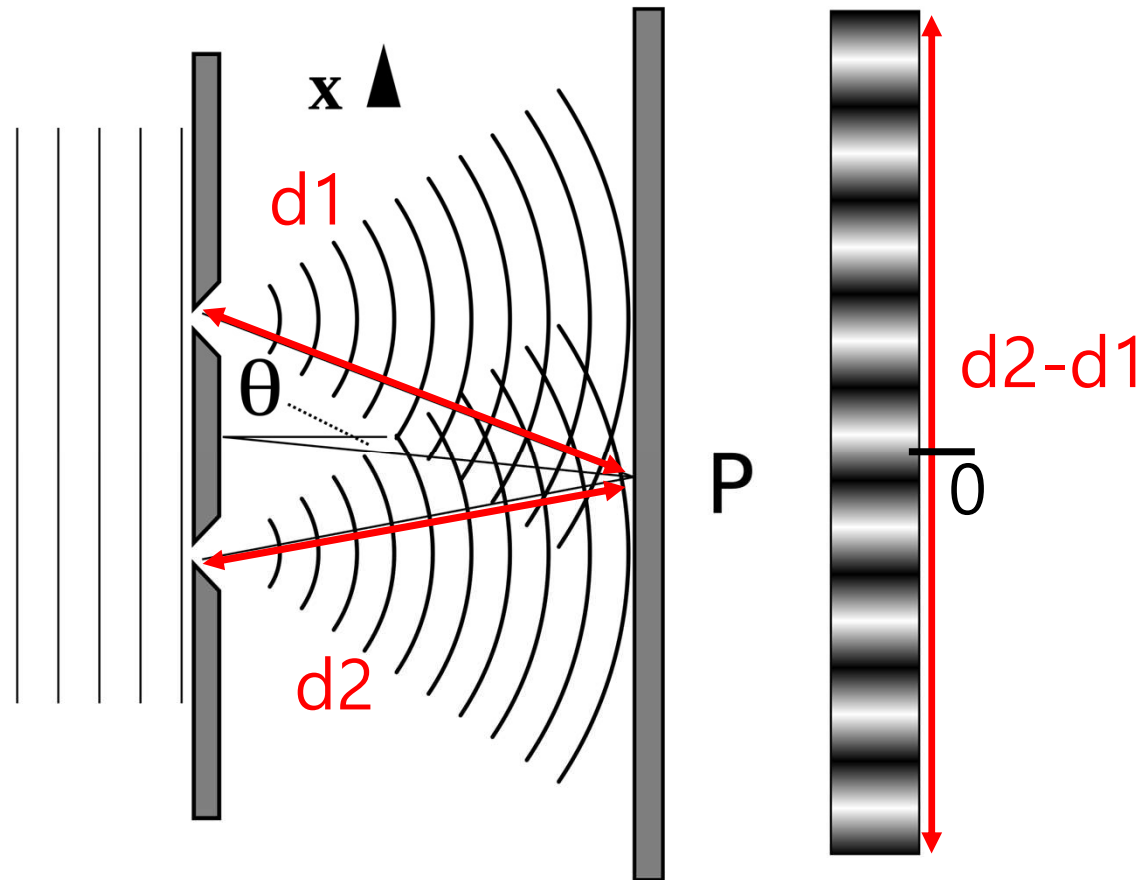
# Double slit experiment



# Interference

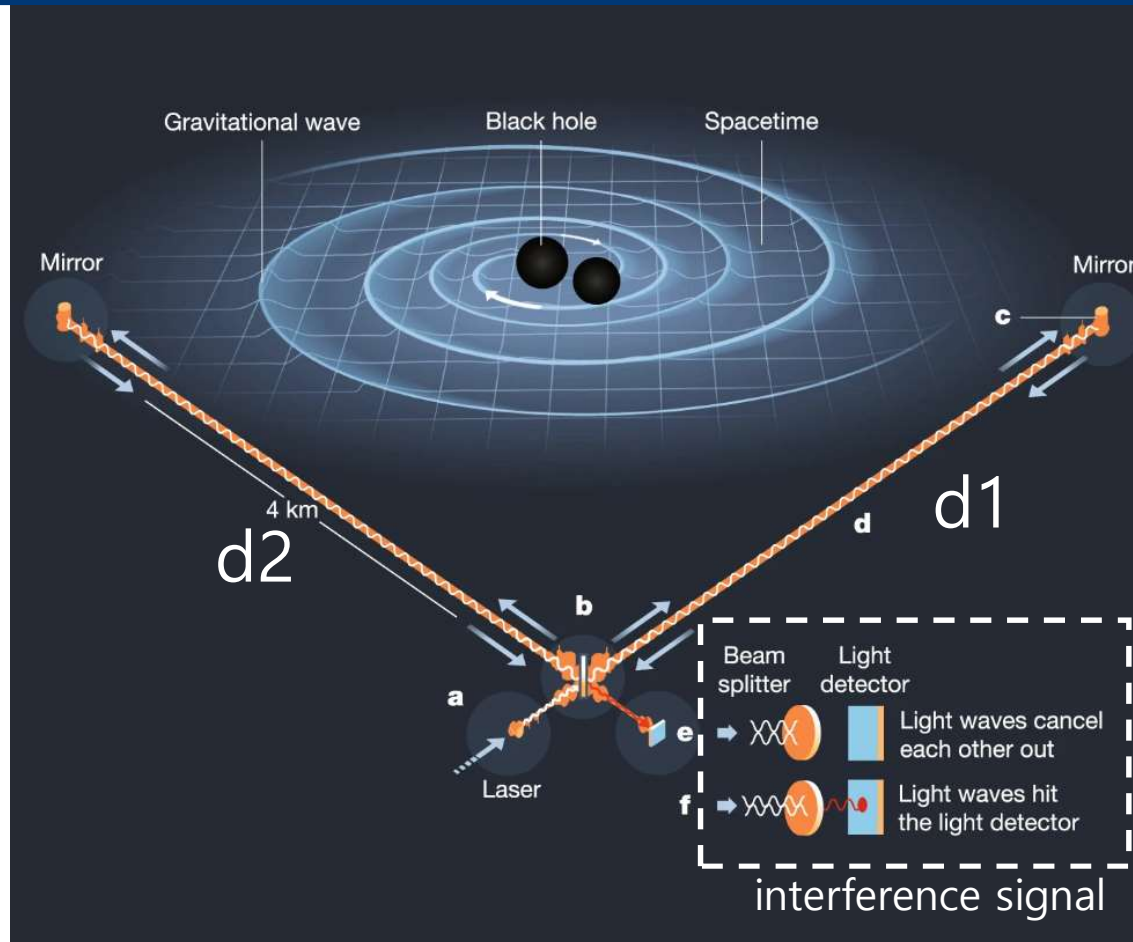


# Interference





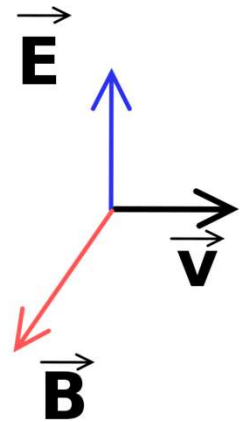
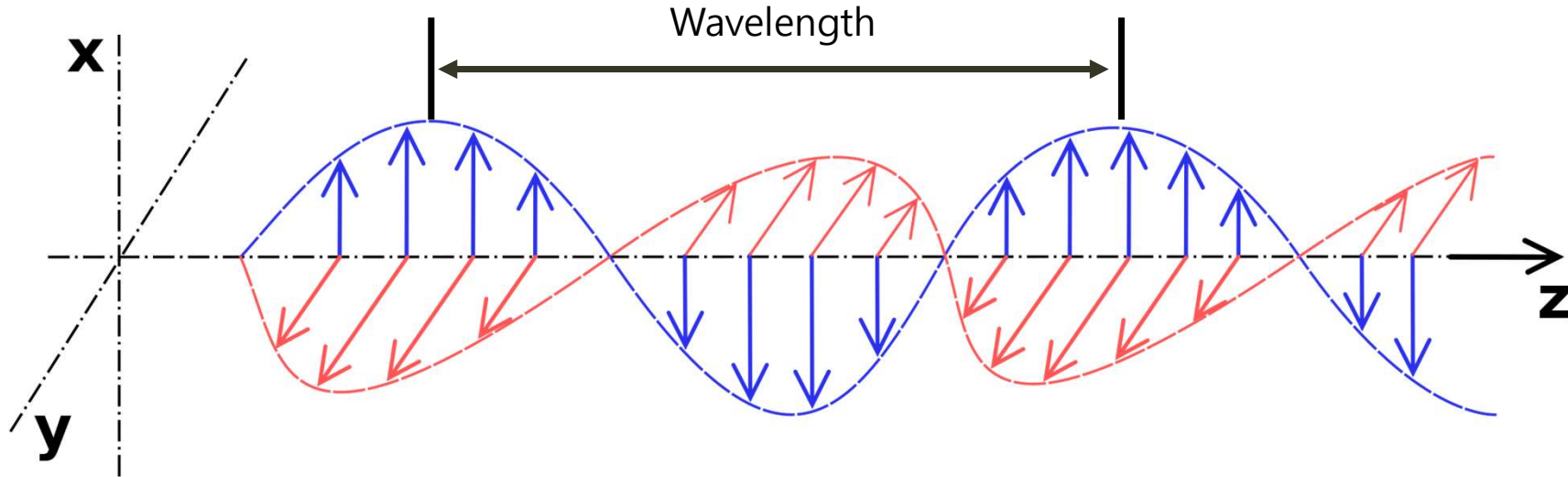
# Gravitational wave detector



- Ground scale Michelson interferometer
- 4 km vacuum tunnel arm
- Over 1000 km interaction length

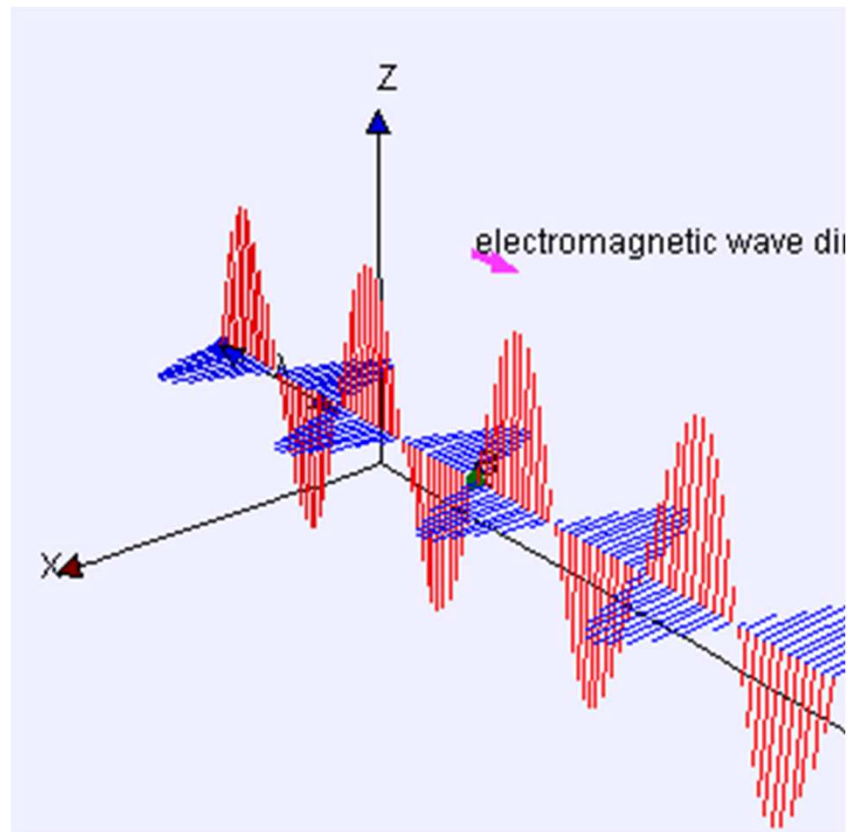
Miller, M.C., Yunes, N. The new frontier of gravitational waves. *Nature* **568**, 469–476 (2019)

# Light(Electromagnetic wave)

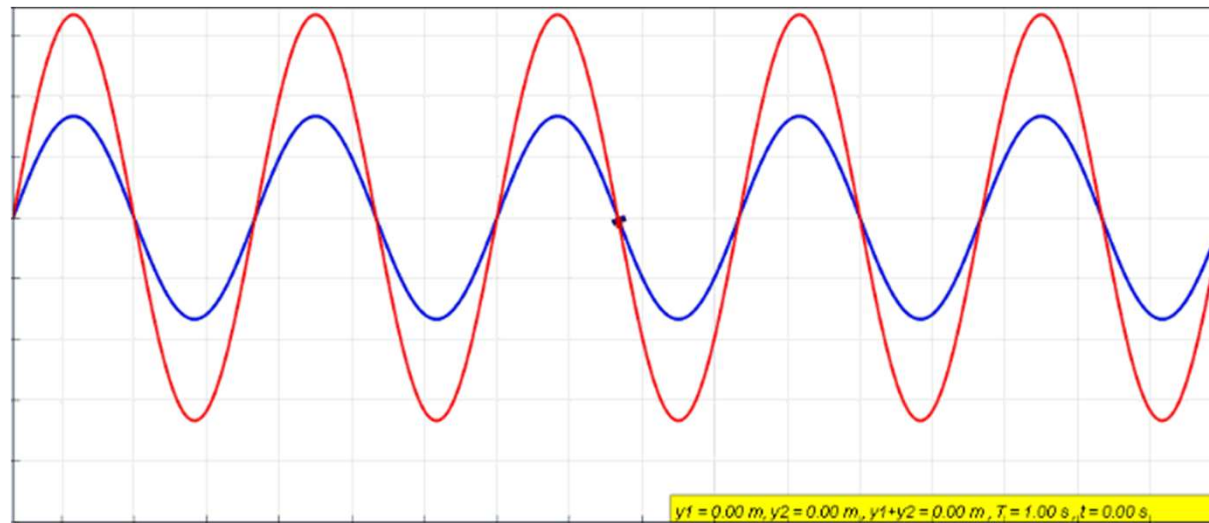


$$|\vec{E}| \propto \text{Intensity of Light}$$

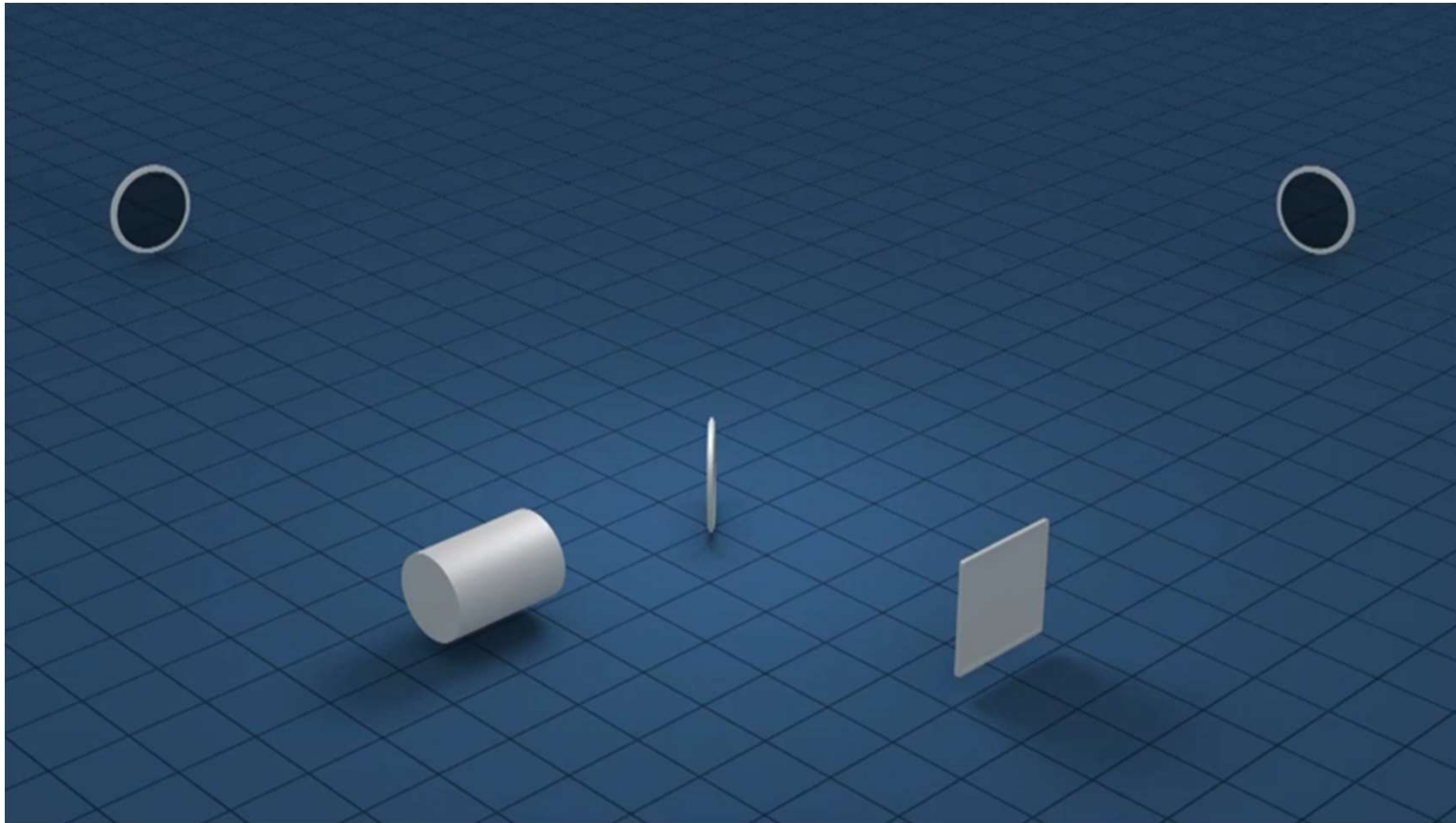
# Light(Electromagnetic wave)



# Interference

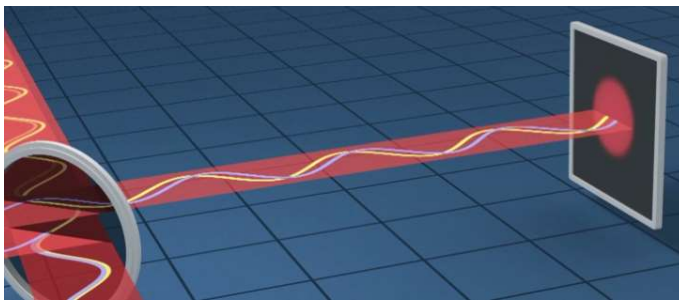
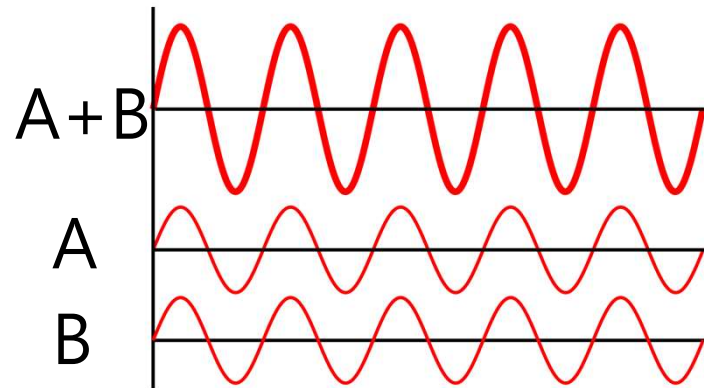


# Michelson interferometer

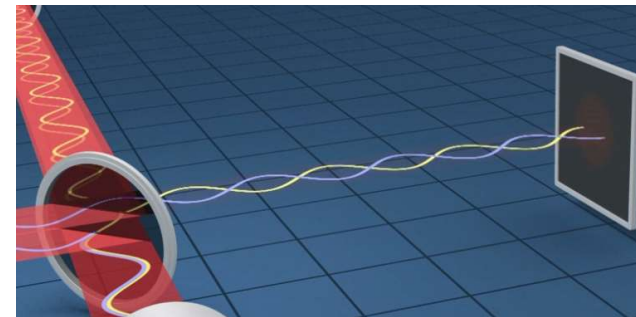
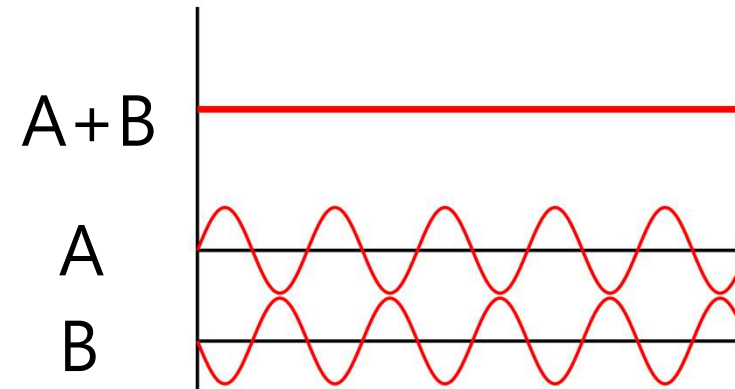




# Interference

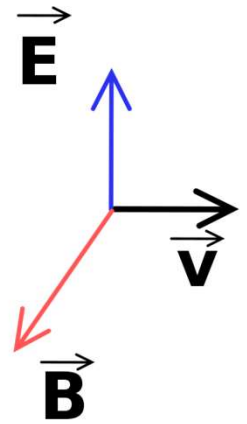
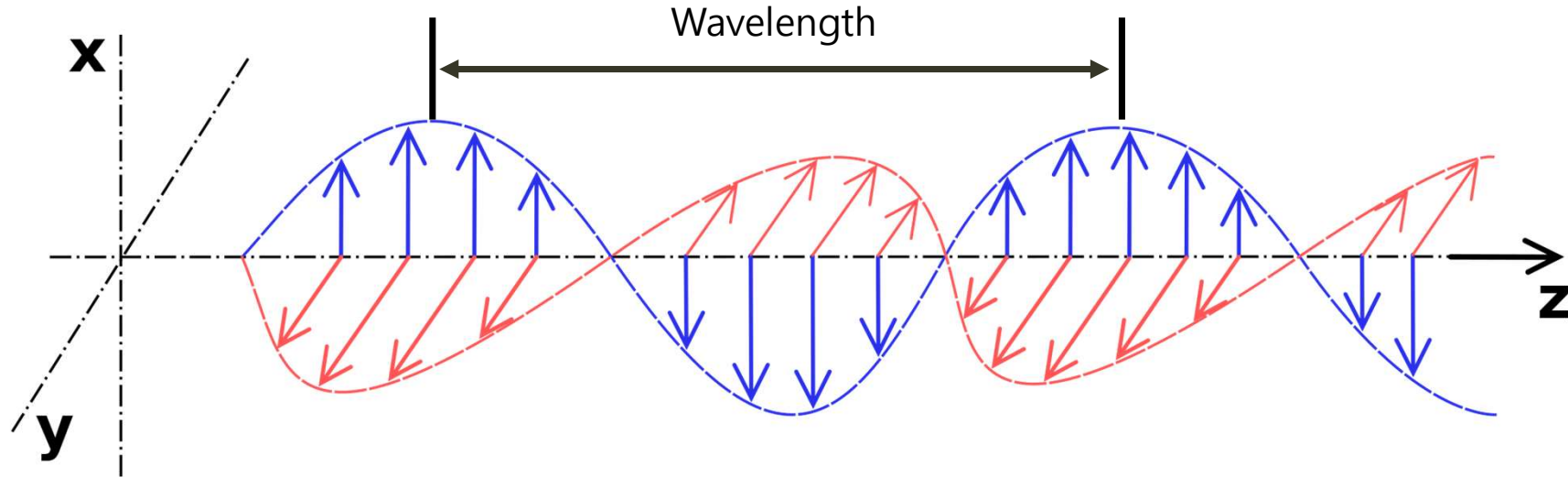


Constructive



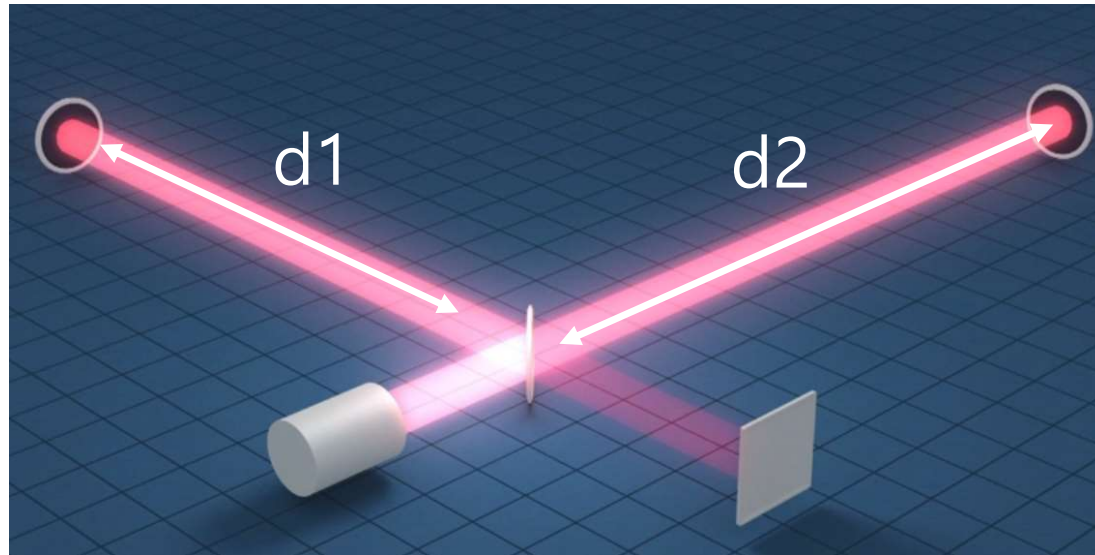
Destructive

# Light(Electromagnetic wave)



$$\vec{E} = E_0 \cos(kx - \omega t + \varphi)$$

# Interference signal

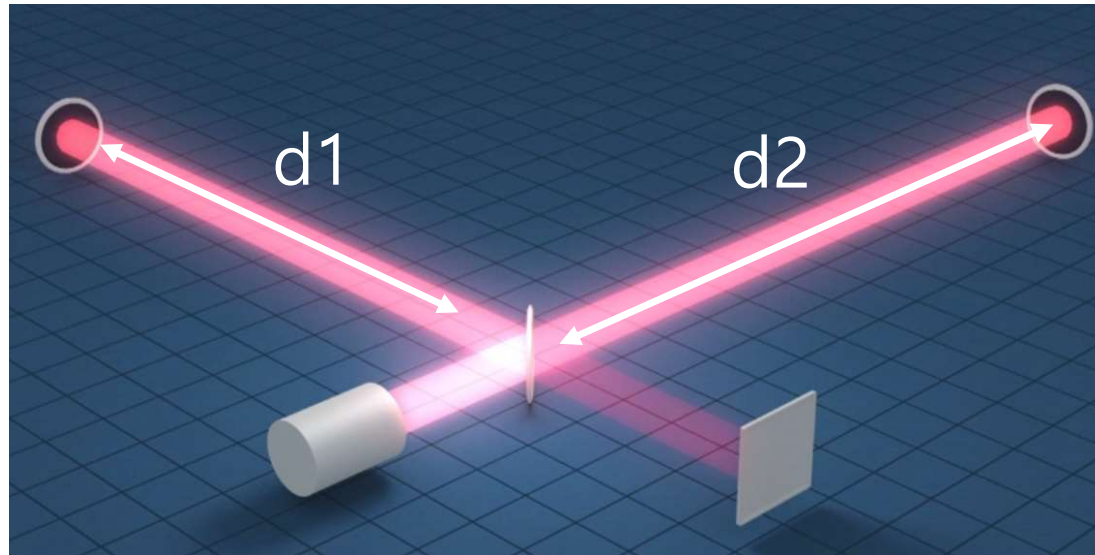


$$\vec{E} = E_0 \cos(kx - \omega t + \varphi)$$

$$\varphi \propto d1 - d2 \text{ (in some condition)}$$

$$k = \frac{2\pi}{\lambda} \quad \lambda : \text{wavelength}$$

# Interference signal

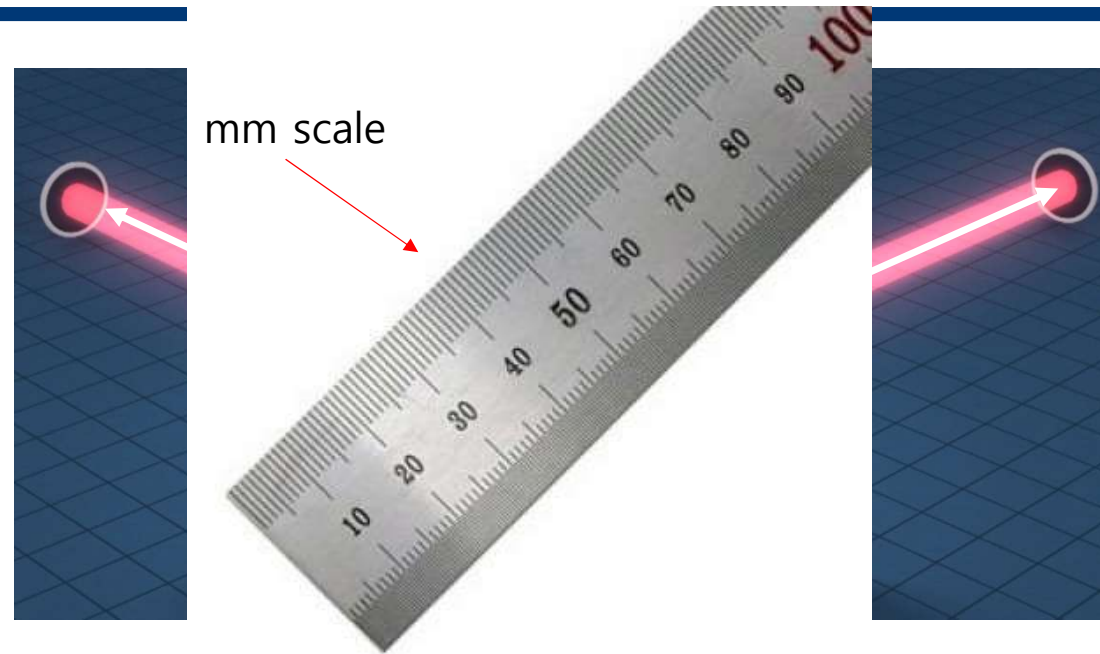


$$\vec{E} = E_0 \cos(kx - \omega t + \varphi) \quad \varphi \propto d1 - d2 \text{ (in some condition)}$$

$$k = \frac{2\pi}{\lambda} \quad \lambda : 0.1\mu m \sim 10\mu m$$

wavelength

# Interference signal

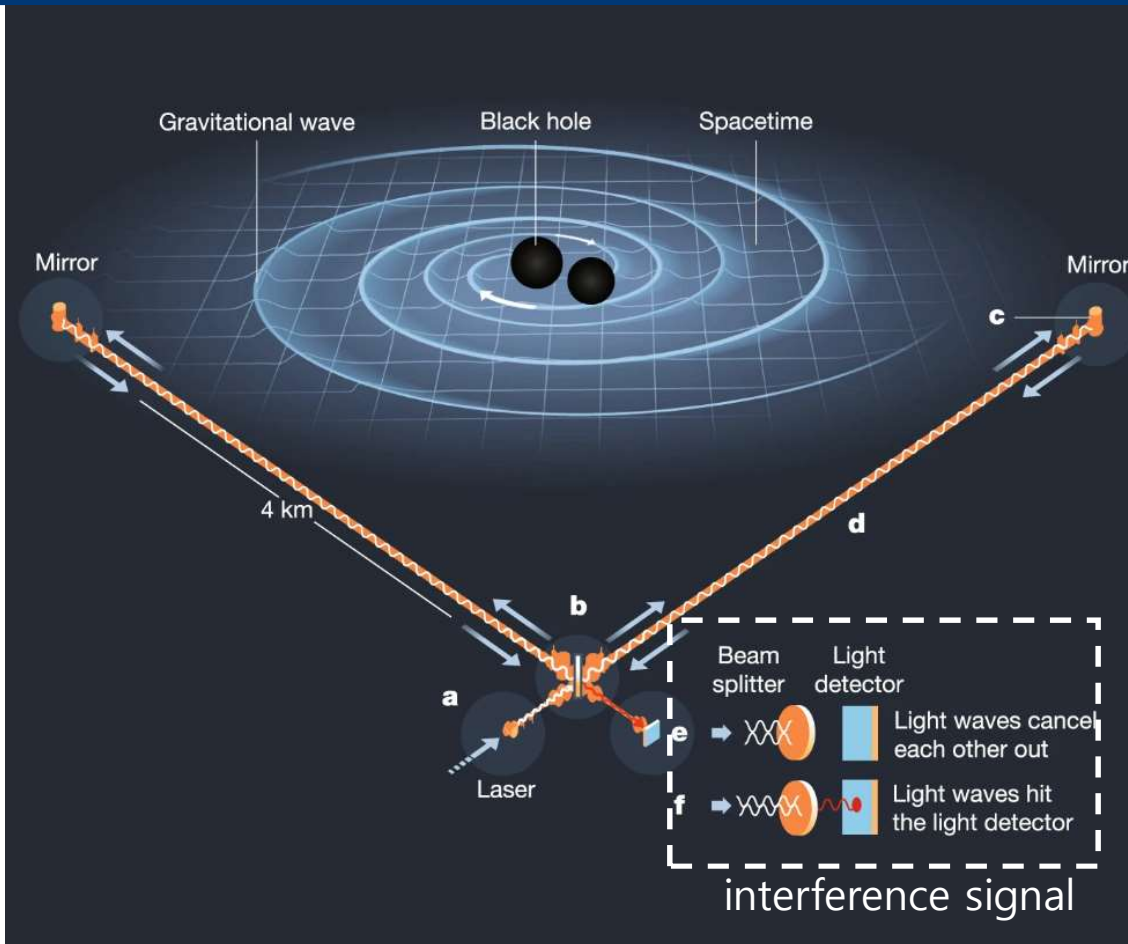


$$\vec{E} = E_0 \cos(kx - \omega t + \varphi) \quad \varphi \propto d_1 - d_2 \text{ (in some condition)}$$

$$k = \frac{2\pi}{\lambda} \quad \lambda : \text{wavelength} \quad 0.1\mu\text{m} \sim 10\mu\text{m}$$



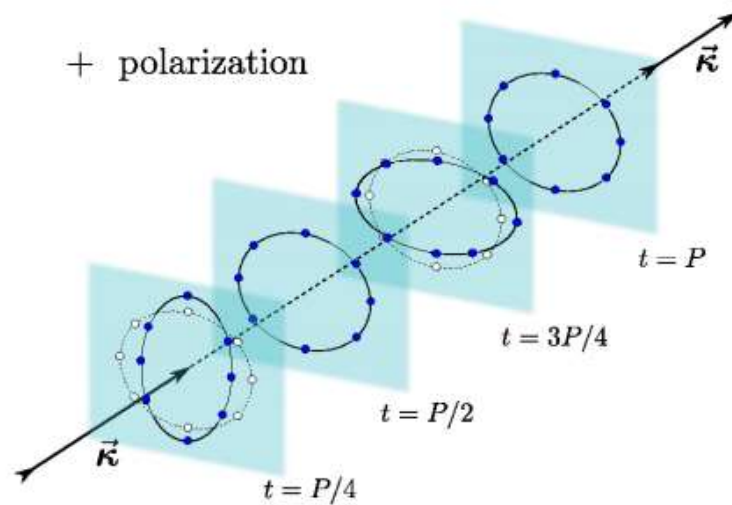
# Gravitational wave detector



- Ground scale Michelson interferometer
- 4 km vacuum tunnel arm
- Over 1000 km interaction length

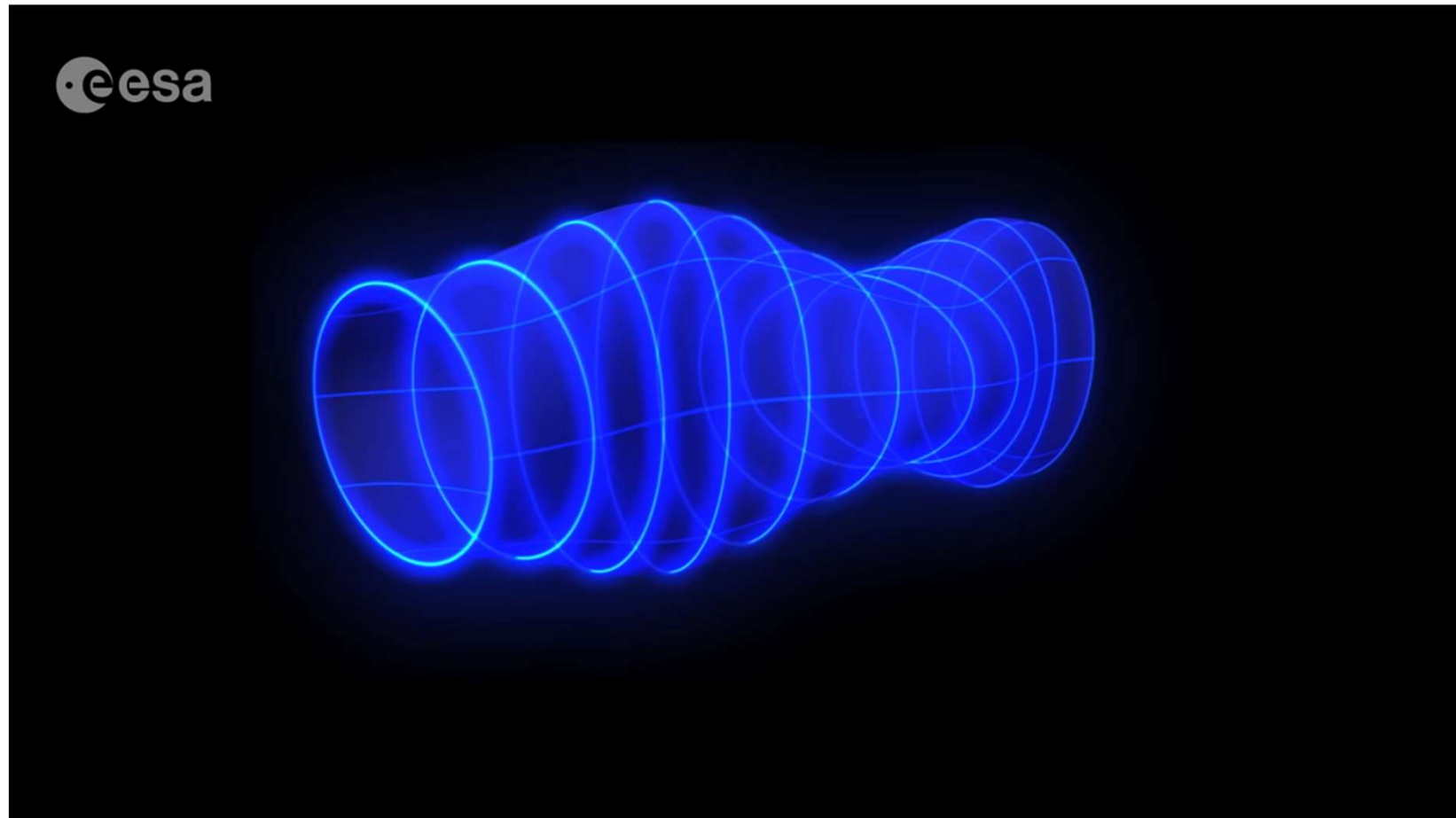
Build most sensitive system  
using most simple arrangement

# Gravitational wave

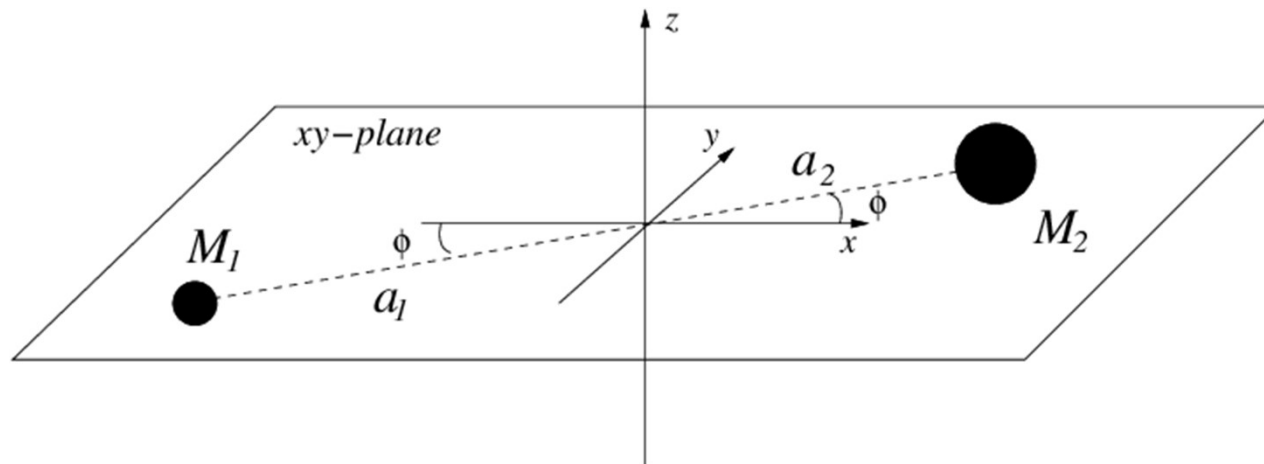


MPA Lectures on Gravitational Waves in  
Cosmology  
Azadeh Maleknejad  
Max-Planck-Institute for Astrophysics

# Gravitational wave



# Compact binary system



Andersson, Nils & Kokkotas, Kostas. (2004).  
Gravitational-wave astronomy: the high-frequency window.

Mass dipole : Center of mass

$$h = \frac{2G}{c^4} \frac{1}{r} \frac{\partial^2 Q}{\partial t^2}$$

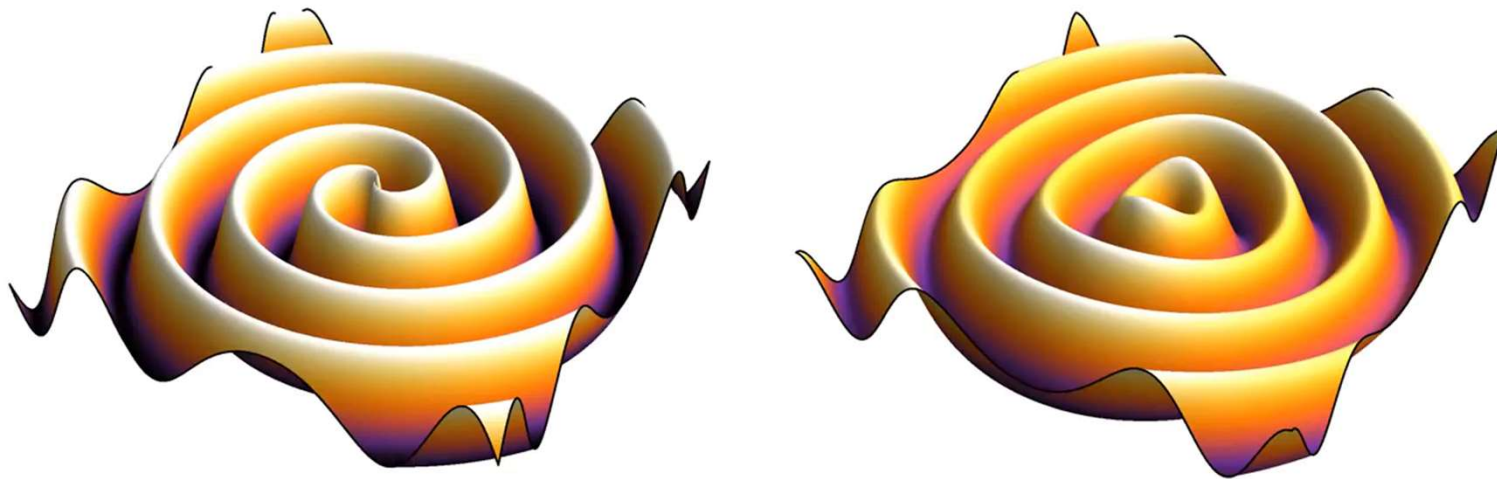
Amplitude of gravitational wave

Conserved

First derivative is momentum

Conserved

# Gravitational wave quadrupole radiation





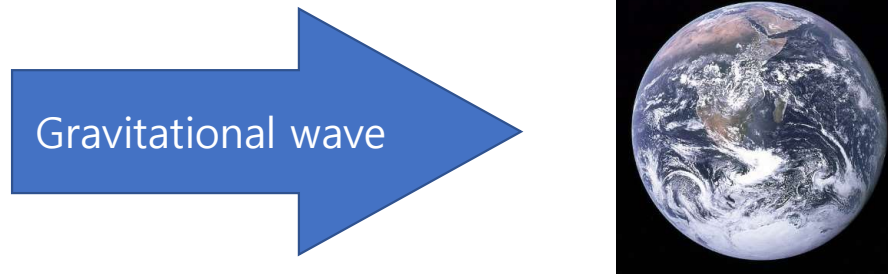
# Effect of gravitational wave



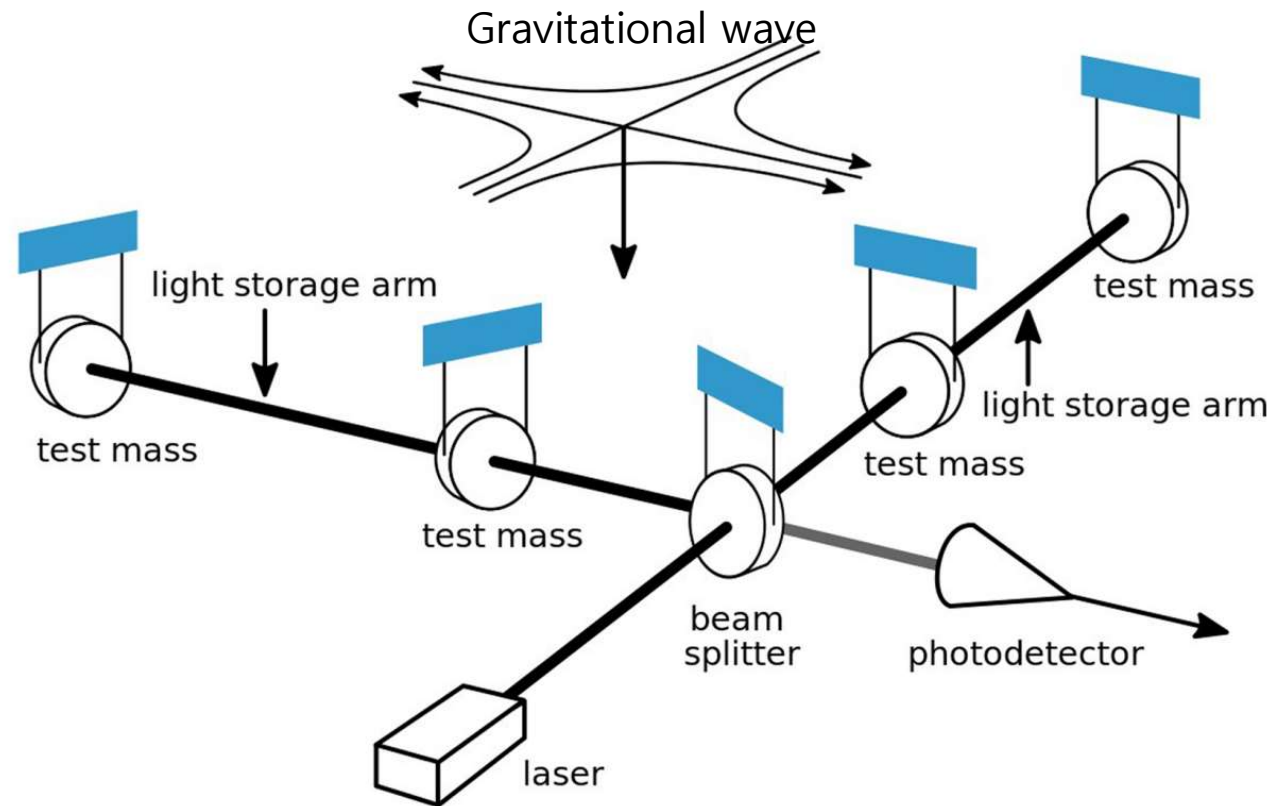
Gravitational wave



# Effect of gravitational wave

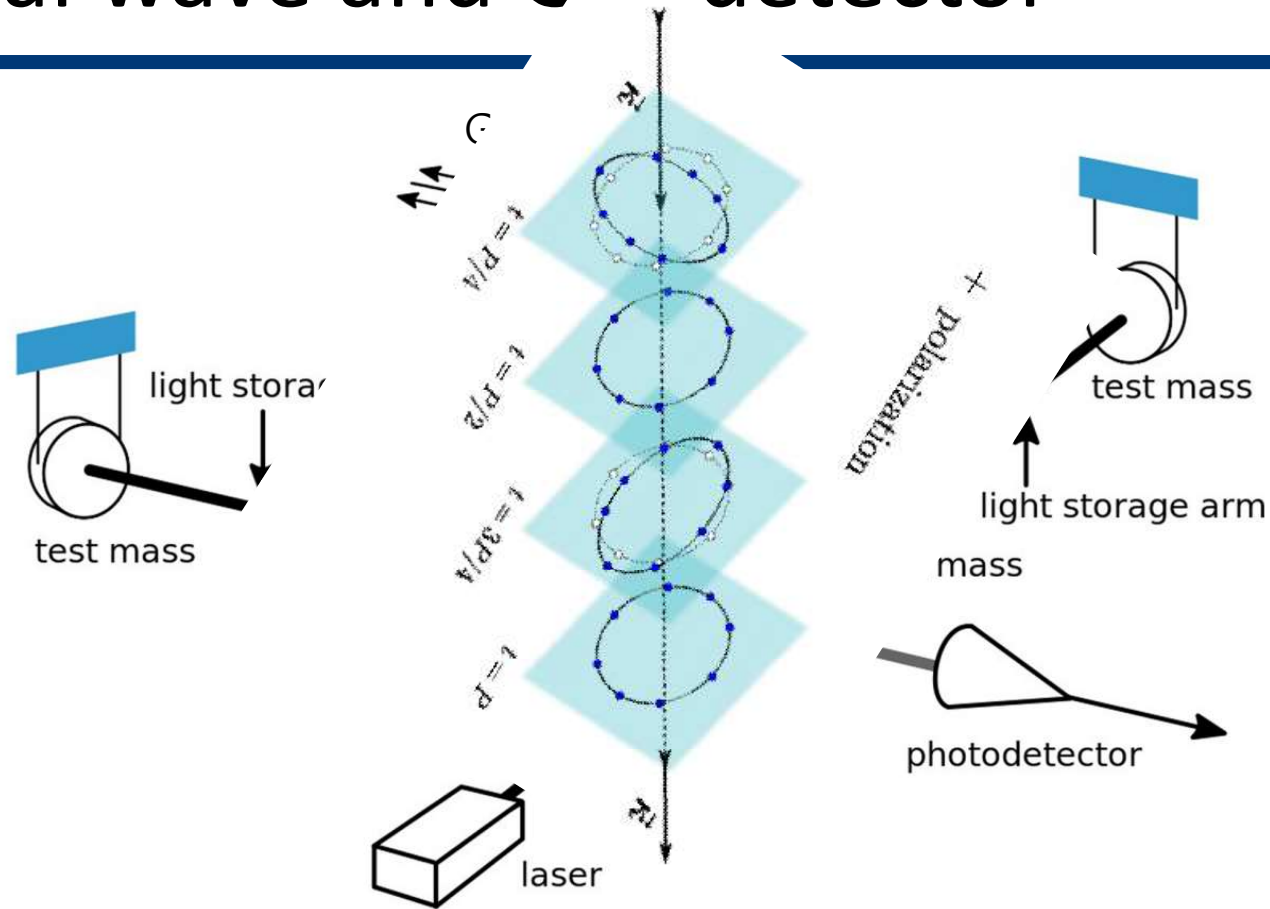


# Gravitational wave and GW detector



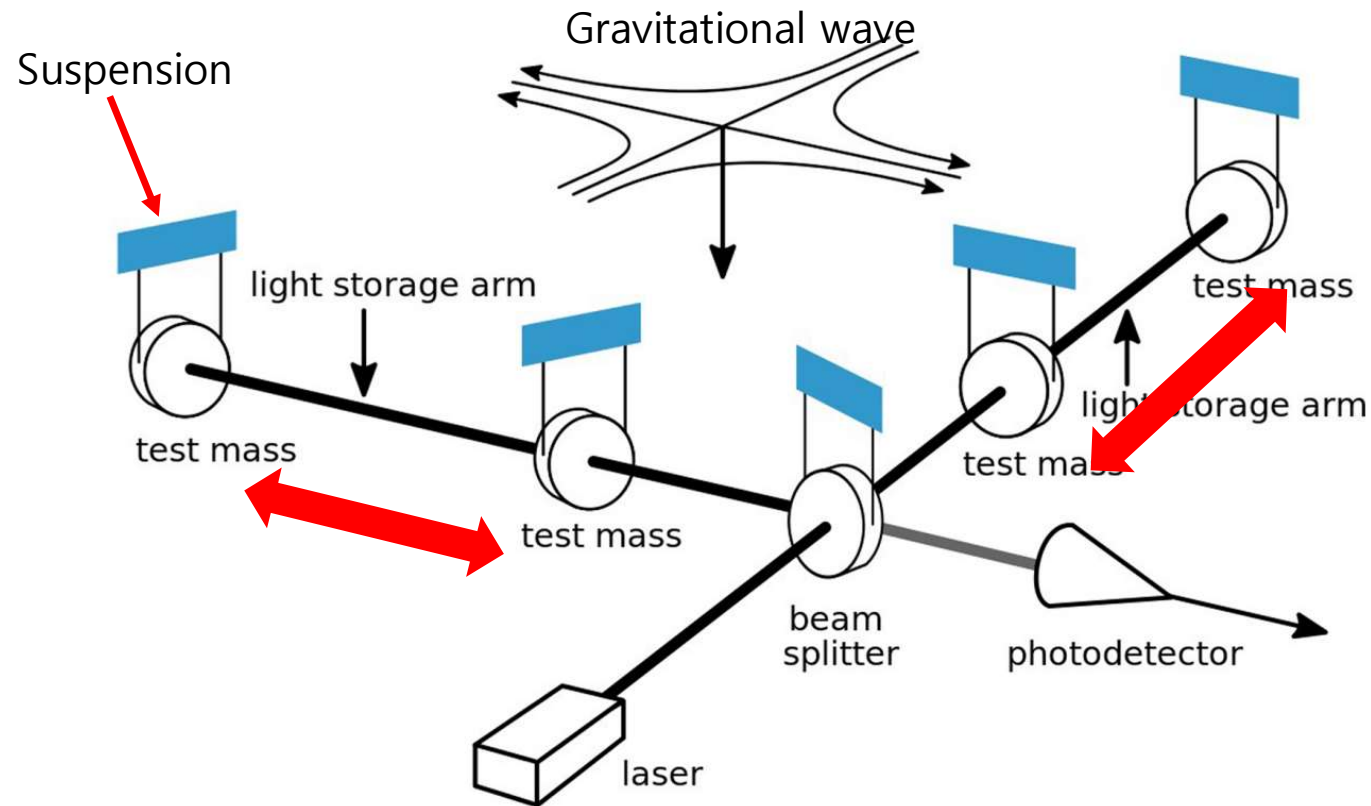
<https://www.ligo.caltech.edu/>

# Gravitational wave and G<sup>o</sup> detector



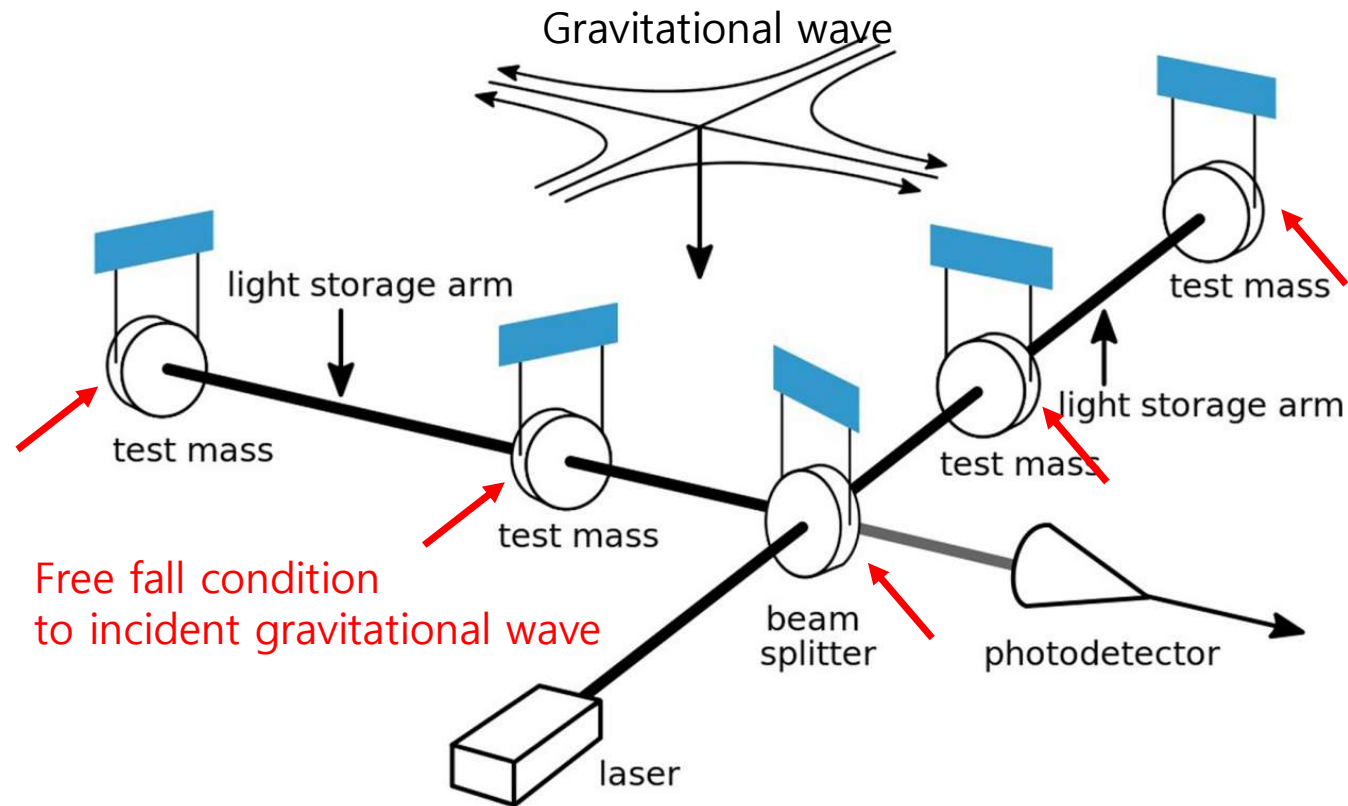
<https://www.ligo.caltech.edu/>

# Gravitational wave and GW detector



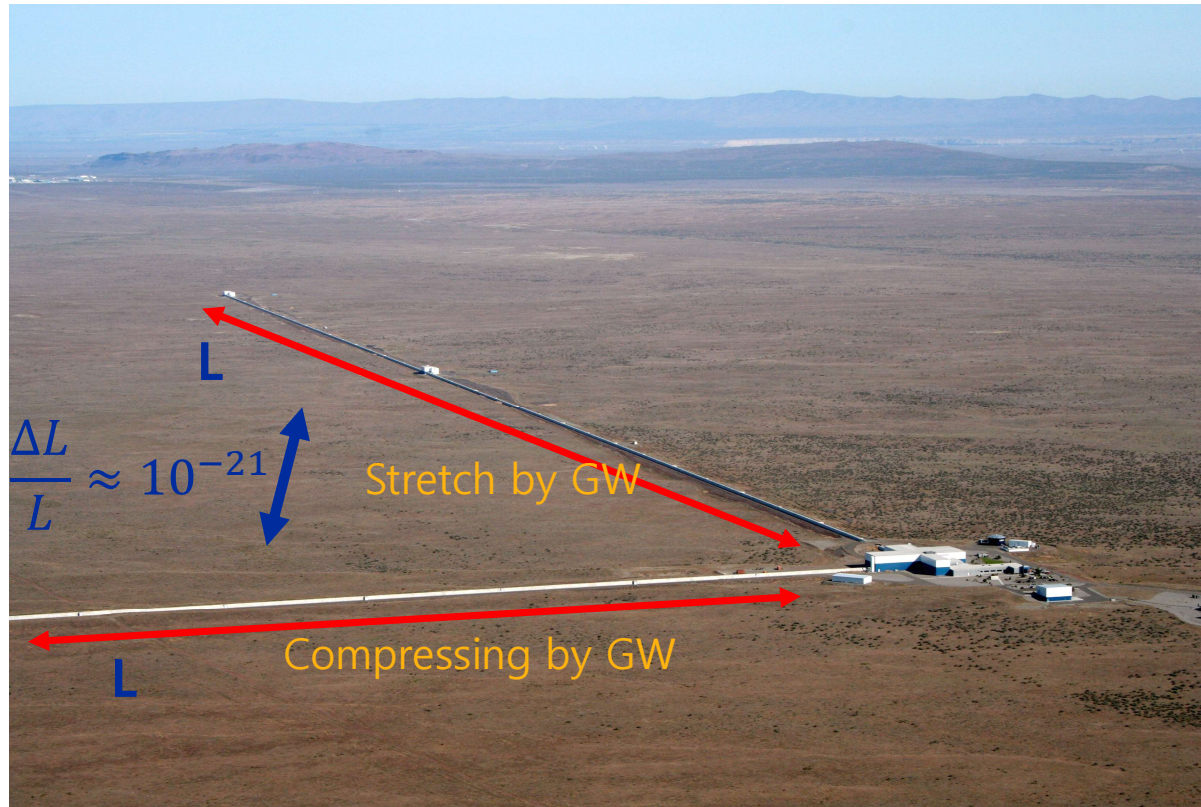
<https://www.ligo.caltech.edu/>

# Gravitational wave and GW detector





# Strain sensitivity

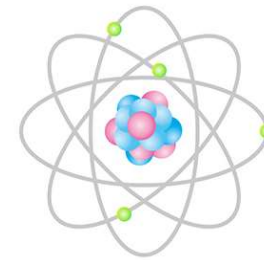
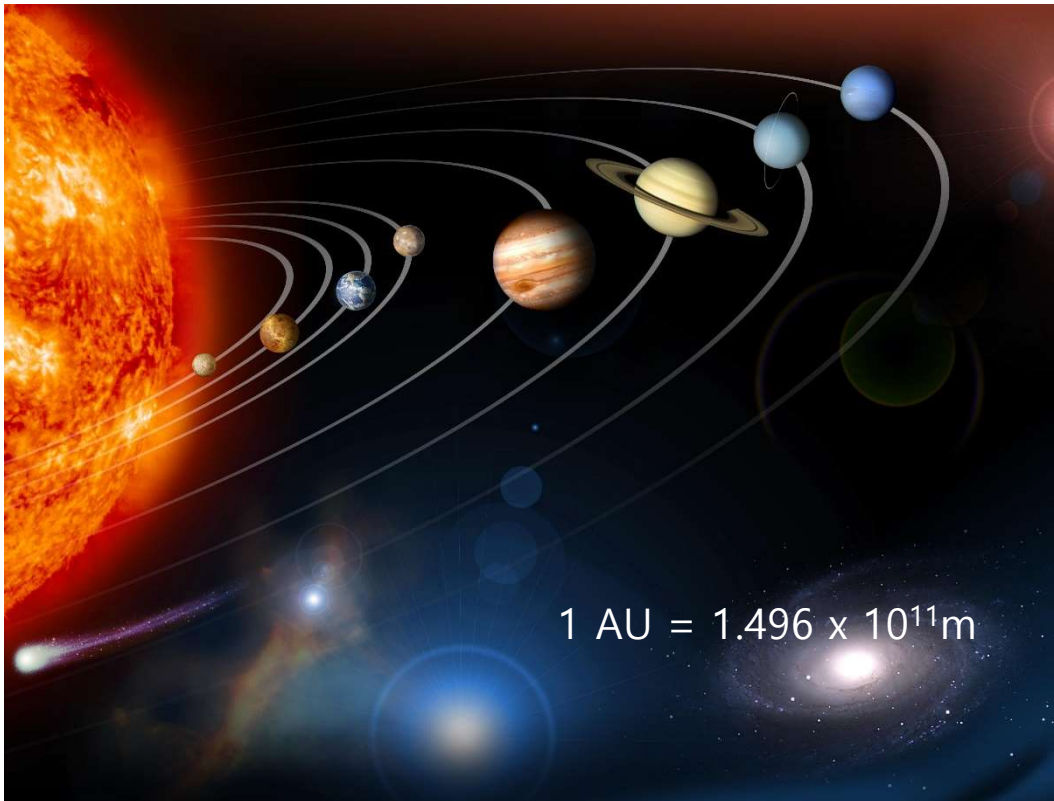


Minimum  
sensitivity

$$\frac{\Delta L}{L} \approx 10^{-21}$$



# Strain due to gravitational wave

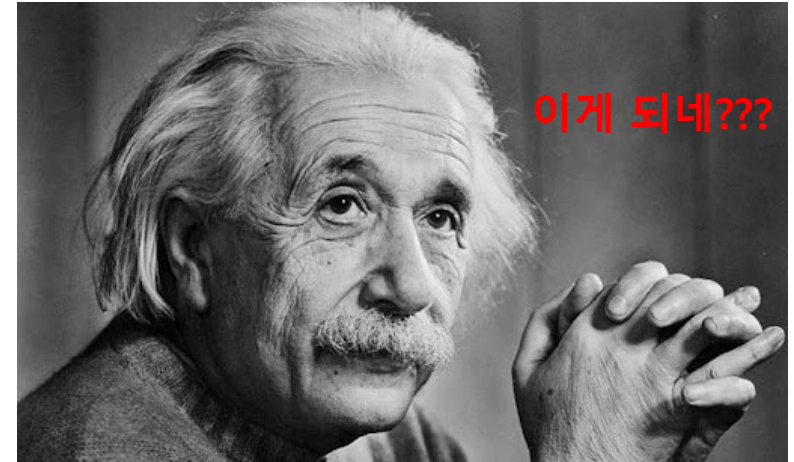
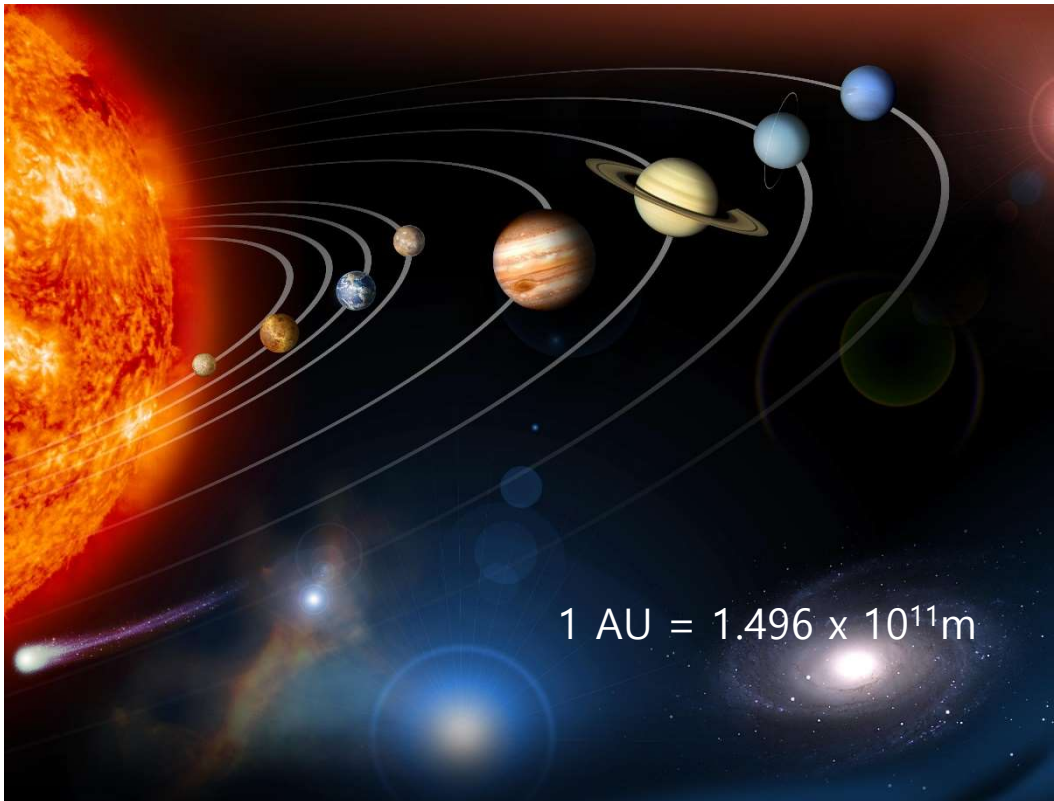


Size of atom =  $1 \times 10^{-10}$  m

$$\frac{\Delta L}{L} \approx 10^{-21}$$

Detect existence of a single atom

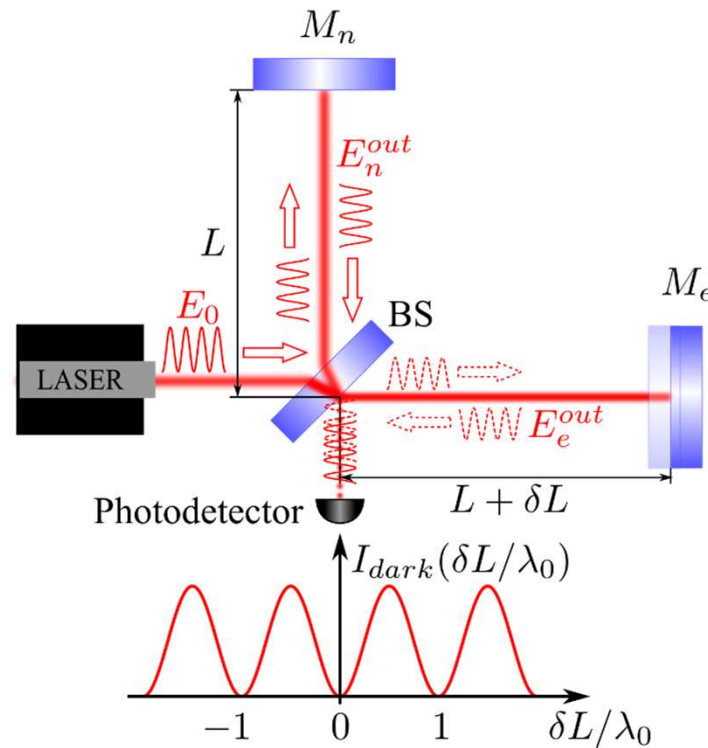
# Strain due to gravitational wave



$$\frac{\Delta L}{L} \approx 10^{-21}$$

Detect existence of a single atom

# Sensitivity of michelson interferometer

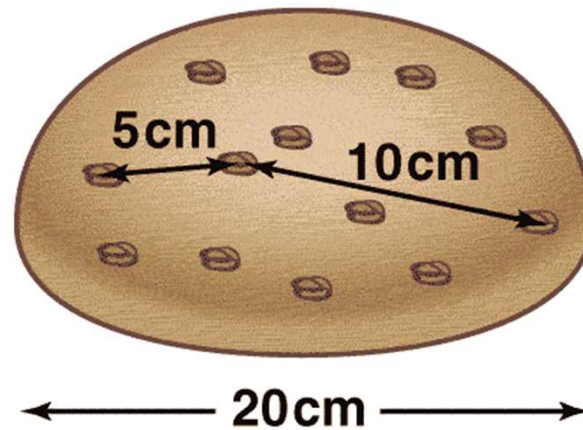


When  $L = 1 \text{ m}$

$$\frac{\Delta L}{L} \approx 10^{-16}$$

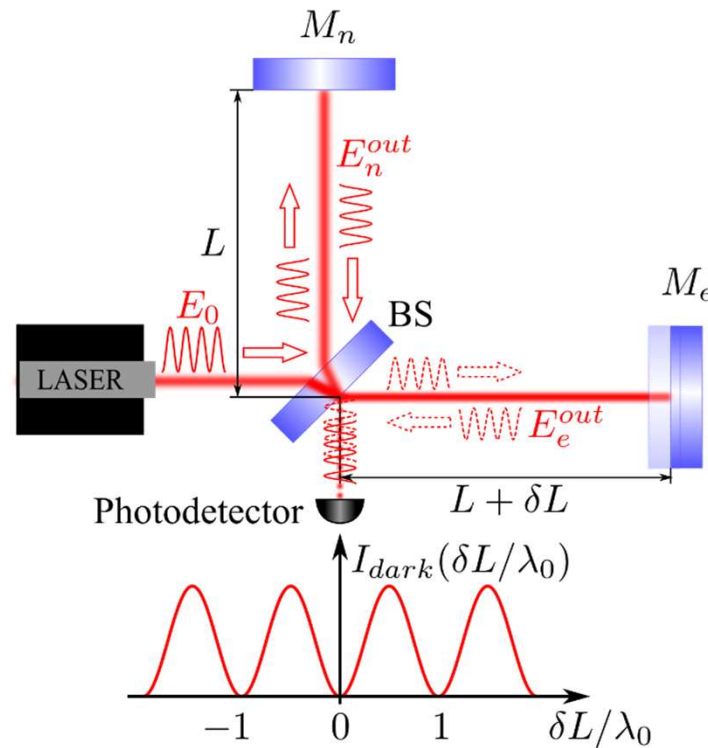
Danilishin, Stefan L. et al. Living Rev.Rel. 15 (2012) 5  
arXiv:1203.1706

# 길이팽창



MAP990404

# Sensitivity of michelson interferometer



When  $L = 1 \text{ m}$

$$\frac{\Delta L}{L} \approx 10^{-16}$$

IF  $L = 1000 \text{ km}$

$$\frac{\Delta L}{L} \approx 10^{-21}$$

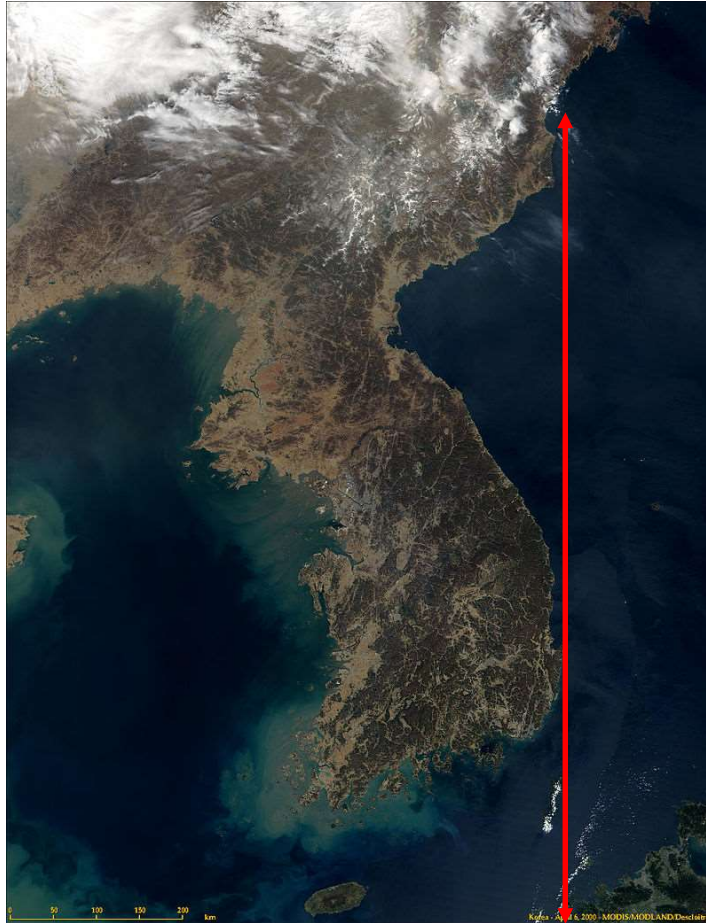
오 1000 km 간섭계를 만들면 되겠다

Danilishin, Stefan L. et al. Living Rev.Rel. 15 (2012) 5  
arXiv:1203.1706





# 1000 km interferometer



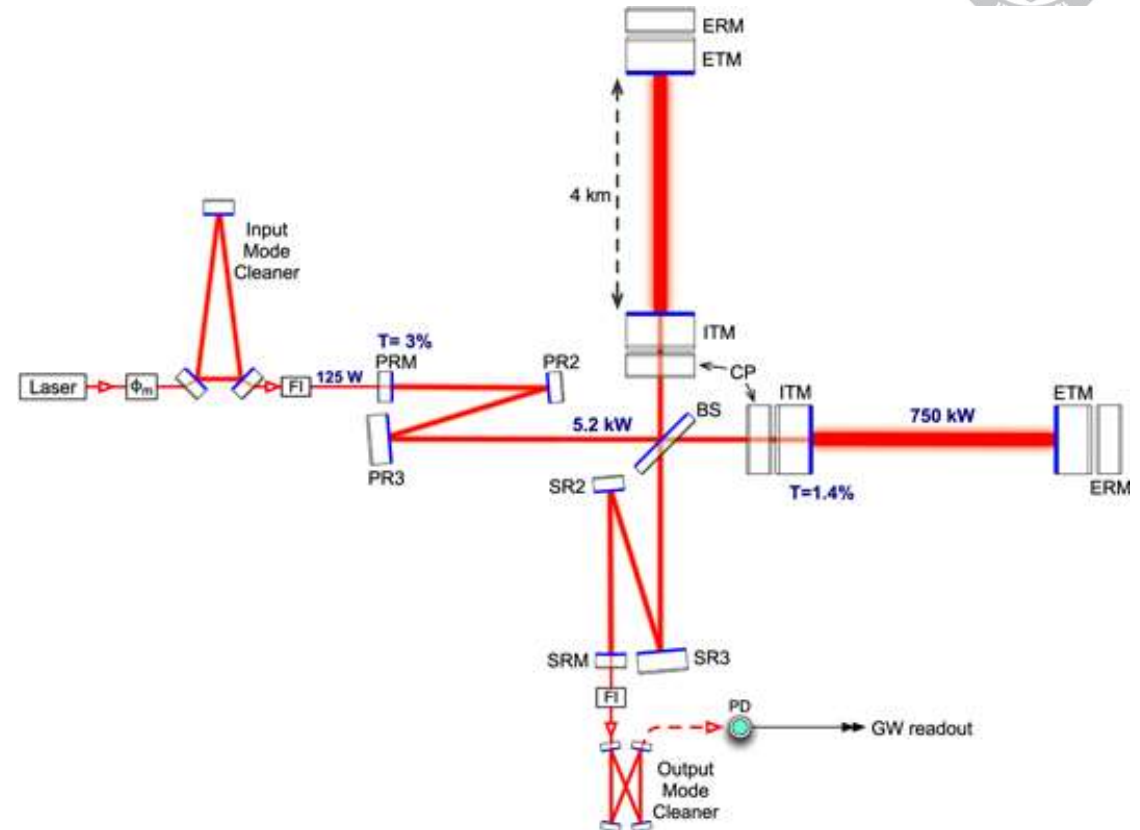
~1100km





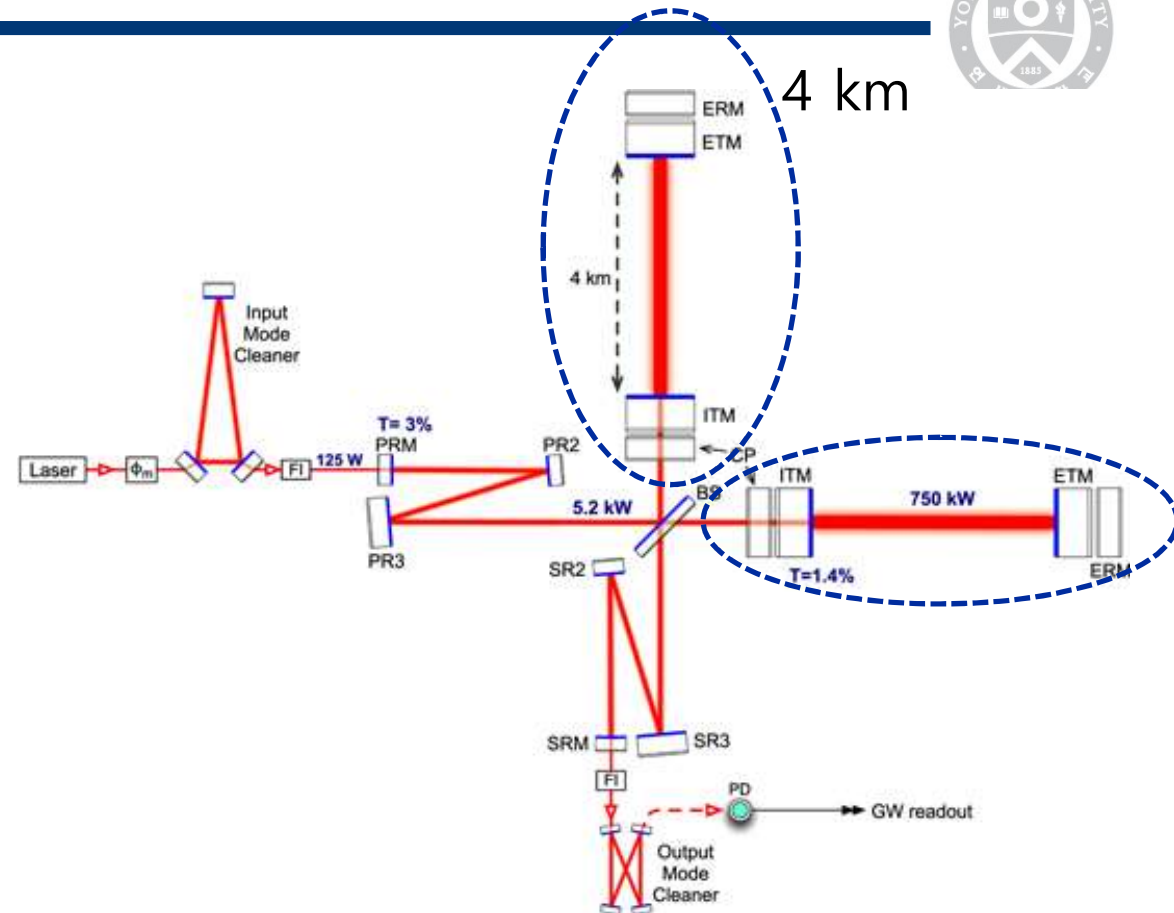


# LIGO interferometer



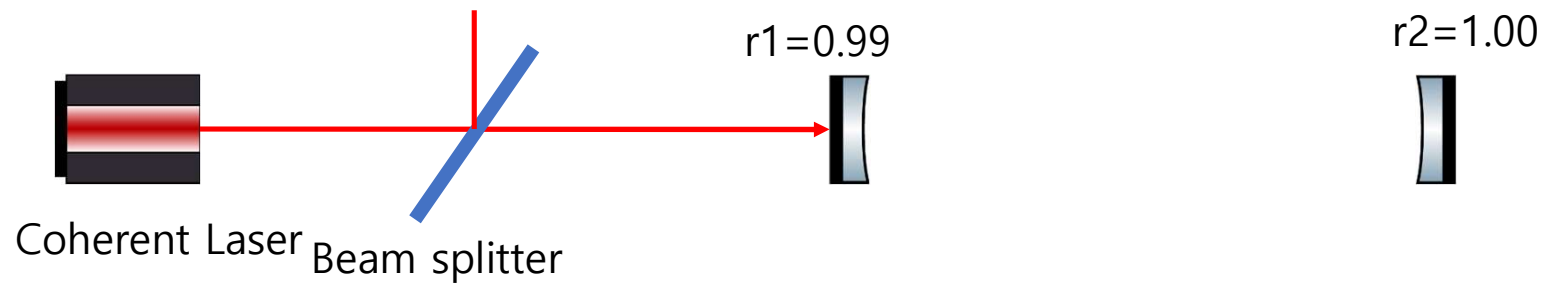
LIGO interferometer / Livingston

# LIGO interferometer

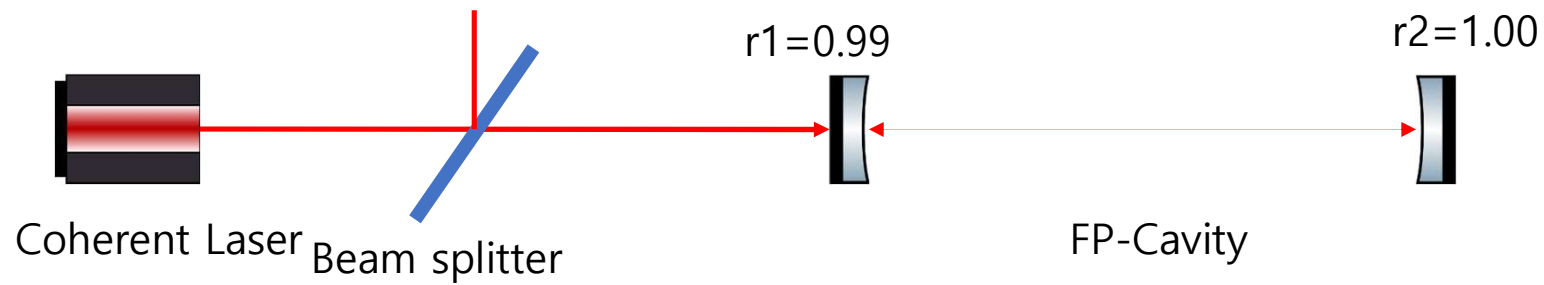


LIGO interferometer / Livingston

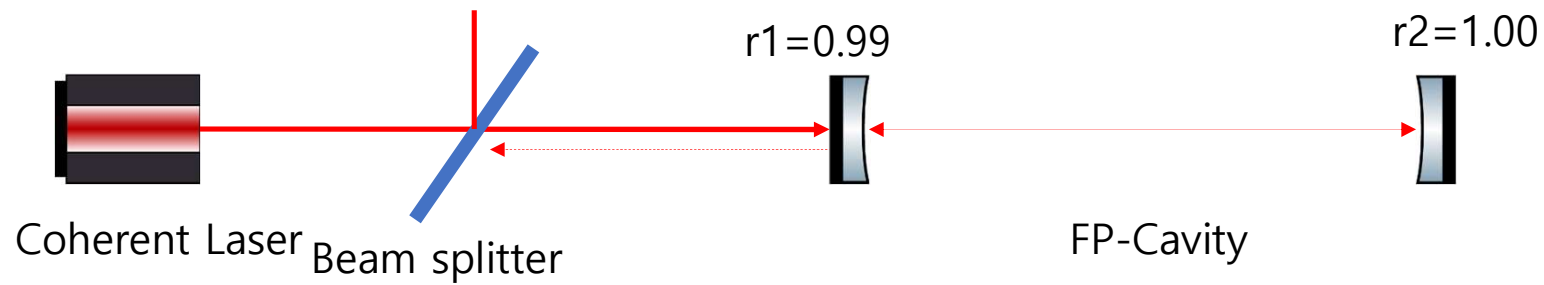
# Fabry-perot cavity



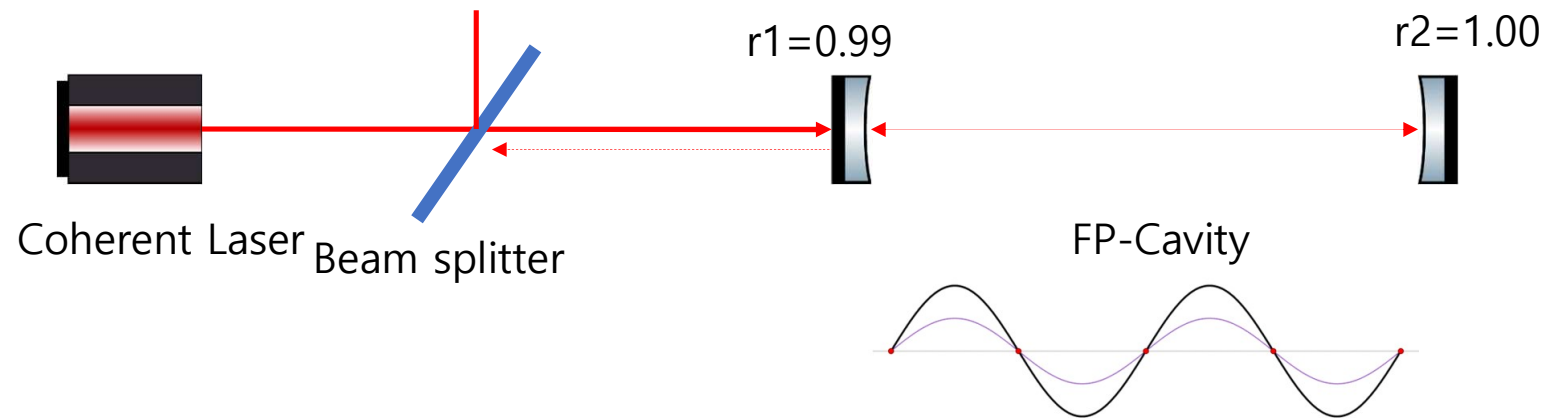
# Fabry-perot cavity



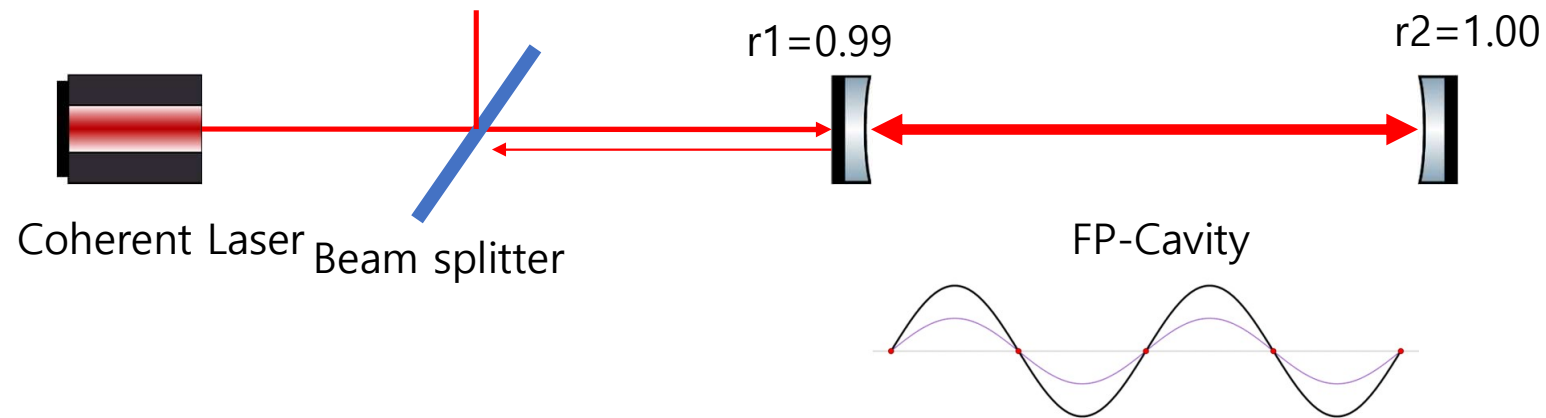
# Fabry-perot cavity



# Fabry-perot cavity

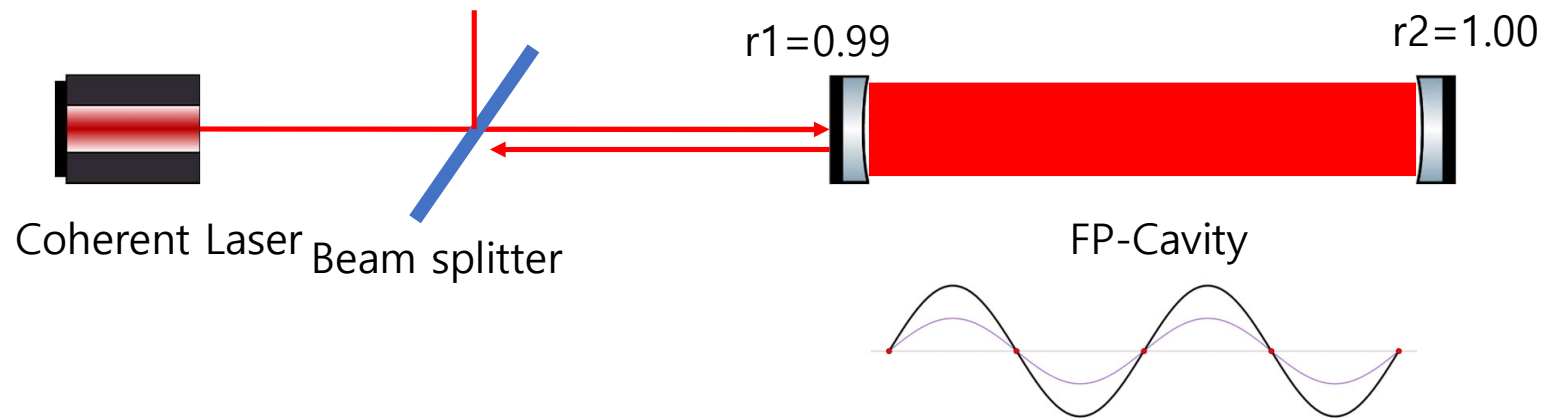


# Fabry-perot cavity





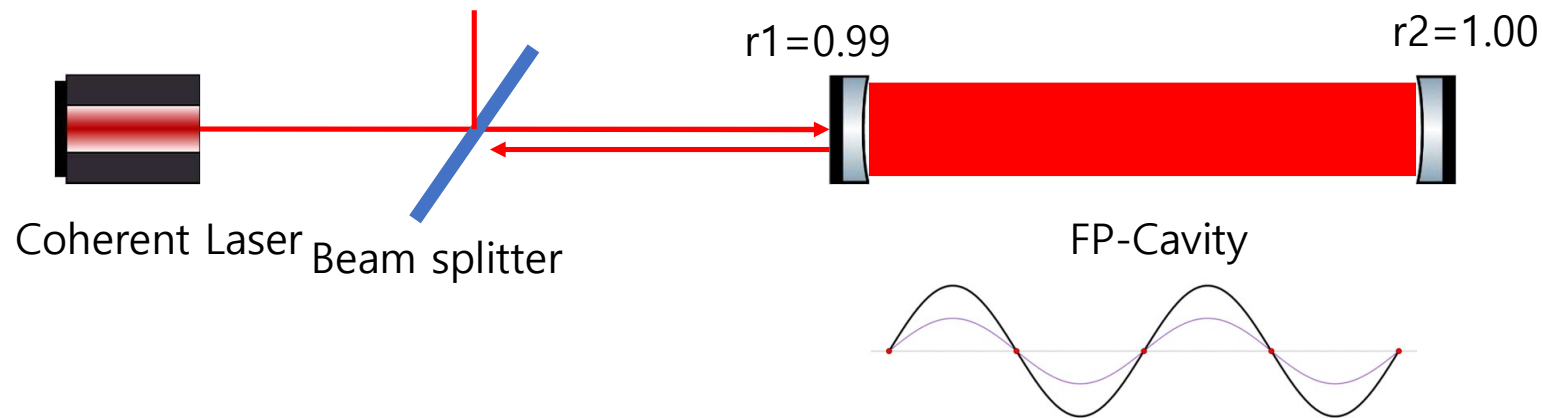
# Fabry-perot cavity



# Fabry-perot cavity



Number of round trip  $> 250$

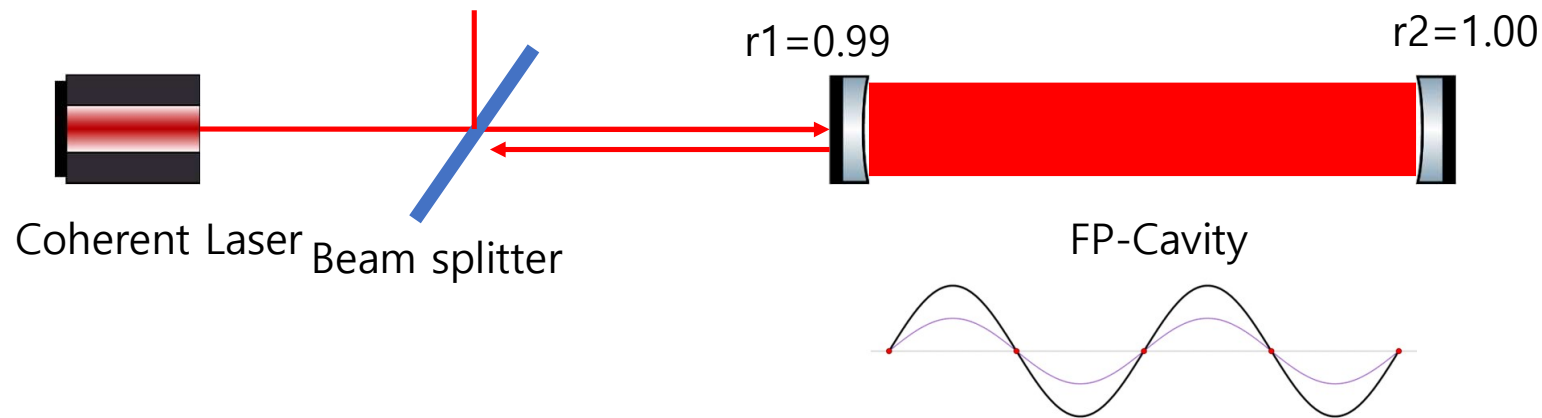


# Fabry-perot cavity

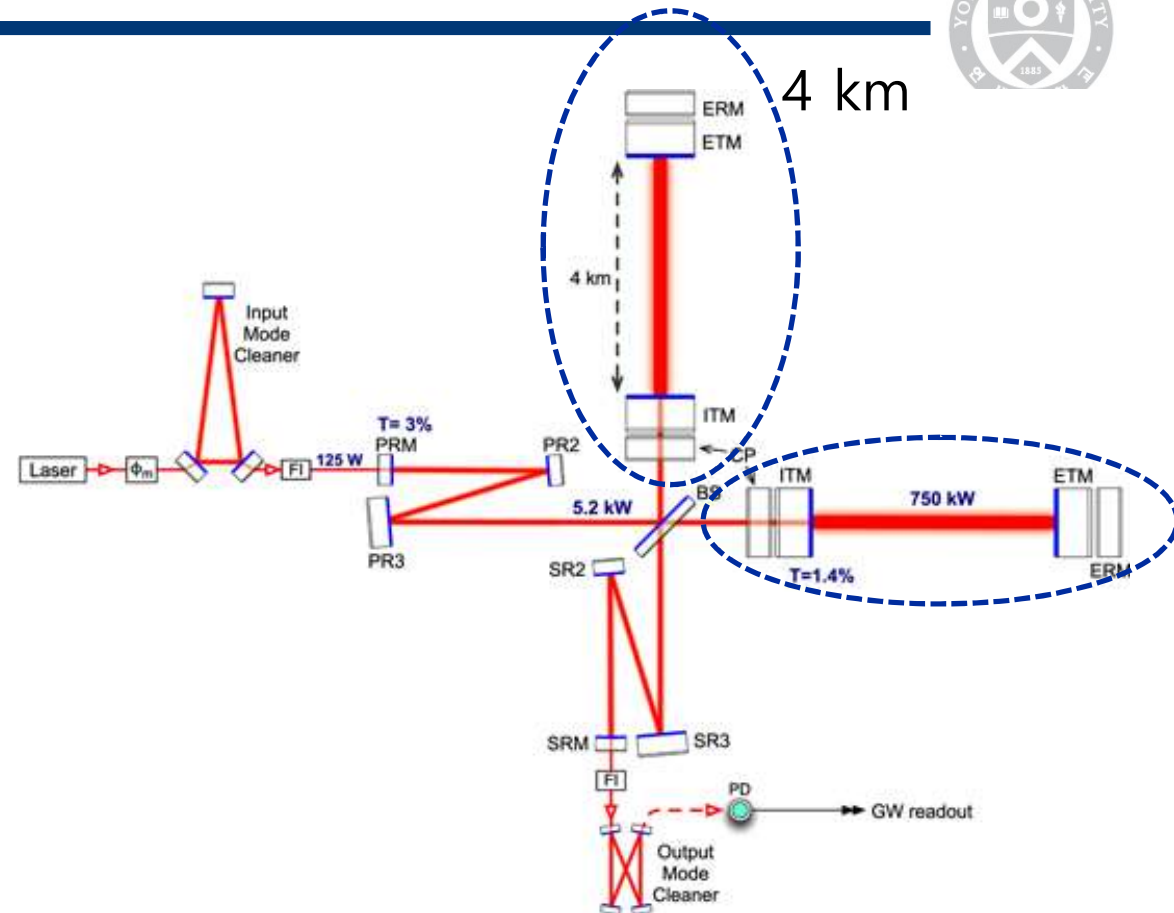


Number of round trip  $> 250$

4 km x 250 ~ 1000 km

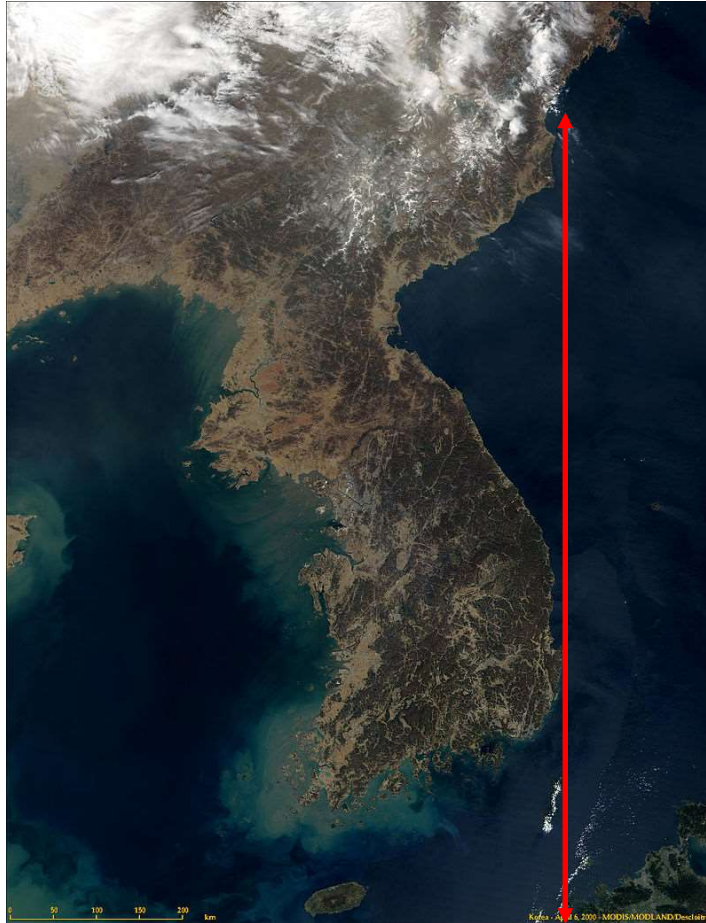


# LIGO interferometer



LIGO interferometer / Livingston

# 1000 km interferometer

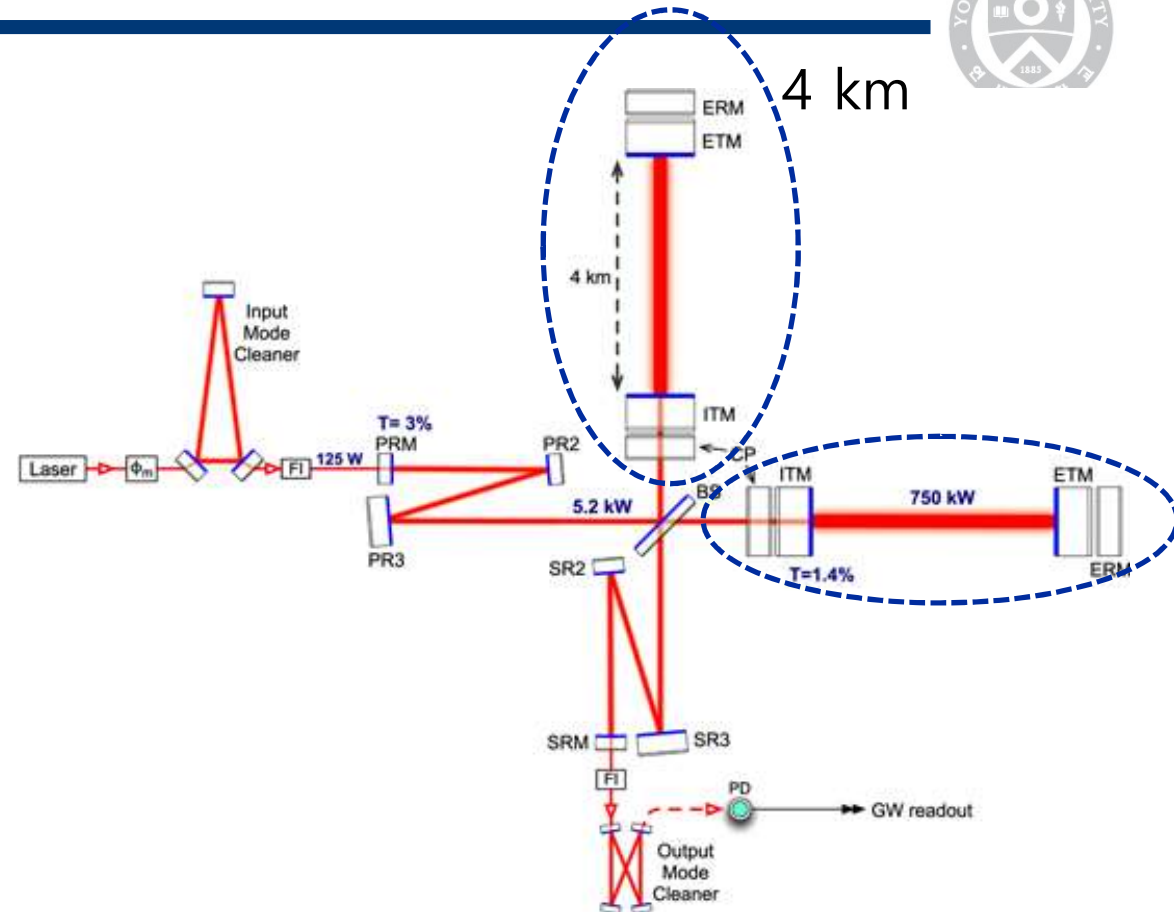


~1100km



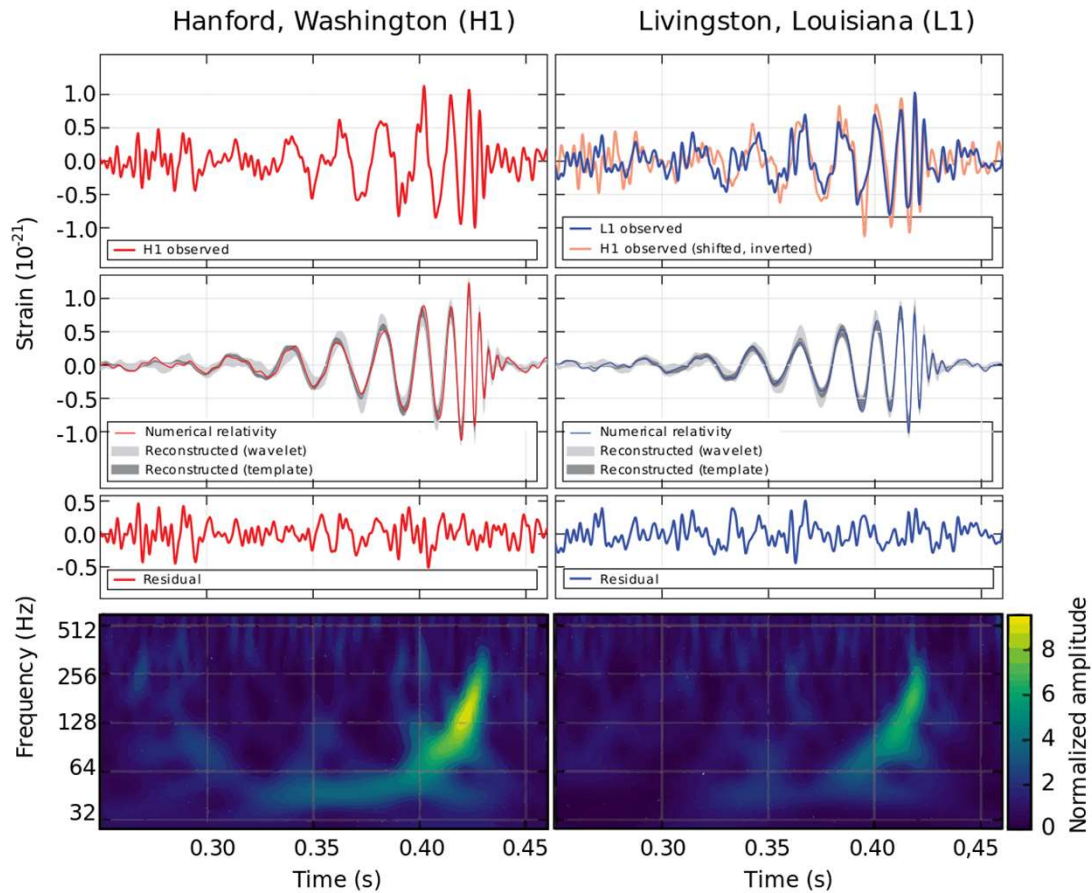


# LIGO interferometer



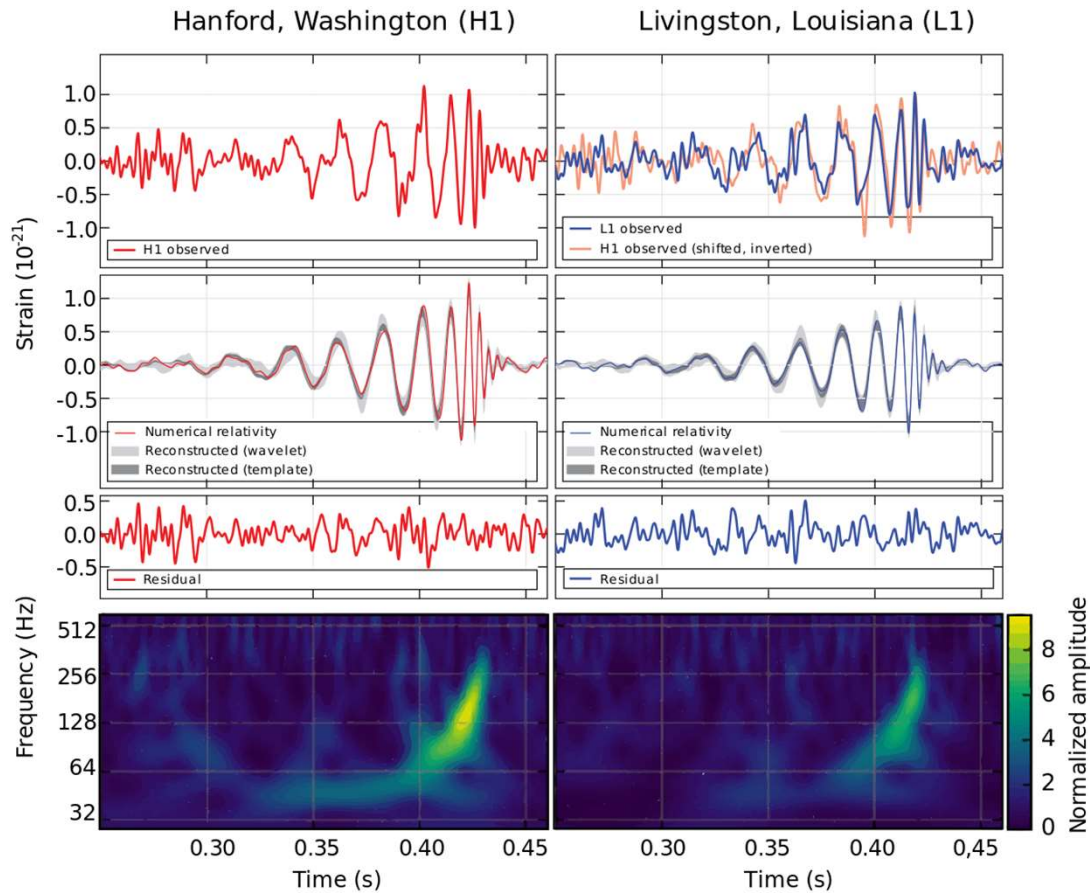
LIGO interferometer / Livingston

# First detection of gravitational wave

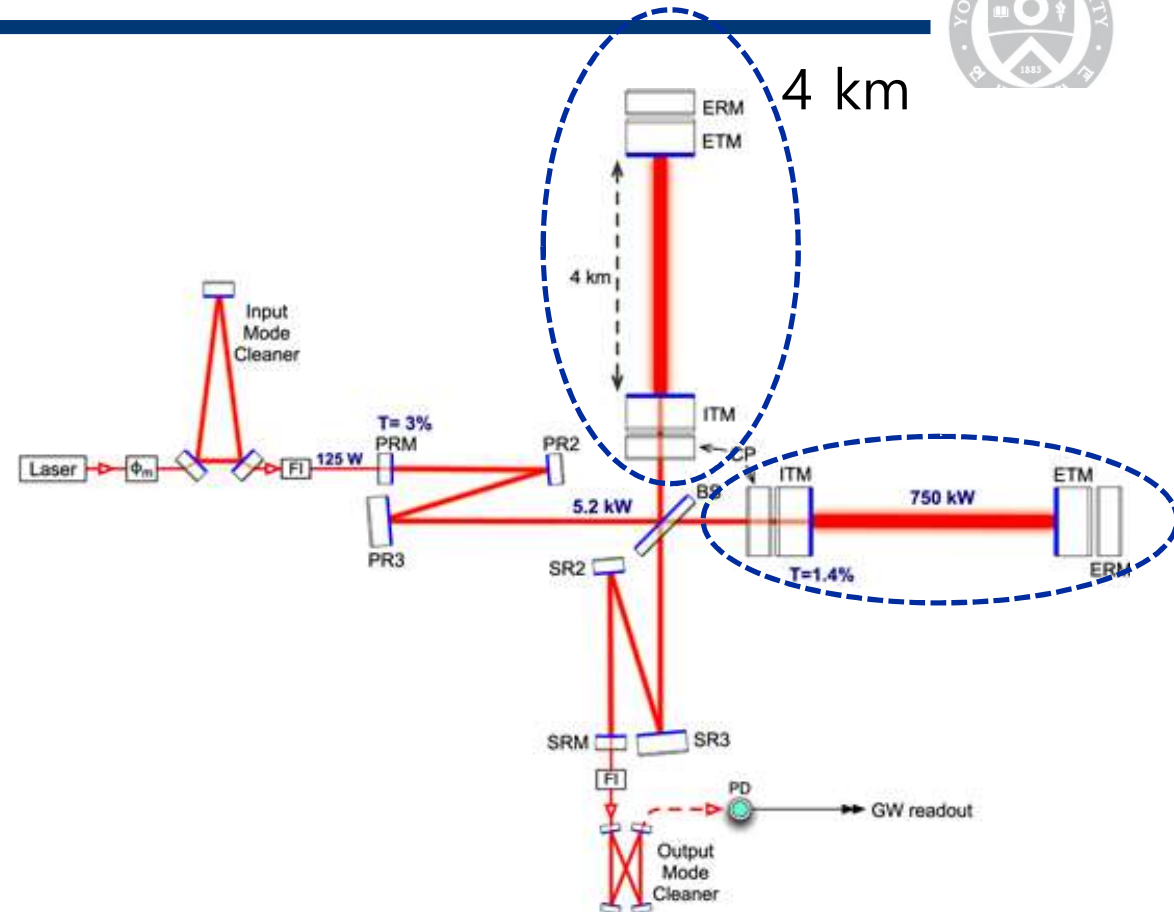




# First detection of gravitational wave

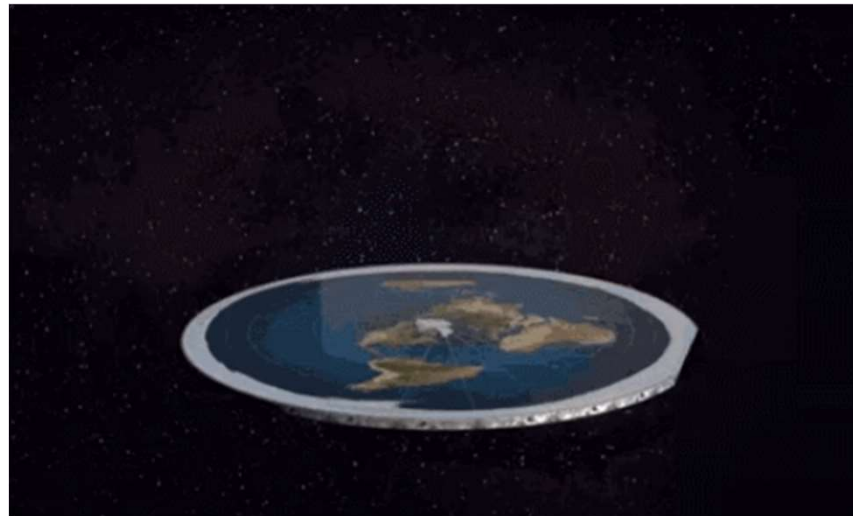


# LIGO interferometer



LIGO interferometer / Livingston

# 4 km interferometer



# 4 km interferometer

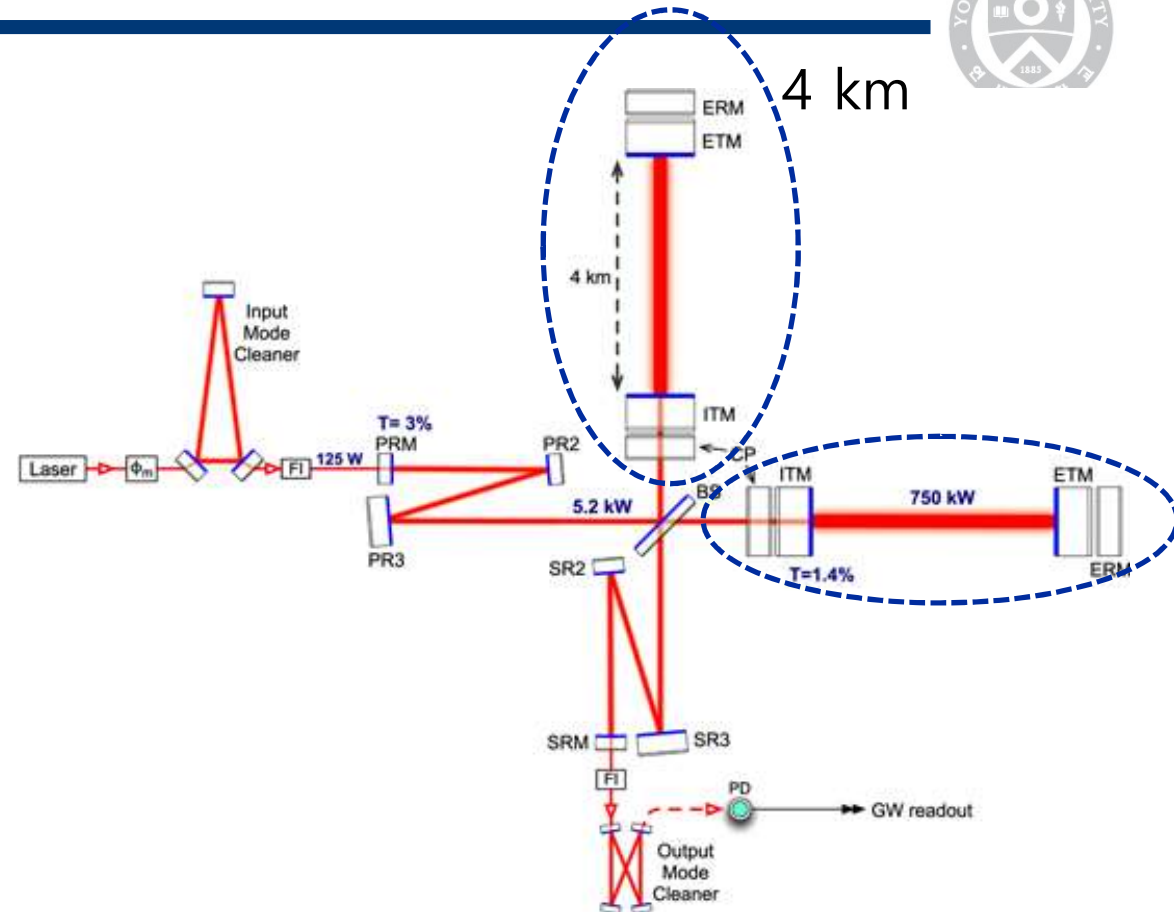


# 4 km interferometer





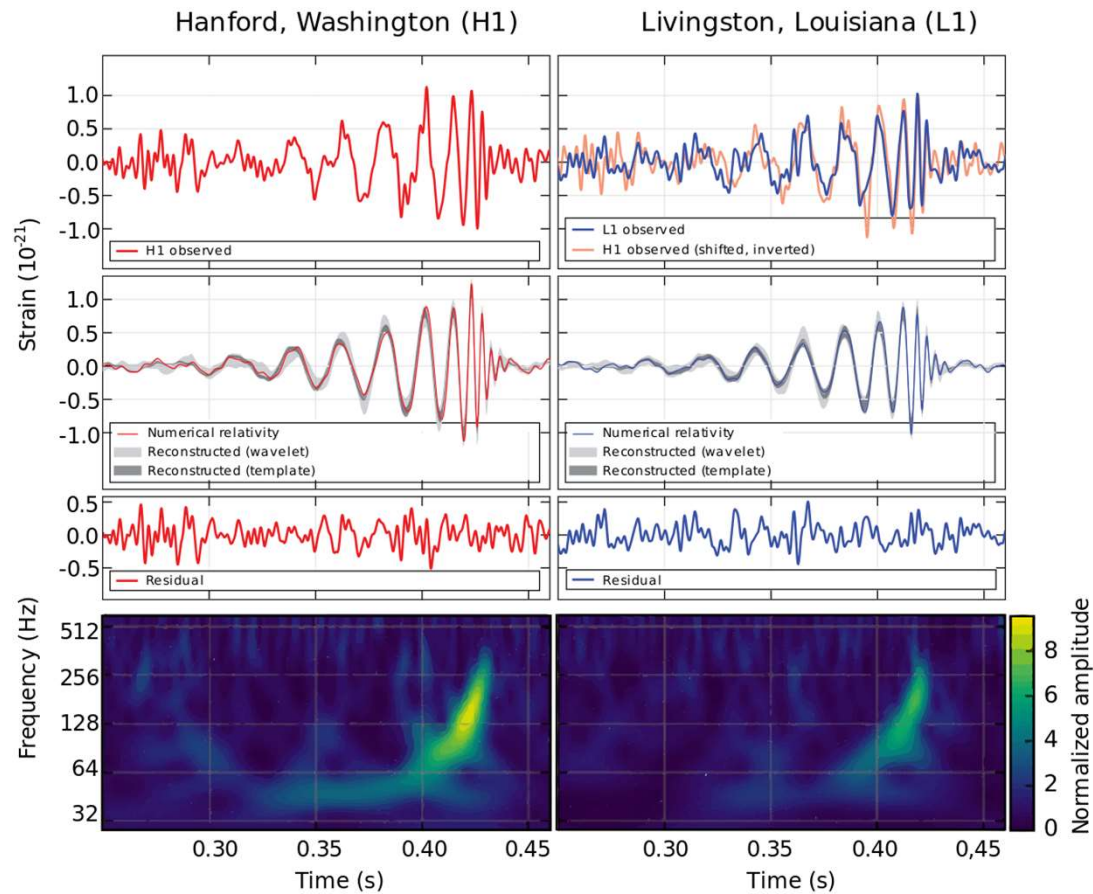
# LIGO interferometer



LIGO interferometer / Livingston



# First detection of gravitational wave

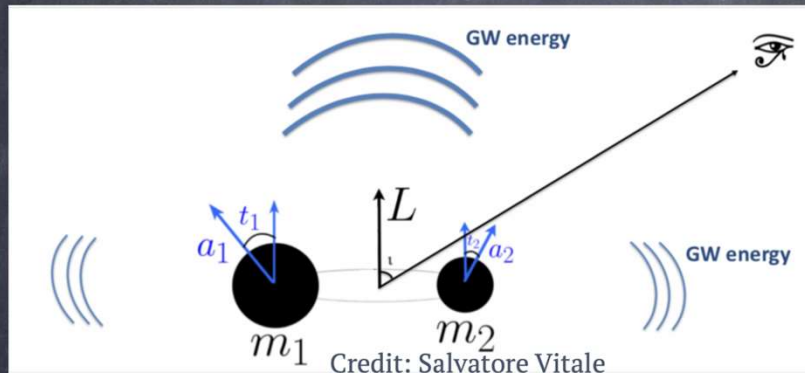


# Parameter estimation



## Compact Binary Coalescences (CBCs) parameters

- For CBCs, the astrophysical contribution is a waveform that depends on 17 parameters



- Intrinsic:** Component masses, Component spins
- Extrinsic:** Sky-location, Distance, Inclination, Polarization, Reference phase, Time at coalescence

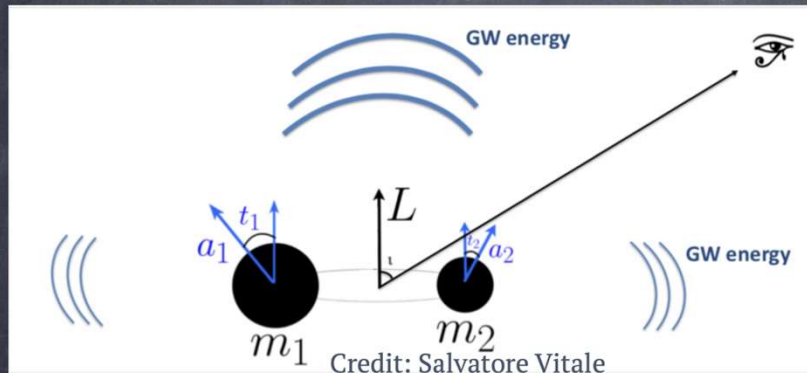
Shu-Wei Yeh (NTHU)

# Parameter estimation



## Compact Binary Coalescences (CBCs) parameters

- For CBCs, the astrophysical contribution is a waveform that depends on 17 parameters



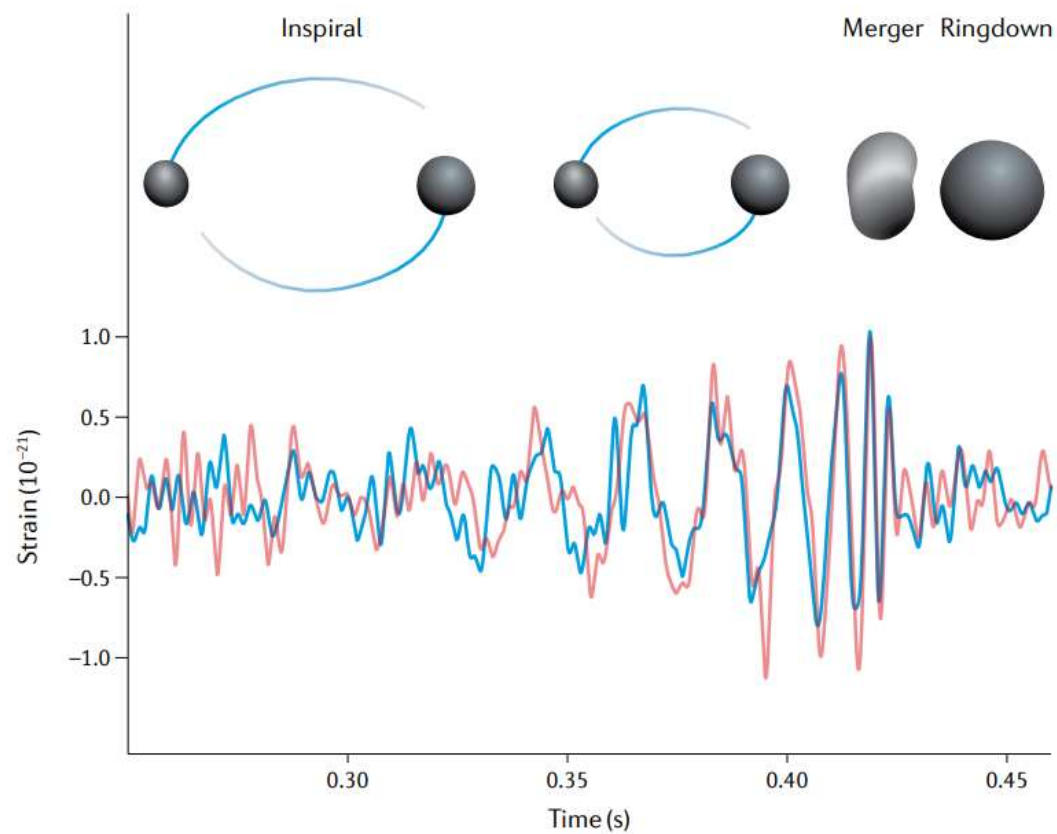
- Intrinsic:** Component masses, Component spins
- Extrinsic:** Sky-location, Distance, Inclination, Polarization, Reference phase, Time at coalescence

## Black Hole

- High mass
- High density(point source)
- No hair

Shu-Wei Yeh (NTHU)

# Inspiral-Merger-Ringdown



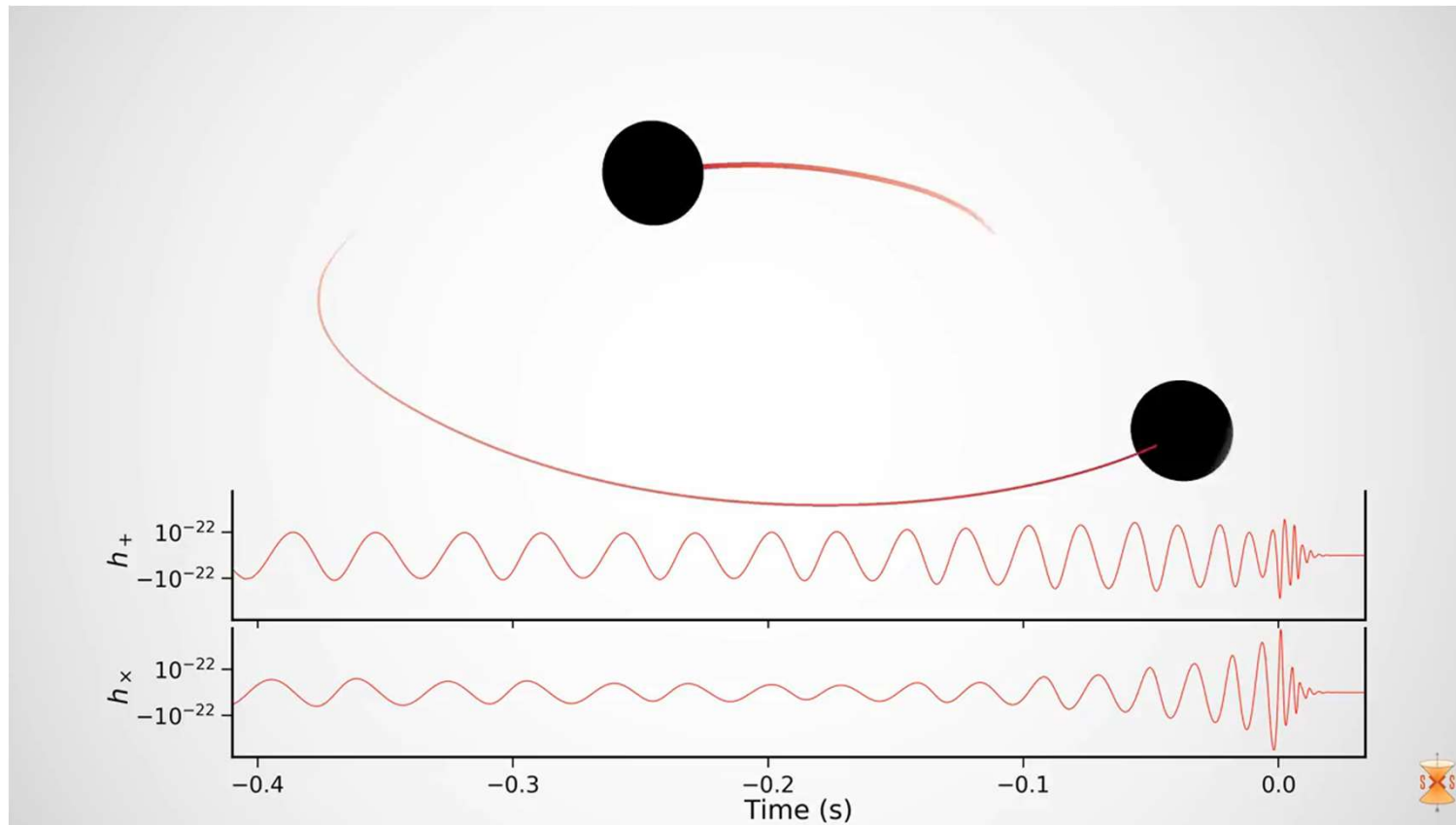
[Nature Reviews Physics](#) volume 3, pages344–366 (2021)

# GW170104



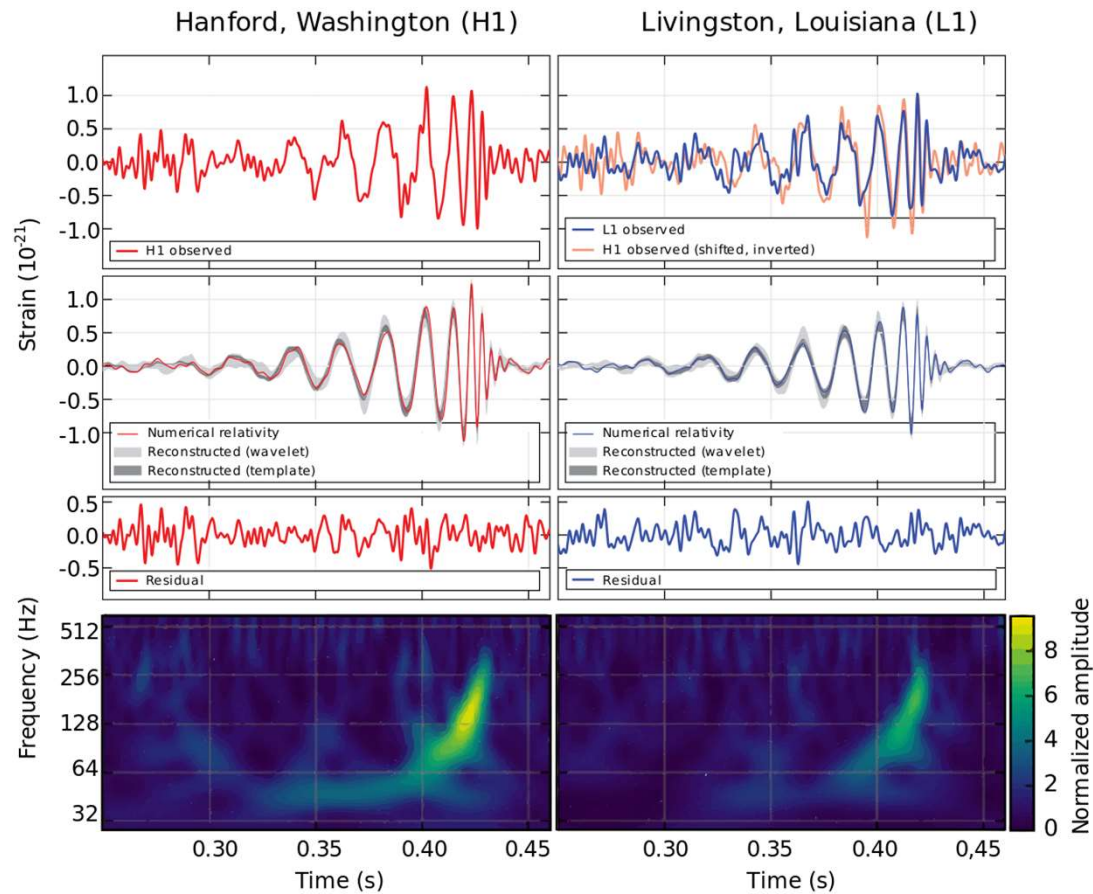
+ pol

x pol





# First detection of gravitational wave





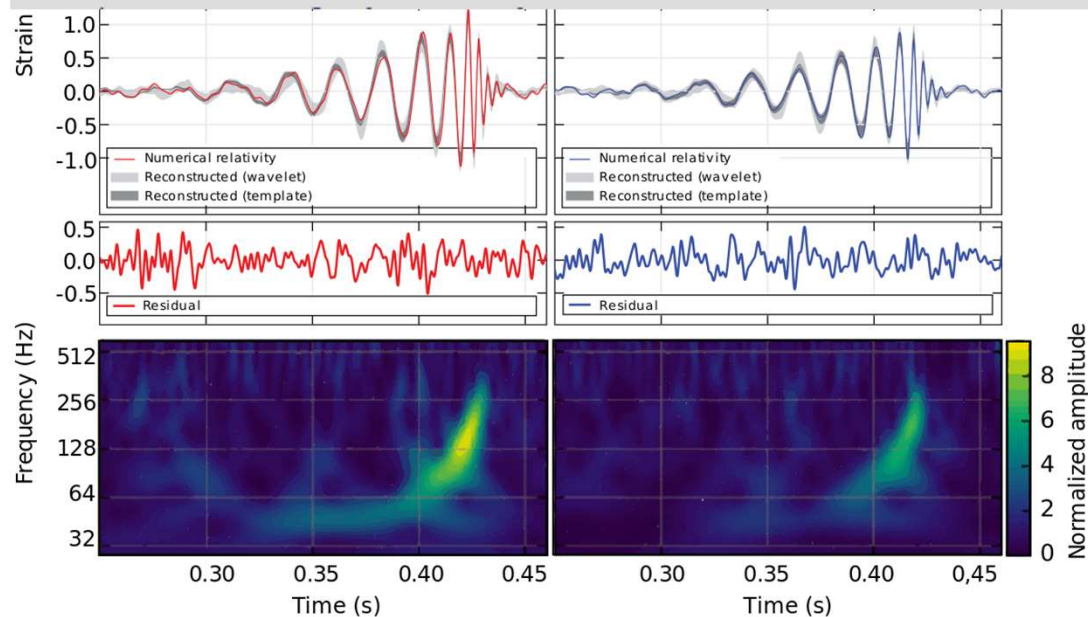
# First detection of gravitational wave



## Gravitational Waves Detected 100 Years After Einstein's Prediction

News Release • February 11, 2016

Visit [The Detection Portal](#)



# First detection of gravitational wave



## Gravitational Waves Detected 100 Years After Einstein's Prediction

News Release • February 11, 2016

Visit [The Detector](#)

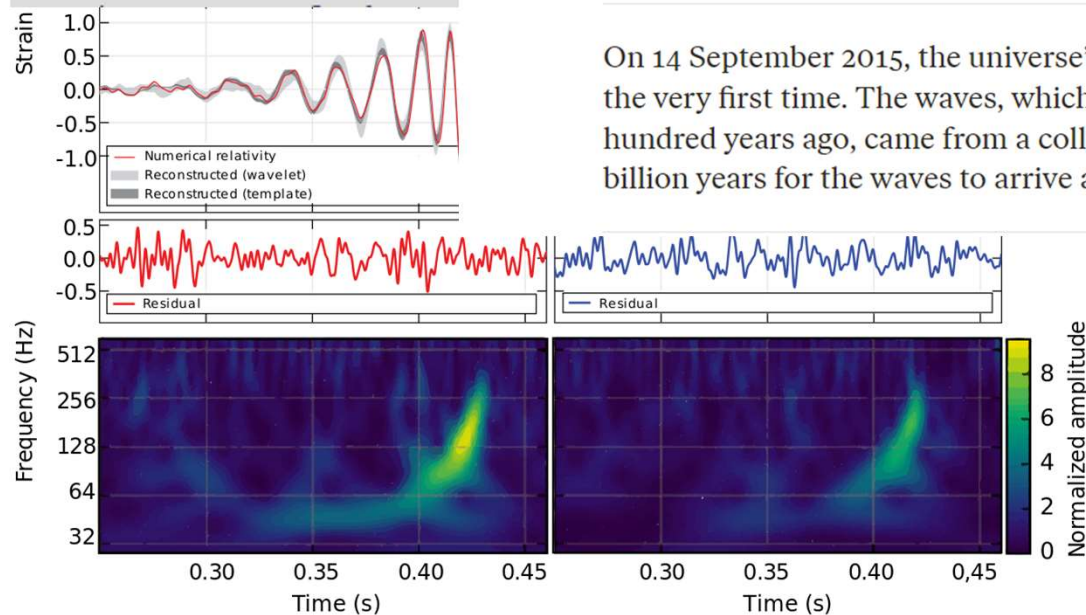
The Nobel Prize in Physics 2017

Press Release

*"for decisive contributions to the LIGO detector and the observation of gravitational waves"*

### Gravitational waves finally captured

On 14 September 2015, the universe's gravitational waves were observed for the very first time. The waves, which were predicted by Albert Einstein a hundred years ago, came from a collision between two black holes. It took 1.3 billion years for the waves to arrive at the LIGO detector in the USA.

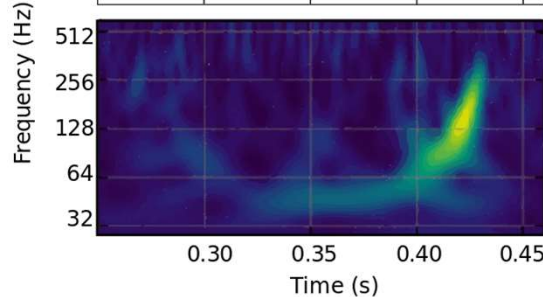
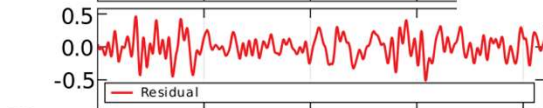
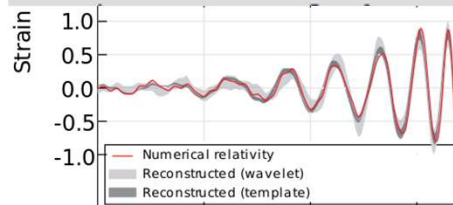


# First detection of gravitational wave

## Gravitational Waves Detected 100 Years After Prediction

News Release • February 11, 2016

Visit [The Detection](#)



The Nobel Prize in Physics 2017

Press Release

*“for decisive contributions*

*to understanding the universe and for the first direct detection of ripples in space-time.*



Three physicists who had leading roles in the first direct detection of gravitational waves have won the 2017 Nobel Prize in Physics.

# First detection of gravitational wave



## Gravitational Waves Detected 100 Years After Prediction

News Release • February 11, 2016

The Nobel Prize in Physics 2017

Press Release

*"for decisive contributions to the discovery of gravitational waves"*

*Rainer Weiss, Barry Barish and Kip Thorne share the 2017 prize for their work at LIGO to*



health

Life, But Better

Fitness

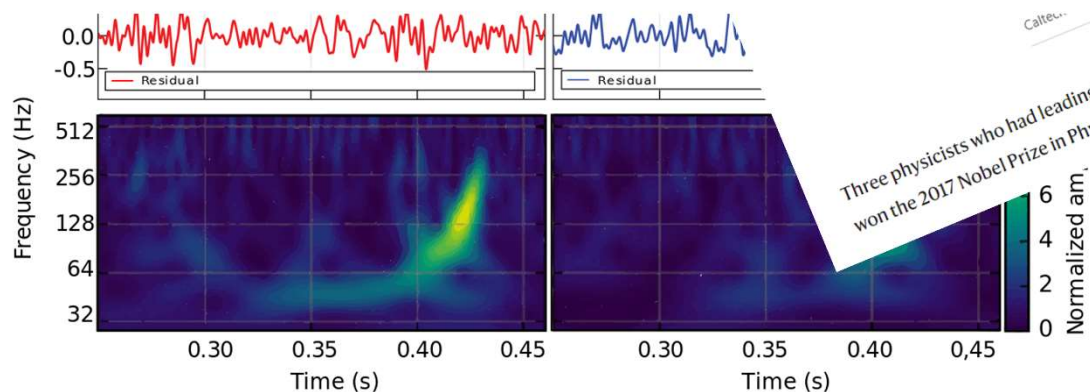
Food

Sleep

Mindfulness

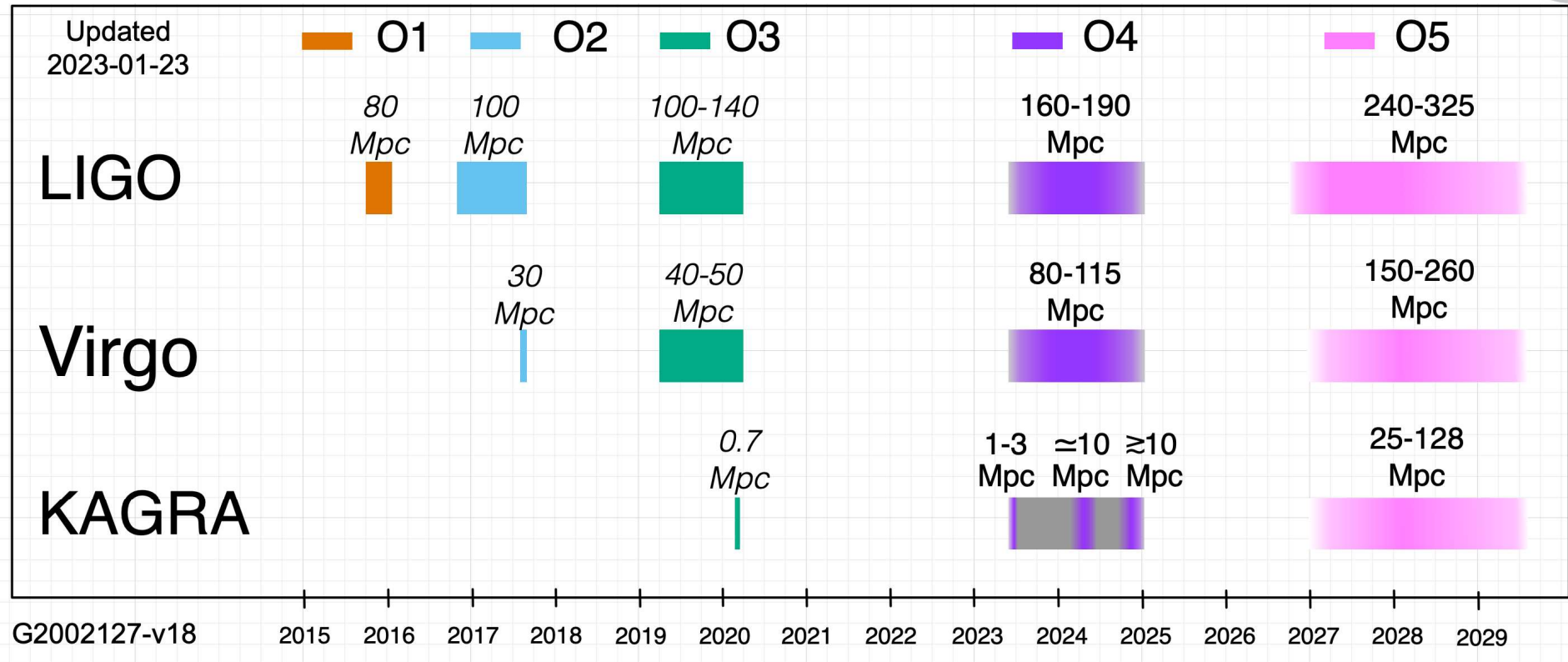
Relationships

## Nobel Prize in Physics goes to 'black hole telescope' trio





# LVK Observation plan



# BNS range



- Binary-neutron-star range
- Common benchmark of sensitivity
- Made up of two 1.4 solar mass neutron stars
- Signal-to-noise ratio of 8

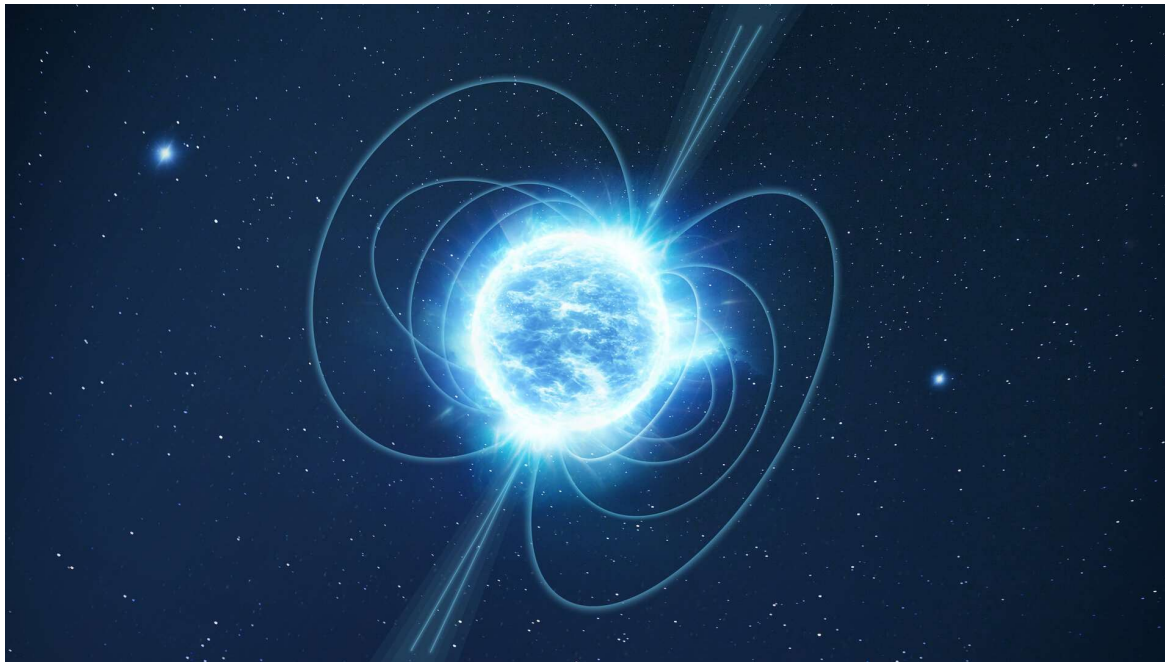


<https://svs.gsfc.nasa.gov/10543>

Maximum distance at which an event can be detected



# Neutron star



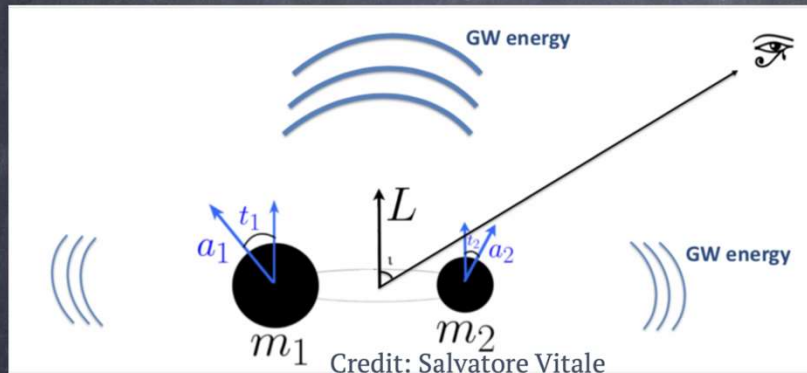
Why binary neutron star?

# Parameter estimation



## Compact Binary Coalescences (CBCs) parameters

- For CBCs, the astrophysical contribution is a waveform that depends on 17 parameters



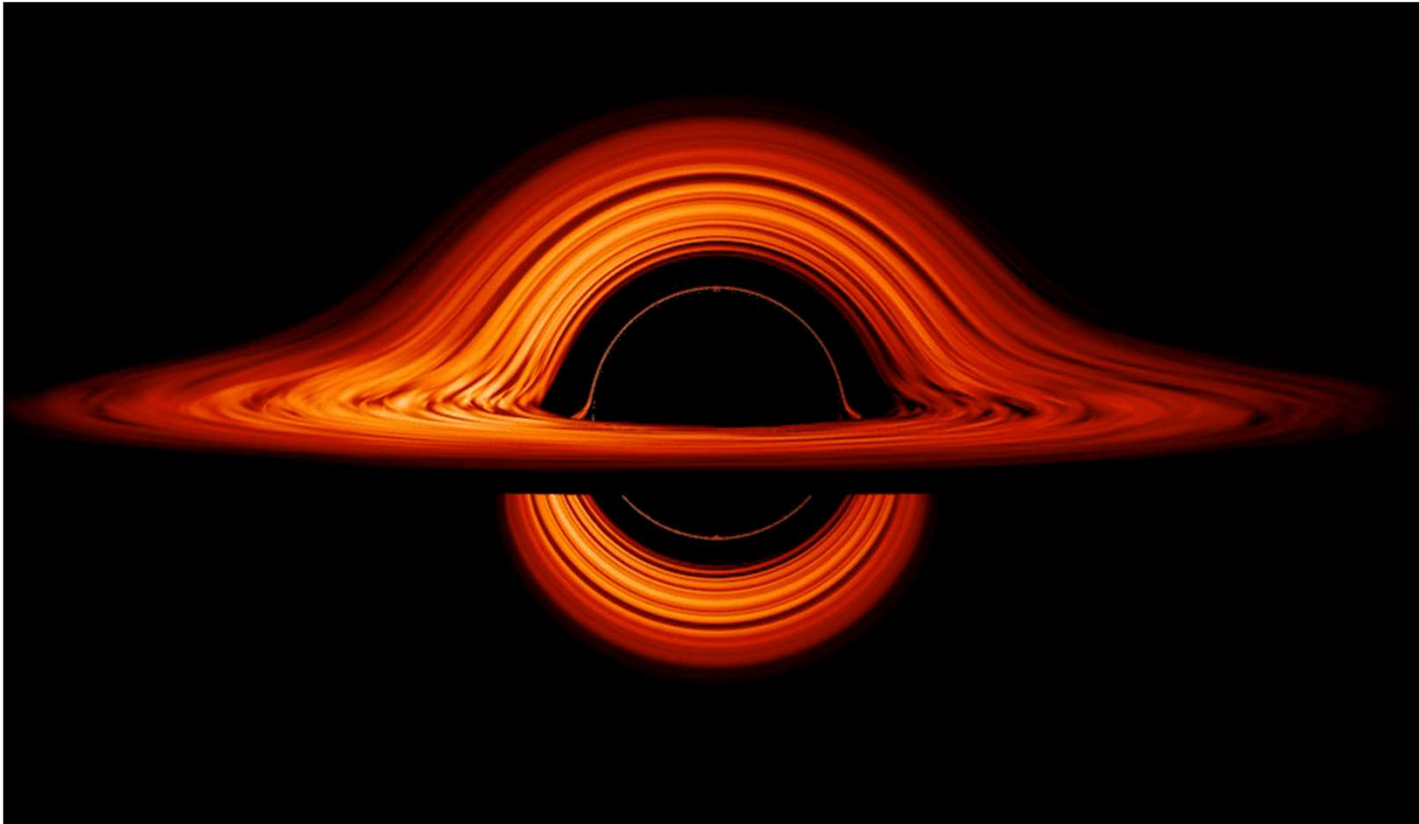
- Intrinsic:** Component masses, Component spins
- Extrinsic:** Sky-location, Distance, Inclination, Polarization, Reference phase, Time at coalescence

## Black Hole

- High mass
- High density(point source)
- No hair

Shu-Wei Yeh (NTHU)

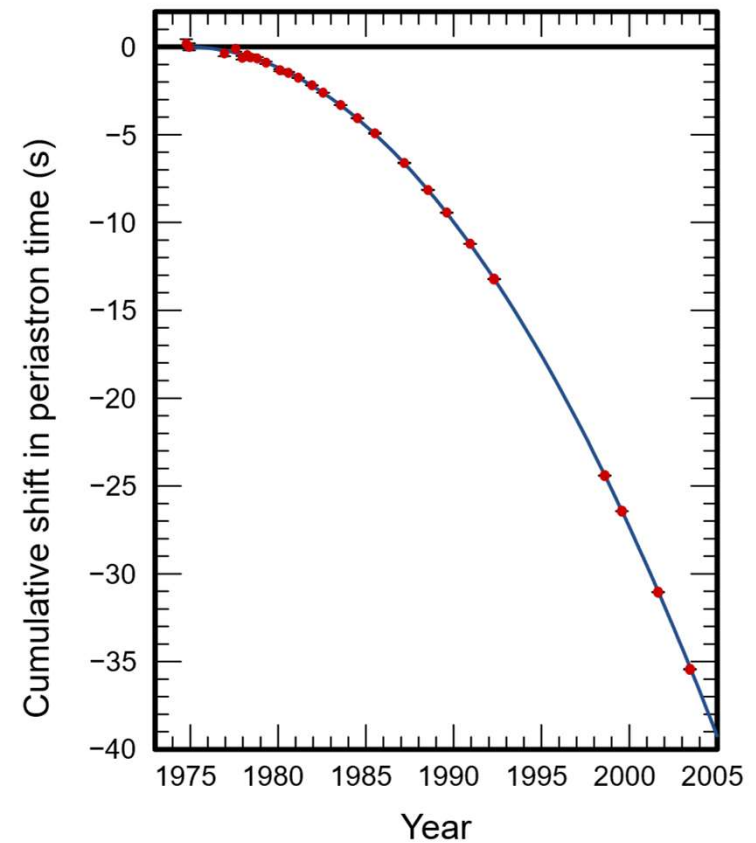
# Event Horizon



# Test of General Relativity



# Hulse-Taylor Pulsar

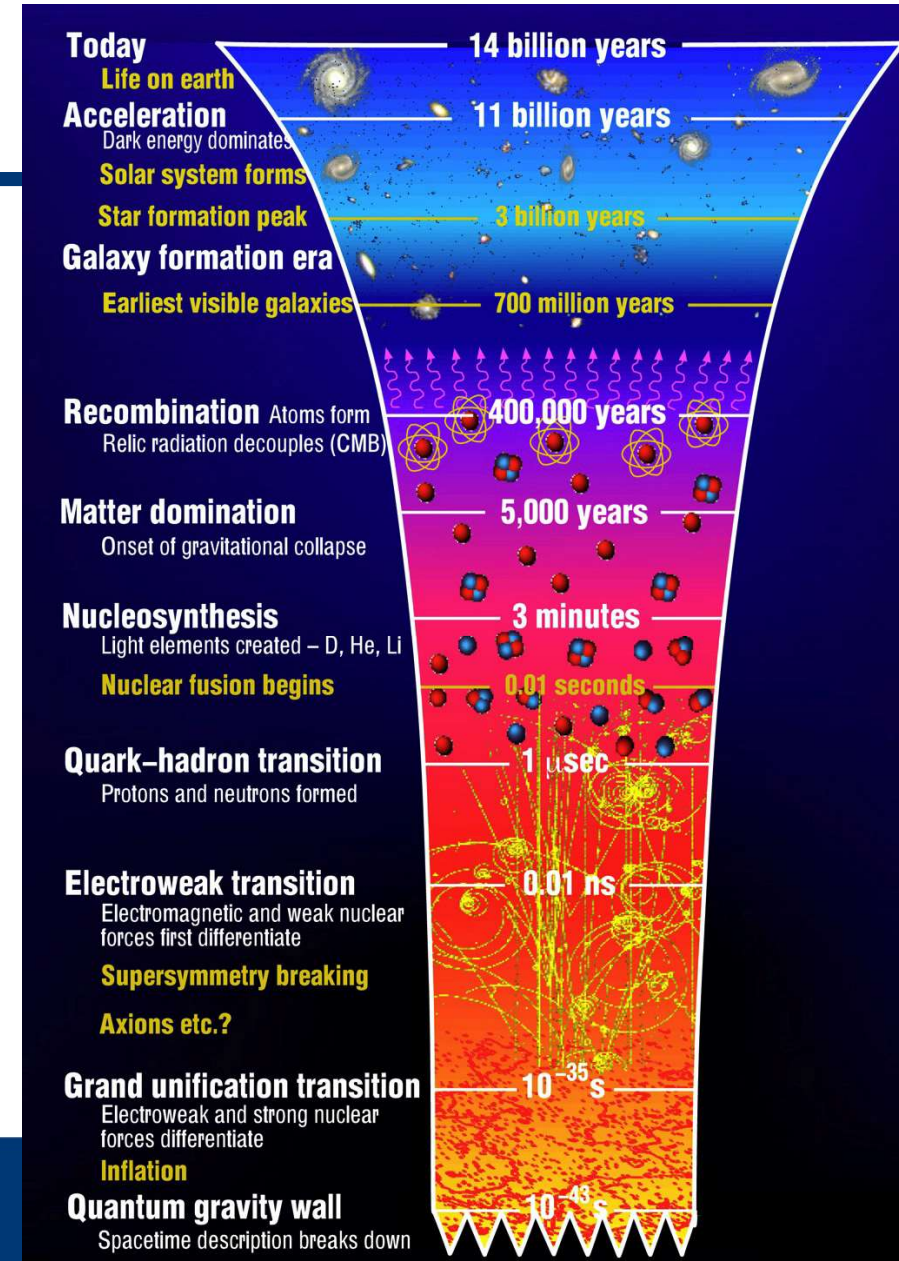




# Bigbang Timeline

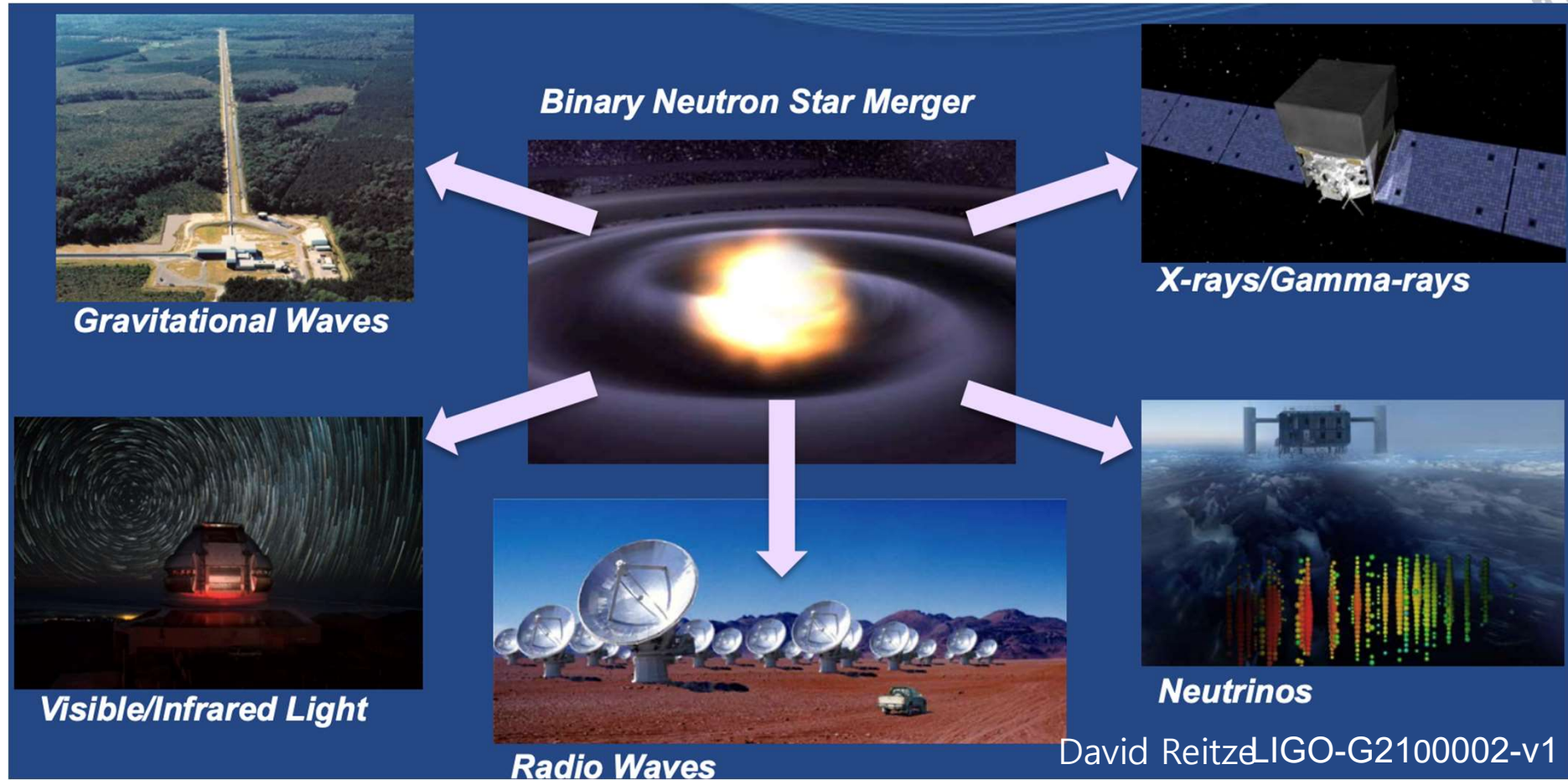
Group →	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
Period ↓	1 1 H																	2 He	
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
6	55 Cs	56 Ba	*	71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	*	103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
			*	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb		
			*	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No		
			*																

중력파 여름학교 2025

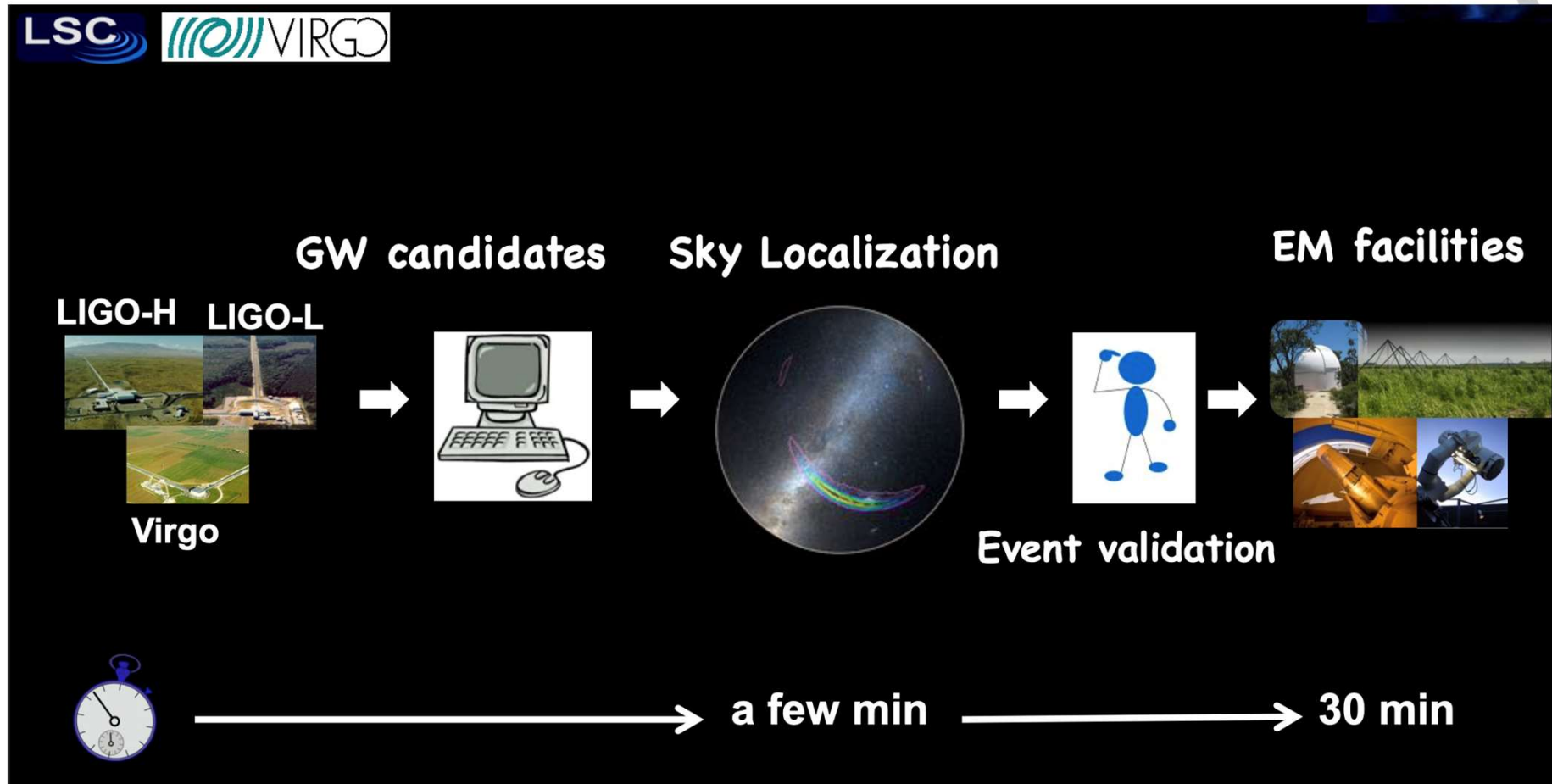




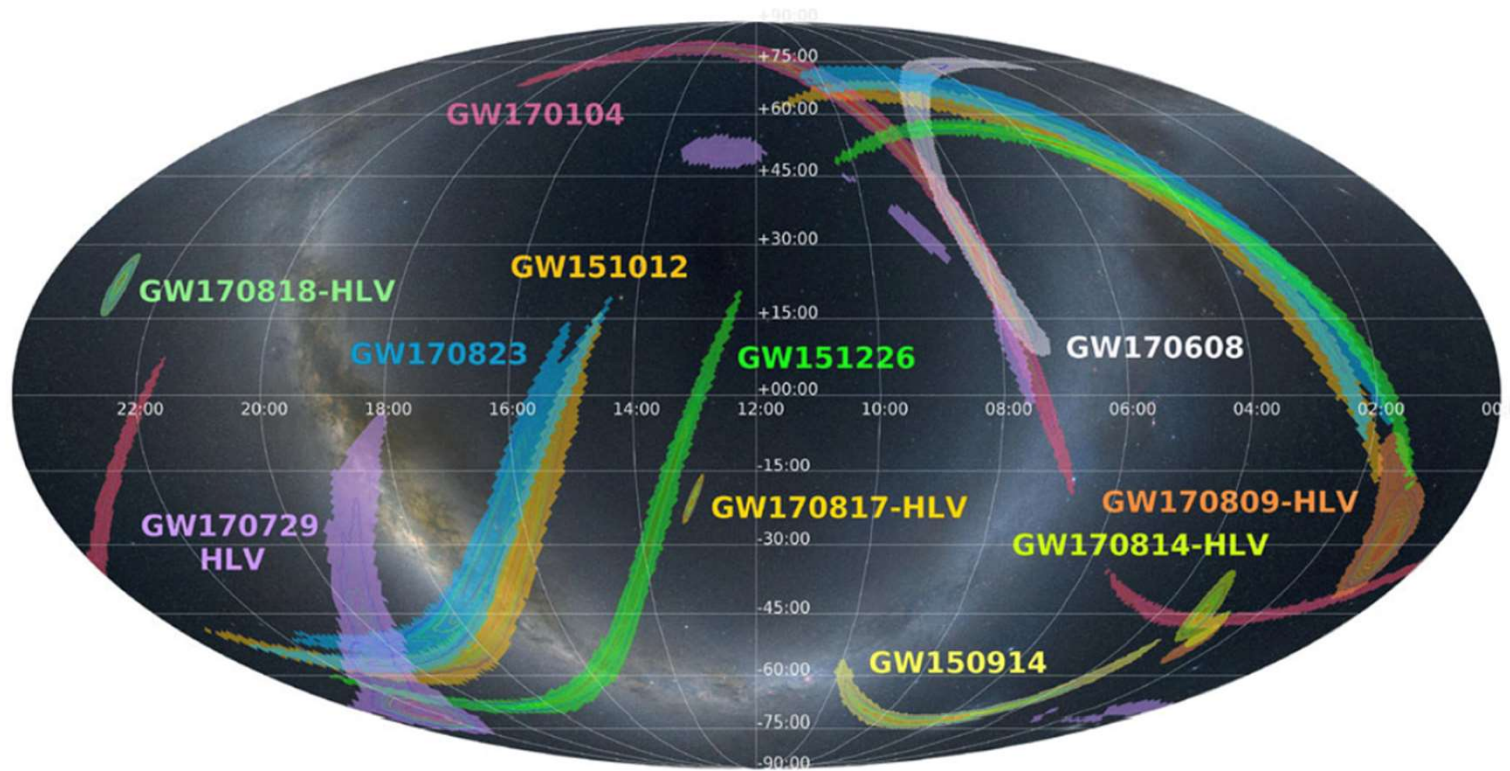
# Multi-messenger astronomy with gravitational waves



# GW alert procedure



# Sky localization





# Localizing Gravitational-wave Events



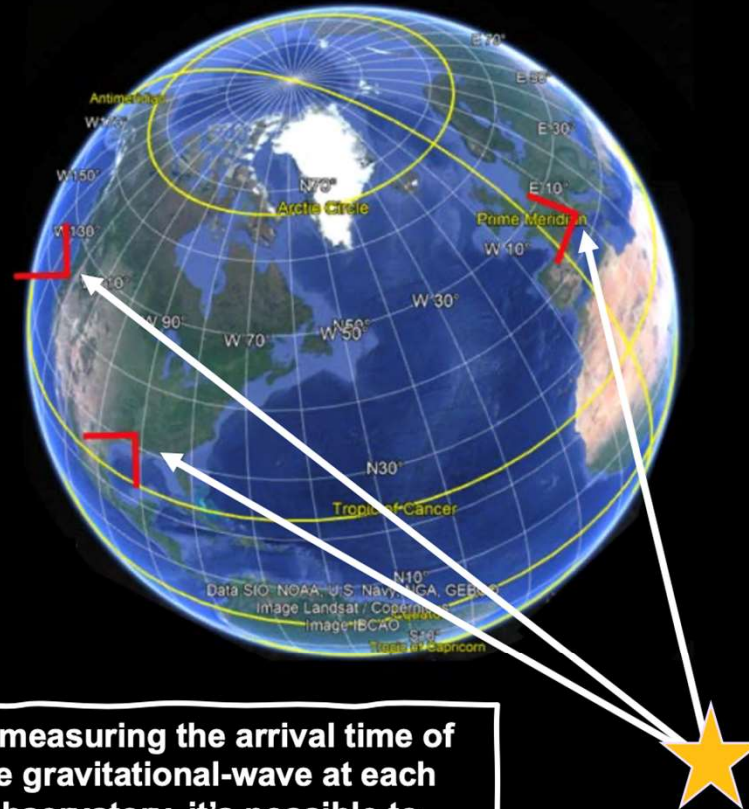
Virgo, Cascina, Italy



LIGO, Livingston, LA



LIGO, Hanford, WA



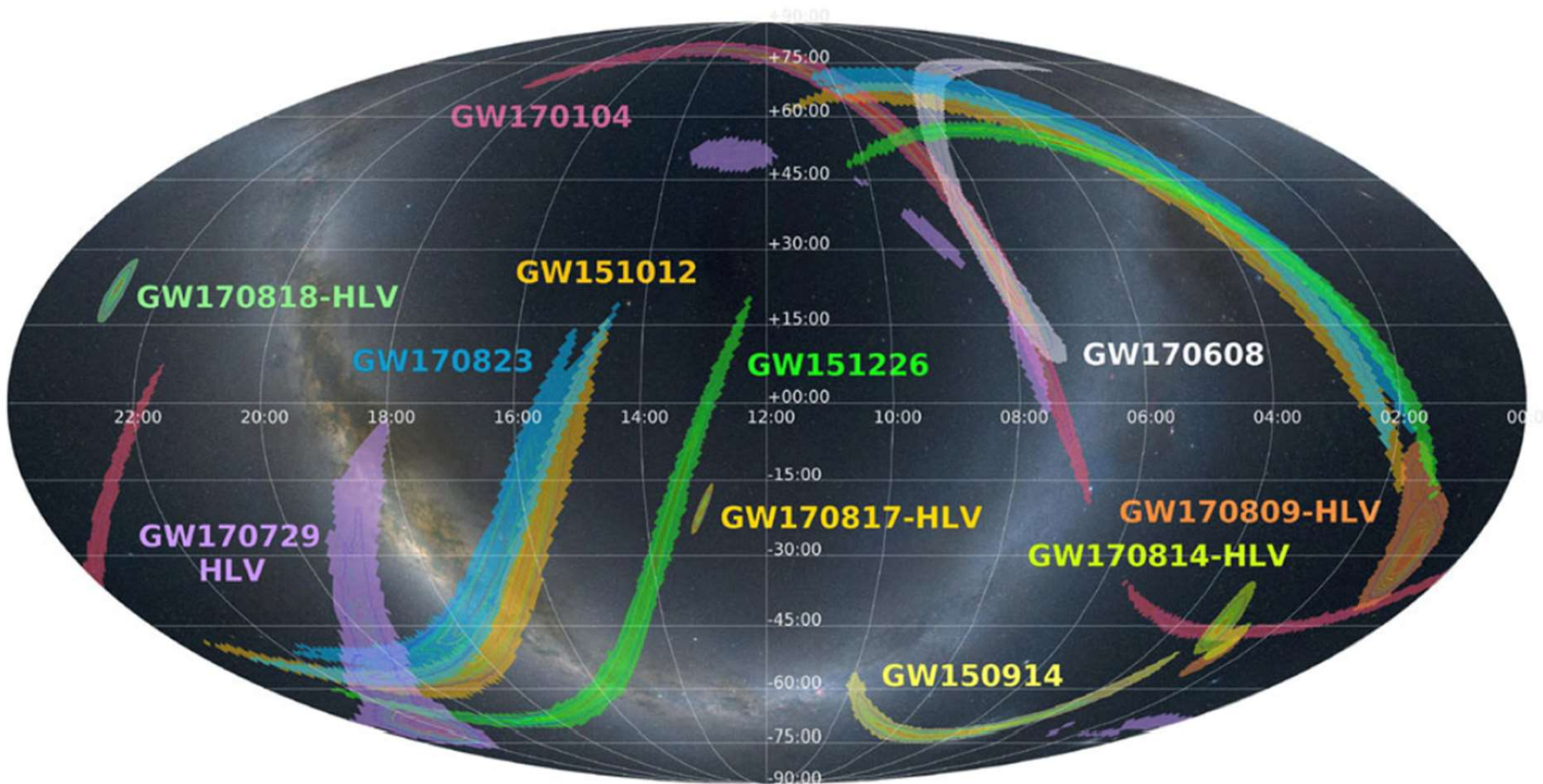
By measuring the arrival time of the gravitational-wave at each observatory, it's possible to identify its location on the sky

A single GW observatory is mostly insensitive to the sky location; we want two and preferably three or more observatories

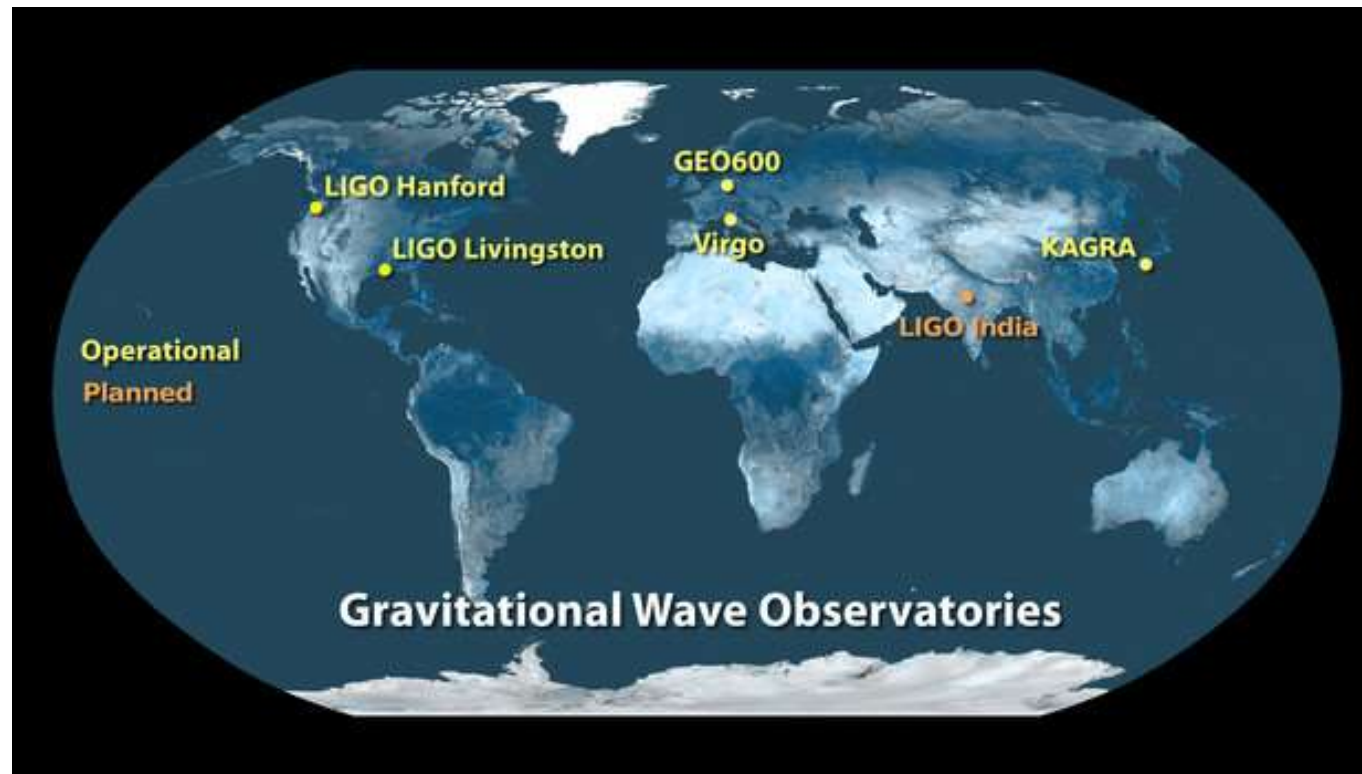
Yokohama GRB 201  
Marci Branchesi

YONSEI UNIVERSITY

# Sky localization



# Gravitational wave observatories



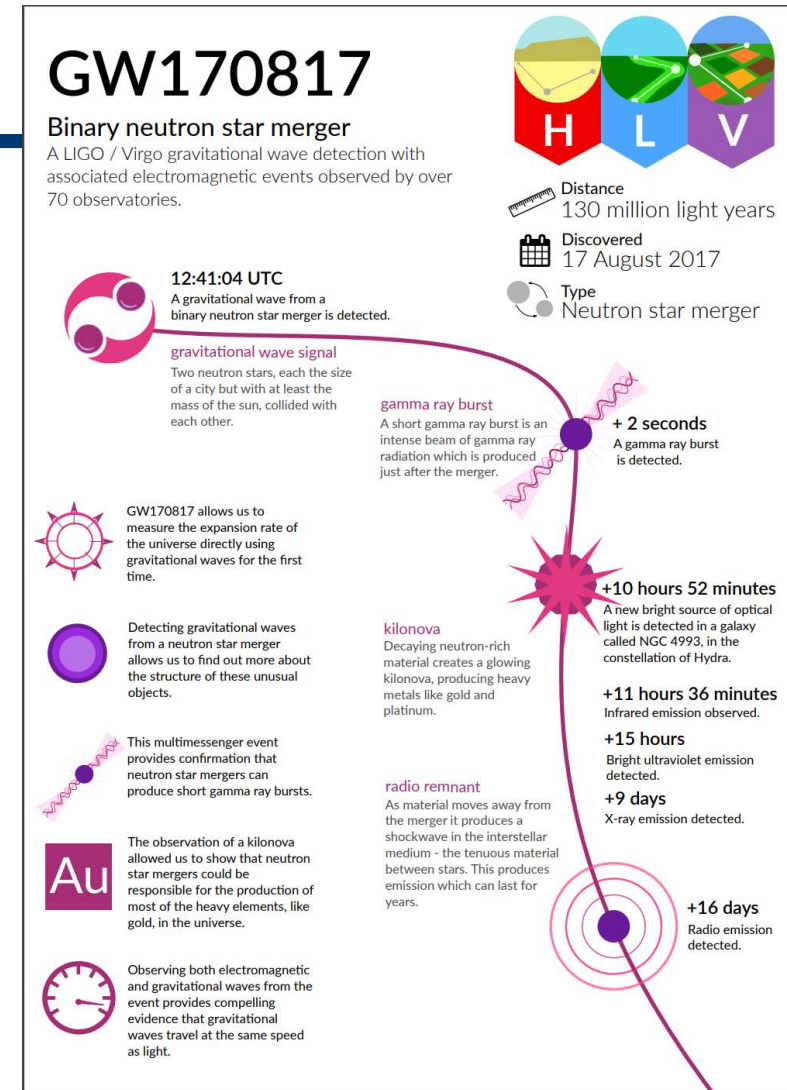


# First multi-messenger detection

Binary neutron star merger

Gamma ray burst detection with Fermi-detector

Kilonova detection using telescope

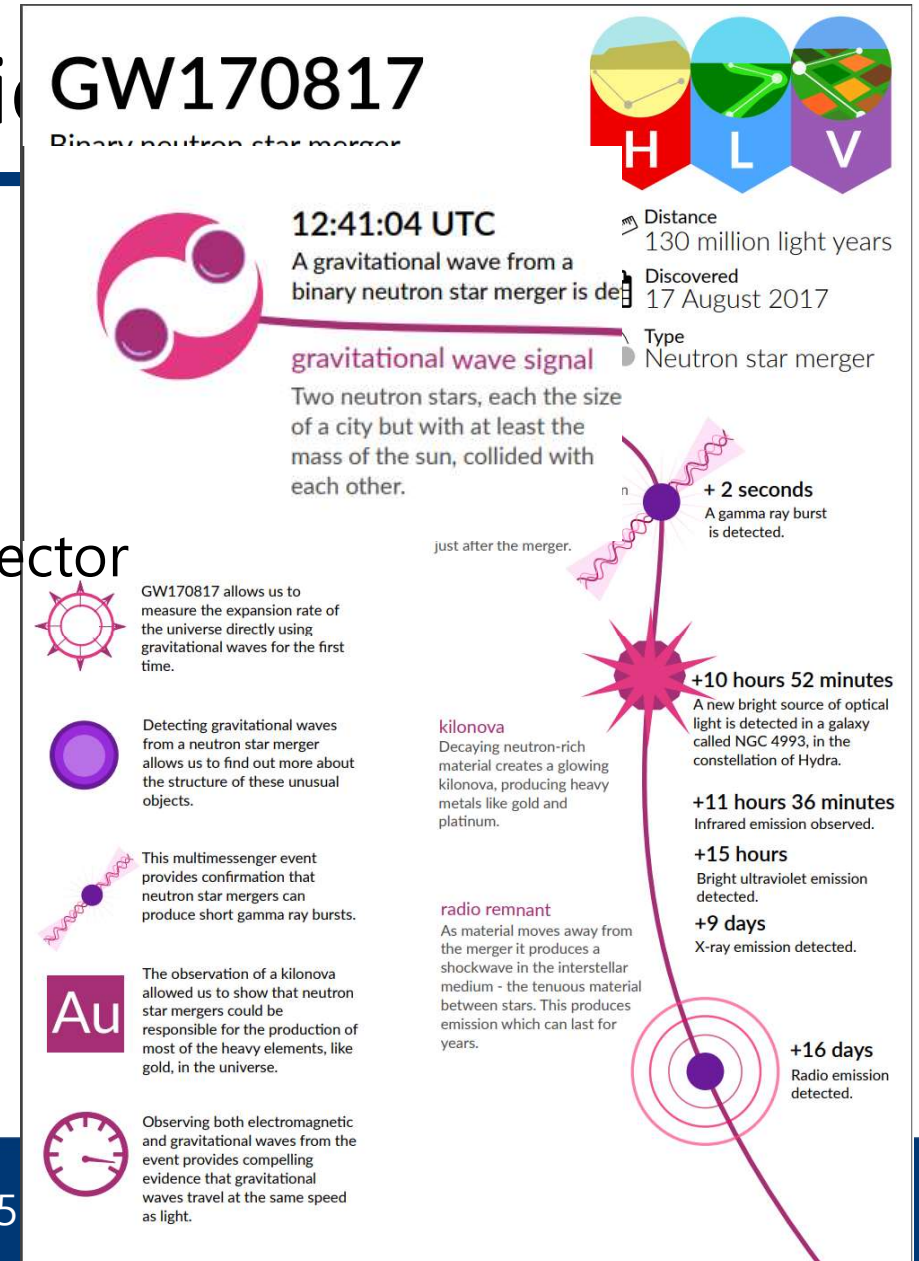


# First multi-messenger detection GW170817

Binary neutron star merger

Gamma ray burst detection with Fermi-detector

Kilonova detection using telescope

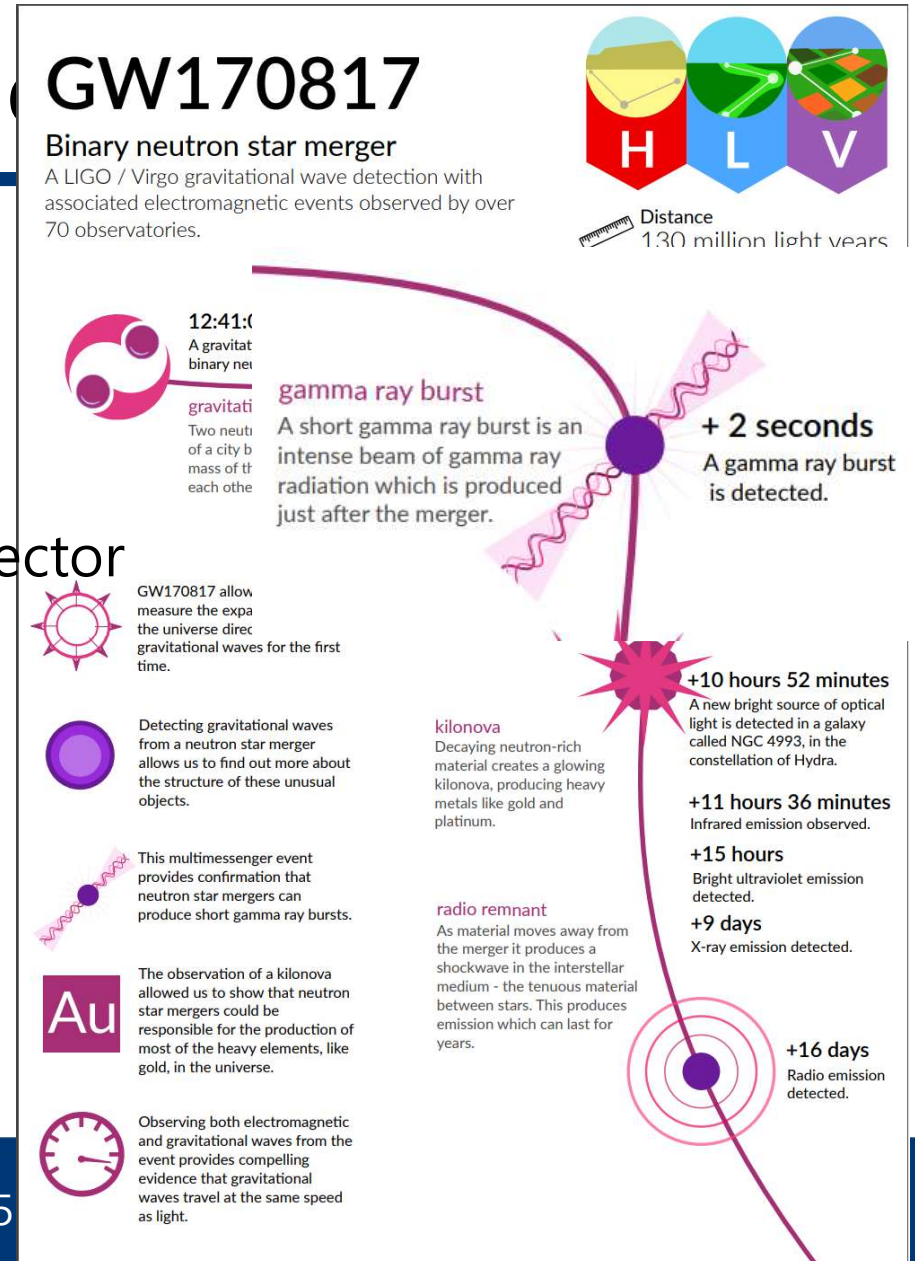


# First multi-messenger detection GW170817

Binary neutron star merger

Gamma ray burst detection with Fermi-detector

Kilonova detection using telescope

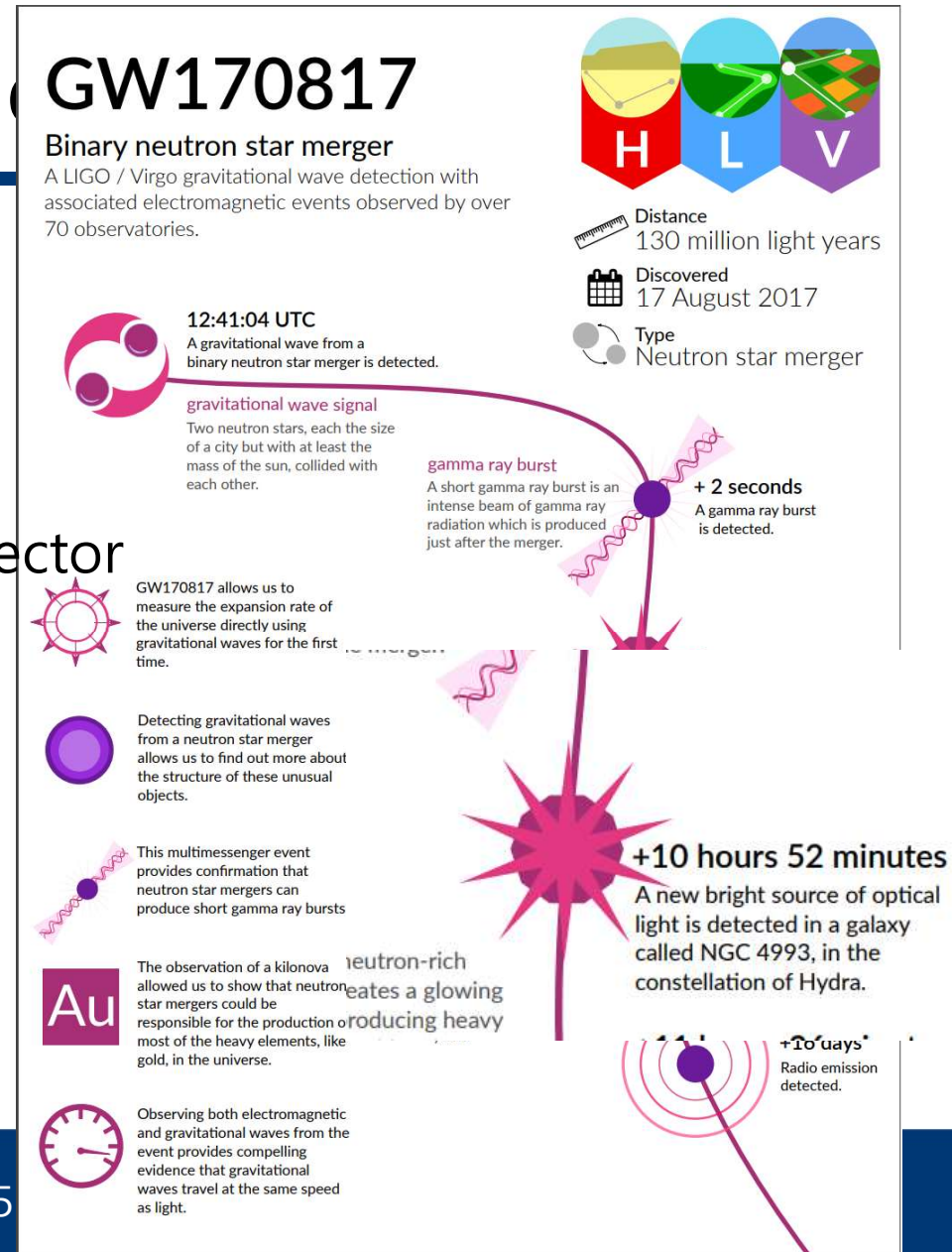


# First multi-messenger detection GW170817

Binary neutron star merger

Gamma ray burst detection with Fermi-detector

Kilonova detection using telescope



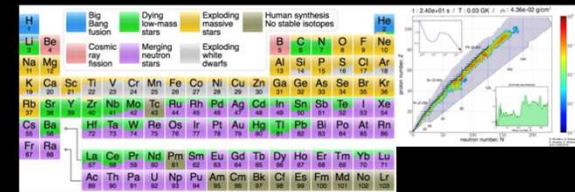


# Radioactively powered transients

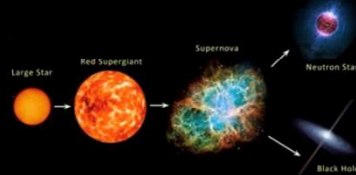
## Relativistic astrophysics



## Nucleosynthesis and enrichment of the Universe



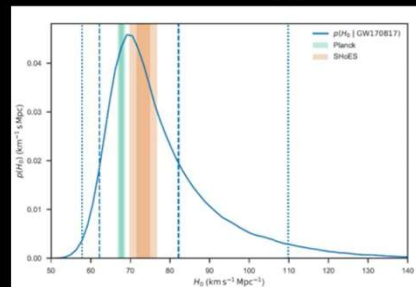
## Compact object formation and evolution



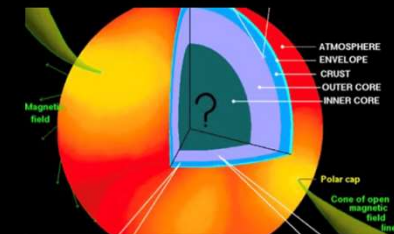
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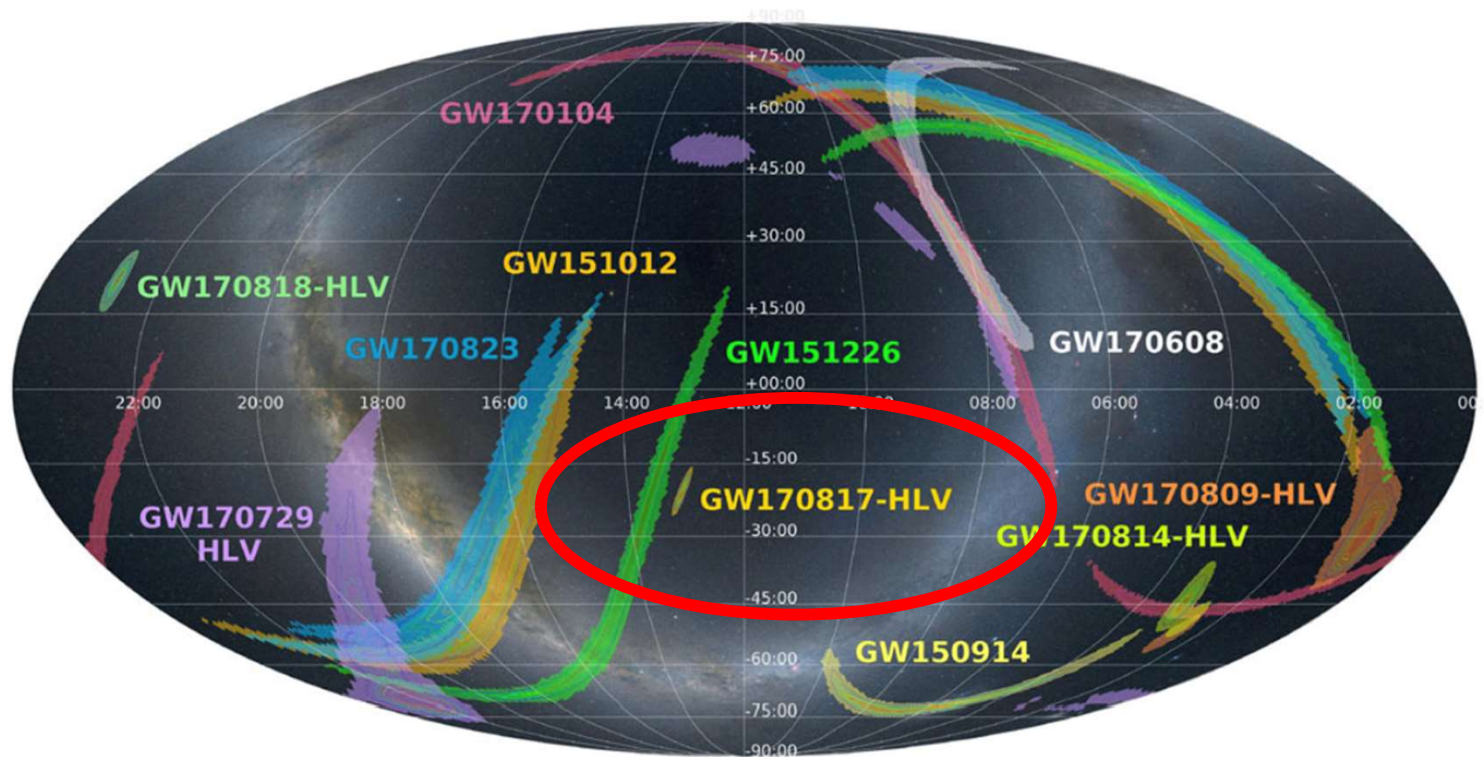
## Cosmology



## Nuclear matter physics

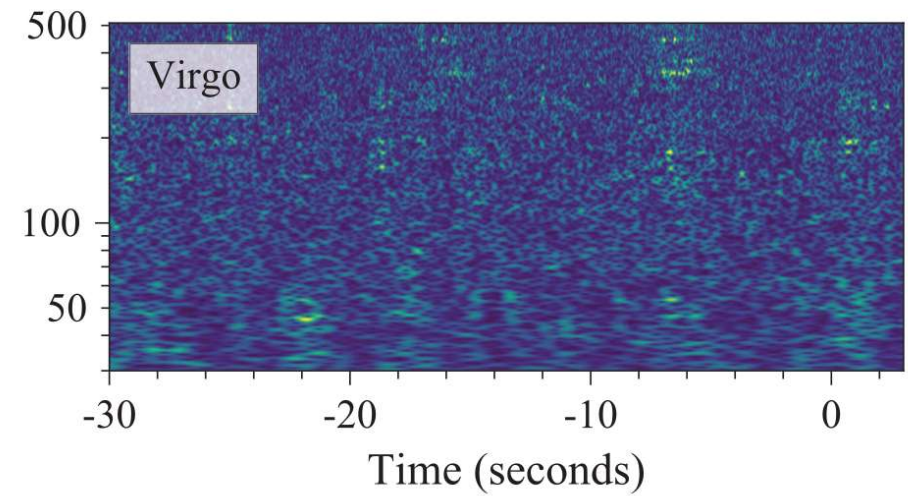
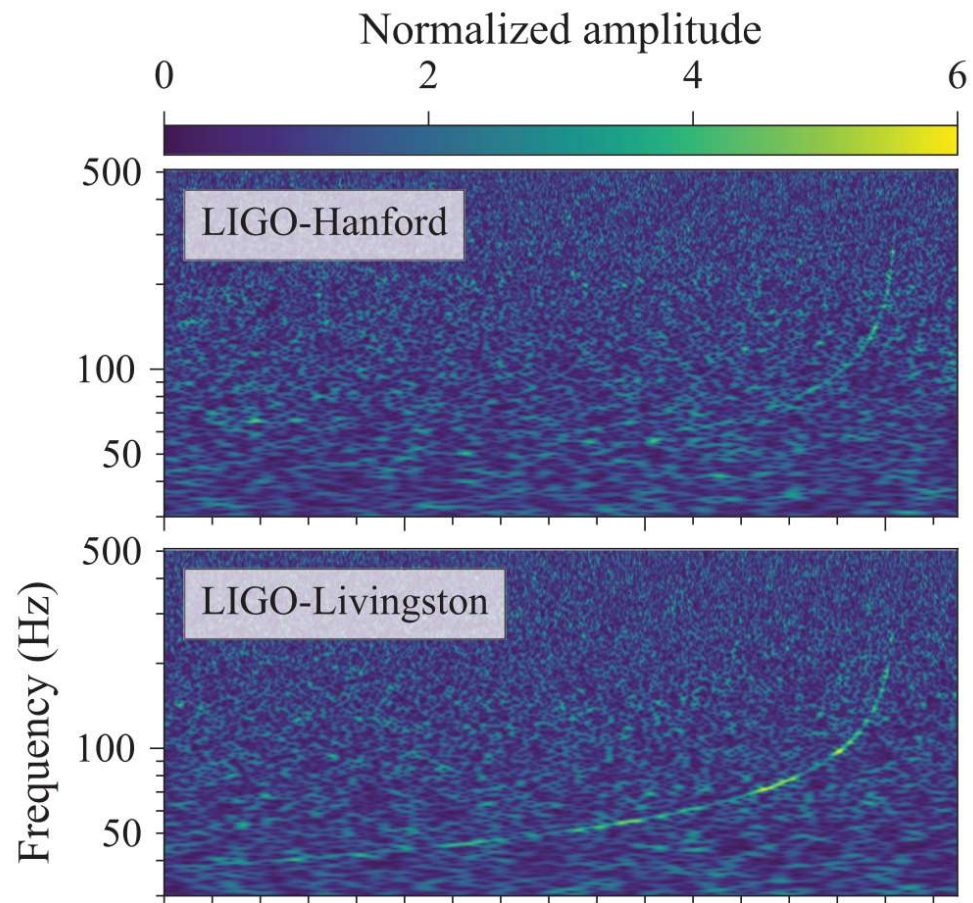


# Sky localization



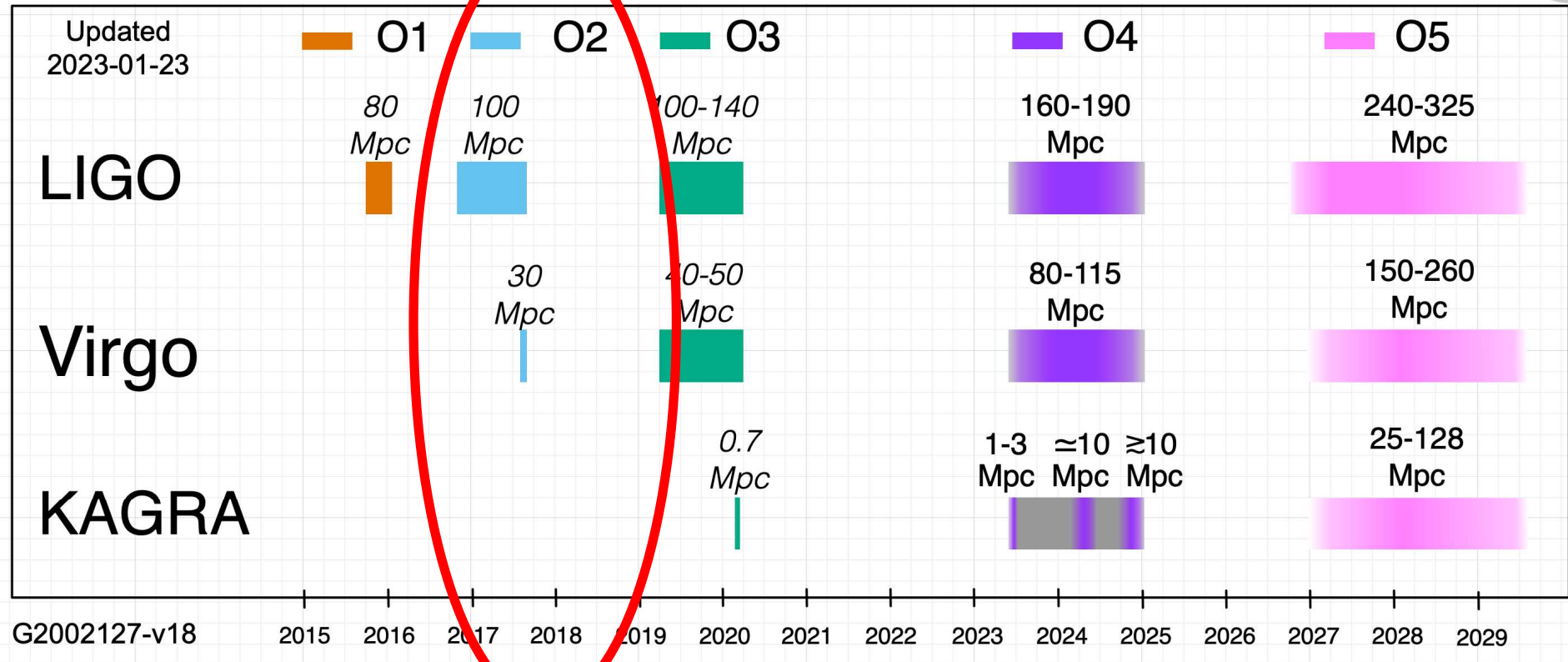


# GW170817



No Signal at VIRGO ???

# LVK Observation plan



# LIGO-VIRGO joint observation



## VIRGO JOINS LIGO FOR THE O2 DATA-TAKING PERIOD

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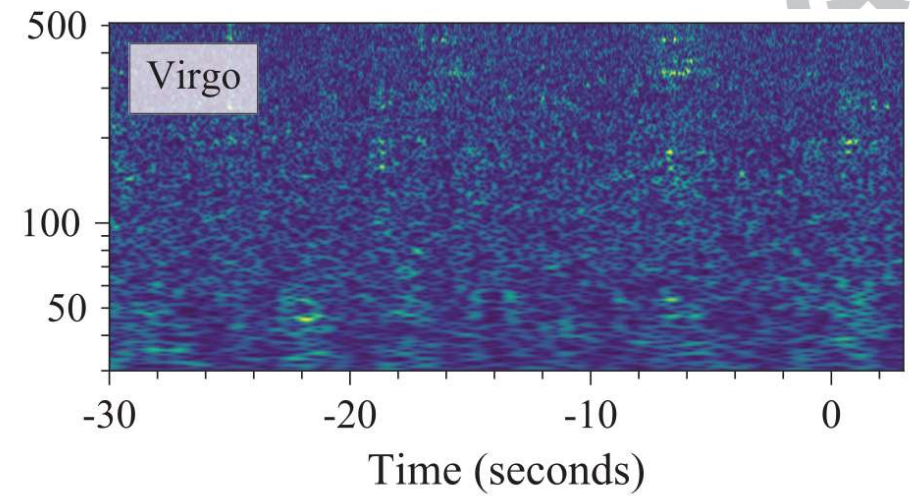
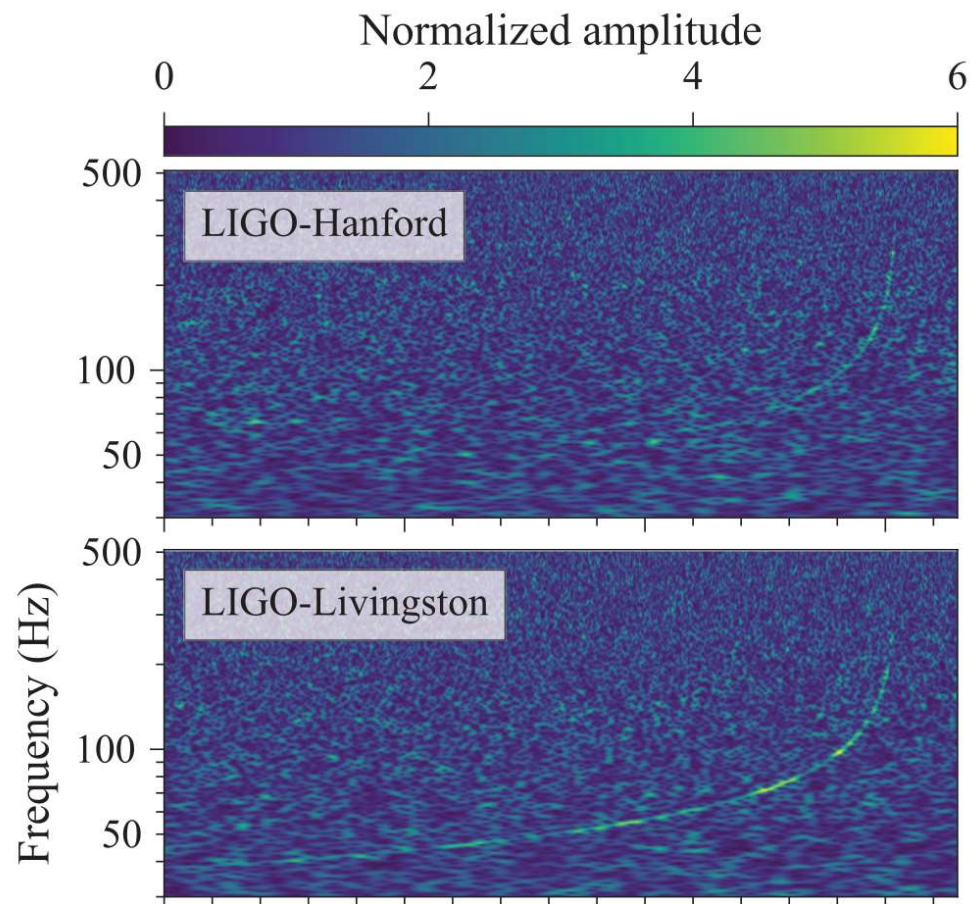
By Massimiliano Razzano August 1, 2017 News, Press Releases

VIRGO joins LIGO for the “Observation Run 2” (O2) data-taking period

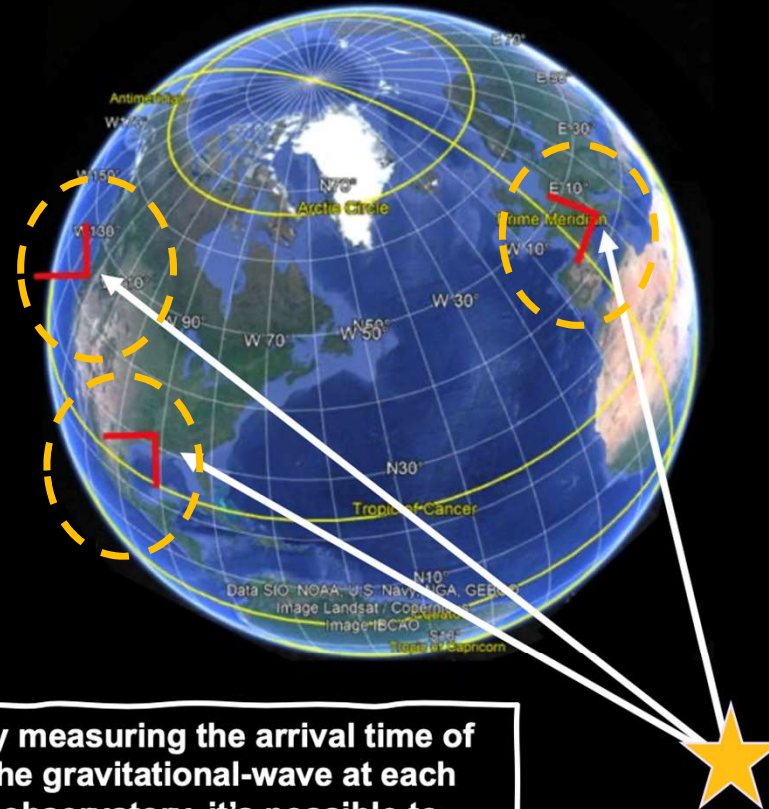
Today, **Tuesday August 1st 2017**, the VIRGO detector based in Europe has officially joined “Observation Run 2” (O2) : based twin LIGO detectors.



# GW170817



# Localizing Gravitational-wave Events



By measuring the arrival time of the gravitational-wave at each observatory, it's possible to identify its location on the sky

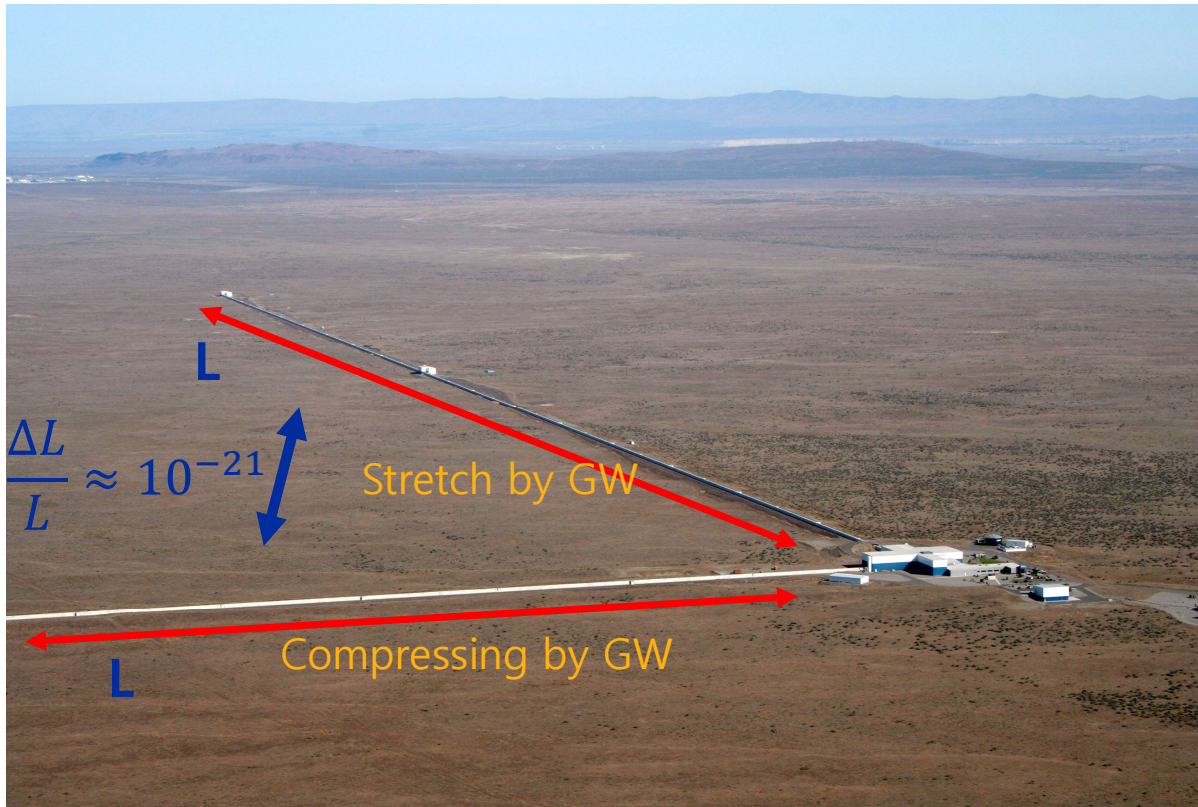
A single GW observatory is mostly insensitive to the sky location; we want two and preferably three or more observatories

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Marci Branchesi

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# Strain sensitivity

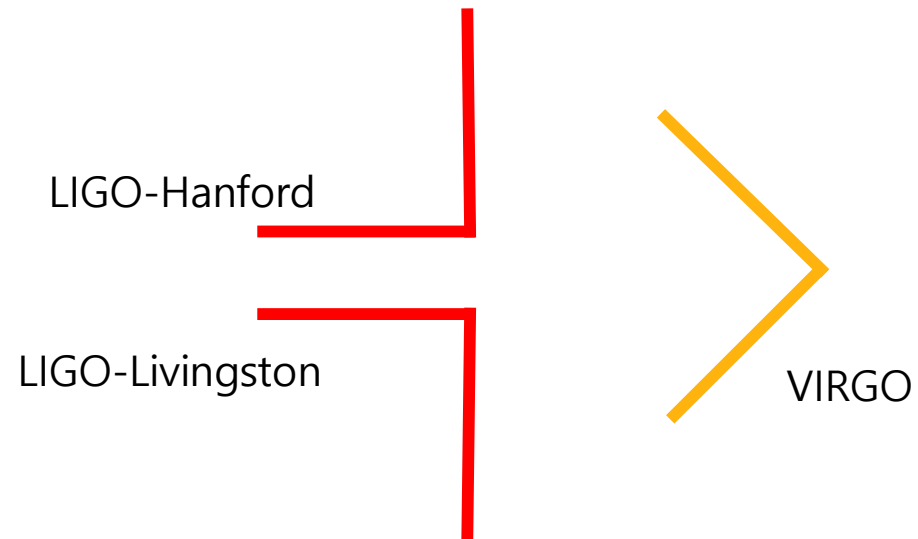


Minimum  
sensitivity

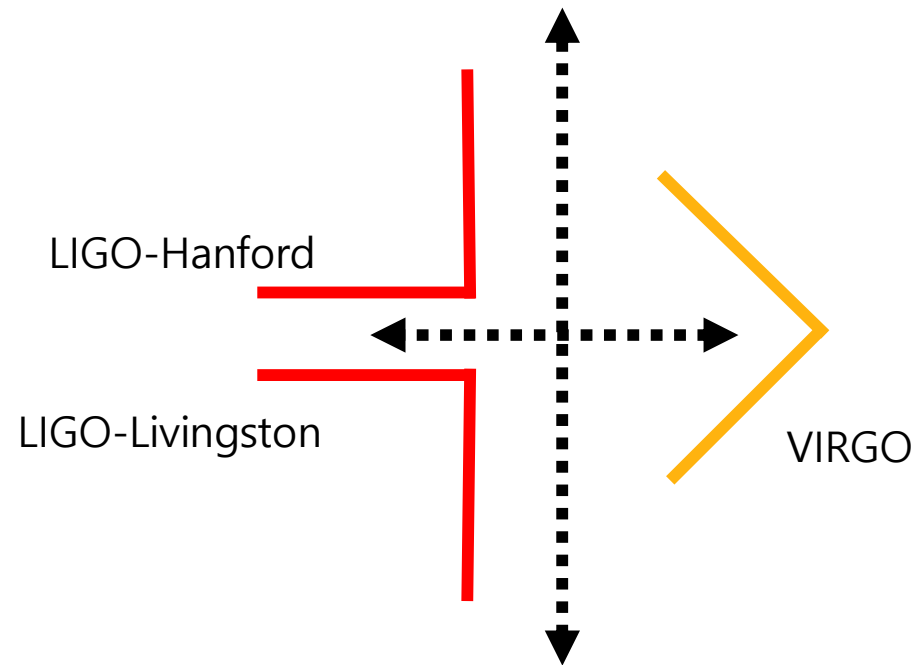
$$\frac{\Delta L}{L} \approx 10^{-2}$$



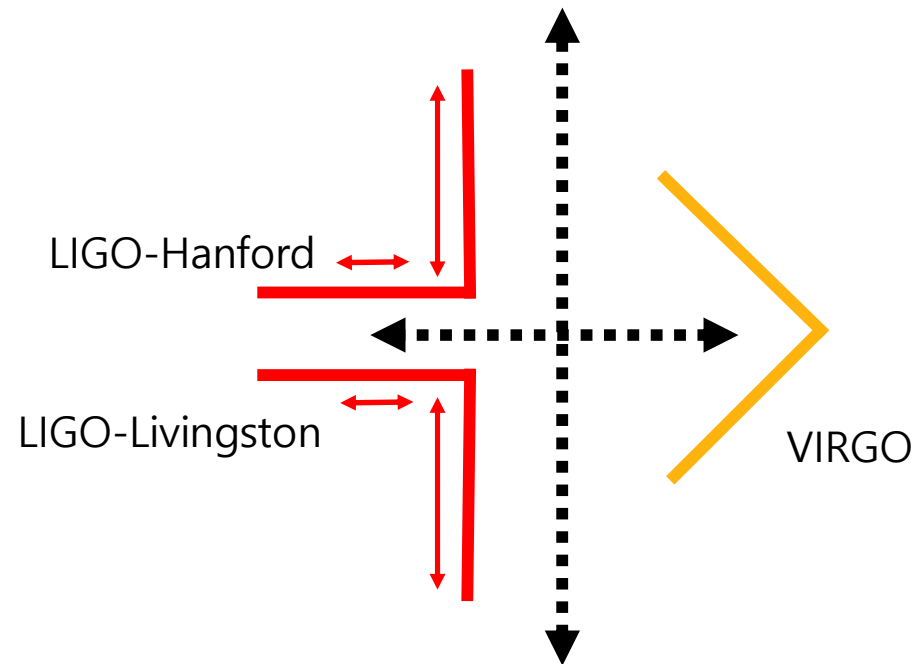
# Polarization of gravitational wave



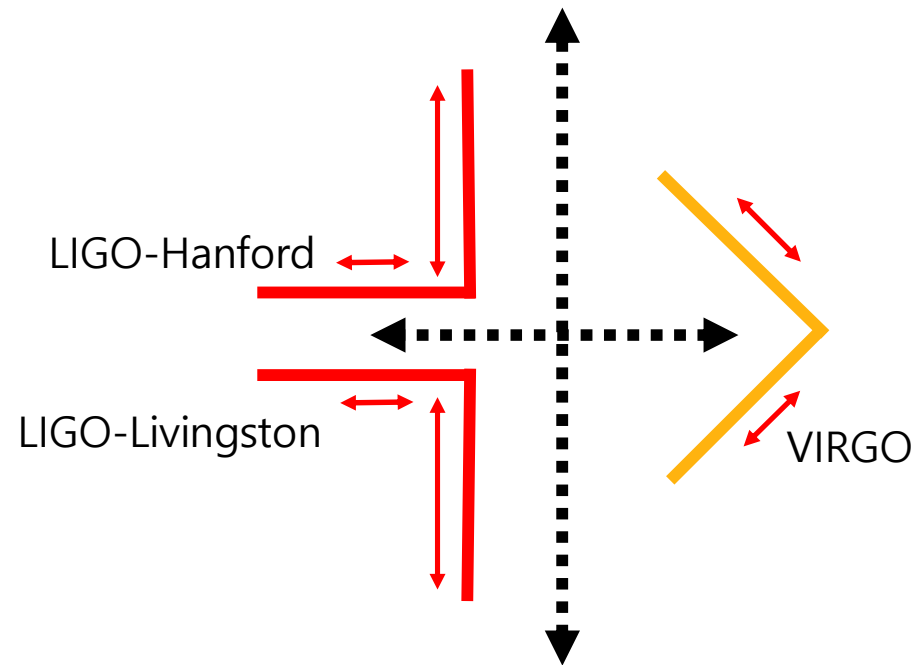
# Polarization of gravitational wave



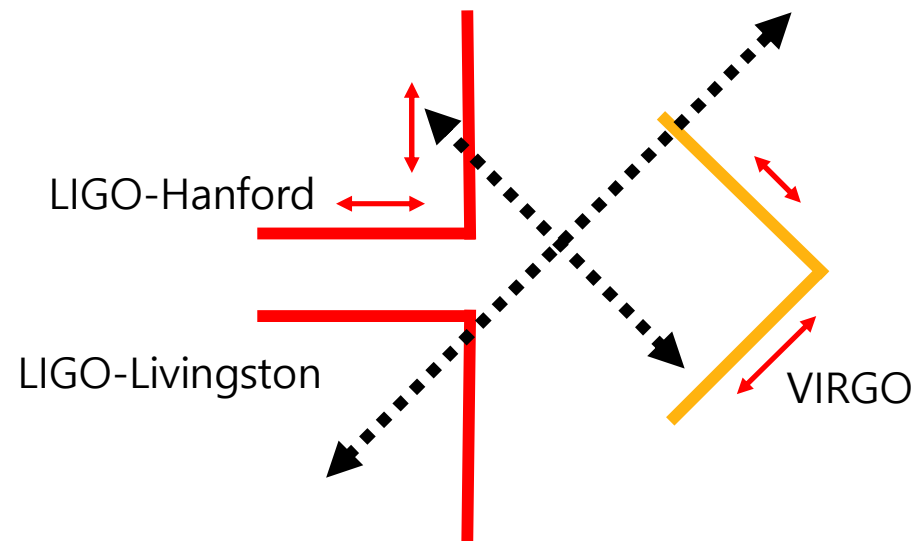
# Polarization of gravitational wave



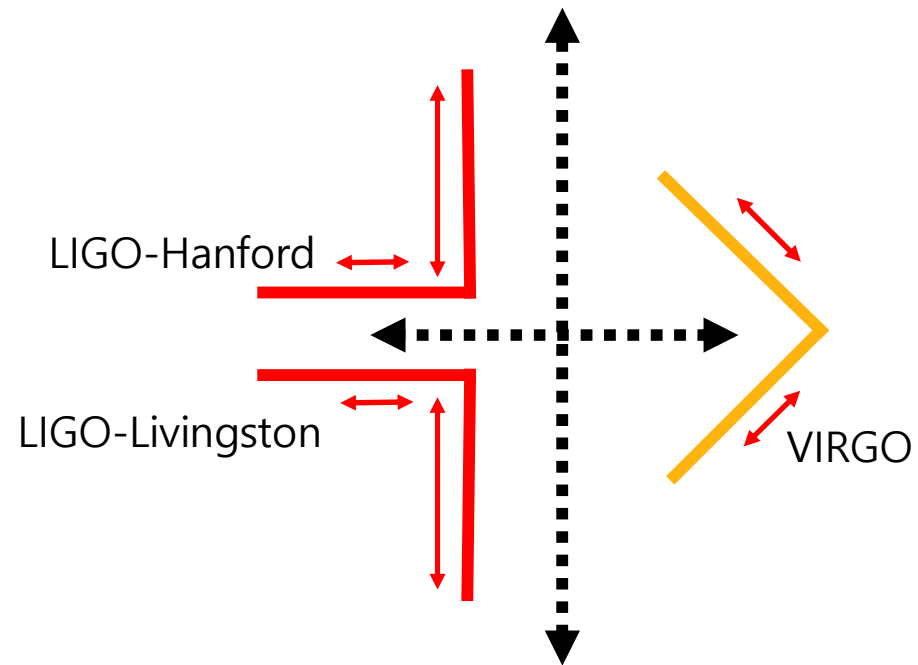
# Polarization of gravitational wave



# Polarization of gravitational wave



# Polarization of gravitational wave

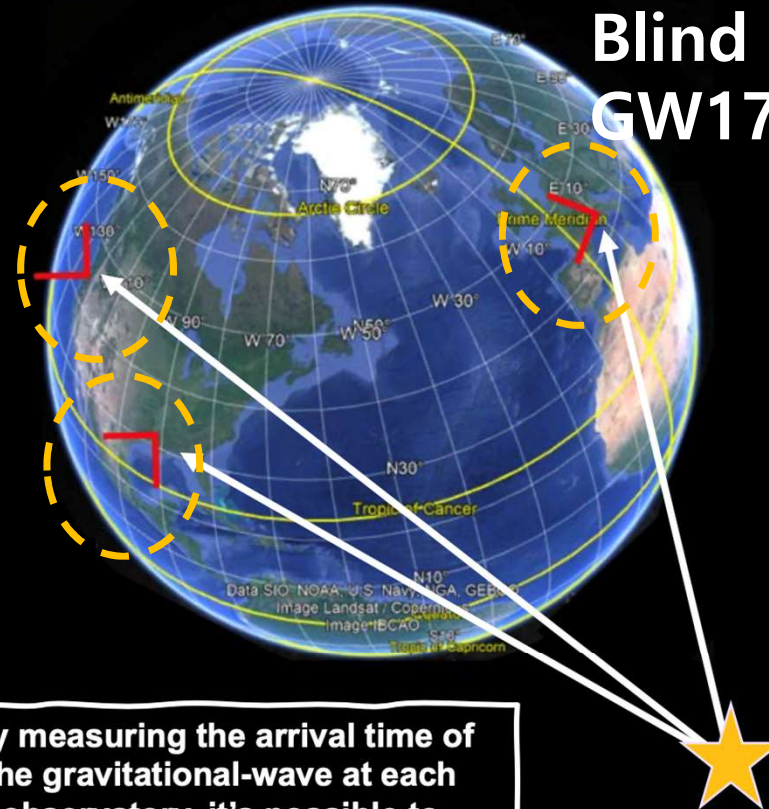






## Localizing Gravitational-wave Events

# Blind spot of GW170817



**By measuring the arrival time of the gravitational-wave at each observatory, it's possible to identify its location on the sky**

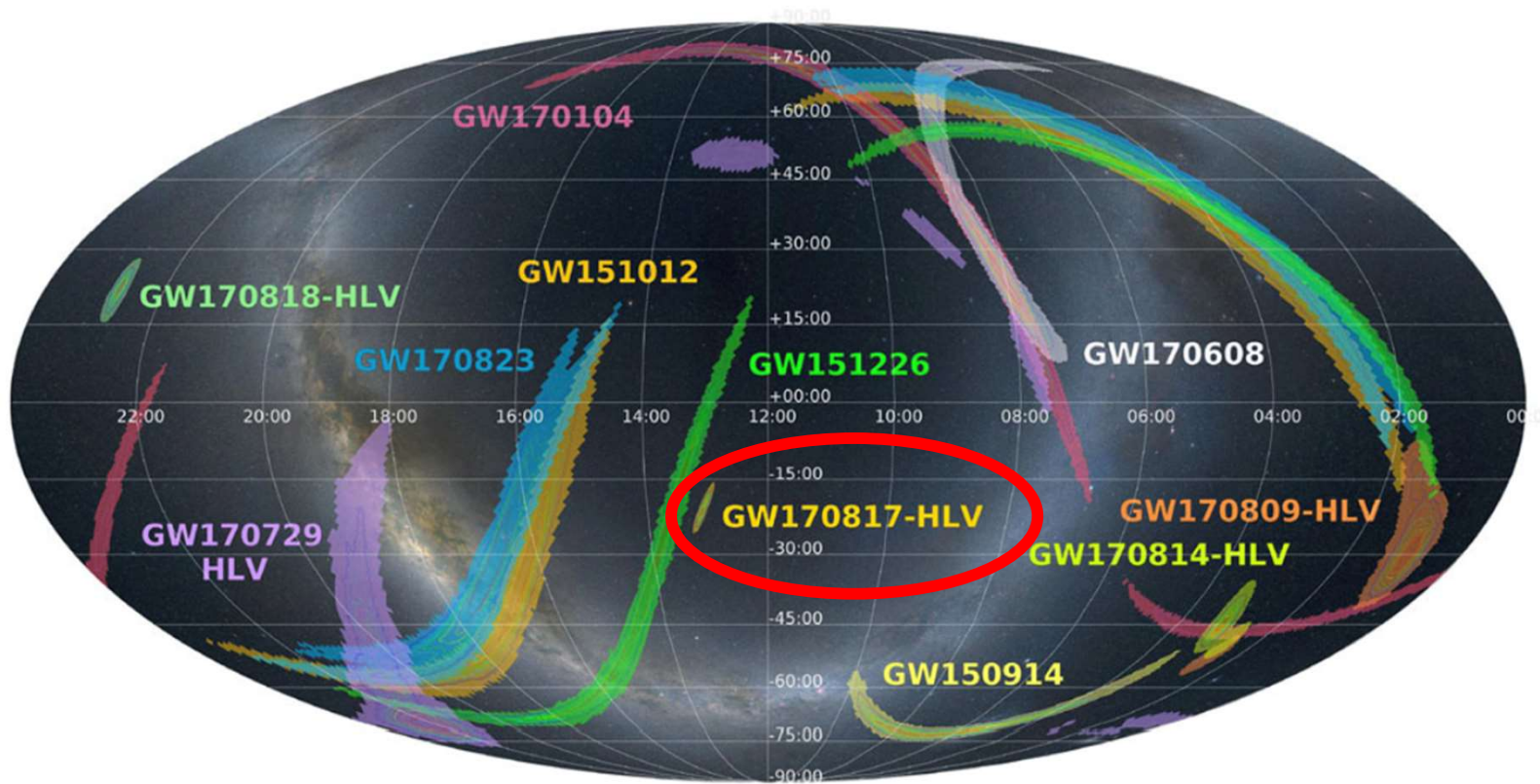
**A single GW observatory is mostly insensitive to the sky location;  
we want two and preferably three or more observatories**



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# Sky localization

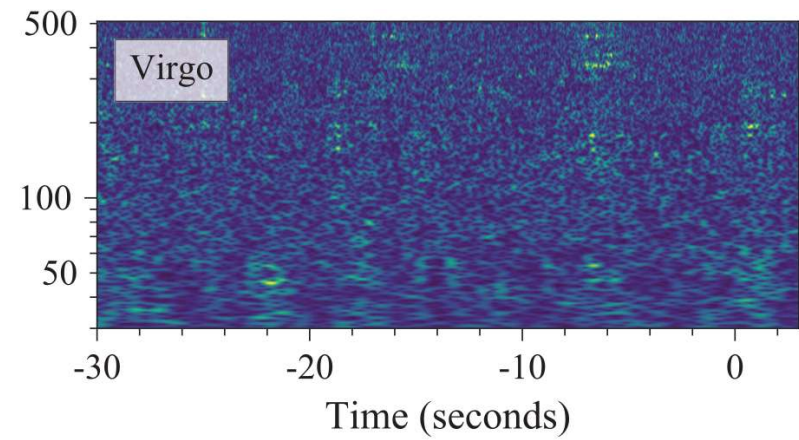
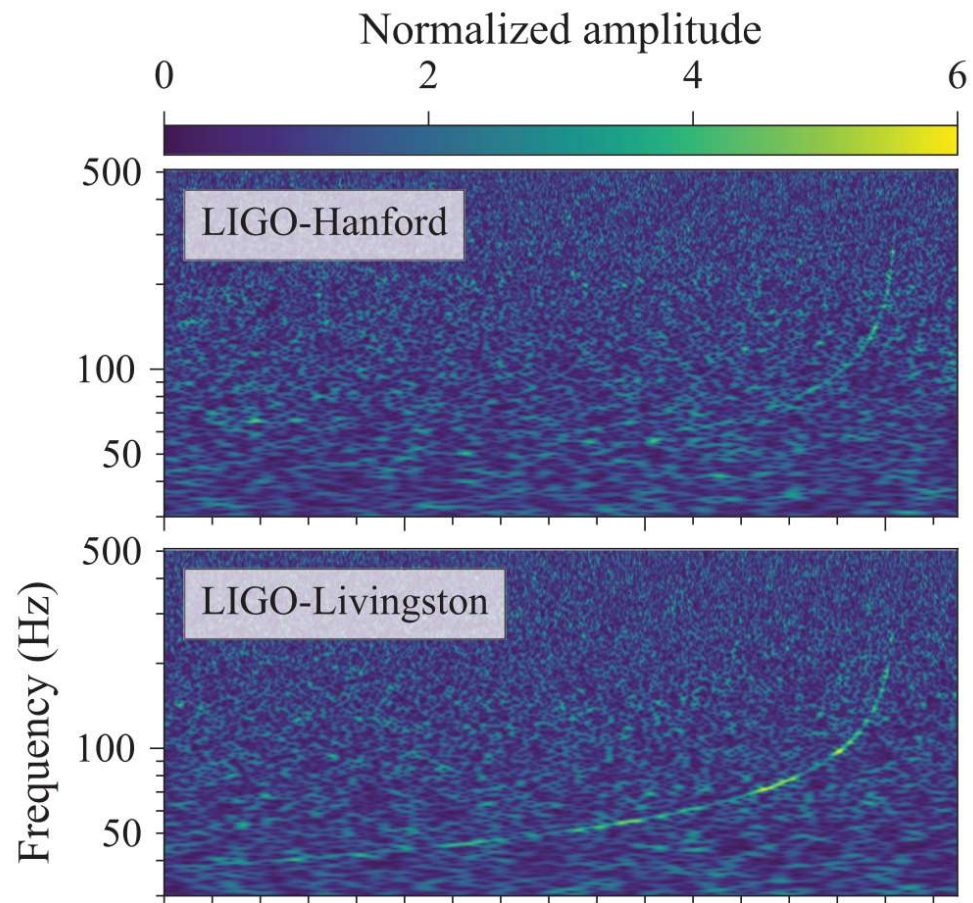


# GW170817





# GW170817

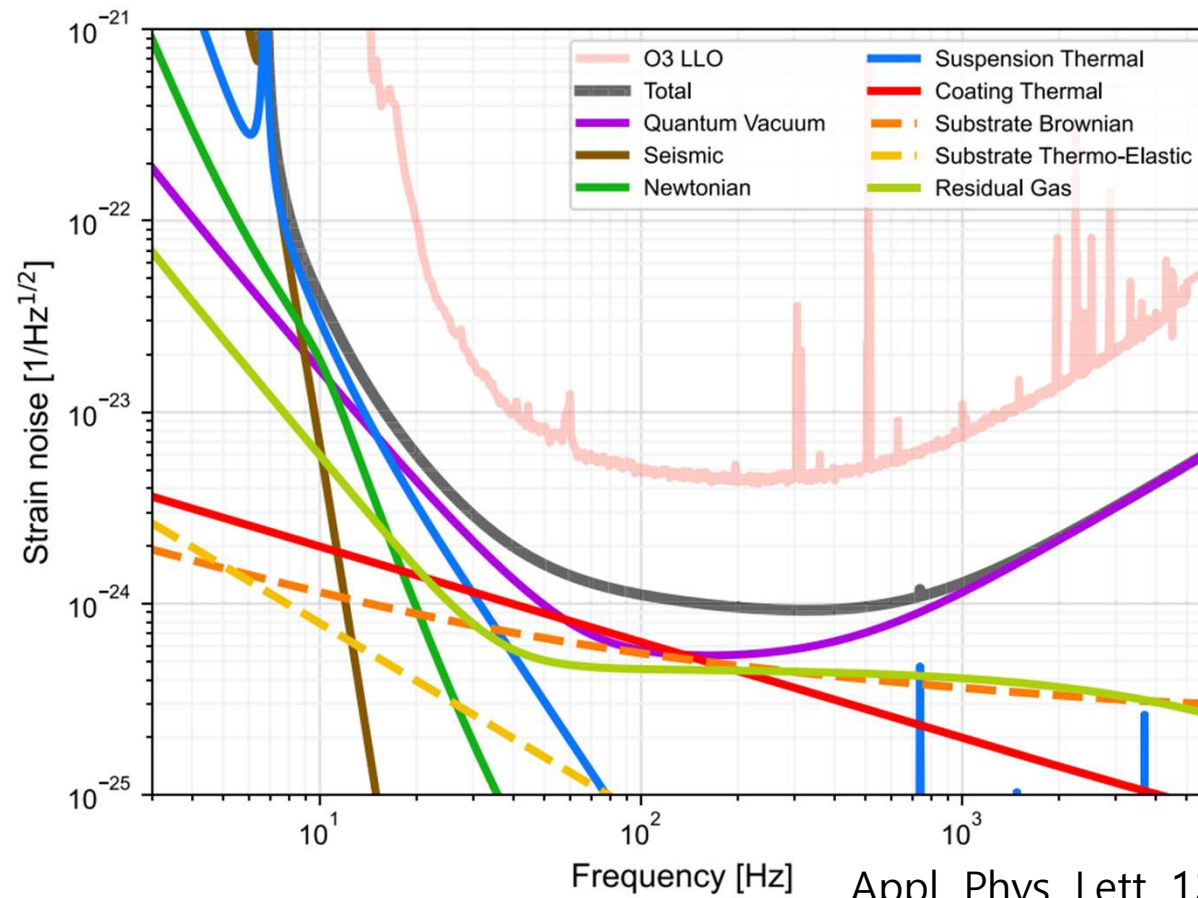




## 2. Sensitivity curve of gravitational wave detector

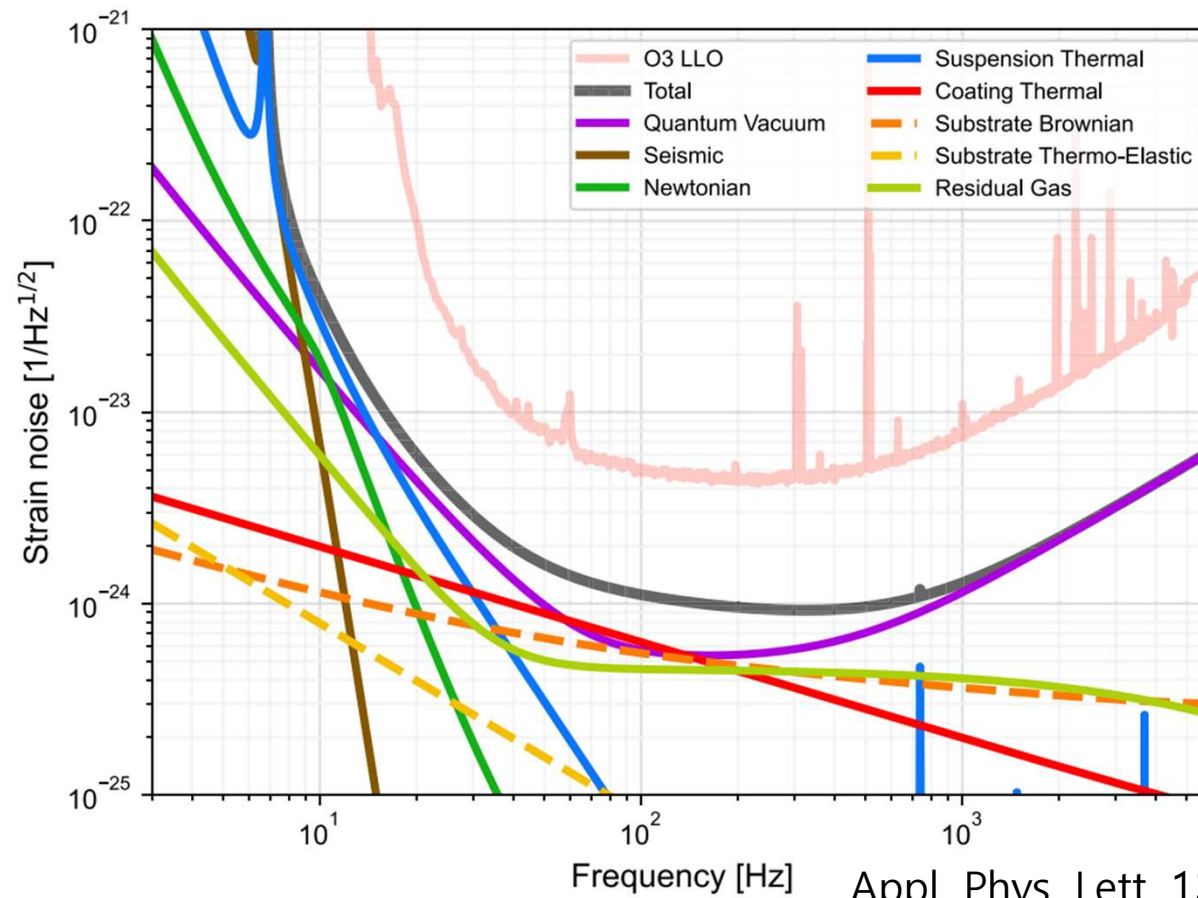


# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)

# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)



# Spectral density

🌐 19 languages ▾

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From Wikipedia, the free encyclopedia

*This article is about signal processing and relation of spectra to time-series. For further applications in the physical sciences, see [Spectrum \(physical sciences\)](#).*

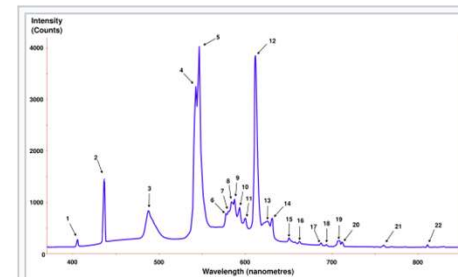
*"Spectral power density" redirects here; not to be confused with [Spectral power](#).*



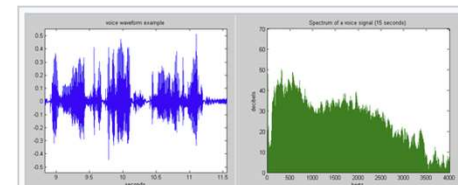
This article **may be too technical for most readers to understand**. Please [help improve it to make it understandable to non-experts](#), without removing the technical details. (June 2024) ([Learn how and when to remove this message](#))

In [signal processing](#), the power spectrum  $S_{xx}(f)$  of a [continuous time signal](#)  $x(t)$  describes the distribution of [power](#) into frequency components  $f$  composing that signal.<sup>[1]</sup> According to [Fourier analysis](#), any physical signal can be decomposed into a number of discrete frequencies, or a spectrum of frequencies over a continuous range. The statistical average of any sort of signal (including [noise](#)) as analyzed in terms of its frequency content, is called its [spectrum](#).

When the energy of the signal is concentrated around a finite time interval, especially if its total energy is finite, one may compute the **energy spectral density**. More commonly used is the **power spectral density** (PSD, or simply **power spectrum**), which applies to signals existing over *all* time, or over a time period large enough (especially in relation to the duration of a measurement) that it could as well have been over an infinite time interval. The PSD then refers to the



The spectral density of a [fluorescent light](#) as a function of optical wavelength shows peaks at atomic transitions, indicated by the numbered arrows.



The voice waveform over time (left) has a broad audio power spectrum (right).





In [signal processing](#), the [energy](#) of a signal  $x(t)$  is given by

$$E \triangleq \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

Assuming the total energy is finite (i.e.  $x(t)$  is a [square-integrable function](#)) allows applying [Parseval's theorem](#) (or [Plancherel's theorem](#)).<sup>[6]</sup> That is,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{x}(f)|^2 df,$$

where

$$\hat{x}(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft} x(t) dt,$$

is the [Fourier transform](#) of  $x(t)$  at [frequency  \$f\$](#)  (in [Hz](#)).<sup>[7]</sup> The theorem also holds true in the discrete-time cases. Since the integral on the left-hand side is the energy of the signal, the value of  $|\hat{x}(f)|^2 df$  can be interpreted as a [density function](#) multiplied by an infinitesimally small frequency interval, describing the energy contained in the signal at frequency  $f$  in the frequency interval  $f + df$ .

Therefore, the **energy spectral density** of  $x(t)$  is defined as:<sup>[8]</sup>

$$\bar{S}_{xx}(f) \triangleq |\hat{x}(f)|^2 \text{ (Eq.1)}$$

The function  $\bar{S}_{xx}(f)$  and the [autocorrelation](#) of  $x(t)$  form a Fourier transform pair, a result also known as the [Wiener–Khinchin theorem](#) (see also [Periodogram](#)).

As a physical example of how one might measure the energy spectral density of a signal, suppose  $V(t)$  represents the [potential](#) (in [volts](#)) of an electrical pulse propagating along a [transmission line](#) of [impedance  \$Z\$](#) , and suppose the line is terminated with a [matched](#) resistor (so that all of the pulse energy is delivered to the resistor and none is reflected back). By [Ohm's law](#), the power delivered to the resistor at time  $t$  is equal to  $V(t)^2 / Z$ , so the total energy is found by integrating  $V(t)^2 / Z$  with respect to time over the duration of the pulse. To find the value of the energy spectral density



In **signal processing**, the **energy** of a signal  $x(t)$  is given by

$$E \triangleq \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

Assuming the total energy is finite (i.e.  $x(t)$  is a **square-integrable function**) allows applying **Parseval's theorem** (or **Plancherel's theorem**).<sup>[6]</sup> That is,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{x}(f)|^2 df,$$

where

$$\hat{x}(f) = \int_{-\infty}^{\infty} e^{-i2\pi ft} x(t) dt,$$

is the **Fourier transform** of  $x(t)$  at **frequency**  $f$ . The integral on the left-hand side is the energy of  $x(t)$  multiplied by an infinitesimally small frequency interval  $f + df$ .

Therefore, the **energy spectral density** of  $x$

$$\bar{S}_{xx}(f) \triangleq |\hat{x}(f)|^2 \text{ (Eq.1)}$$

The function  $\bar{S}_{xx}(f)$  and the **autocorrelation** function are related by the **Khinchin theorem** (see also **Periodogram**).

As a physical example of how one might measure the **potential** (in **volts**) of an electrical pulse propagating along a **transmission line** of **impedance**  $Z$ , and suppose the line is terminated with a **matched** resistor (so that all of the pulse energy is delivered to the resistor and none is reflected back). By **Ohm's law**, the power delivered to the resistor at time  $t$  is equal to  $V(t)^2/Z$ , so the total energy is found by integrating  $V(t)^2/Z$  with respect to time over the duration of the pulse. To find the value of the energy spectral density





# Fourier transformation

---



- Fourier series
  - > basis
- Fourier transformation
- Application

# Fourier series



$$f(x) = f(x + T) \quad \text{Periodic function}$$

$$\int_a^b |f|^2 dx \quad \text{Periodic function}$$

$$g_n = \frac{1}{T} \int_{t_0}^{t_0+T} \exp\left(-\frac{2n\pi i x}{T}\right) f(x) dx, \quad t_0 \in \mathbb{R}.$$

$$f(x) = \sum_{n=-\infty}^{\infty} g_n \exp\left(\frac{2n\pi i x}{T}\right).$$

# Fourier series



$$f(x) = \sum_{n=-\infty}^{\infty} g_n \exp\left(\frac{2n\pi i x}{T}\right).$$

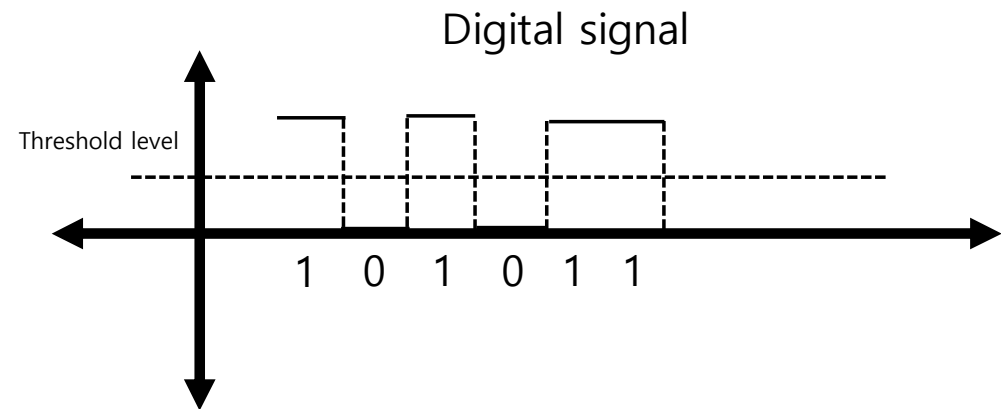
$$f(x) = g_0 \exp(2\pi i n_0 x) + g_1 \exp(2\pi i n_1 x) \dots$$

# Fourier series

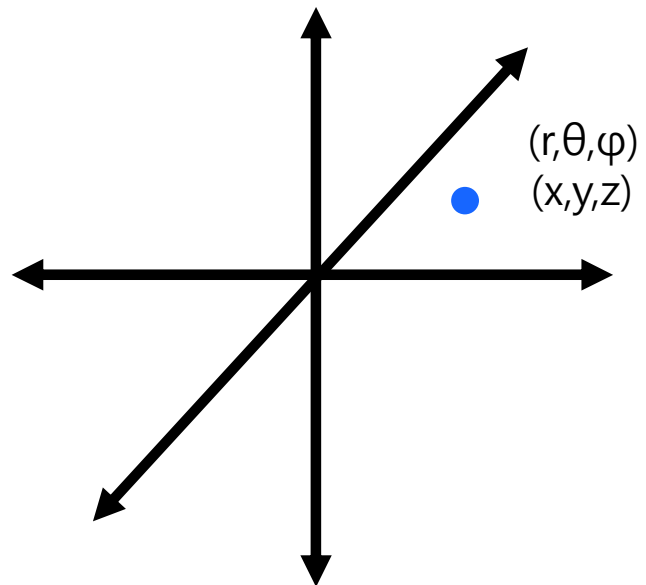


$$f(x) = \sum_{n=-\infty}^{\infty} g_n \exp\left(\frac{2n\pi i x}{T}\right).$$

$$f(x) = g_0 \exp(2\pi i n_0 x) + g_1 \exp(2\pi i n_1 x) \dots$$



# Basis



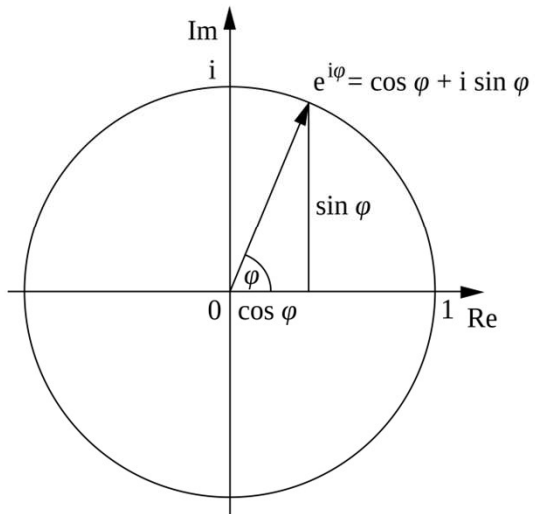
Orthogonal basis



# Fourier series



$$f(x) = \sum_{n=-\infty}^{\infty} g_n \exp\left(\frac{2n\pi i x}{T}\right).$$



# Fourier transform



$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$$

Transform

\* We use a caret (^) to denote the Fourier transform.

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi$$

Inverse transform

# Fourier transform



Fourier transformation with the angular frequency  $k$

$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{f}(k) e^{ikx} dk$$

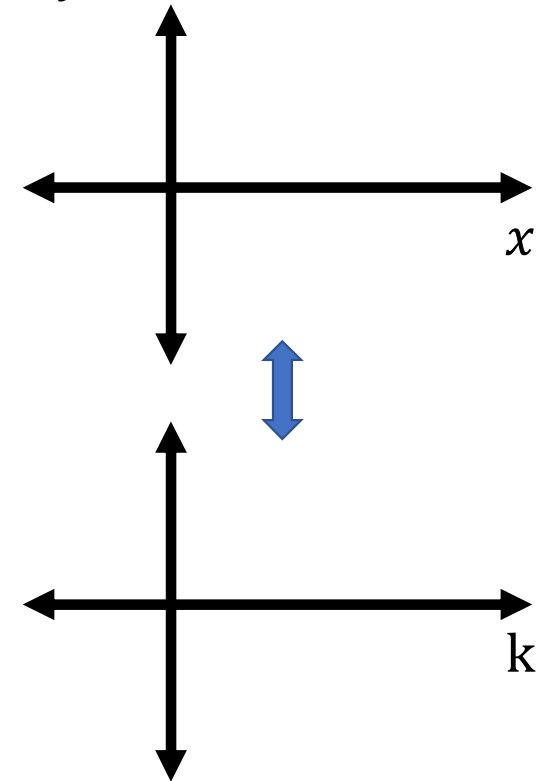
# Fourier transform



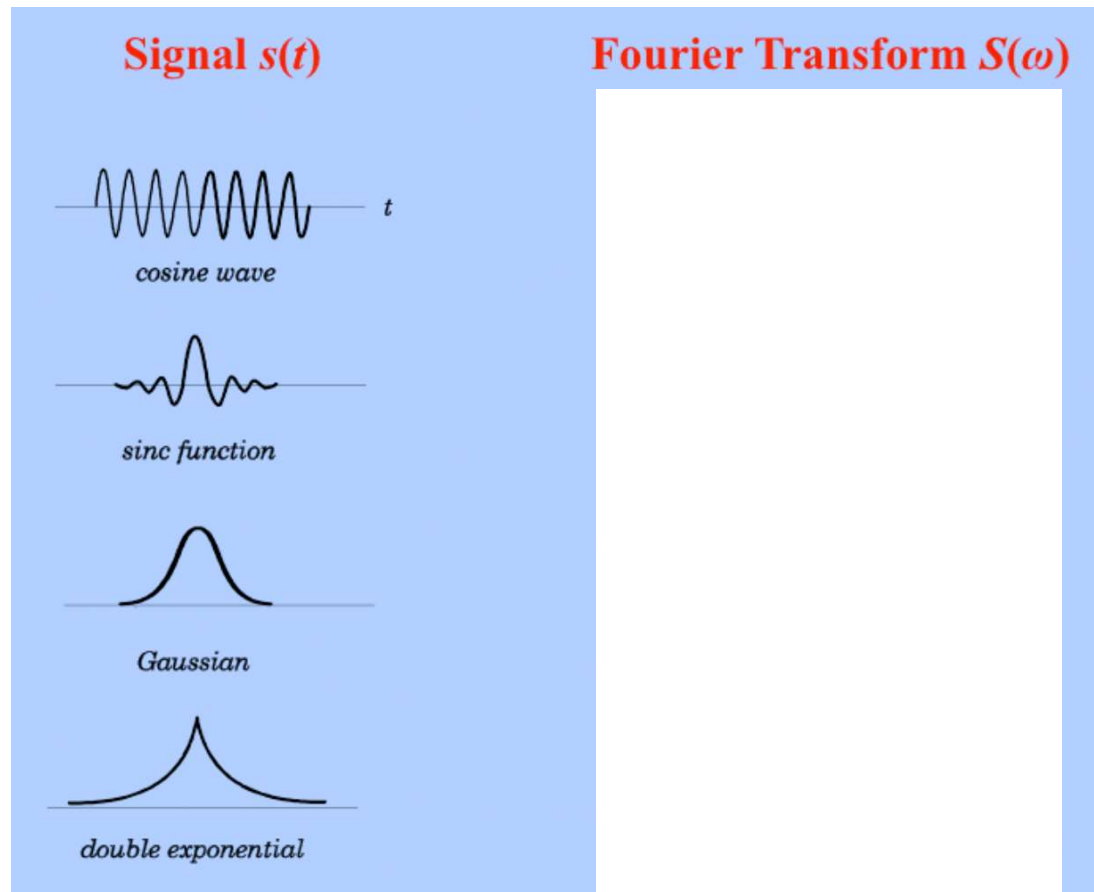
Fourier transformation with the angular frequency  $k$

$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{f}(k) e^{ikx} dk$$

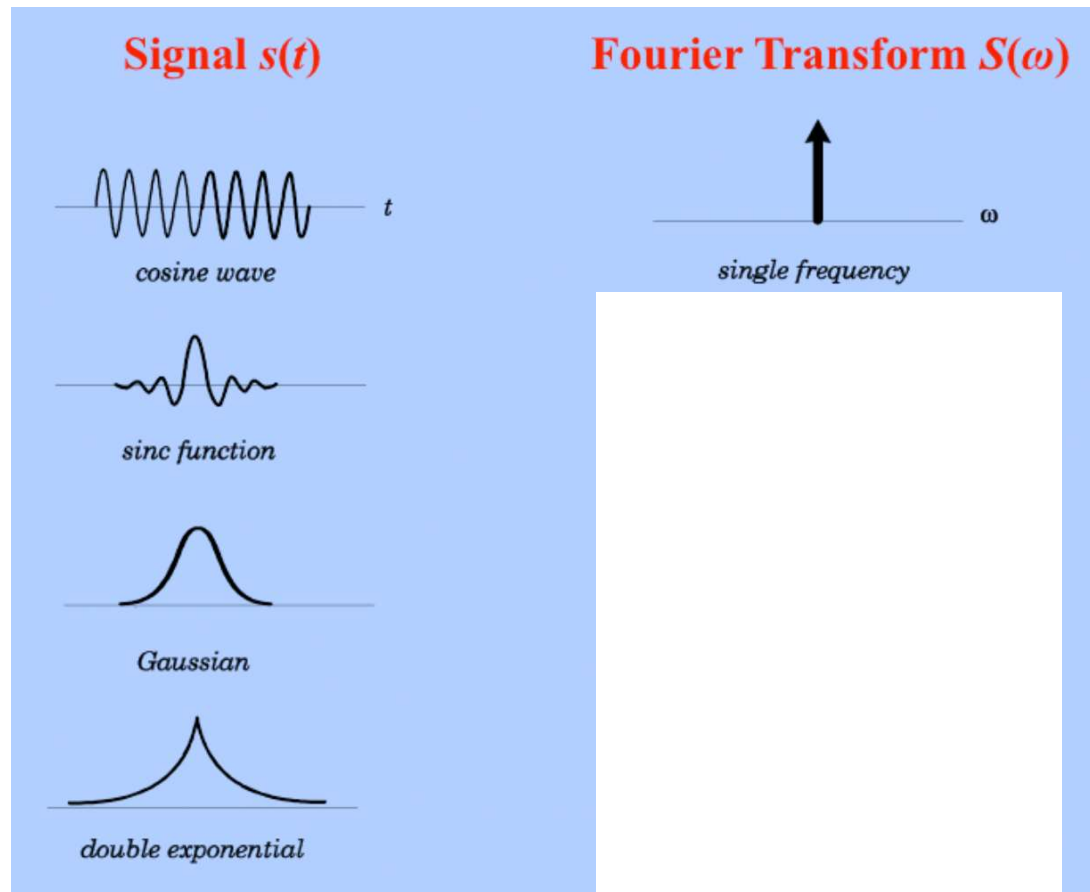


# Fourier transform

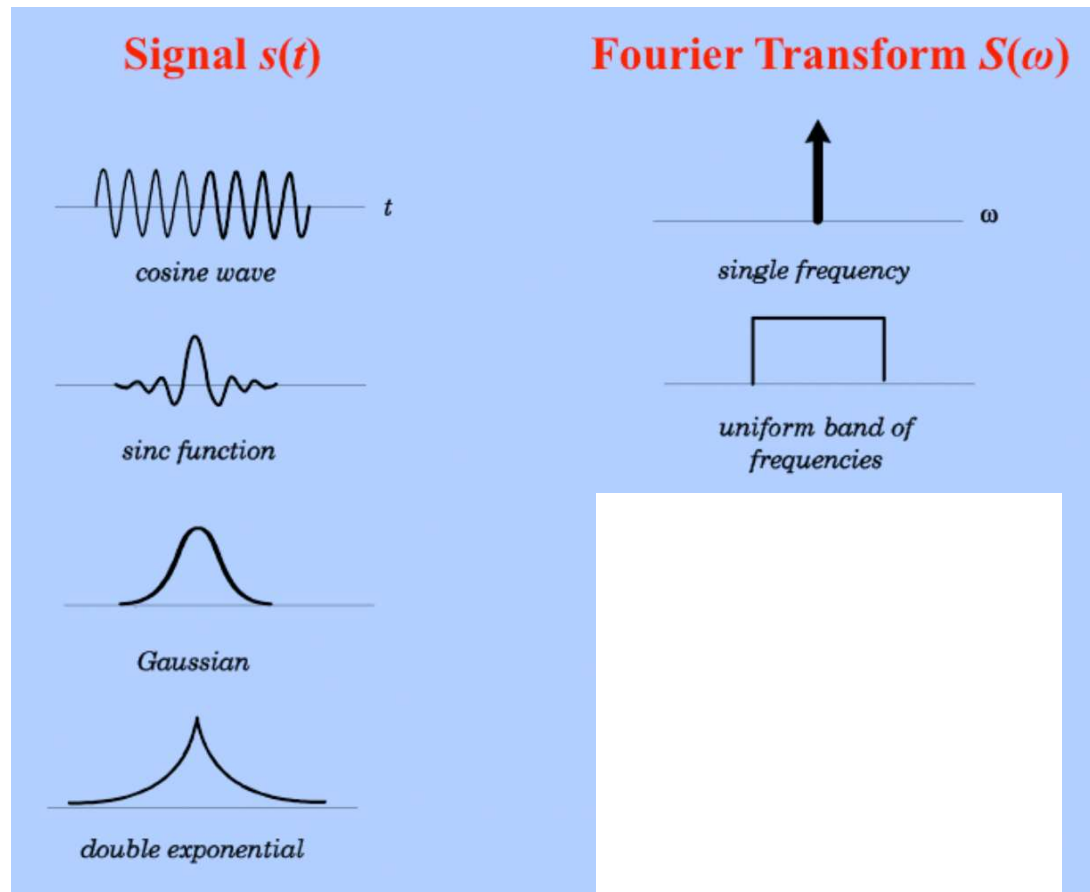




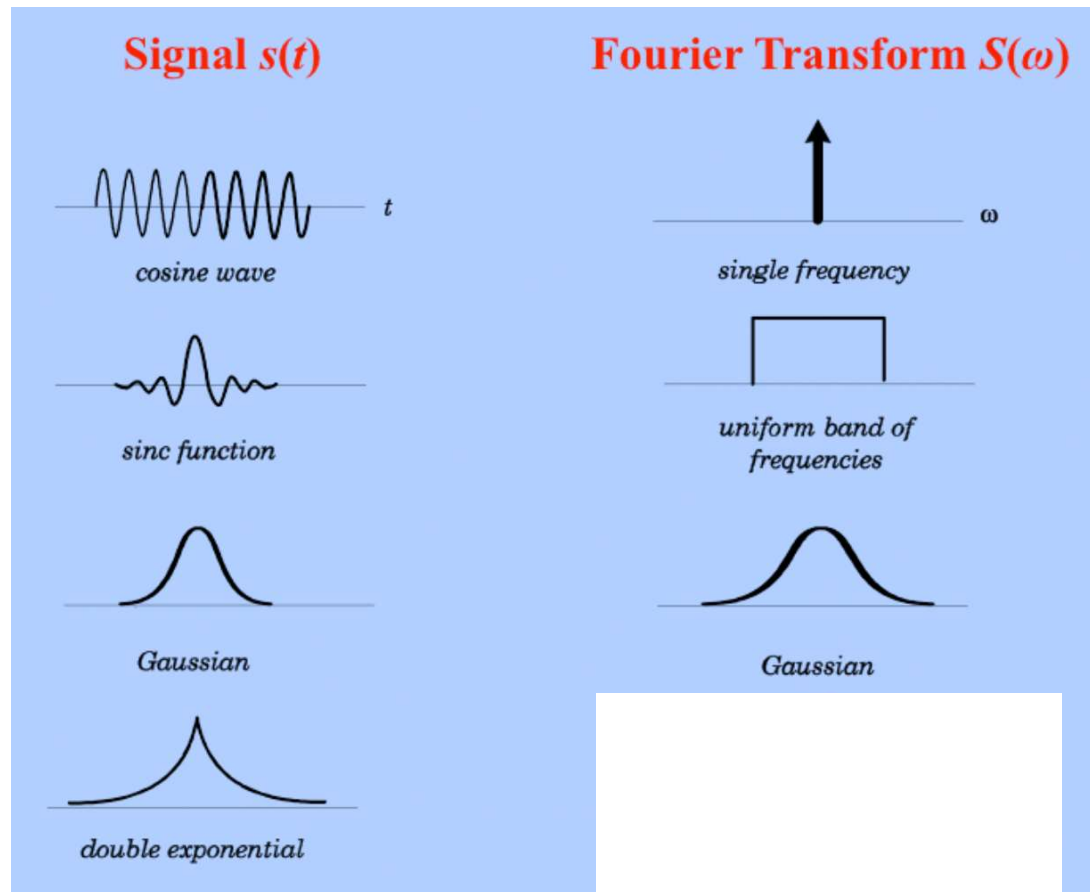
# Fourier transform



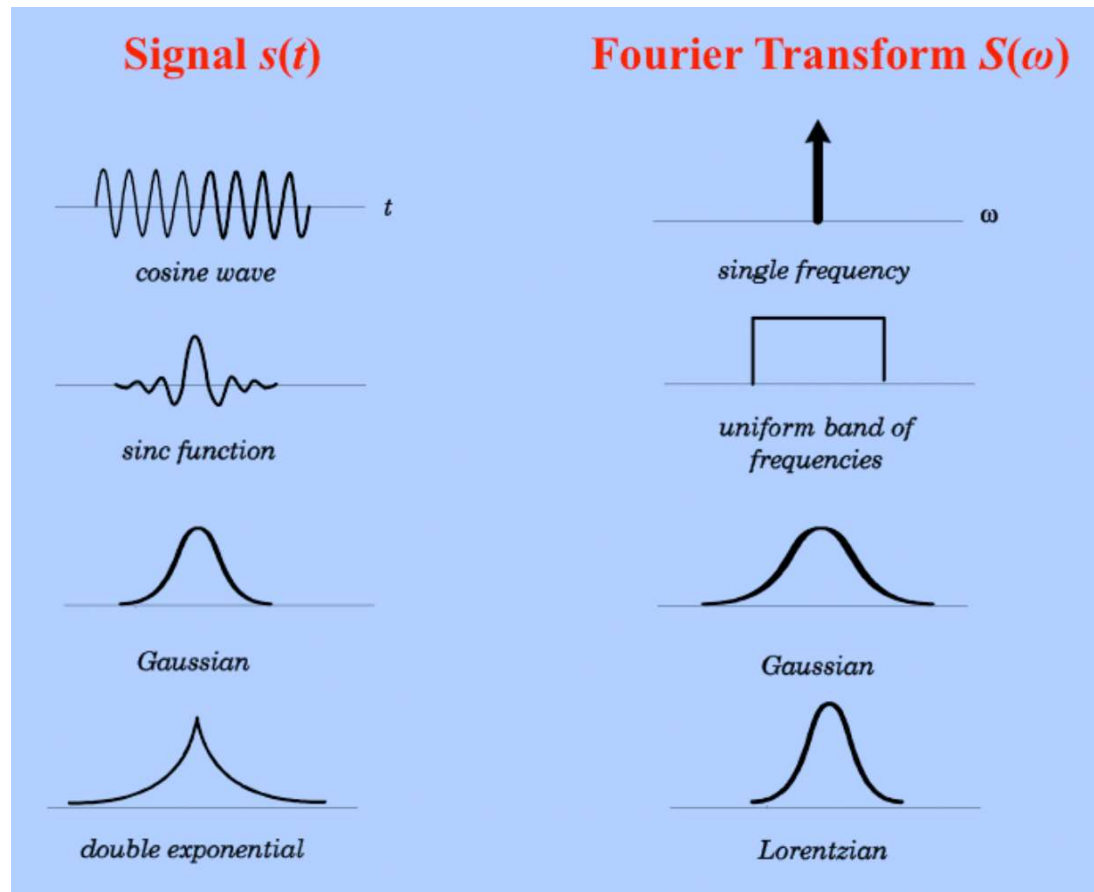
# Fourier transform



# Fourier transform



# Fourier transform



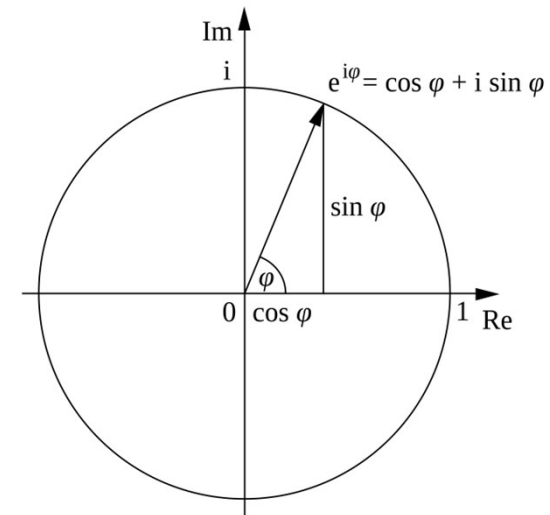
# Fourier expansion



$$\hat{f}(k) = \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

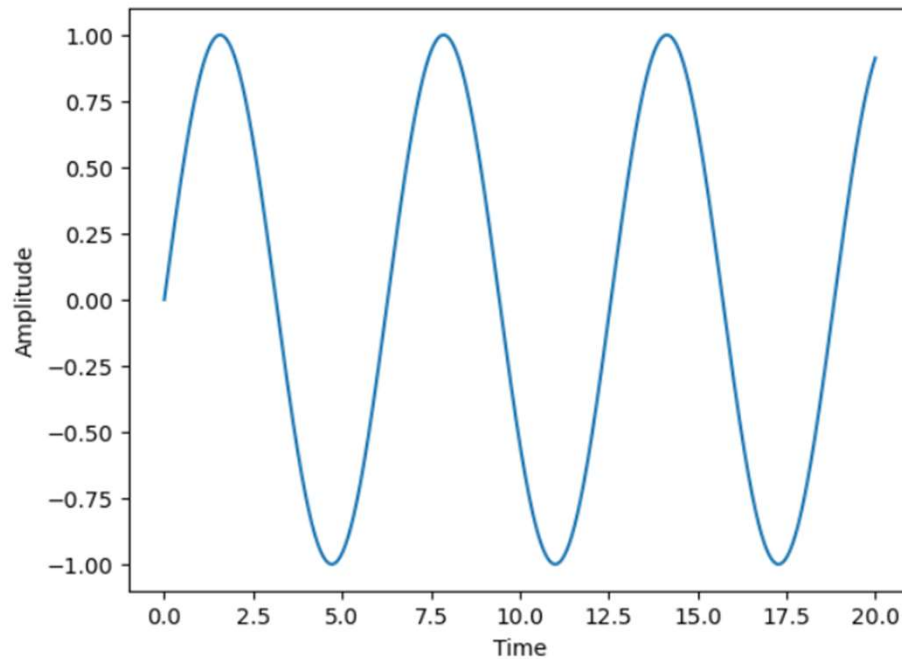


$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$

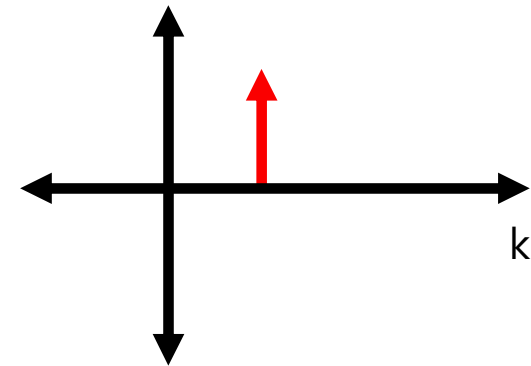




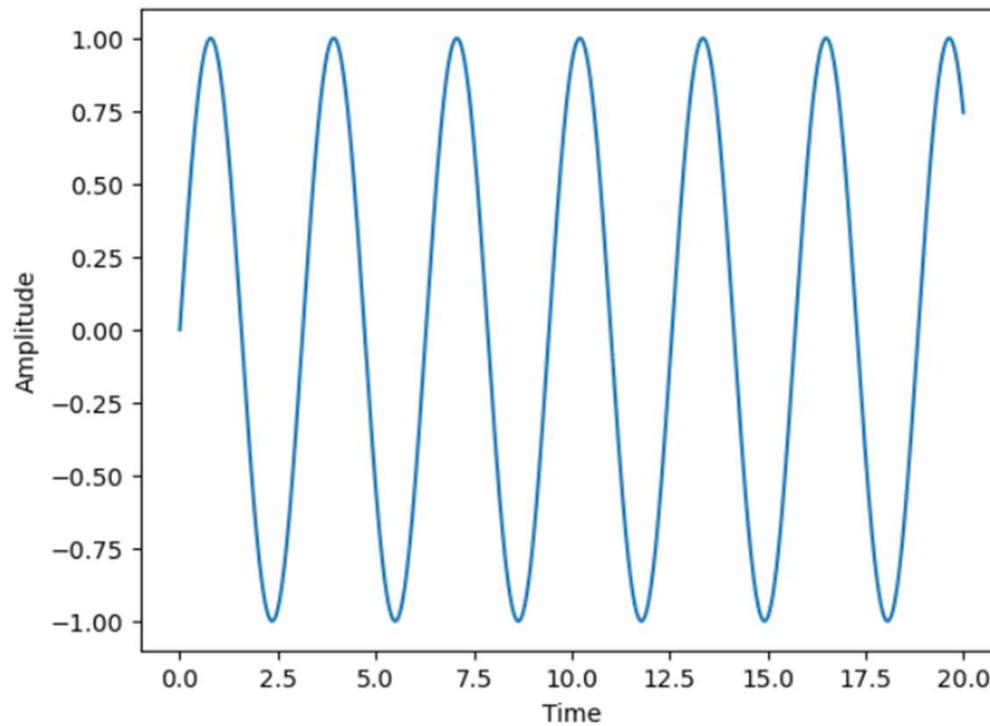
# Fourier expansion



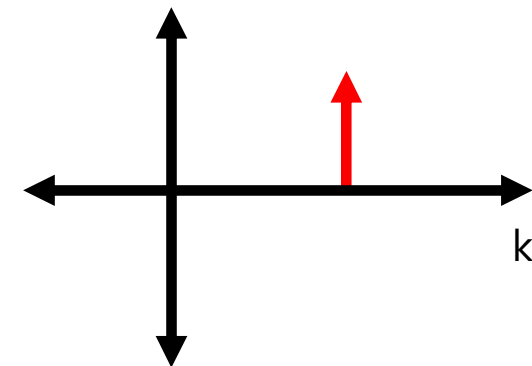
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$
$$= a \sin kx$$



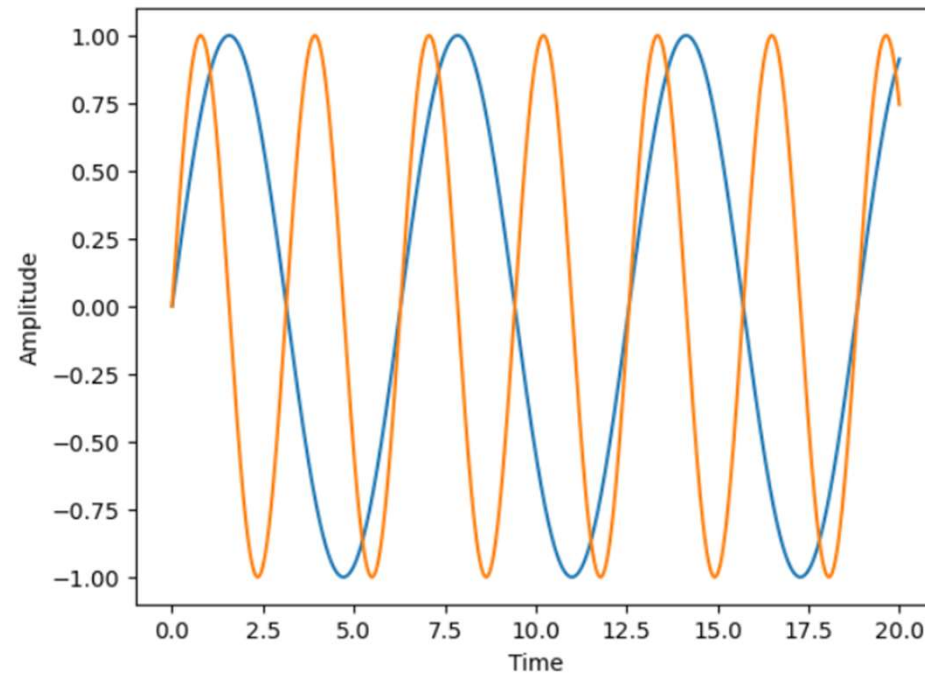
# Fourier expansion



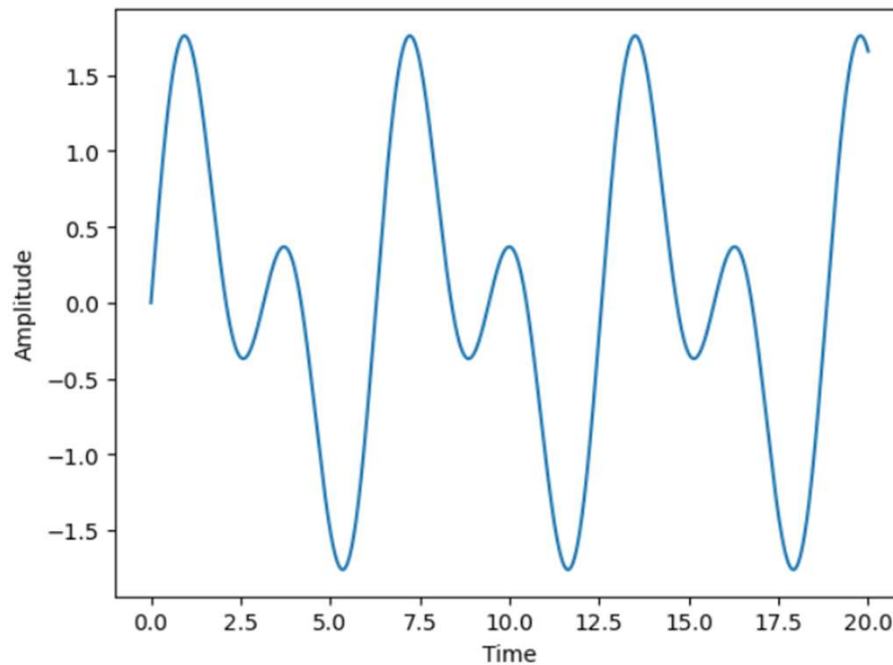
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$
$$= a \sin 2kx$$



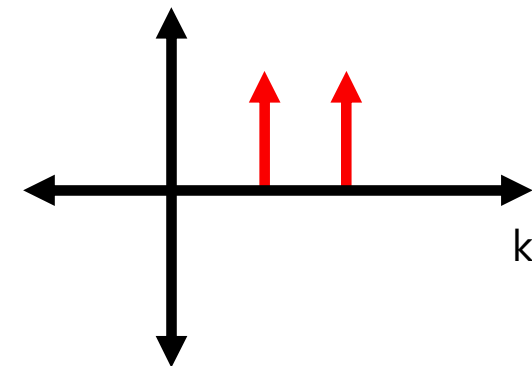
# Fourier expansion



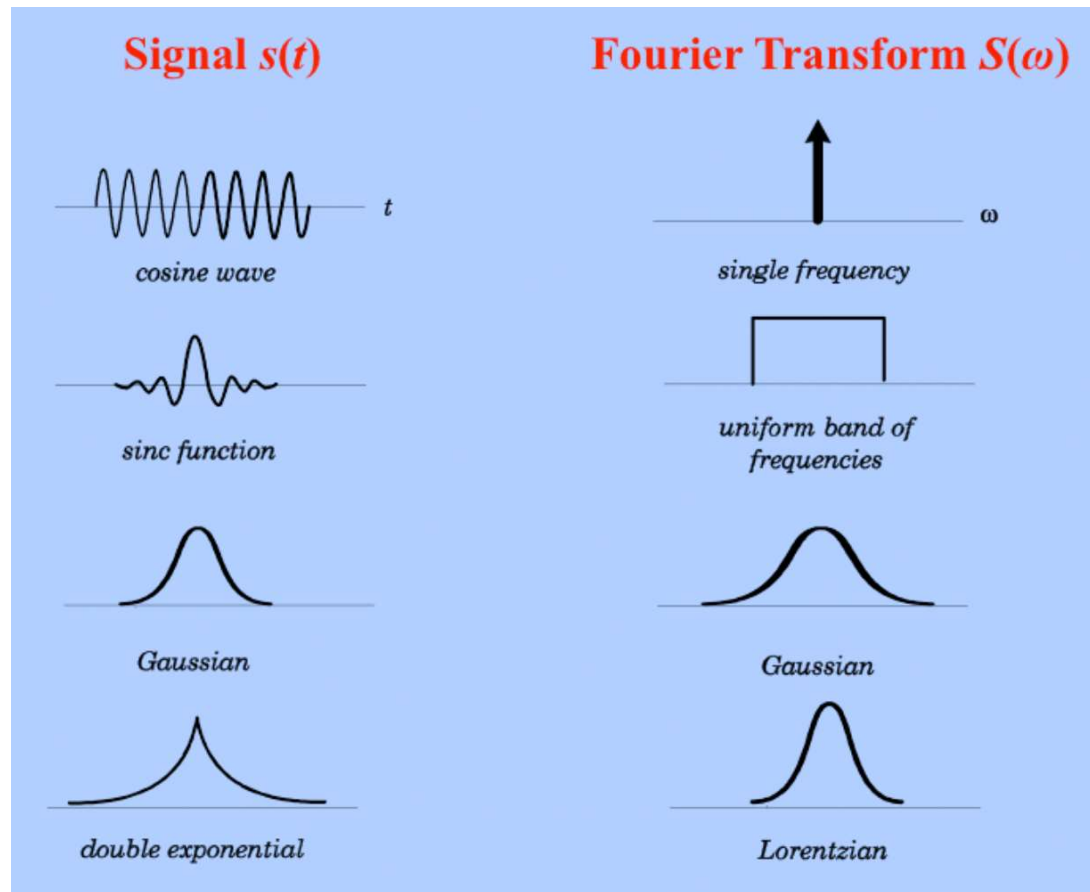
# Fourier expansion



$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$
$$= a \sin kx + a \sin 2kx$$



# Fourier transform

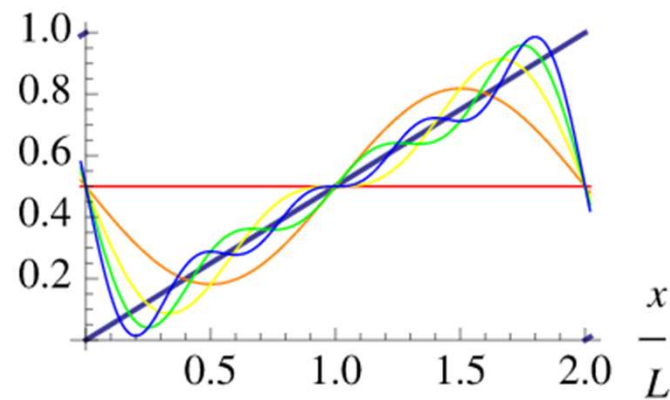




# Fourier expansion



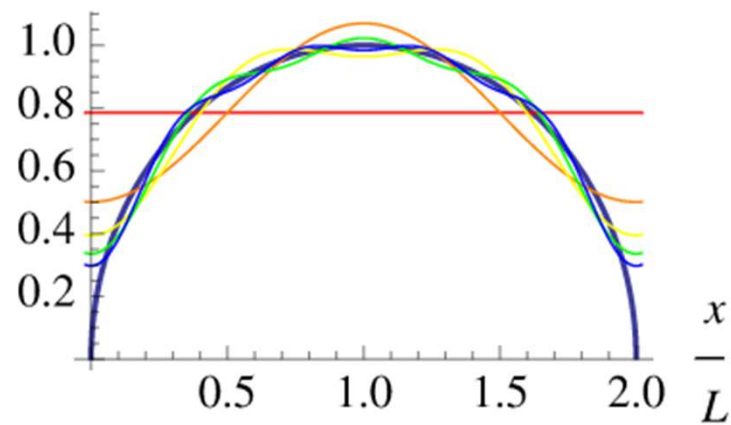
*sawtooth wave*



# Fourier expansion



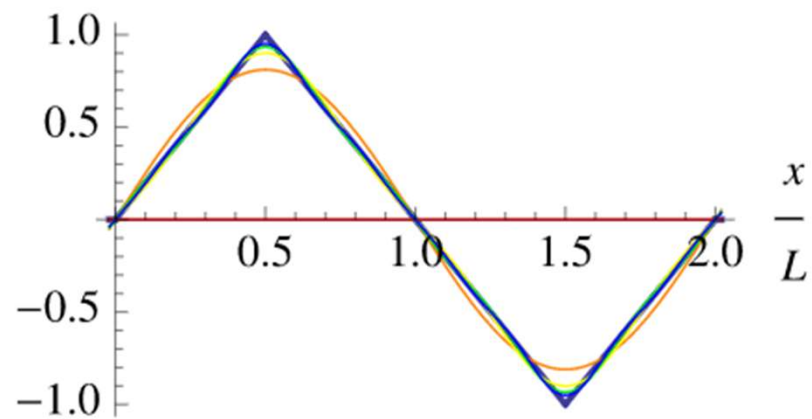
*semicircle*



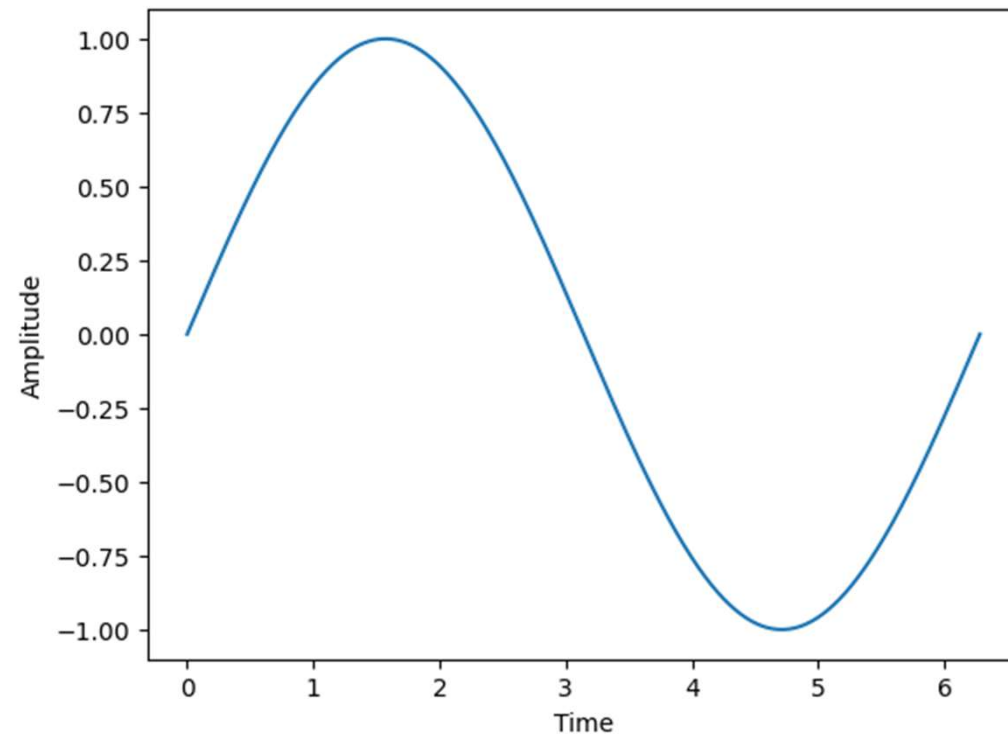
# Fourier expansion



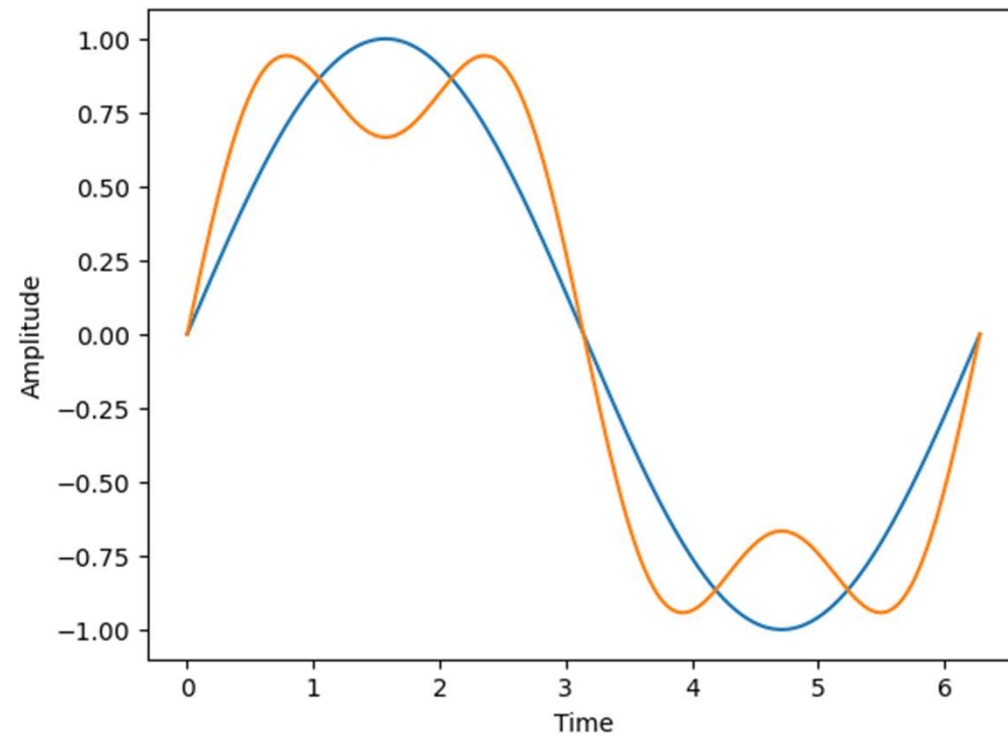
*triangle wave*



# Fourier expansion

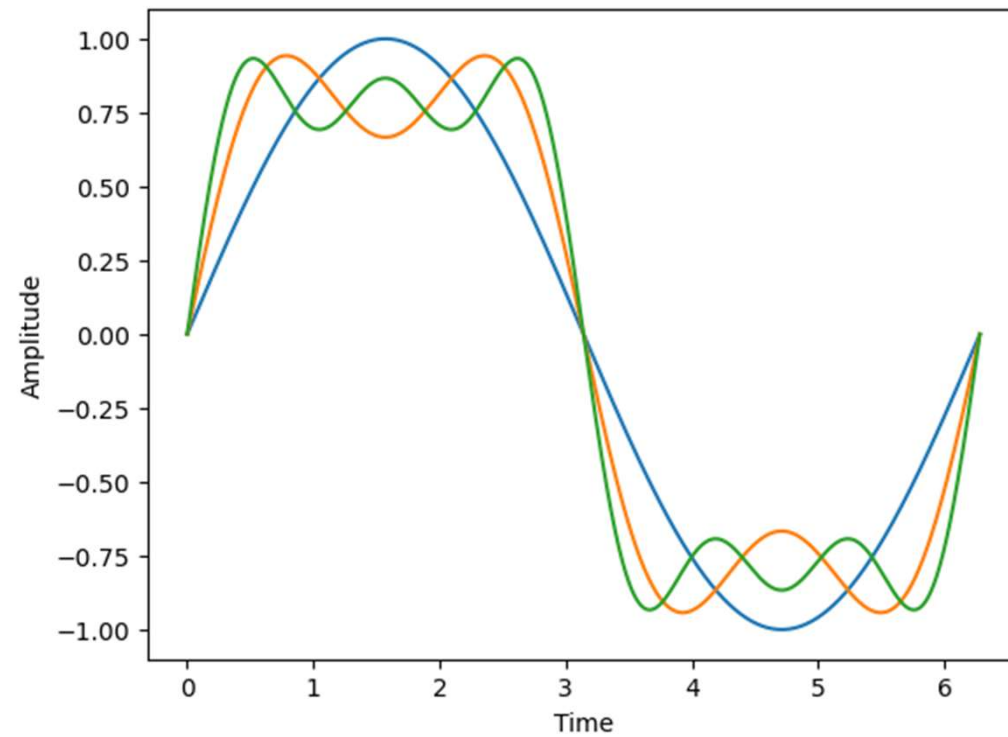


# Fourier expansion

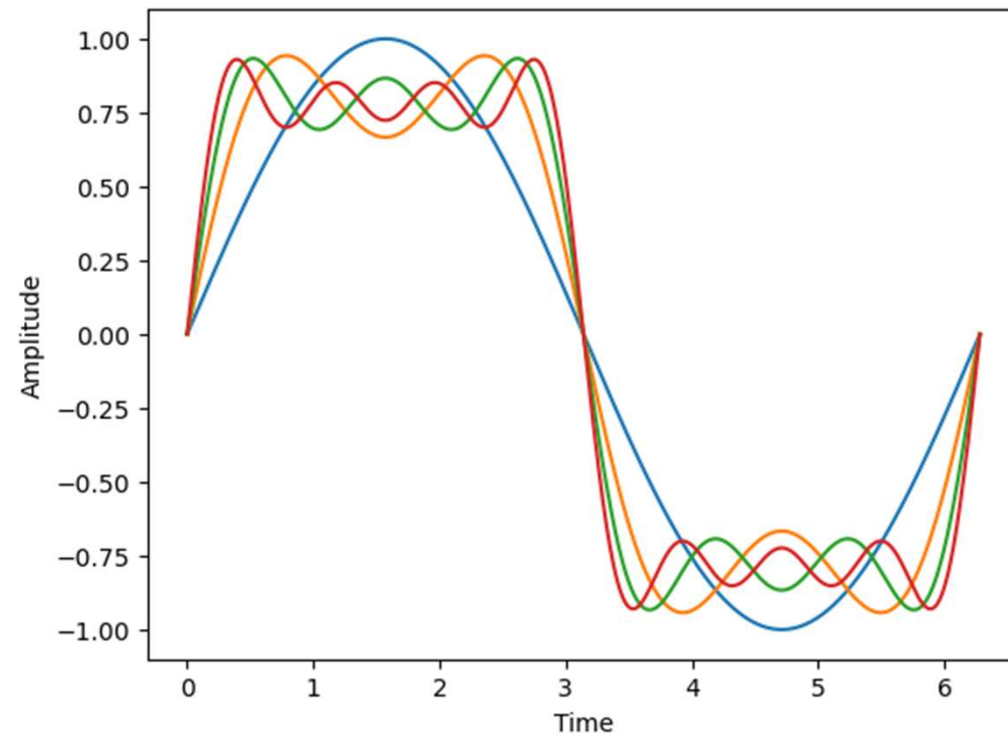




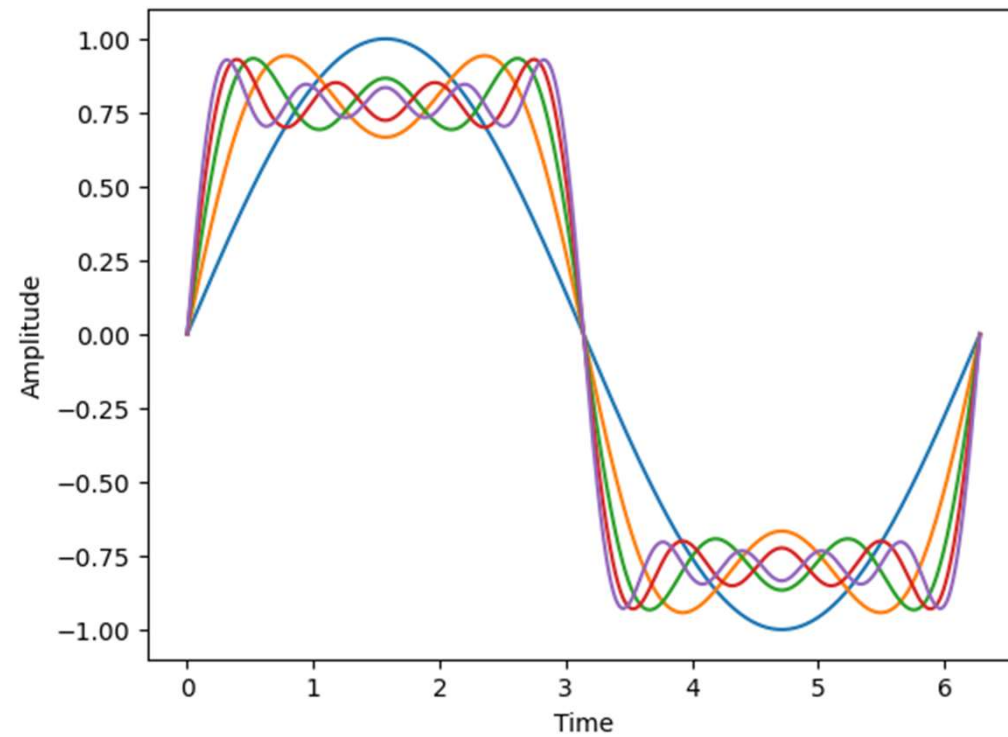
# Fourier expansion



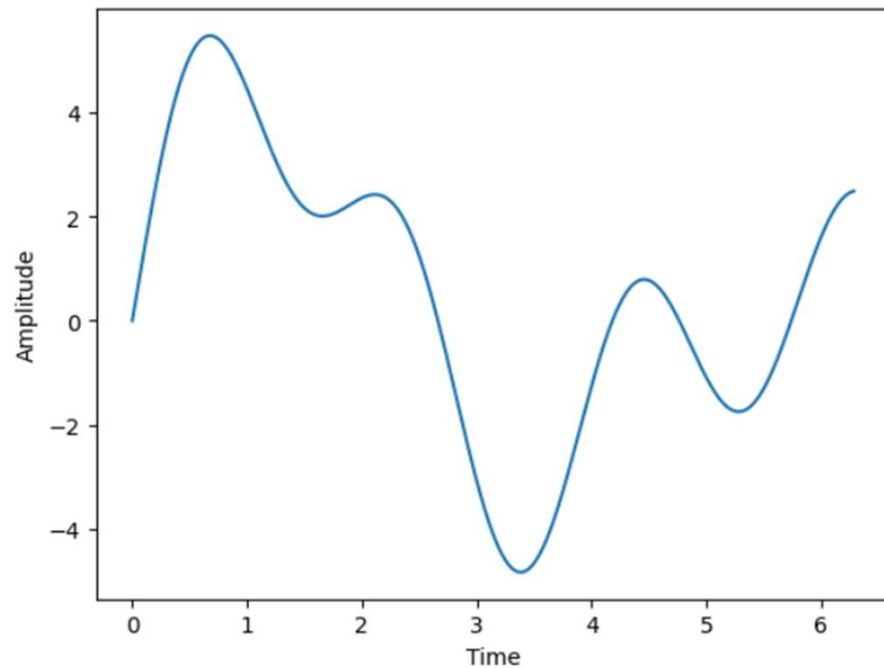
# Fourier expansion



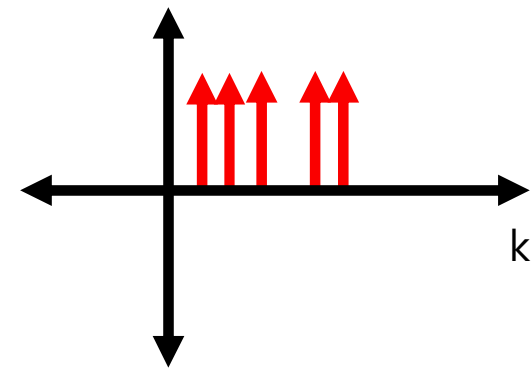
# Fourier expansion



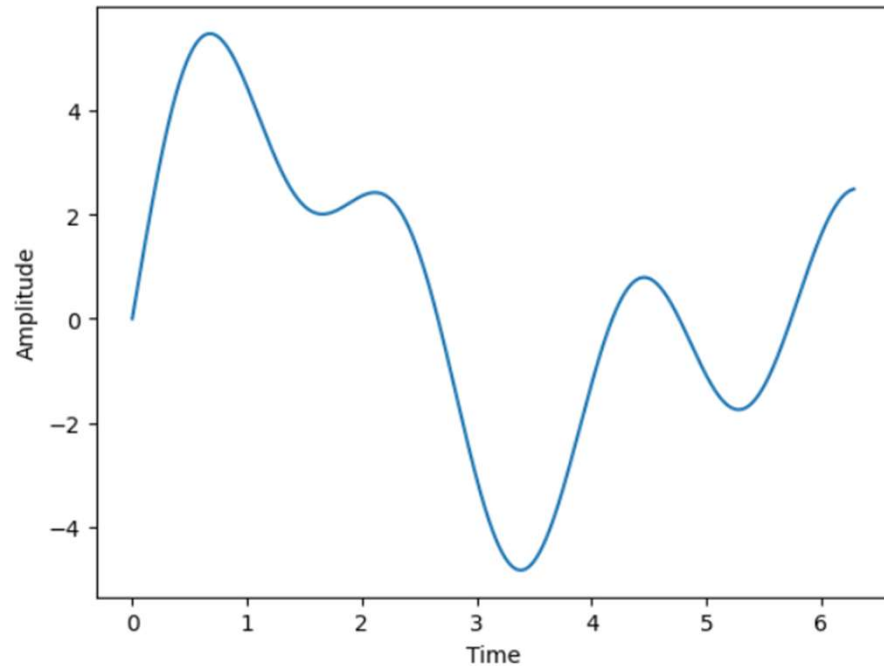
# Application of Fourier transformation



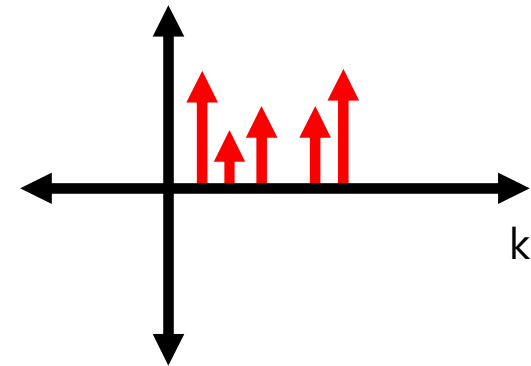
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$
$$= a \sin kx + a \sin 2kx$$



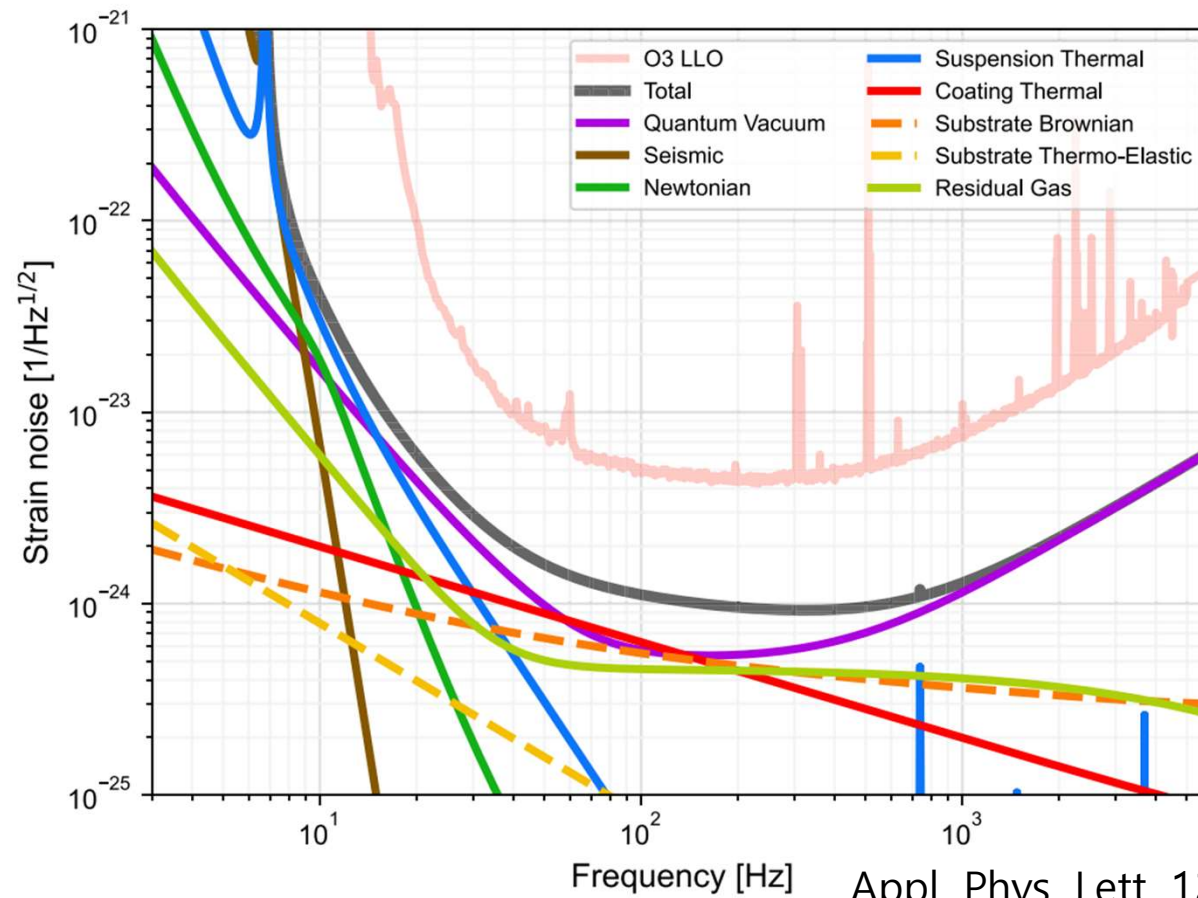
# Application of Fourier transformation



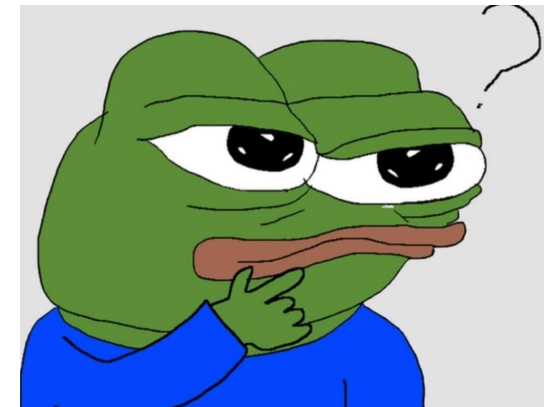
$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{inx}$$
$$= a \sin kx + a \sin 2kx$$



# LIGO sensitivity

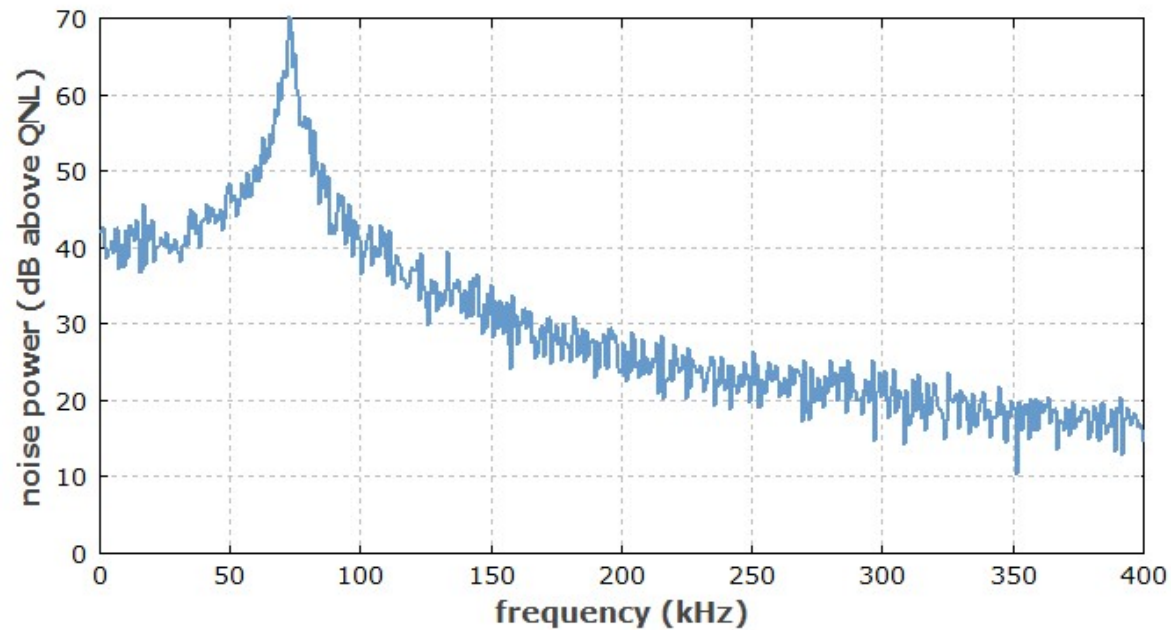


Appl. Phys. Lett. 122, 110502 (2023)

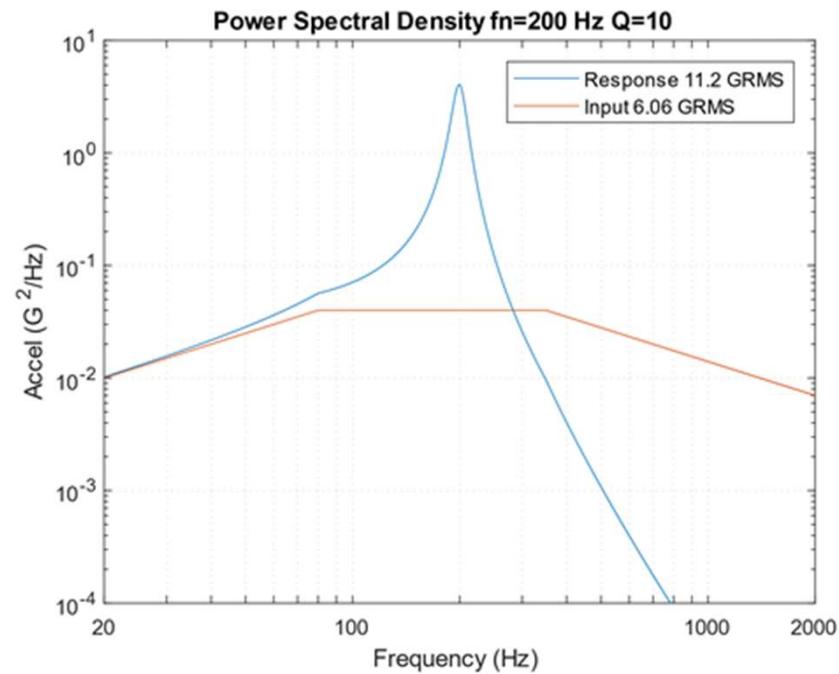




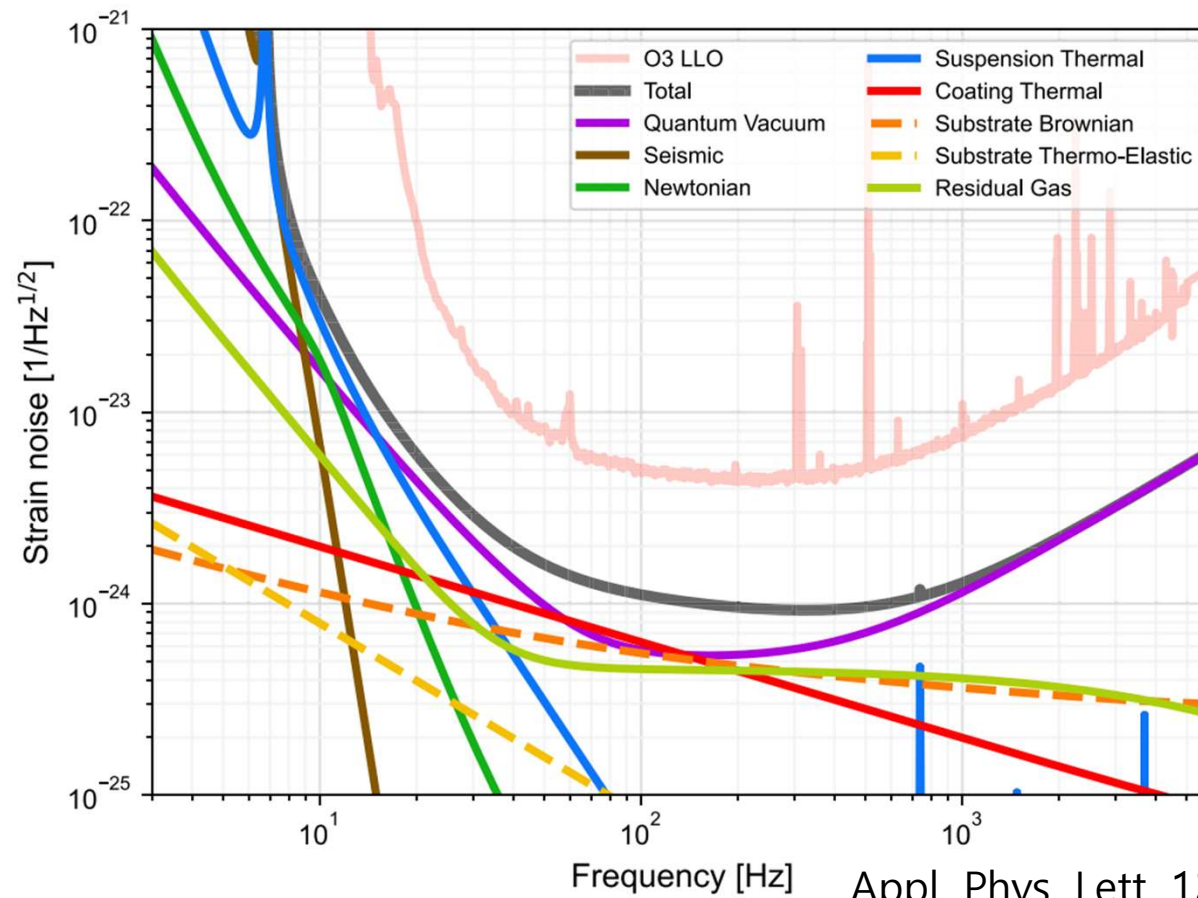
# Power spectral density



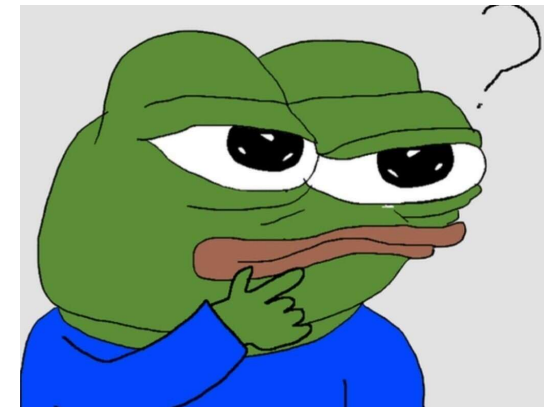
# Power spectral density



# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)



# Signal and noise

---



Signal : 전달하고 자 하는 것

Noise : 시그널을 제외한 나머지



YouTube  
입덕직캠] 아이브 장원영 직캠 4K 'Off The ...



YouTube  
안방1열 직캠4K] 아이브 리즈 'Accendio' (I...



YouTube  
MPD직캠] 아이브 직캠 8K 'I AM' ...



YouTube  
MPD직캠] 아이브 장원영 직캠 4K 'ATTIT...

아이돌 직캠?



tv 네이트 TV  
안방1열 직캠4K] 아이브 레이 'LOVE DIV...



kr.pinterest.com  
K-Fancam] 아이브 안유진 직캠 'I AM' (IV...



번개장터 · 재고 있음  
아이브 직캠 도무송 판매 | 브랜...



kr.pinterest.com  
안방1열 직캠4K] 아이브 장원영 'After LIK...

왜??



# Signal and Noise

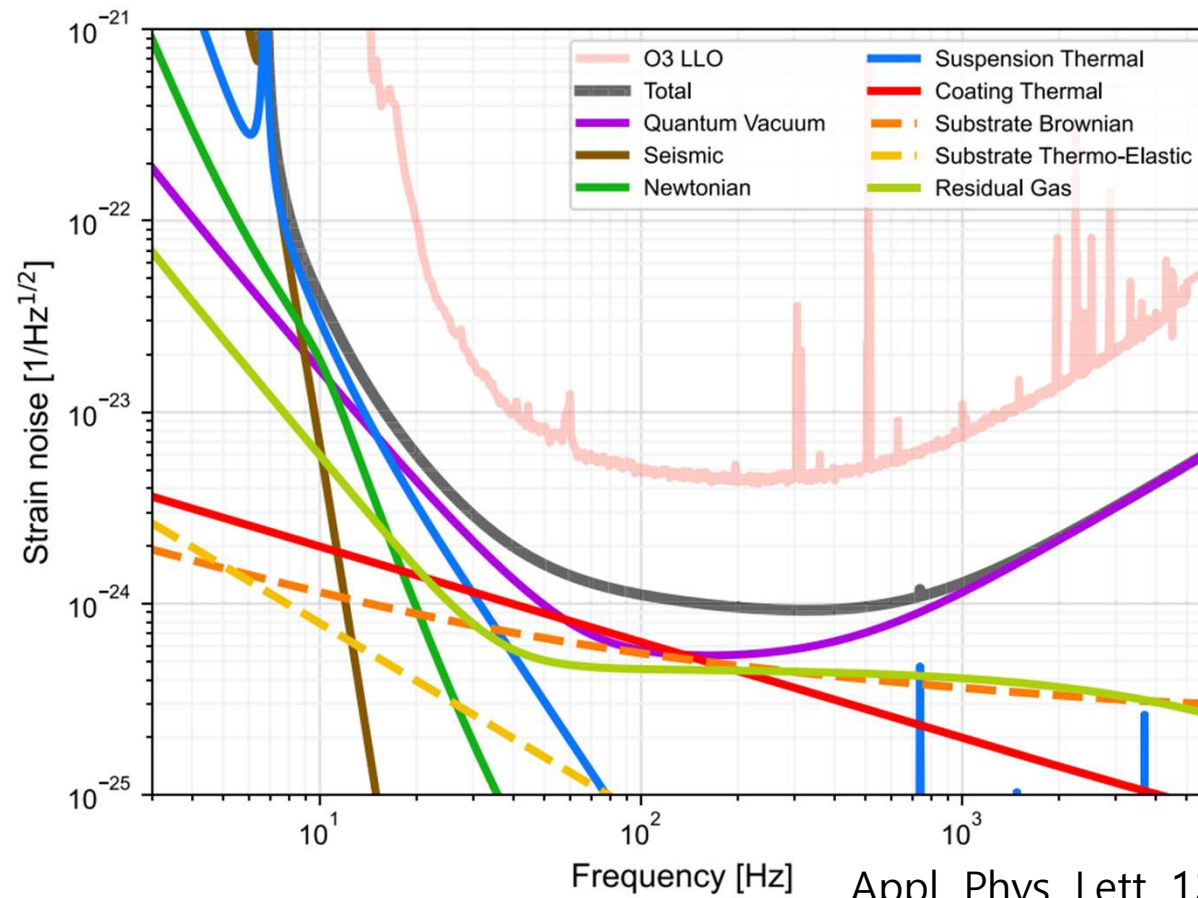


Signal : 내가보고 싶은 멤버

Noise : 내가 보고싶은 멤버  
뺀 나머지

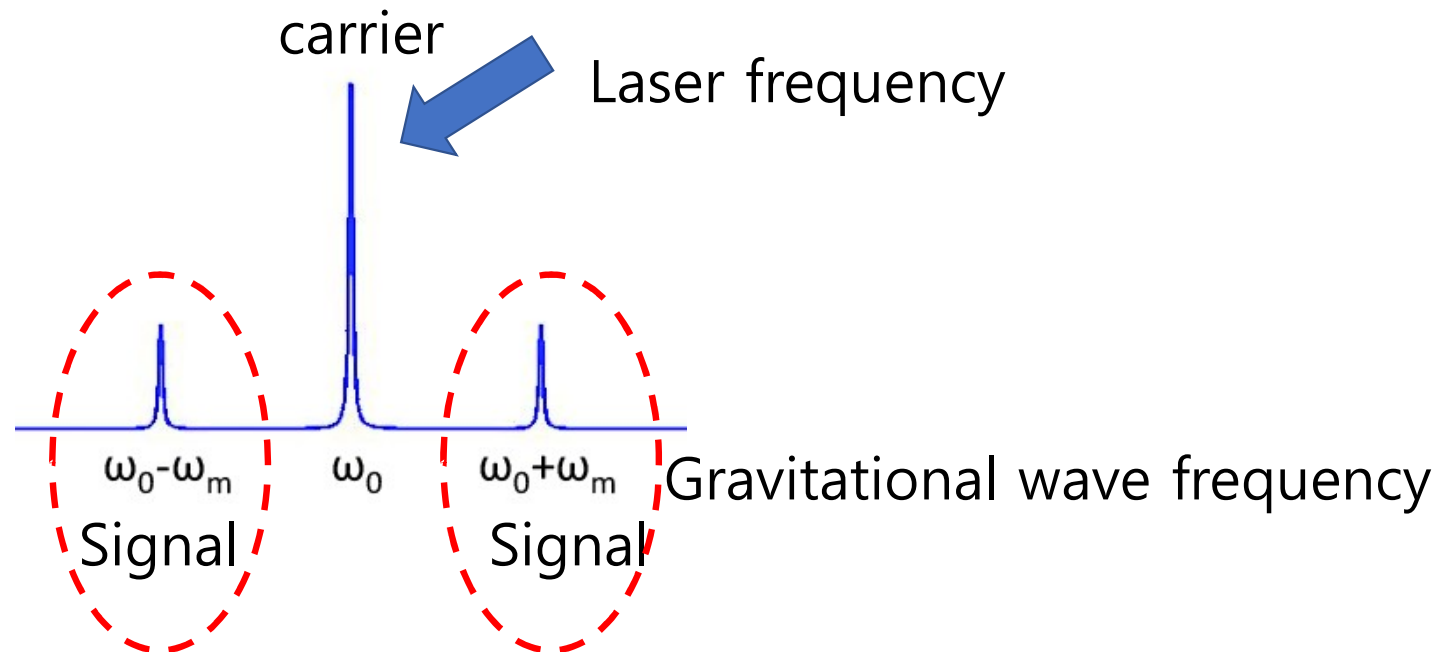


# LIGO sensitivity



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# Side band figure



# Side band figure



We can illustrate the creation of sidebands with one trigonometric identity:

$$\cos(A) \cdot \cos(B) \equiv \frac{1}{2} \cos(A + B) + \frac{1}{2} \cos(A - B)$$

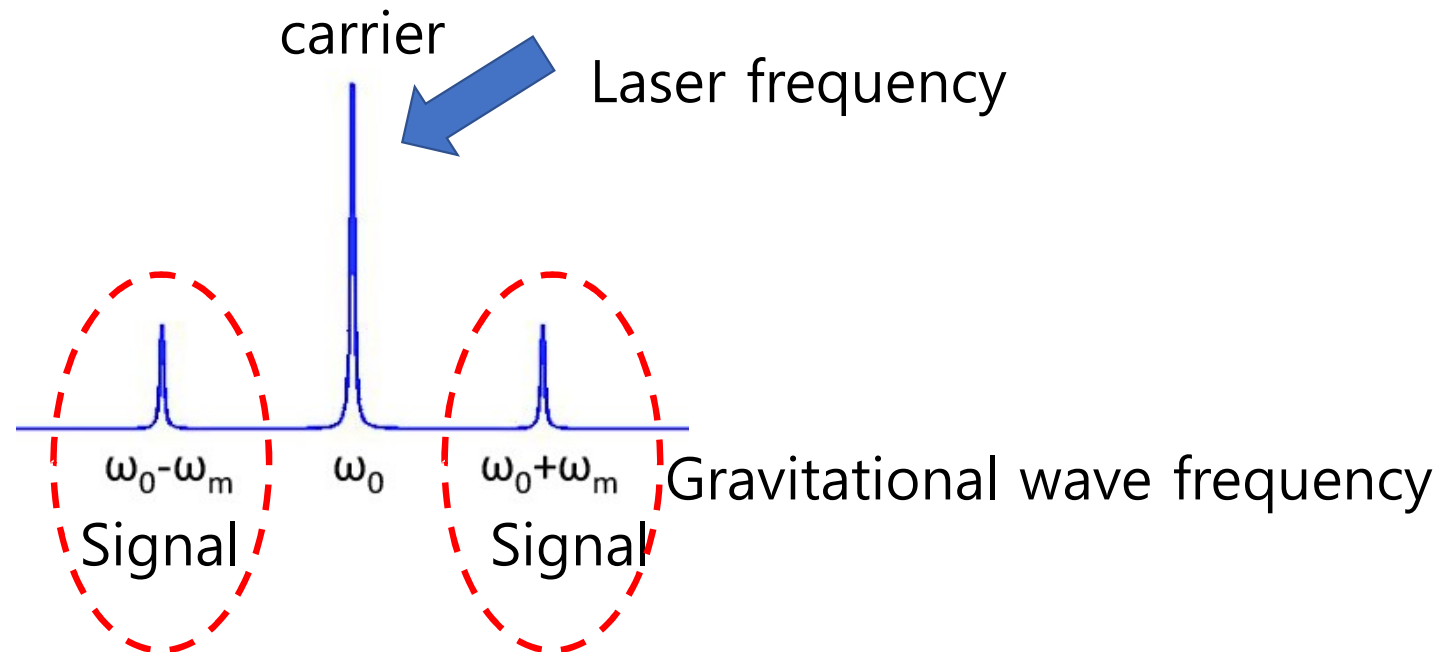
Adding  $\cos(A)$  to both sides:

$$\cos(A) \cdot [1 + \cos(B)] = \frac{1}{2} \cos(A + B) + \cos(A) + \frac{1}{2} \cos(A - B)$$

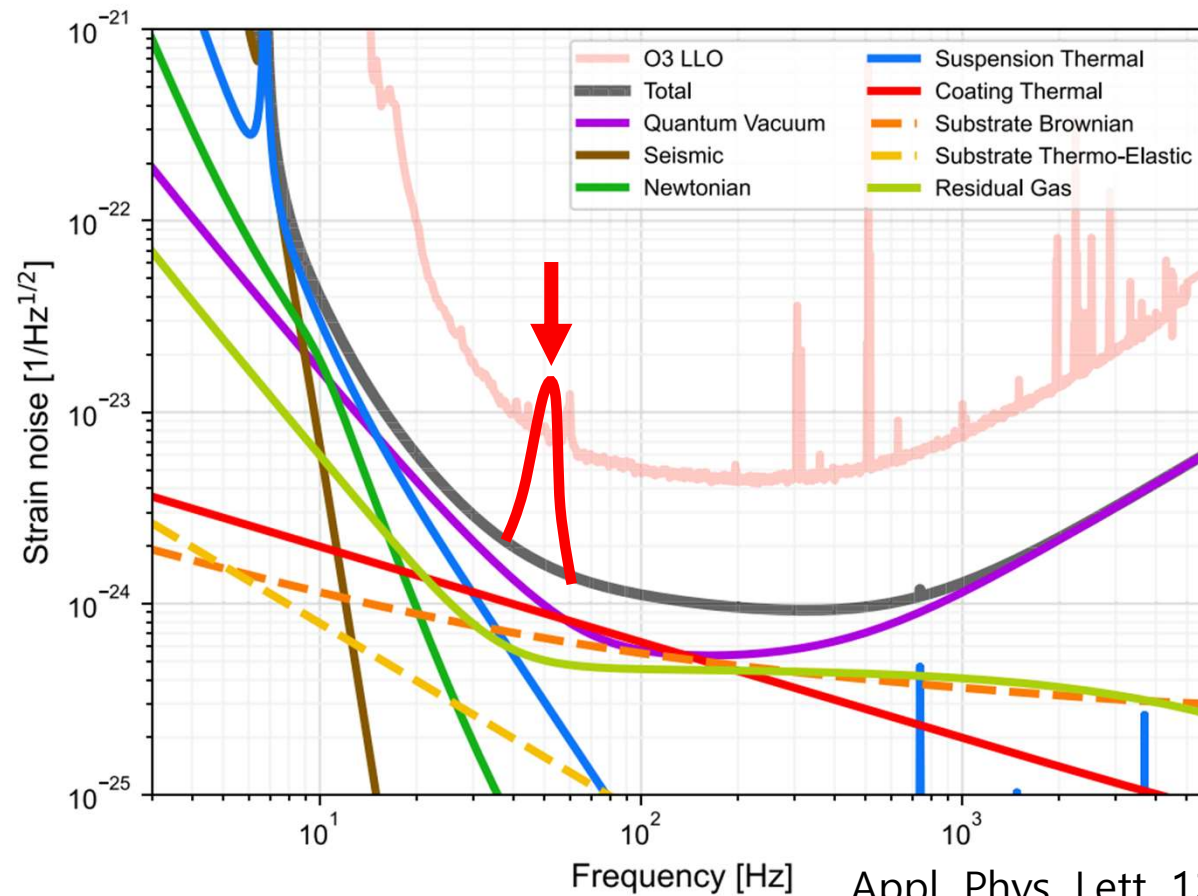
Substituting (for instance)  $A \triangleq 1000 \cdot t$  and  $B \triangleq 100 \cdot t$ , where  $t$  represents time:

$$\underbrace{\cos(1000 t)}_{\text{carrier wave}} \cdot \underbrace{[1 + \cos(100 t)]}_{\text{amplitude modulation}} = \underbrace{\frac{1}{2} \cos(1100 t)}_{\text{upper sideband}} + \underbrace{\cos(1000 t)}_{\text{carrier wave}} + \underbrace{\frac{1}{2} \cos(900 t)}_{\text{lower sideband}}.$$

# Side band figure

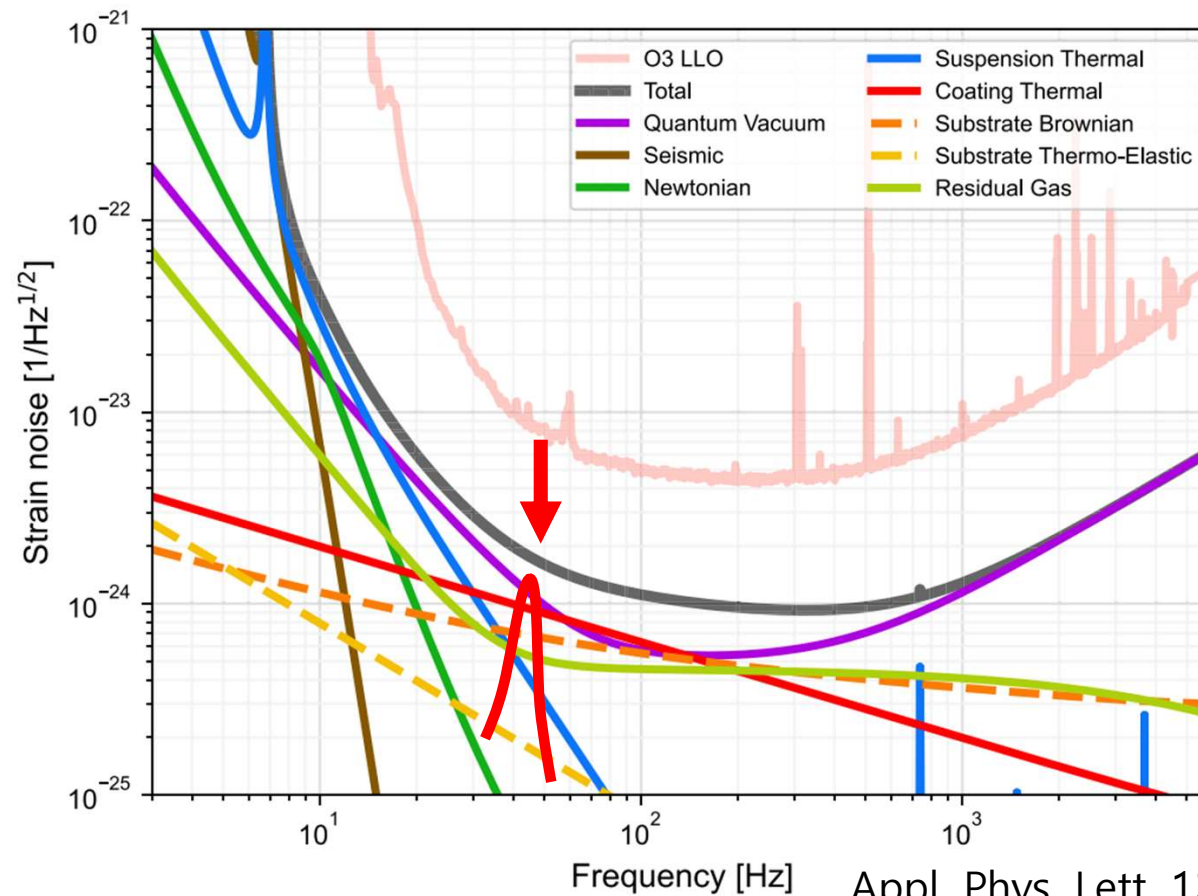


# LIGO sensitivity



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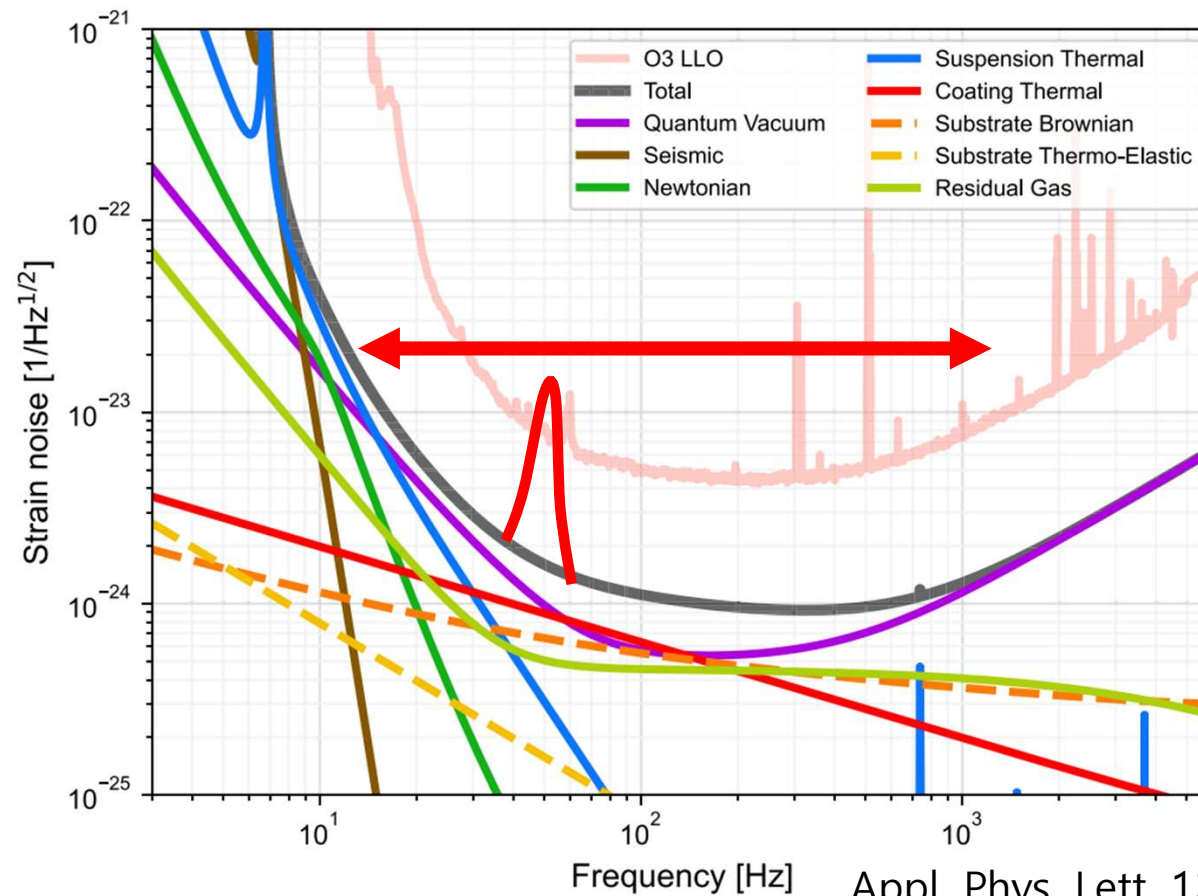
# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)

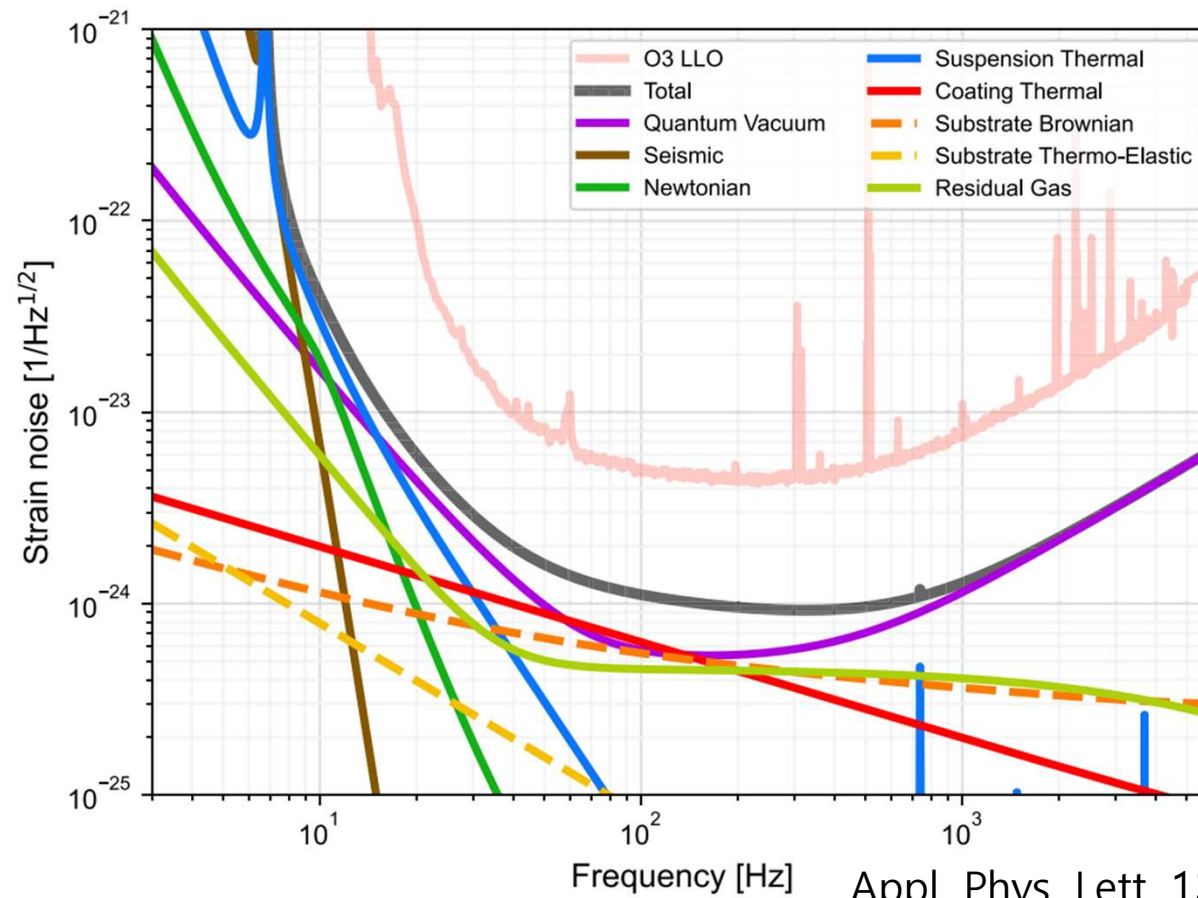


# LIGO sensitivity



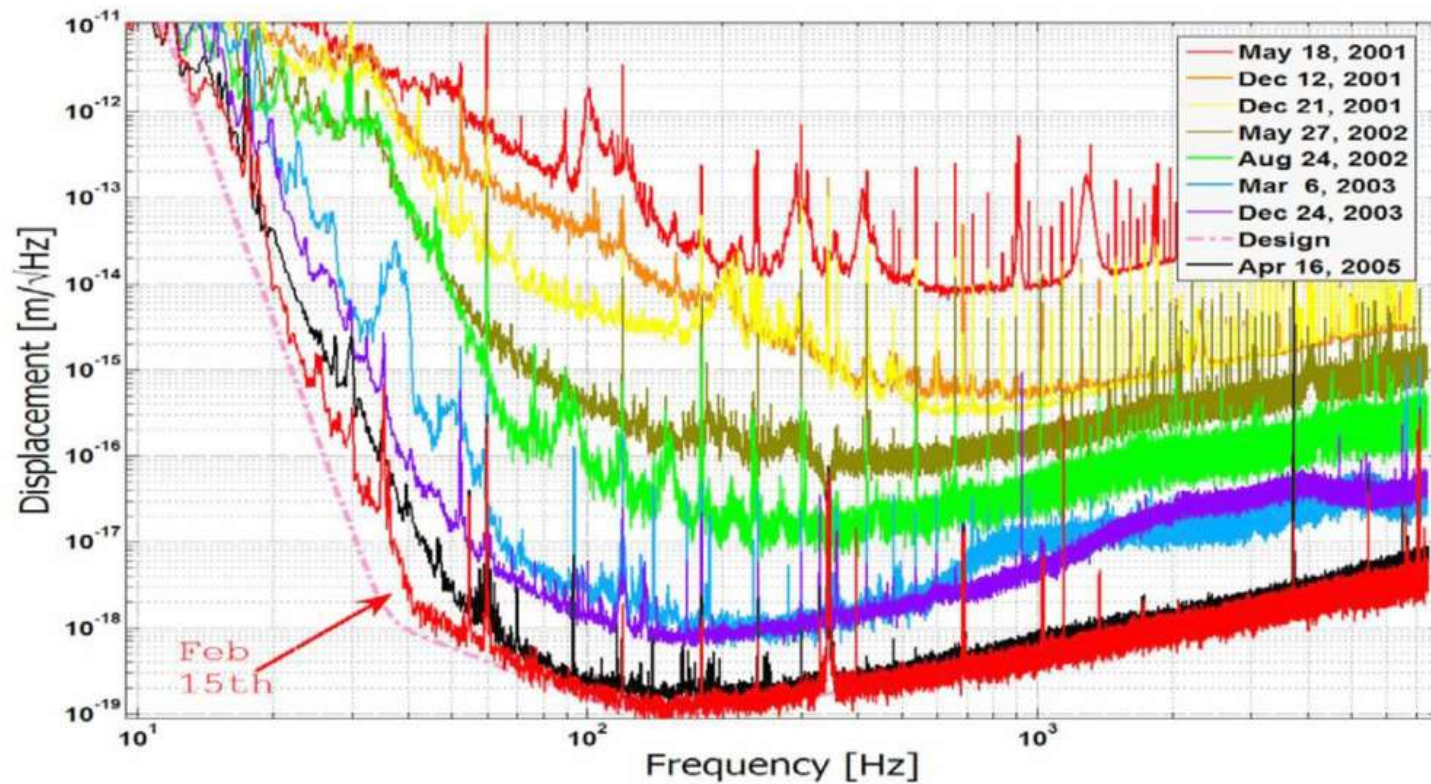
Appl. Phys. Lett. 122, 110502 (2023)

# LIGO sensitivity

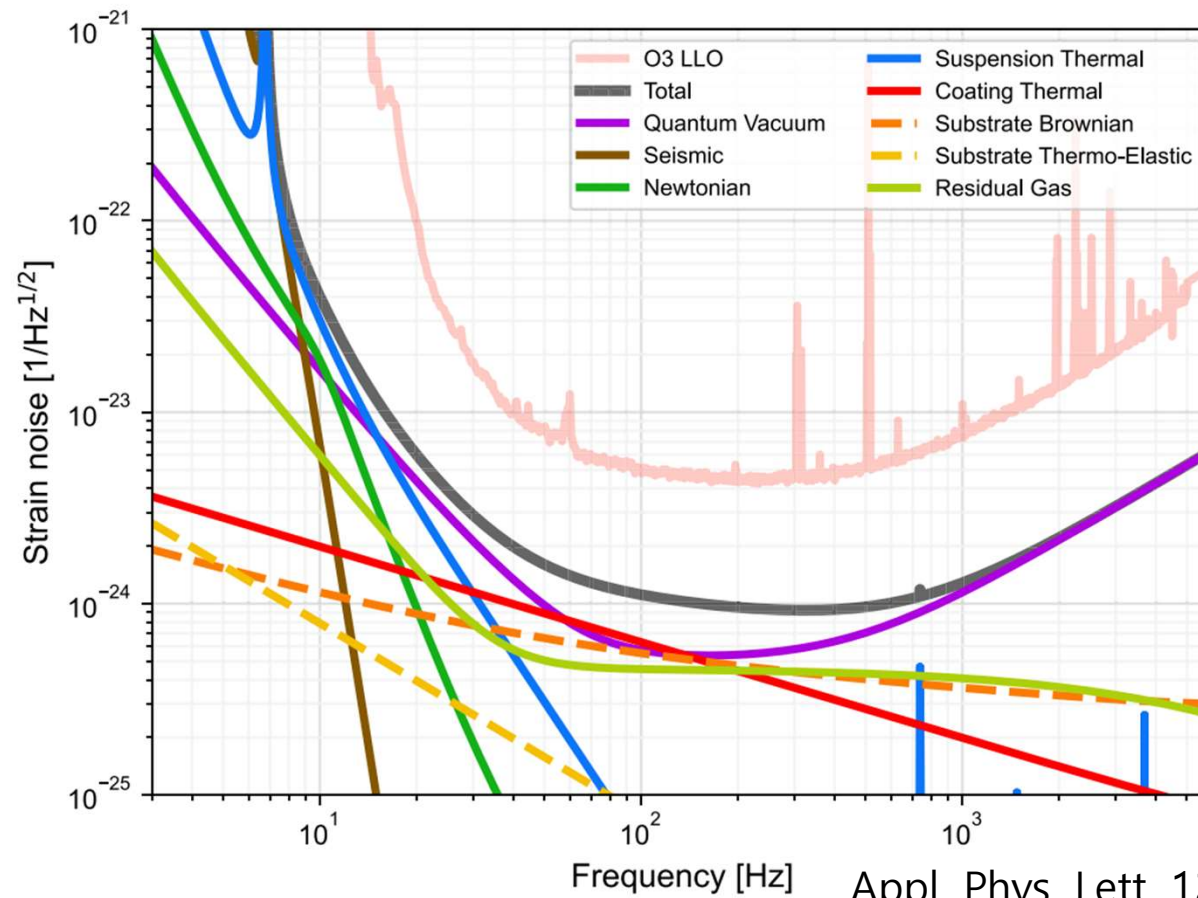


Appl. Phys. Lett. 122, 110502 (2023)

# Sensitivity during science run



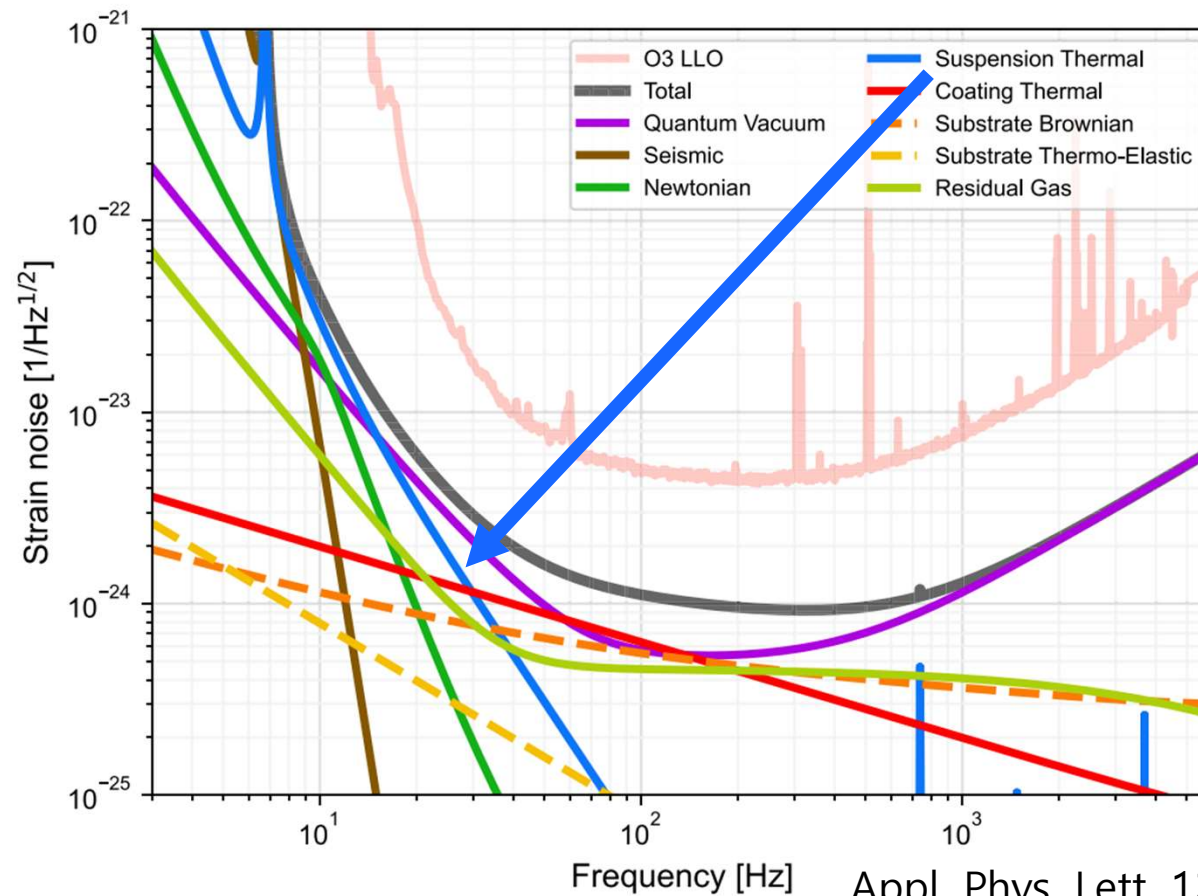
# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)

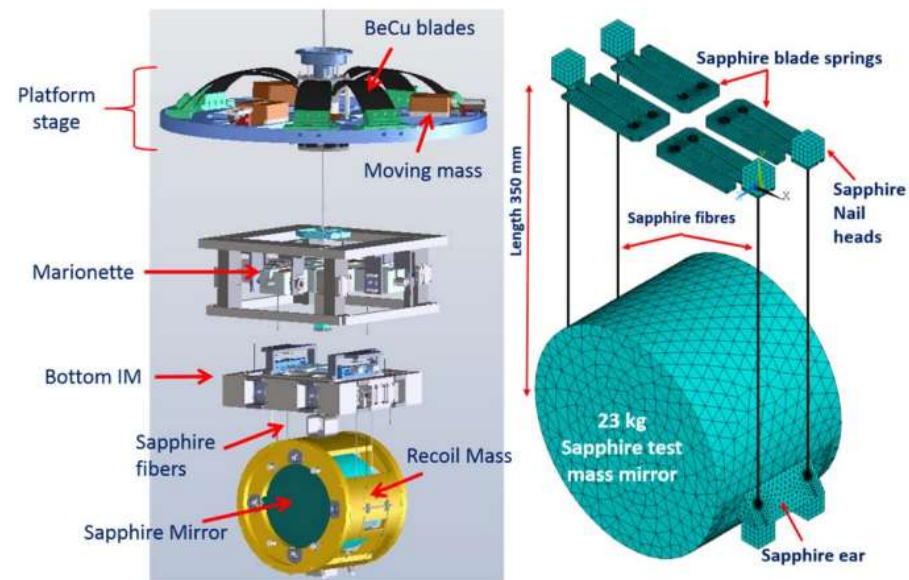


# LIGO sensitivity



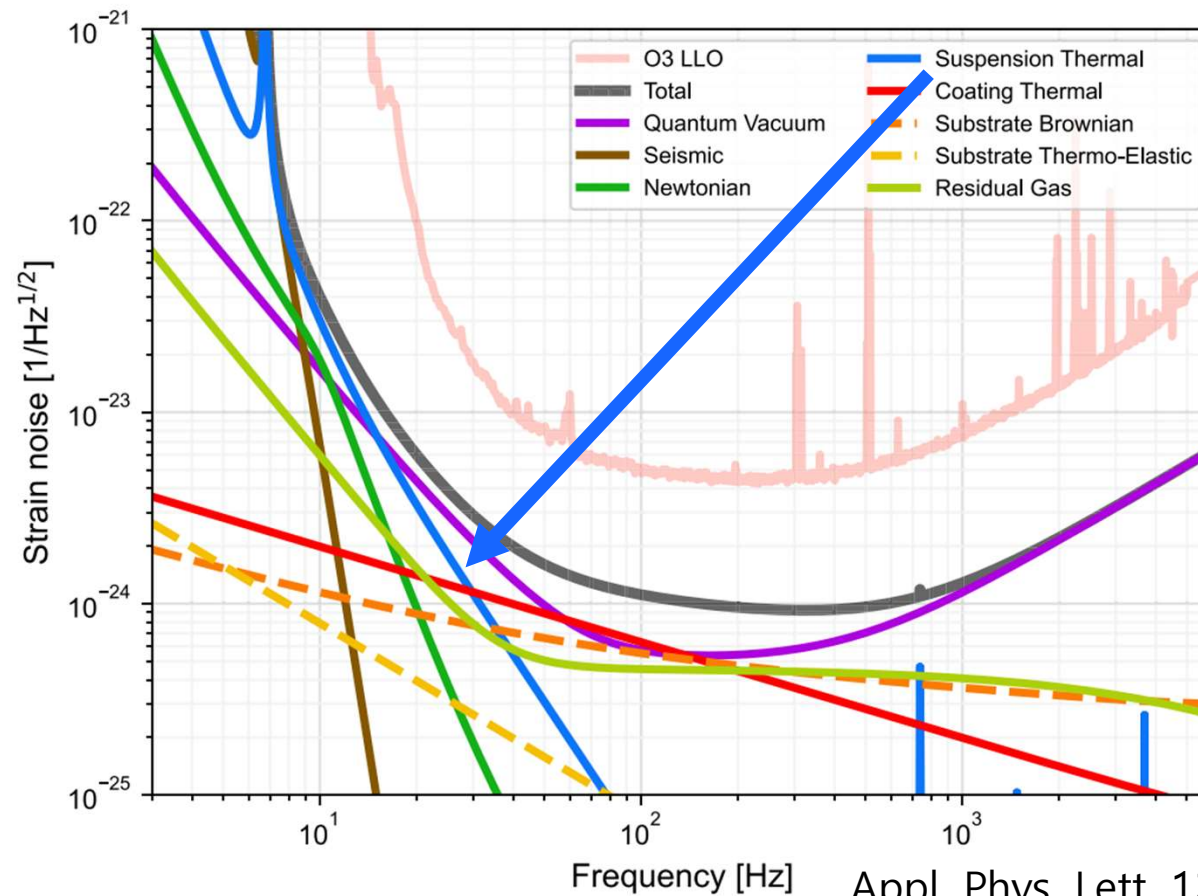
Appl. Phys. Lett. 122, 110502 (2023)

# Suspension



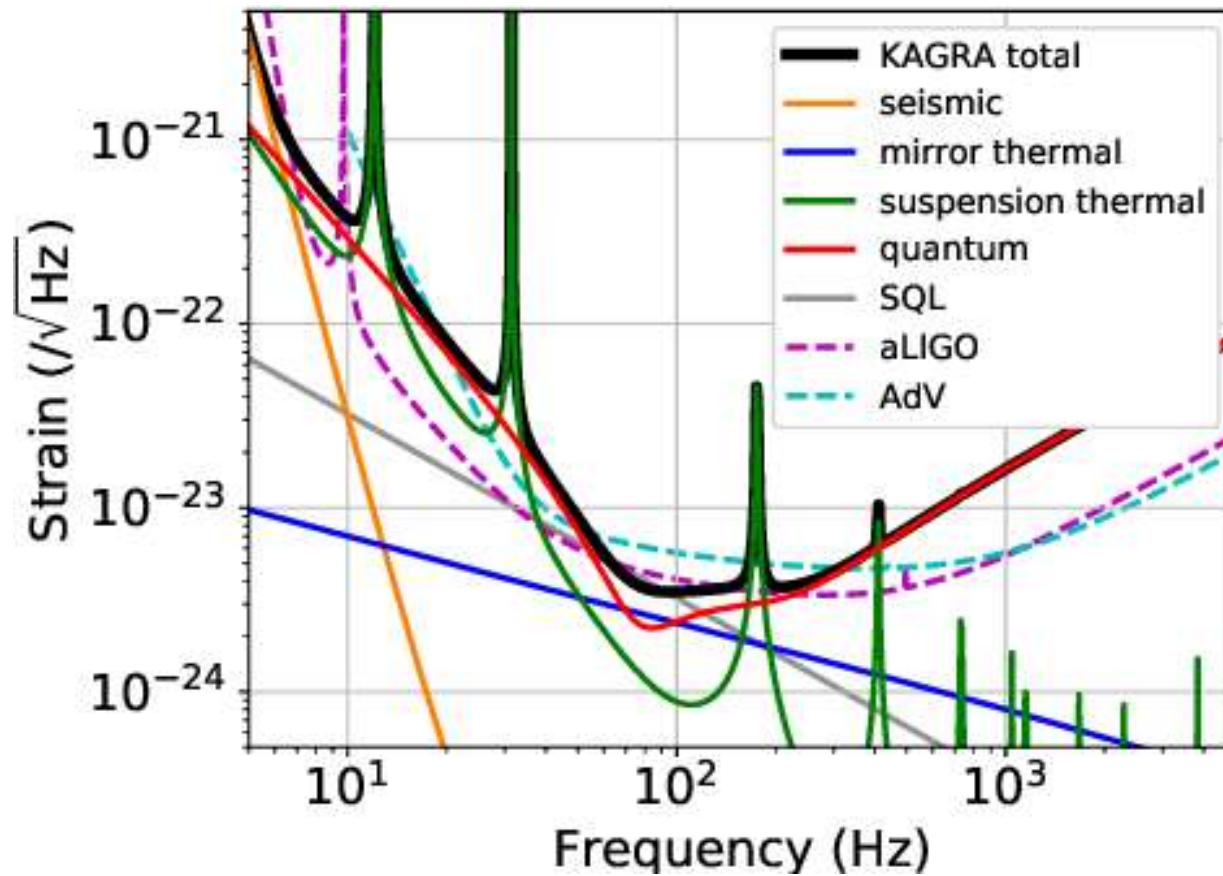


# LIGO sensitivity

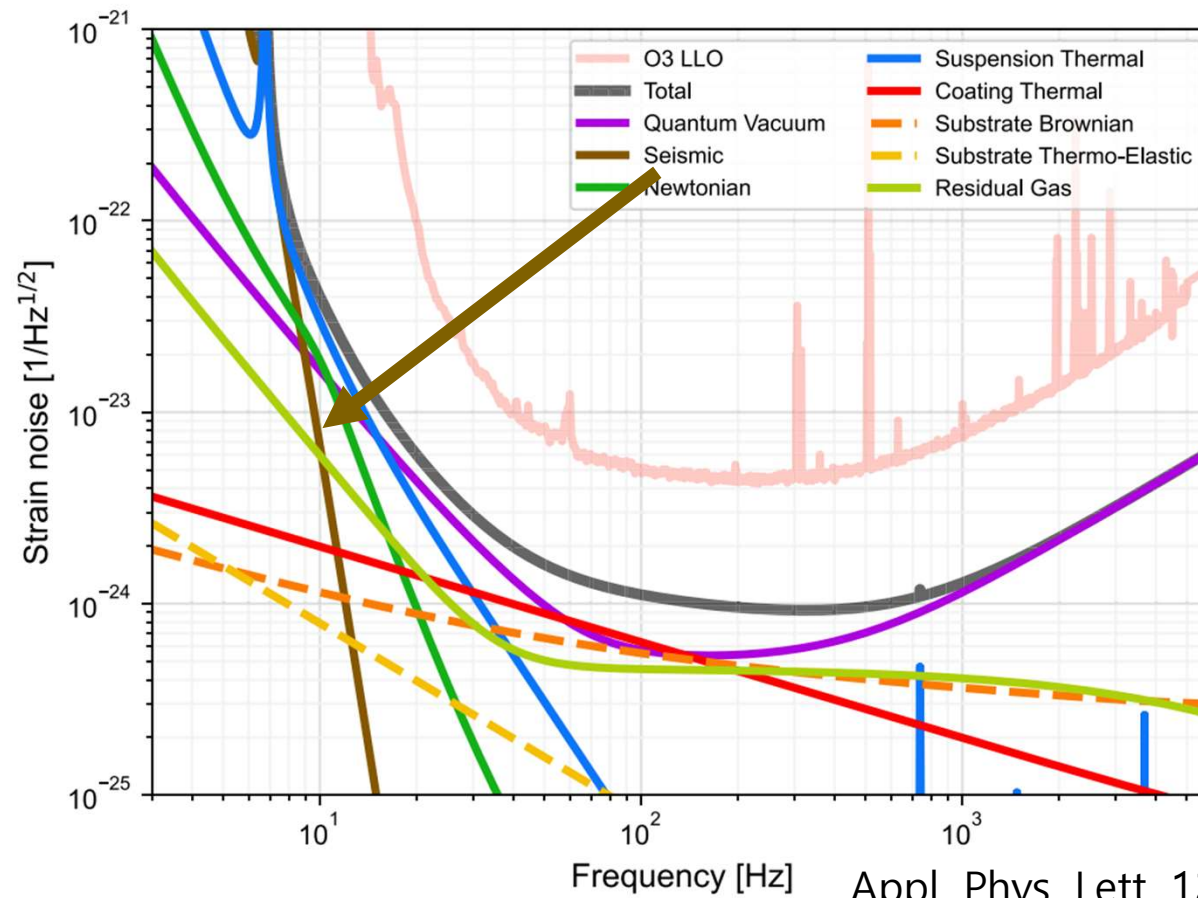


Appl. Phys. Lett. 122, 110502 (2023)

# Sensitivity curve of KAGRA

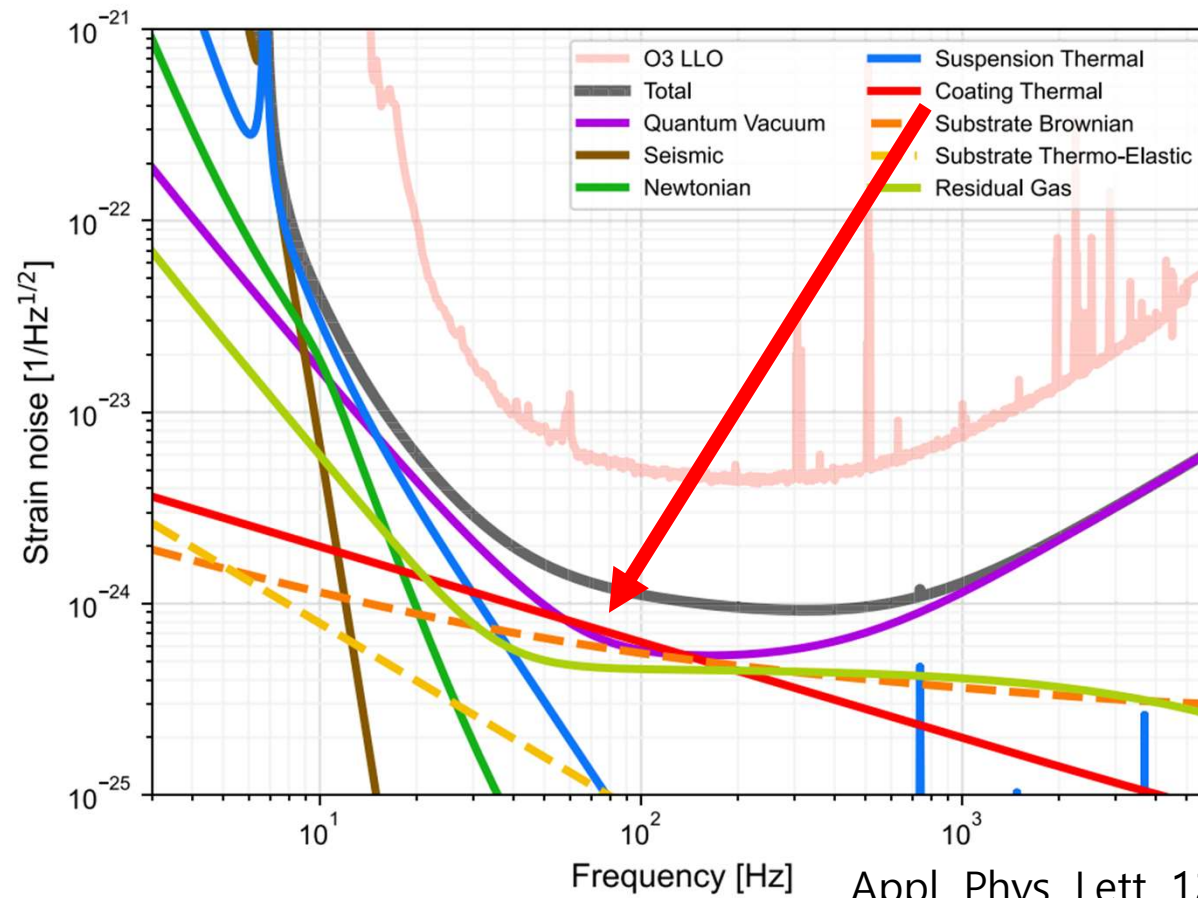


# LIGO sensitivity



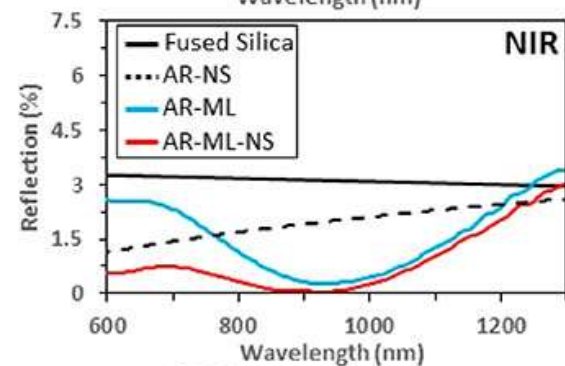
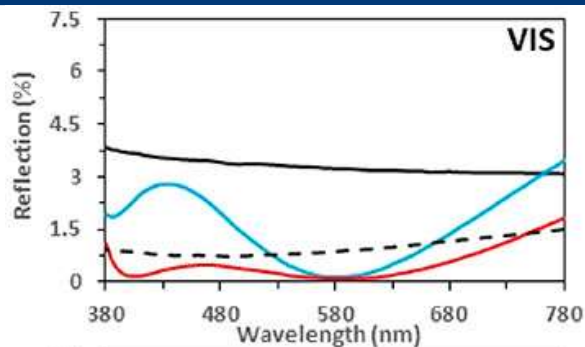
Appl. Phys. Lett. 122, 110502 (2023)

# LIGO sensitivity

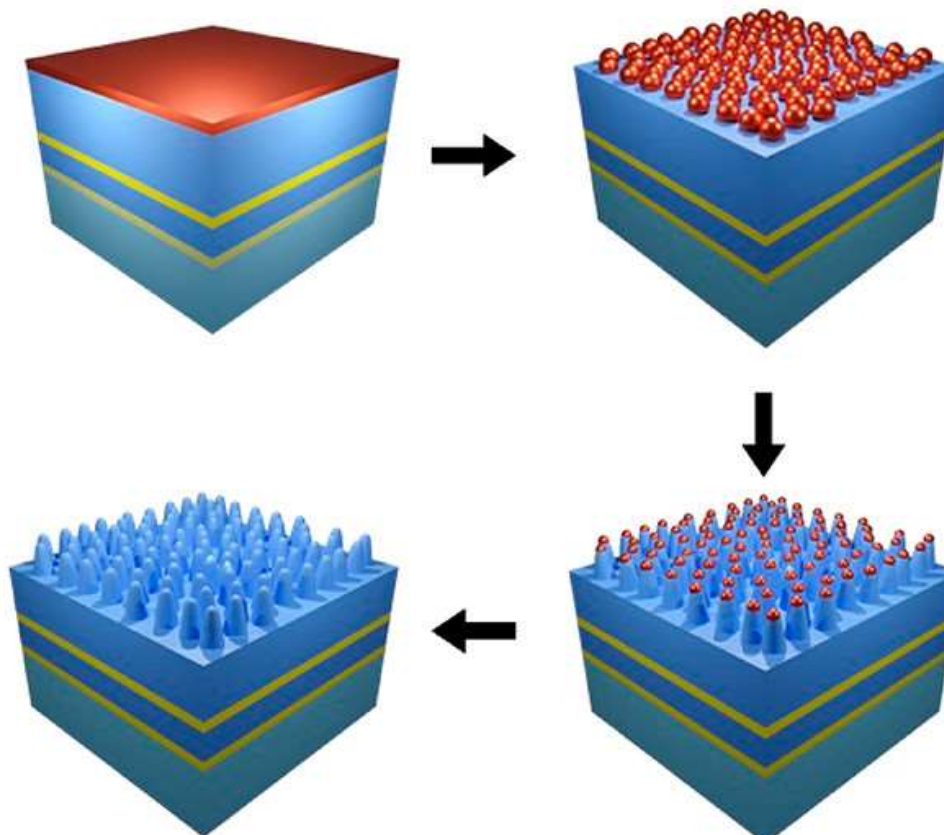


Appl. Phys. Lett. 122, 110502 (2023)

# Optical coating



CA > 150°



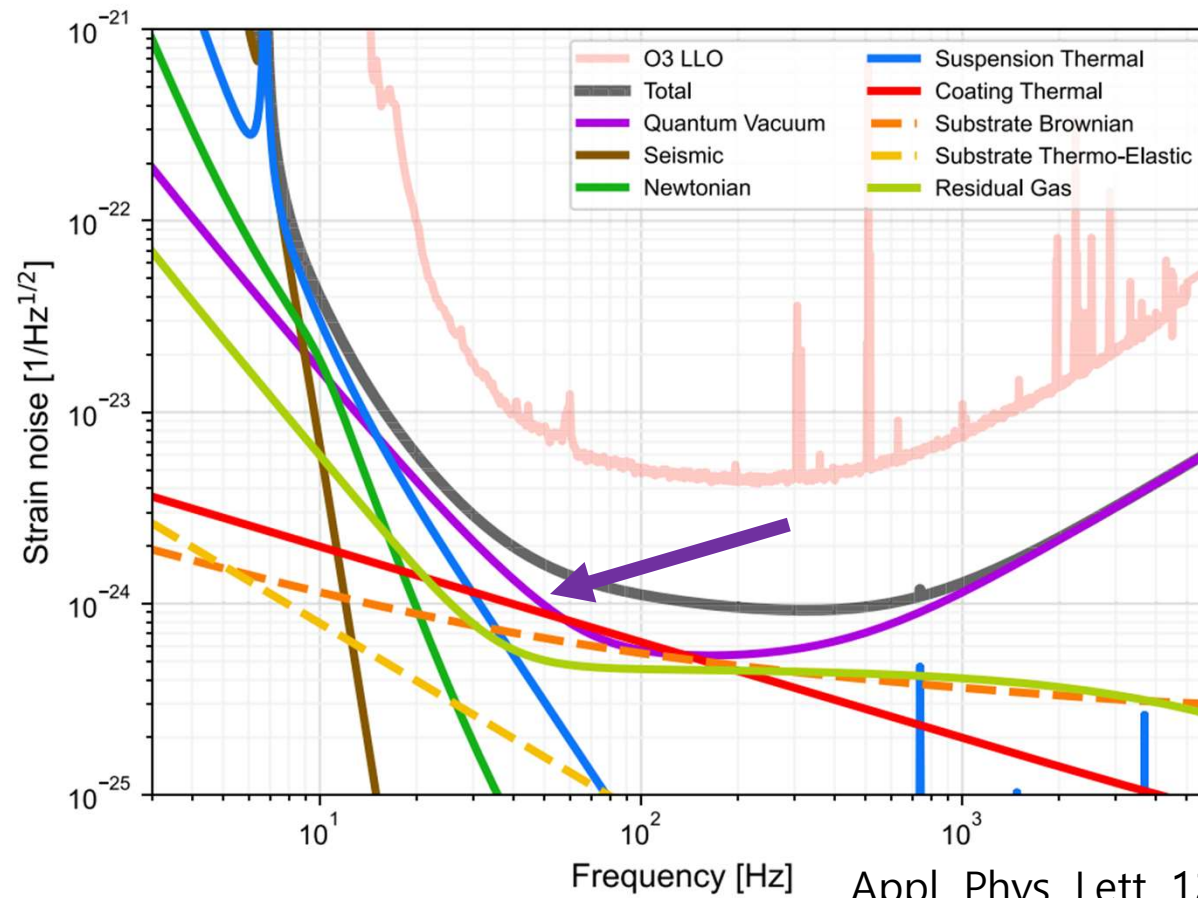
ACS Photonics 2021, 8, 3, 894–900

Publication Date: February 17, 2021

<https://doi.org/10.1021/acsp Photonics.0c01909>



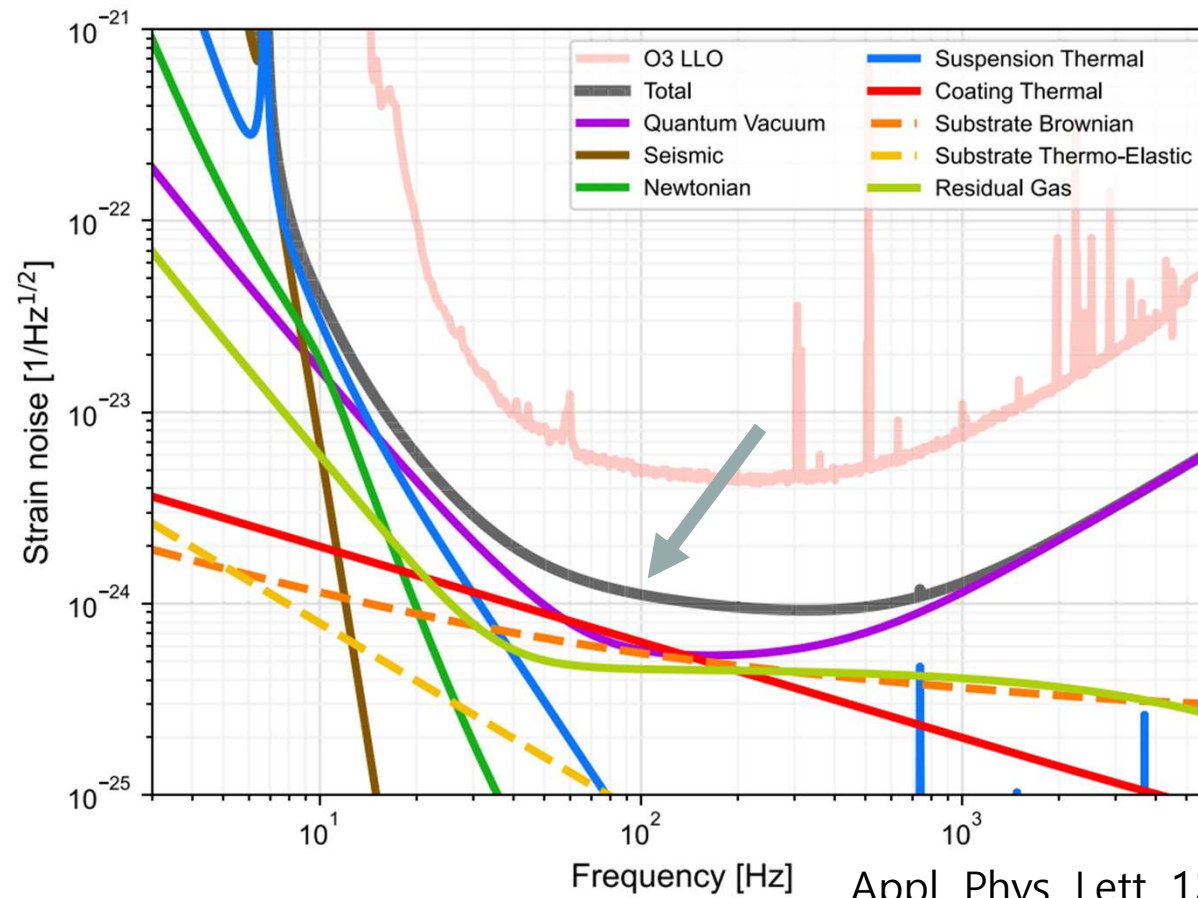
# LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)



# LIGO sensitivity



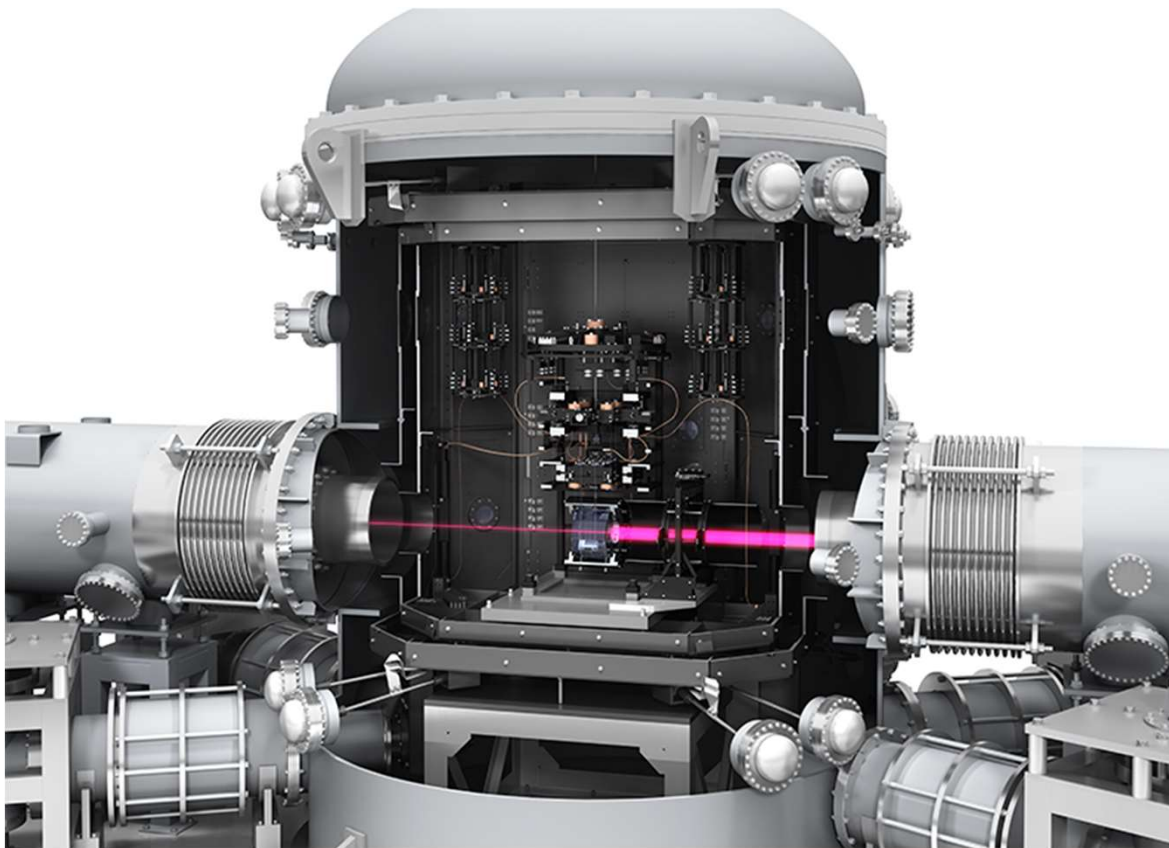
Appl. Phys. Lett. 122, 110502 (2023)

# Vacuum tunnel



Air turbulence, scattering from air

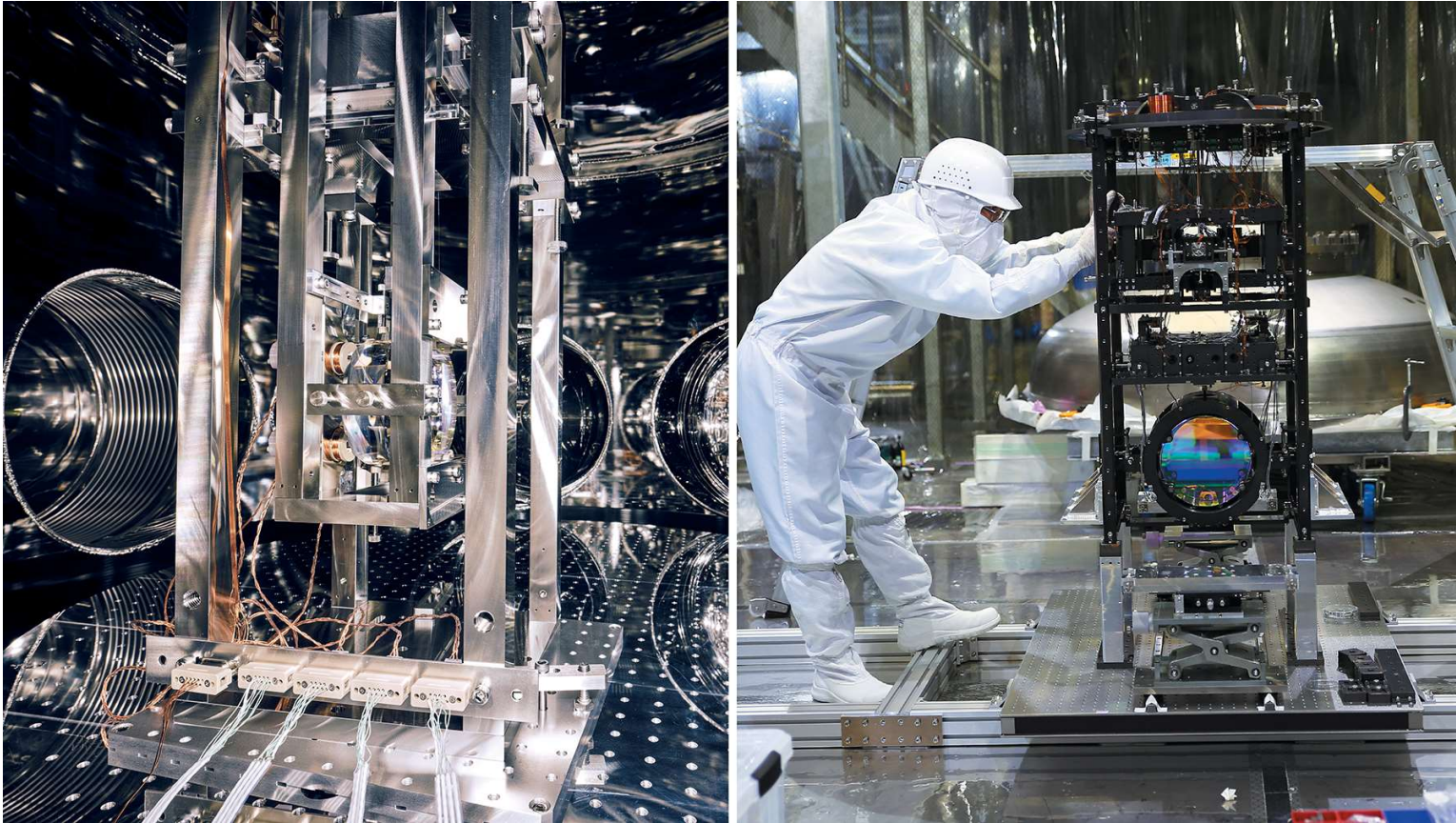
# Test mass chamber



Rey.Hori



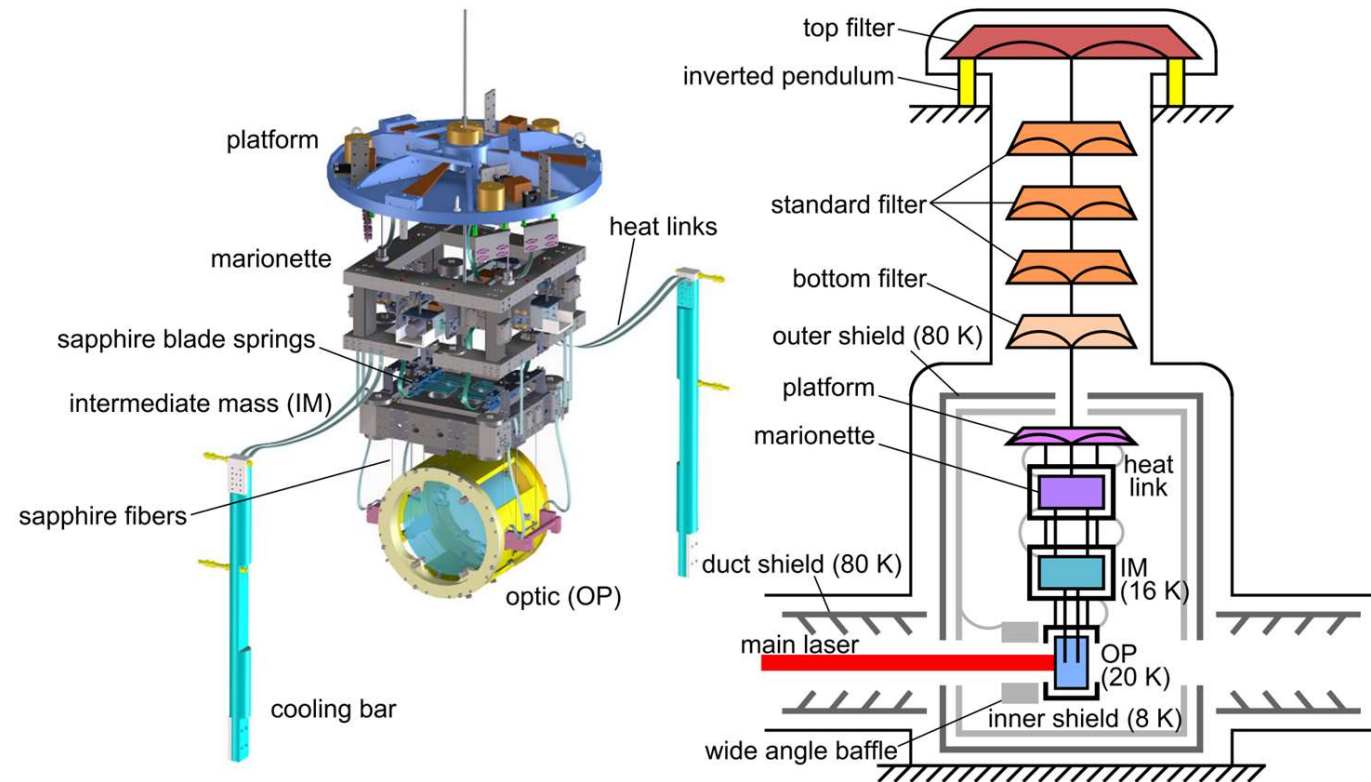
# KAGRA vacuum chamber



# Cryocooler of KAGRA

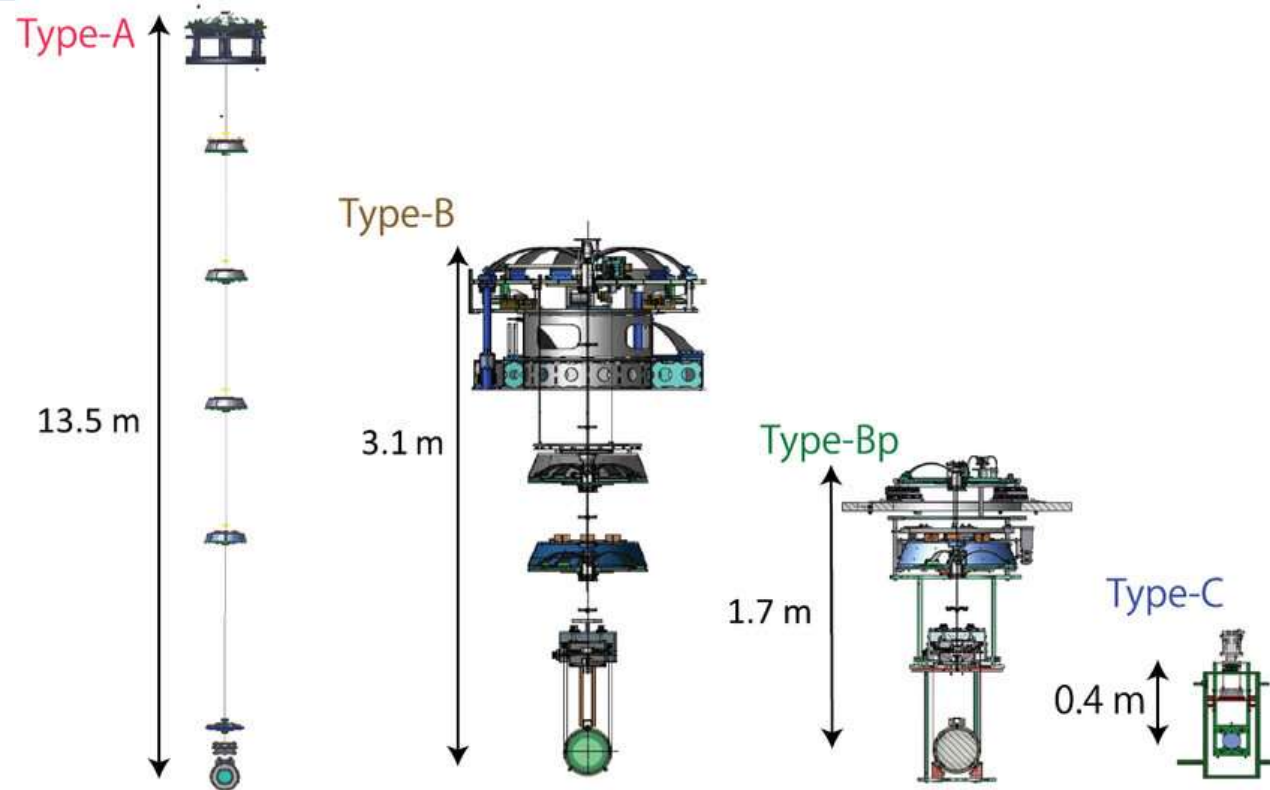


# KAGRA cryostat





# Vibration isolation



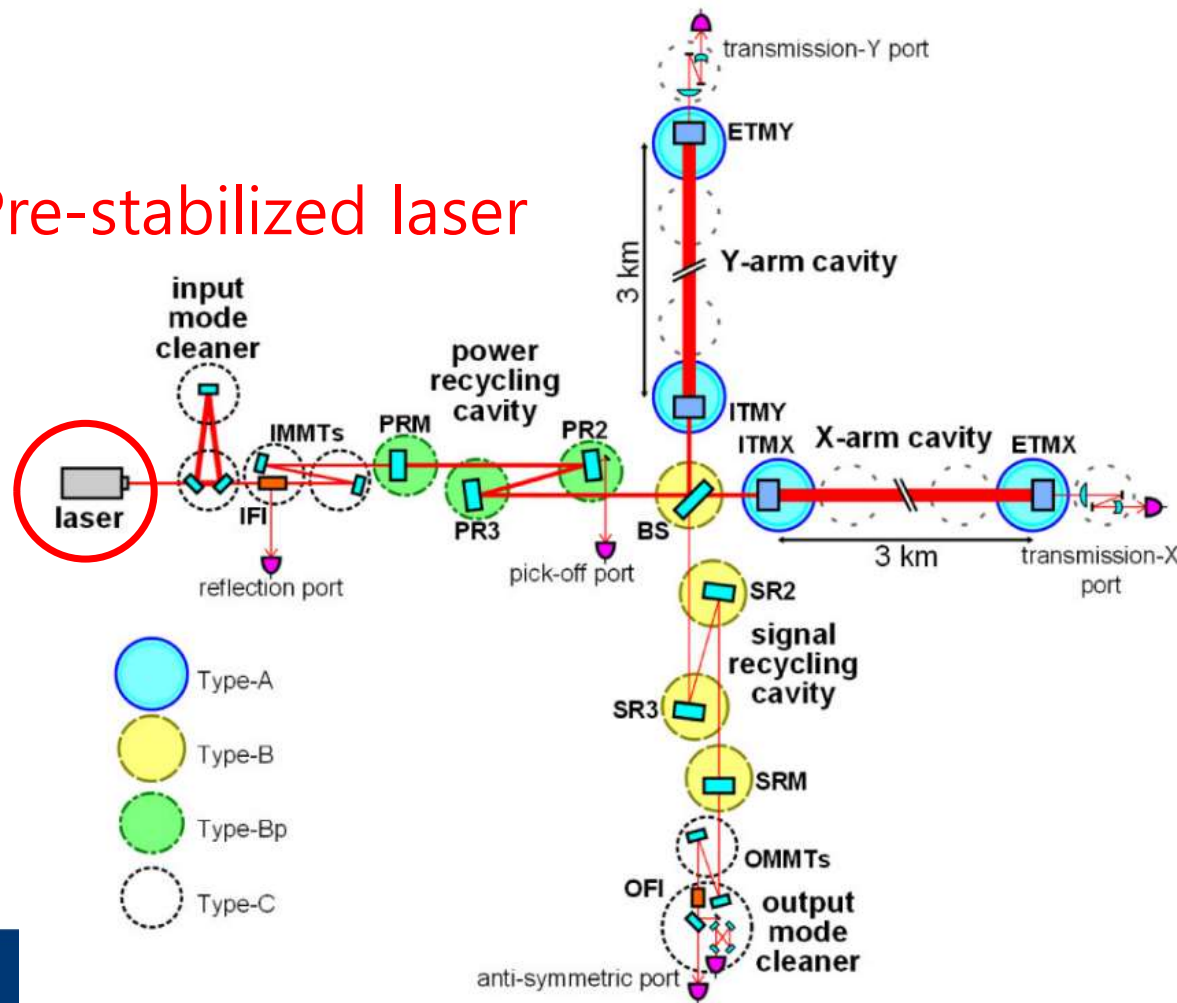
Vibration isolation system with a compact damping system for power recycling mirrors of KAGRA

T.Akuts et al.,

# KAGRA interferometer

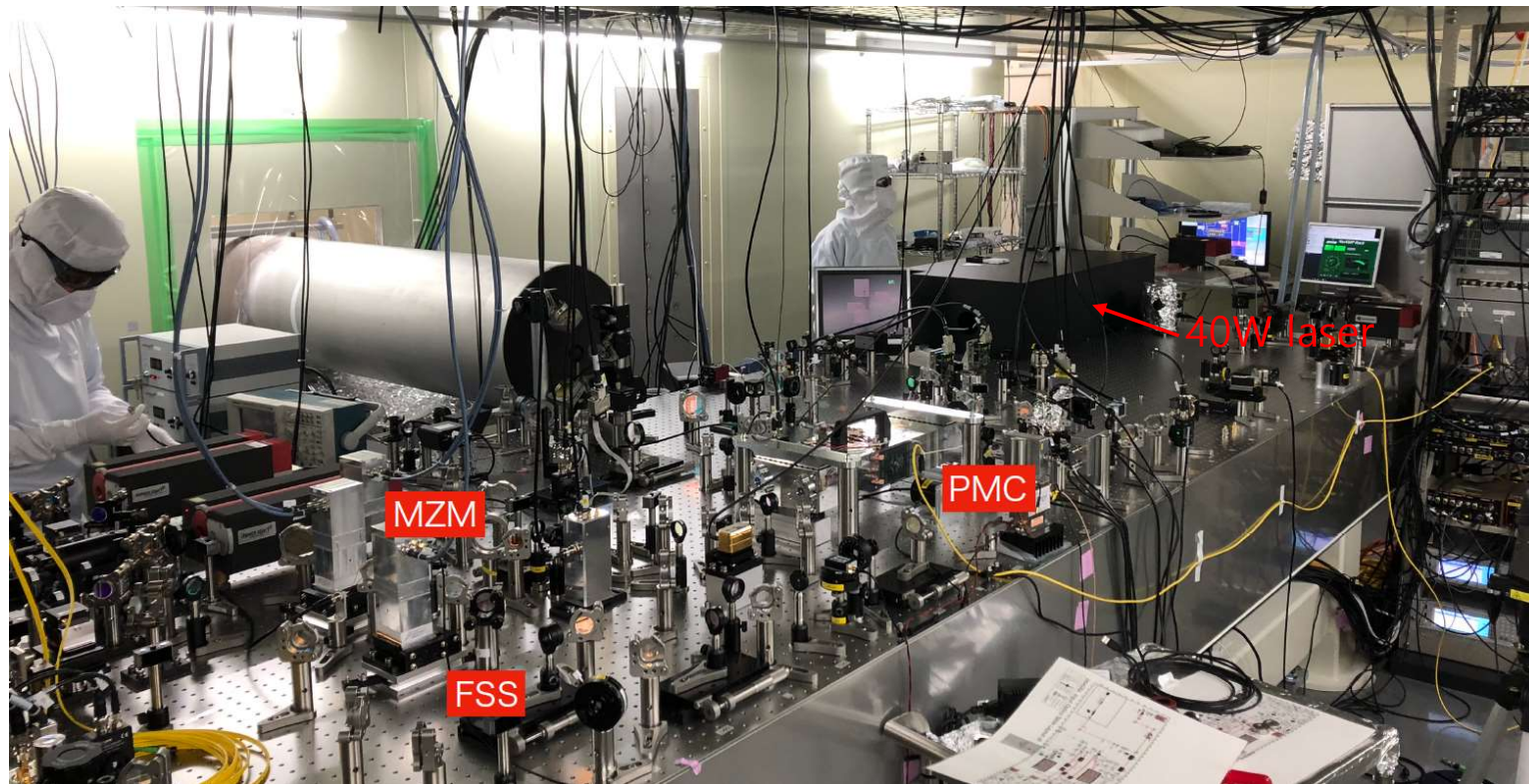


Pre-stabilized laser



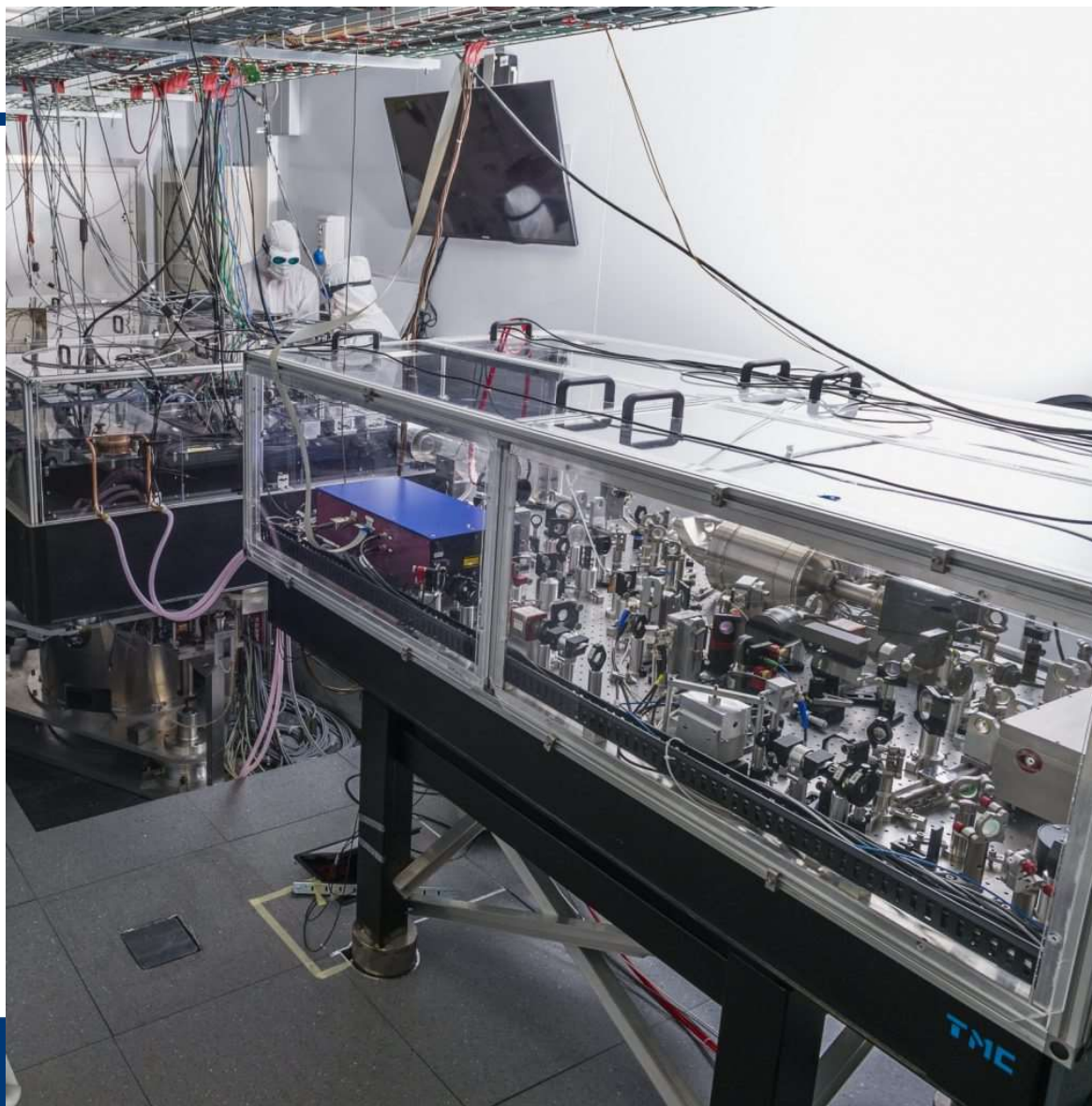
IOO : Input output optics  
 MIF : Main interferometer  
 MIR : Mirror  
 MMT : Mode matching telescope  
 OMC : Output mode cleaner

# PSL room of KAGRA

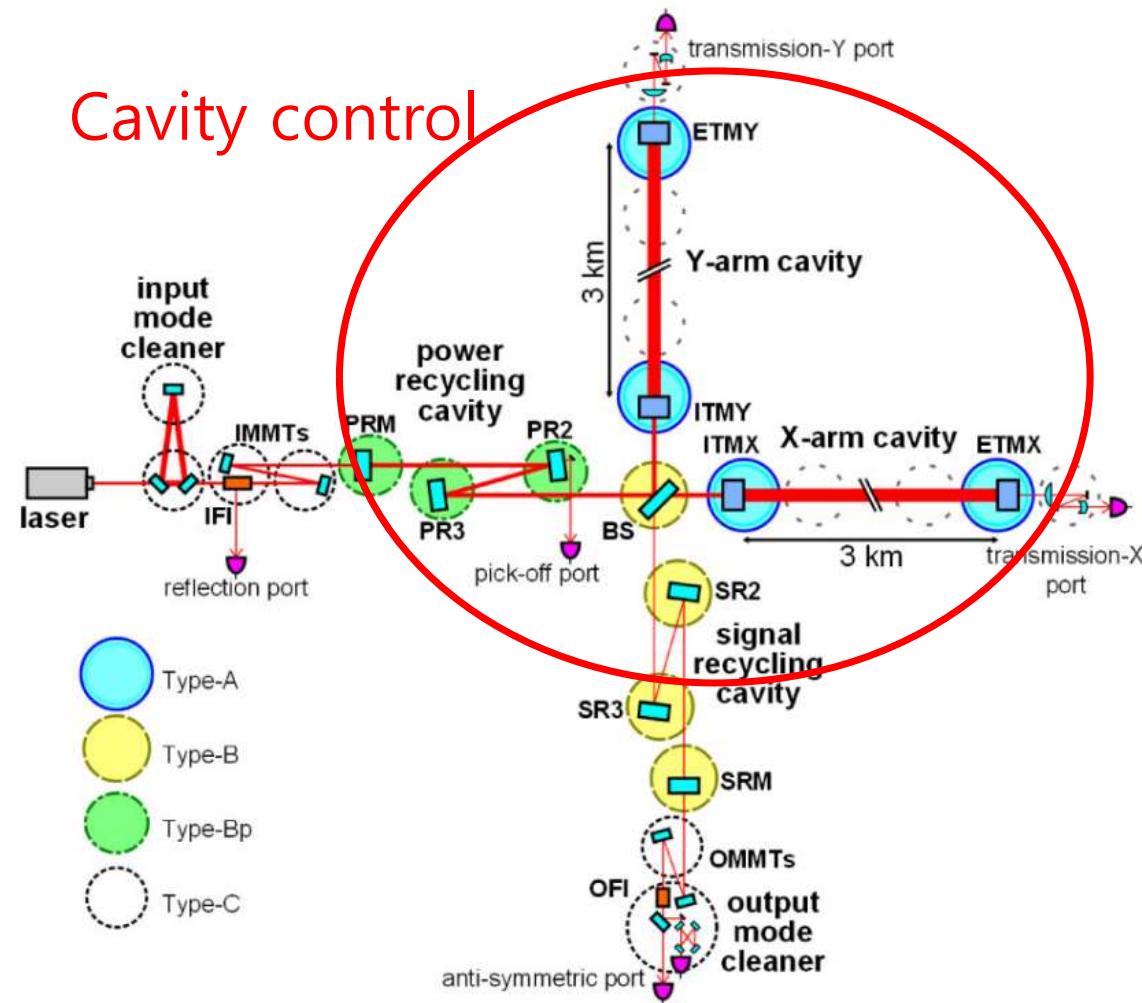


JGW-G1910363 – Masayuki Nakano





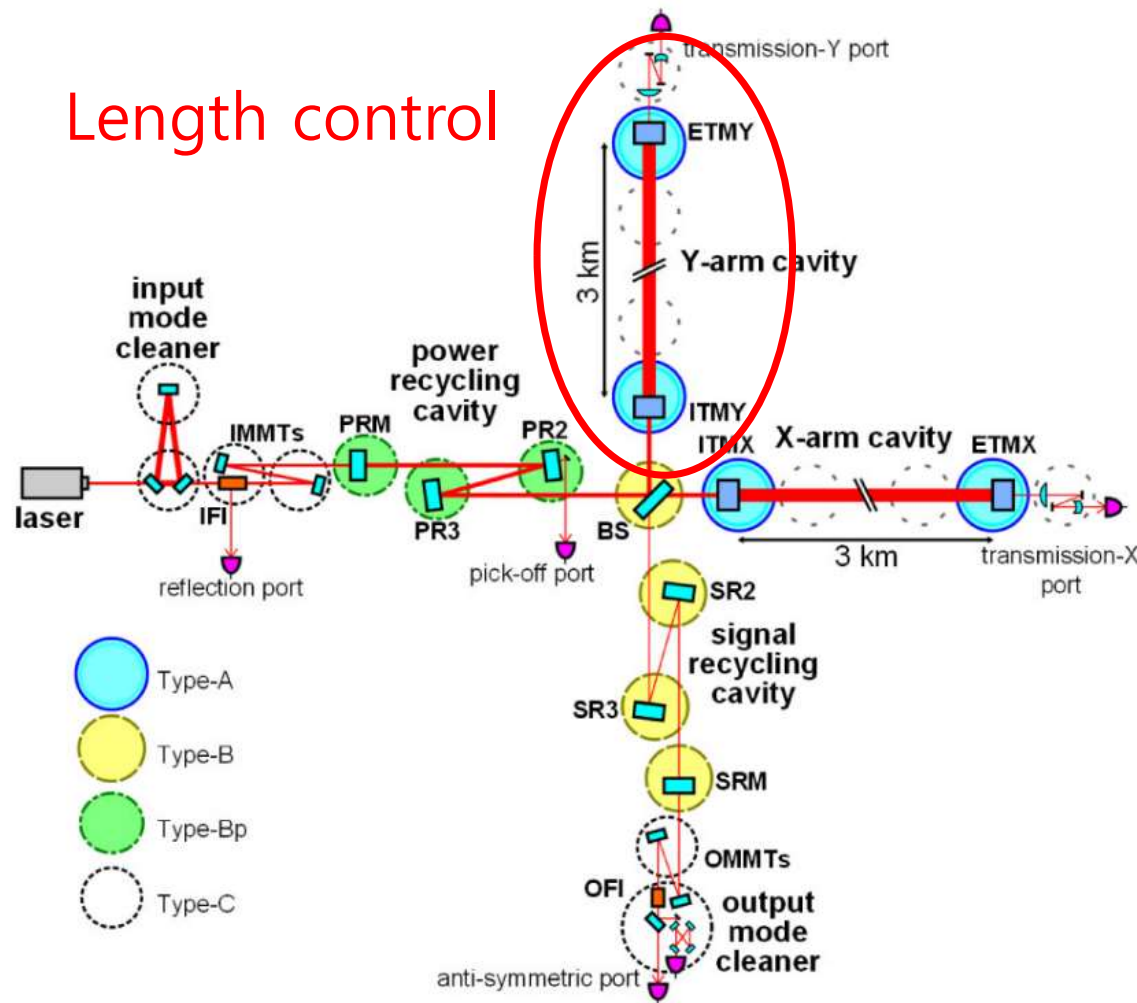
# KAGRA interferometer



IOO : Input output optics  
 MIF : Main interferometer  
 MIR : Mirror  
 MMT : Mode matching telescope  
 OMC : Output mode cleaner

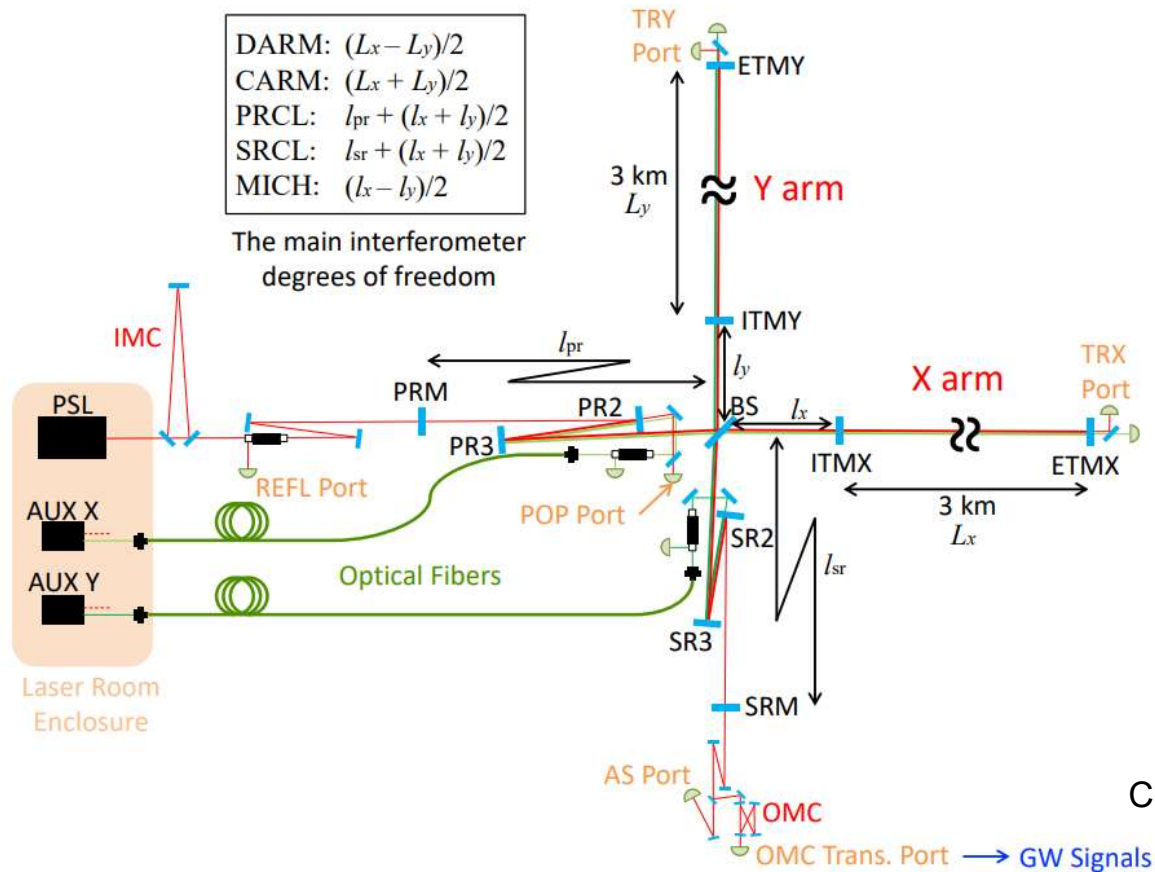


# KAGRA interferometer

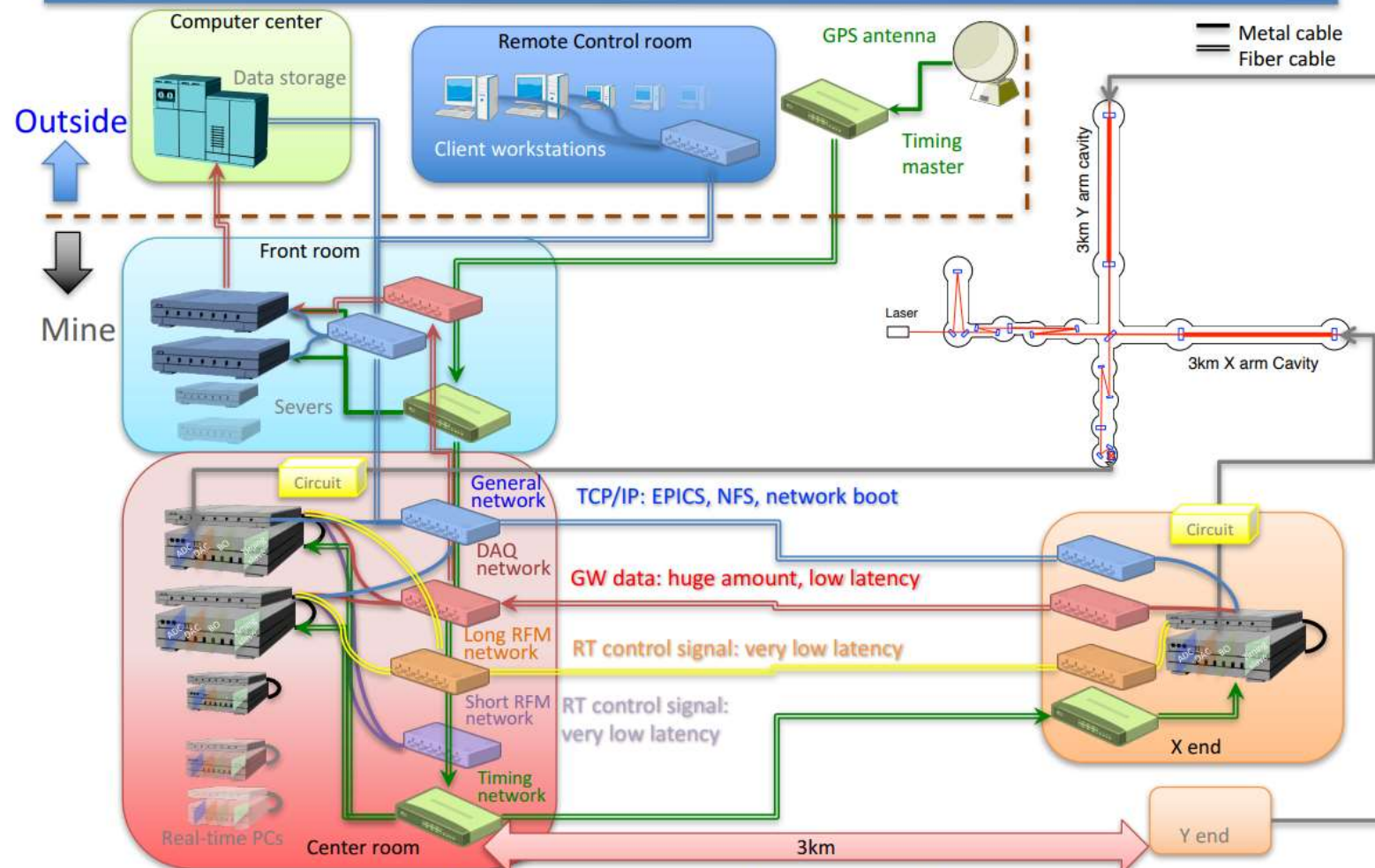


IOO : Input output optics  
 MIF : Main interferometer  
 MIR : Mirror  
 MMT : Mode matching telescope  
 OMC : Output mode cleaner

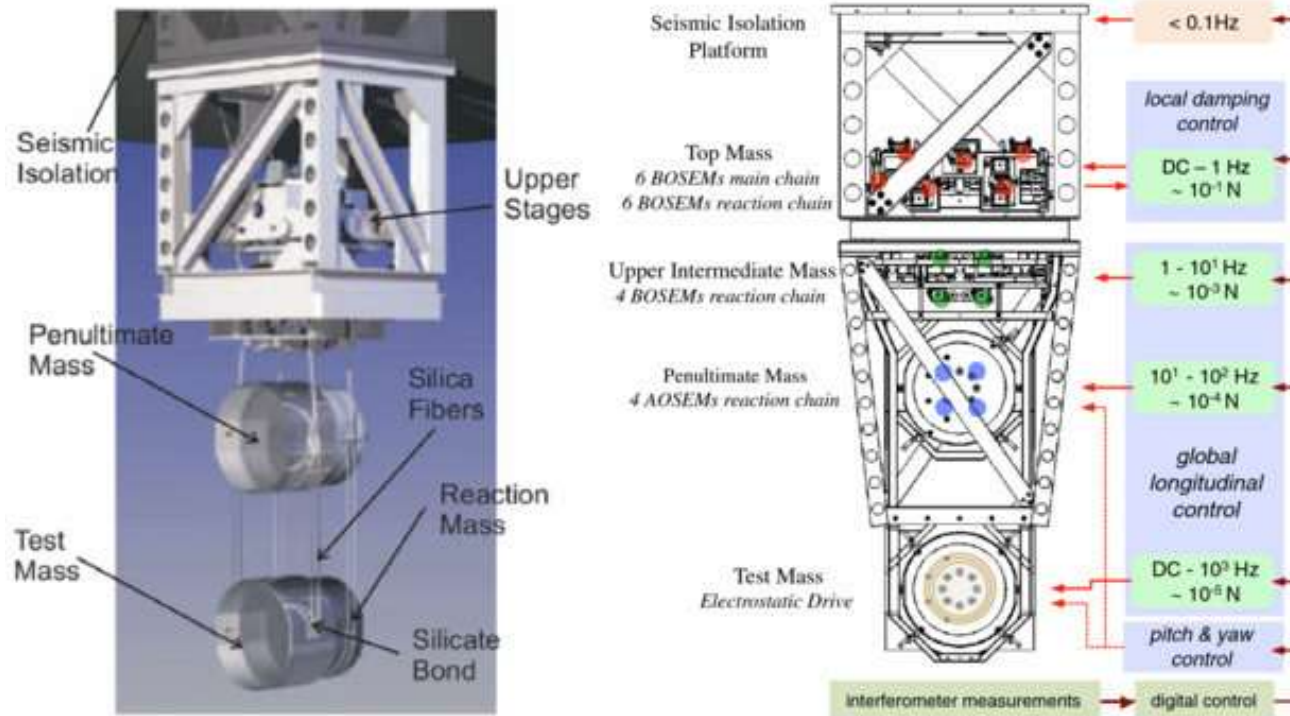
# Arm length stabilization of KAGRA



Class. Quant. Grav. 37 (2020) 035004

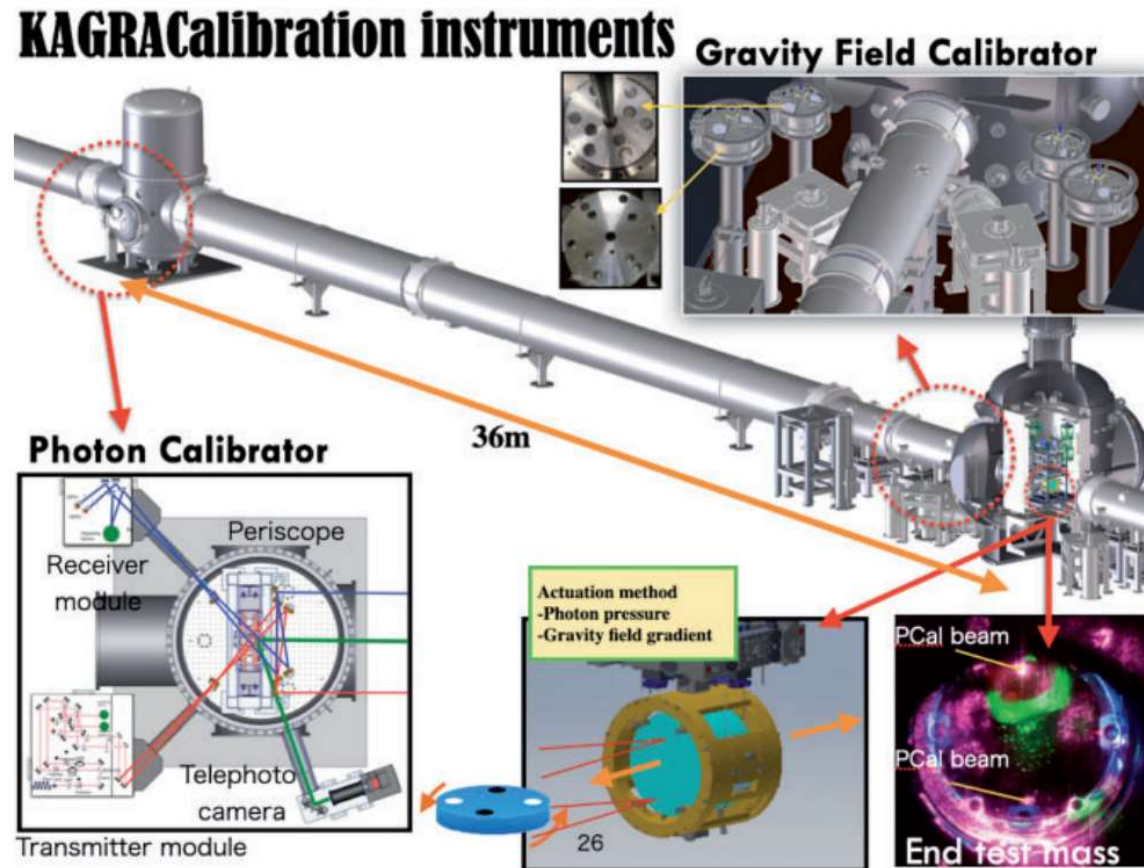


# Sensors in gravitational wave detector





# Calibration instrument

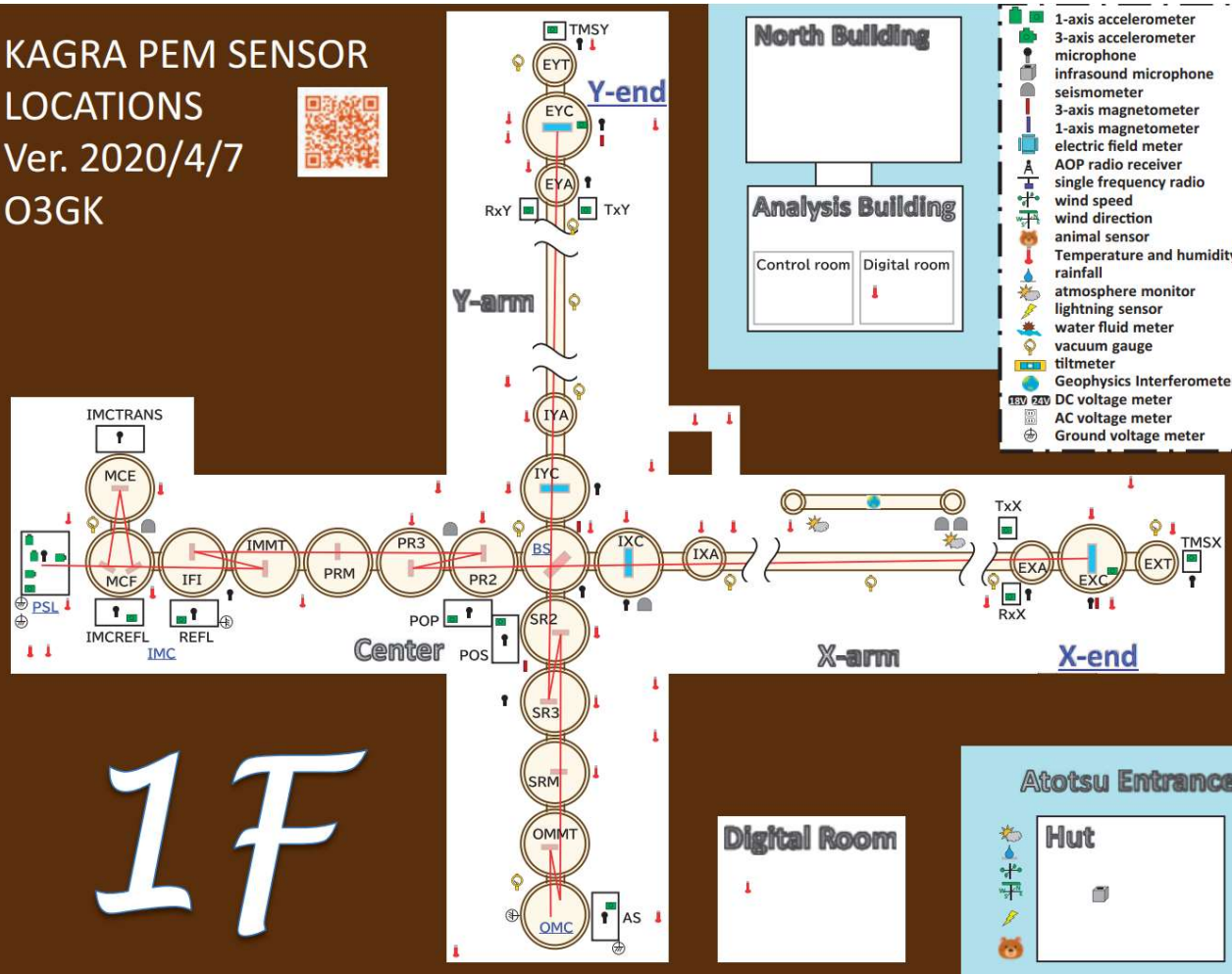




# Sensors in KAGRA



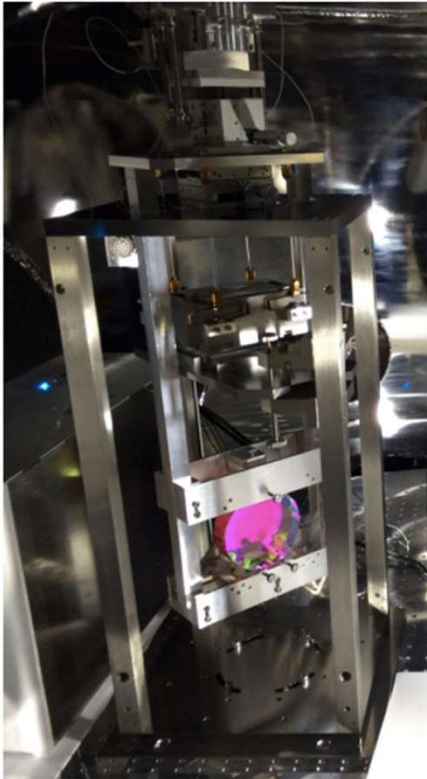
KAGRA PEM SENSOR  
LOCATIONS  
Ver. 2020/4/7  
03GK



1F

온누리 이모그림 2020

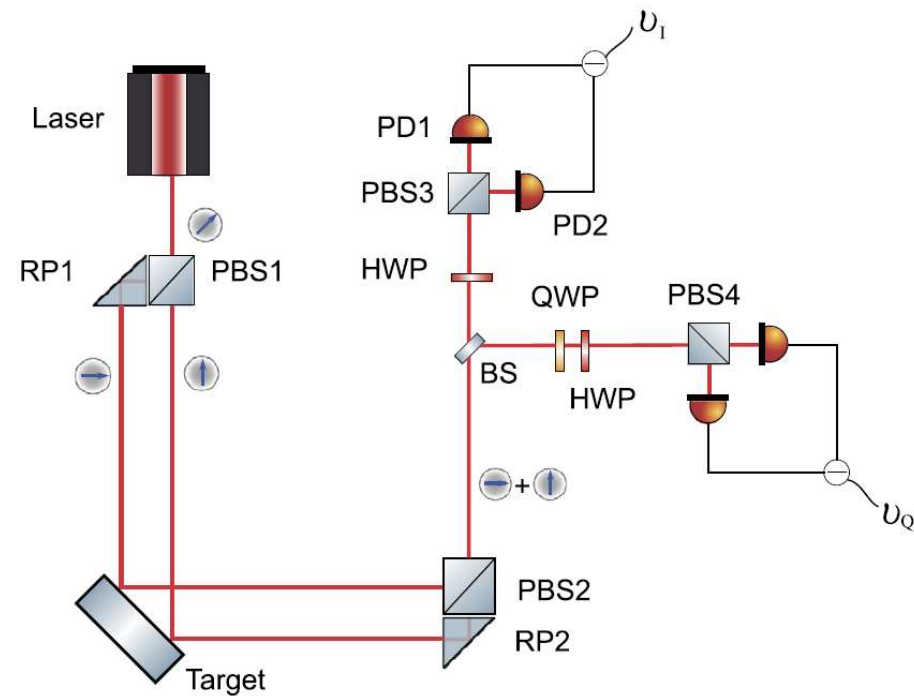
# Sensors in gravitational wave detector



Required specification of tilt sensor

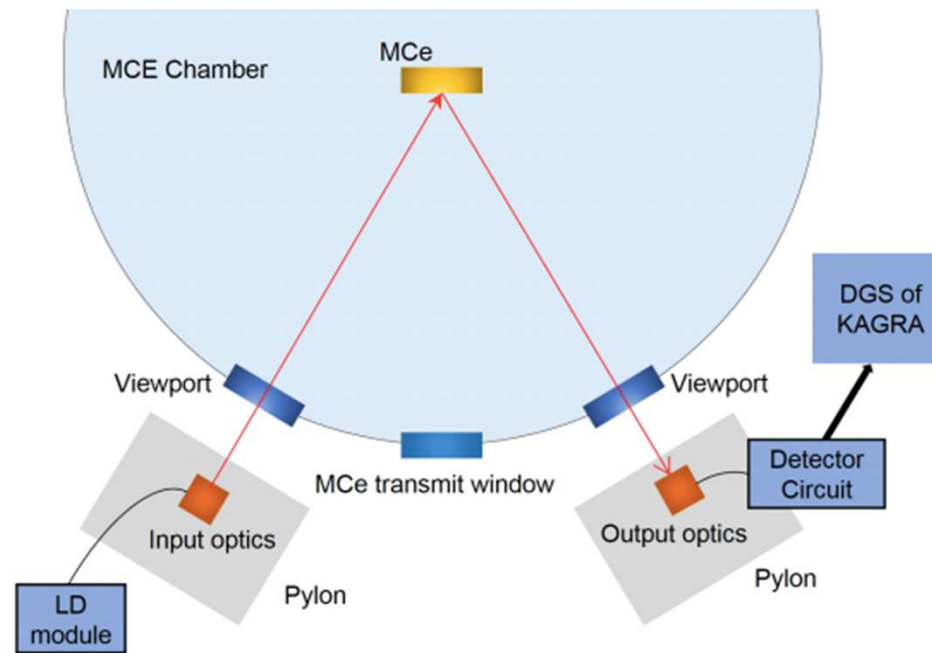
- Resolution :  $0.1 \mu\text{rad}$  for cavity  
 $1 \mu\text{rad}$  for BS
- Drift :  $<10 \mu\text{rad/day}$
- Measurement Range:  $>1\text{mrad}$ .

# Folded mach-zehnder interferometer

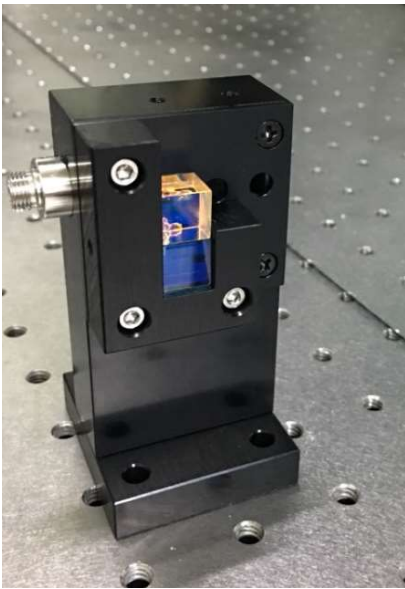


Physics Letters A, 382, 29, 1950-1955 (2018)

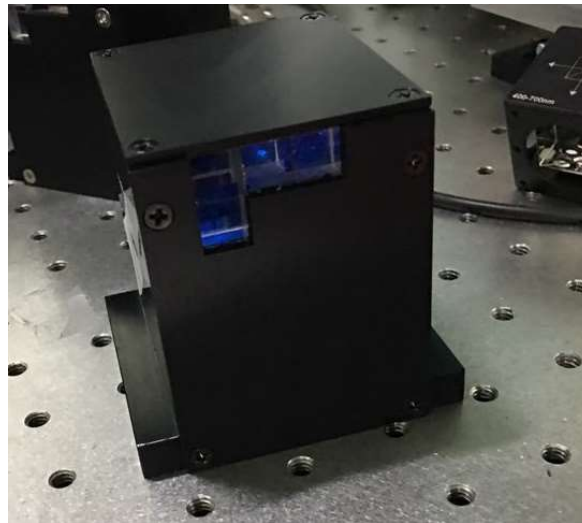
# Demonstration of 2-axis tilt sensor



# Interferometric tilt sensor



Input optics



Detector optics



Controller



# Demonstration of 2-axis tilt sensor



# KAGRA O3 Commissioning

