


Quantum noise of ground base interferometer gravitational wave detector

June Gyu Park



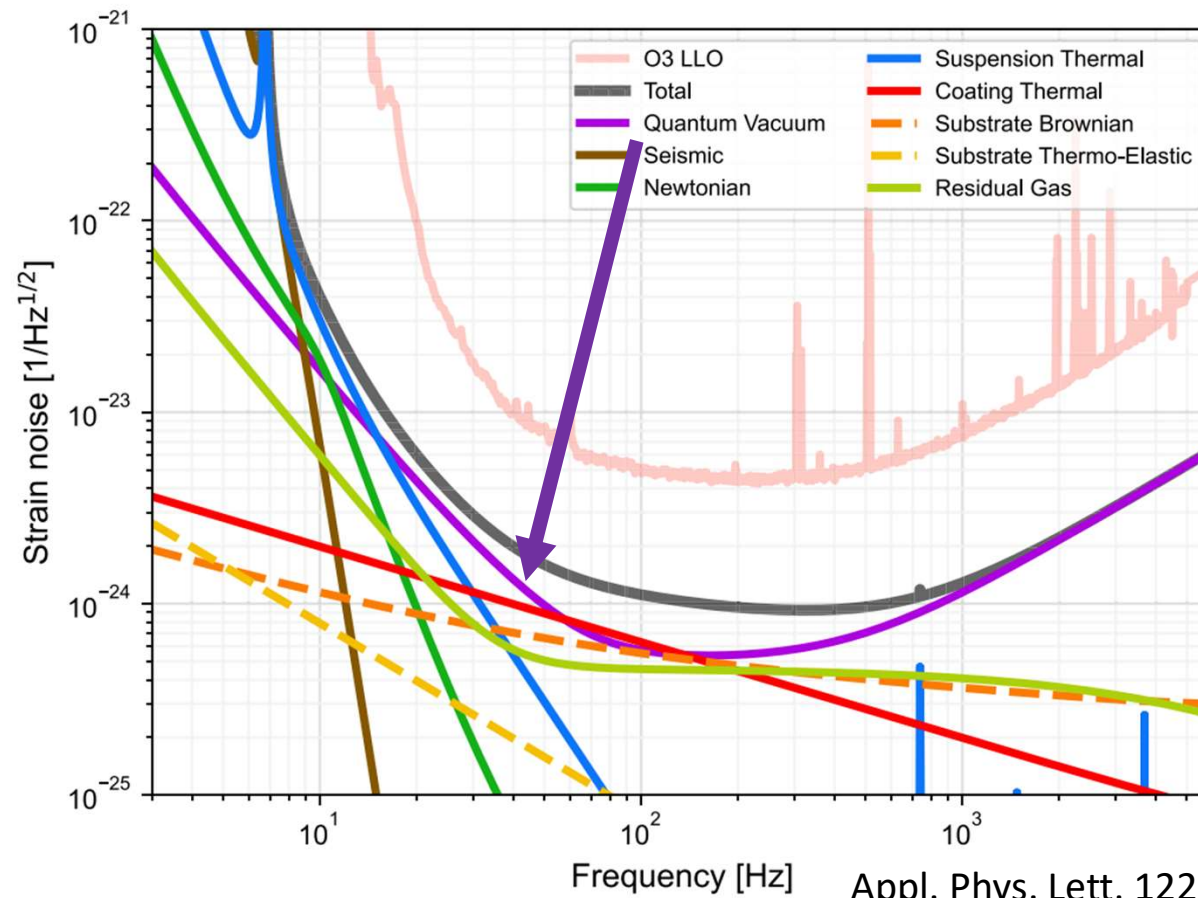
연세대학교
YONSEI UNIVERSITY



1. Standard quantum limit of gravitational wave detector

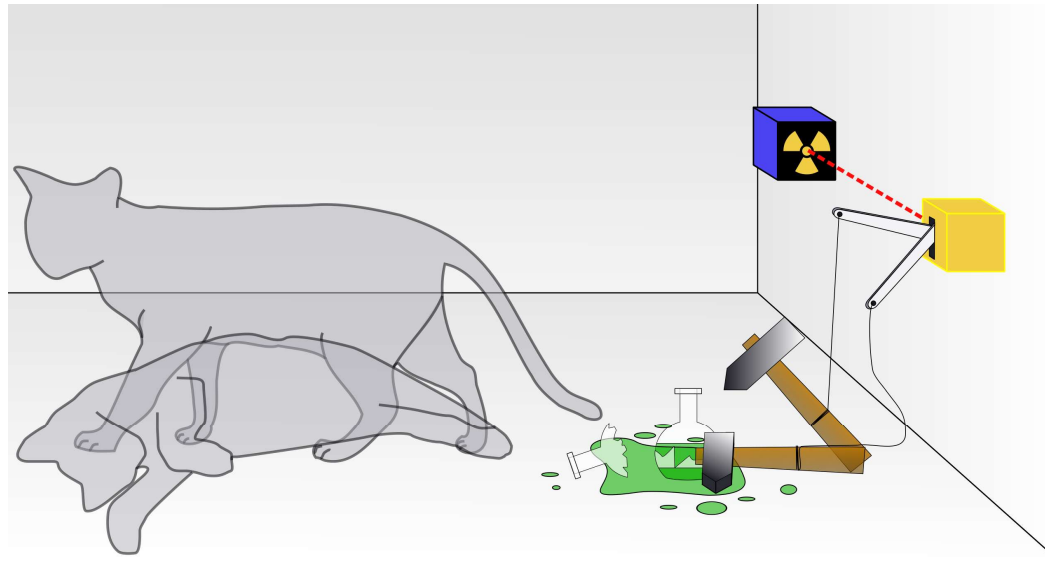


LIGO sensitivity

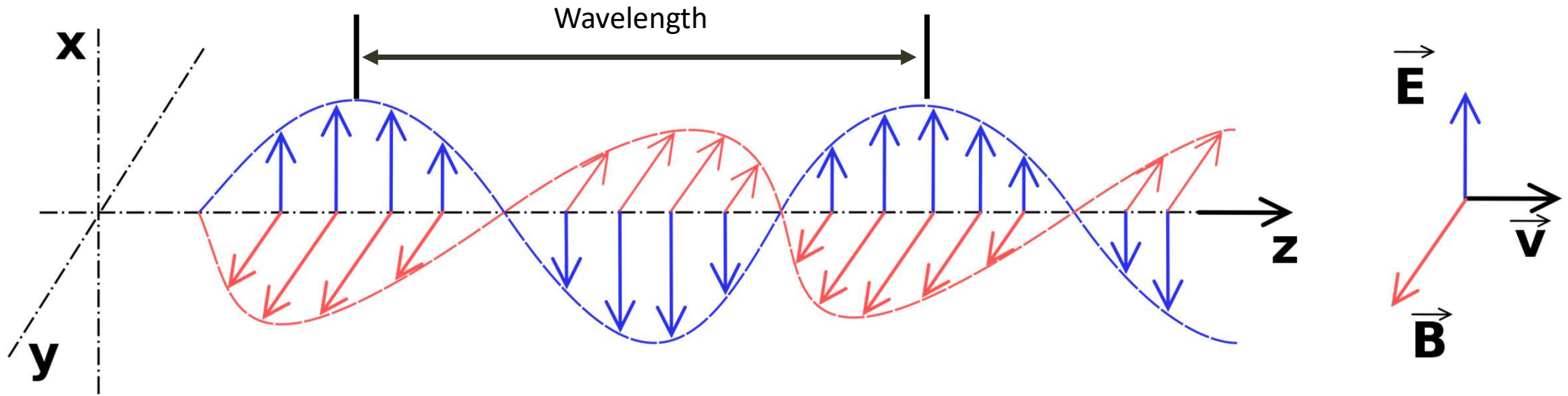


Appl. Phys. Lett. 122, 110502 (2023)

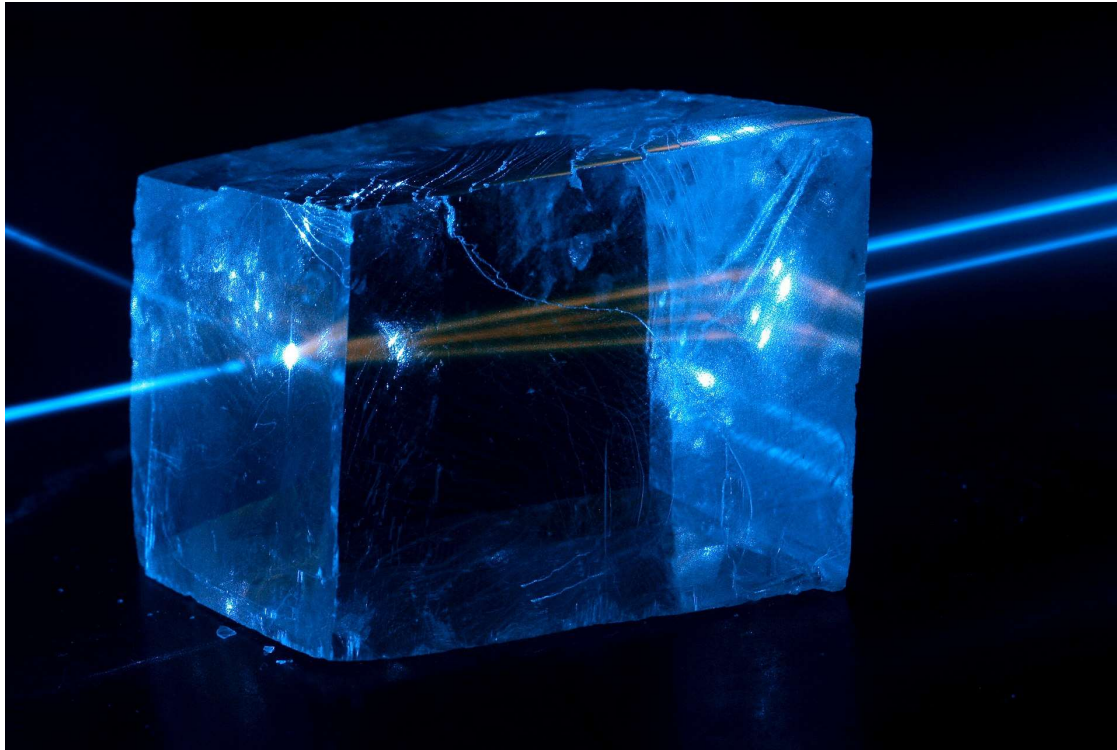
Quantum mechanics



Light (Electromagnetic wave)



Wave-particle duality



Photoelectric effect

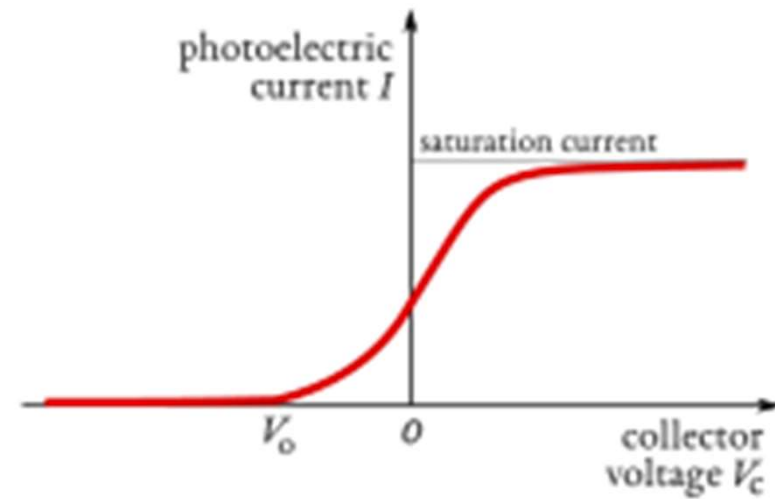
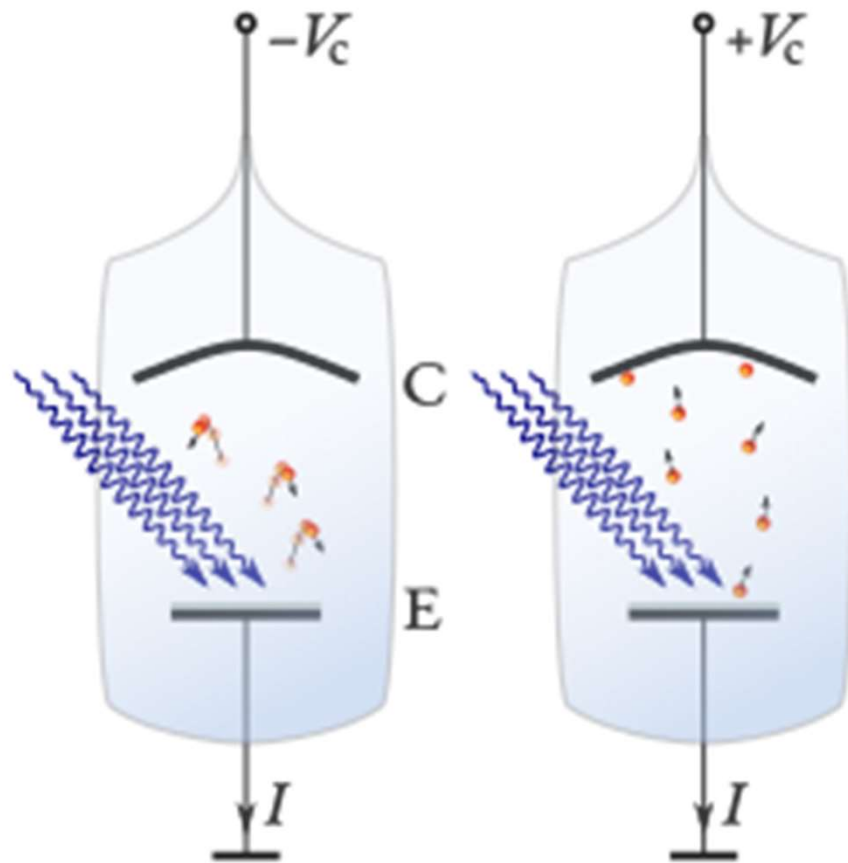


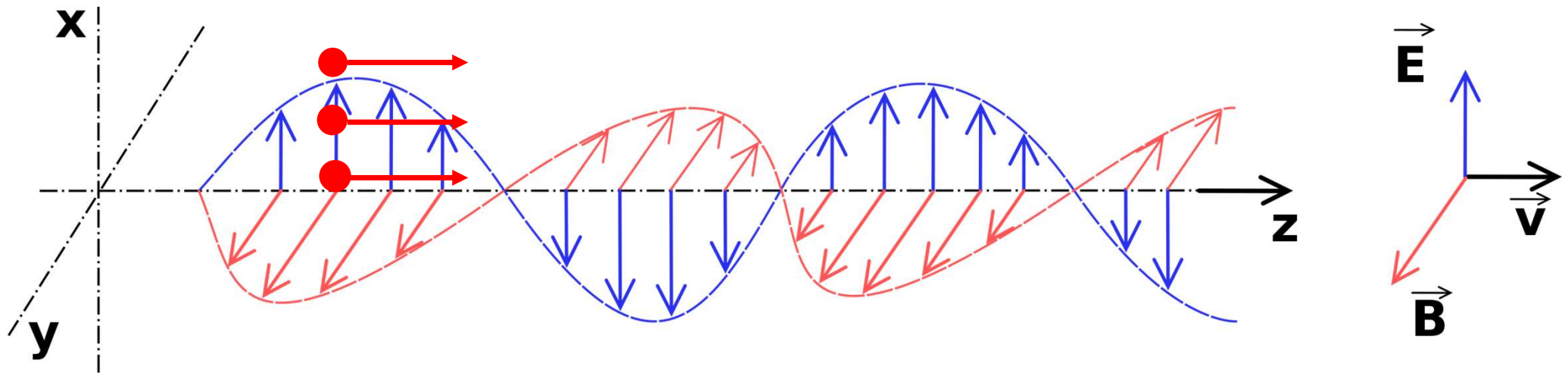
Photo detector

Amplitude of electricfield
 \propto *Number of photon*

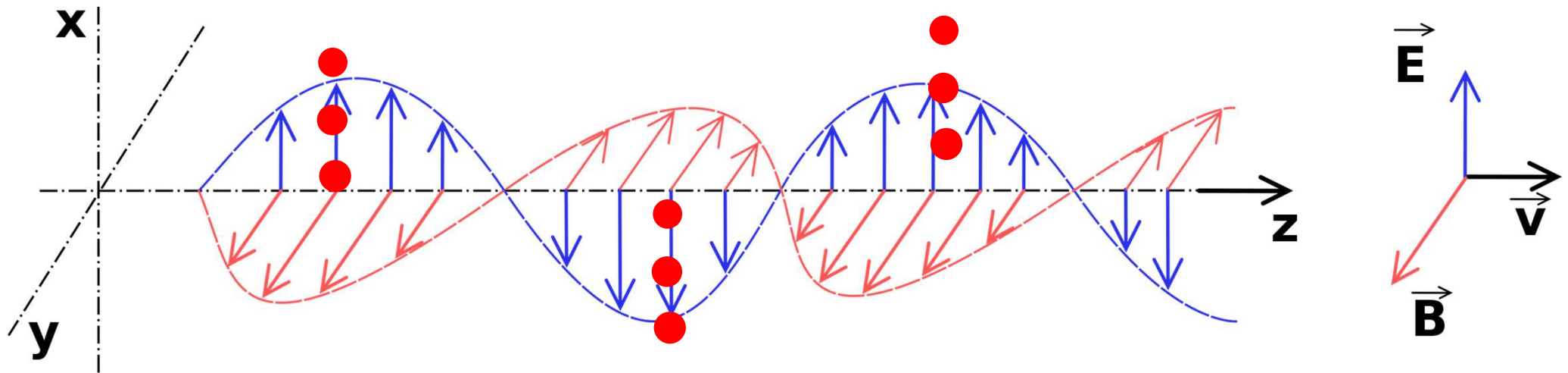
Measuring the intensity of light
= Counting the number of photon



Wave-particle duality

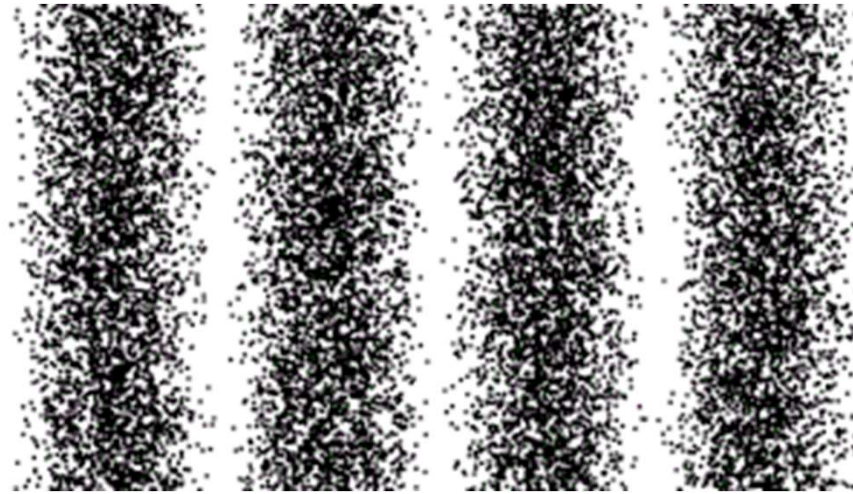


Wave-particle duality

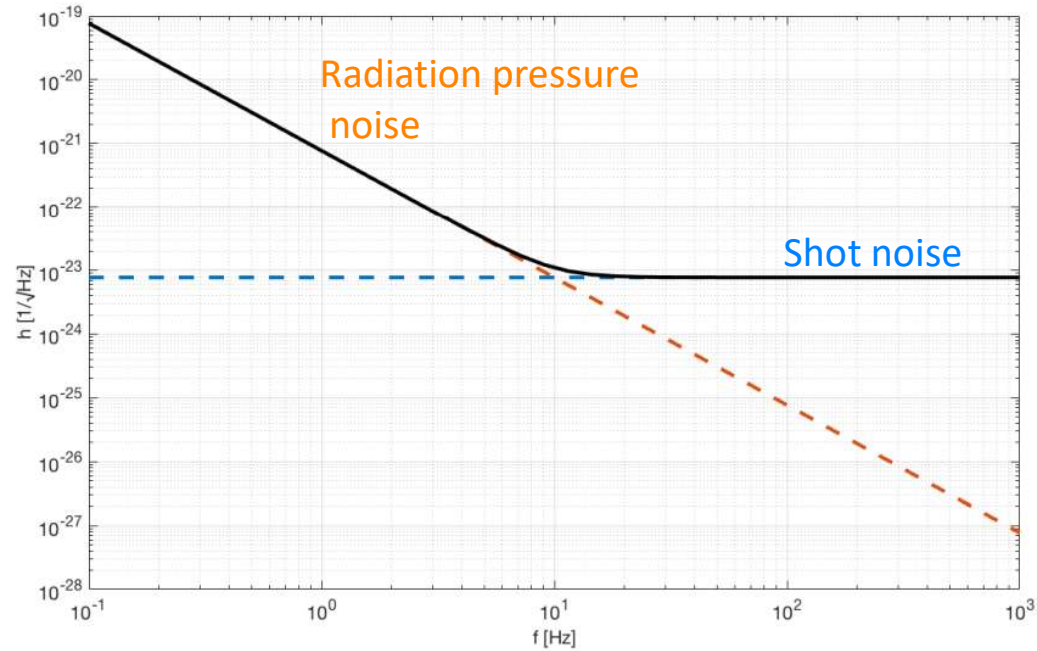


Number of **observed photon** is proportional to intensity of electric field

Double slit interference



Standard quantum limit of GW detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

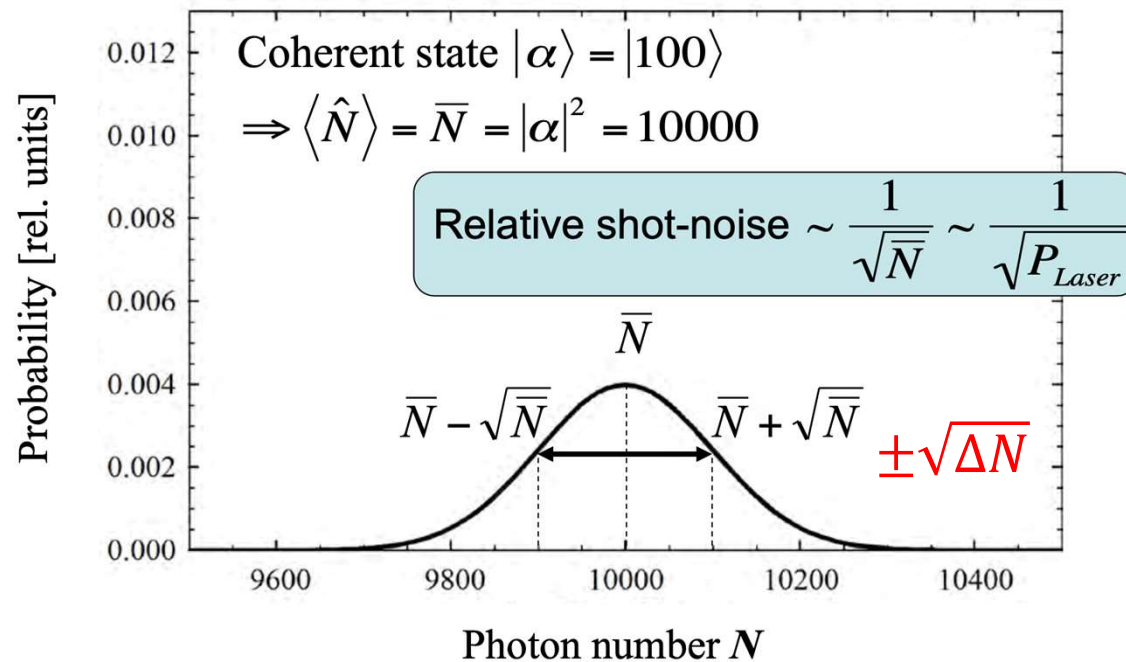
Shot noise



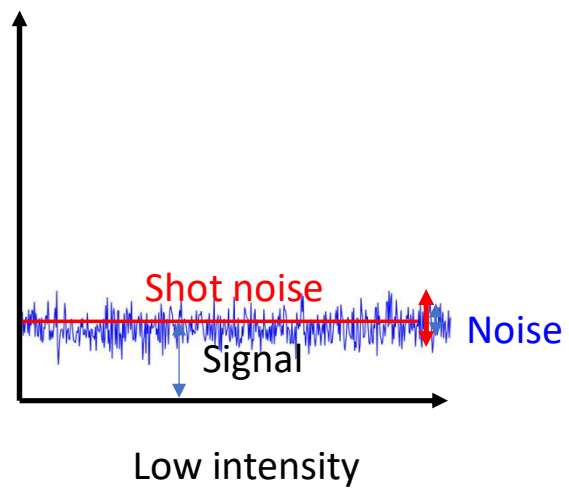
Shot noise of interferometer

Photon Counting Statistics

$$h = 6.62607015 \times 10^{-34} \text{ J}\cdot\text{Hz}^{-1}$$

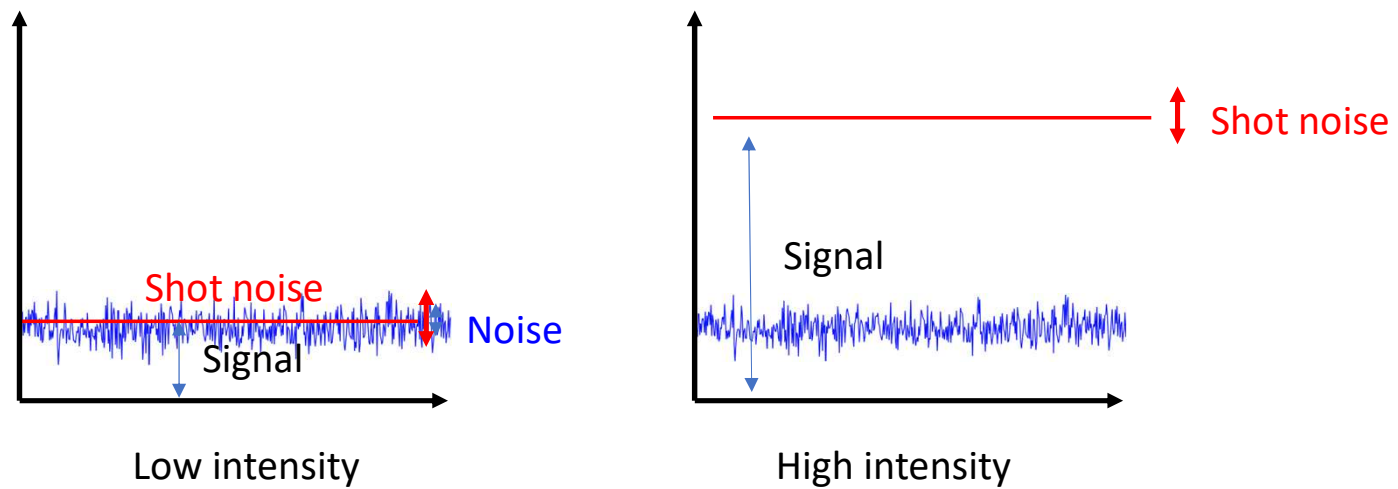


Shot noise of interferometer



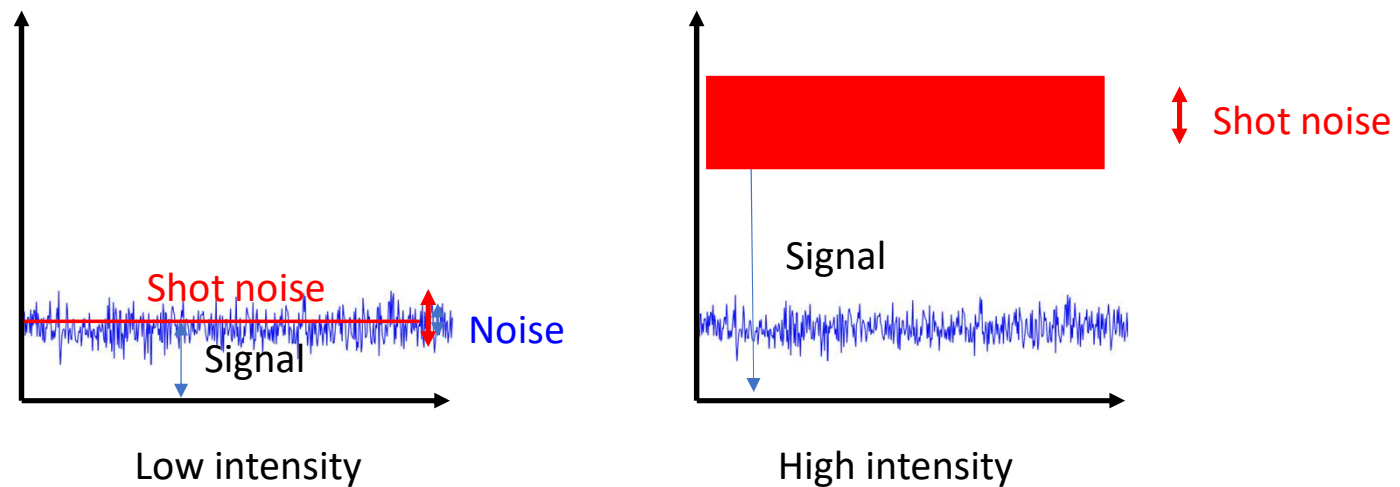
$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

Shot noise of interferometer



$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

Shot noise of interferometer

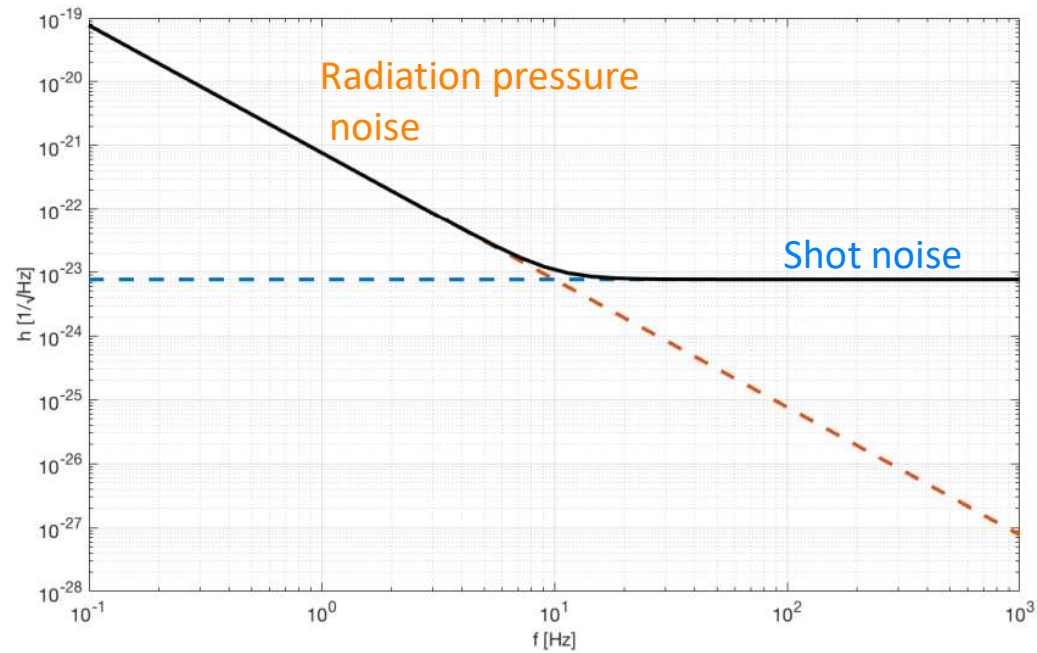


$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

N=100 , SNR =10
N=10000 , SNR =100

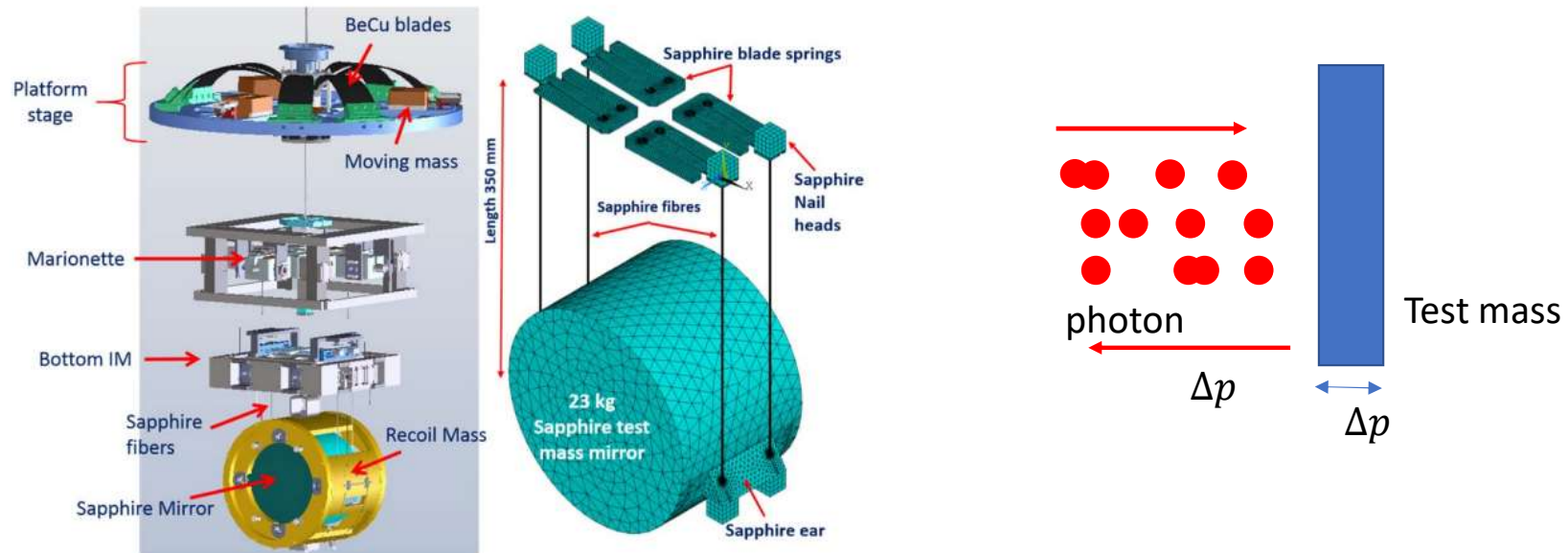
If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)
We say that it has **shot noise limit sensitivity**

Standard quantum limit of GW detector



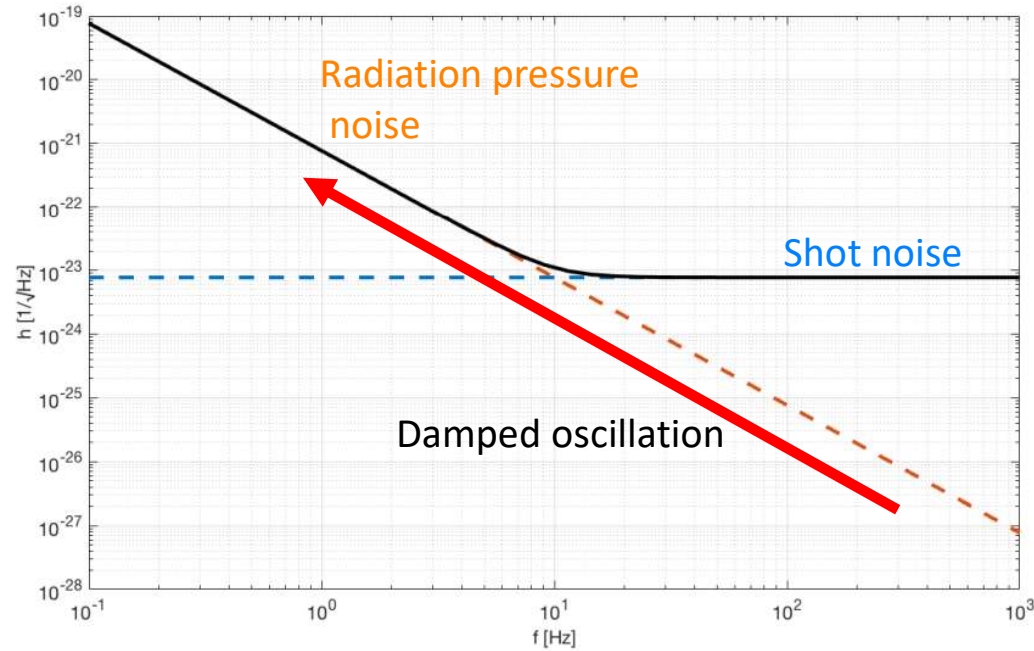
Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

Radiation pressure noise



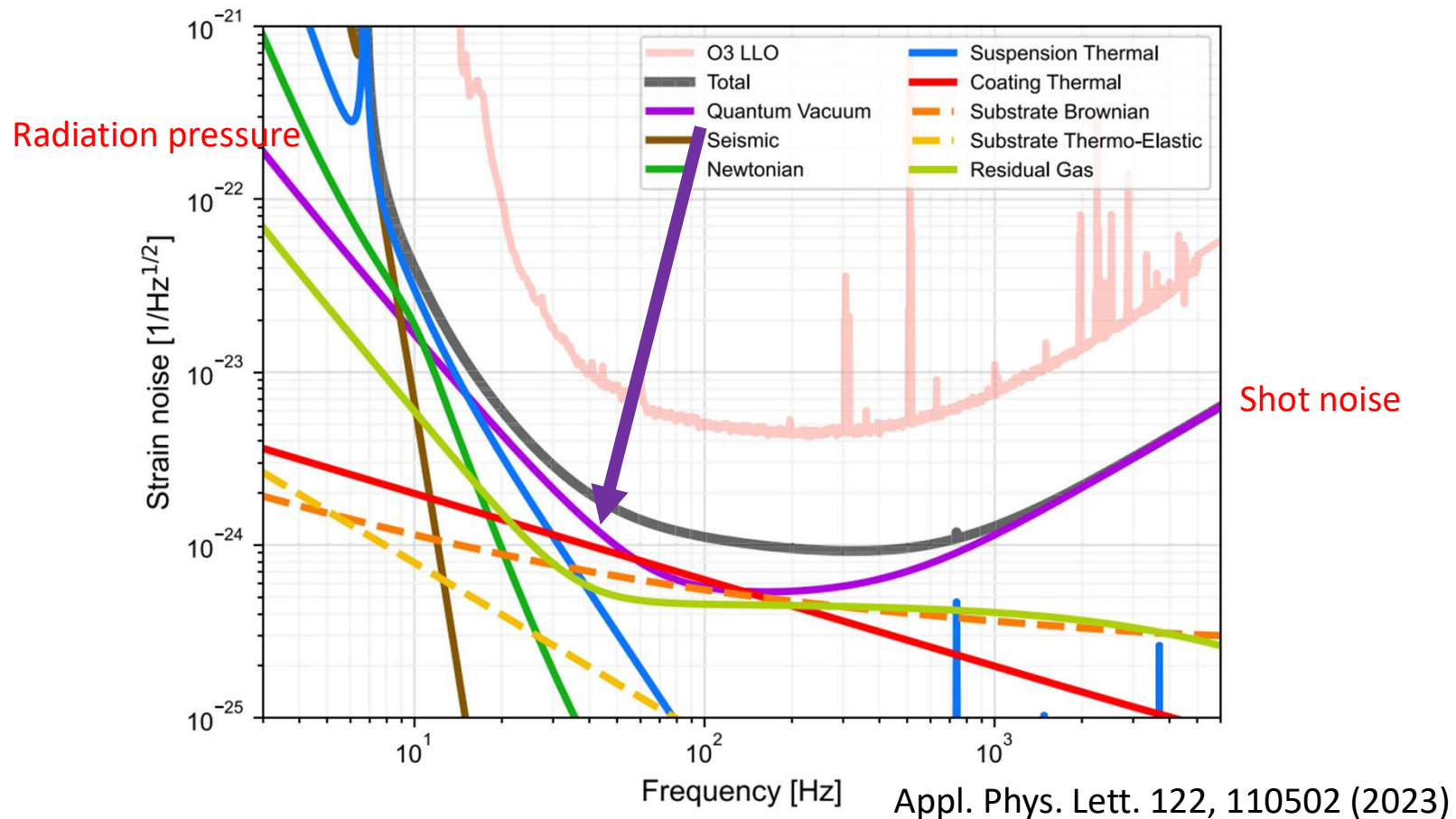
- Stored energy is very high (750 kW)
- Desired sensitivity is very high ($10^{-21} \sim 10^{-24}$)

Standard quantum limit of GW detector

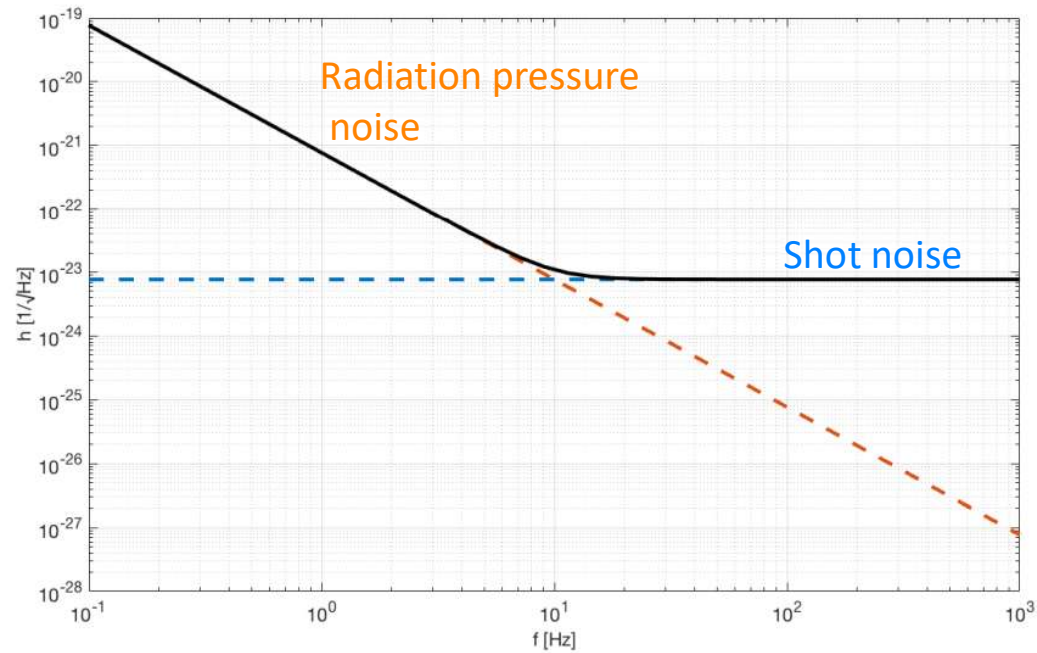


Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

LIGO sensitivity

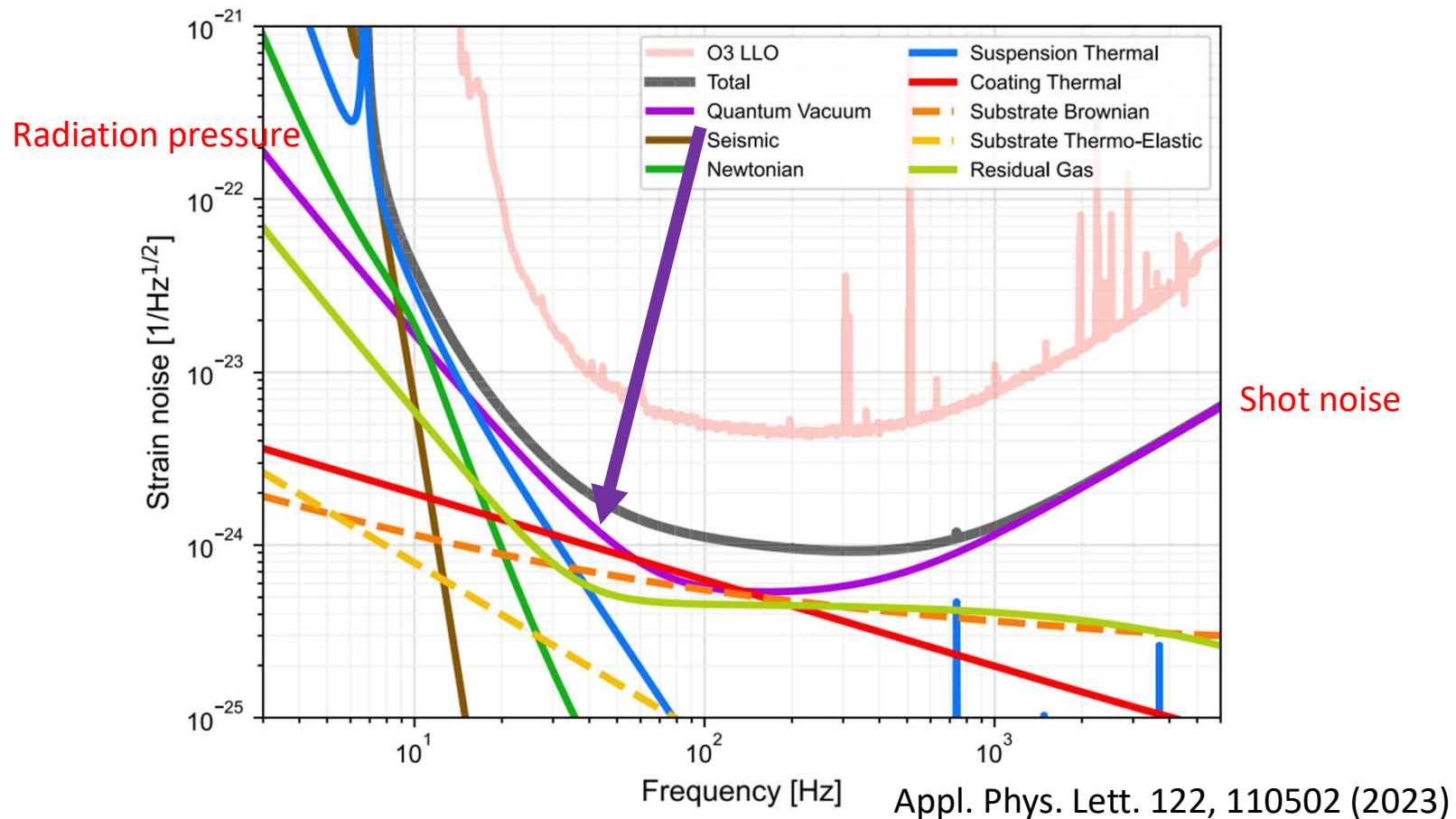


Standard quantum limit of GW detector



Standard quantum limit of gravitational wave detector
Shot noise + Radiation pressure noise

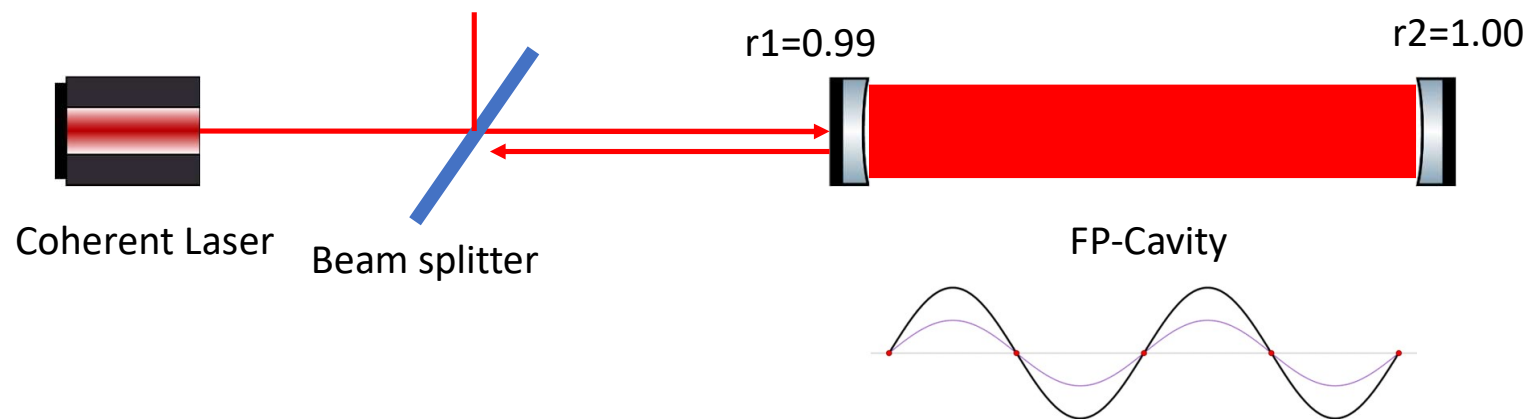
LIGO sensitivity



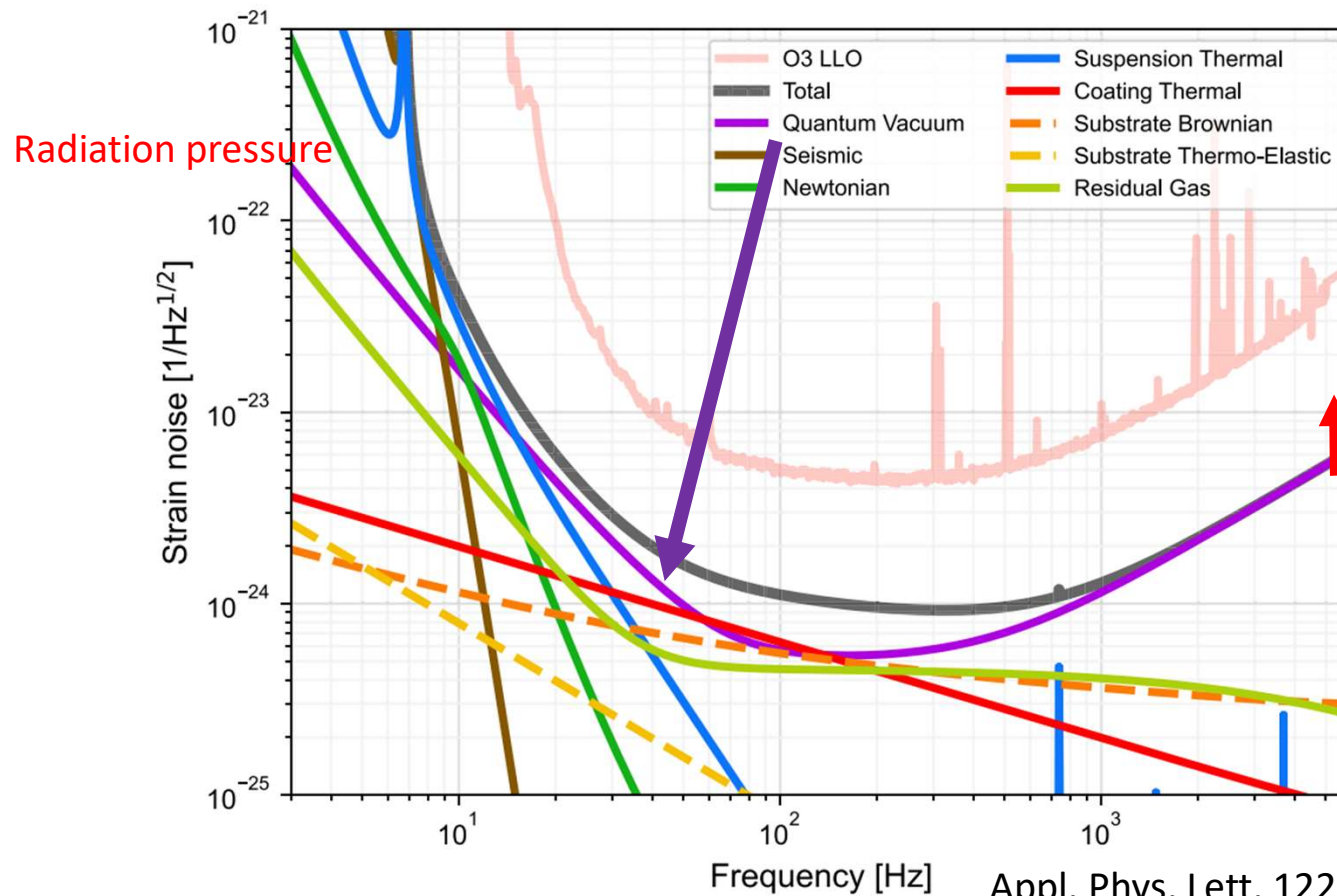
Fabry-perot cavity

Number of round trip > 250

4 km x 250 \sim 1000 km



LIGO sensitivity



Frequency = $1/s$ (Hz)

1 Hz signal : 1 sec signal

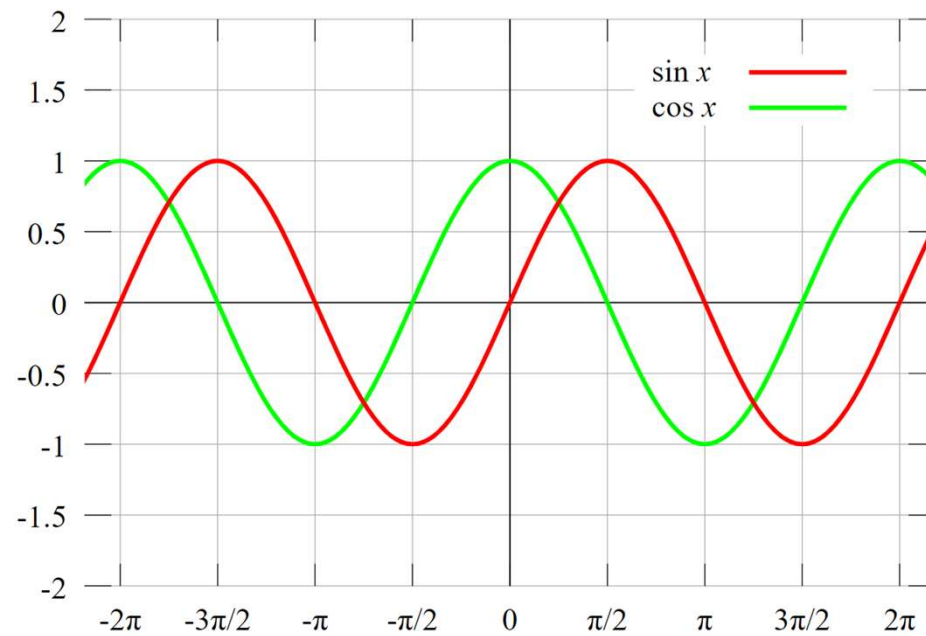
1000 Hz signal : 1/1000 sec signal

Number of round trip is different

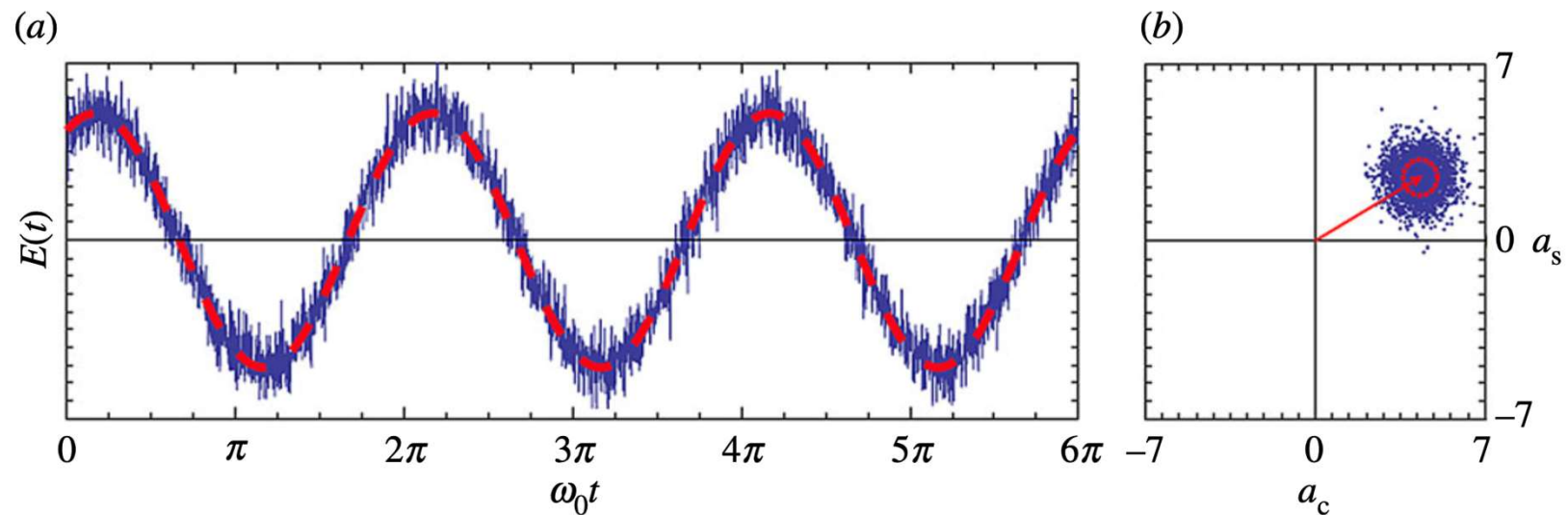
Shot noise

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Classical electromagnetic wave



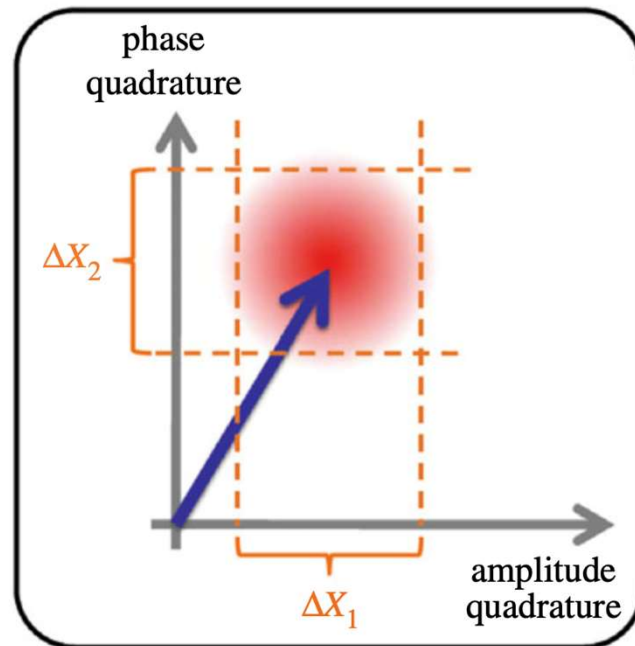
Quantum noise of coherent light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. *Phil. Trans. R. Soc. A* 376: 20170289.

Phase and amplitude quadrature

(a) Shot noise

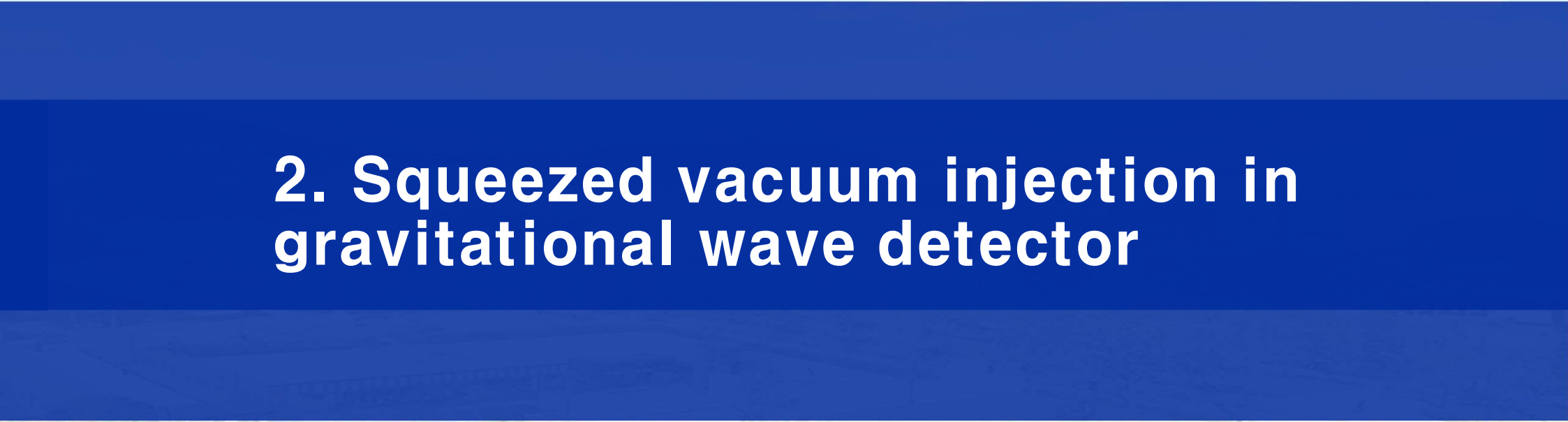


$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Uncertainty principle

Radiation pressure noise

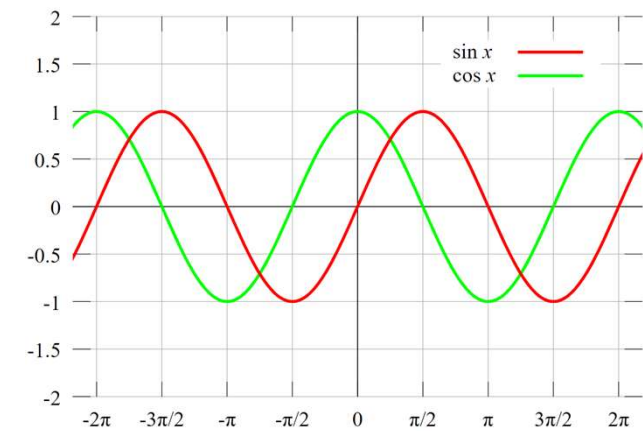
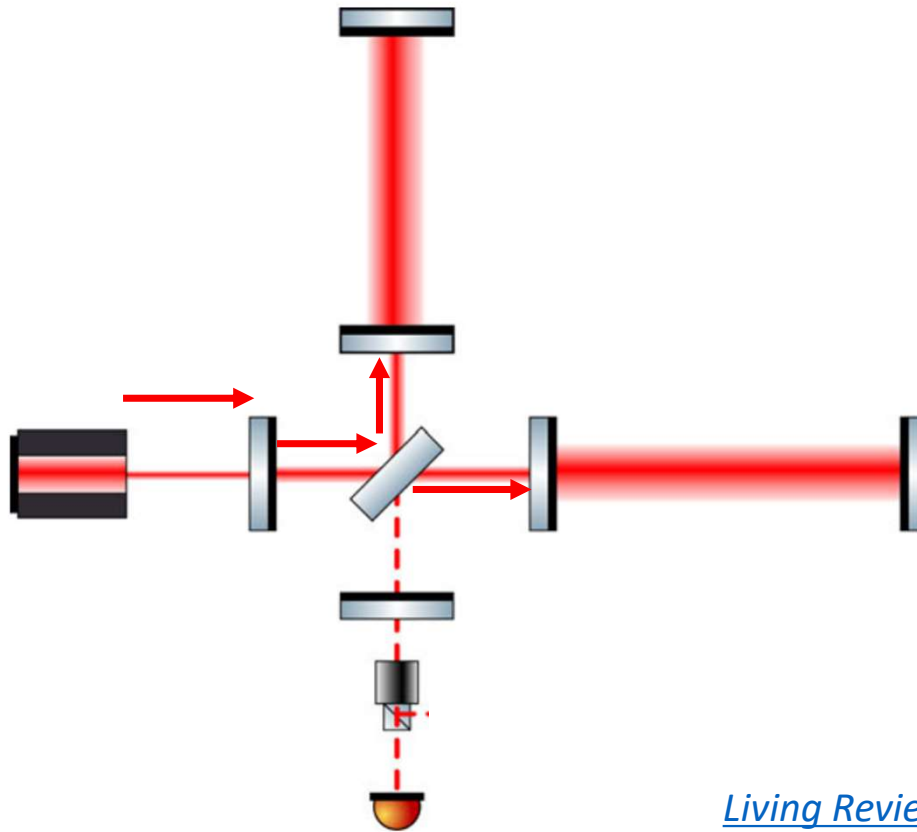
Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.



2. Squeezed vacuum injection in gravitational wave detector

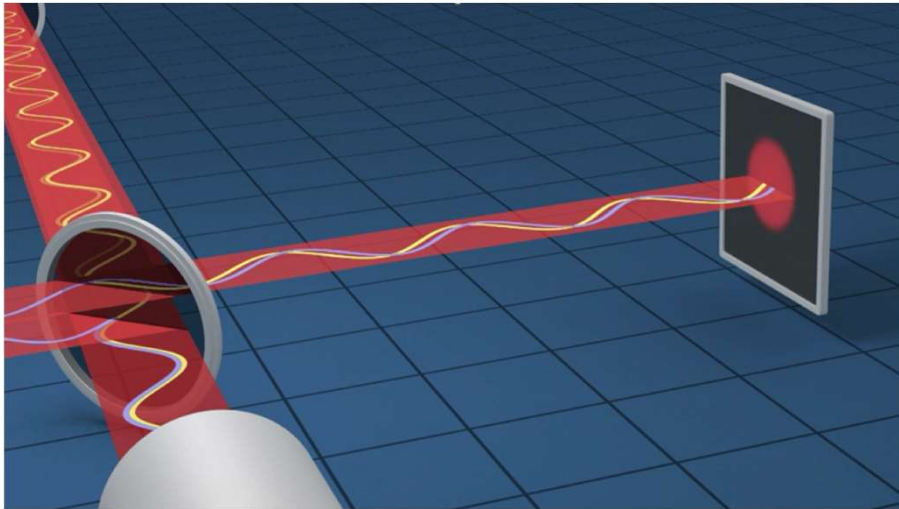


Gravitational wave detector

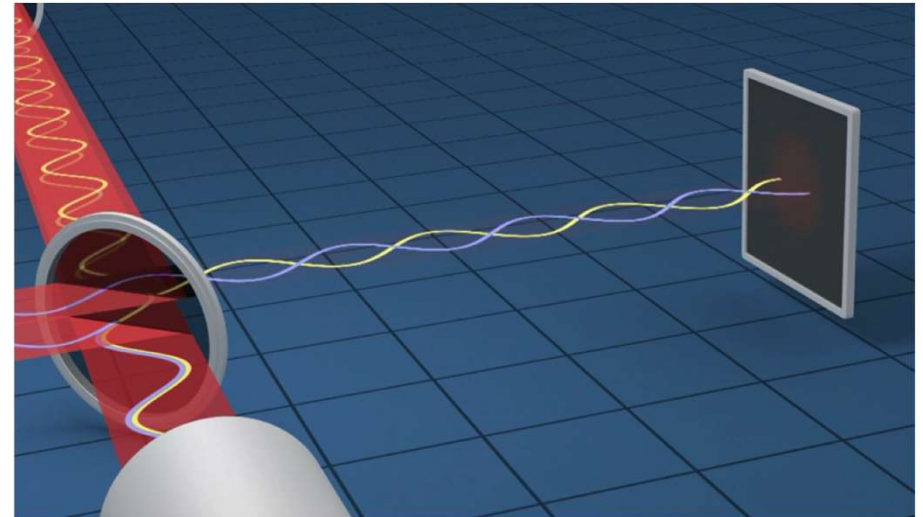


Living Reviews in Relativity volume 22,
Article number: 2 (2019)

Bright port and dark port

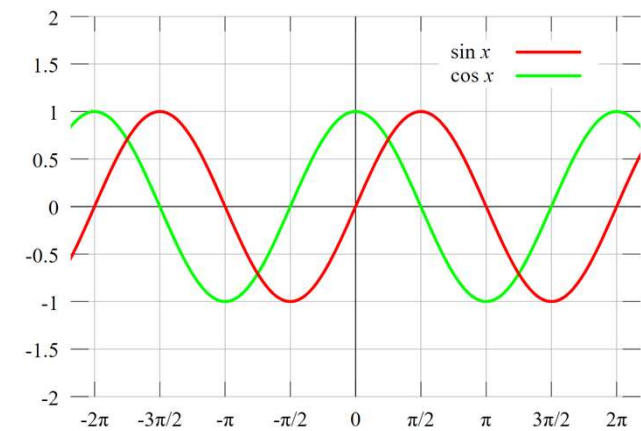
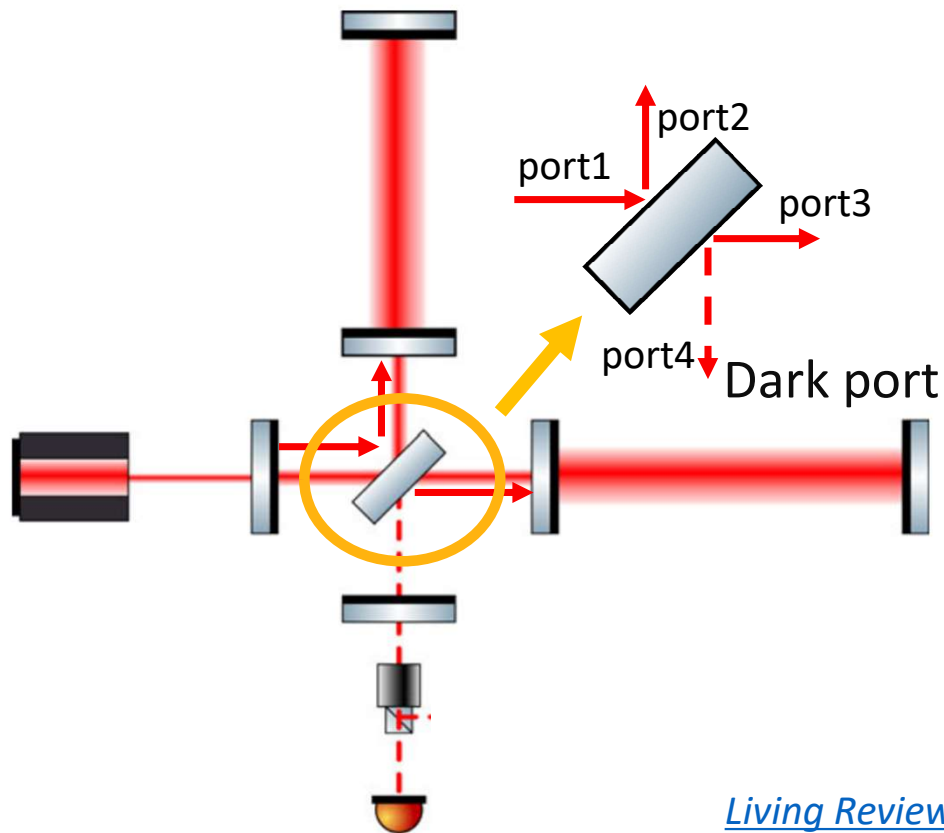


Bright



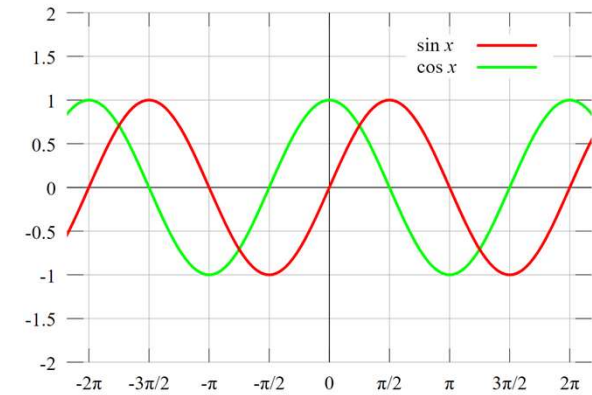
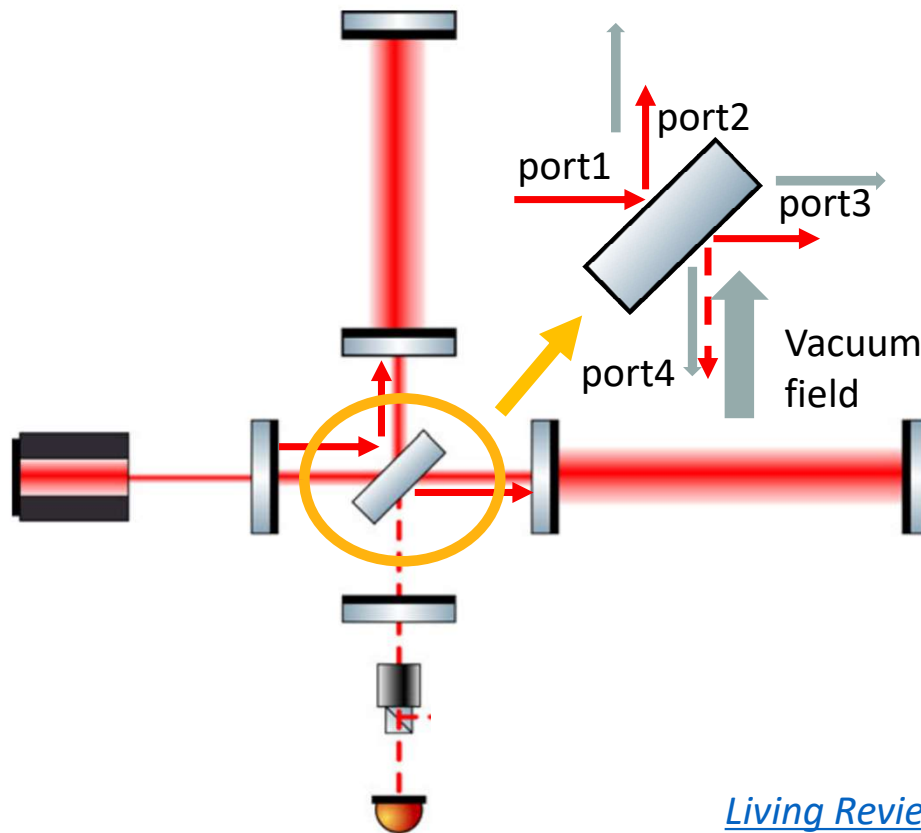
Dark

Gravitational wave detector



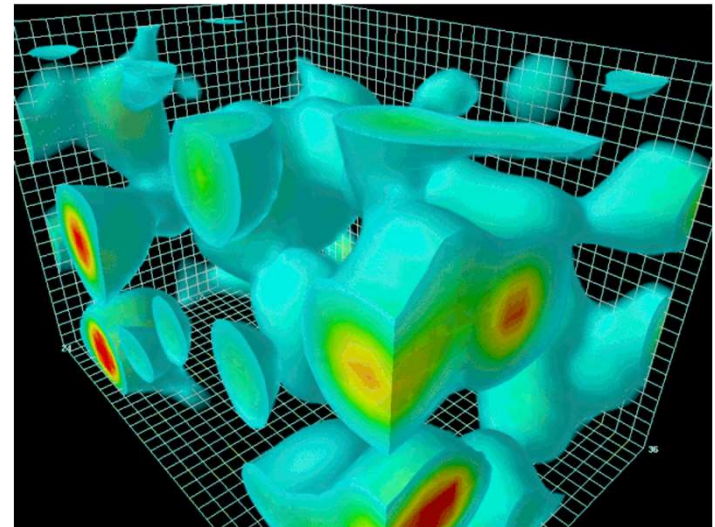
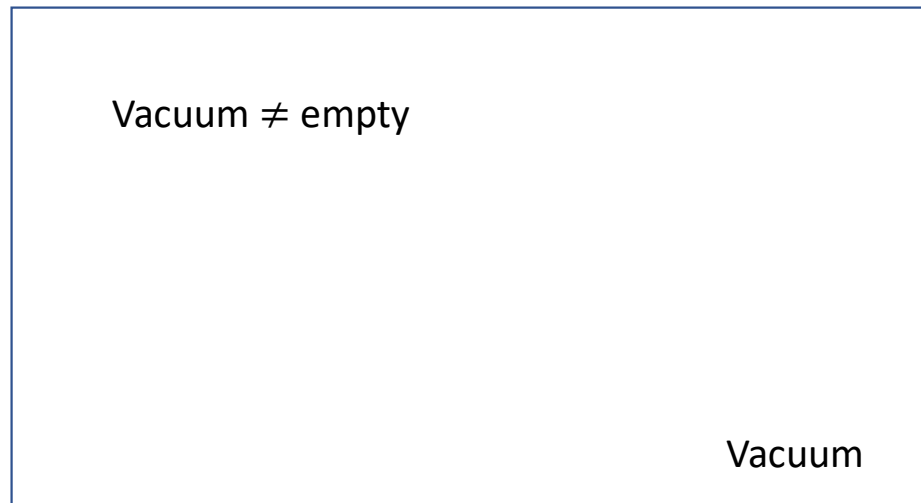
Living Reviews in Relativity volume 22,
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Quantum noise of gravitational wave detector



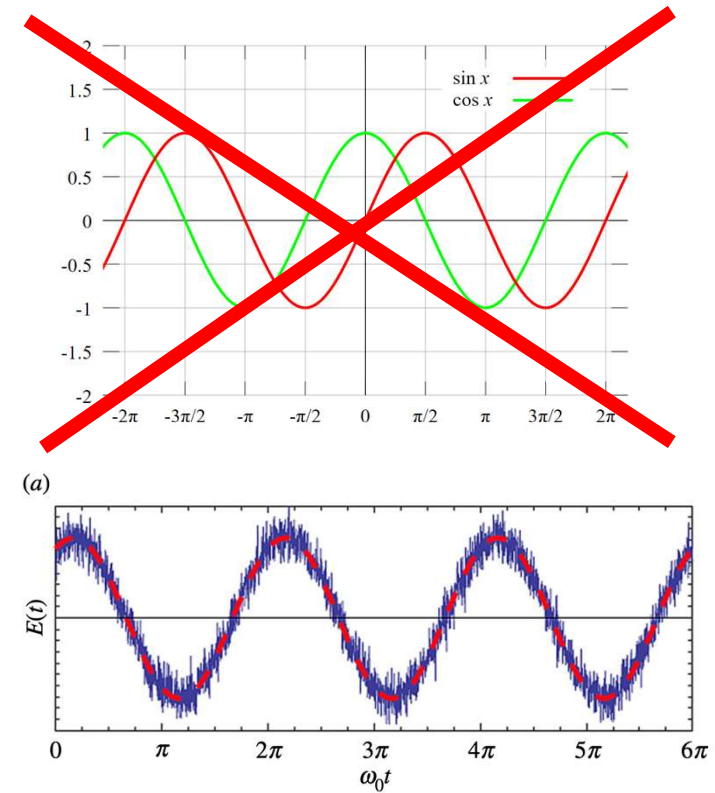
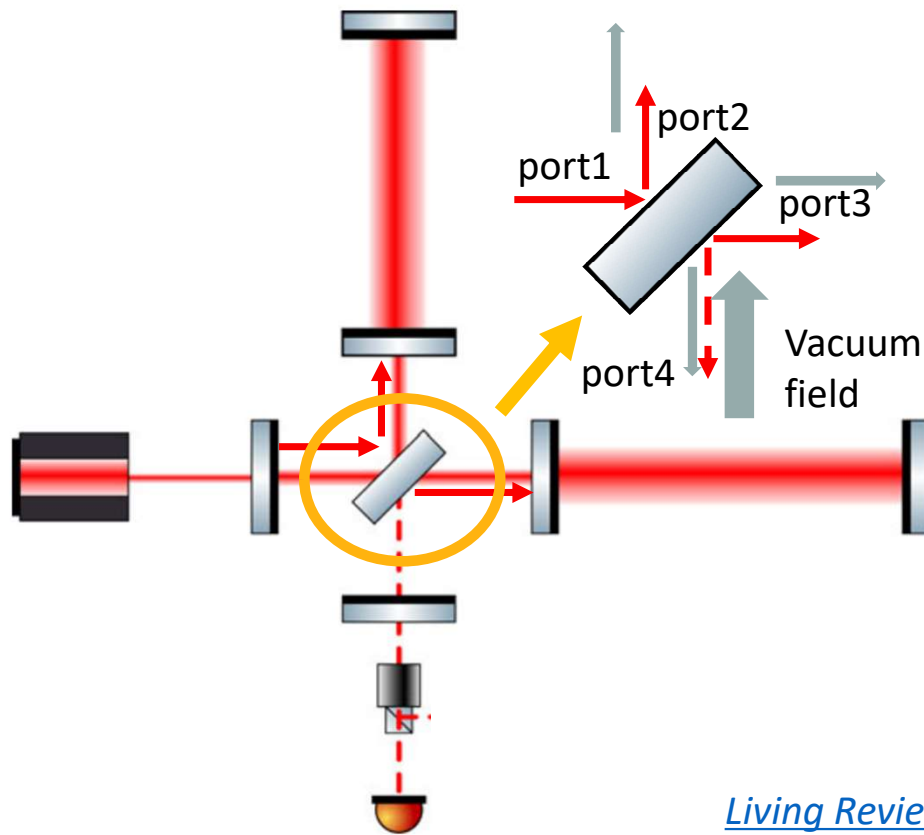
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Quantum vacuum fluctuation



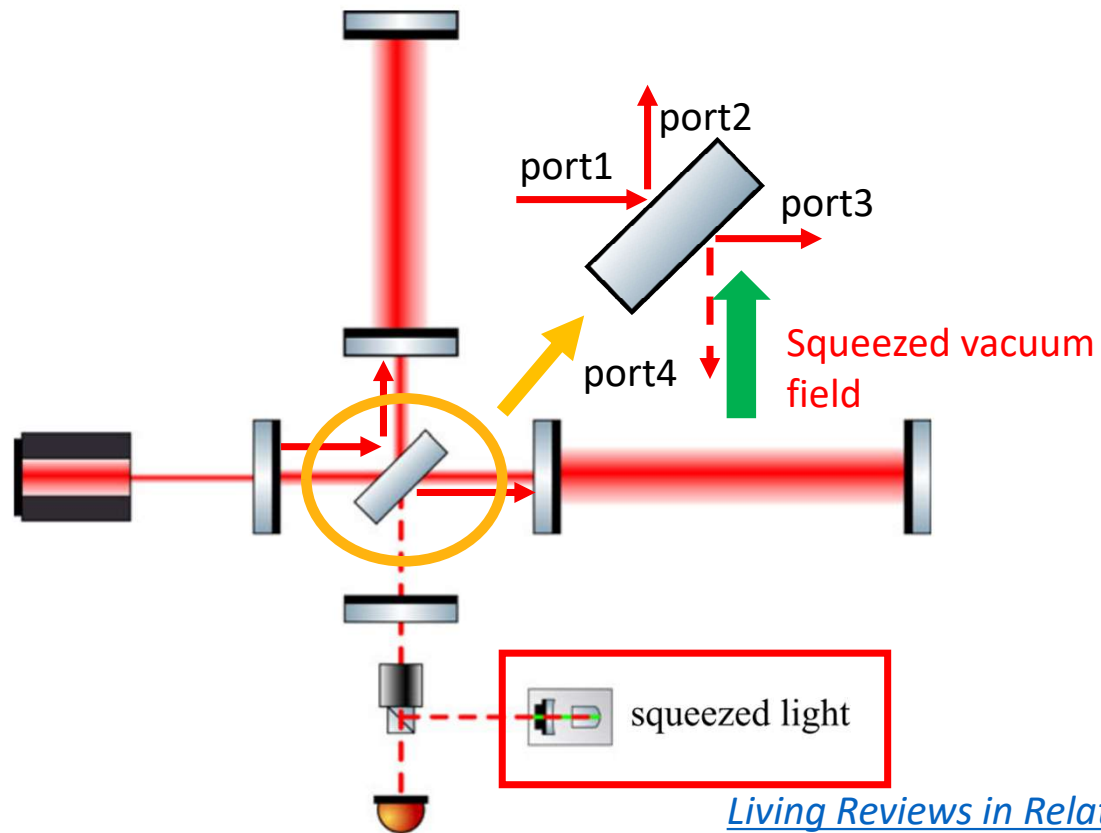
Average energy = zeropoint energy

Quantum noise of gravitational wave detector



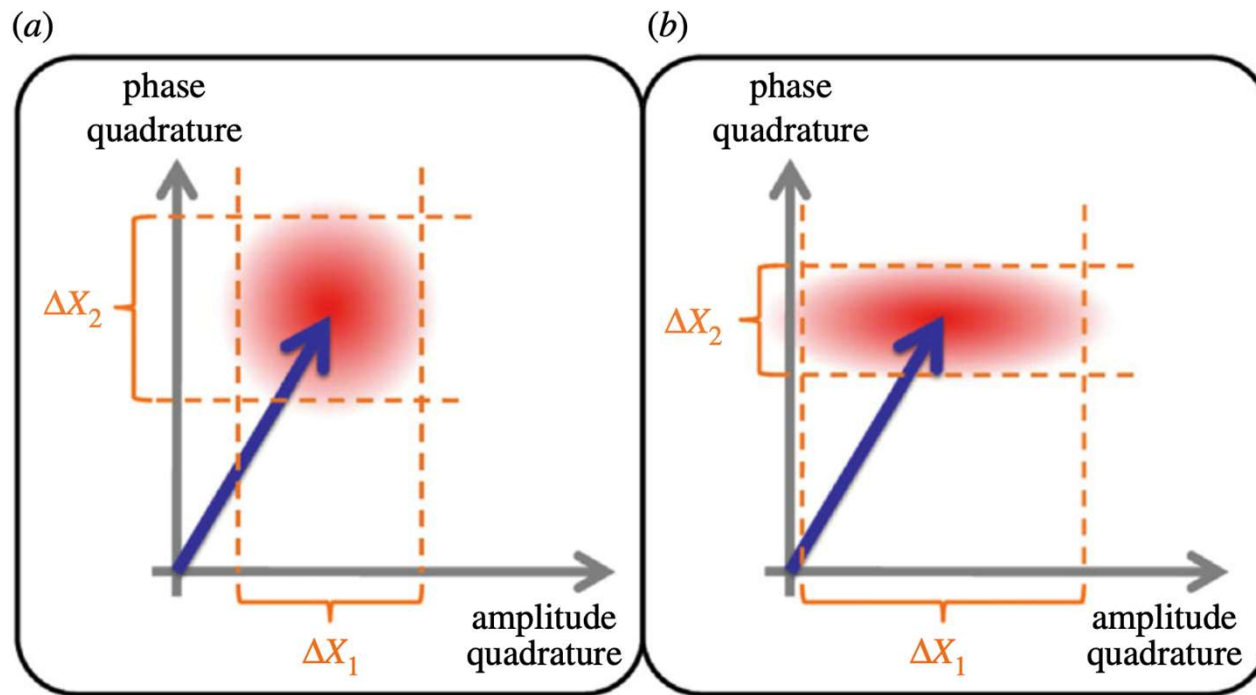
Living Reviews in Relativity volume 22,
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Gravitational wave detector

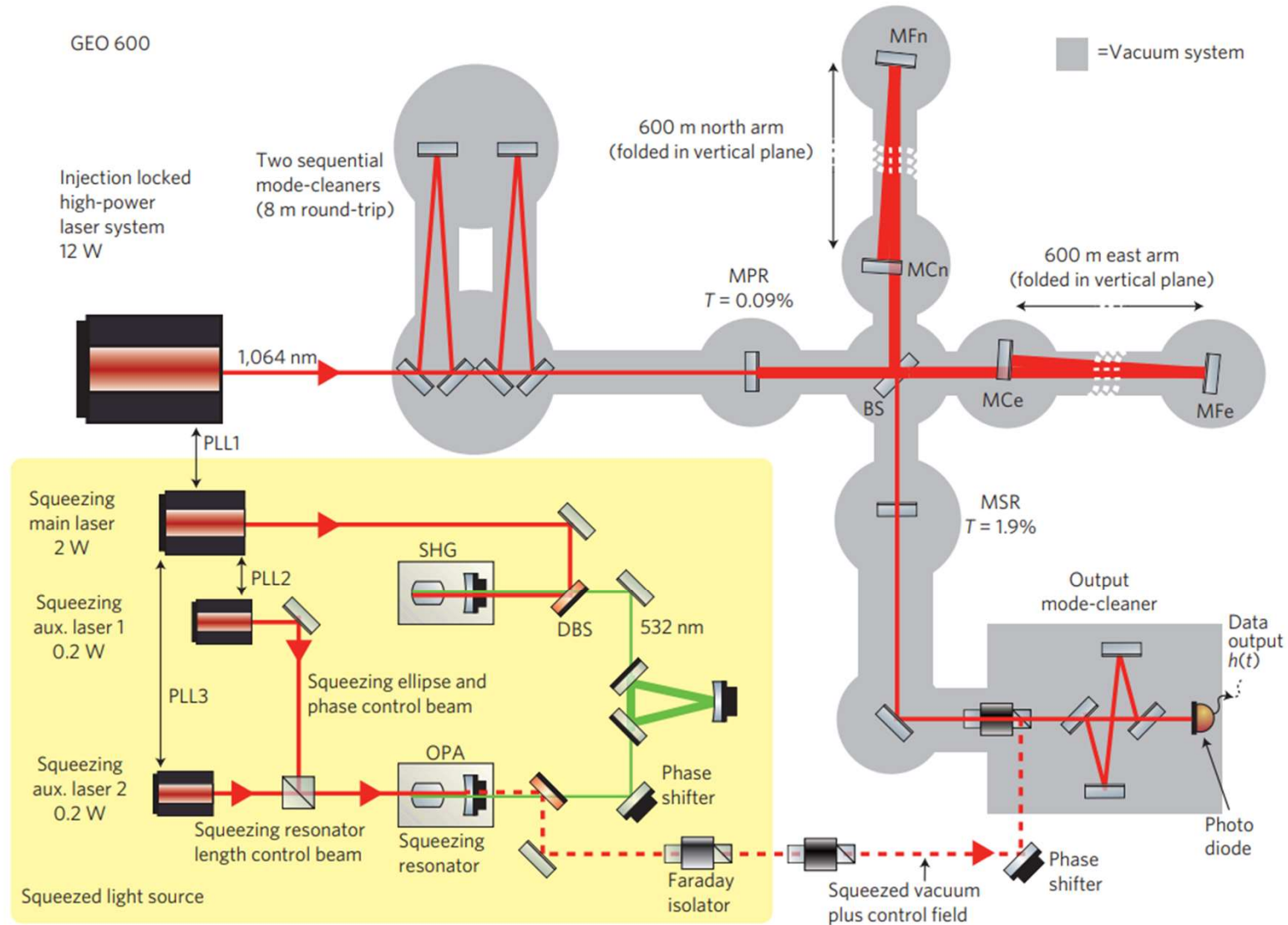


Living Reviews in Relativity volume 22,
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Squeezed light



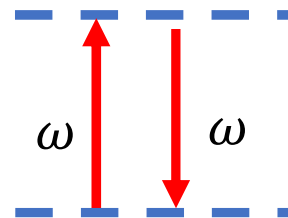
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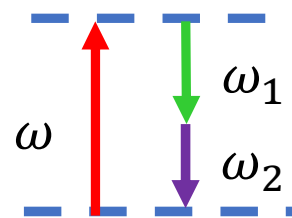
Non linear crystal



Non-linear crystal

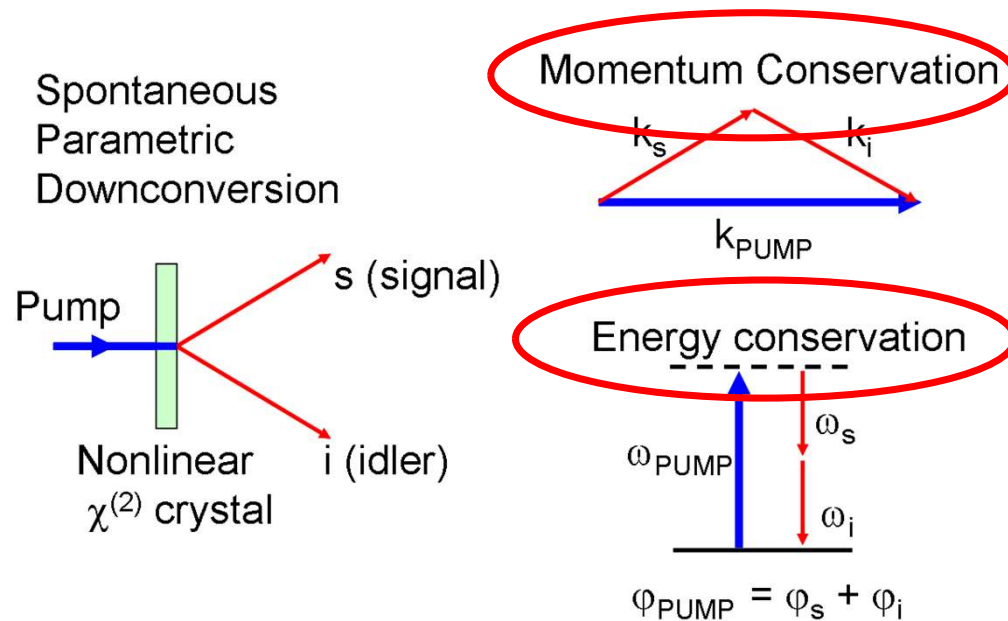


linear optics



Non linear optics

Parametric down conversion

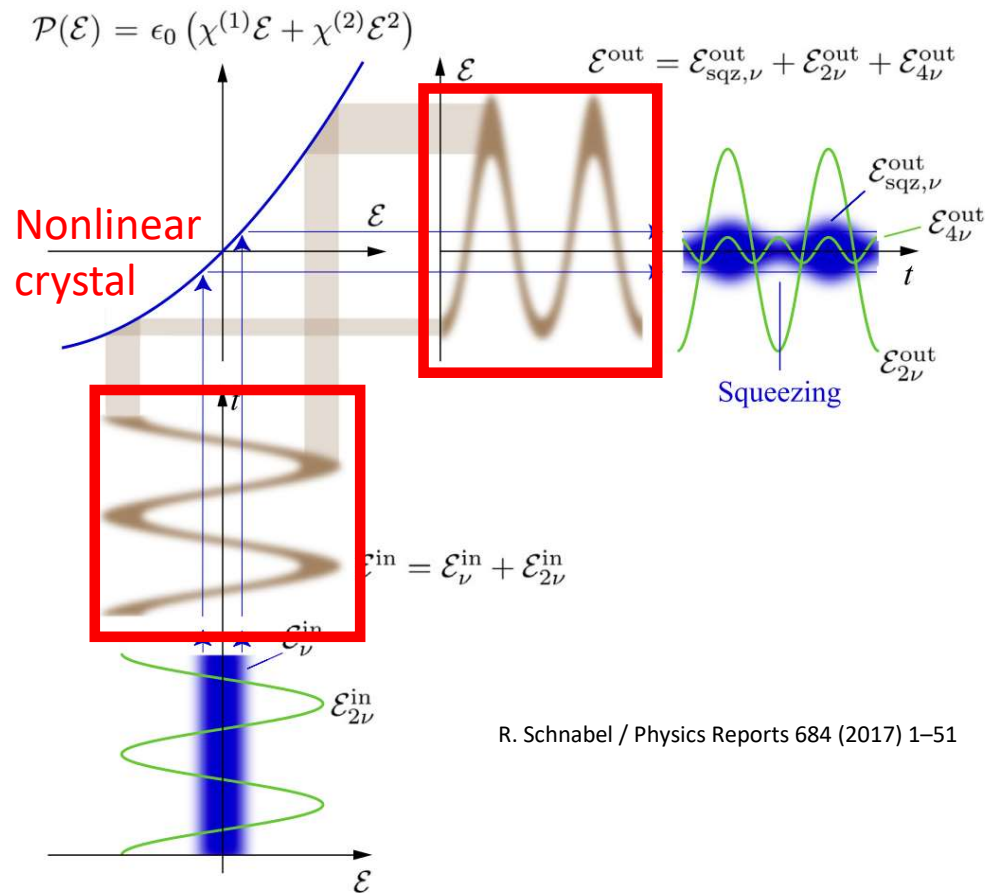


Every field which is participated in Non-linear conversion process, Must obey energy, momentum conservation.

Even for vacuum field!!

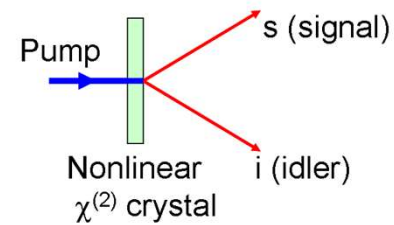
[J S Lundeen](#)

Parametric down conversion

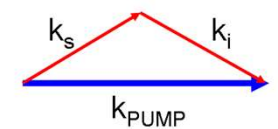


R. Schnabel / Physics Reports 684 (2017) 1–51

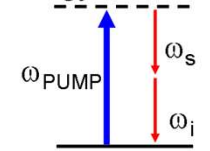
Spontaneous Parametric Downconversion



Momentum Conservation



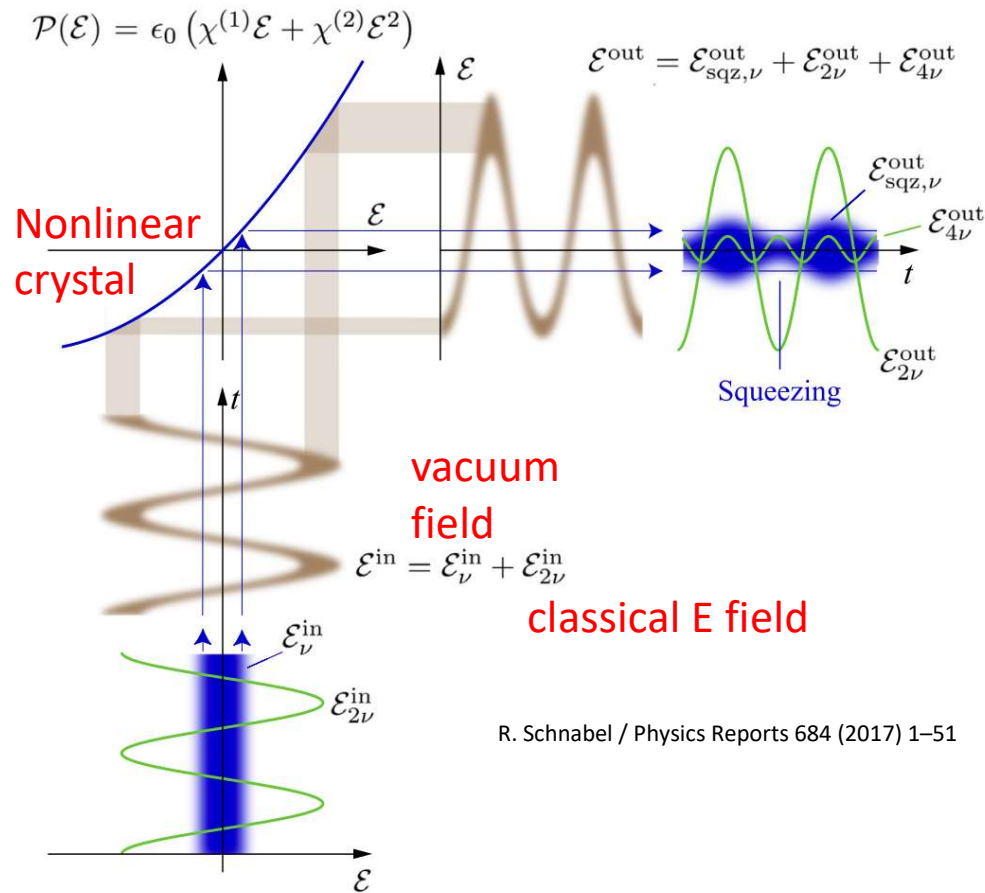
Energy conservation



$\Phi_{\text{PUMP}} = \Phi_s + \Phi_i$

Boundary condition

Parametric down conversion



vacuum field
classical E field

State of vacuum is changed during conversion process due to momentum and energy conservation

Squeezed vacuum

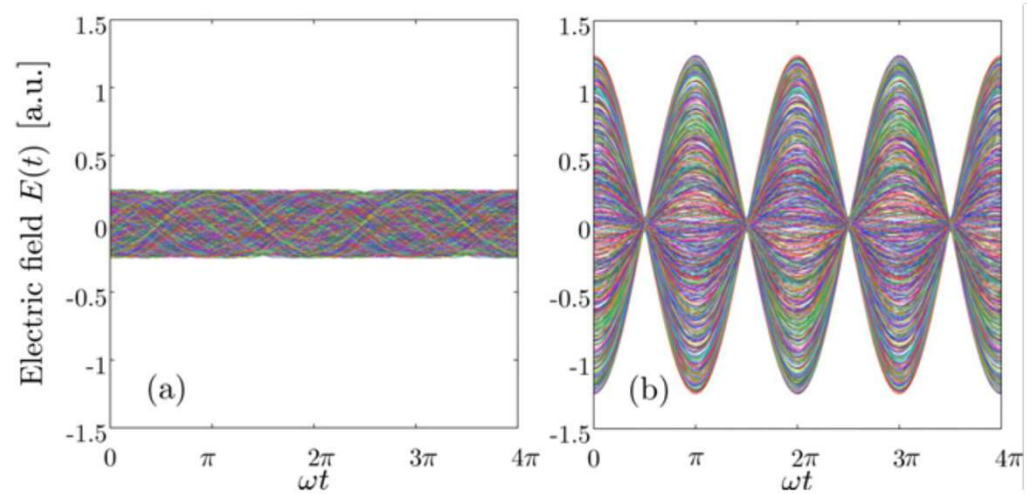
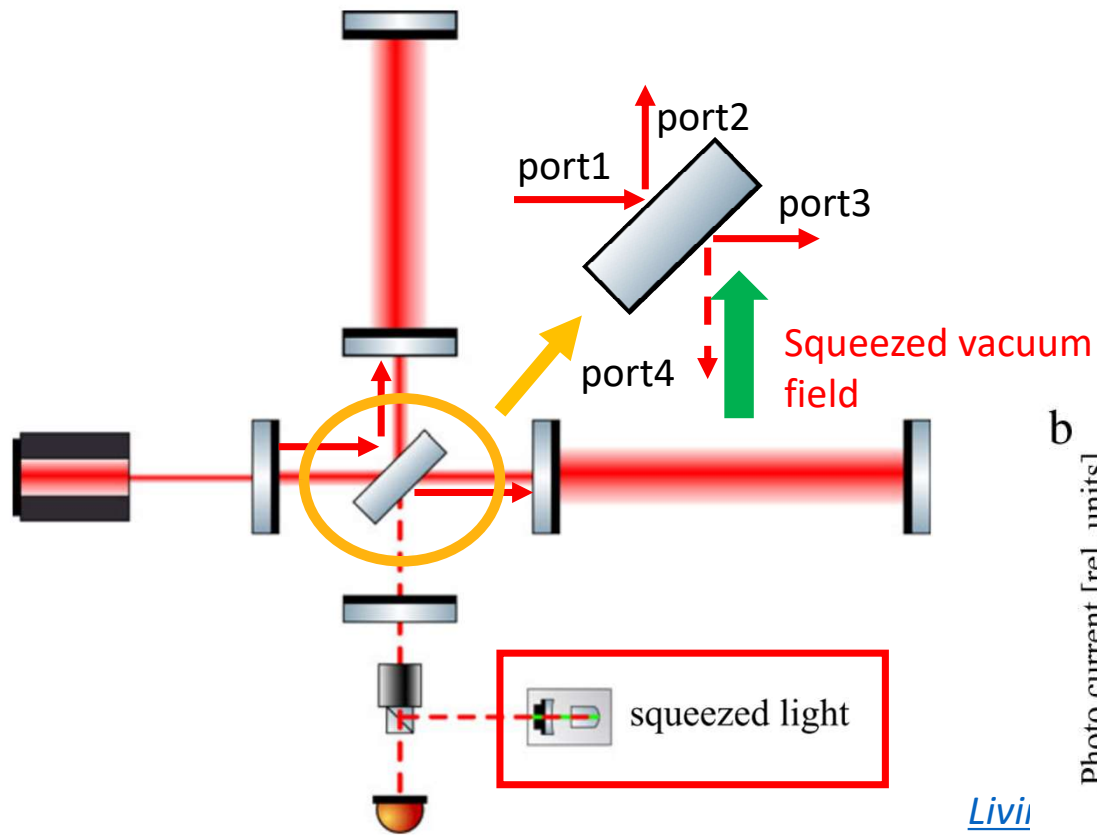
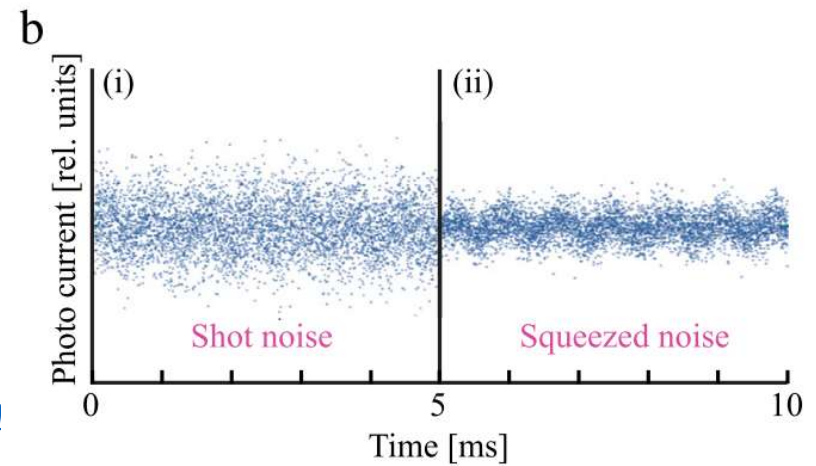
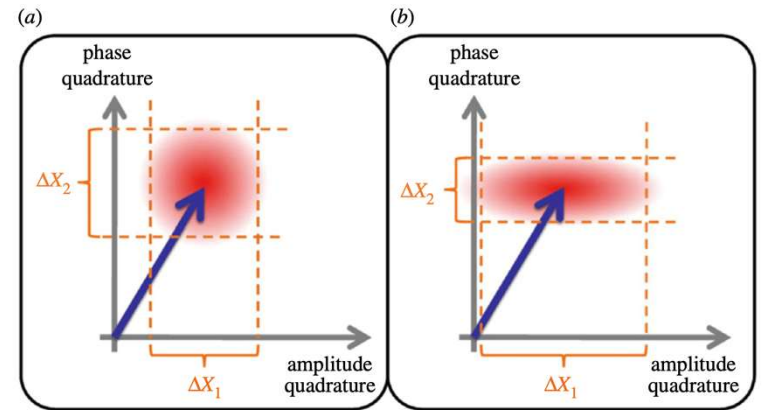


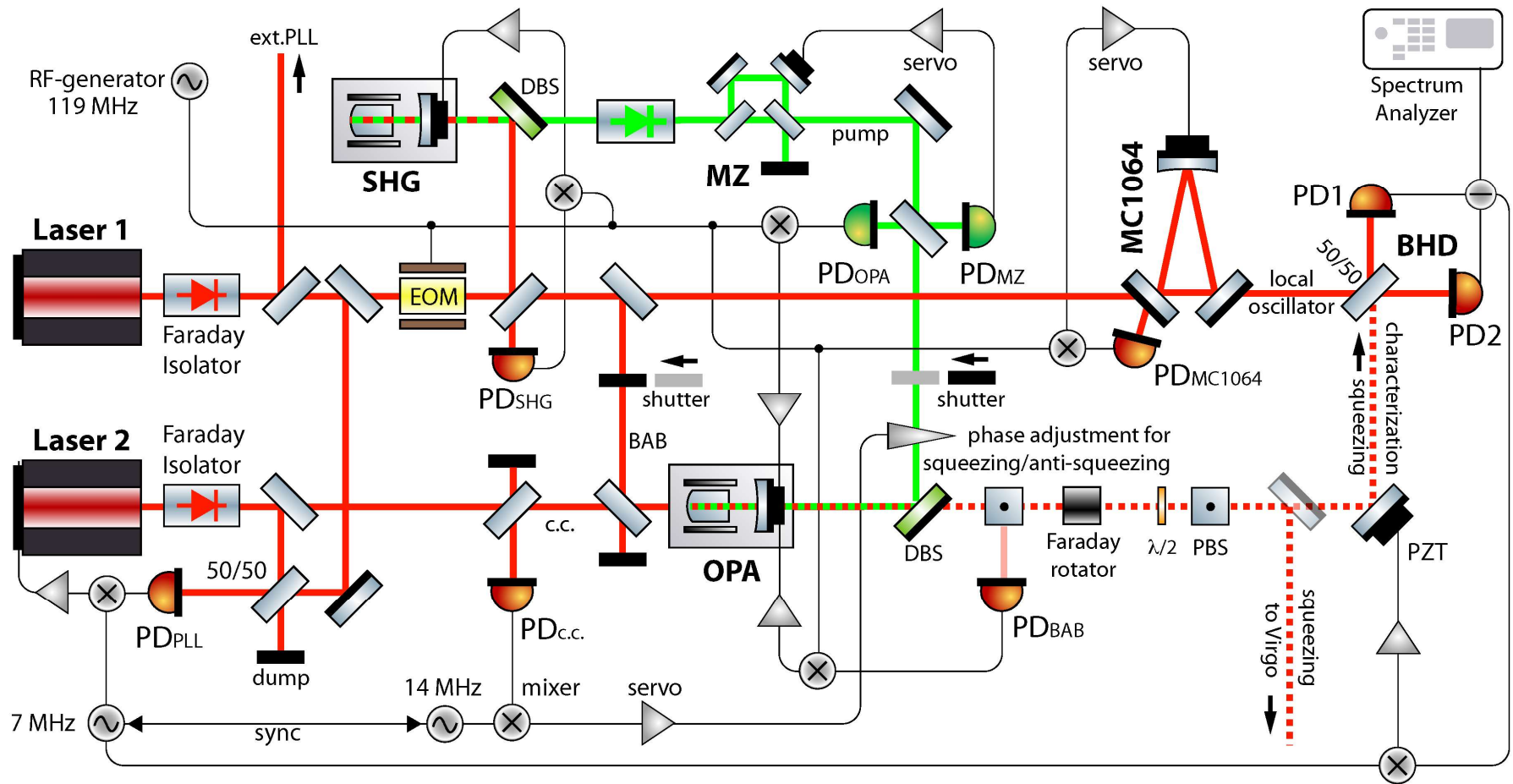
Figure 1.6: Simulation of electric field in time for (a) vacuum state and for (b) squeezed vacuum.

Gravitational wave detector

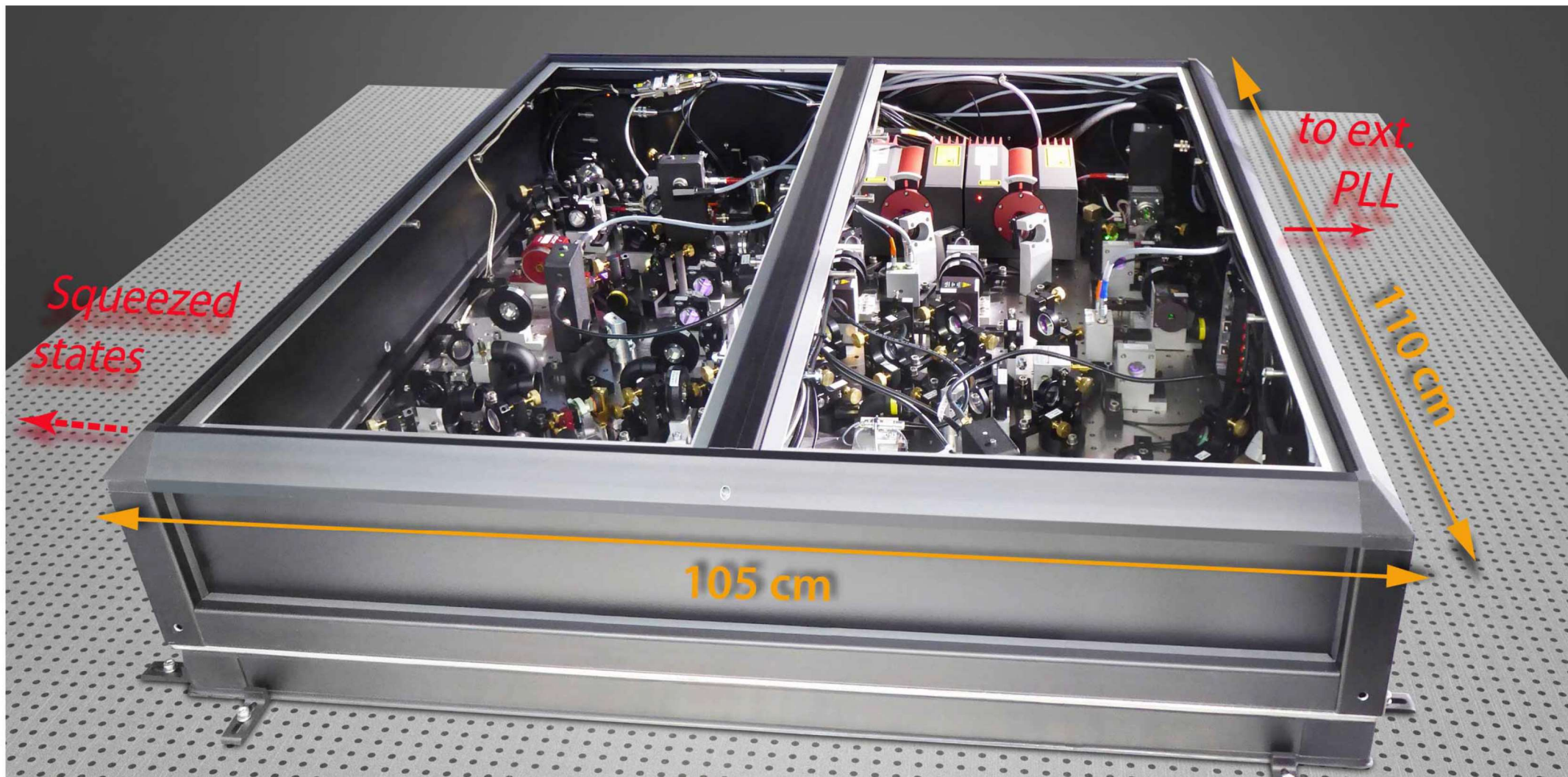


Livi
Arti









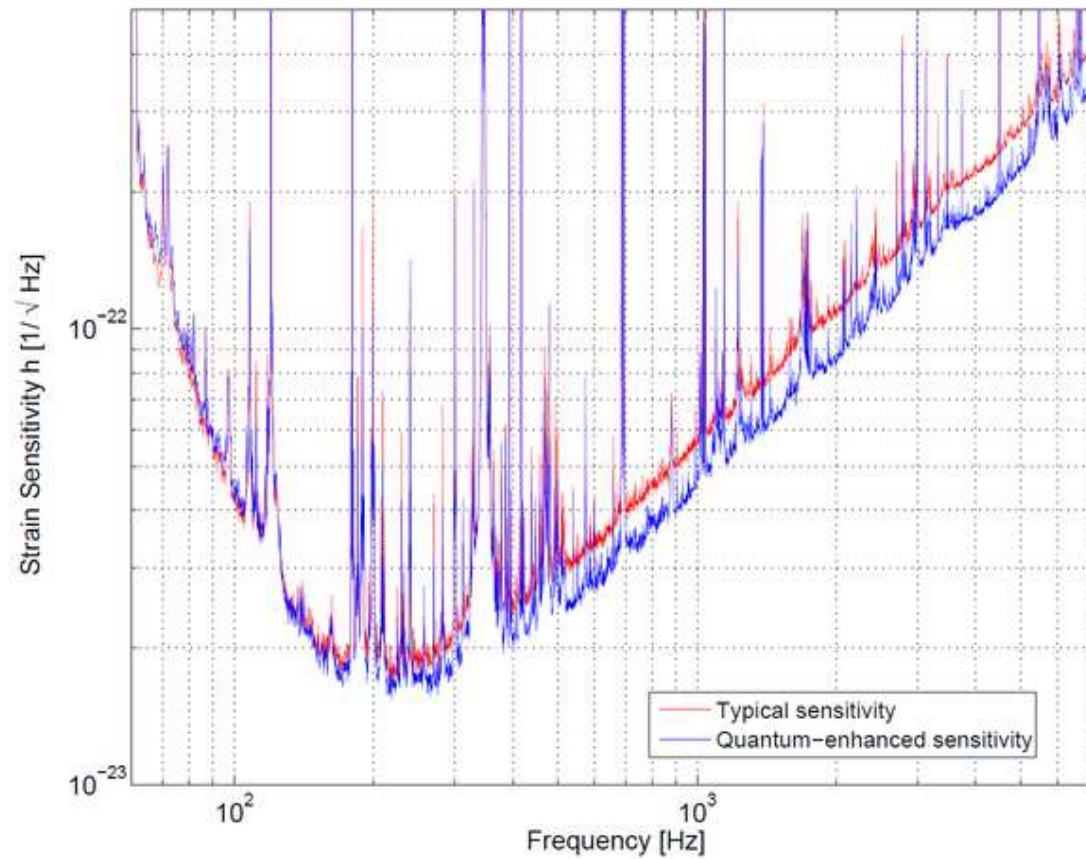
Squeezed states

105 cm

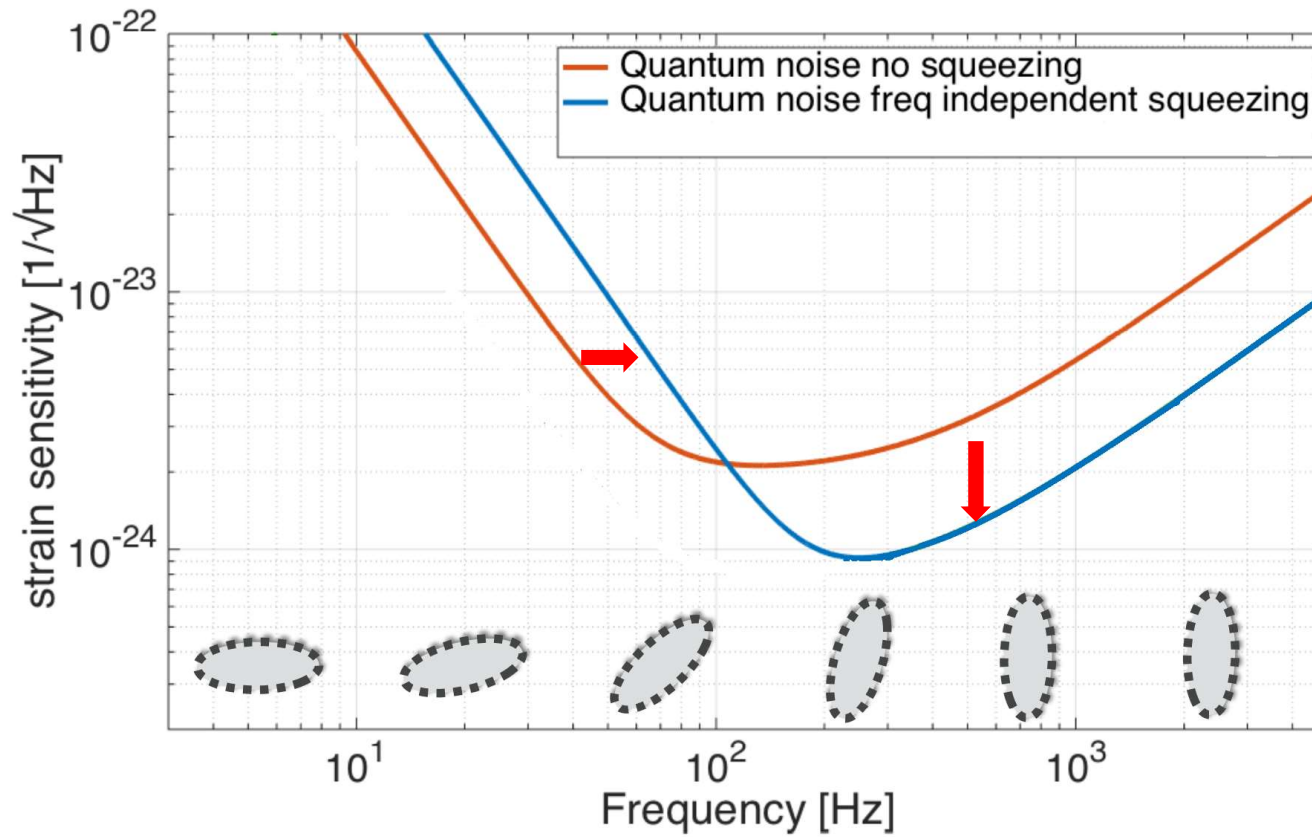
to ext. PLL

110 cm

Quantum noise enhancement of LIGO

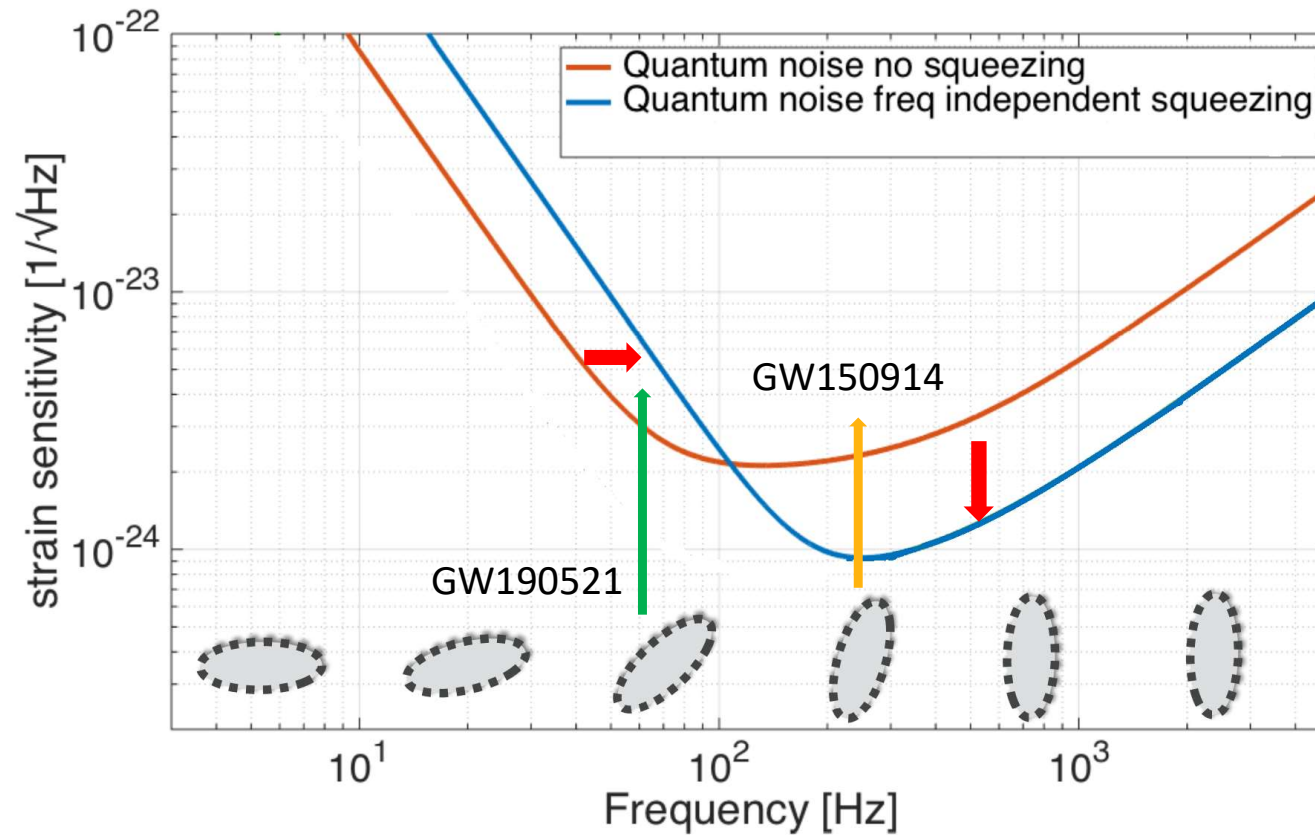


Frequency independent squeezing



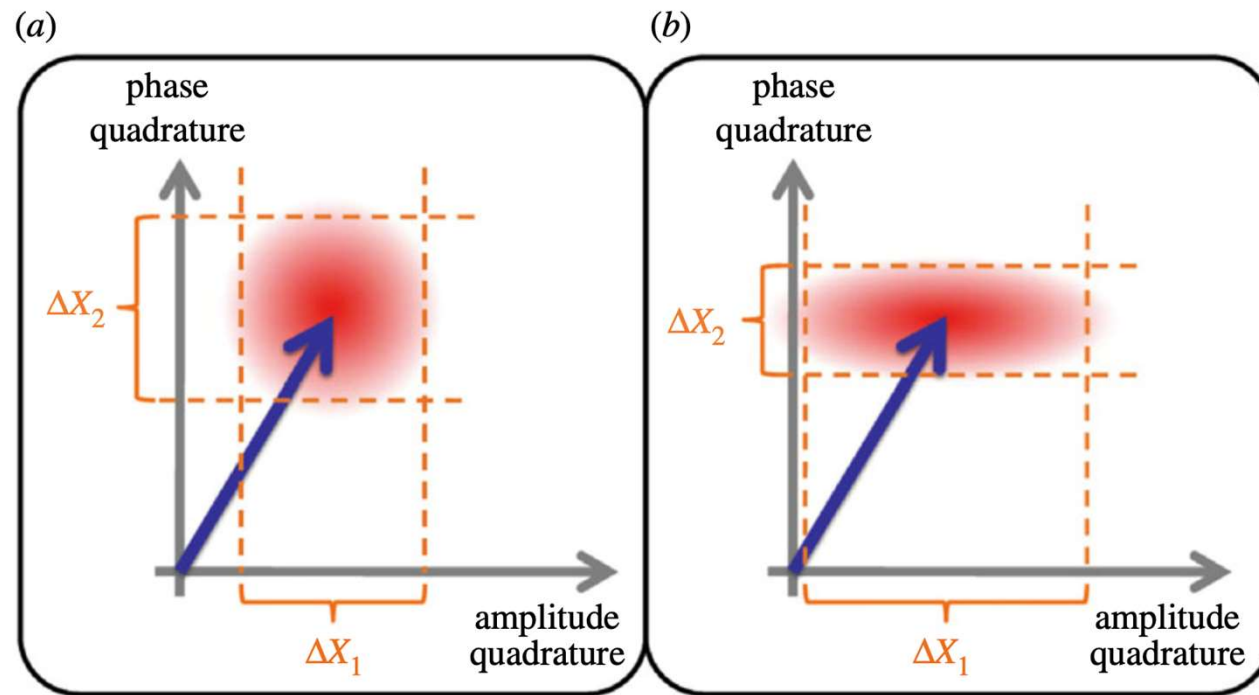
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Frequency independent squeezing



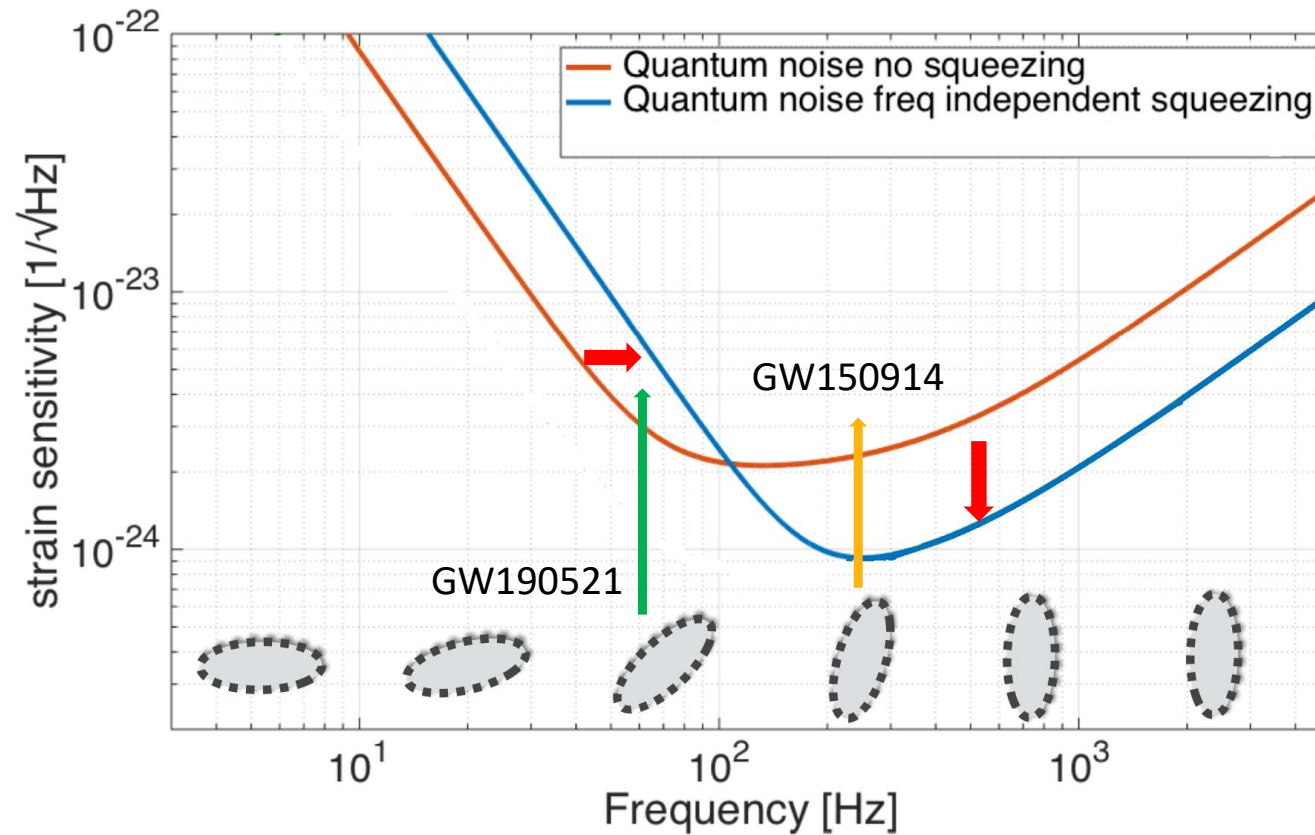
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Squeezed light



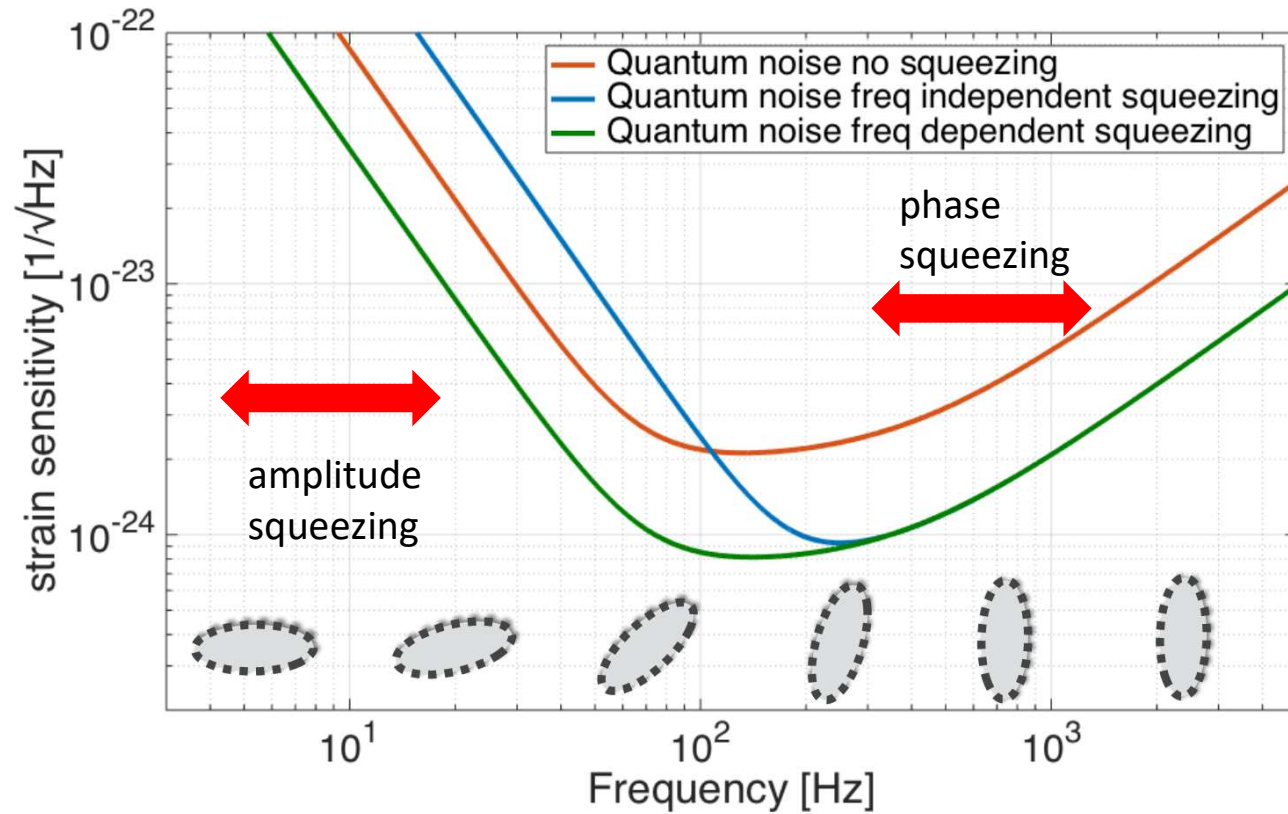
Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. *Phil. Trans. R. Soc. A* 376: 20170289.

Frequency independent squeezing



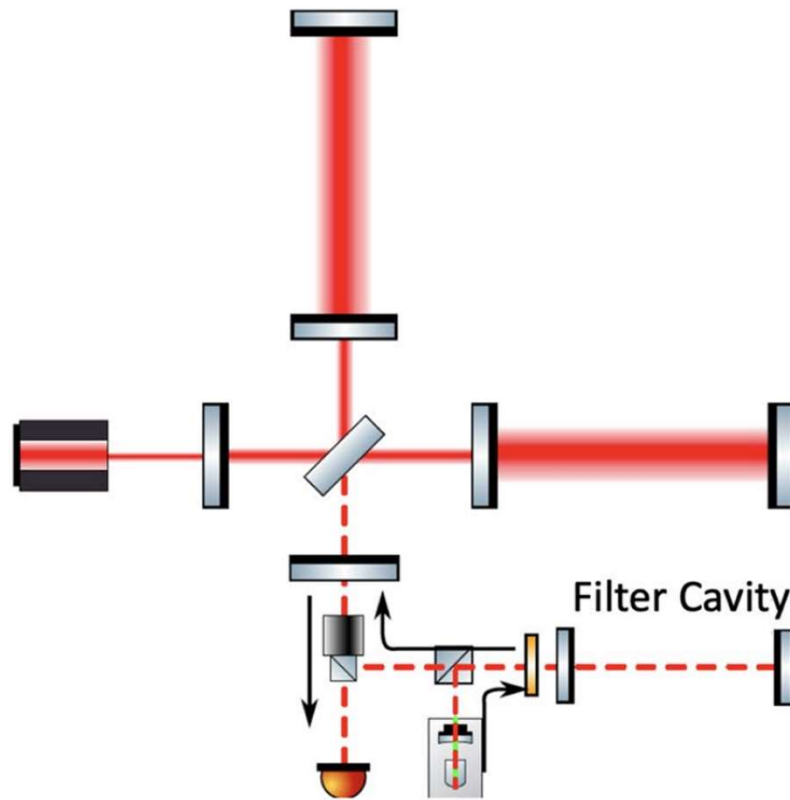
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Frequency dependent squeezing(FDS)

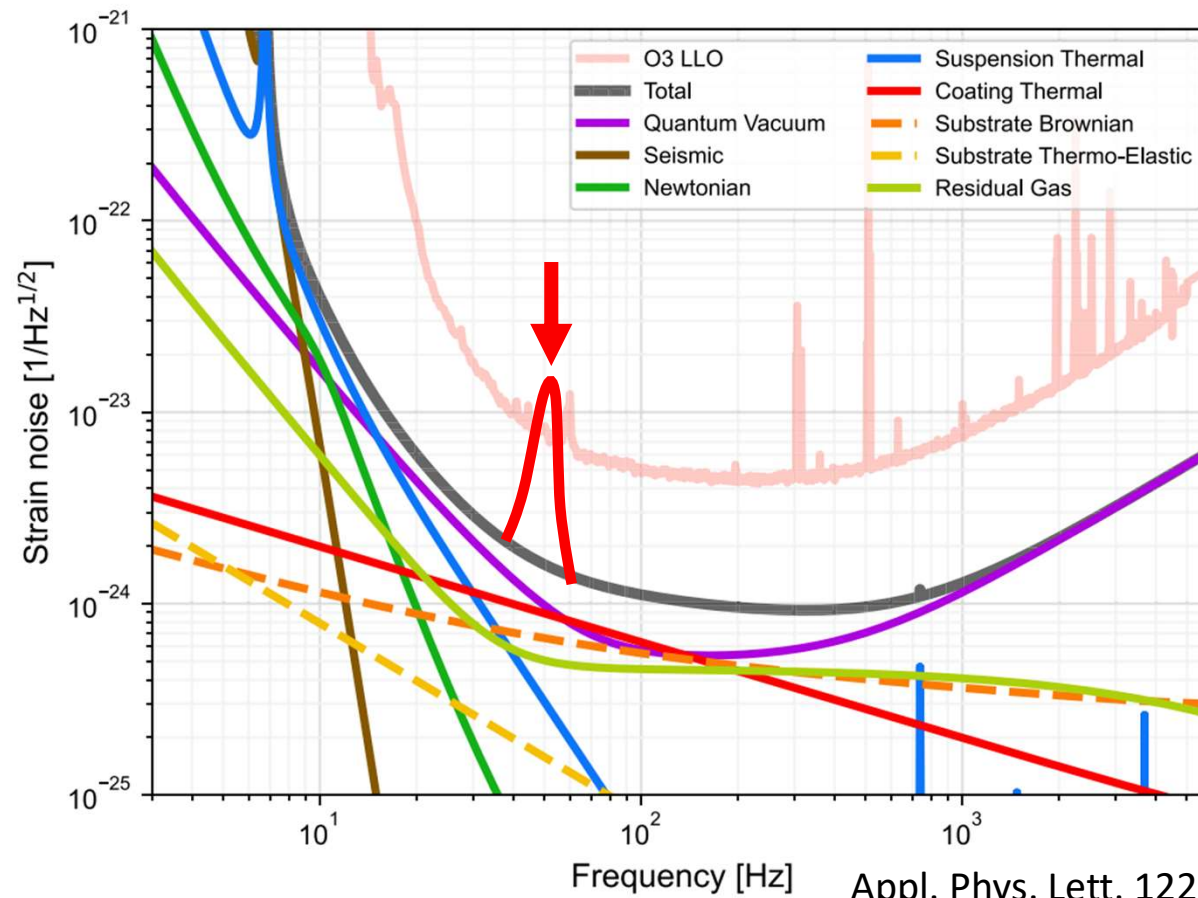


Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

Frequency dependent squeezing using filter cavity

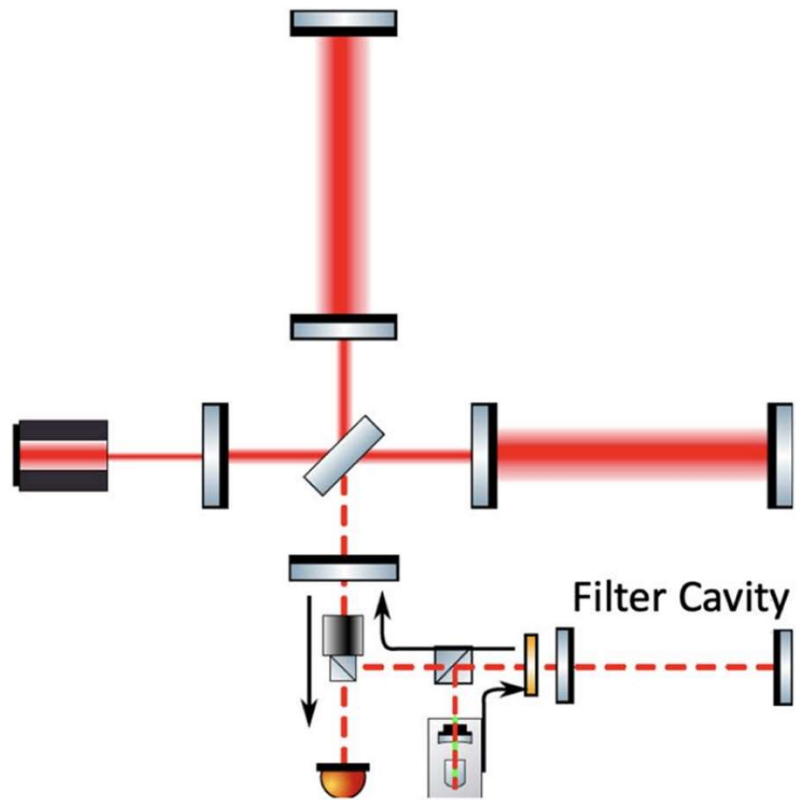


LIGO sensitivity



Appl. Phys. Lett. 122, 110502 (2023)

Frequency dependent squeezing using filter cavity



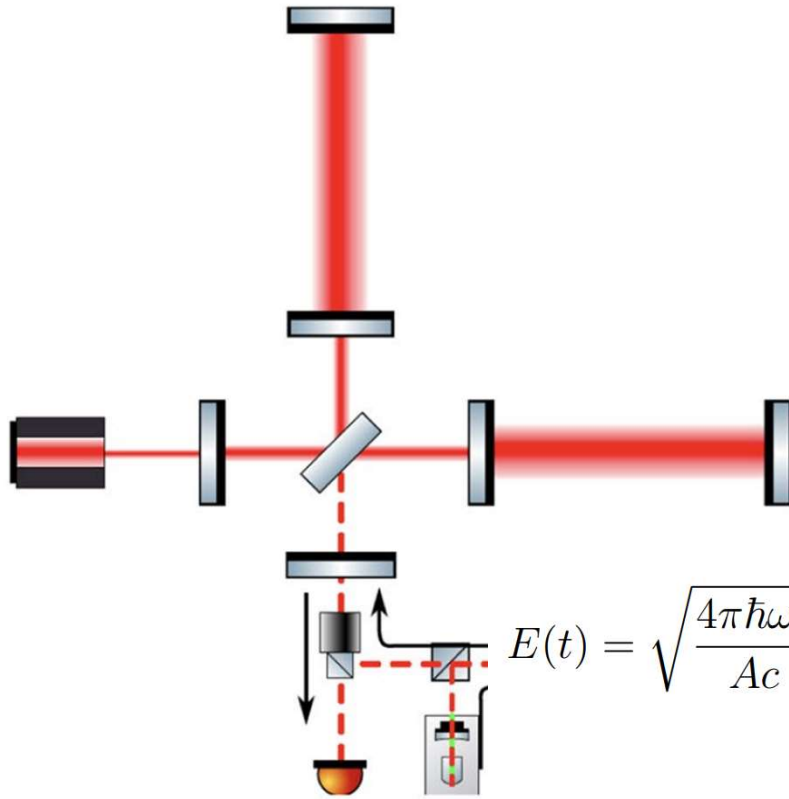
$$E^{(+)}(t) = \int_0^{\infty} E(\omega) e^{-i\omega t} \frac{d\omega}{2\pi}$$

$$E^{(+)}(t) = e^{-i\omega_0 t} \int_0^{\infty} (E(\Omega) e^{-i\Omega t} + E(-\Omega) e^{+i\Omega t}) \frac{d\Omega}{2\pi}$$

$$\omega = \omega_0 + \Omega$$

Laser frequency GW signal frequency

Frequency dependent squeezing using filter cavity



$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} [a_1(t) \cos(\omega_0 t) + a_2(t) \sin(\omega_0 t)]$$

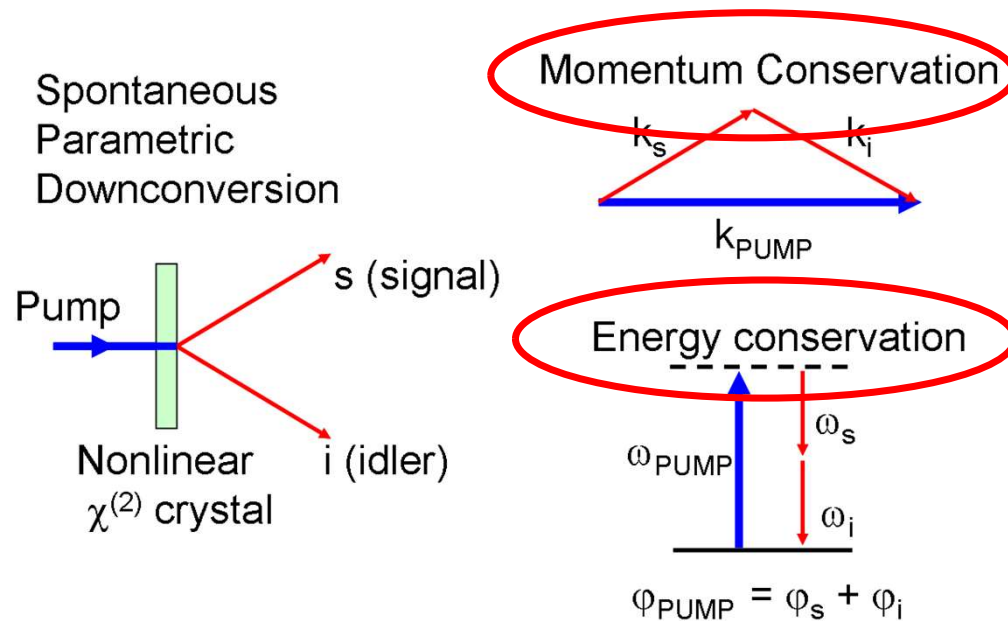
$$\text{Amplitude Quadrature : } a_1(t) = \frac{a(t) + a^\dagger(t)}{\sqrt{2}}$$

$$\text{Phase Quadrature : } a_2(t) = \frac{a(t) - a^\dagger(t)}{i\sqrt{2}}$$

$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} \left[\cos(\omega_0 t) \int_0^\infty (a_1(\Omega)e^{-i\Omega t} + a_1^\dagger(\Omega)e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right.$$

$$\left. + \sin(\omega_0 t) \int_0^\infty (a_2(\Omega)e^{-i\Omega t} + a_2^\dagger(\Omega)e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right]$$

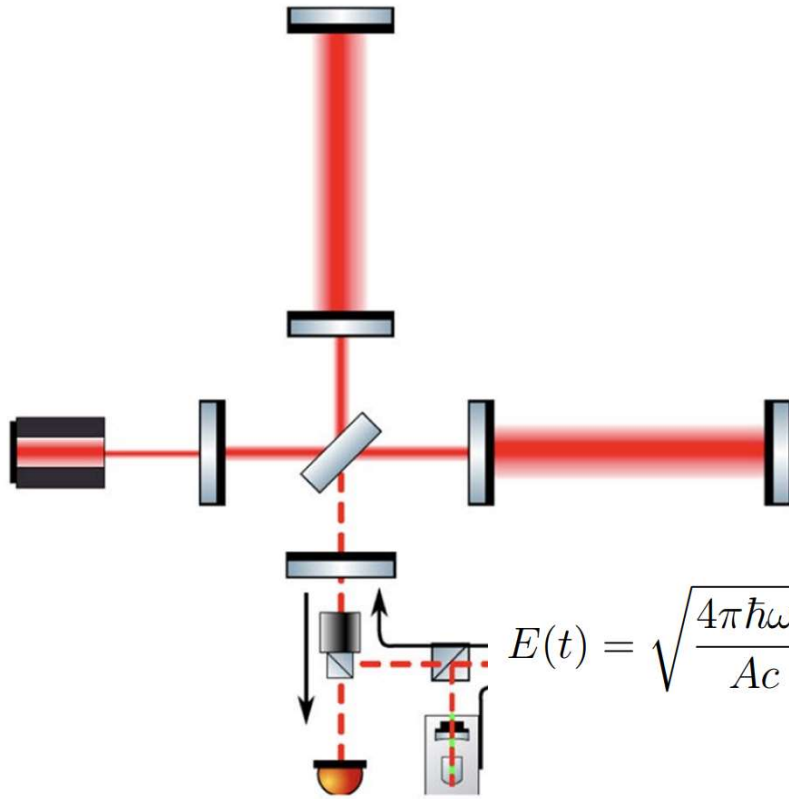
Parametric down conversion



Every field which are participated in Non-linear conversion process, Must obey energy, momentum conservation.

[J S Lundeen](#)

Frequency dependent squeezing using filter cavity



$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} [a_1(t) \cos(\omega_0 t) + a_2(t) \sin(\omega_0 t)]$$

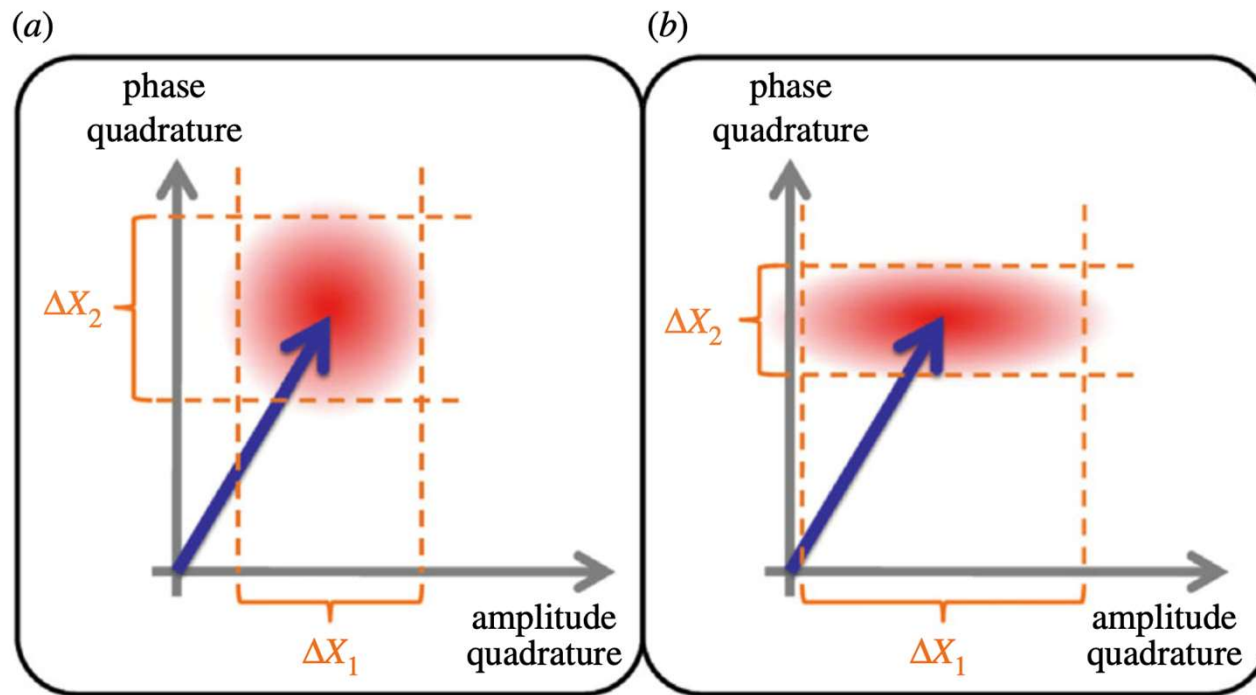
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$$E(t) = \sqrt{\frac{4\pi\hbar\omega_0}{Ac}} \left[\cos(\omega_0 t) \int_0^\infty (a_1(\Omega)e^{-i\Omega t} + a_1^\dagger(\Omega)e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right.$$

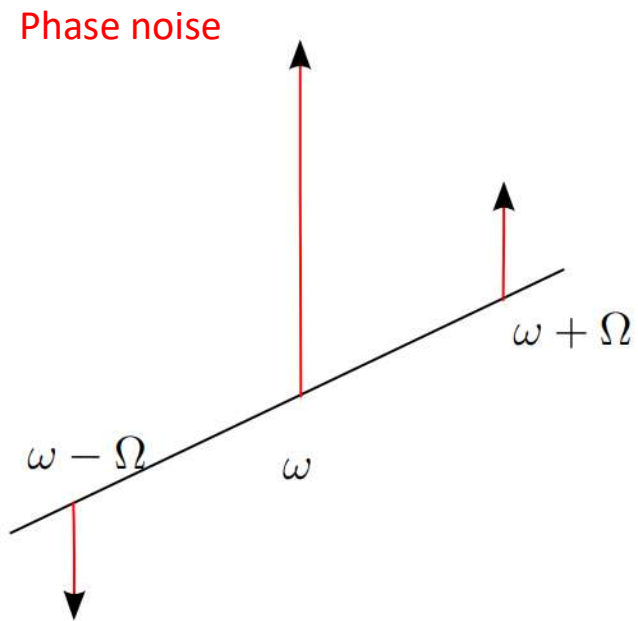
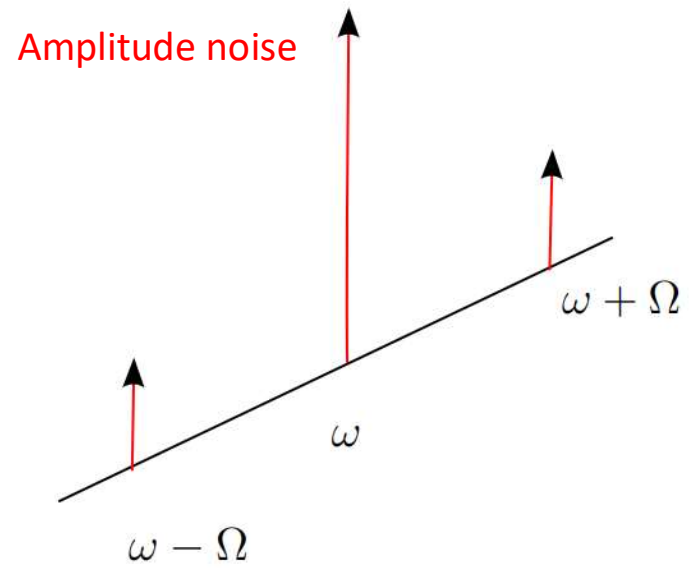
$$\left. + \sin(\omega_0 t) \int_0^\infty (a_2(\Omega)e^{-i\Omega t} + a_2^\dagger(\Omega)e^{+i\Omega t}) \frac{d\Omega}{2\pi} \right]$$

Squeezed light

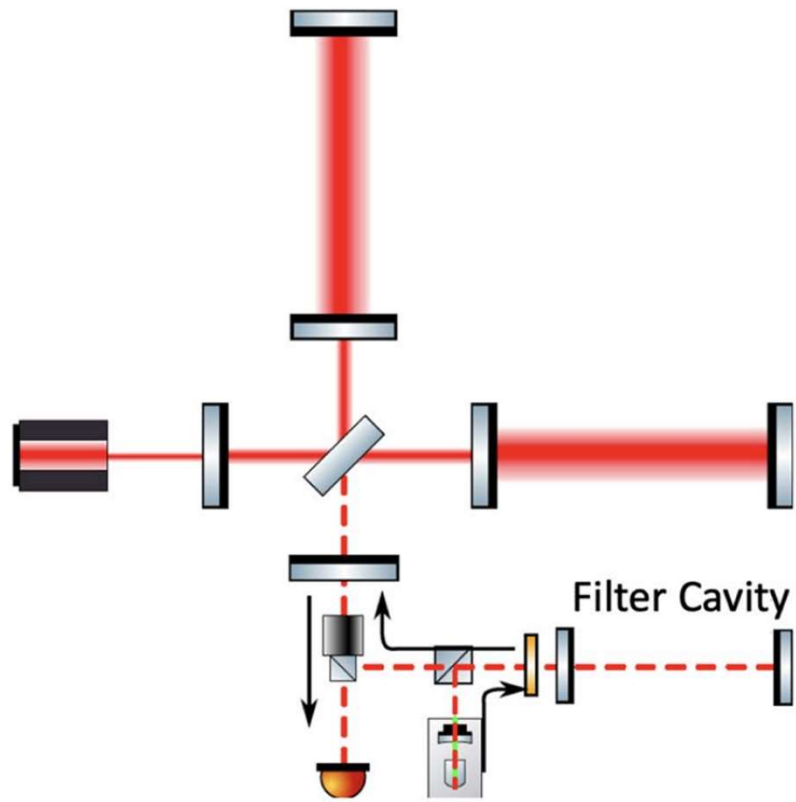


Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

Quantum noise side band figure



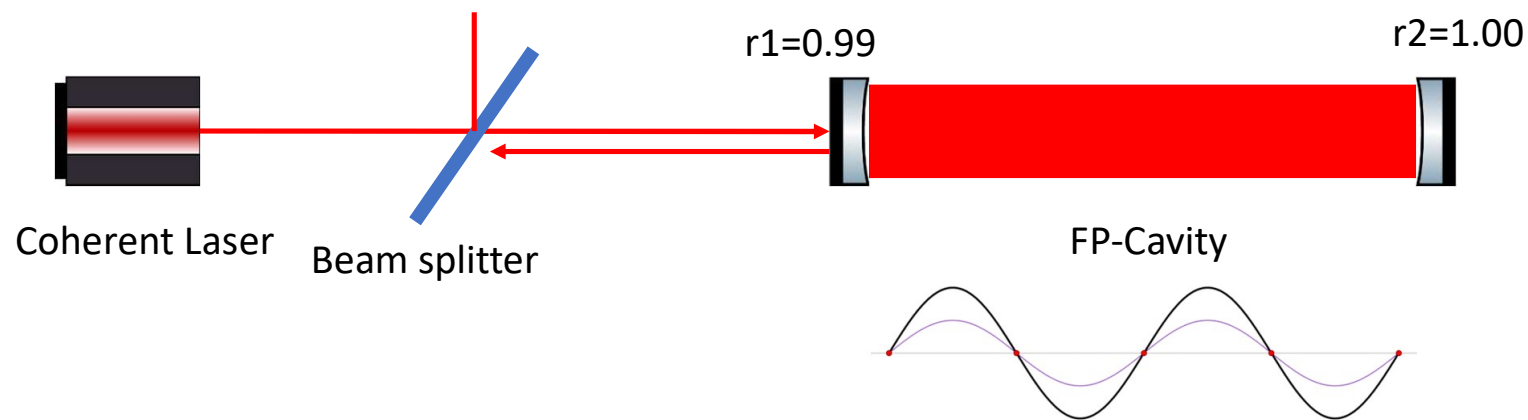
Frequency dependent squeezing using filter cavity



Fabry-perot cavity

Number of round trip > 250

4 km x 250 \sim 1000 km



Detuned cavity



Simple picture

Stefan Hild et al, "Detuned arm cavities", 3rd GEO simulation workshop, Hannover, June 2007

B:

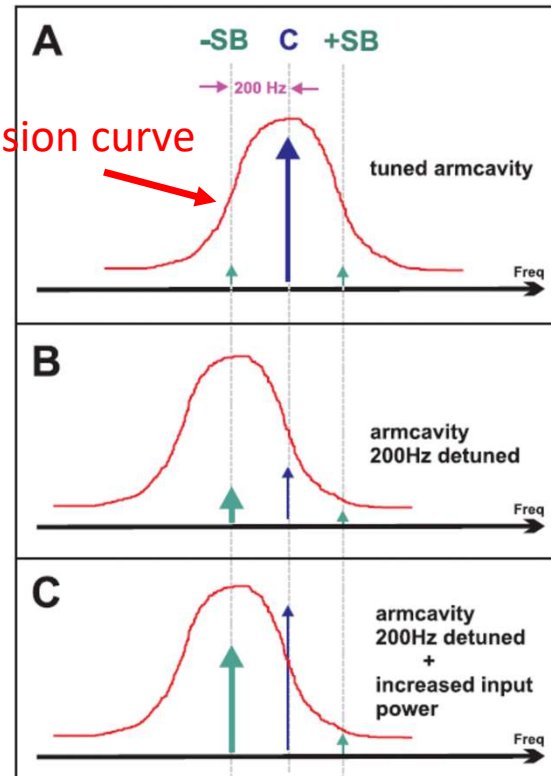
- less carrier light in cavity => less GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced.

=> Smaller GW signal

C:

- optical power is restored in the cavity by larger PR-gain.
 - Same amount of GW sidebands are produced.
 - Since one GW sideband is resonant, it gets enhanced.
- Overall we win GW signal.

=> Larger GW signal



Quantum noise side band figure

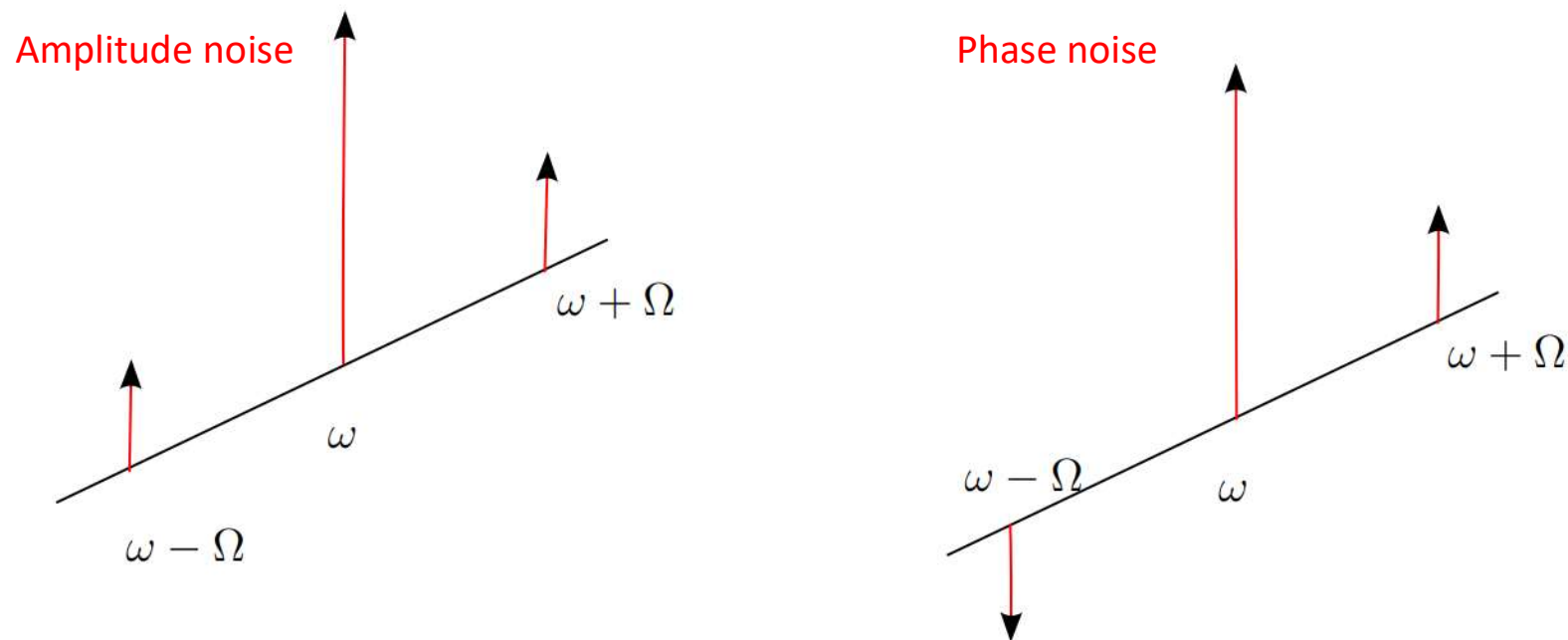
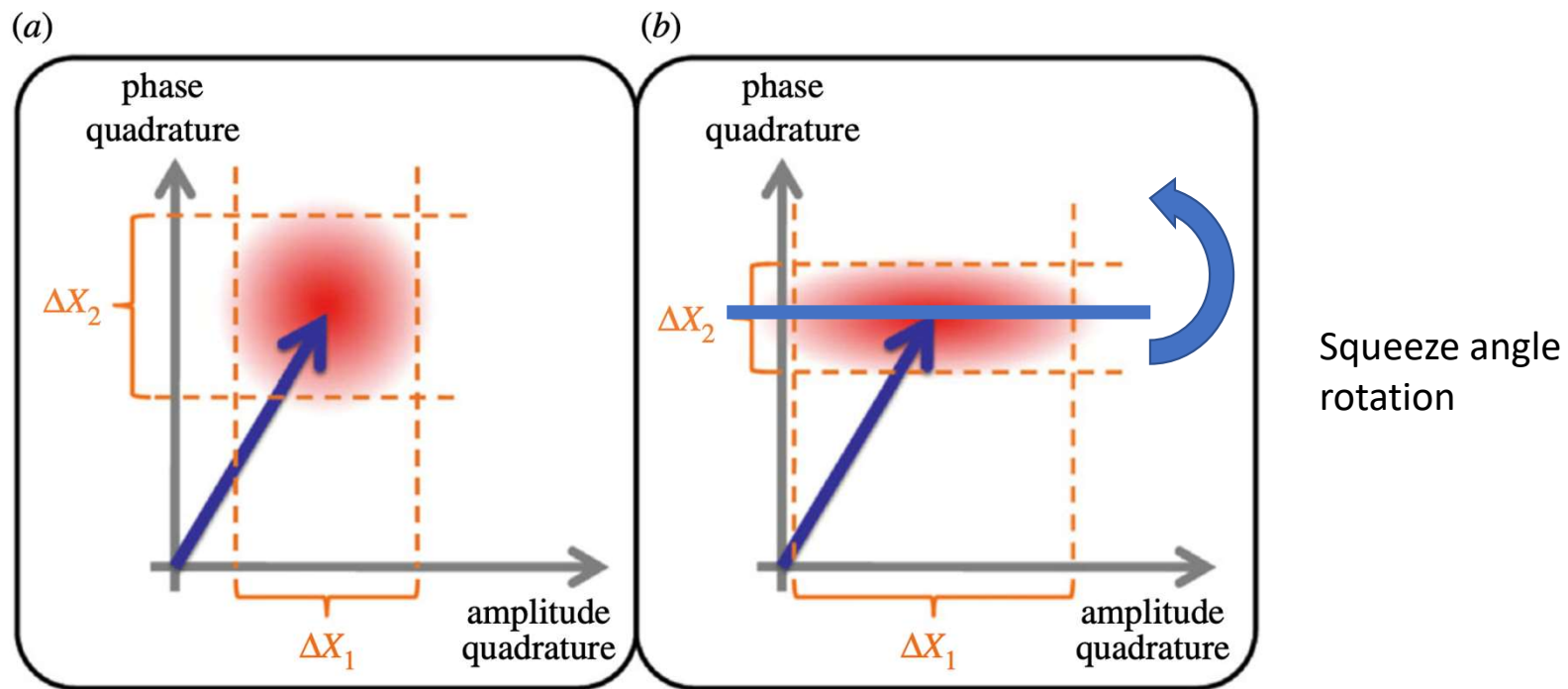


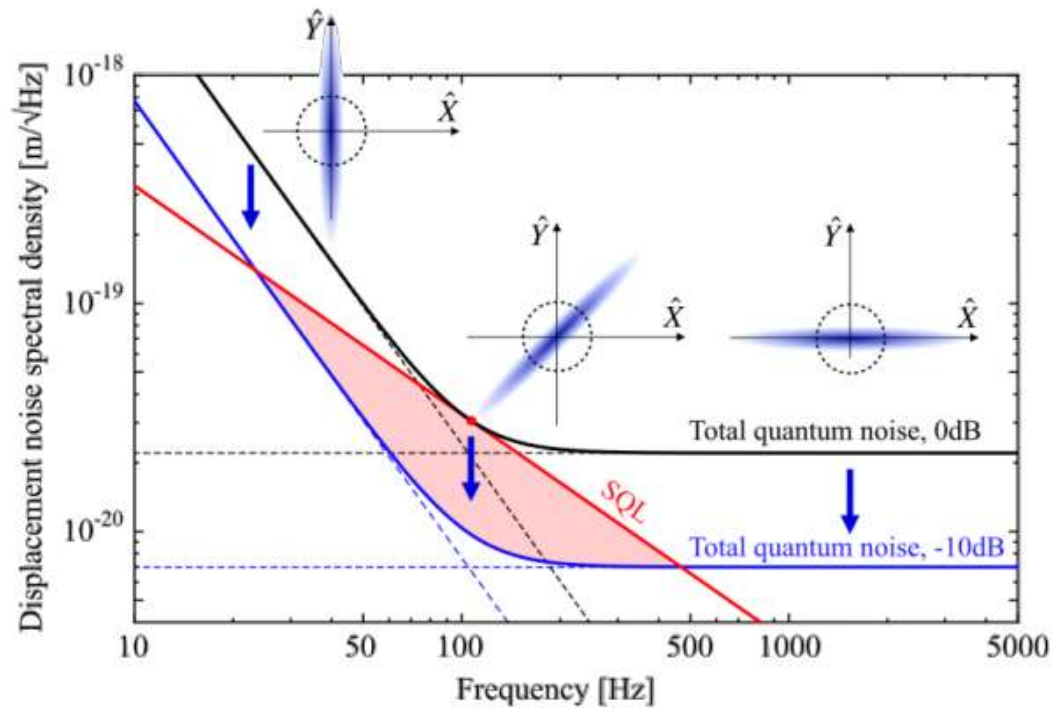
Figure 1-2: Phasors of amplitude noise (left) and phase noise (right) in the sideband picture. In the frame rotating at the carrier frequency ω the carrier is still in these diagrams while the sidebands rotate at Ω , the signal at $\omega + \Omega$ rotating clockwise while the idler at $\omega - \Omega$ rotates counter clockwise. (Sidebands have equal amplitudes)

Squeezed light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

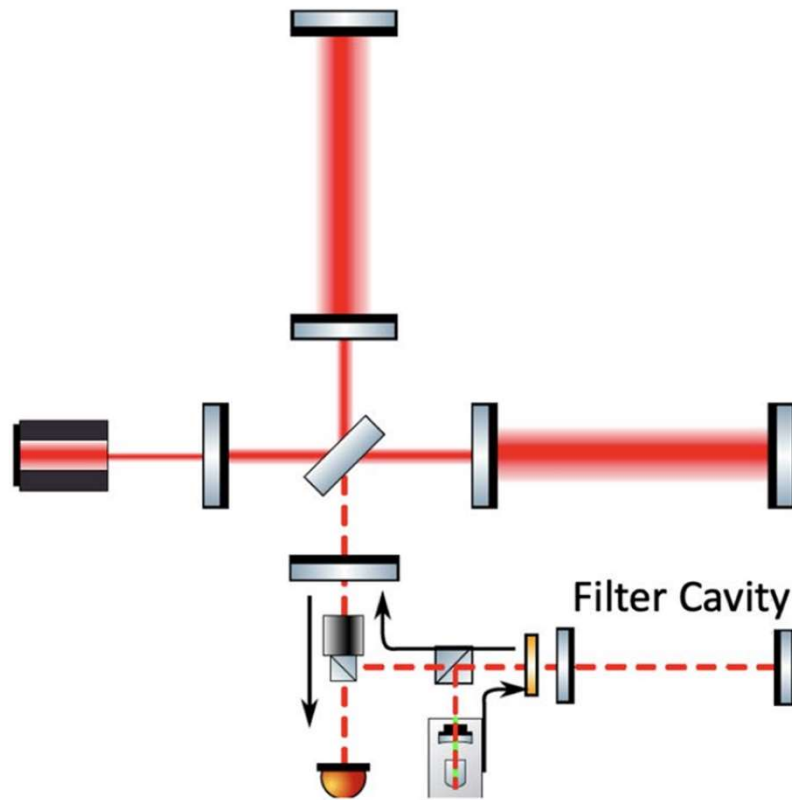
Squeeze angle rotation



$$\alpha_p = \arctan \left(\frac{2\gamma_{fc}\Delta\omega_{fc}}{\gamma_{fc}^2 - \Delta\omega_{fc}^2 + \Omega^2} \right)$$

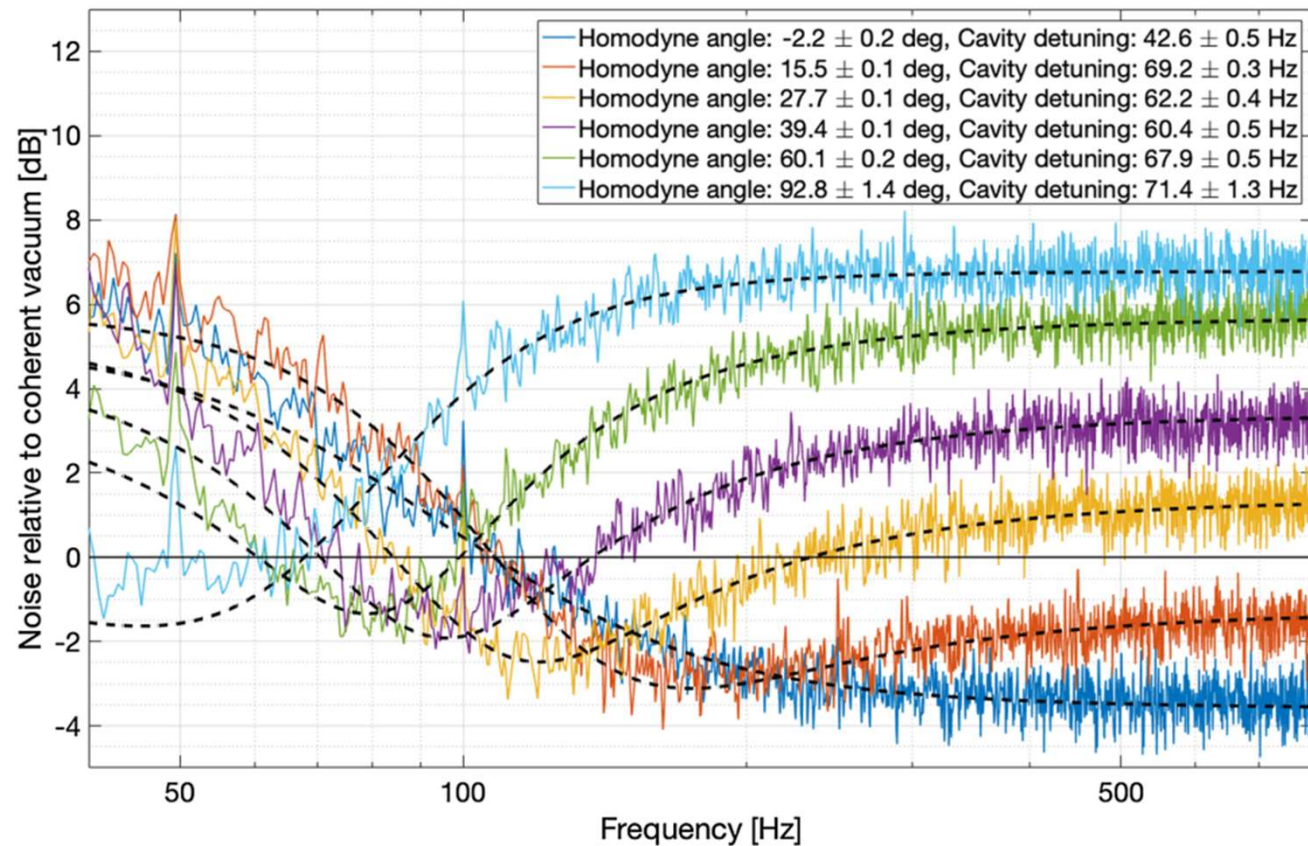
$$t_{\text{st}} = \frac{1}{\gamma_{fc}} = \frac{\sqrt{2}}{\Omega_{\text{SQL}}} \simeq 3 \text{ ms}$$

Frequency dependent squeezing using filter cavity

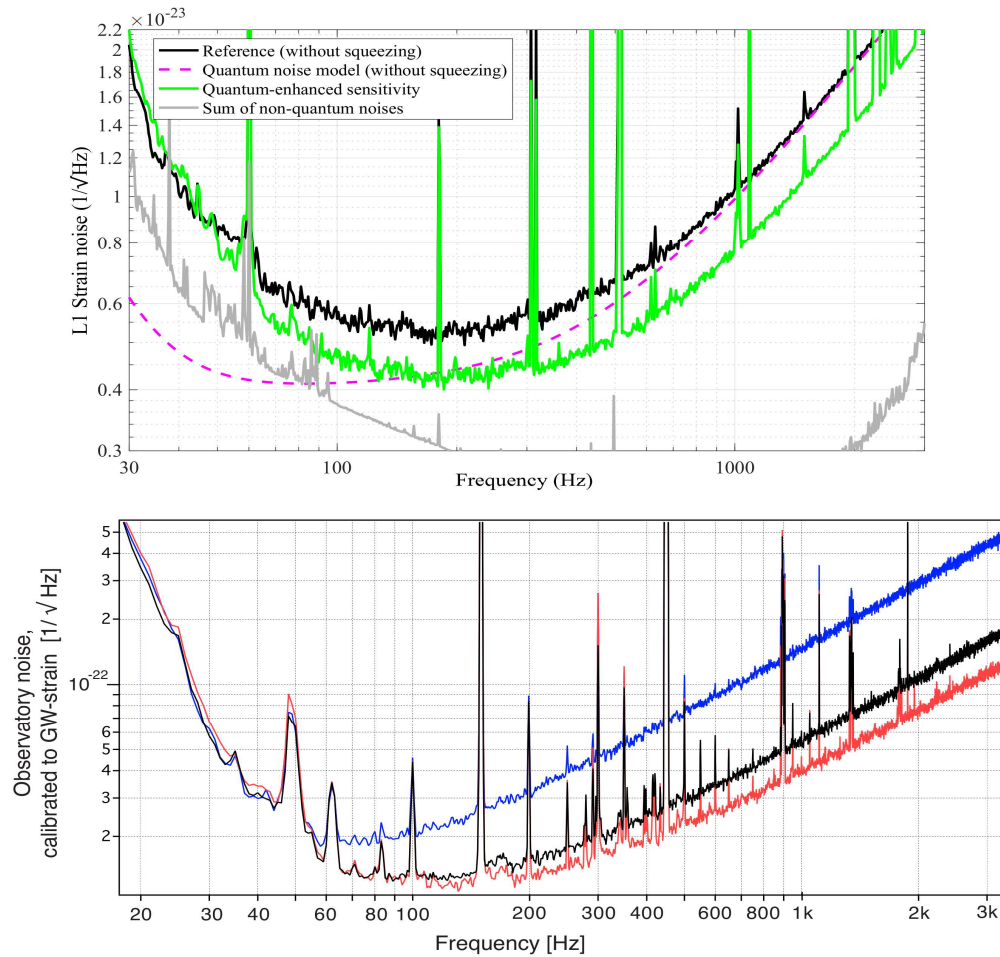


Frequency dependent squeezing – KAGRA

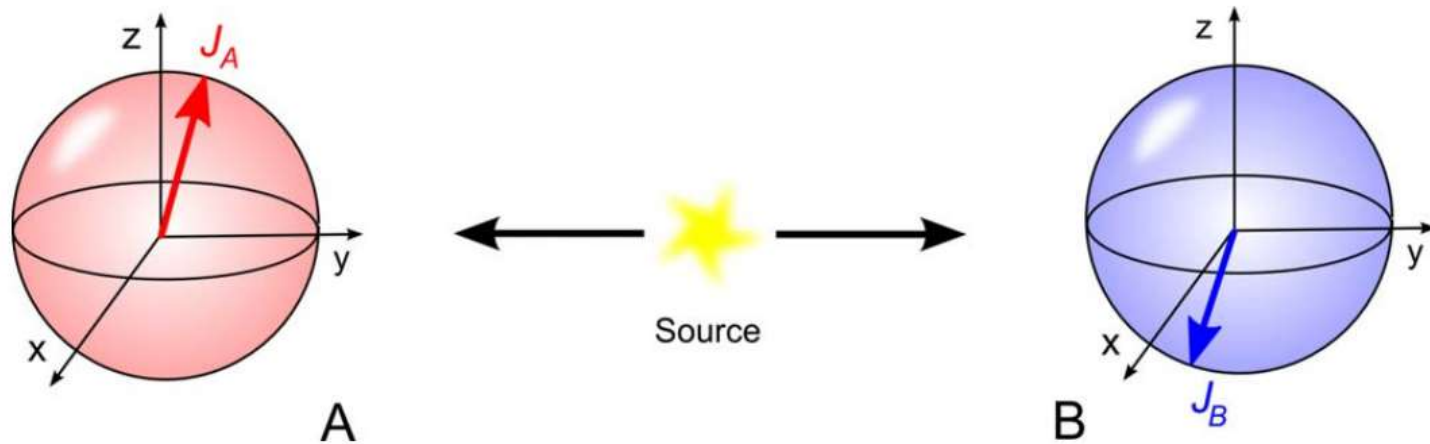
PHYSICAL REVIEW LETTERS **124**, 171101 (2020)



Quantum noise of gravitational wave detector



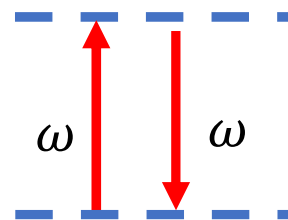
Einstein-Podolsky-Rosen paradox



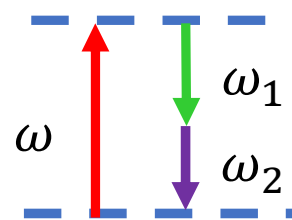
Non linear crystal



Non-linear crystal

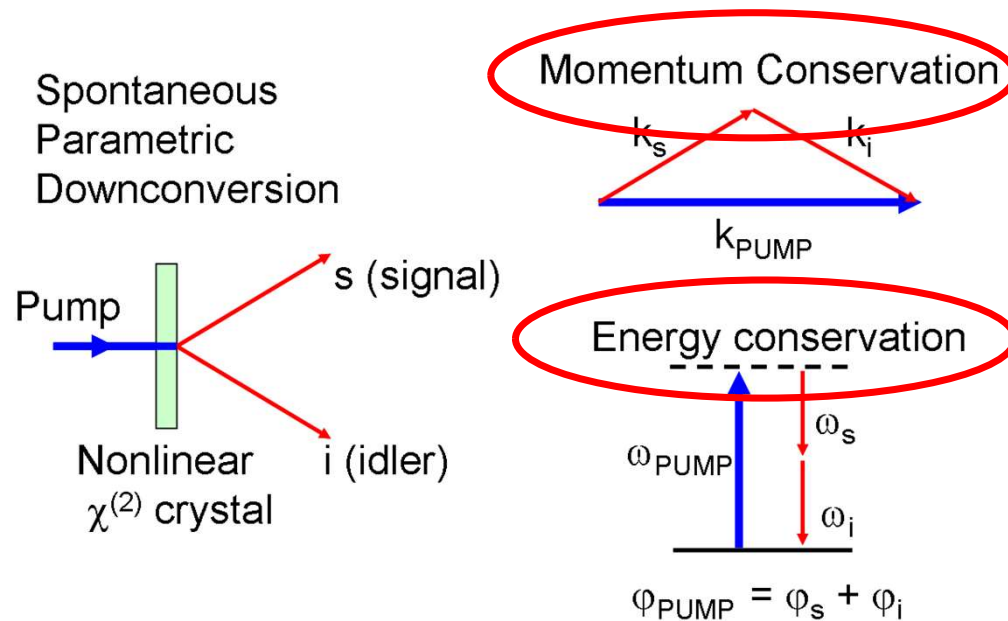


linear optics



Non linear optics

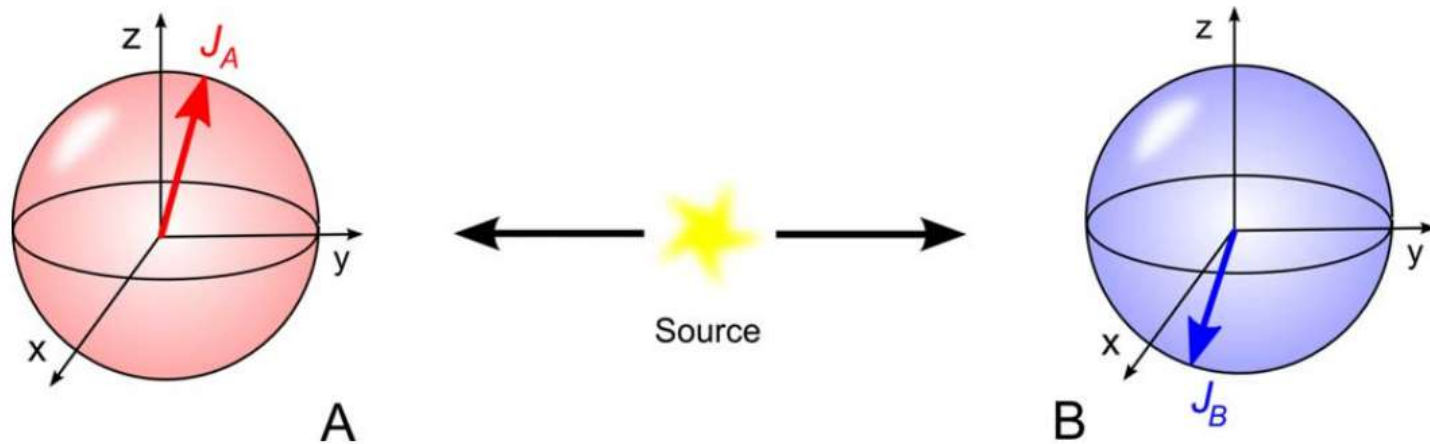
Parametric down conversion



Every field which are participated in Non-linear conversion process, Must obey energy, momentum conservation.

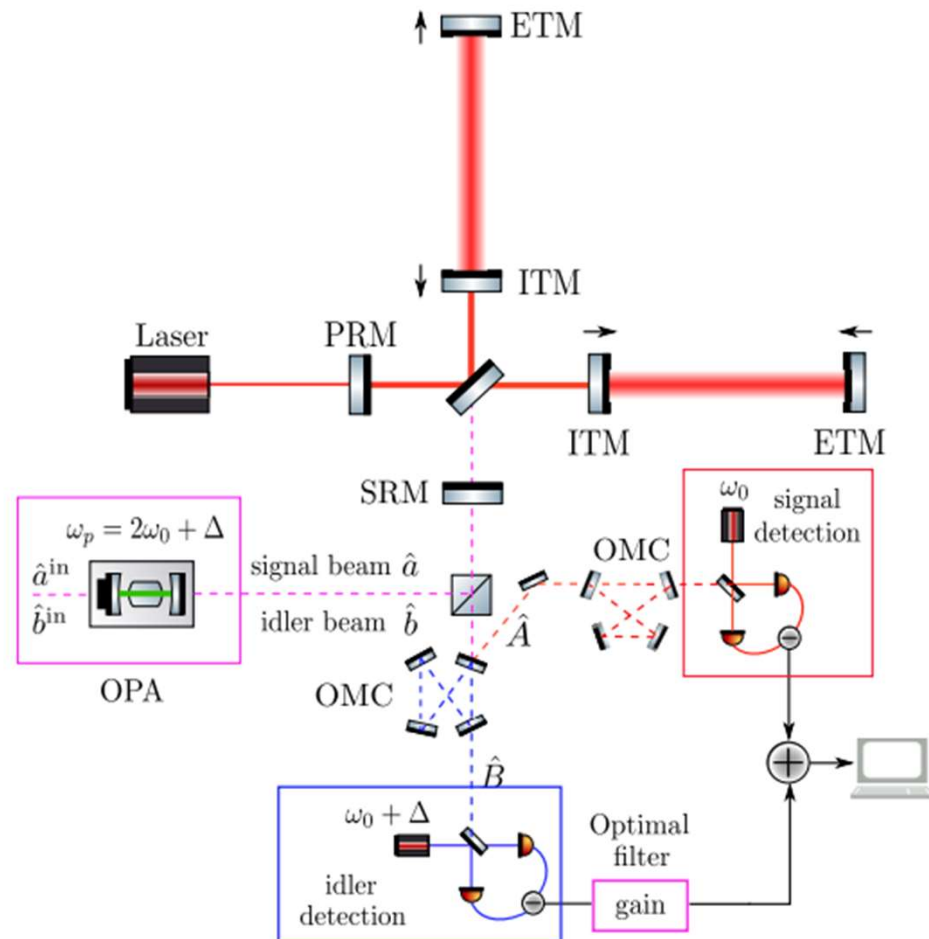
[J S Lundeen](#)

Einstein-Podolsky-Rosen paradox



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DECEMBER 2009

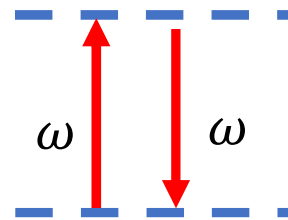
EPR squeezing for gravitational wave detector



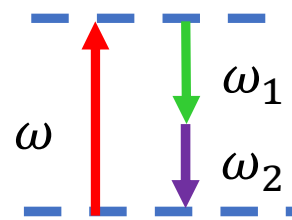
Non linear crystal



Non-linear crystal

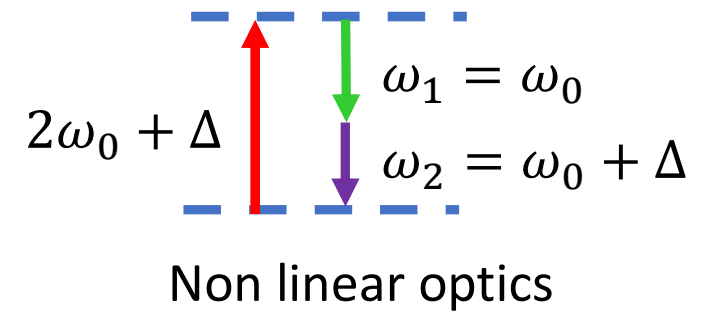
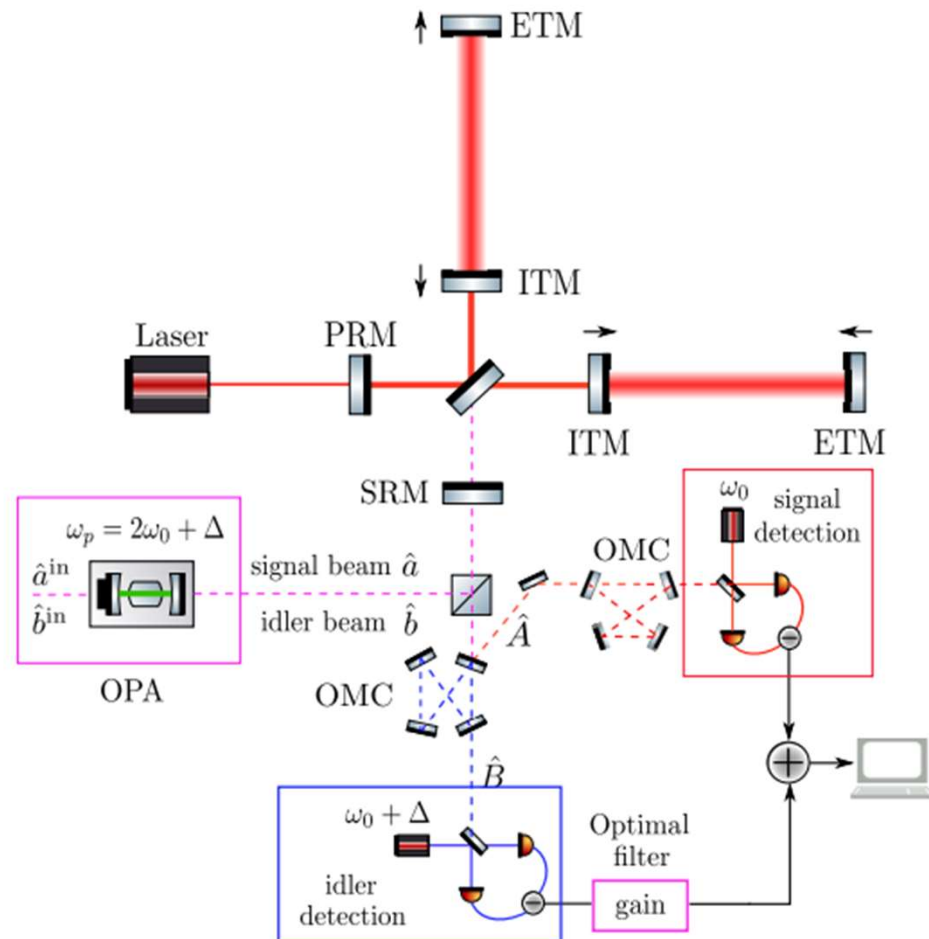


linear optics

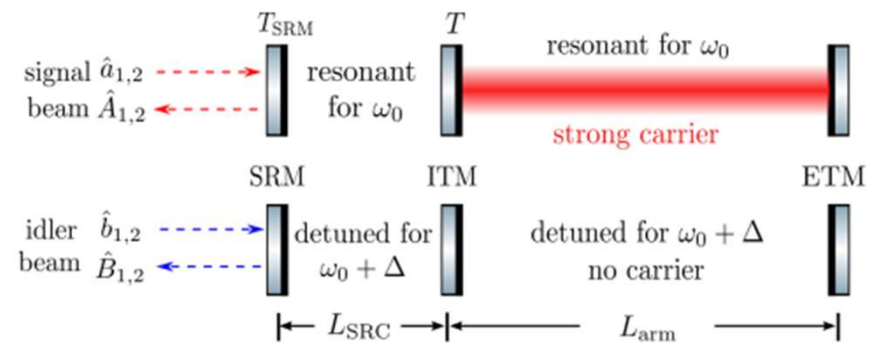
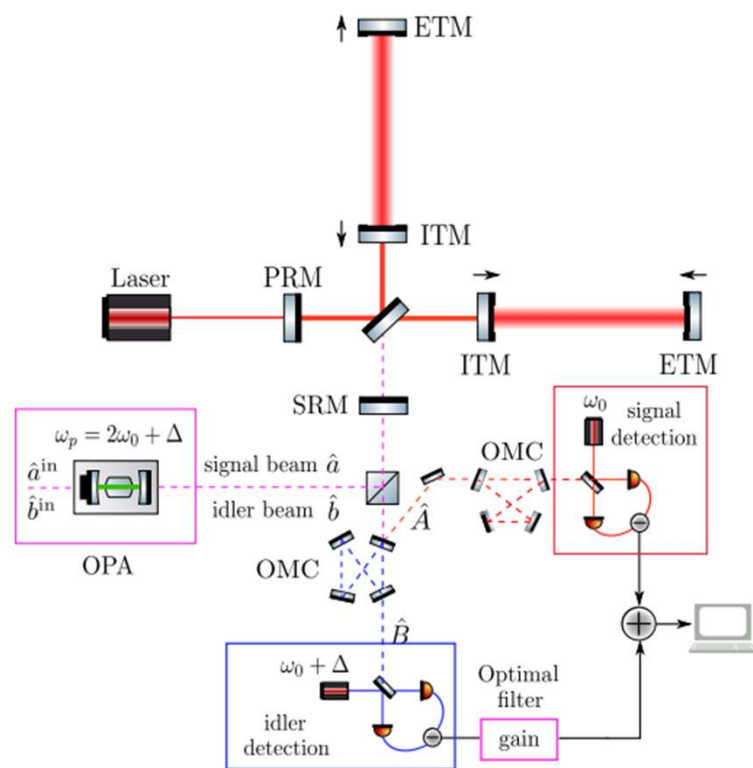


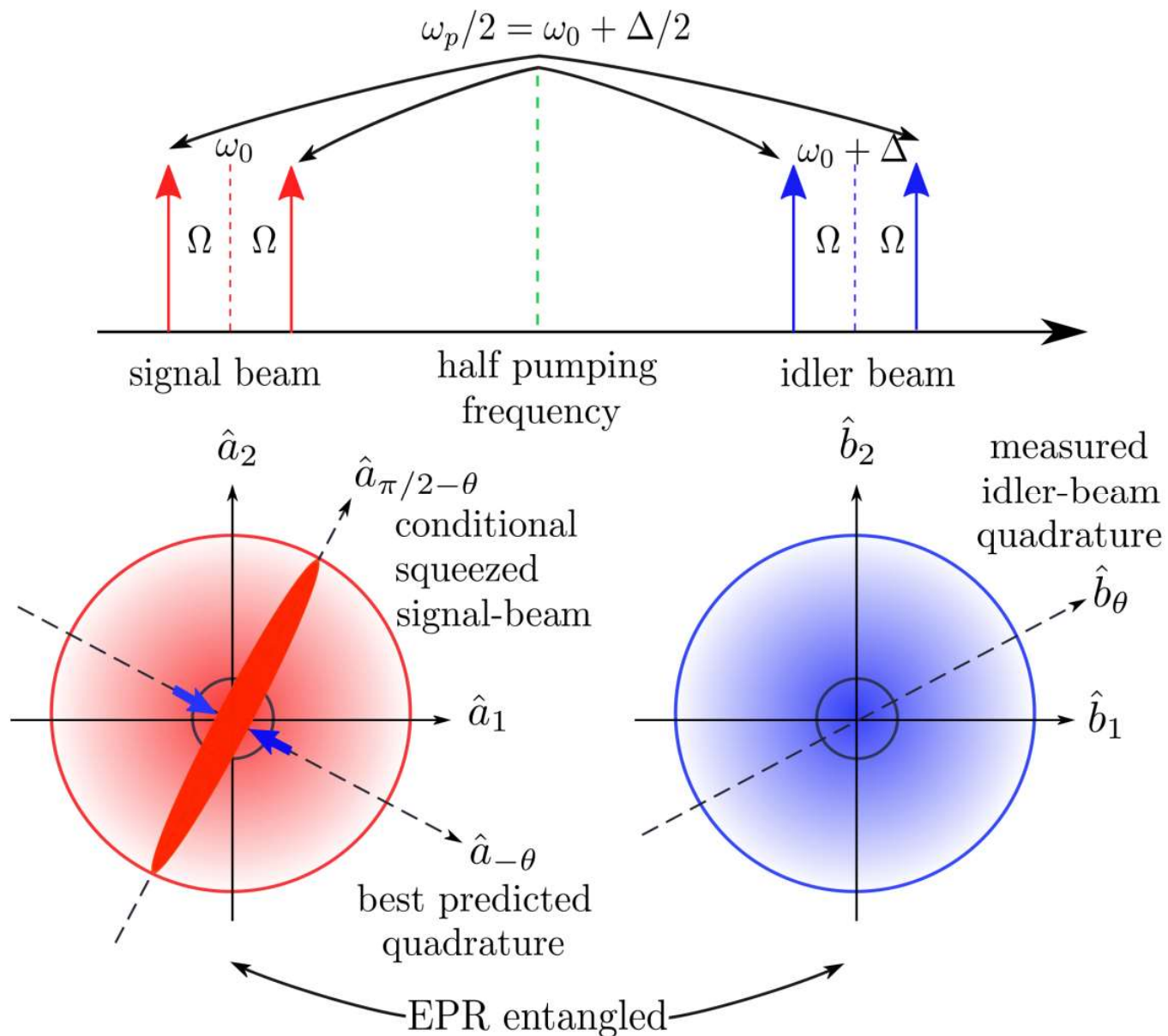
Non linear optics

EPR squeezing for gravitational wave detector

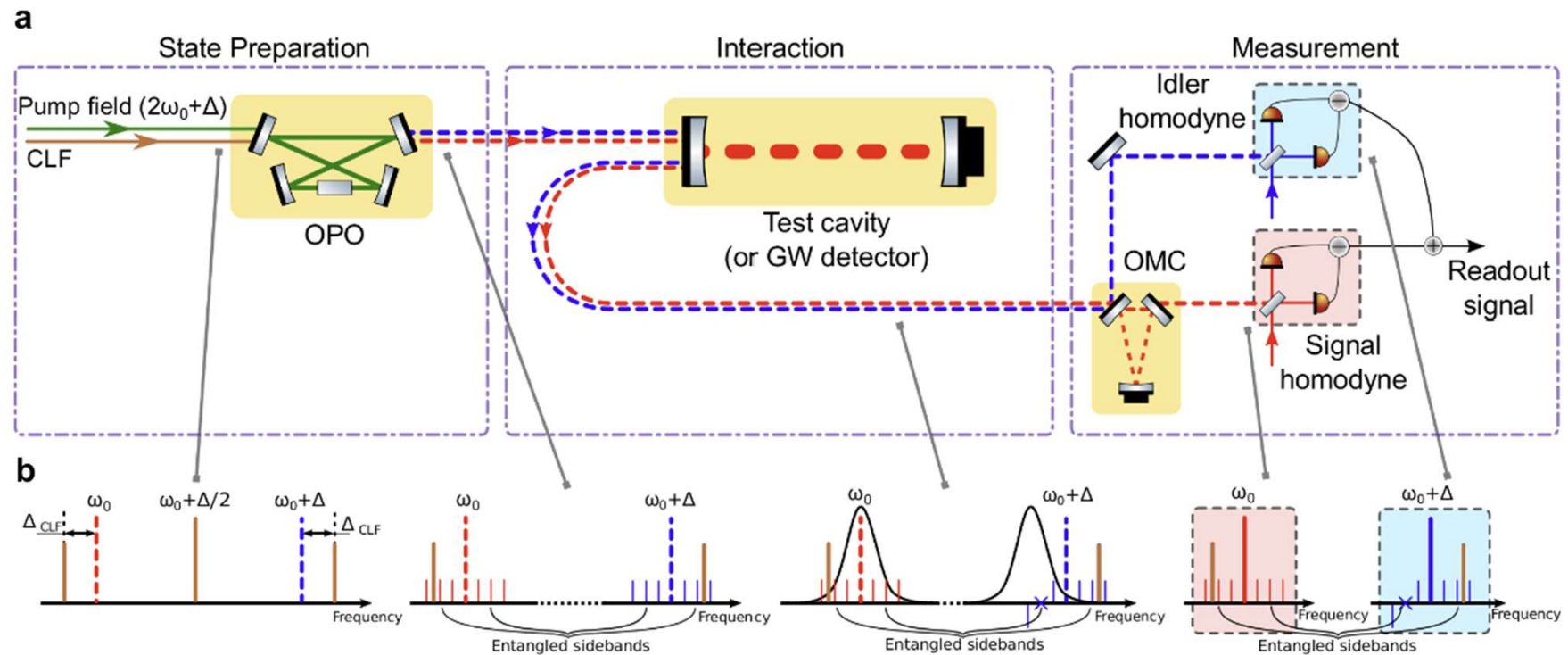


EPR squeezing for gravitational wave detector





Generation and control of frequency dependent squeezing via EPR entanglement





Thank you

