Gravitational Wave Cosmology

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(2024년 7월 29 - 8월 2일)

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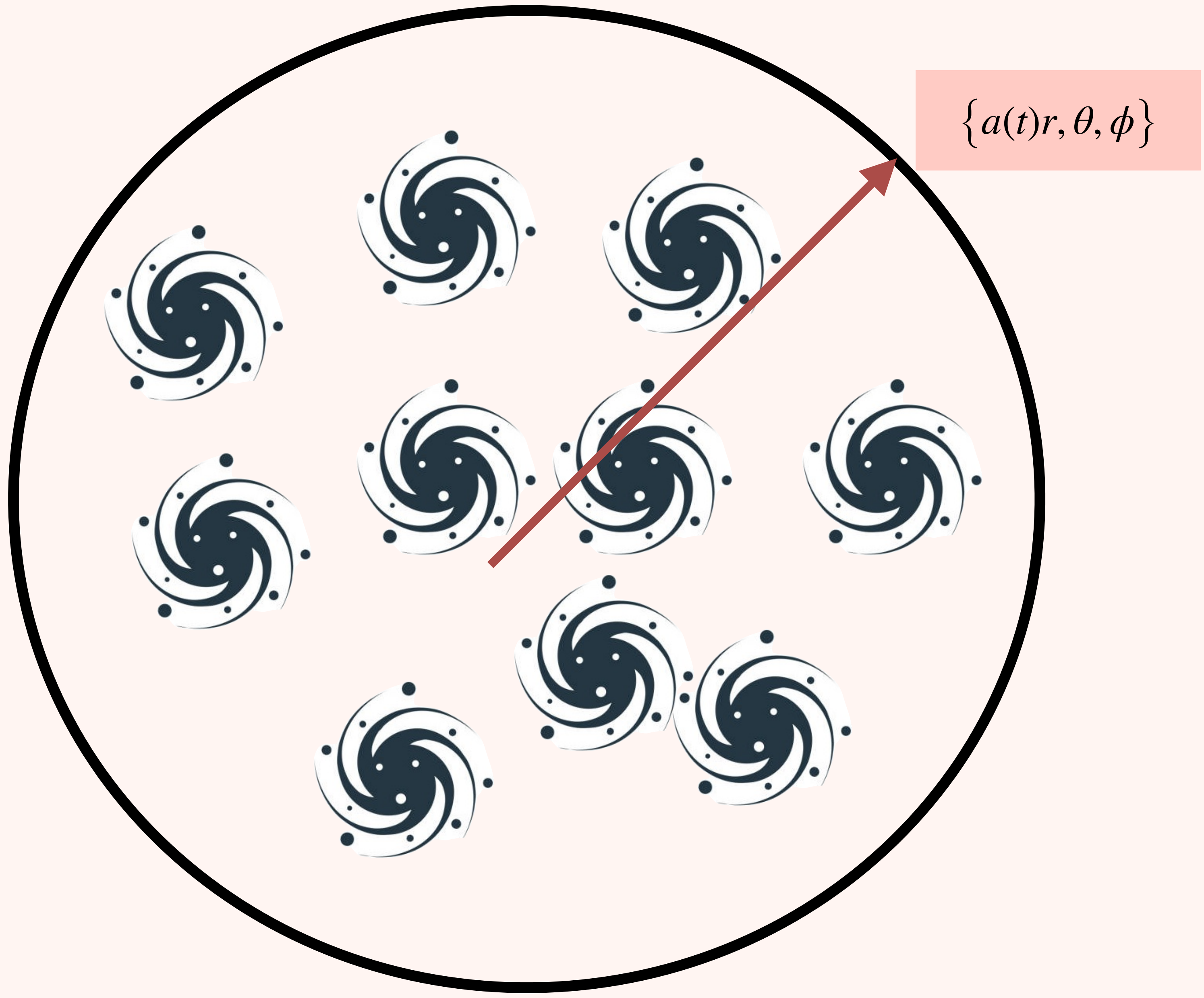
Topics

- Cosmological models and parameters
- Hubble constant and Hubble Tension
- Measurements of distances to GW sources
- Multi-messenger astronomy for Hubble constant
- Future prospects of dark siren measurements

Big Bang Cosmology



- Cosmological Principle
  - Universe is homogenous and isotropic
- Einstein's equations :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu}$$

where λ is the cosmological constant

- Friedman-Robertson-Walker metric for homogenous and isotropic universe

$$ds^2 = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right\}$$

where $(r, \overrightarrow{\Omega})$ is the coming coordinates and $a(t)$ is the scale factor.

Expansion of the universe

- The energy-momentum tensor $T_{\mu\nu}$
  - $T_{\mu\nu} = pg_{\mu\nu} + (p + \rho)U_\mu U_\nu$, U: velocity four vector of the Hubble flow
- The Einstein's equations become

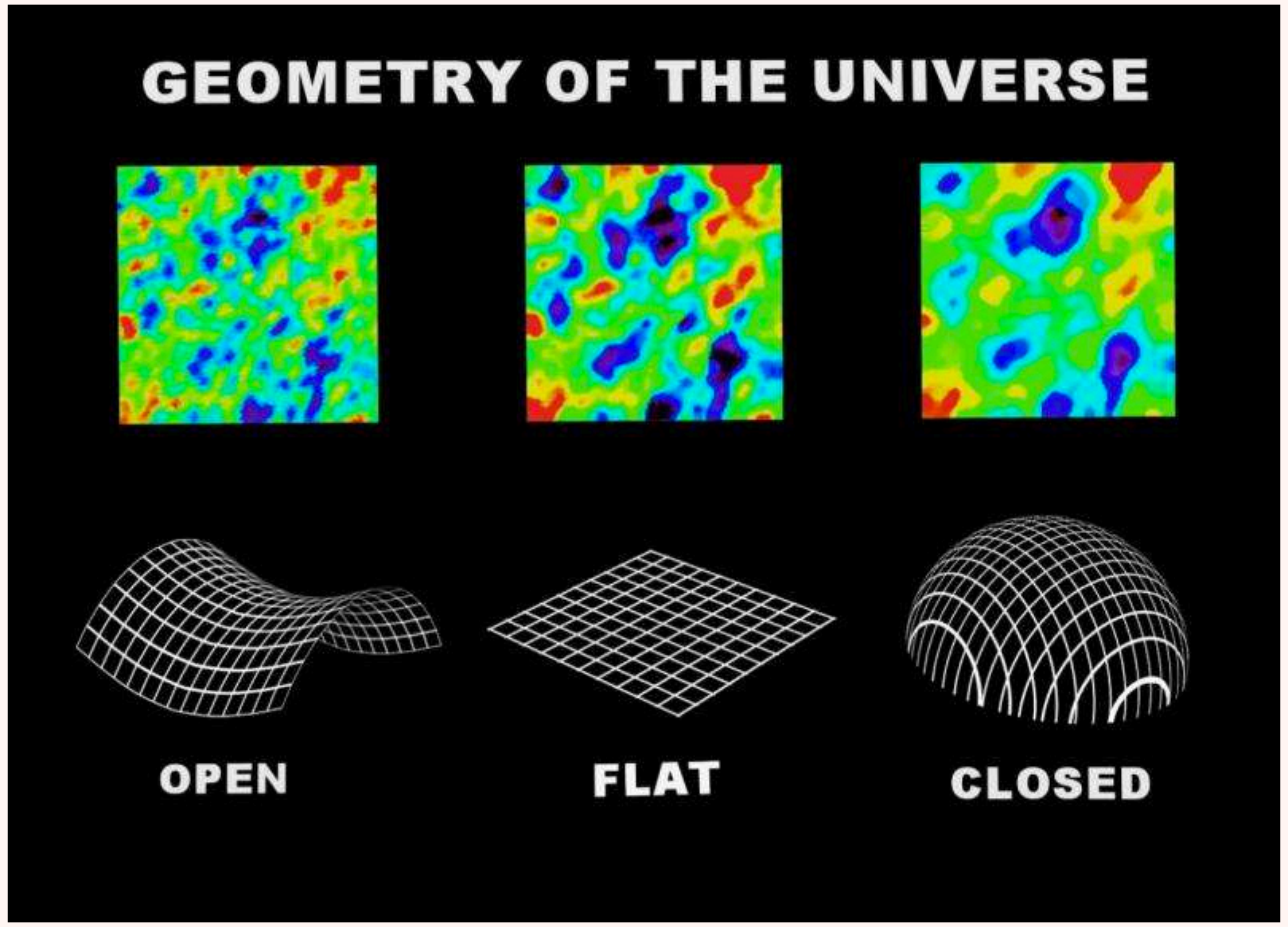
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \qquad \frac{d}{da}(\rho a^3) = -3pa^2$$

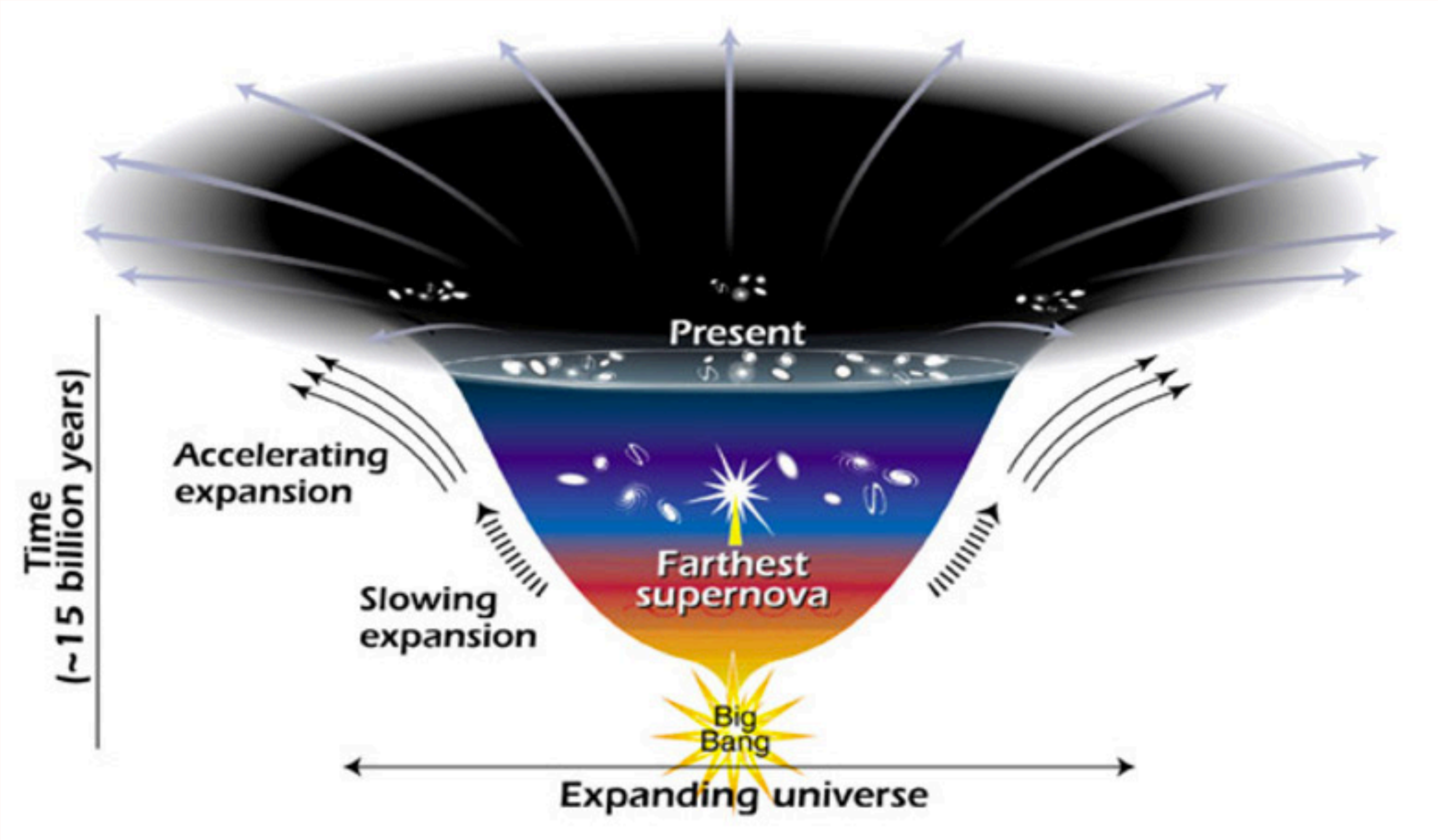
p : Pressure, ρ : energy density

- Constituents of the Universe
  - Matter: $p = 0, (p \ll \rho)$. $\rightarrow \rho \propto a^{-3}, \ddot{a} < 0$ (deceleration)
  - Radiation (or relativistic matter): $p = \frac{1}{3}\rho$, $\rightarrow \rho \propto a^{-4}$, $\ddot{a} < 0$ (deceleration)
  - Dark Energy: $p = -w\rho$, $w > \frac{1}{3}$, $\ddot{a} > 0$ (deceleration if $w > 1/3$)

Cosmological Parameters

- Hubble parameter
  - $H(t) = \frac{\dot{a}}{a}$, $H_0 = H(t_0)$: Hubble constant
- Density parameters: $\Omega_0 = \frac{\rho_0}{\rho_{crit}}$
  - $\Omega < 1$: Open,
  - $\Omega > 1$: Closed
  - $\Omega = 1$: Flat
- $\Omega_m = \Omega_b + \Omega_{DM}$ (Baryon, Dark matter)
- If the density of the universe is dominated by matter+ratiation, deceleration
- If the density of the universe is dominated by dark energy, acceleration





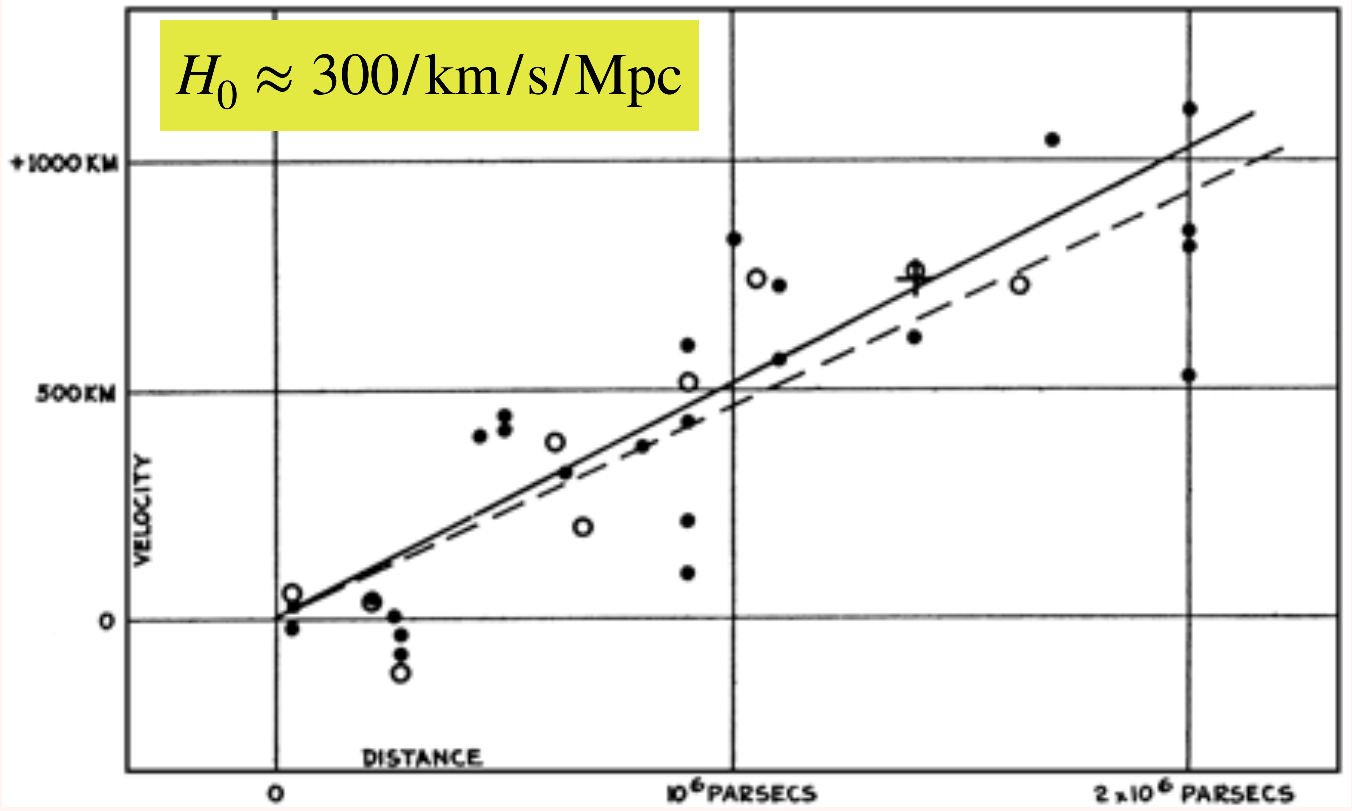
Accelerating universe: $\Omega_\Lambda > \Omega_m$

How the Hubble parameter is measured?

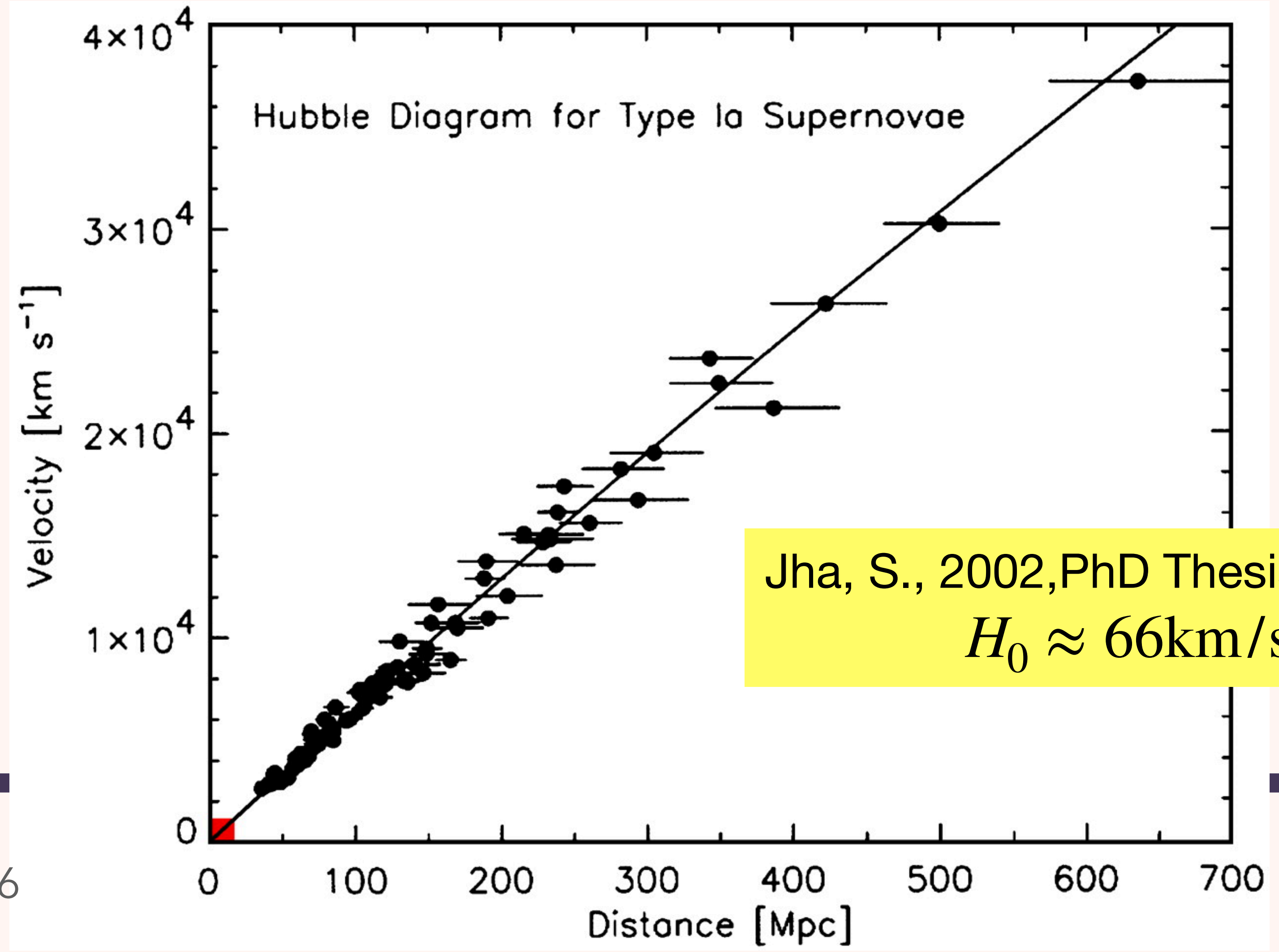
- Hubble's law;

  $v_r = H_0 d$

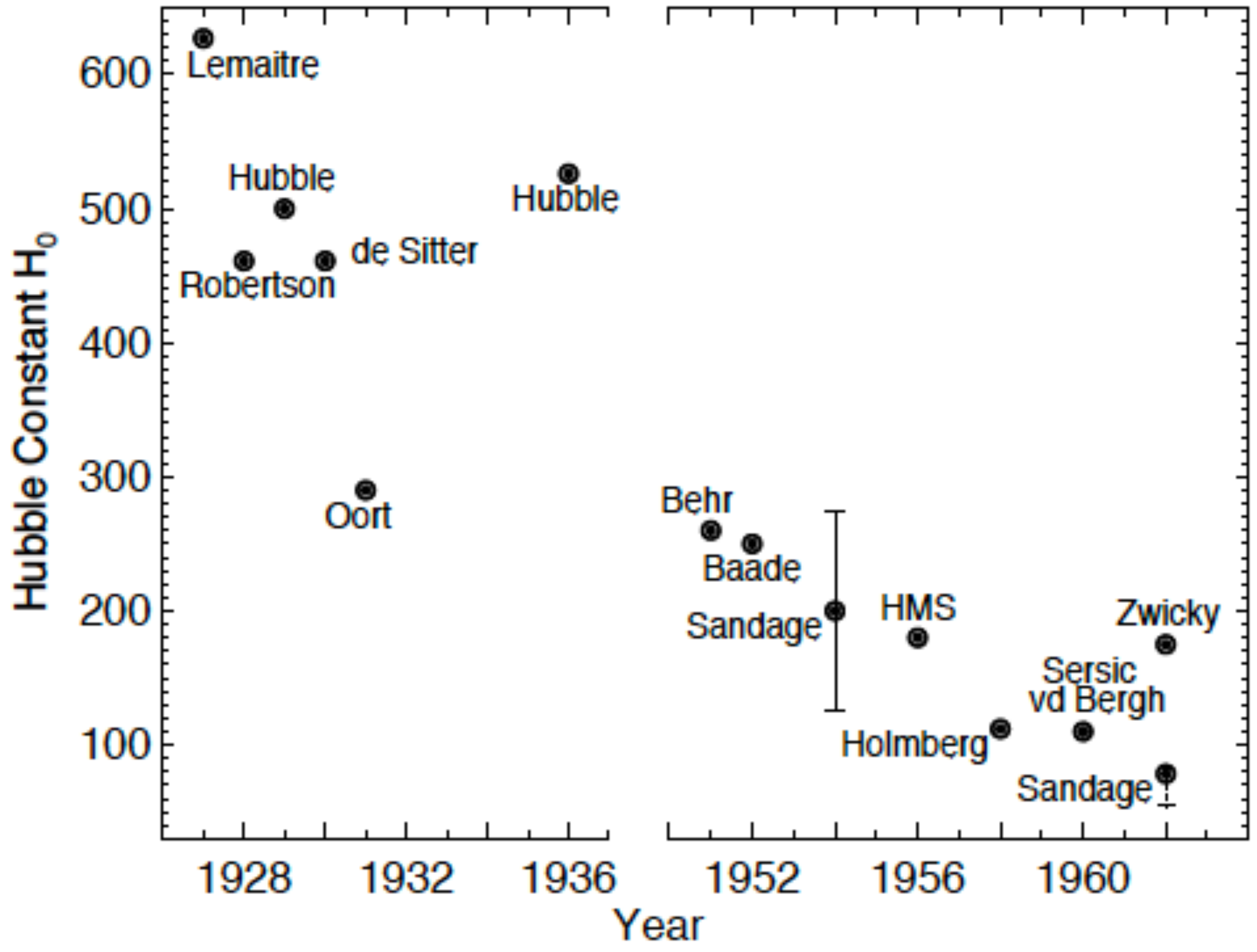
- We need to measure radial velocity (v_r, or redshift z) and distance (d) simultaneously for large number of galaxies.
- Galaxies should be not too close (because of the peculiar motion) and not too far (because we are looking at th past).
- Distance measurements are difficult, especially for distant galaxies



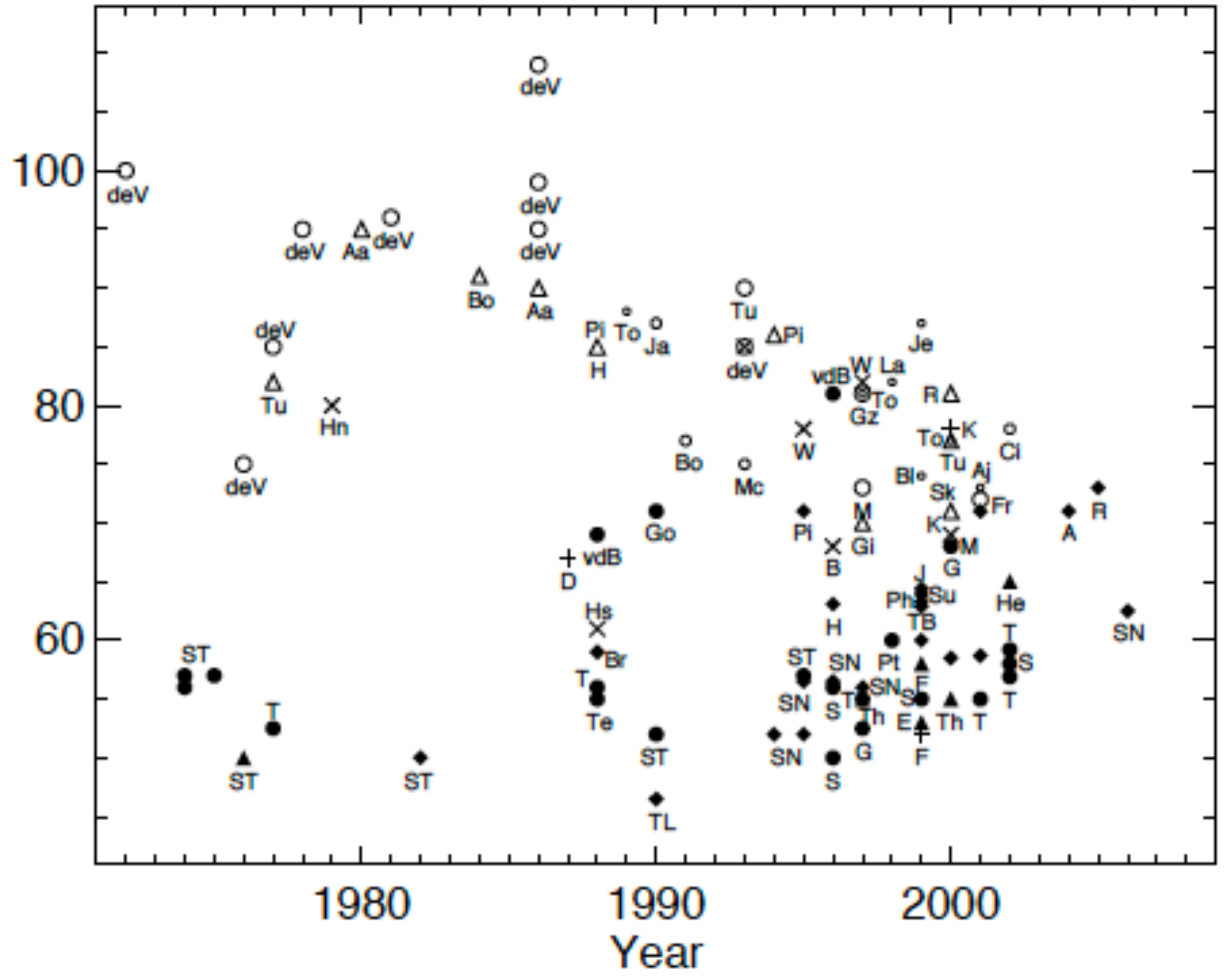
Hubble, E. P., 1929, Prof. Nat. Acad. Sci., 15, 168



H_0 has changed over time



From 1927 to 1962
(Tamman 2005)



From 1975
(Tamman 2005)

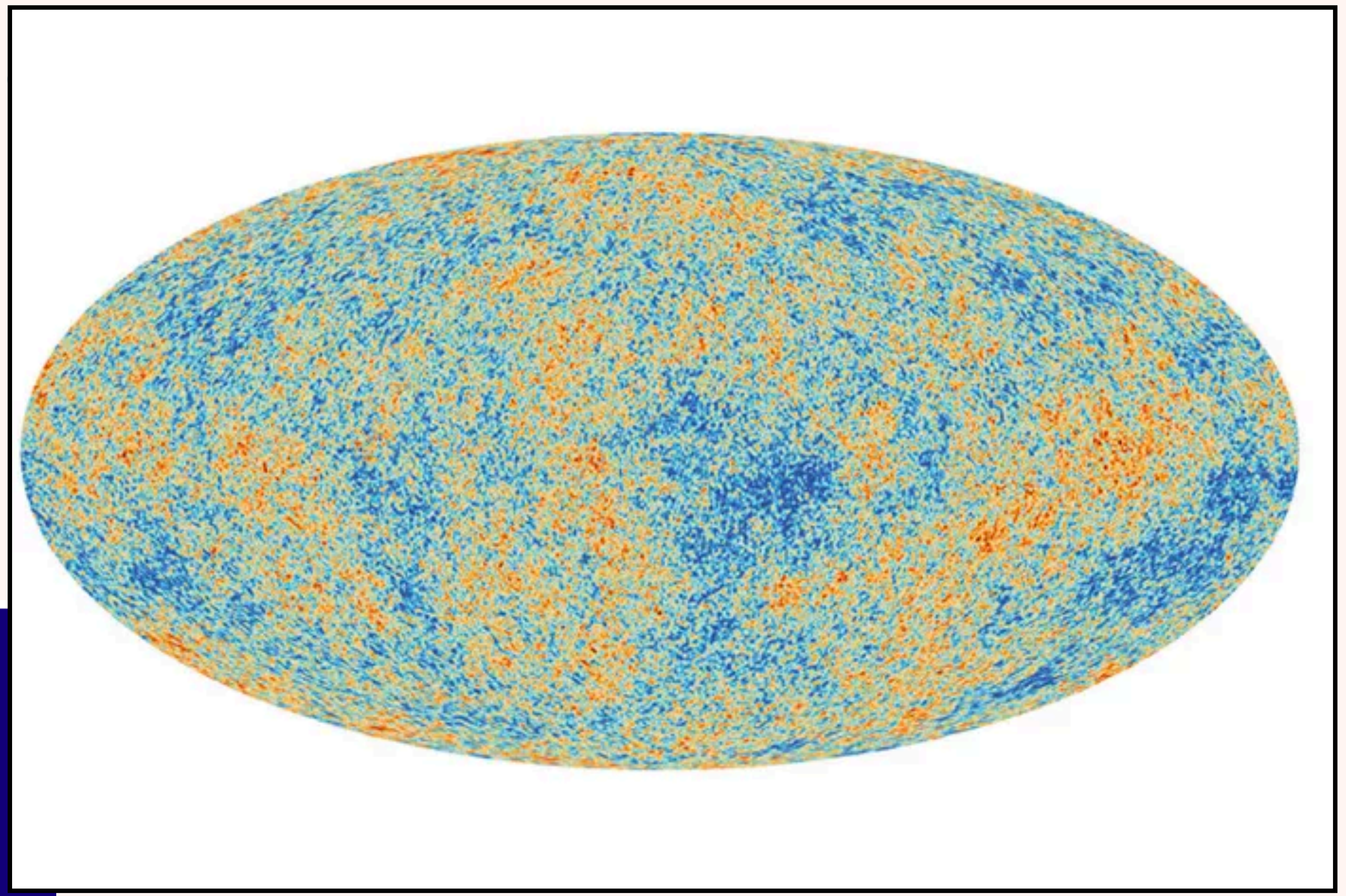
Summary of local measurement: Hubble Key Project

(Freedman et al., 2001, ApJ, 553, 47)

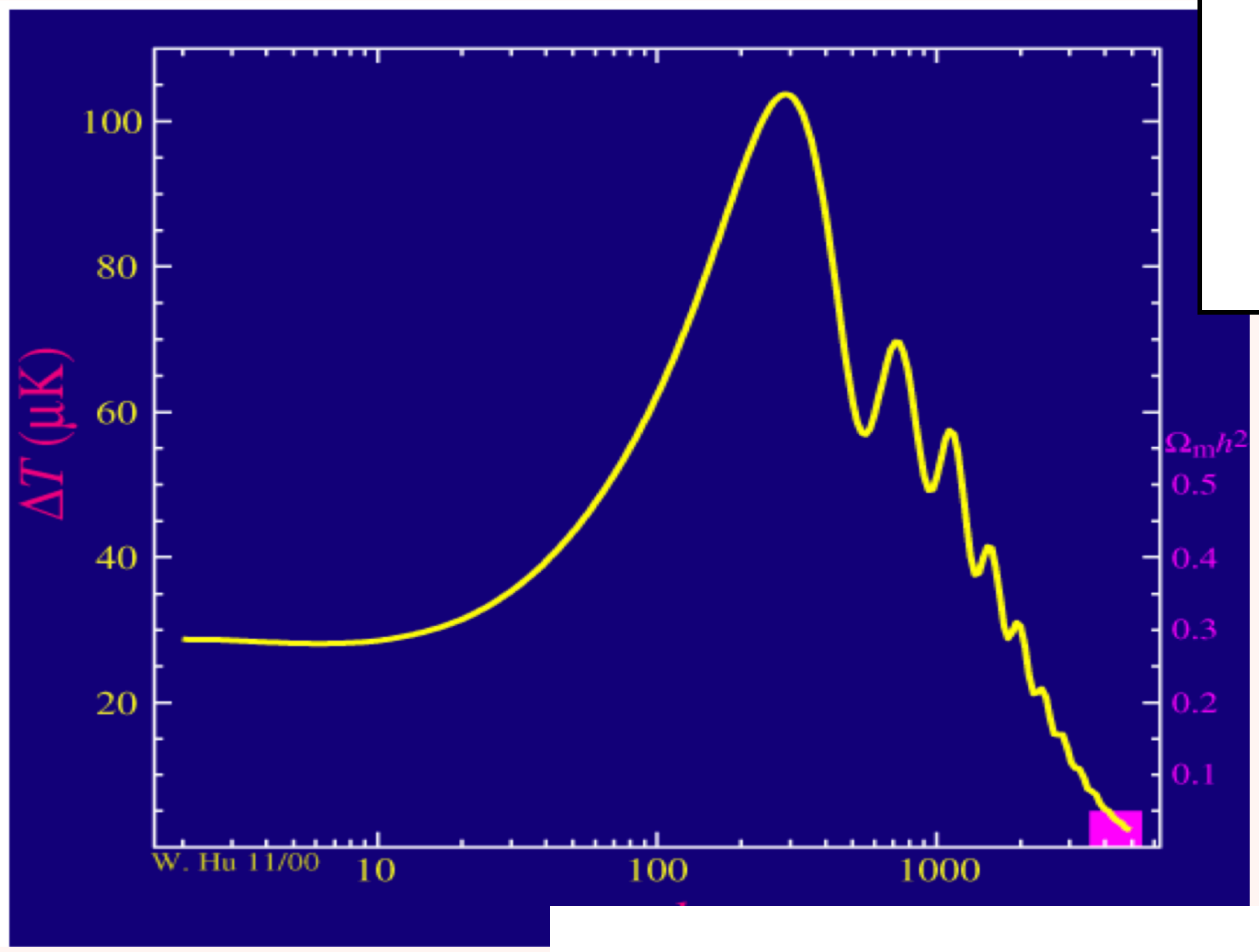
- Cepheid Hubble Diagram (up to 25 Mc): $H_0 = 75 \pm 10(r)\text{km/sec/Mpc}$
- Relative distance methods (60 - 400 Mpc)
    - Type Ia SN: $H_0 = 71 \pm 2(r) \pm 6(s)\text{km/sec/Mpc}$
    - Tully-Fisher Relation $H_0 = 71 \pm 3(r) \pm 7(s)\text{km/sec/Mpc}$
    - Fundamental Plane: $H_0 = 70 \pm 5(r) \pm 6(s)\text{km/sec/Mpc}$
    - Surface Brightness Fluctuation $H_0 = 82 \pm 6(r) \pm 9(s)\text{km/sec/Mpc}$
    - Type II SN $H_0 = 72 \pm 9(r) \pm 7(s)\text{km/sec/Mpc}$
- Combination:

$$H_0 = 72 \pm 8\text{km/sec/Mpc}$$

Cosmic Microwave Background contains Rich Inform



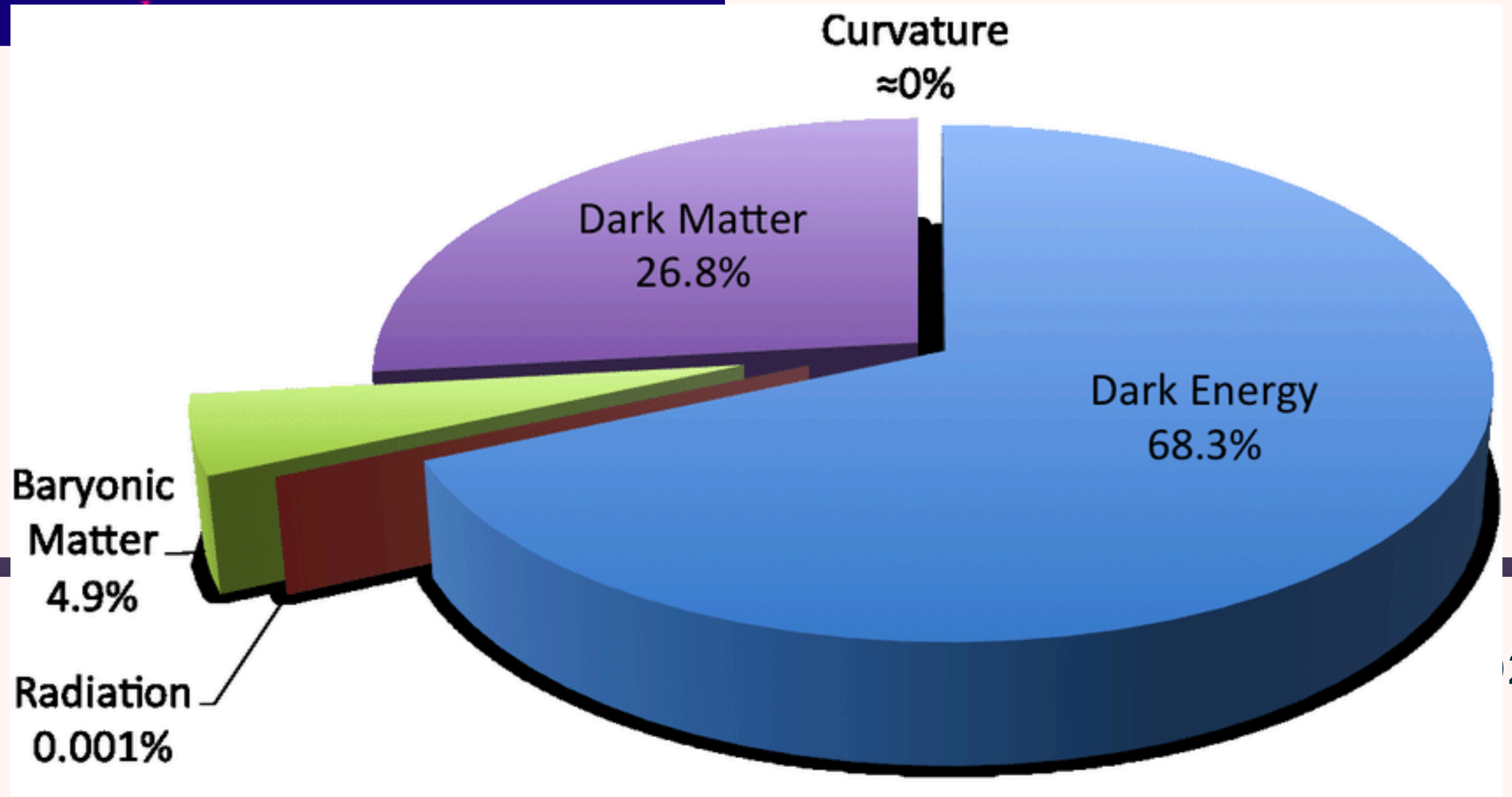
- CMB power spectrum depends on various parameters: Ω_0, Ω_b, Ω_m and H_0.
- For flat universe,
  - Peak positions: ~ $\Omega_m h^3$
  - Height of the second peak ~ $\Omega_b h^2$
  - Peak Ratios ~ $\Omega_m h^2$



$$h = H_0/100 \text{ km/s}$$

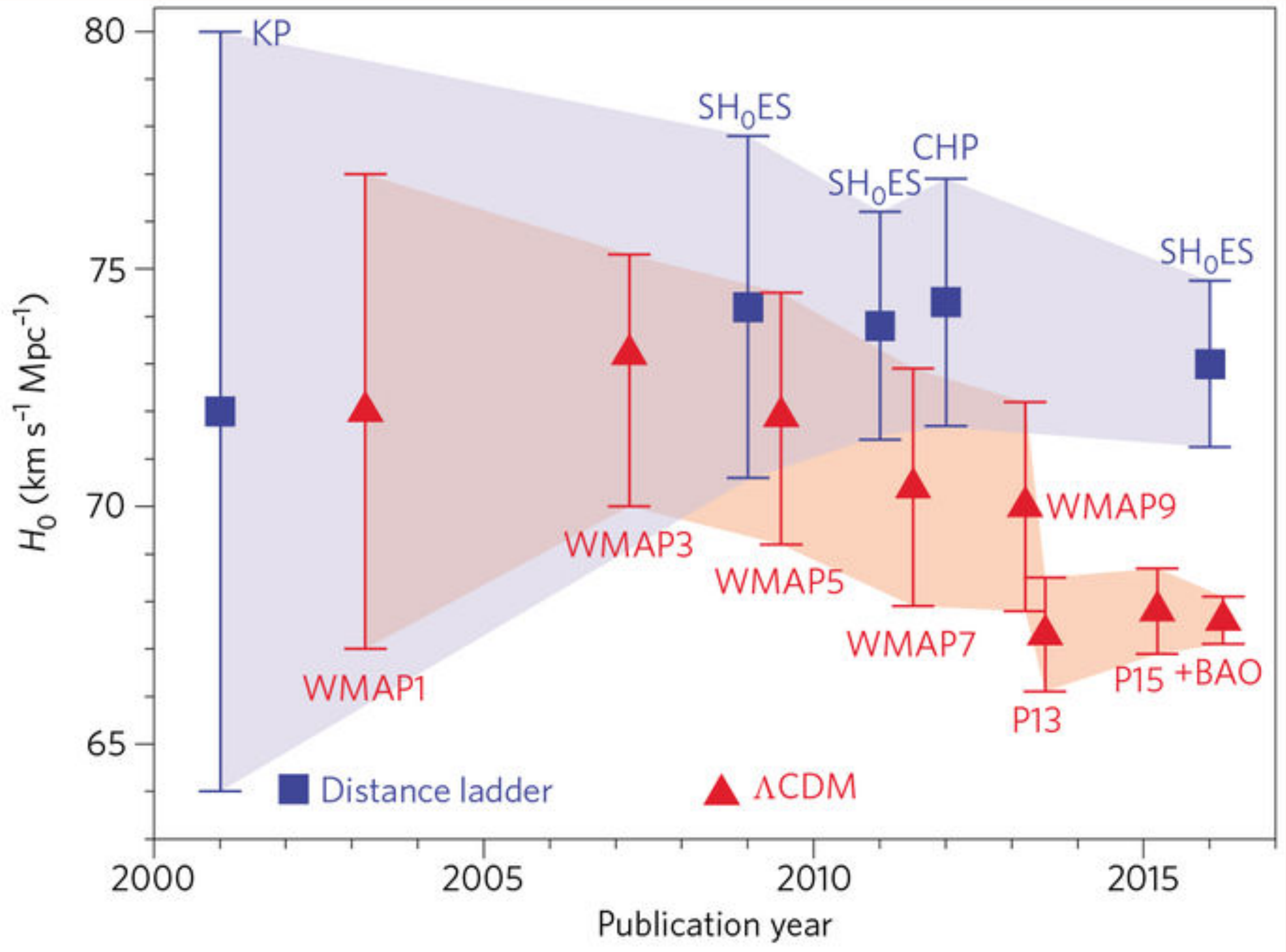
Results from Planck (2018)

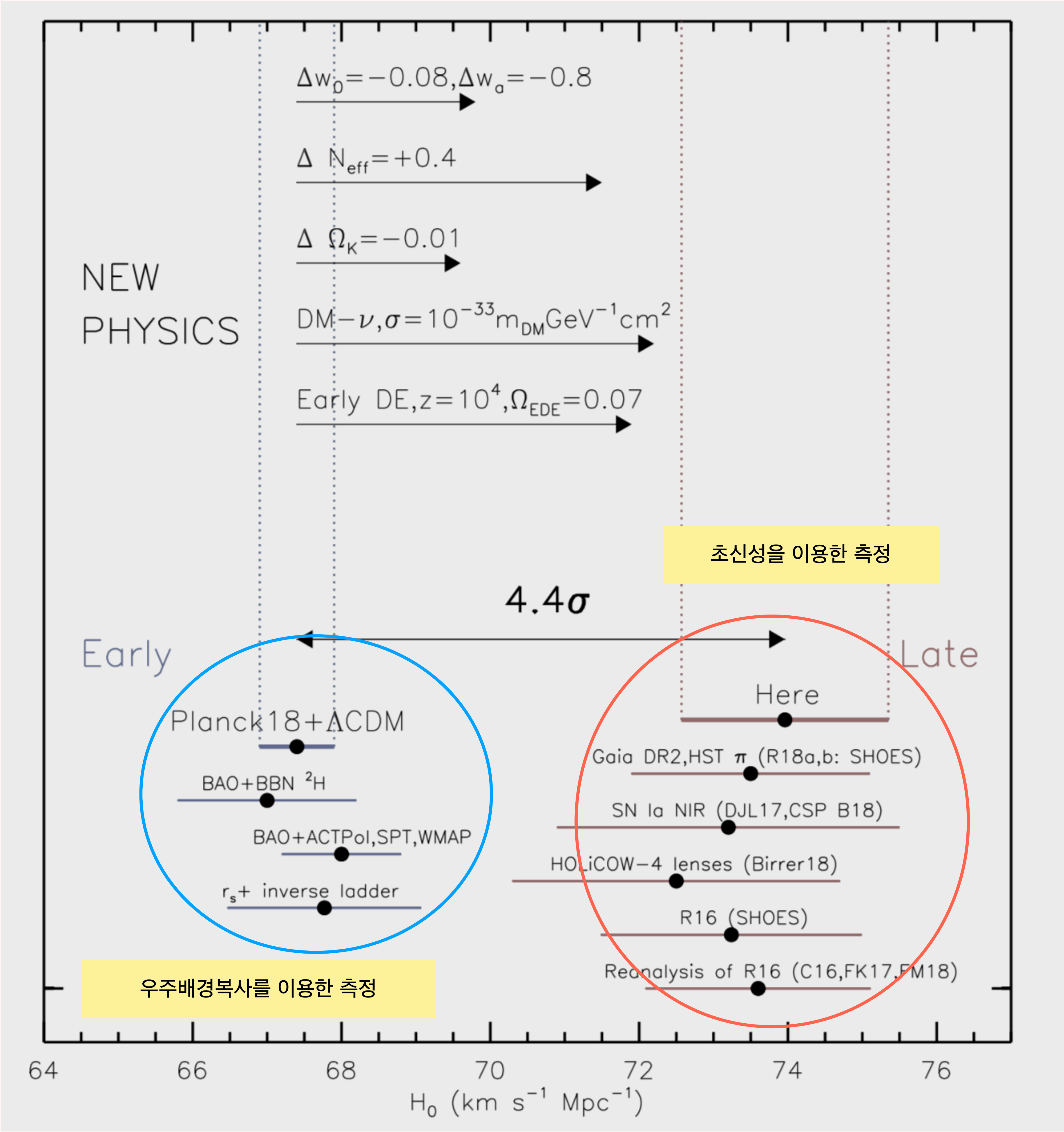
$$H_0 = 67.4 \pm 0.5 \text{ km/sec/Mpc}$$



Hubble tension

- SH0ES: SNe, H0
- WMAP: Wilkinson Microwave Probe
- P: Planck
- BAO: Baryonic Acoustic Oscillation





Riess et al. (2019) ApJ, 876, 85

W. Freedman, 2017, Nature Astronomy, 1, 0121

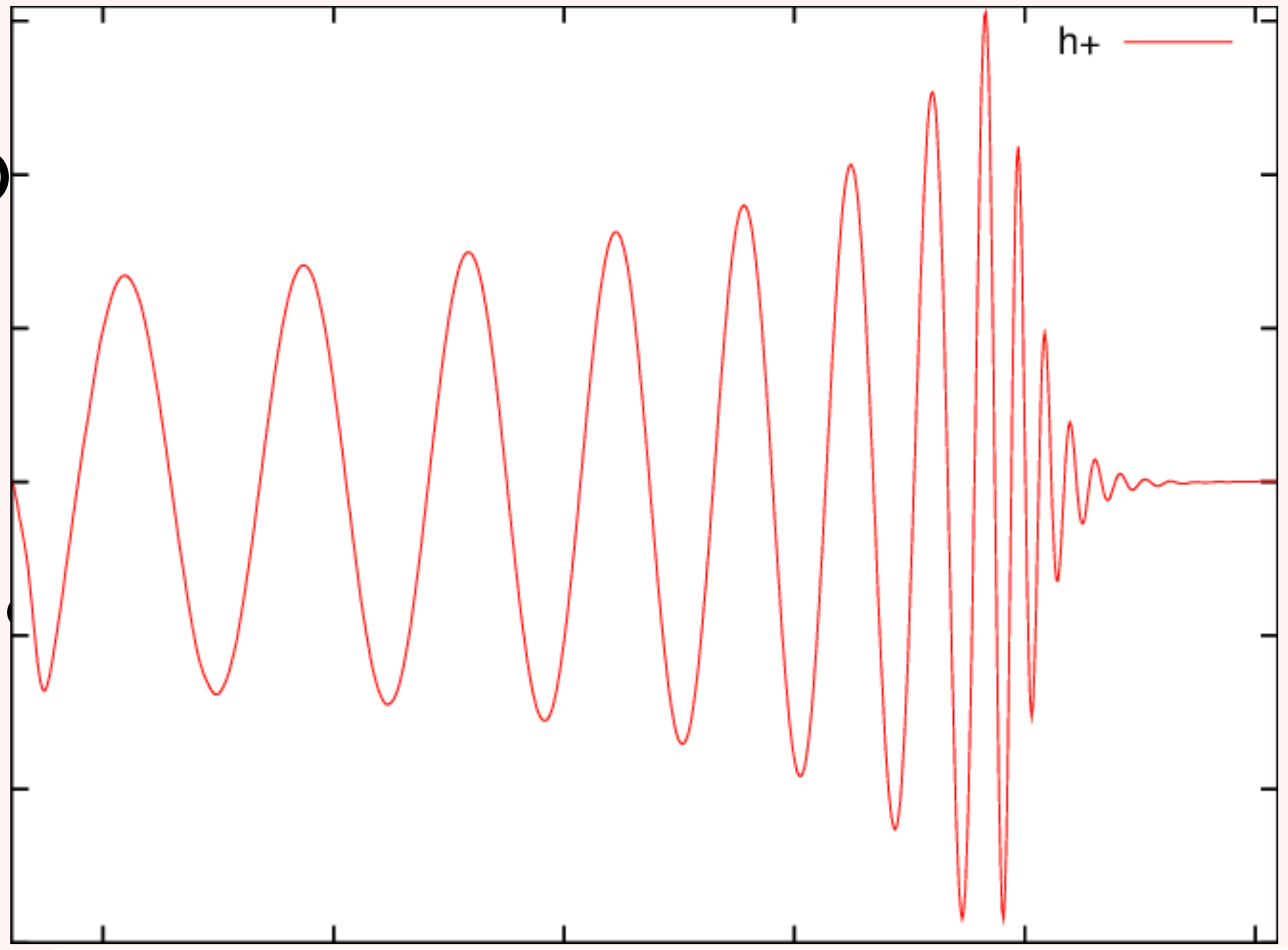
Cosmological Research with GWs

- Reconstruction of expansion history of the universe through direct measurements of distances
  - GWs from compact binaries are 'standard sirens' as the distances can be estimated from the detected waveforms and amplitudes
    - Bright sirens [optical counterparts] vs. Dark sirens [no optical counterparts]
  - Need to identify the host galaxies in order to measure the redshifts in order to construct Hubble diagram

Estimation of Parameters from GW Observations

- What we measure is the waveforms form for certain duratio

$$h(t) = F_+(\alpha, \delta, \psi, t_c) h_+(t) + F_\times(\alpha, \delta, \psi, t_c) h_\times(t)$$

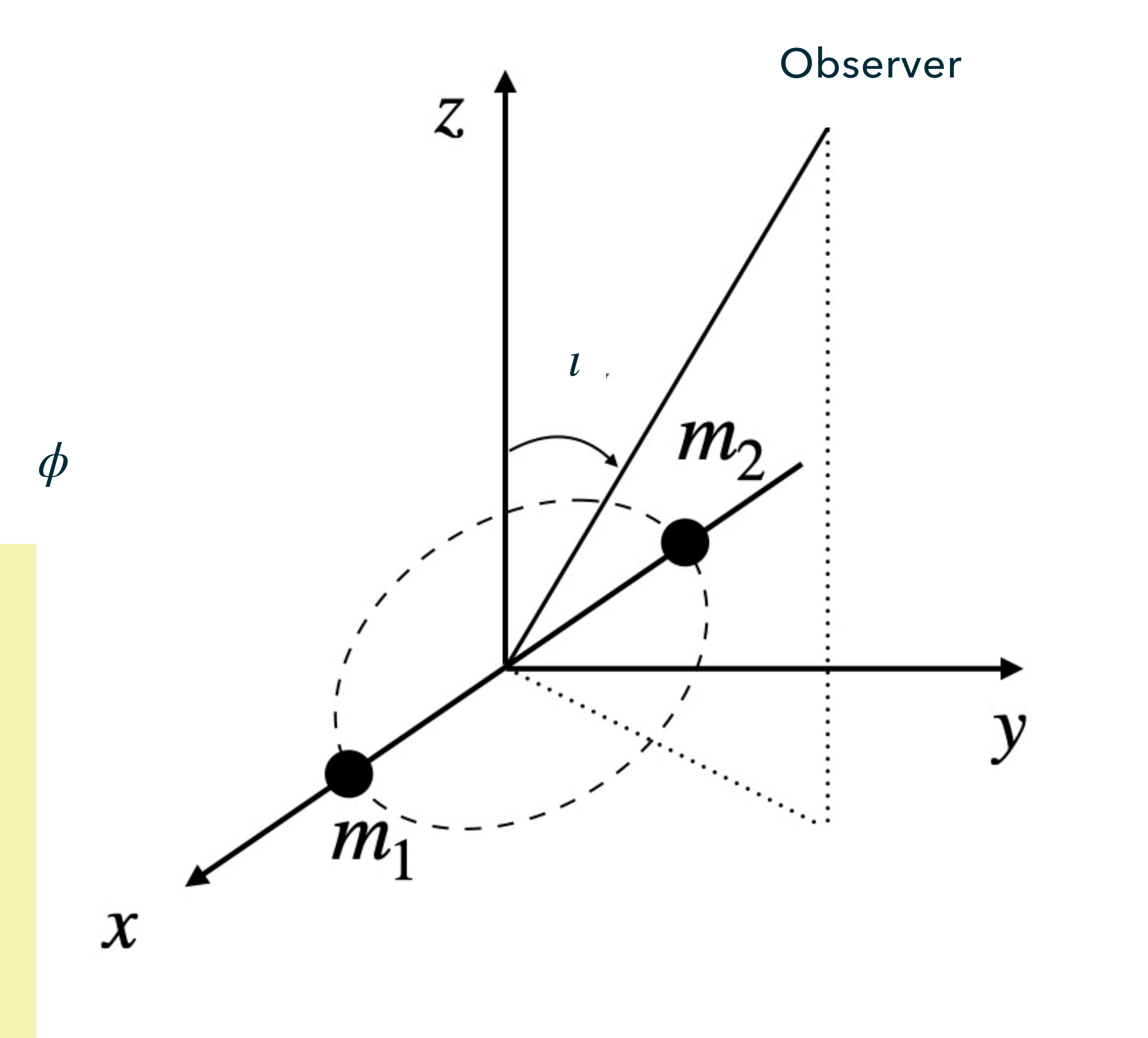


- For circular binaries, 15 parameters are imprinted on the w
  - Intrinsic: $m_1, m_2, \mathbf{s_1}, \mathbf{s_2}$ (8)
  - Extrinsic: $\mathbf{\Omega} = (\alpha, \delta), \iota, d, \psi, t_c, \phi_c$ (7)
  - If we ignore spins, the number of parameters reduces to 9

Polarization angle

Coleascence Phase

- Duration of the observations from frequency f until merger (



$$T = \frac{f}{\dot{f}} = \frac{5}{96} \pi^{-8/3} \frac{c^5}{\eta (GM)^{5/3}} f^{-8/3}$$

For ground based detectors, $f_0 \sim$ 30 Hz
- $T < 1$ second for BBH
- $T \sim$ minute for BNS

For lower frequency detectors, T could become very long (weeks to years)

Estimation of Distances

- The shape of the waveforms does not depend on distances →masses & spins
- The amplitude of the GW signal is **inversely proportional** to the distance.

$$d_L = \frac{5}{96\pi^2}\frac{c}{h}\frac{\dot{f}_{GW}}{f_{GW}^3}$$
$$= 512\frac{1}{h_{21}}\left(\frac{0.01s}{\tau}\right)\left(\frac{100\text{Hz}}{f_{GW}}\right)^2 \text{ Mpc}$$

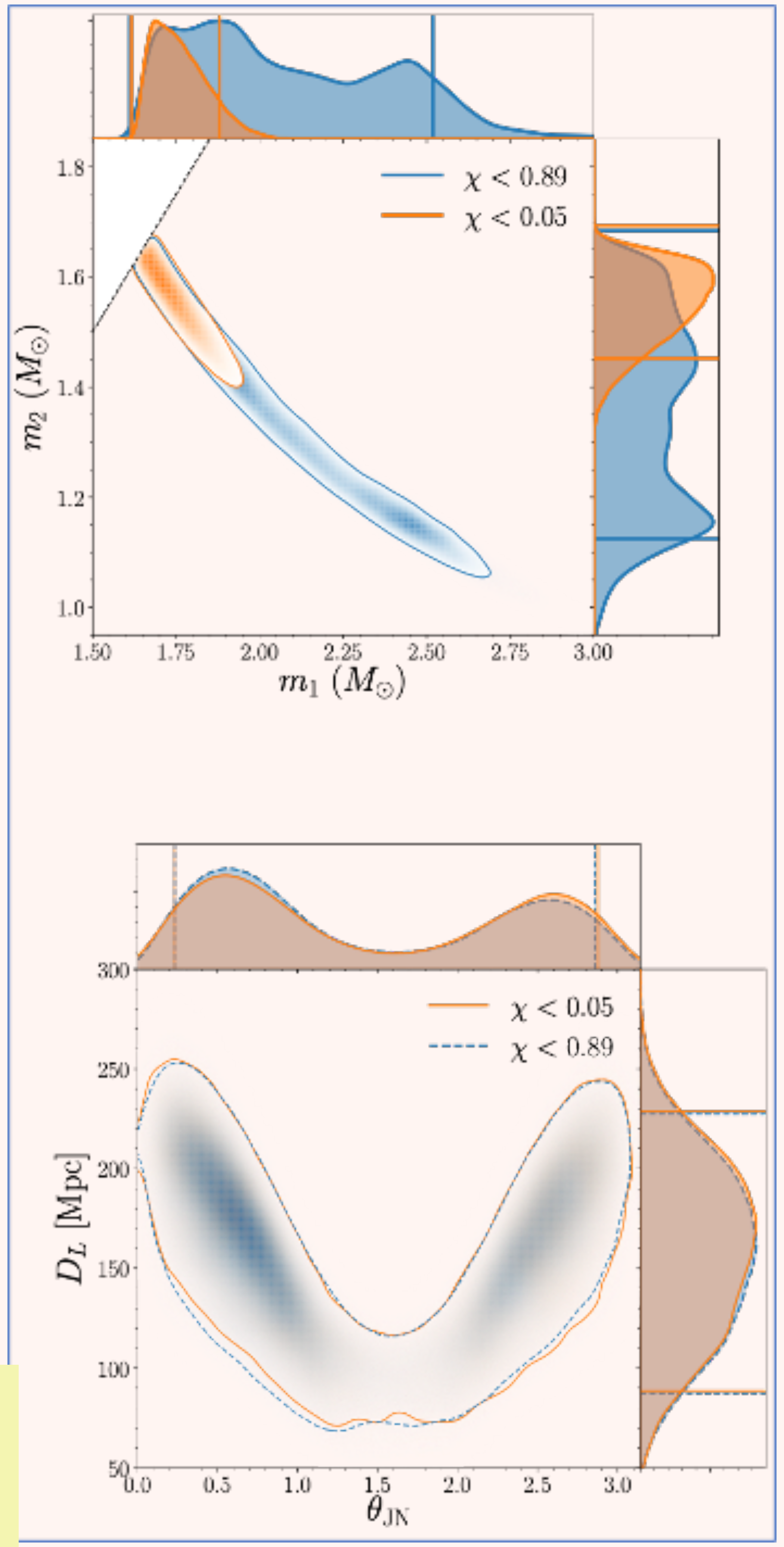
Schutz 1982

- The distance estimation from GWs does not suffer from **systematic uncertainty**,
- However, individual GW distance estimation is subject to large **statistical uncertainty**

$$h_+ = \frac{h_c}{d}(1+\cos^2\iota),\ h_c \equiv 2\mu(M\Omega^{2/3})\cos(2[\Omega t - \phi_0])$$

$$h_\times = \frac{h_s}{d}\cos\iota,\ h_s \equiv 2\mu(M\Omega^{2/3})\sin(2[\Omega t - \phi_0])$$

Ω : Orbital frequency

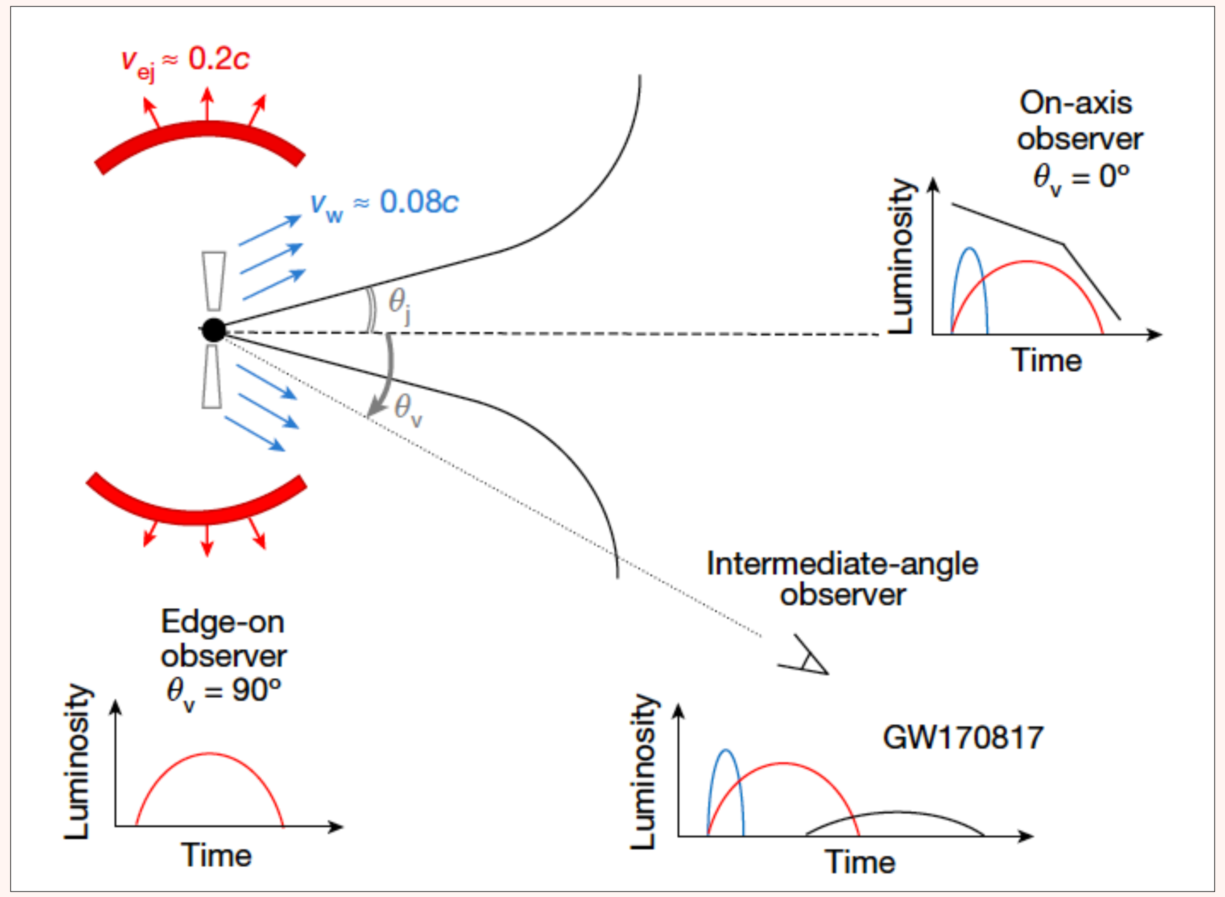


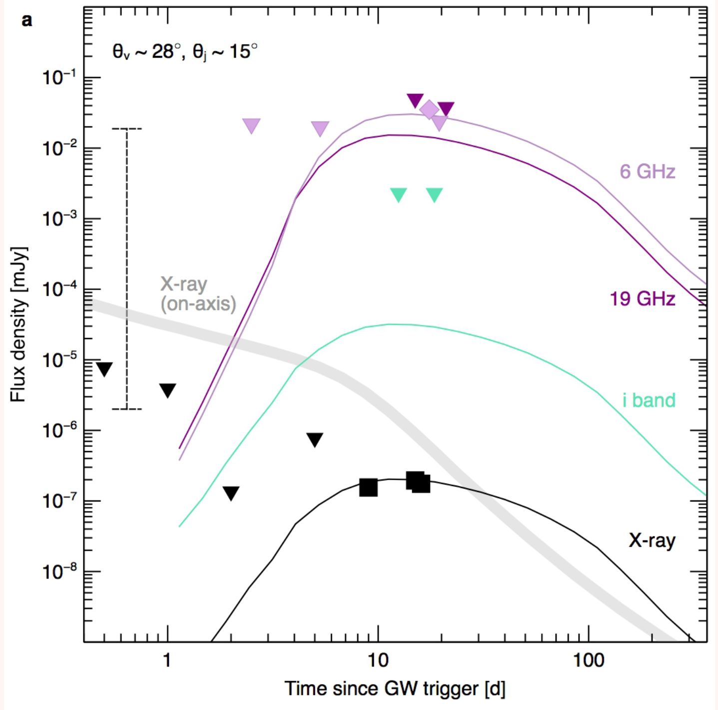
Estimated masses (upper) and distance (lower) to GW190425, a binary neutron star merger event.(Figure from Abbott et al. 2020, ApJL, 892, L3)

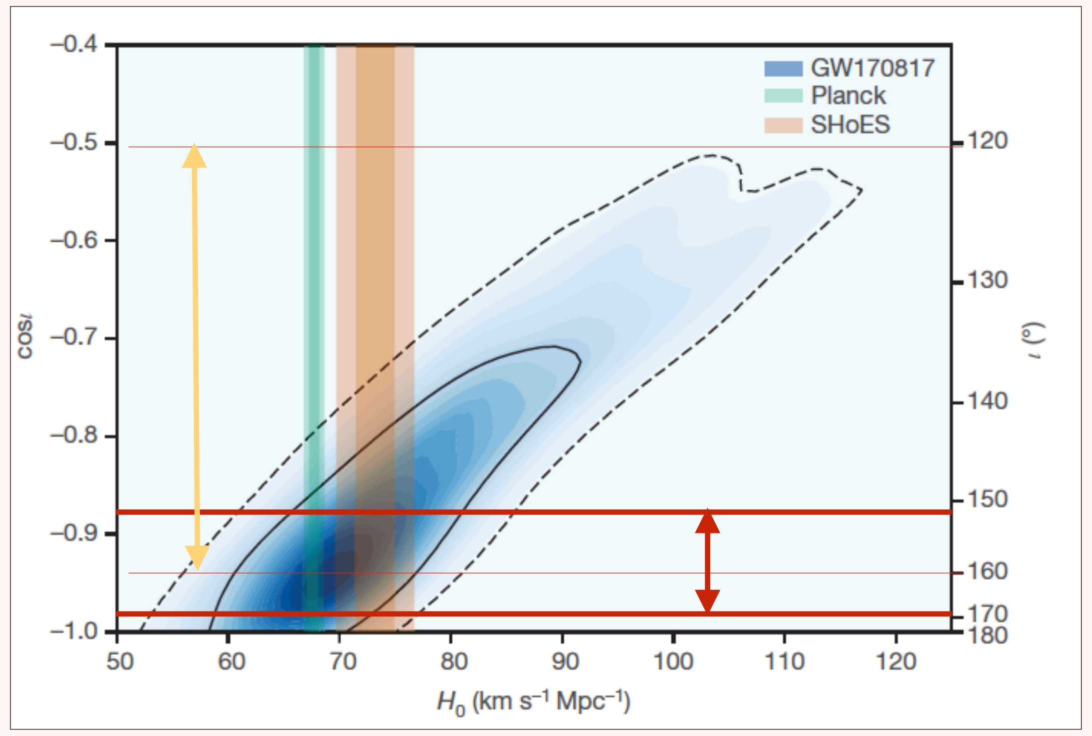
Can we constrain the viewing angle?

Troja et al. 2017, Nature

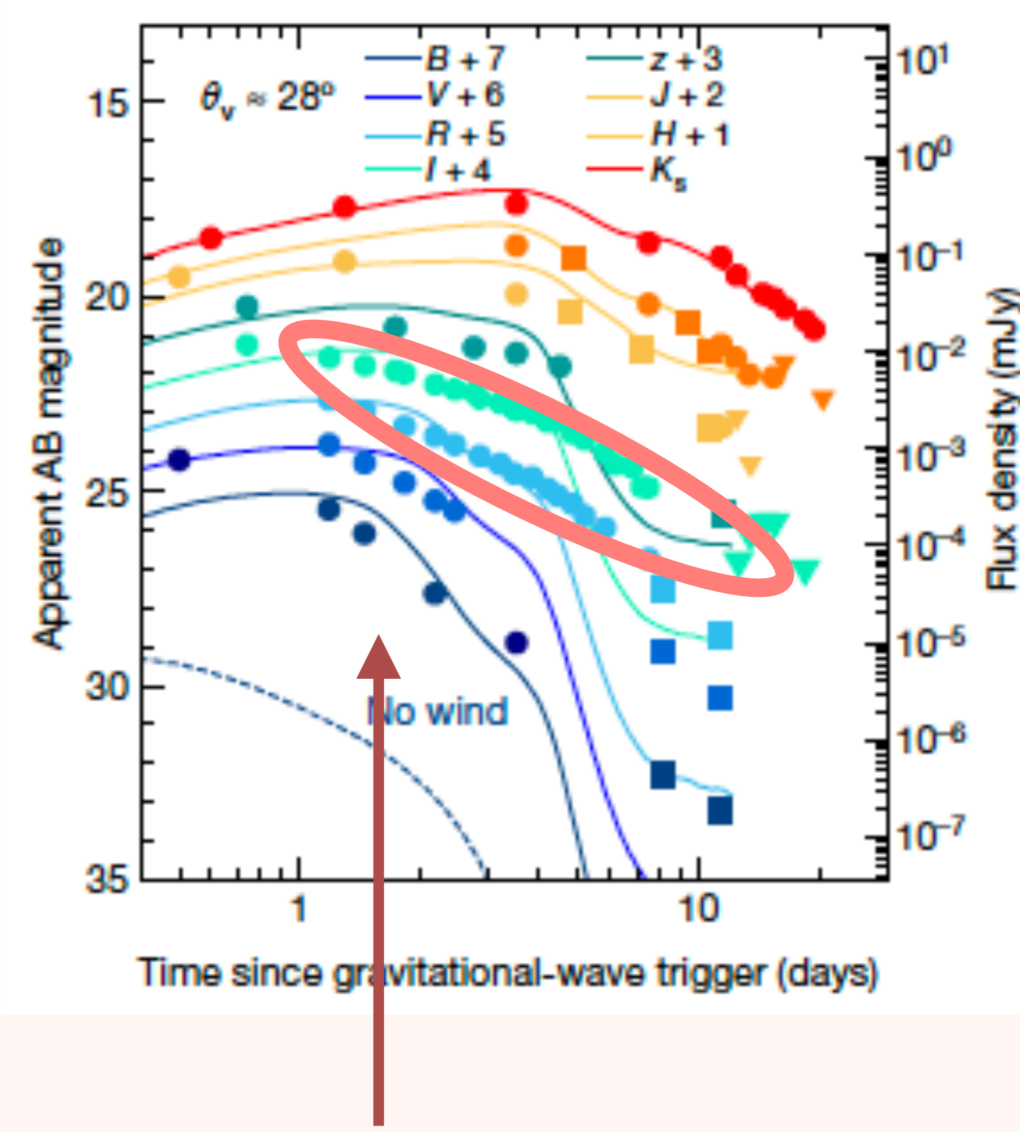
- If the GRB is observed, together with the GWs, the l-o-s should lie within the opening angle (θ_j) of the jet $\iota < \theta_j$
- Radio synchrotron emission contains the information on ι
- However, kilonova lightcurve in optical/IR is not sensitive to ι.
- GW170817:
  - Early constraint: $20^\circ \lesssim \iota \lesssim 60^\circ$ mostly from radio data (Troja et al. 2017)
  - Discovery of the superluminal radio jet (Mooley et al. 2019) is claimed to give the range of the viewing angle more tightly ($14^\circ \lesssim \iota \lesssim 28^\circ$).
  - Optical superluminal jet (Mooley et al. 2022) gave tighter constraints the viewing angle ($19^\circ \lesssim \iota \lesssim 25^\circ$)







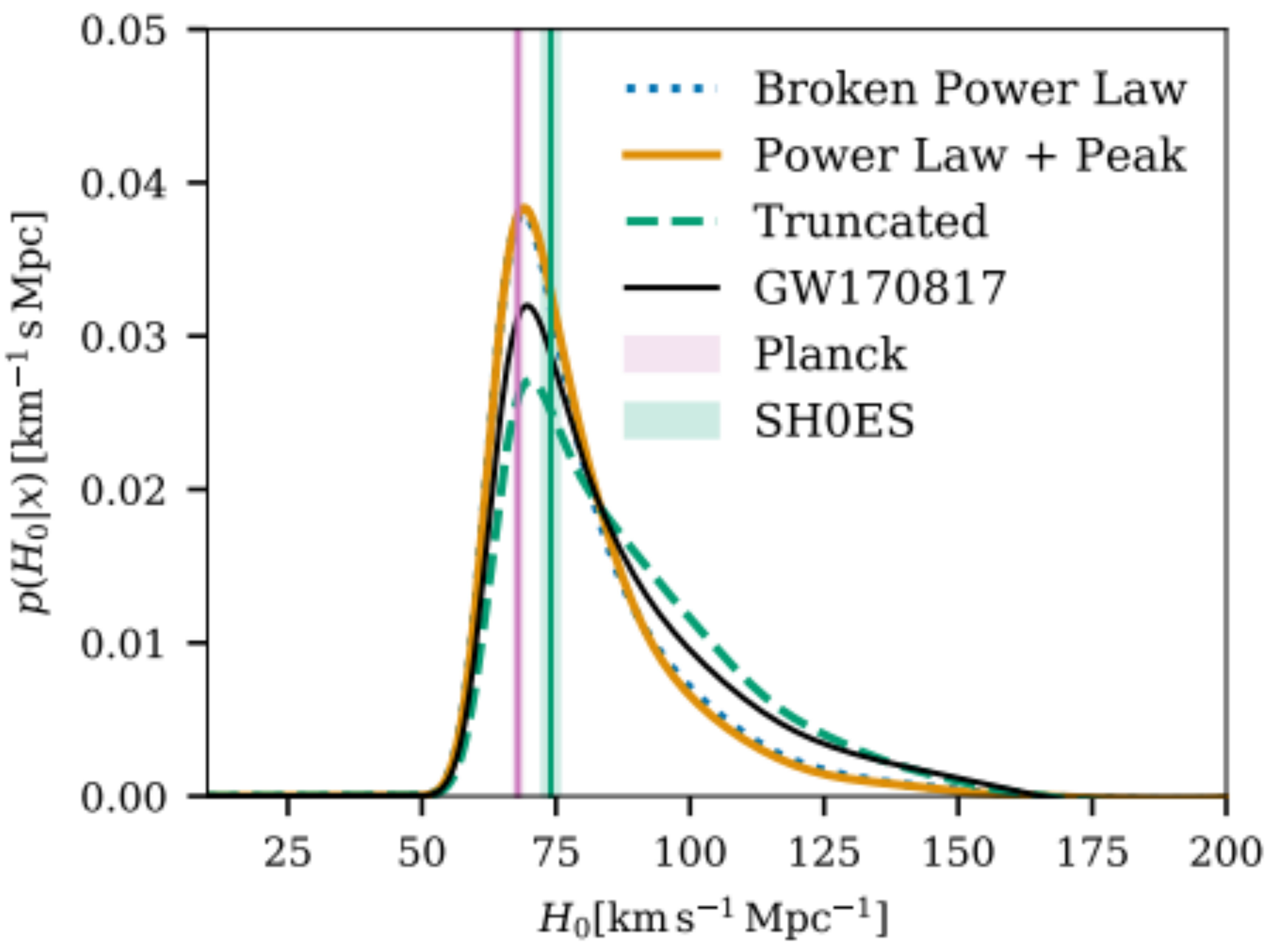
LSC (2017) Nature



KMTNet

Measurements of H_0 with GW170817

arXiv:2111.03604V1

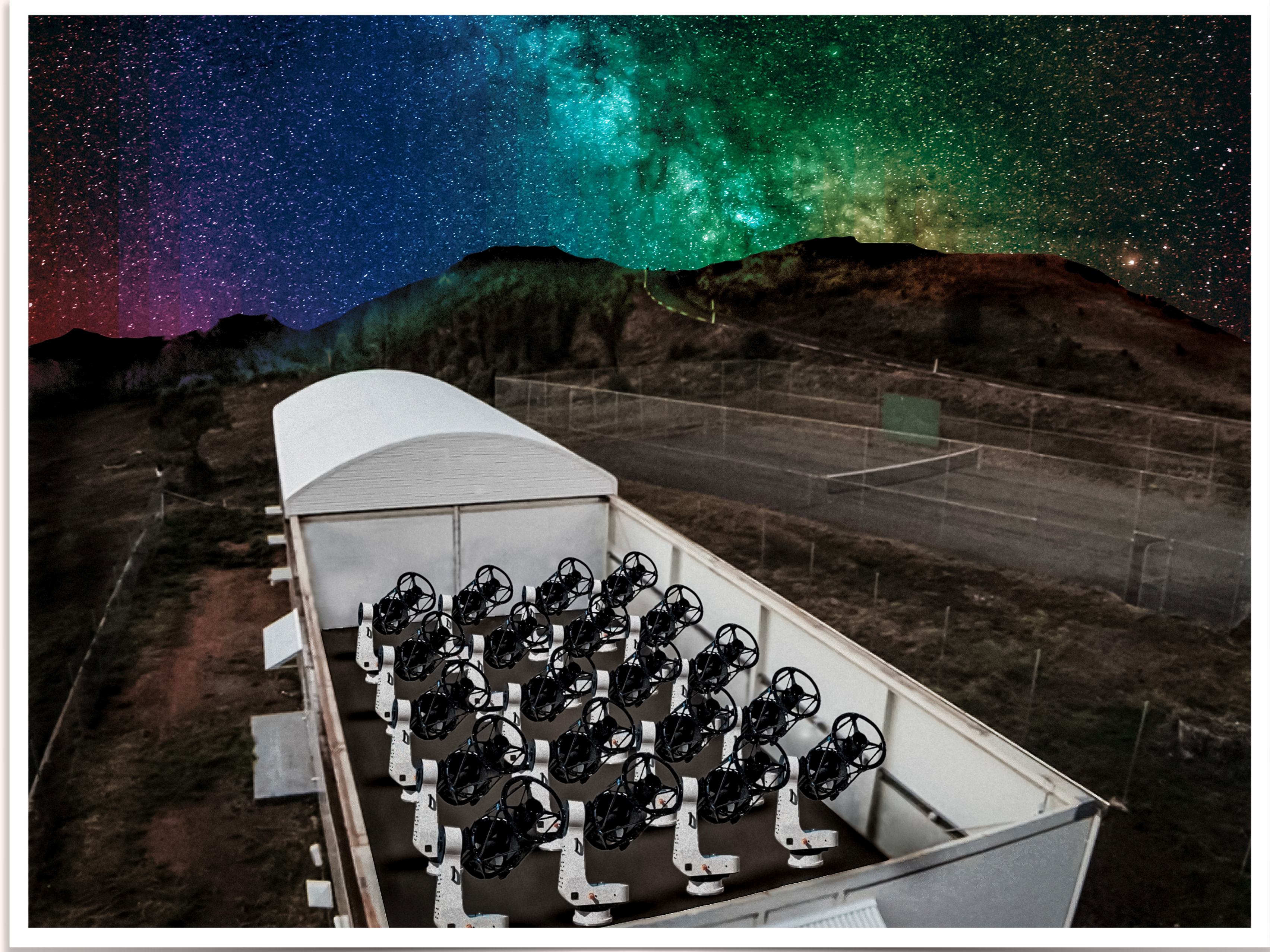


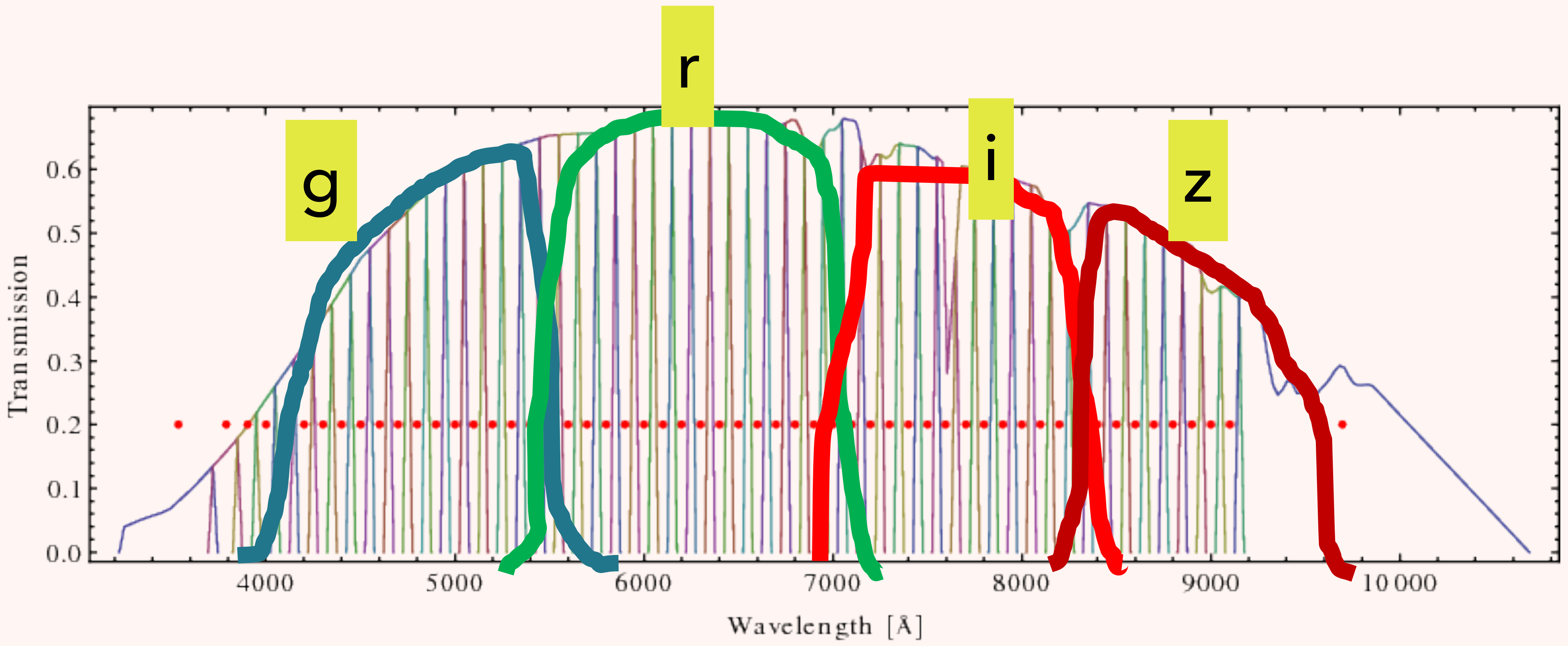
- GW170817 alone: $H_0 = 69^{+17}_{-8}$ km/sec/Mpc (LSC, 2017)
- GW170817 (Bright Siren) + 42 BBH (Dark Sirens) $H_0 = 68^{+12}_{-7}$ km/sec/Mpc (LSC, 2021, arXiv:2111.03604V1)
- With radio and optical superluminal jets $H_0 = 72.5 \pm 4.6$ km/s/Mpc (Mooley et al. 2022)

- Accuracy of Hubble constant can be improved if we have many BNS events with EM counterparts.
- There will be a few more such events in during O4.
- Since the EM emission fades away very quickly, we need to identify the EM counterpart rather quickly.

A new facility (7DT) for rapid followup of transients

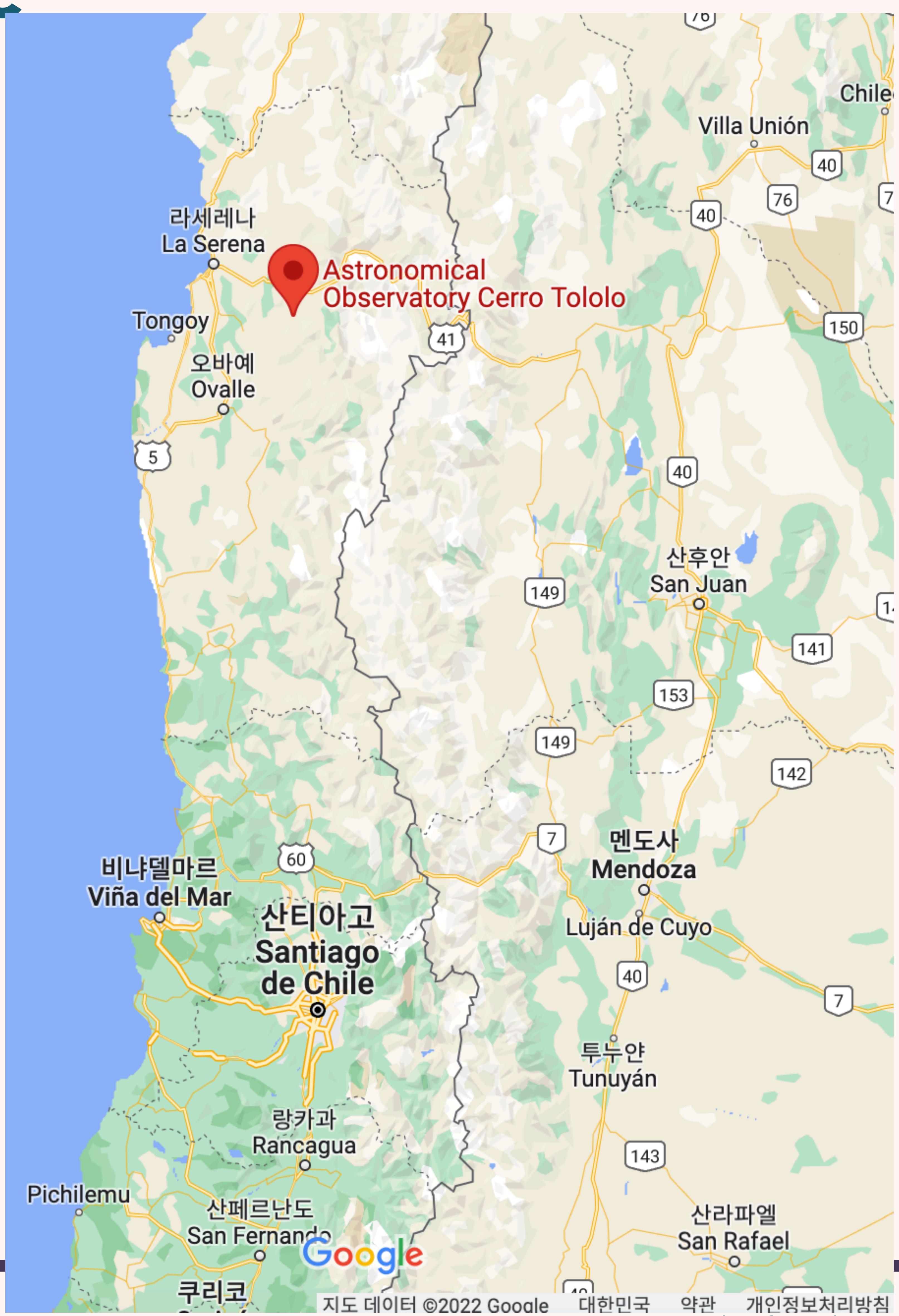
- We are building a system of telescope composed of 20 telescopes of 50 cm aperture (~Effective D~2.2 m if all telescopes are combined)
- Imaging wide field with 40 medium band filters (each telescope has 2) —> **low resolution spectroscopy** for every pixel in the field of view
- It can cover **large area** of sky repeatedly:
  - Wide-field, time domain, spectroscopic telescope
  - Suitable for the survey of transients such as GRB and Kilonovae





Telescope site

- Chile, Rio Hurtado (near CTIO/Cerro Pachon)
- Altitude: 1700 m
- 320 clear nights
- <1” Seeing

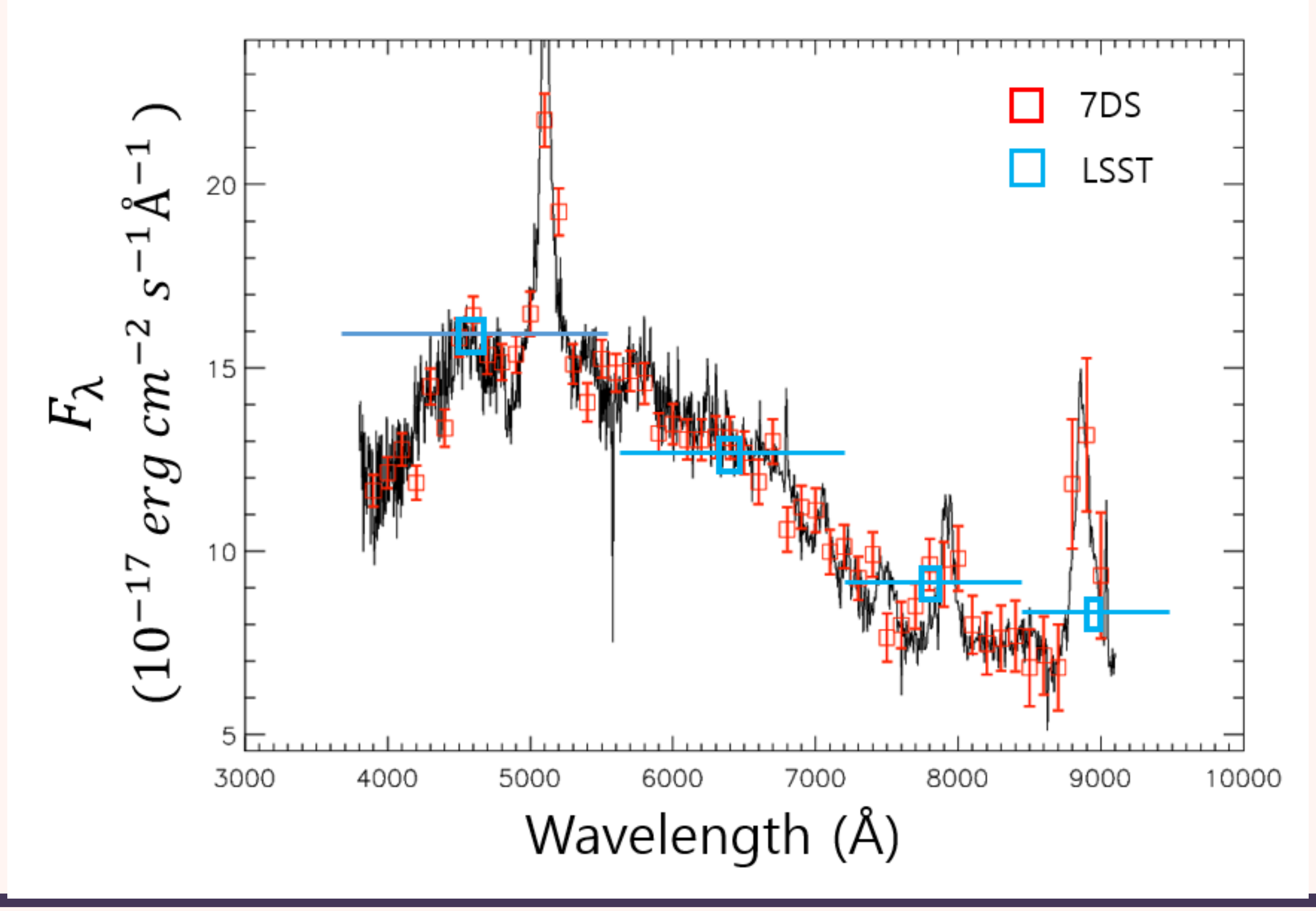


Current Status of 7DT

- 12 telescopes (out of planned 20) telescopes are installed and are being commissioned.
  - Current Field of View 1.25 sq. deg. (1.4 deg x 0.9 deg)
- Twenty 25nm medium band filters + Sloan Broad-band filters
- Computers for data storage and processing (GPU) are ready
- A GPU-based data process server with 2 NVIDIA A100 cards, is ready.
- Network synchronization with KREONET has been tested.
  - Data rate = 400 Mbps
  - Data per day ~ 1 PBytesSlide courtesy: Myungshin Im

Advantage of Medium Band Spectrum

- Best suited for broad continuum features and broad emission lines
- Photometric redshift: < 0.3% – 1% accuracy
- Emission line/continuum can be separated well, and thus characterize the nature of the transients.



z = 0.822,
i = 18.3 quasar spectrum
(black: SDSS
red: 7DS
blue: LSST)

Separation between Kilonova and Supernova

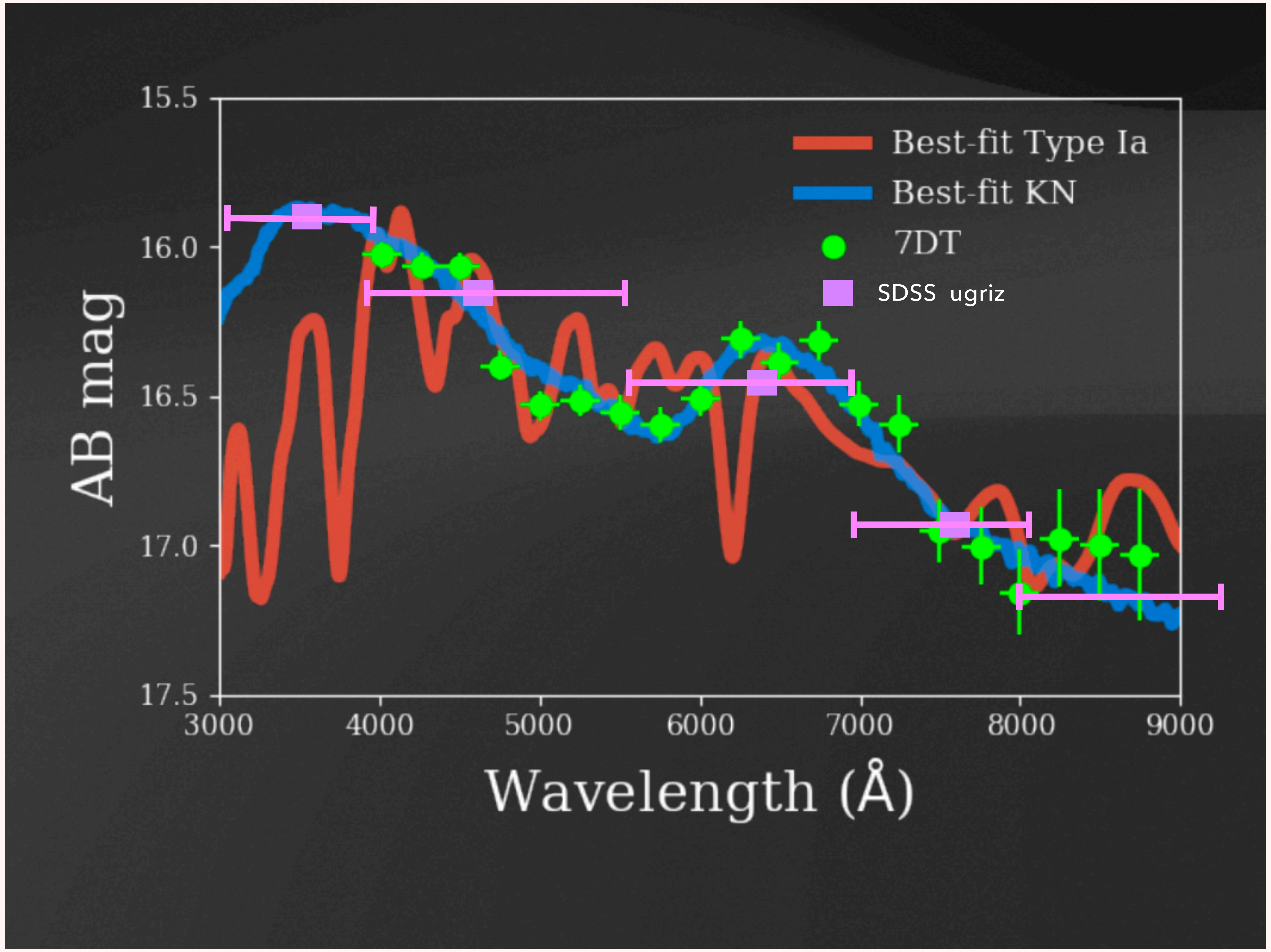


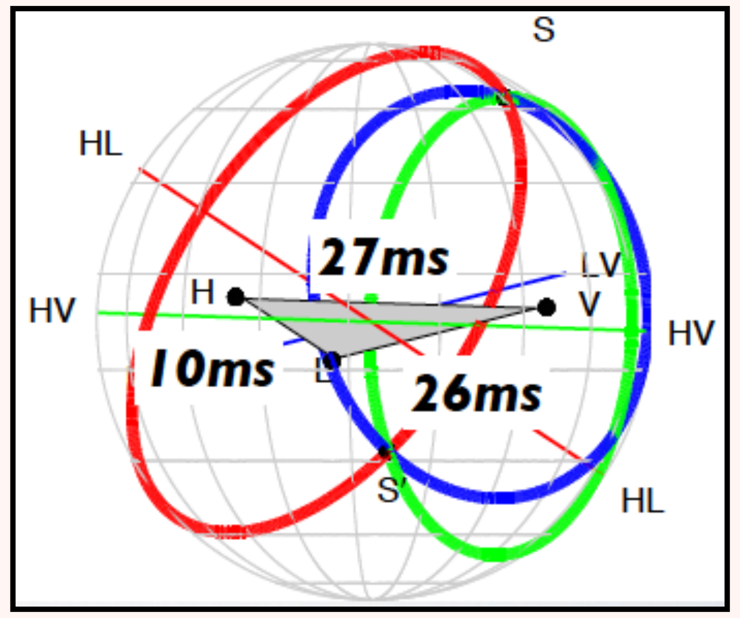
Figure Credit: Gregory Paek (SNU)

Dark sirens

- BBH does not emit EM radiation, and distant BNS may emit EM radiation that is too faint to observe. Such objects are called **dark sirens**
- Unless the angular resolution of the dark sirens becomes very good, it would be very difficult to uniquely identify their host galaxies with ground-based detectors.
- One can still use dark sirens to constrain cosmological parameters (Hubble constant, DE equation of state, etc.) using dark sirens statistically.
  - Photometric redshifts of galaxies within Ω and d_L range (e.g., Soares-Santos et al. 2021)
  - Cross-correlation with galaxies (Mukherjee et al. 2018, 2021)
  - Identification of host galaxies with future detectors

Why host identification is so difficult?

- Most of the GW sources do not emit electromagnetic (EM) radiation (e.g., BBH)
  - BNS or NS-BH can emit EM radiation, but they are rare and generally rather faint.
- Sky localization is done by triangulation using differences in the arrival times to different detectors.
  - Accuracy depends on the signal-to-noise ratio



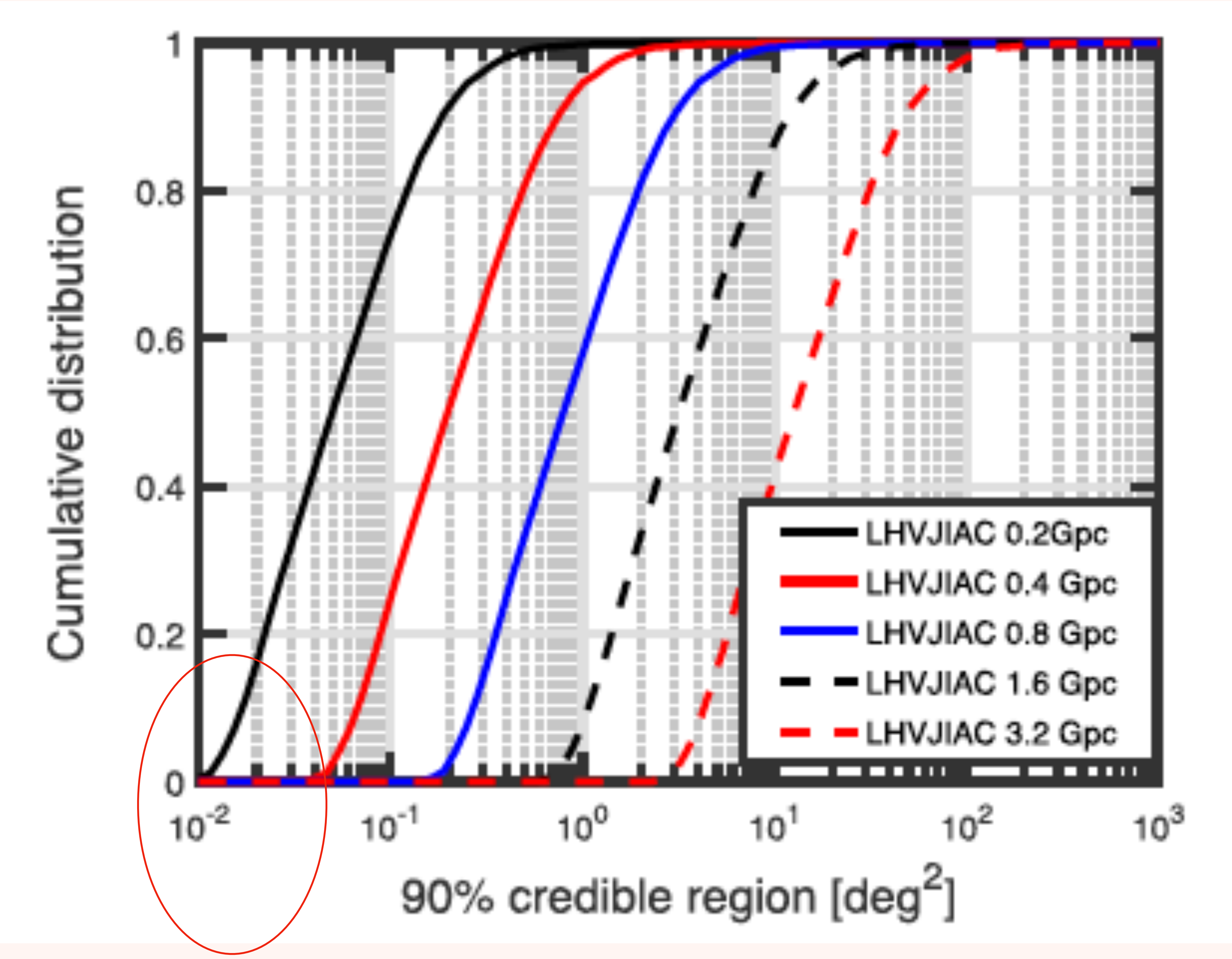
$$\sin\theta d\theta = \frac{\sqrt{\sigma_1^2 + \sigma_2^2}}{\Delta t} \qquad \sigma_t = \frac{1}{2\pi\rho\sigma_f}$$

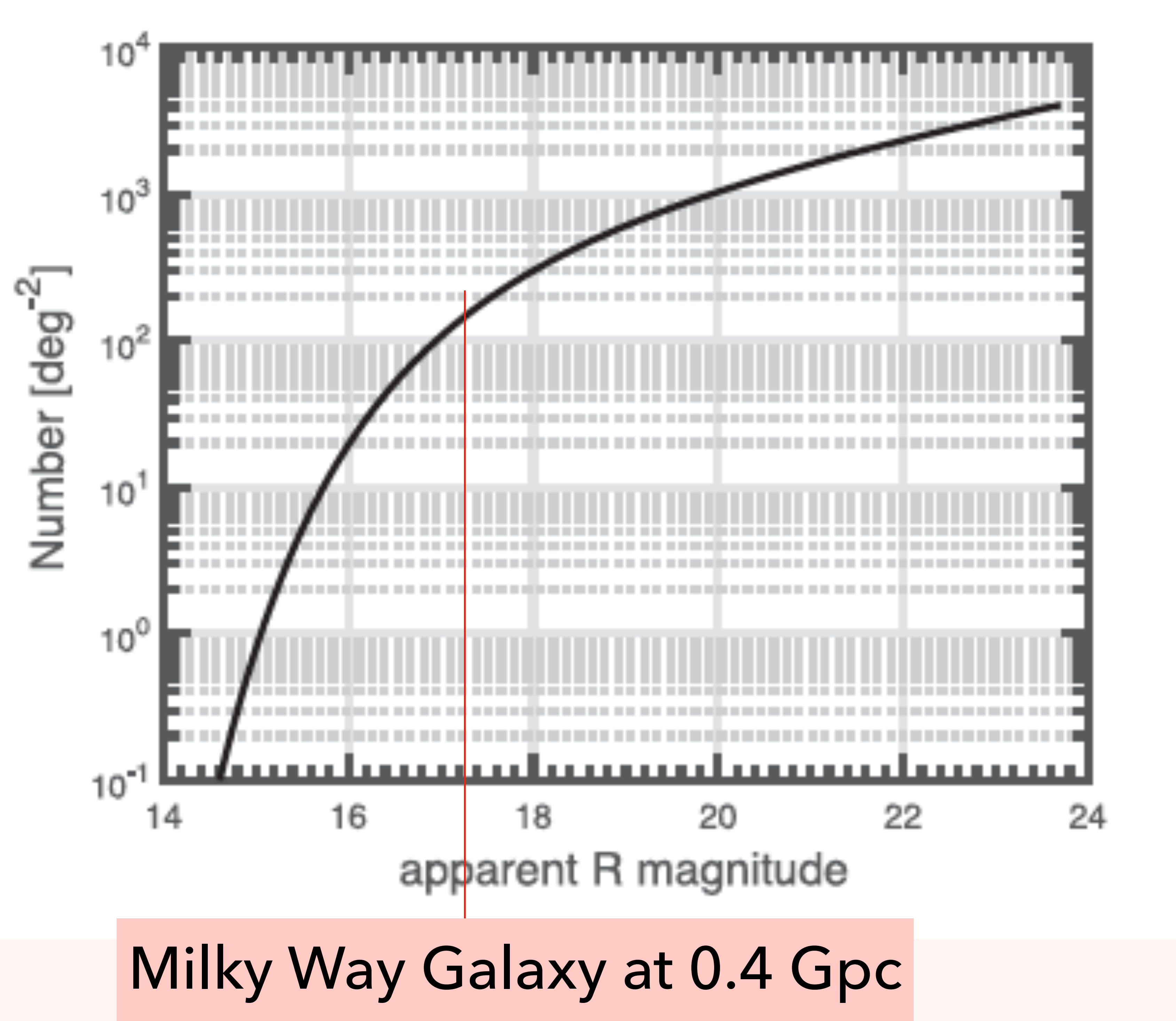
$d\theta$: width of the ring, Δt: baseline, ρ: signal-to-noise ratio, σ_f: effective **bandwidth of the source**, (~100 Hz for NS binaries, smaller for BH binaries)

  - $\Delta\Omega$ is very large (100 - 1000 sq. deg.)
  - Localization accuracy improves with number and sensitivity of detectors

Can BBH hosts be identified with ground based detectors?

Howell, … Lee, … et al. 2018

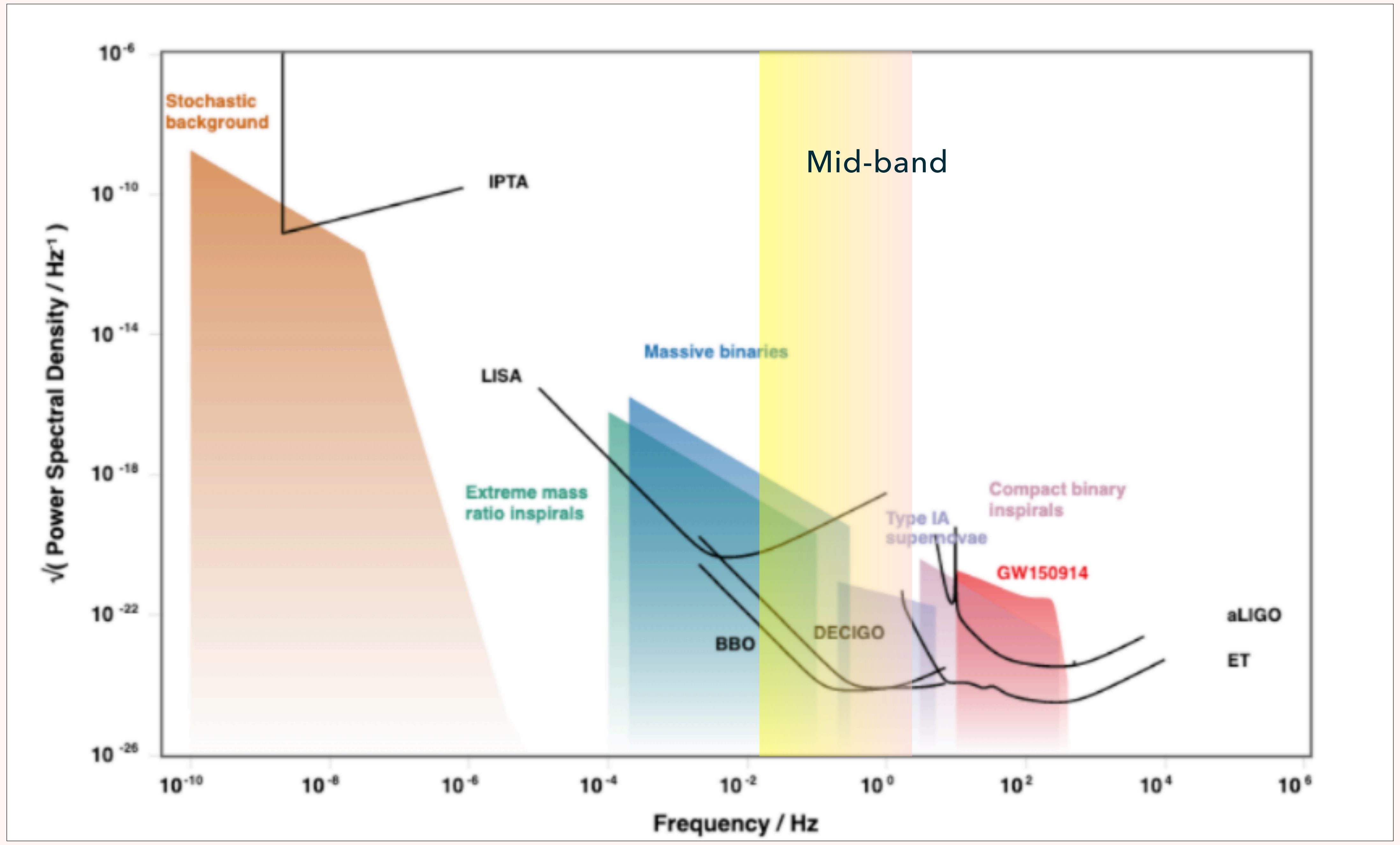


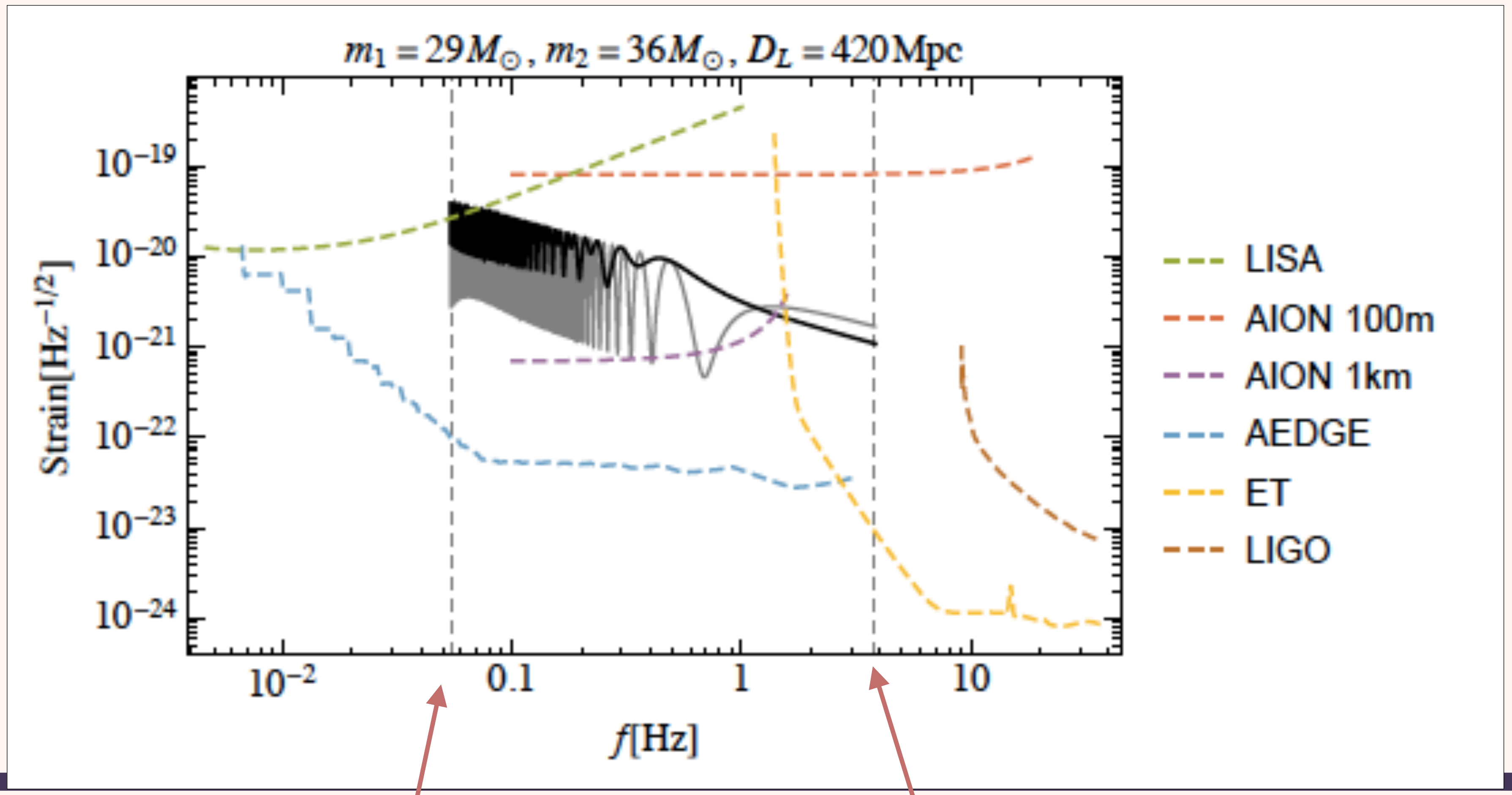


- With network of 5 advanced detectors and two additional detectors with better sensitivity, ~20% of the BBHs at 400 Mpc can be localized within 0.1 sq. deg.
- There could be 10-20 galaxies within the 0.1 sq. deg.
- **BBH host identification by ground based detectors is very challenging!**

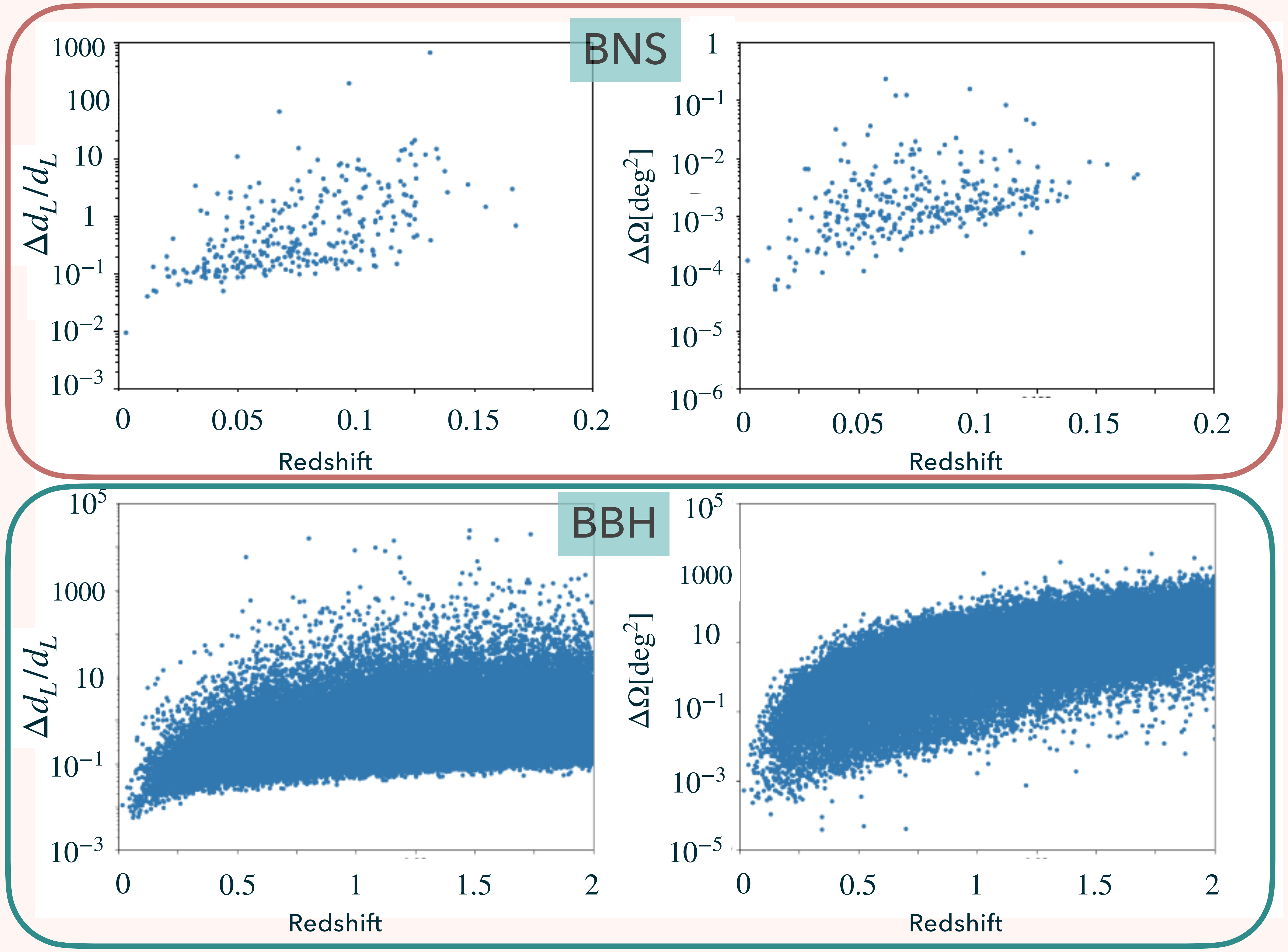
Improvement of localization in 3D volume with mid-band detectors

- Detectors operating at lower frequencies can observe the merging binaries for a long time (**days to years**)
- The source position and inclination angle are encoded in the measured signal through
  - Relative amplitudes and phases of the two polarization components,
  - **Periodic Doppler shift** imposed on the signal by the detector's motion around the Sun,
  - Further modulation of the signal caused by the detector's **time-varying orientation**
- Accuracies of Ω and d_L can be significantly improved





A case study: Simulation of BBH and BNS observations with AEDGE (Yang, Lee+, 2022, JCAP [arXiv:2110.9967v1])



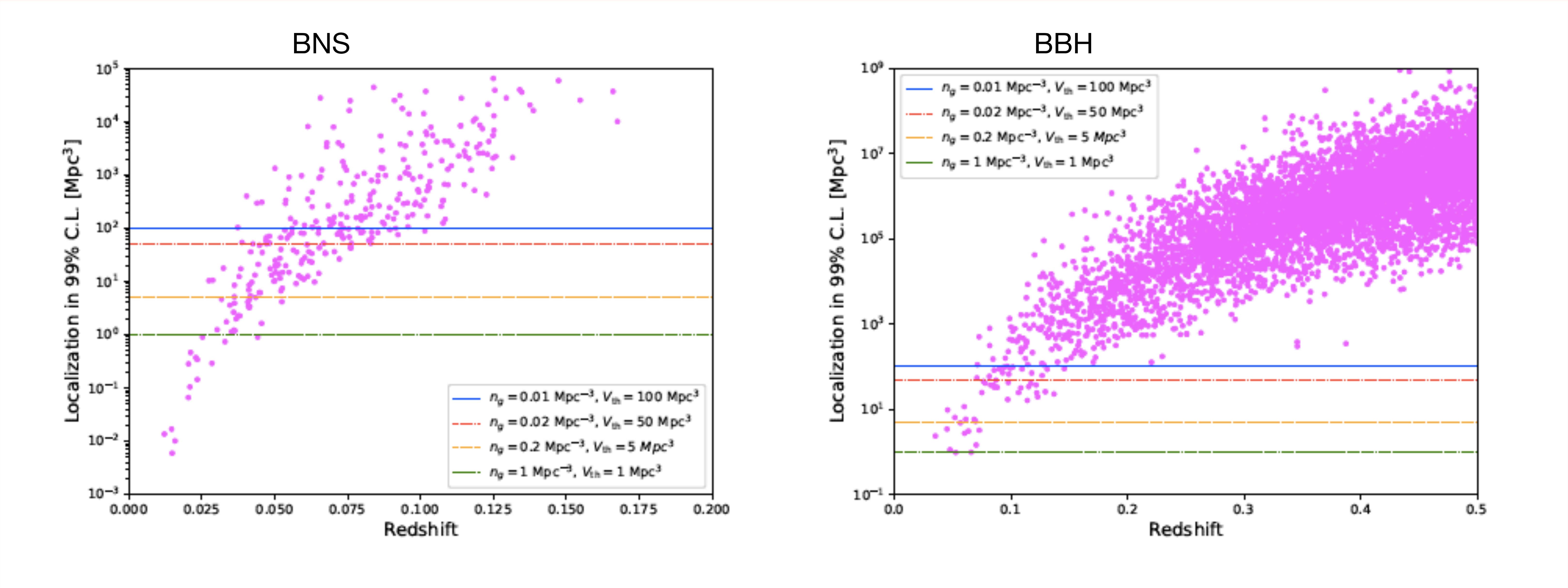
$$\Delta\Omega = 2\pi|\sin\theta|\sqrt{\Gamma^{-1}_{\theta\theta}\Gamma^{-1}_{\phi\phi} - (\Gamma^{-1}_{\theta\phi})^2}$$

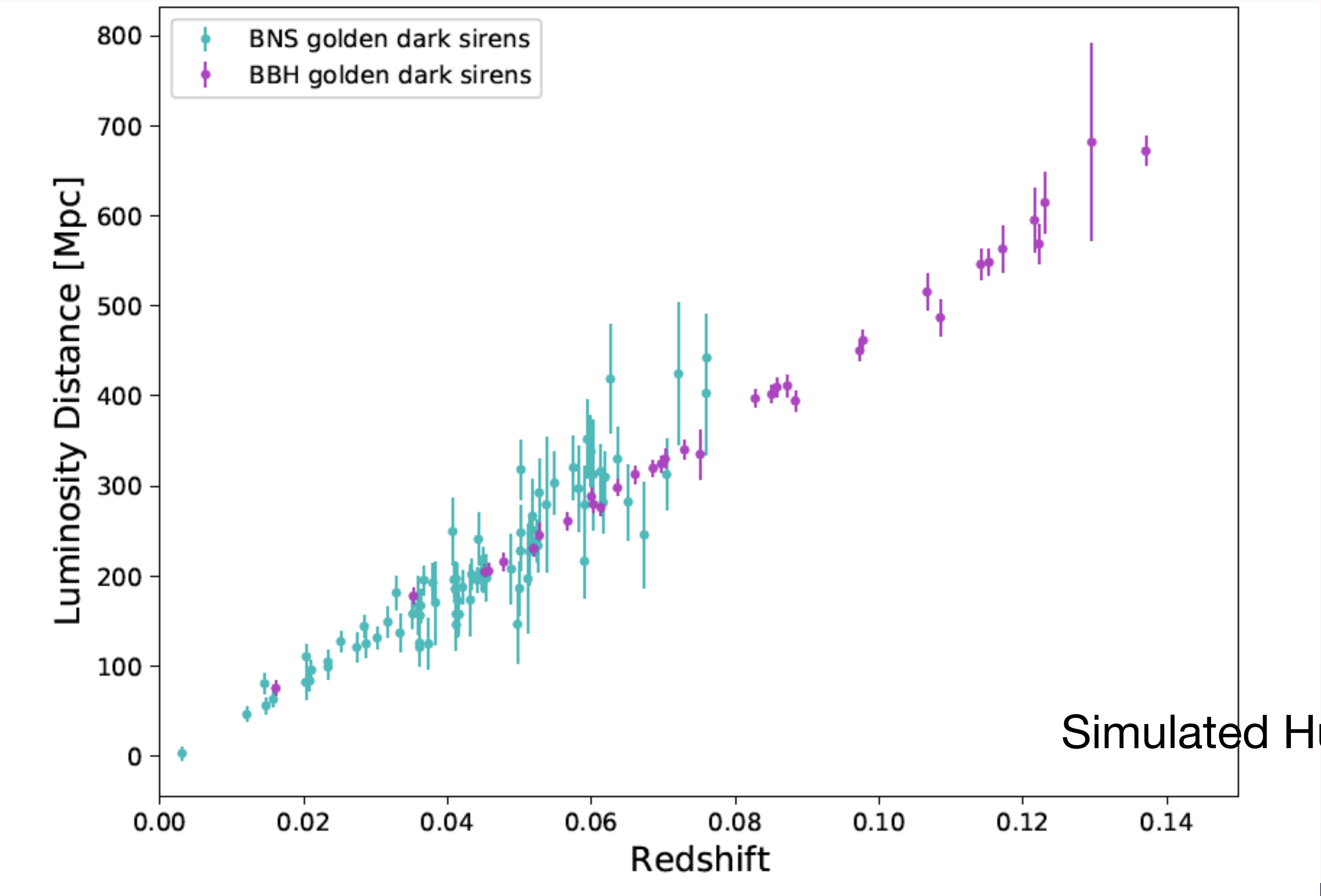
where $\Gamma_{ij} = \left(\frac{\partial h}{\partial \lambda_i}, \frac{\partial h}{\partial \lambda_j}\right)$ is Fisher matrix.

Simulated results for 5 year run of AEDGE assuming GWTC-3 population

AEDGE is a proposed mid-band detector in space,

Localization volume and Hubble Diagram

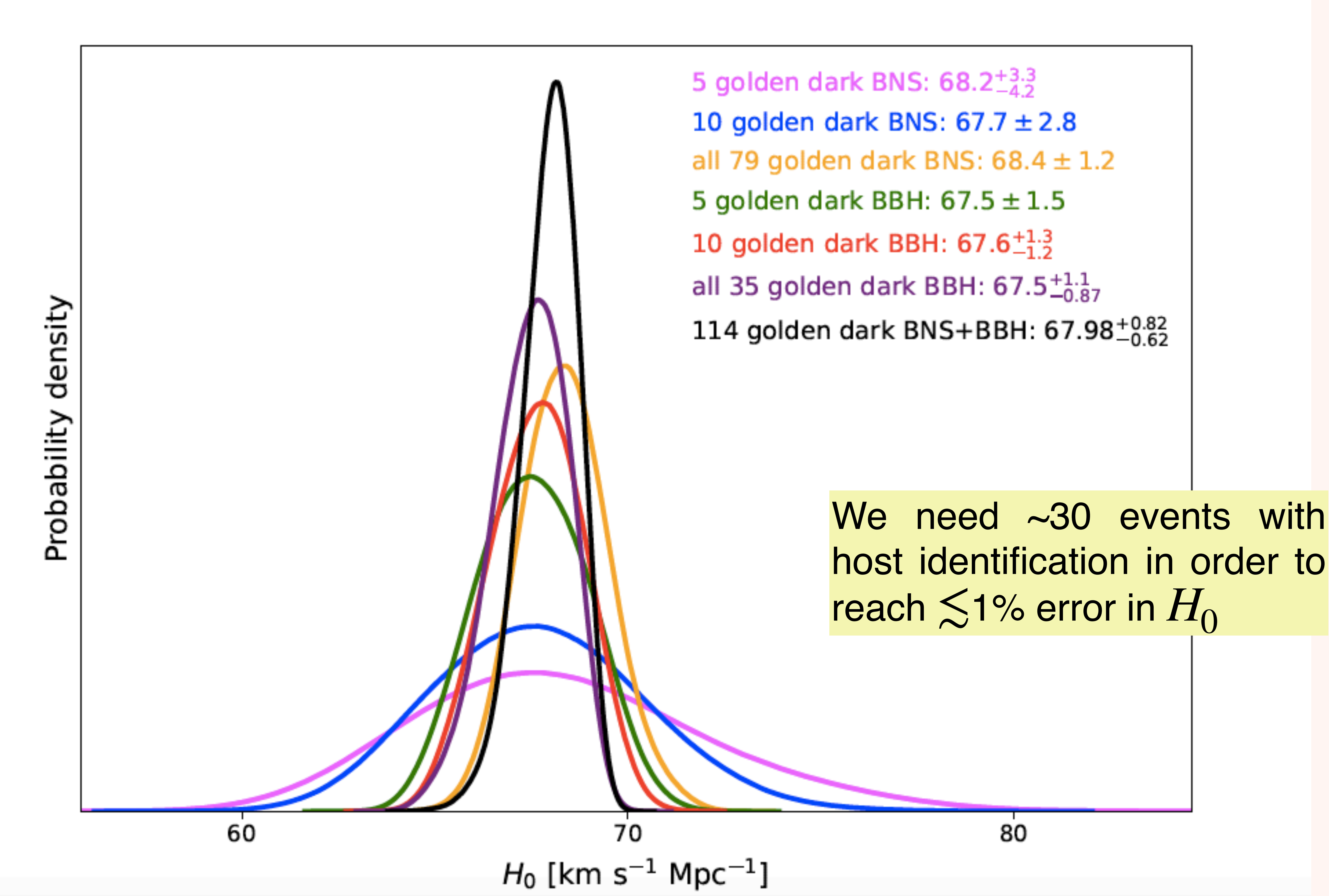




Simulated Hubble Diagram

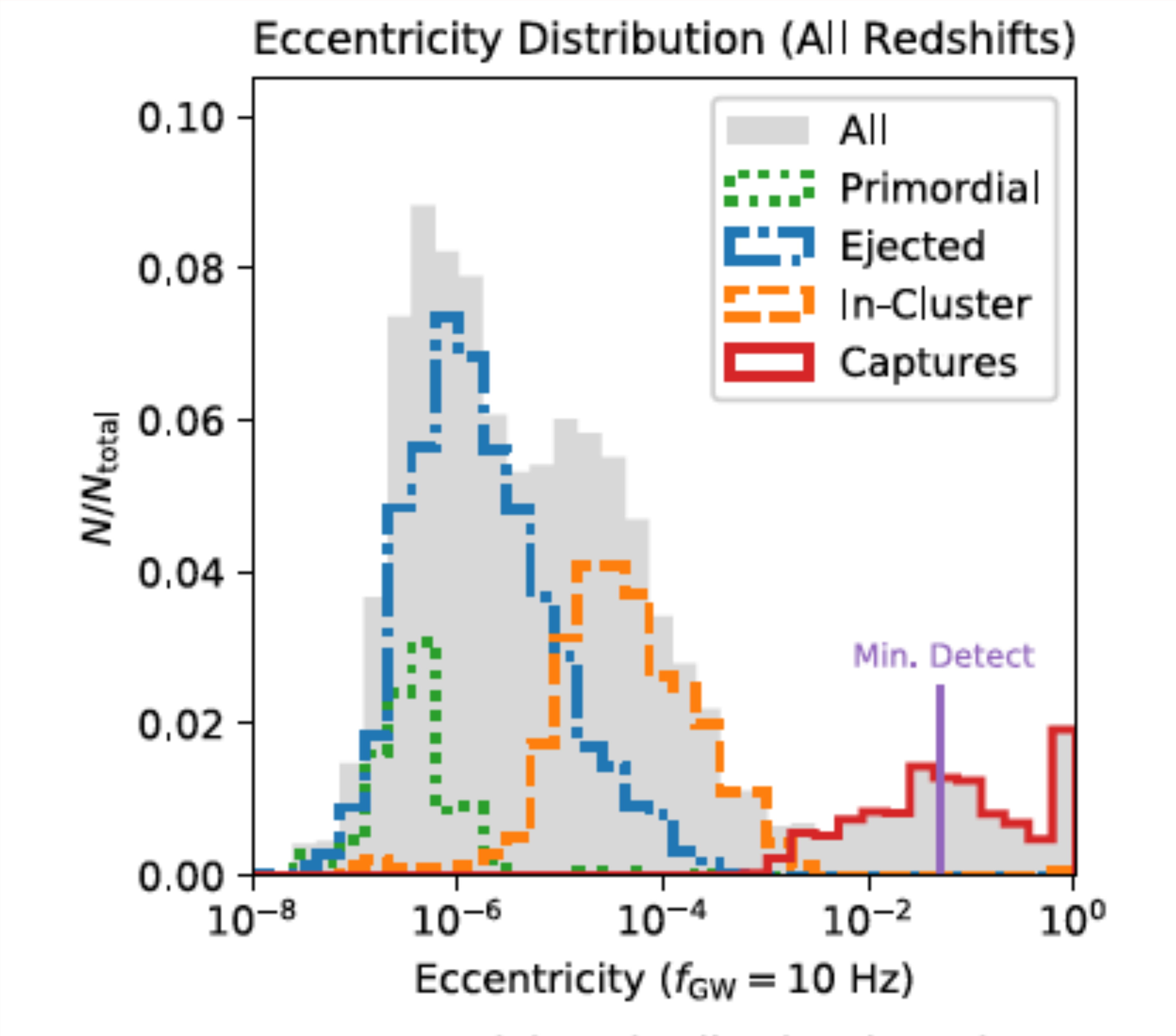
Various cuts are assumed galaxy number densities: below these lines, we can uniquely identify host galaxies within 5 year observation

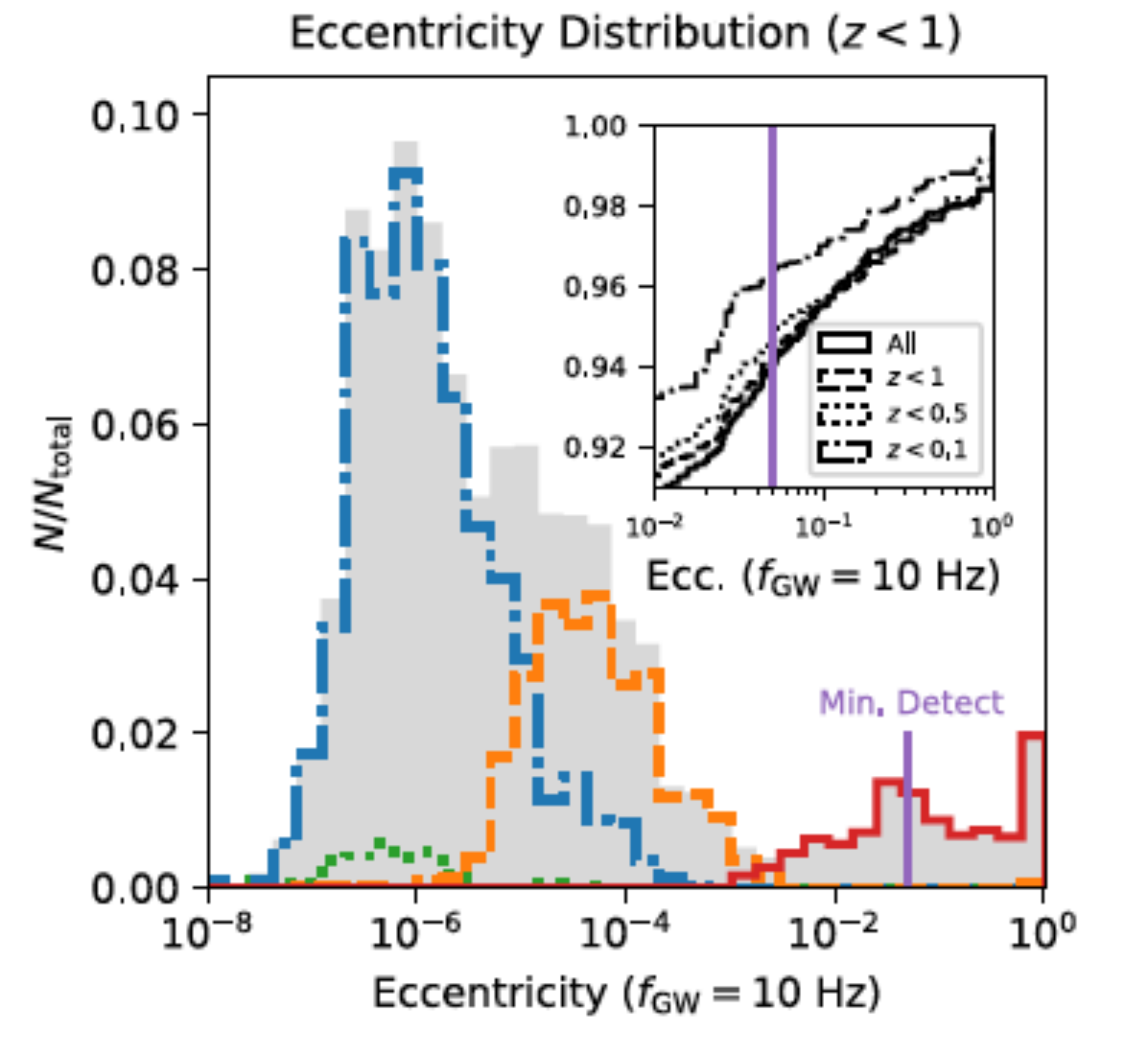
Simulation of Hubble Constant Estimation with Dark Sirens



We need ~30 events with host identification in order to reach $\lesssim$1% error in H_0

So far we assumed circular binaries, but dynamical processes produce eccentric binaries



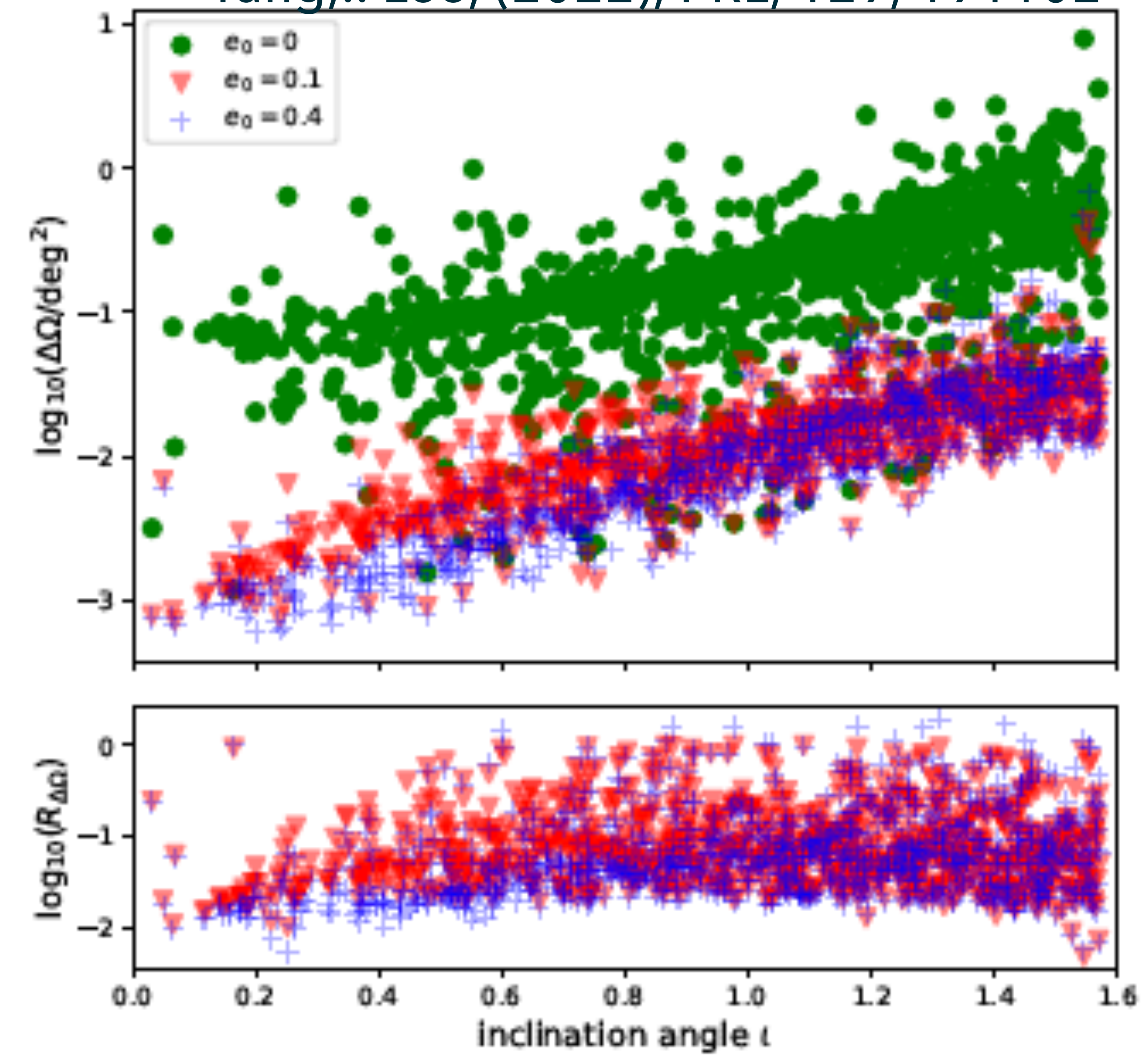


Rodriguez et al., PRD 98. 123005 (2018)

Further improvements of estimated parameters for eccentric binaries

- In mid-frequency band, some binaries may have significant eccentricity (i.e., $e > 0.1$)
- The eccentric waveforms have more features than circular ones, and thus enable us to break some of the degeneracies during the inspiral phase → more accurate parameters can be inferred
- A case study with B-DECIGO:
  - $\Delta d_L / d_L$ can be improved near $\iota = 0$.
  - $(\Delta\Omega)_{e=0.1} \lesssim (\Delta\Omega)_{e=0}$
  - More improvement for larger e.

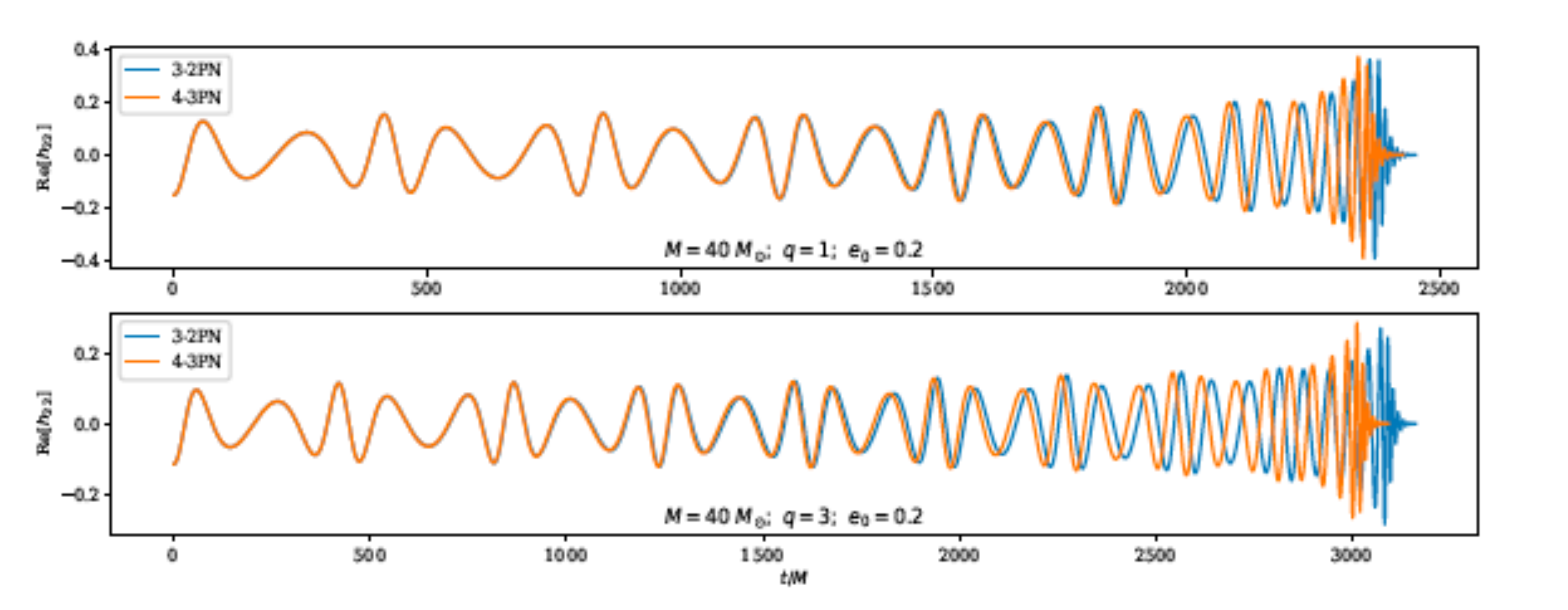
Yang,.. Lee, (2022), PRL, 129, 191102



Accurate Waveforms for longer duration

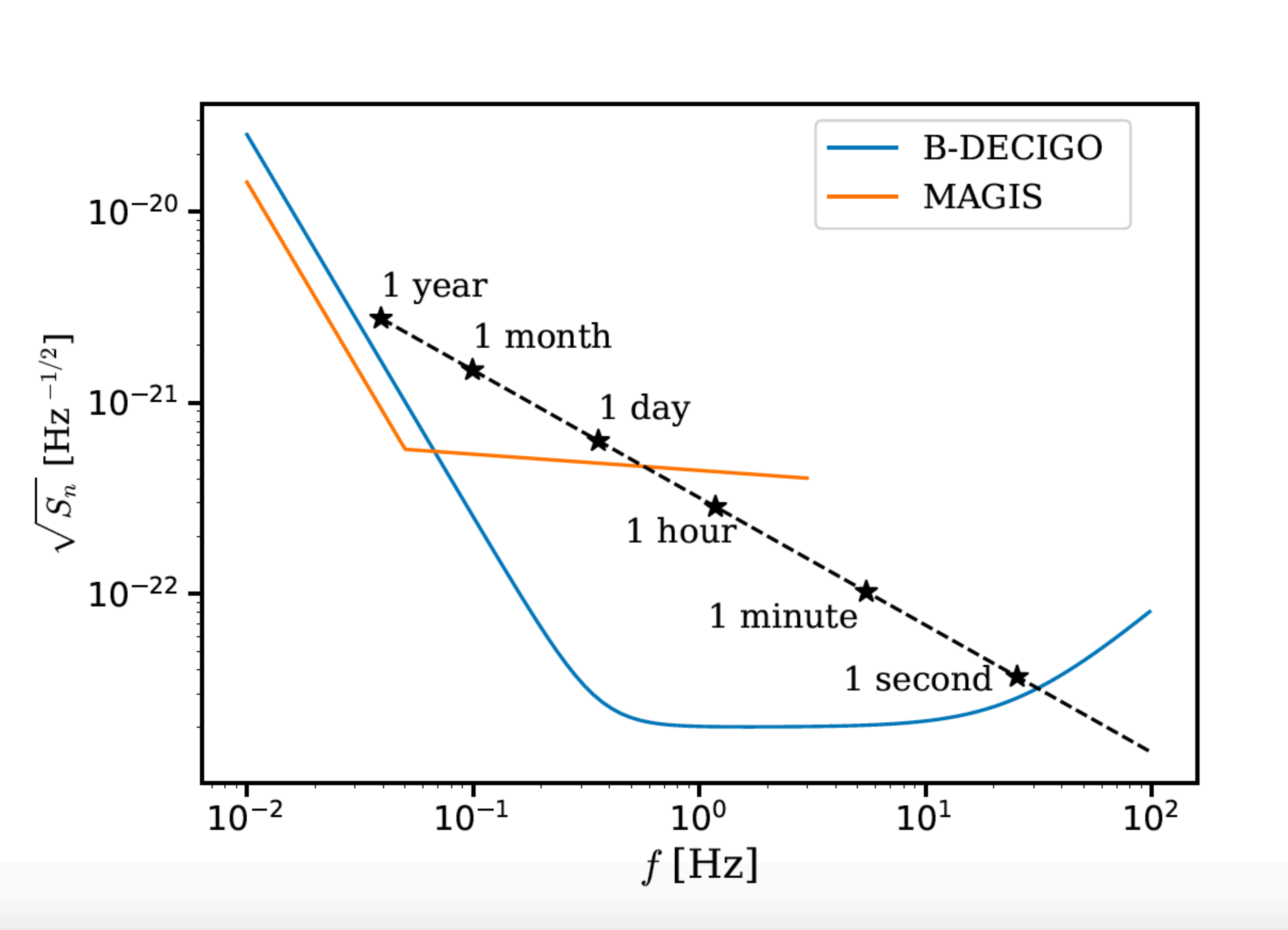
- In order to fully utilize the long duration observation data with mid-frequency, we need accurate waveforms from binaries with eccentricity and spins.
- Current status:
  - Time domain waveforms can be computed up to 4 PN. (Cho et al. 2022) for binaries with arbitrary eccentricity.
  - We need to transform the TD waveform into freq. domain: issue of higher modes.
- Spin:
  - Waveforms for spinning black hole binaries are not well understood yet.

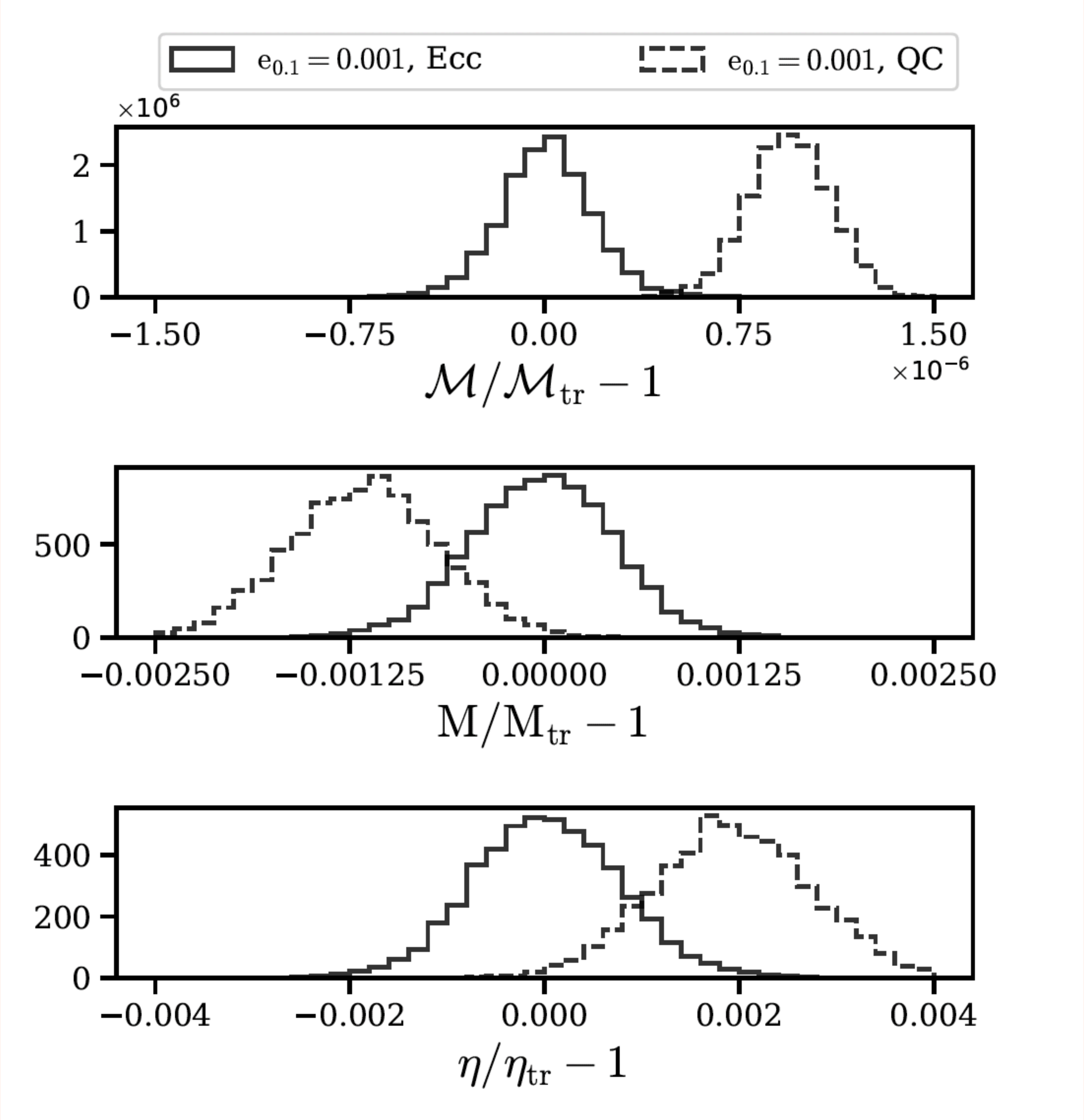
Cho et al. 2022, PRD, 105, 064010



Systematic Errors due to uncertain waveforms (Choi, Yang & Lee, PRD, 2024

- We made detailed parameter estimation for assumed B-DECIGO and MAGIS Detectors for GW150914-like BBH
- The injected waveform is eccentric model (TaylorF2ECC) but analyzed with Quasi-circular model
- Systematic Errors in mass estimations (thus eventually distance) greater than statistical errors arise.





Eccentricities

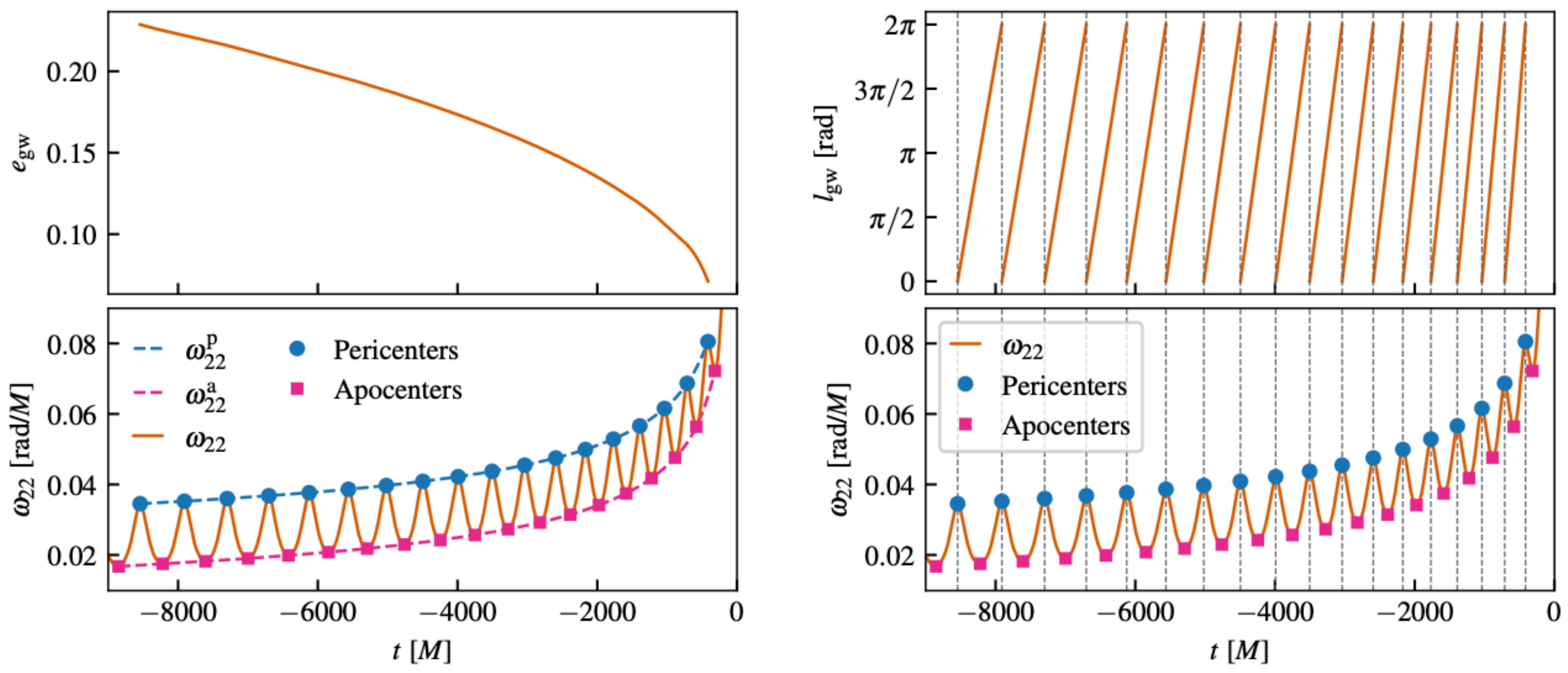
- What is the eccentricity?
  - It is uniquely defined in Keplerian orbits only.
  - In PN theory, there are three eccentricities, e_r, e_ϕ and e_t with e_t being dependent on gauge.
  - Different models may be using eccentricity that is defined differently.
- Recently, Shaikh et al. (2023) proposed definition of guage independent eccentricity from the GW frequency evolution.

$$h_{22}(t) = A_{22}(t)e^{-i\phi_{22}(t)}, \quad \omega_{22}(t) = \frac{d\phi_{22}(t)}{dt}$$

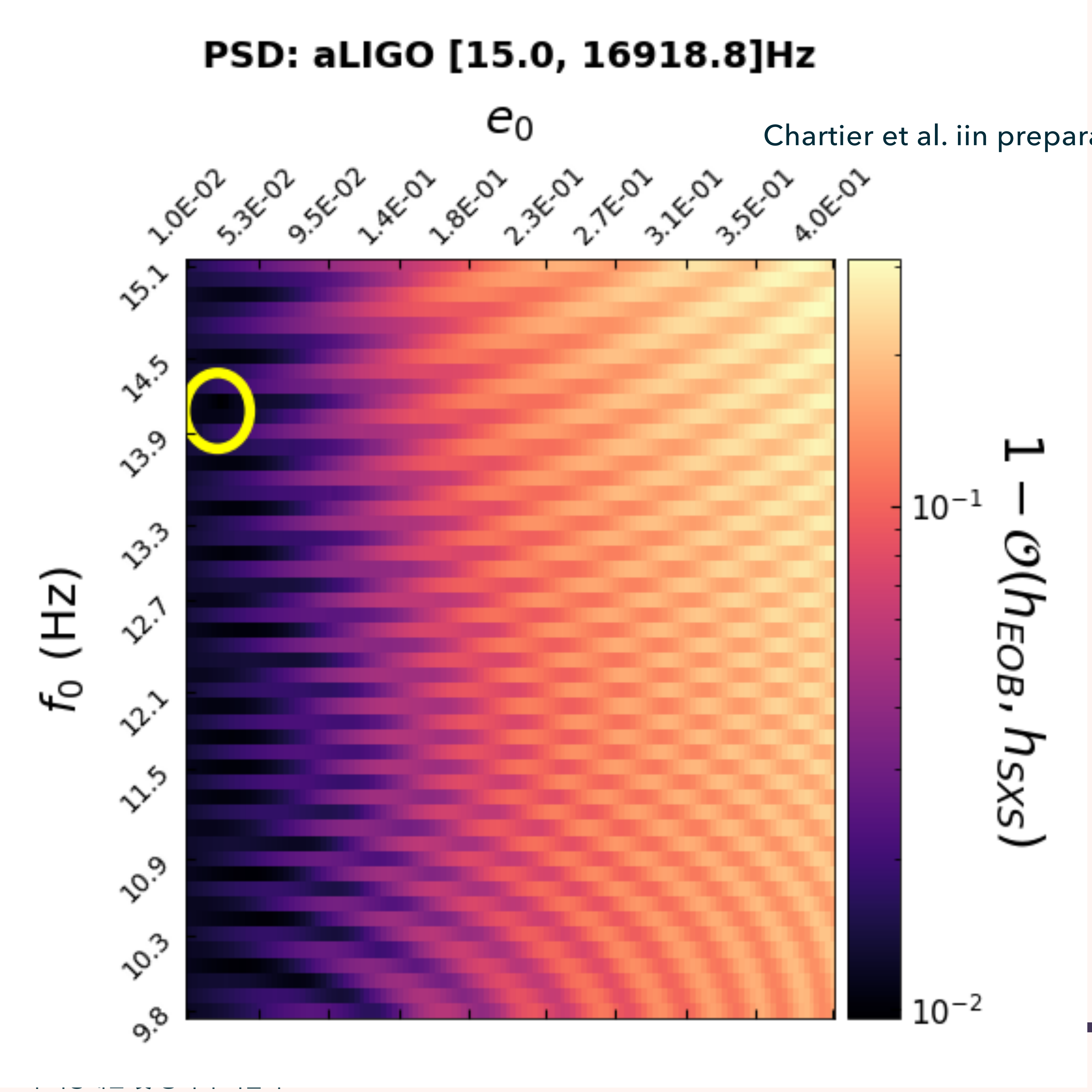
$$e_{\omega_{22}} = \frac{\omega_{22}^p - \omega_{22}^a}{\omega_{22}^p + \omega_{22}^a} \quad \Psi = \arctan\left(\frac{1 - e_{\omega_{22}}^2}{2e_{\omega_{22}}}\right)$$

$$e_{gw} = \cos(\Psi/3) - \sqrt{3}\sin(\Psi/3)$$

Shaikh et al, 2023, PRD



Are eccentric models consistent each other (Chartier et al. in prep)

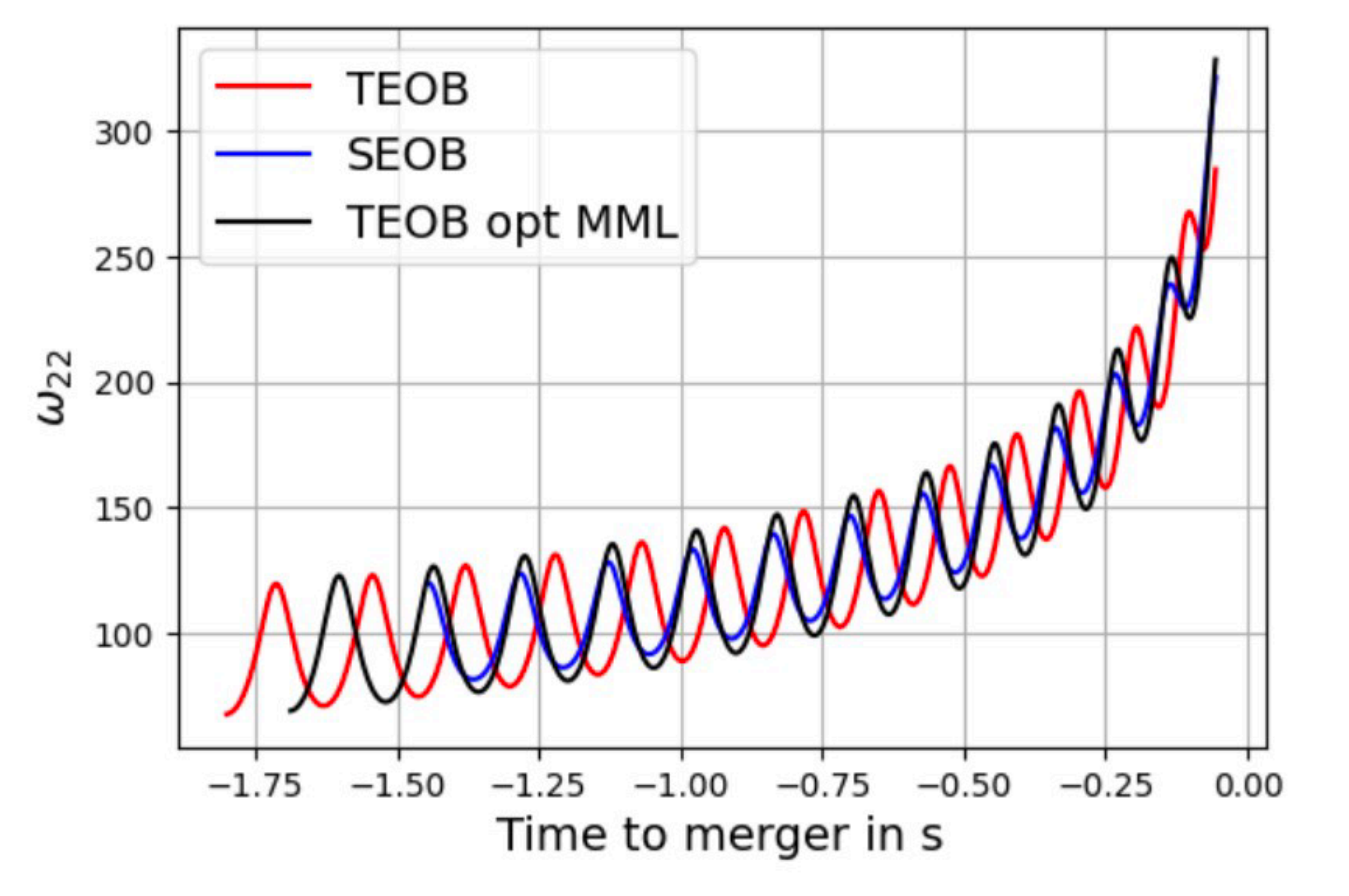


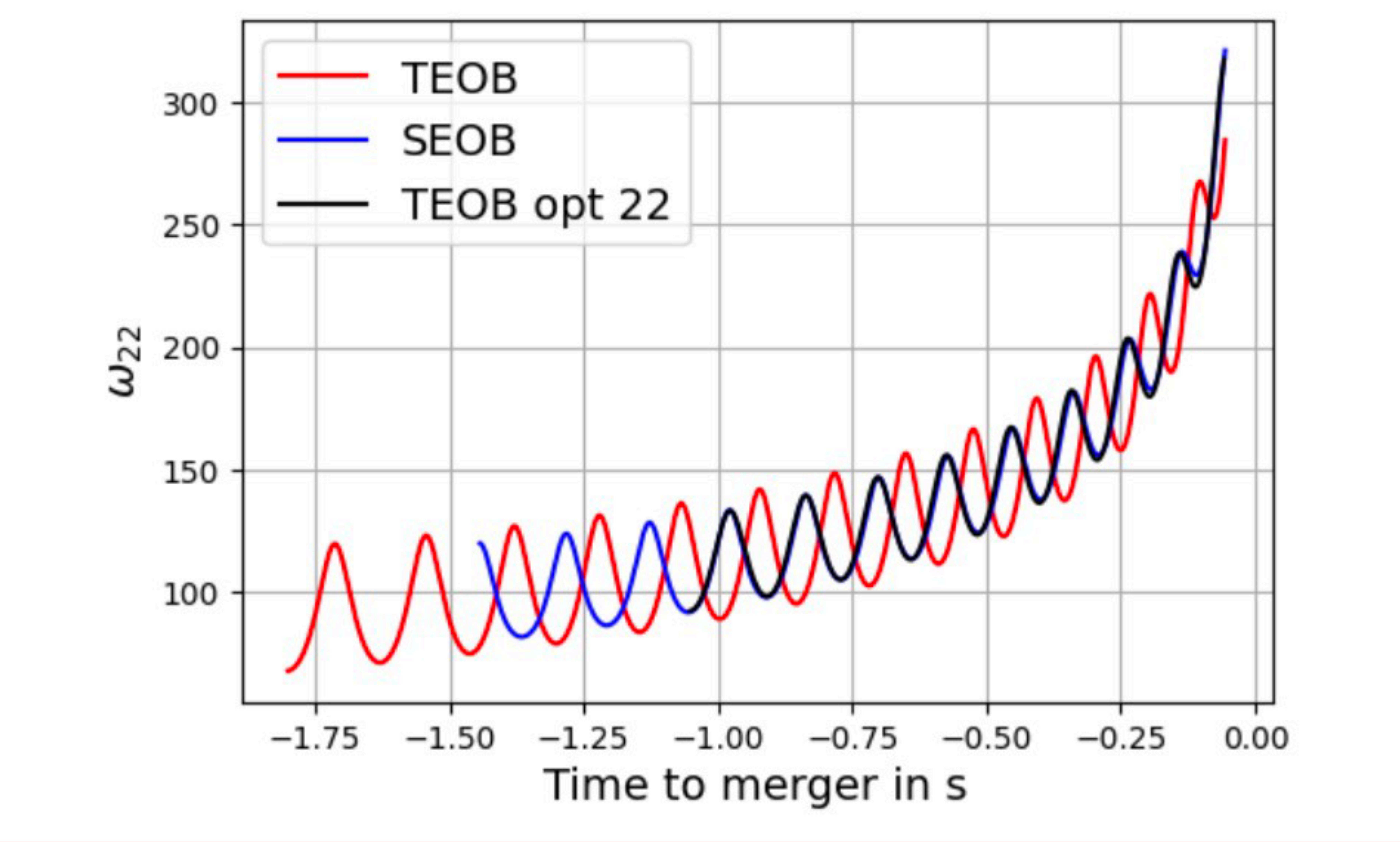
$$O(h_1, h_2) = \frac{< h_1 h_2 >}{|h_1||h_2|}$$

- Comparison between EOB model and NR shows that there is a significant mismatch for e>0.05 at high $f_0 \sim 10$ Hz.
- Further, the mismatch seems to have oscillating behavior depending on the f_0 where e_0 is defined.
  - This indicates that the definition of eccentricity depends on the phase of the GW waves.

Optimize with the same e_{gw}

Chartier et al. iin preparation





- If we require the same e_{gw} (t) behavior for different models, we sometimes end up with different phase evolution that means significant mismatch.

Summary

- Hubble tension could be due to our lack of understanding of the universe or systematic error in local measurements
- GW sources can be used to make completely independent measurements of the Hubble constant.
- However, individual measurement of distances with GW sources have large random error due to the lack of information on inclination angle of the coalescing binaries.
- BBHs do not emit EM radiation. The pointing accuracy of the ground-based detectors (including the future ones) is very poor for host identification.
- Localization can be significantly improved with future detectors in miid-band (0.1 ~ 1 Hz).
  - If some binaries are eccentric, accuracies of directions and distances can be further improved.
- Cosmological parameters could be precisely constrained with dark sirens alone with mid-band detectors.