중력파 렌징

Gravitational Lensing of Gravitational Waves

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강의 내용

1. 중력렌징 개요 Gravitational lensing in a nutshell
2. 빛의 중력렌징 Gravitational lensing of lights
3. 중력파의 중력렌징 Gravitational lensing of gravitational waves

1. 중력렌징 개요

Gravitational lensing in a nutshell

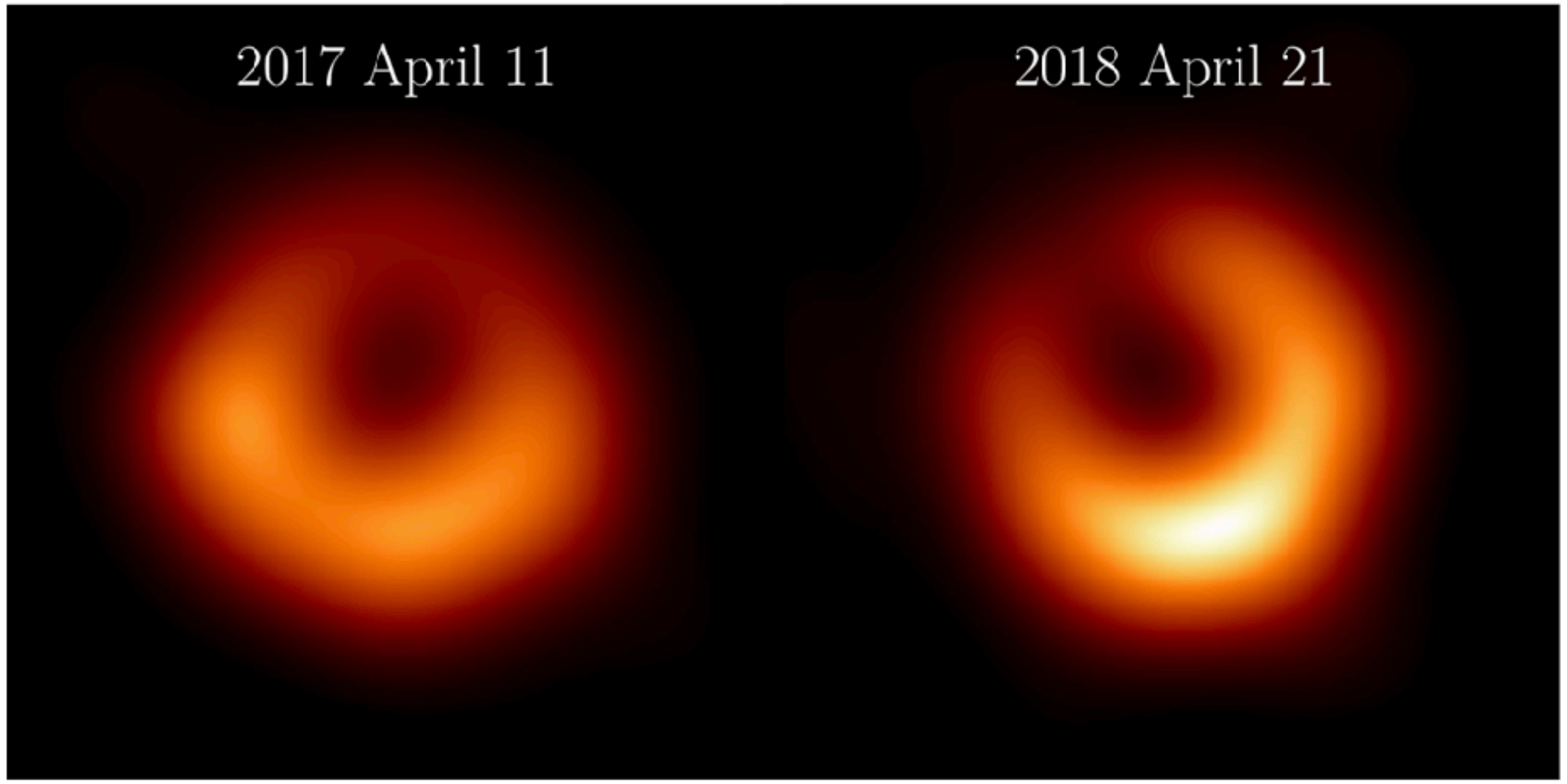
Do you know what this picture is?

Do you know what this picture is?

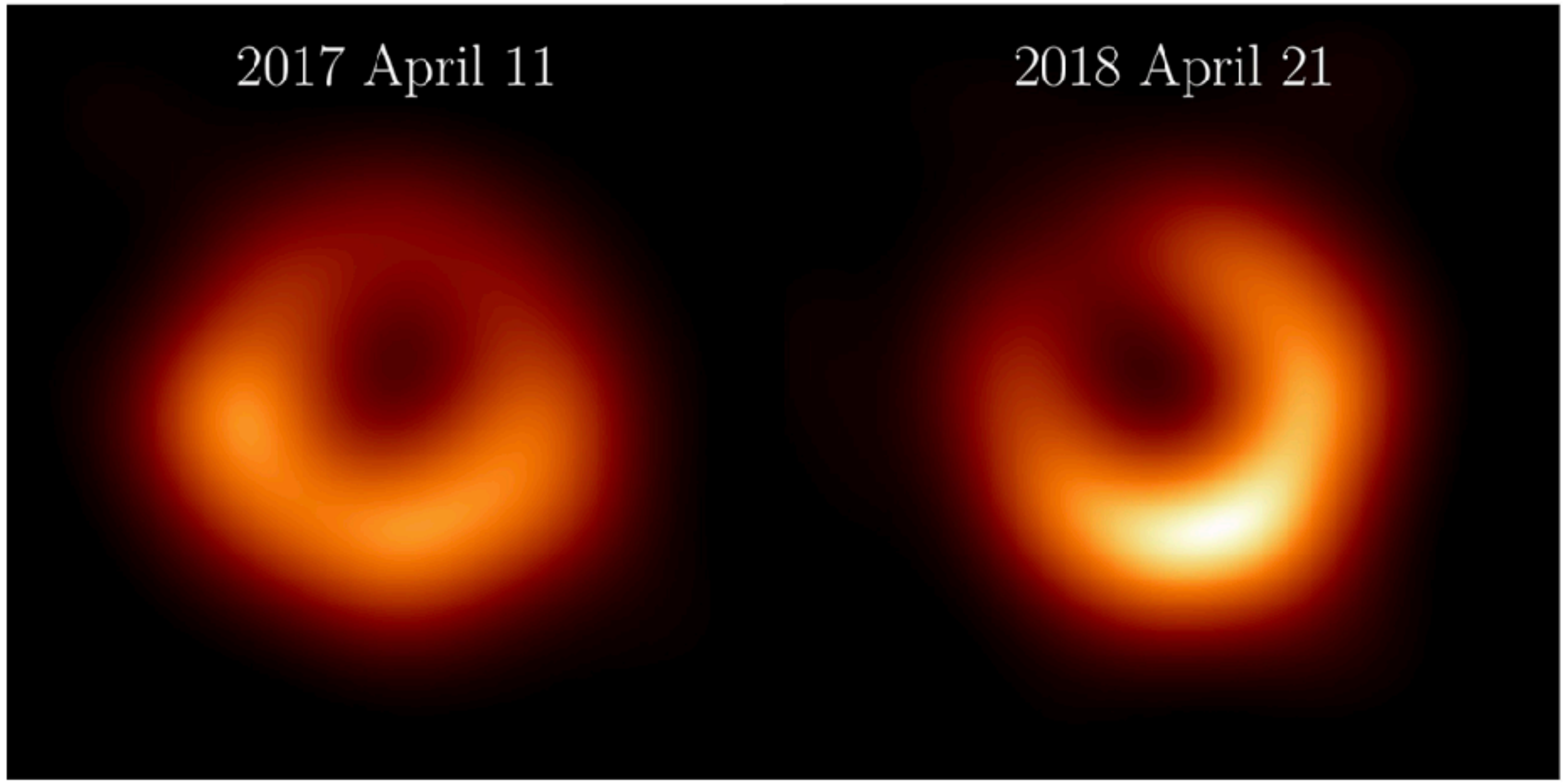
SMACS 0723 은하단 (galaxy cluster)

source:
제임스 웹 우주 망원경
(James Webb Space Telescope)

Do you know what this picture is?



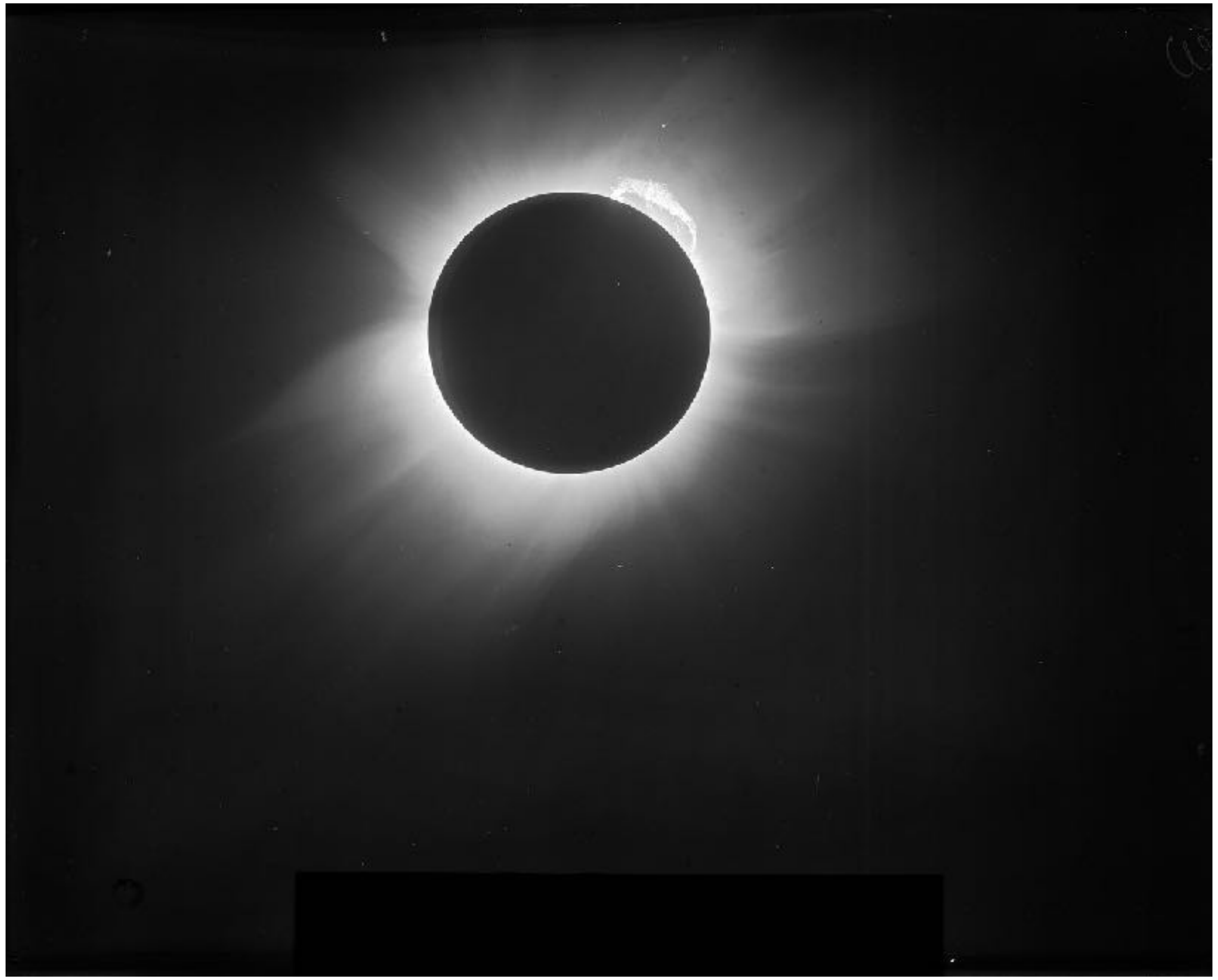
Do you know what this picture is?



M87 은하 중심의 초거대질량 블랙홀
(Supermassive BH in M87)

source:
이벤트 호라이즌 망원경
(Event Horizon Telescope)

Do you know what this picture is?



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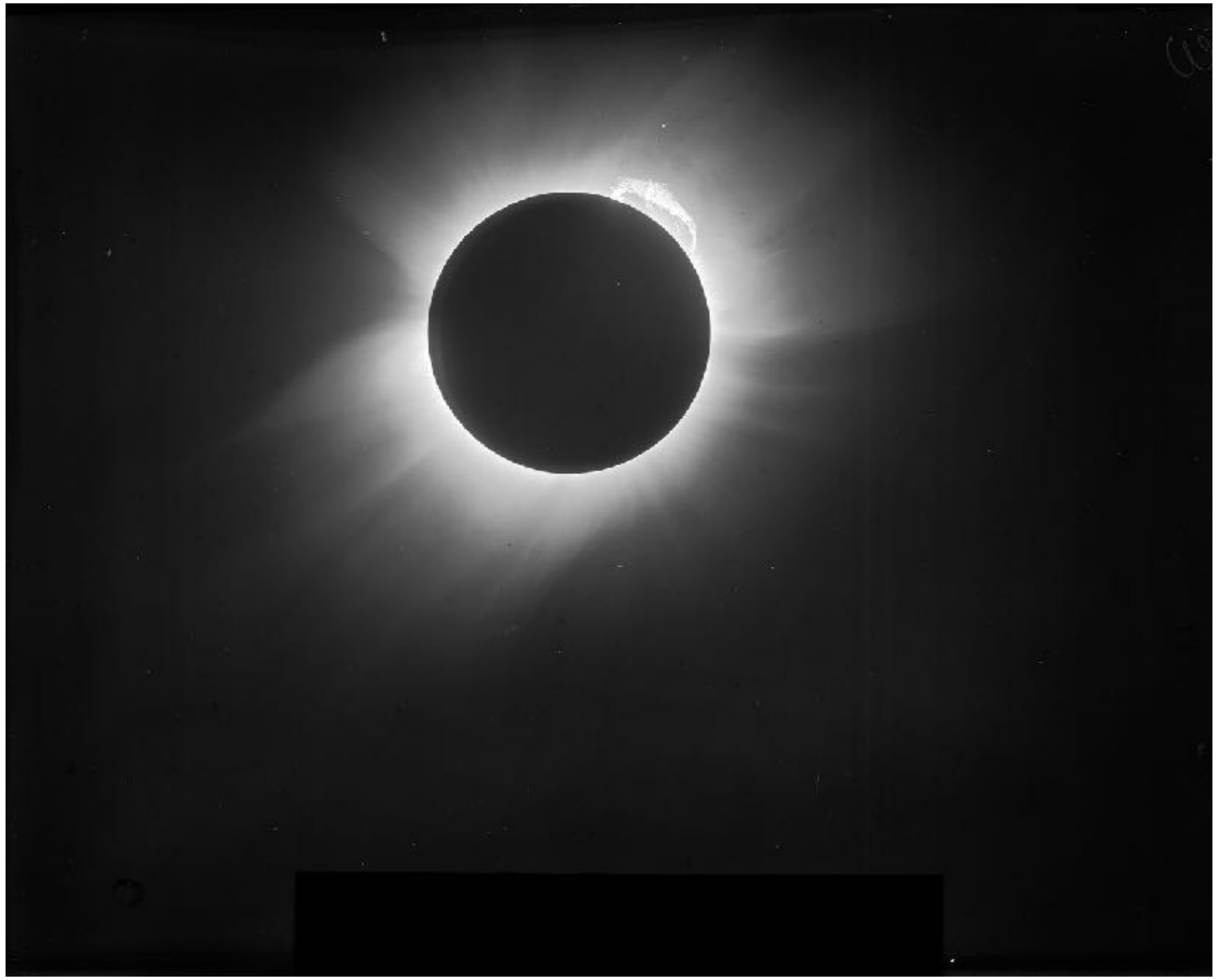
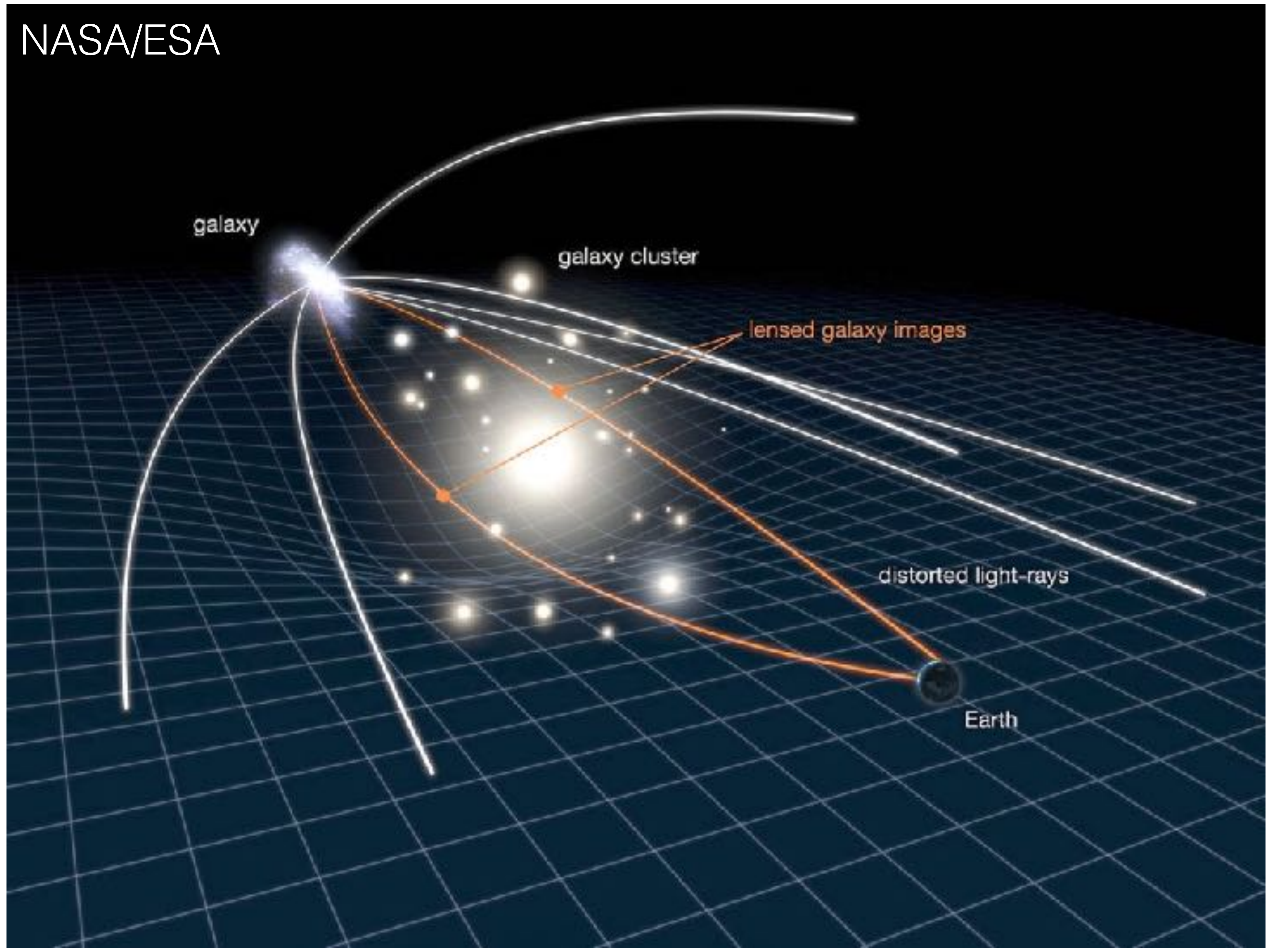


브라질 Sobral에서 관측한
개기일식 (1919년)

source:
ESO/Landessternwarte
Heidelberg-Königstuhl/
F. W. Dyson, A. S. Eddington,
& C. Davidson

Gravitational lensing phenomena

- The deflection of light by massive bodies and the phenomena resulting therefrom.



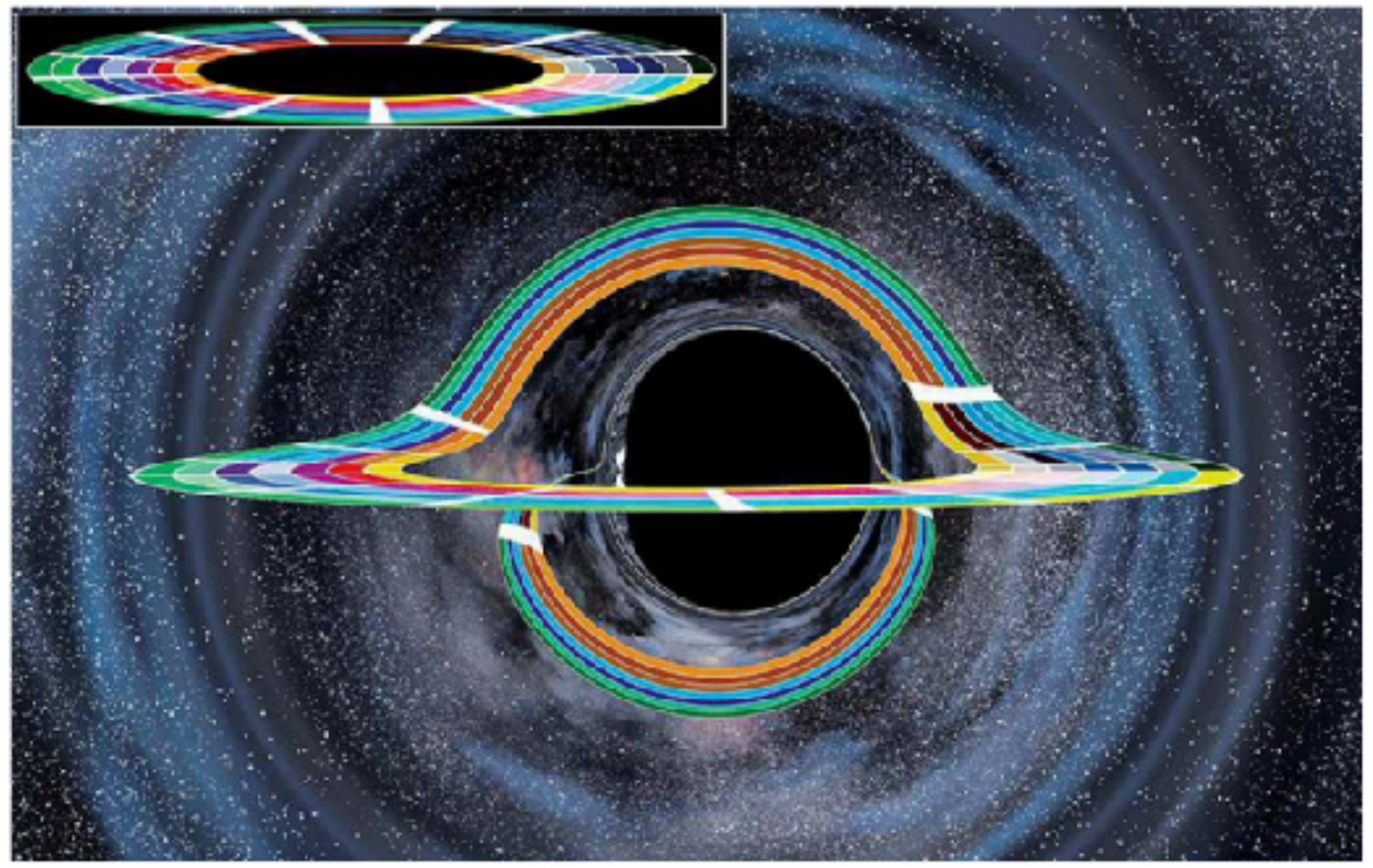


Fig. 9.7. An infinitesimally thin disk in Gargantua's equatorial plane, gravitationally lensed by Gargantua's warped space and time. Here Gargantua spins very fast. *Inset*: The disk in the absence of the black hole. *[From Eugénie von Tunzelmann's artistic team at Double Negative.]*

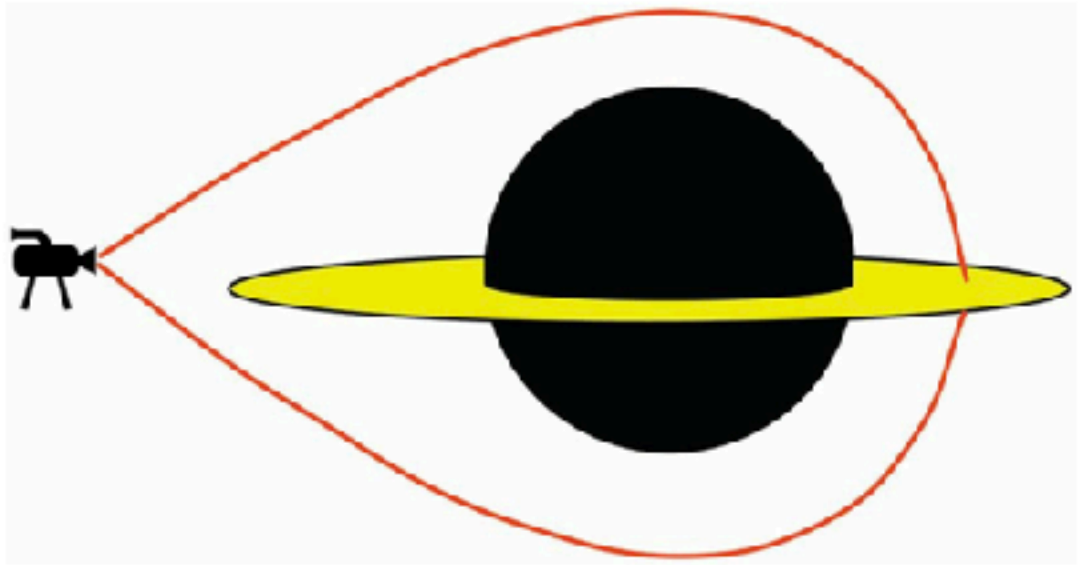
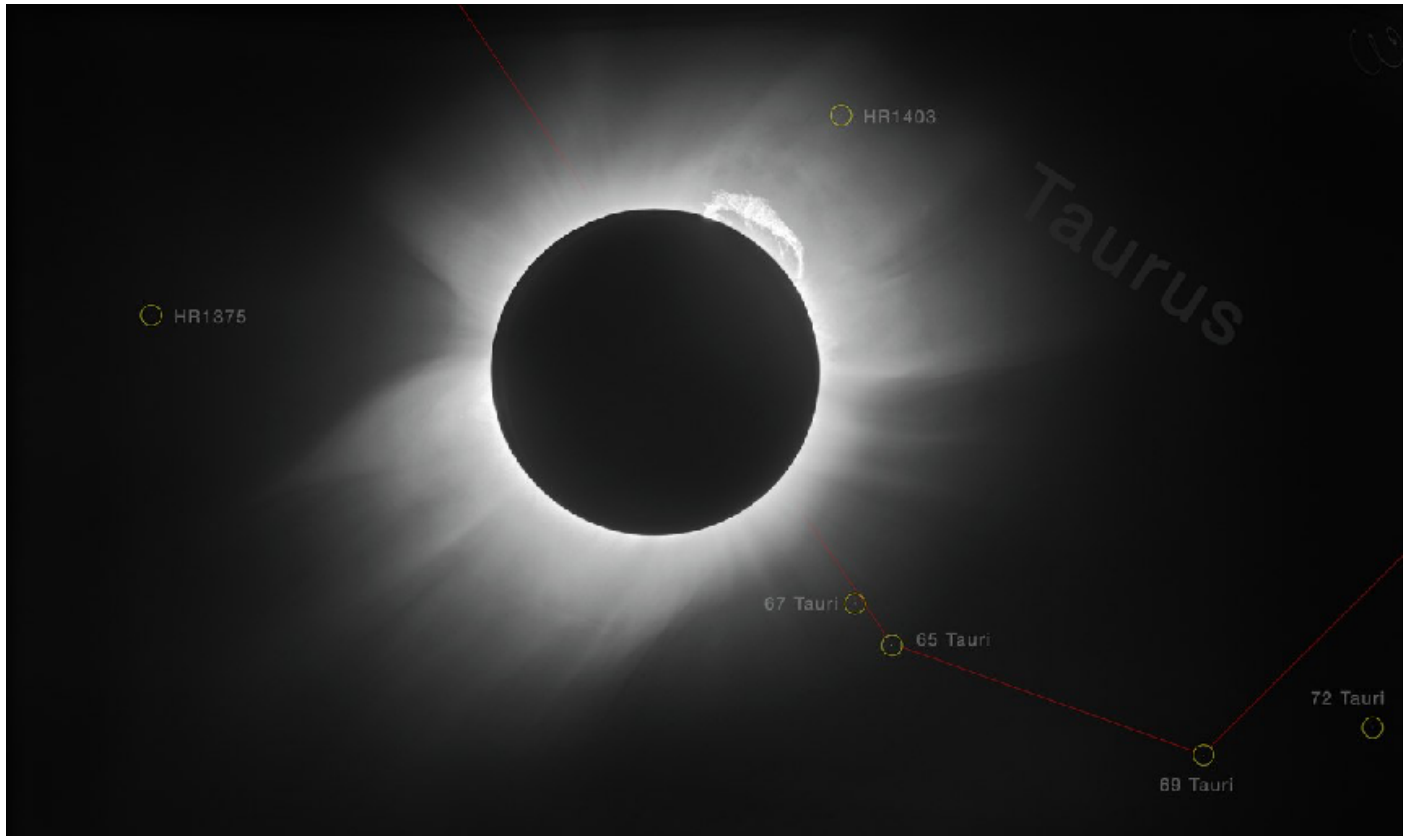


Fig. 9.8. Light rays (*red*) bring to the camera images of the back part of the accretion disk, behind Gargantua: one image above the hole's shadow, the other below the hole's shadow.

Kip Thorne (2014)
"The Science of Interstellar"

Gravitational lensing phenomena

- One of the consequences of Einstein's General Theory of Relativity.
- Confirmed with a ray of light passing close to the limb of the Sun (1919).



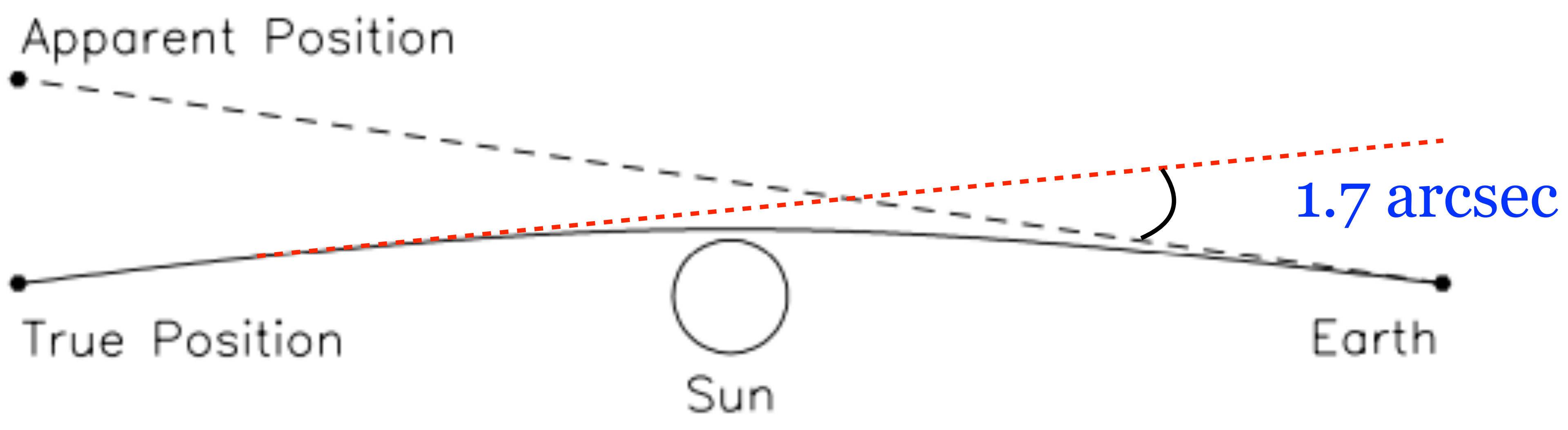


FIG. 1.—Angular deflection of a ray of light passing close to the limb of the Sun. Since the light ray is bent toward the Sun, the apparent positions of stars move away from the Sun.

[figure & caption from Narayan & Bartelmann '96]

- Magnify light from distant sources which would otherwise remain undetected (Zwicky 1937).

2. 빛의 중력렌징

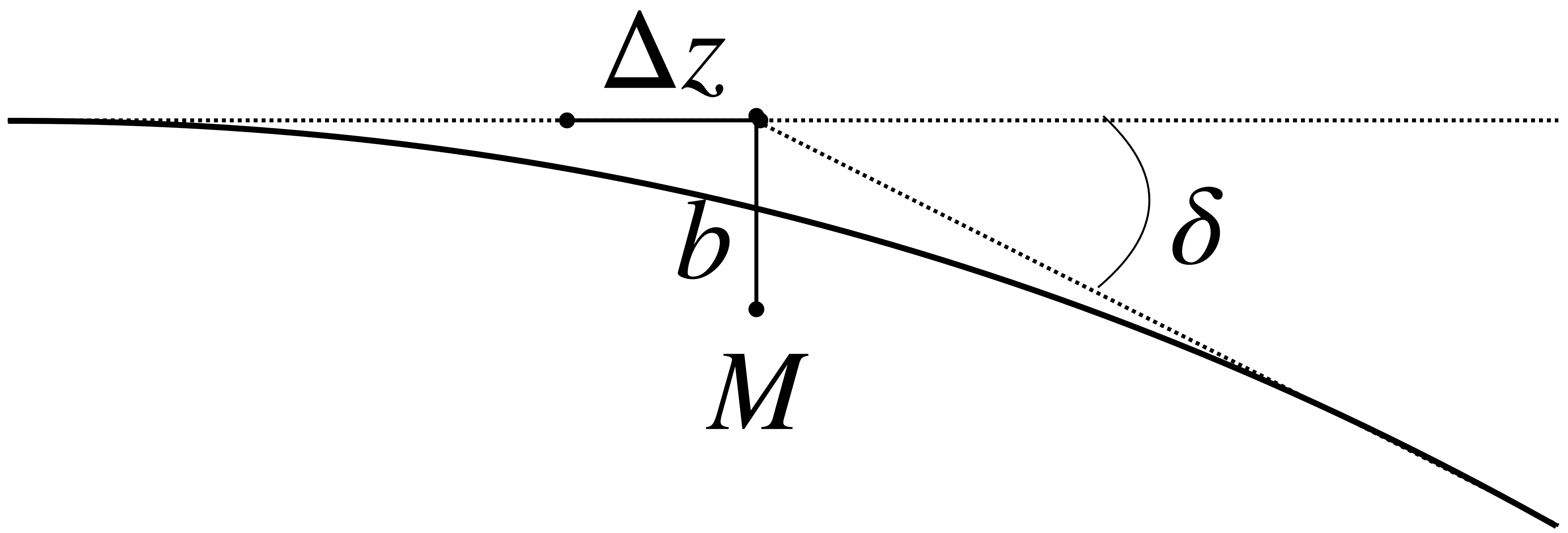
Gravitational lensing of lights

Further reading: textbooks
such as
“***Introduction to Gravitational Lensing with Python Examples***”
by Massimo Meneghetti

Lensing by a point mass

Narayan & Bartelmann (astro-ph/9606001)

- **Let's suppose a lensing by a point mass M**
  - deflection angle, $\delta = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi dz$, where
    - b: impact parameter
    - z: distance along the original light path
    - $\Phi = -GM/(b^2+z^2)^{1/2}$: Newtonian potential

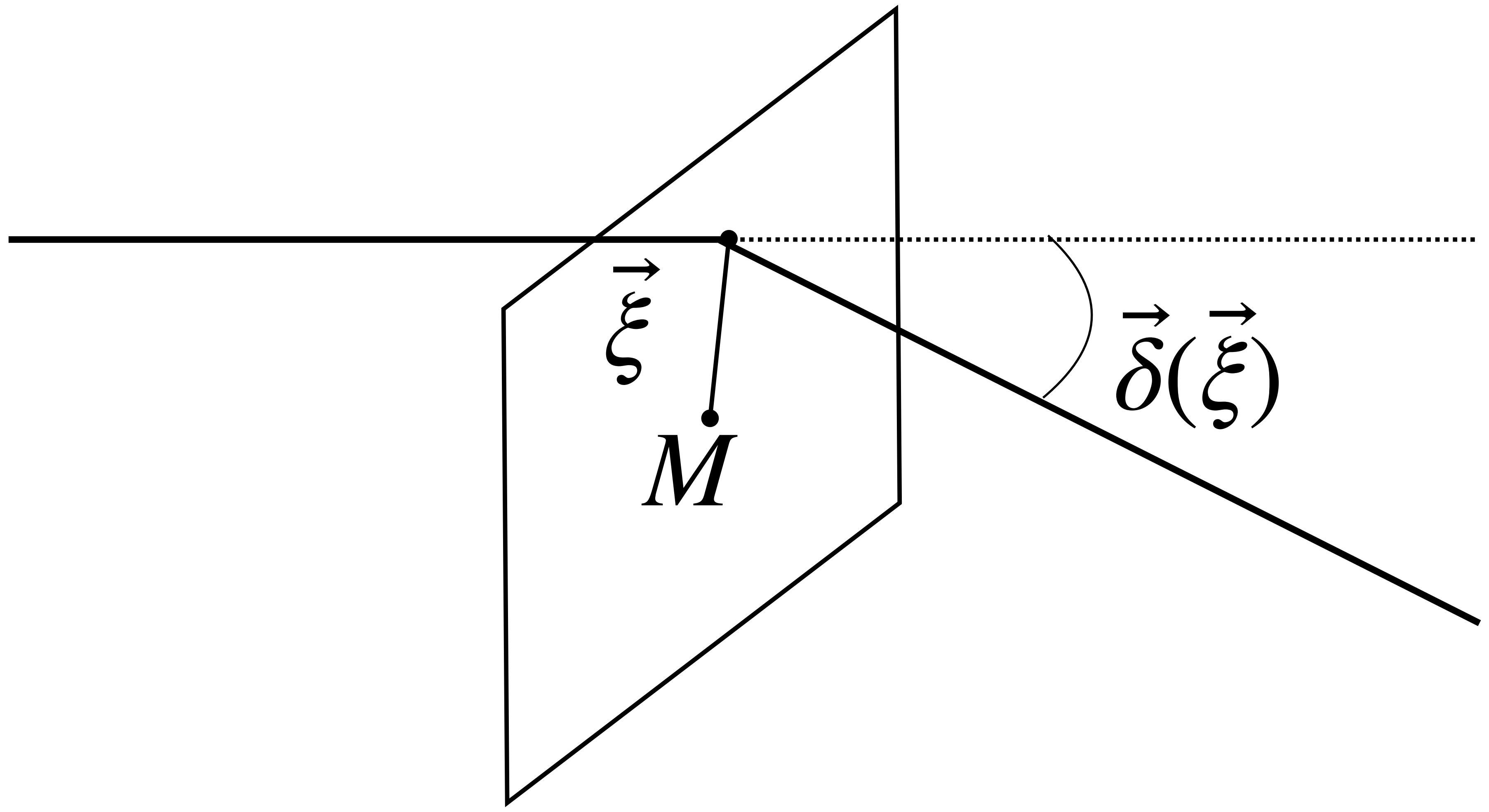


  - most of light deflection occurs within $\Delta z \sim \pm b$ of the point of closest encounter between the light ray and the point mass.
    - Δz is typically much smaller than the distances between (the observer and lens) and between (lens and source).
      → thin lens (screen) approximation!

Thin lens approximation

Narayan & Bartelmann
(astro-ph/9606001)

- deflection angle $\vec{\delta}(\vec{\xi}) = \frac{4G}{c^2}\int \frac{(\overrightarrow{xi} - \vec{\xi}')\Sigma(\vec{\xi}')}{|\vec{\xi} - \vec{\xi}'|^2} d^2\xi'$, where
  - $\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)dz$: surface mass density
  - $\vec{\xi}$: a two-dimensional vector in the lens plane



- for a circularly symmetric lens, deflection angle becomes $\delta(\xi) = \frac{4GM(\xi)}{c^2\xi}$, where
  - ξ: distance from the lens center
  - $M(\xi) = 2\pi\int_0^{\xi} \Sigma(\xi')\xi' d\xi'$: mass enclosed within radius ξ

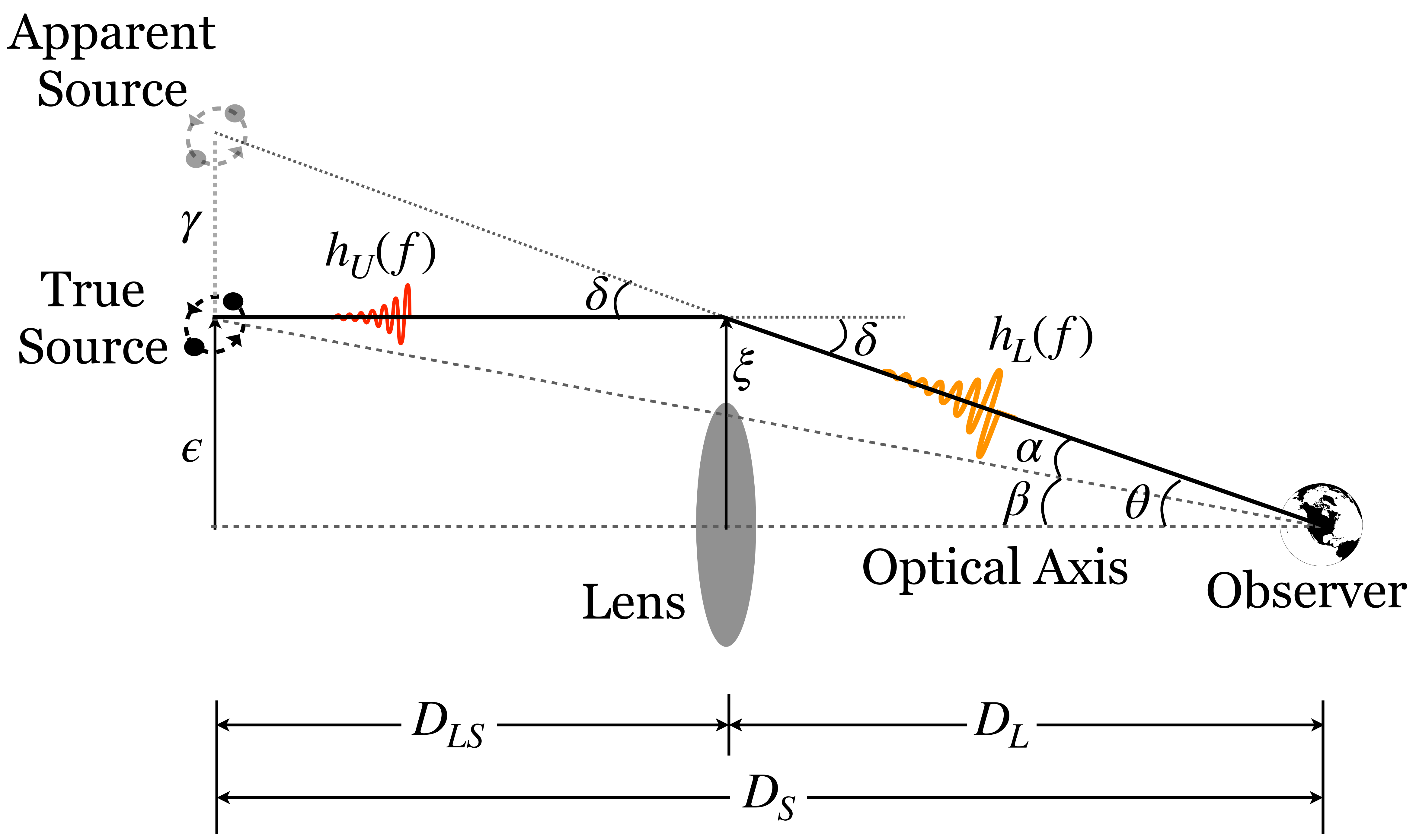
Lensing geometry and lens equation

Narayan & Bartelmann
(astro-ph/9606001)

- reduced deflection angle $\vec{\alpha} = \dfrac{D_{LS}}{D_S}\vec{\delta}$

- We see that $\theta D_S = \beta D_S + \delta D_{LS}$ from the schematic.

  $\rightarrow \vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$: lens equation
  (or, equivalently, lay-tracing equation)



Einstein radius

Narayan & Bartelmann (astro-ph/9606001)

- With the deflection angle for a circularly symmetric lens and its reduced deflection angle, the lens equation reads

$$\beta(\theta) = \theta - \frac{D_{LS}}{D_L D_S}\frac{4GM(\theta)}{c^2\theta}$$

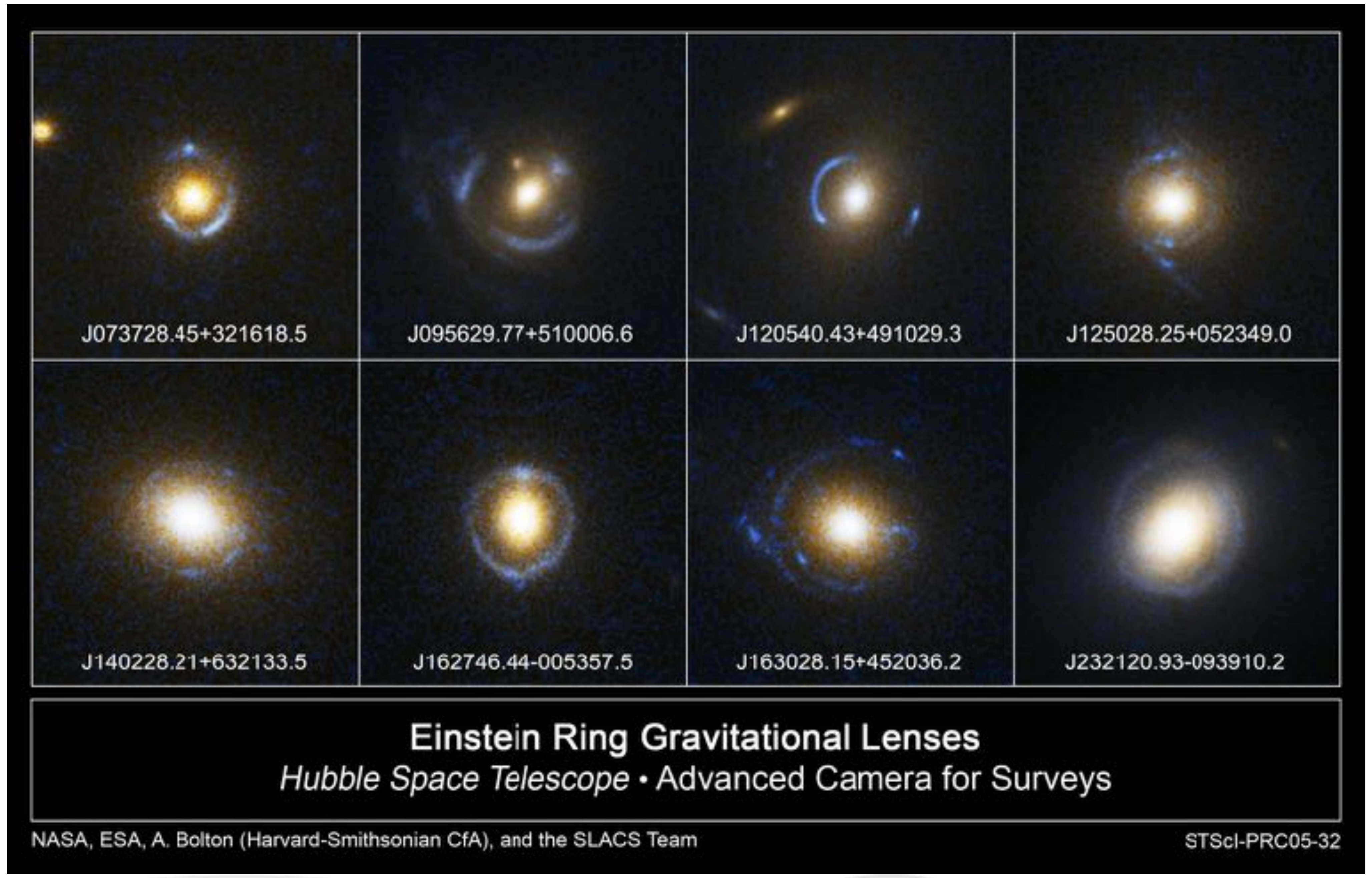
- Due to the rotational symmetry of the lens system, a source which lies exactly on the optic axis ($\beta = 0$) is imaged as a ring having the radius θ_E such as

$$\theta_E = \left[\frac{4GM(\theta_E)}{c^2}\frac{D_{LS}}{D_L D_S}\right]^{1/2},$$

that is referred to as the Einstein radius.

  - For a point mass M, it becomes

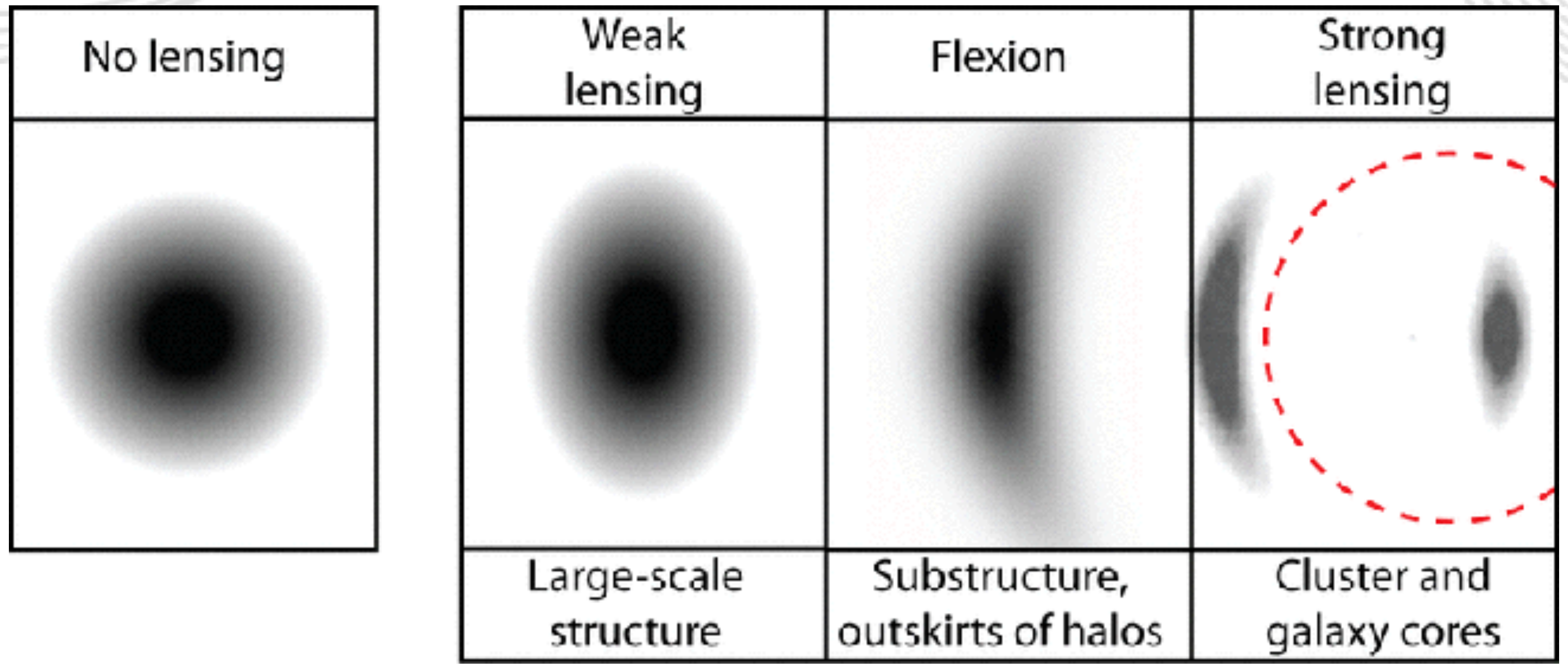
$$\theta_E = \left[\frac{4GM}{c^2}\frac{D_{LS}}{D_L D_S}\right]^{1/2}$$



Types of gravitational lensing

- weak lensing
  - distortion of background source such as galaxy
  - typically produces a single distorted image.
- strong lensing
  - occurs when source, lens, and observer are well positioned.
  - 2+ distorted images
- microlensing
  - occurs by small stellar objects
  - observable by changes in the brightness

[credit: ESA]



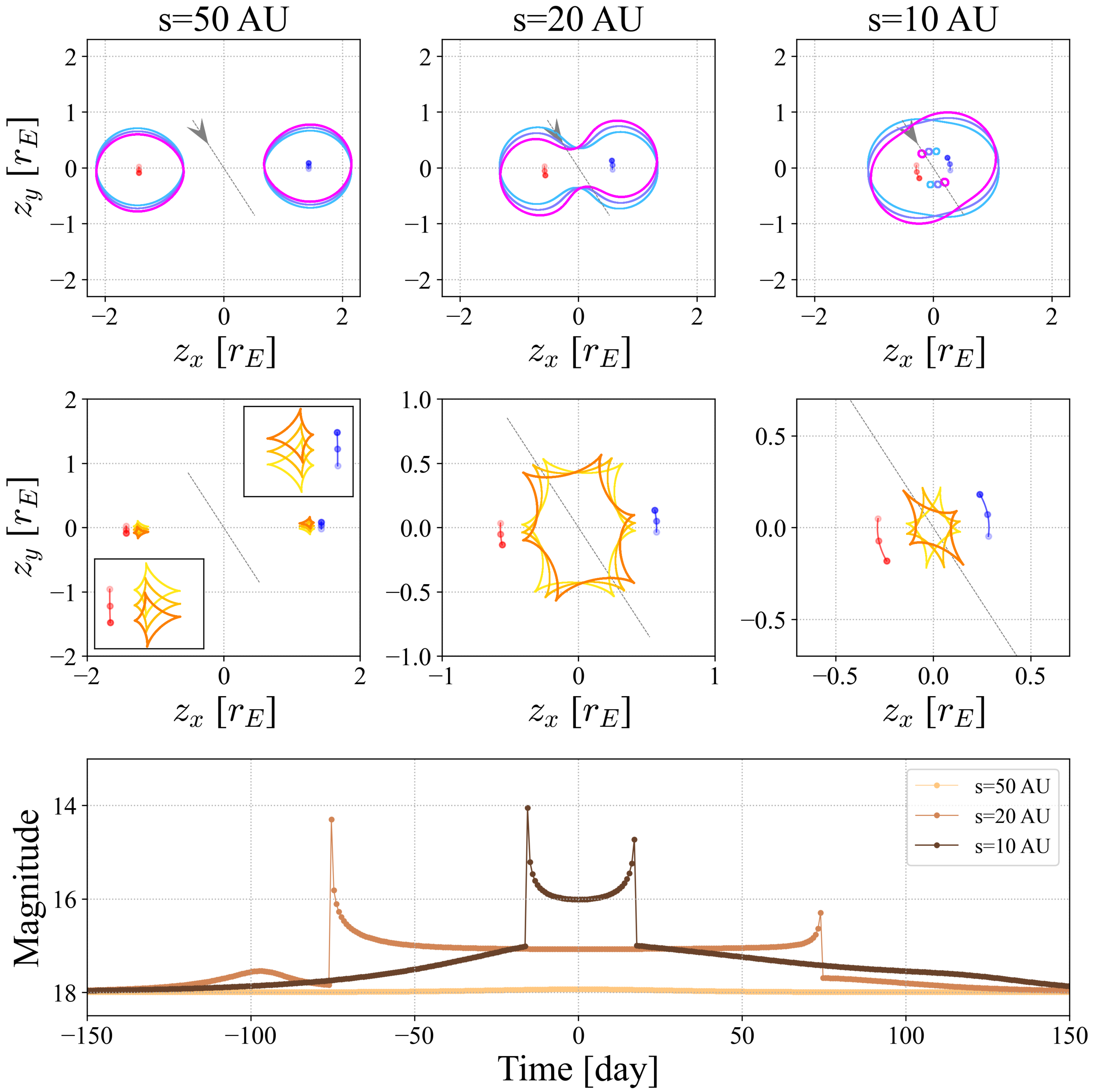
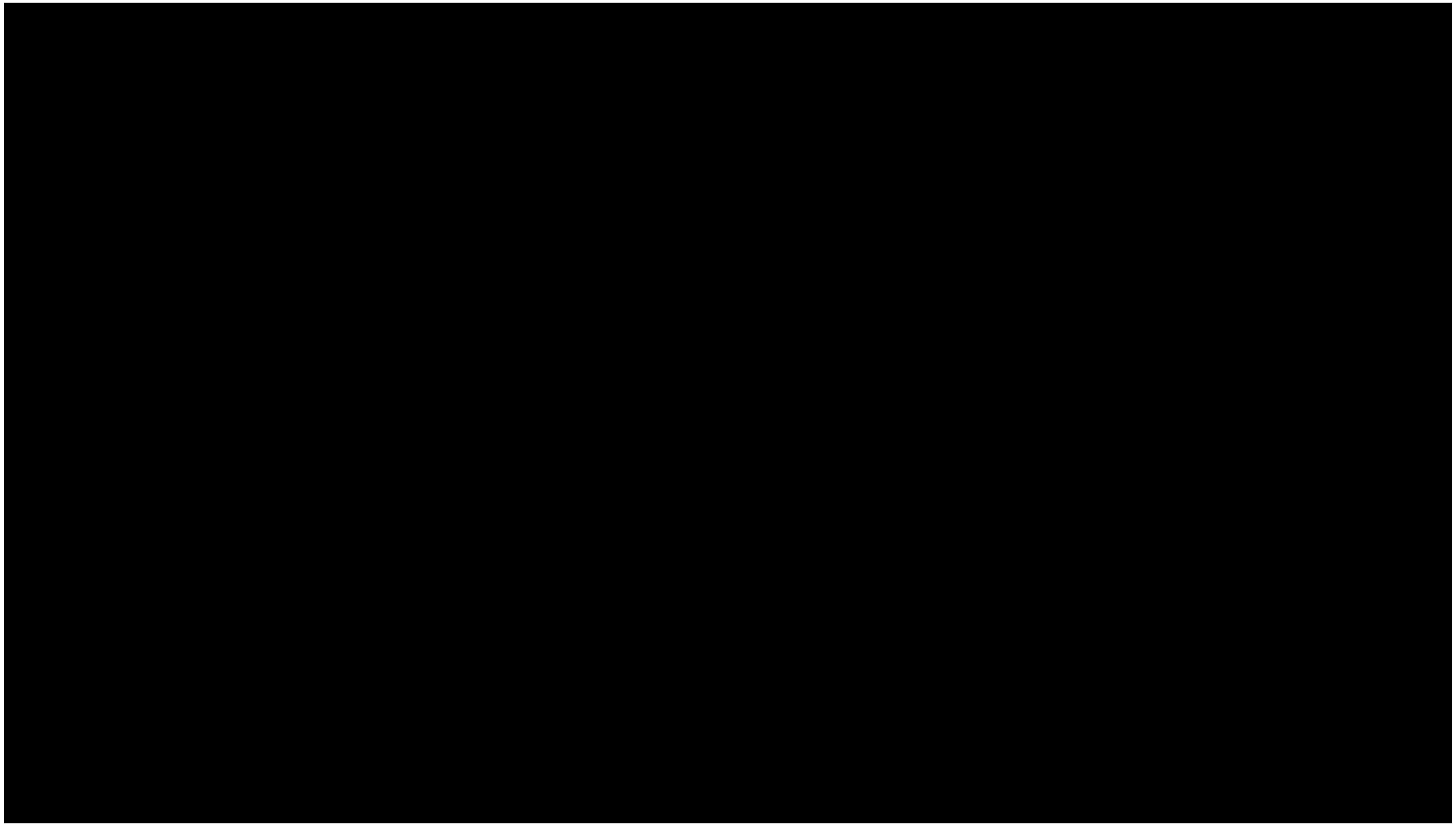
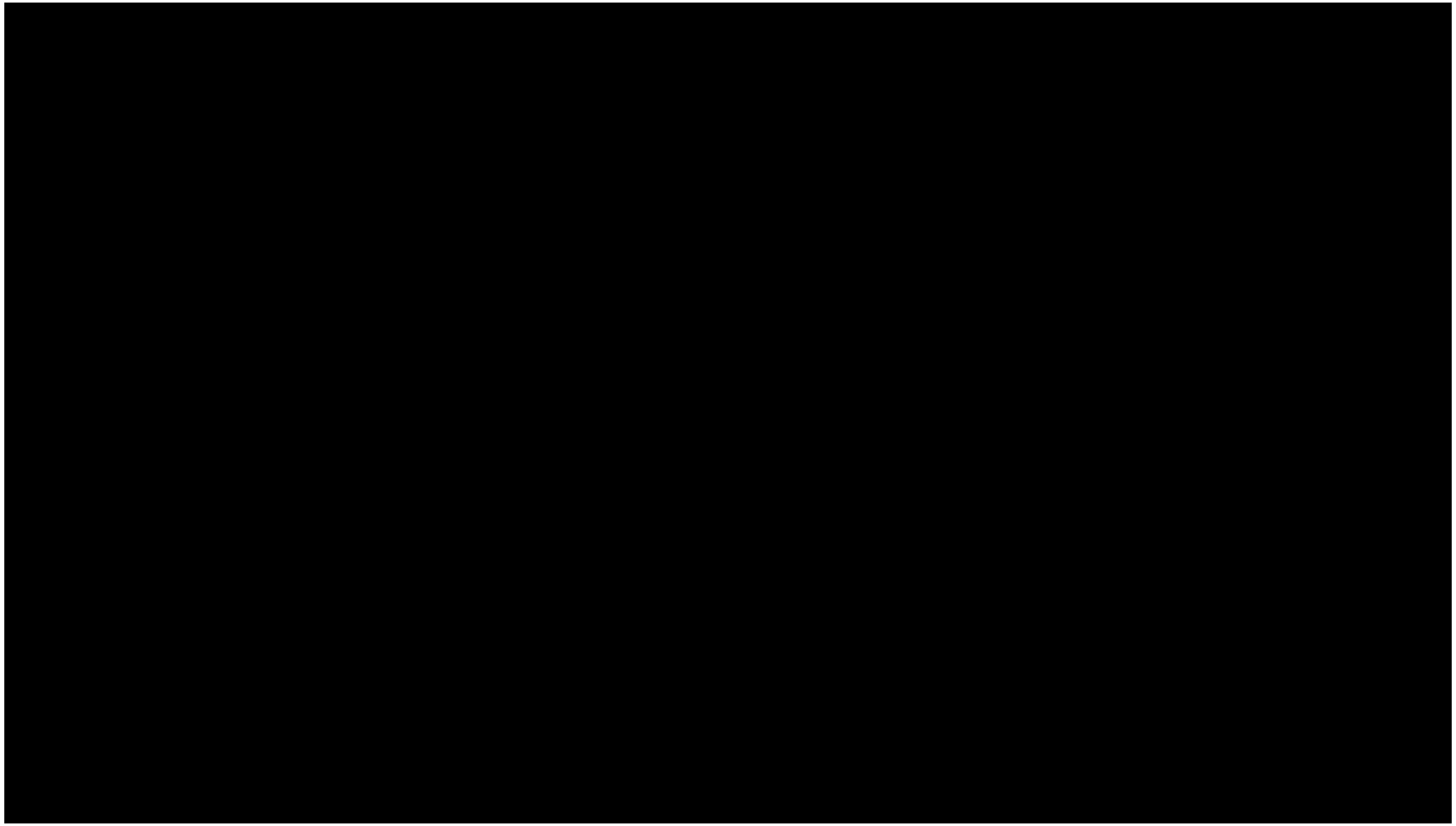


Table-top gravitational lensing of lights



[credit: Perimeter Institute for Theoretical Physics / source: YouTube]

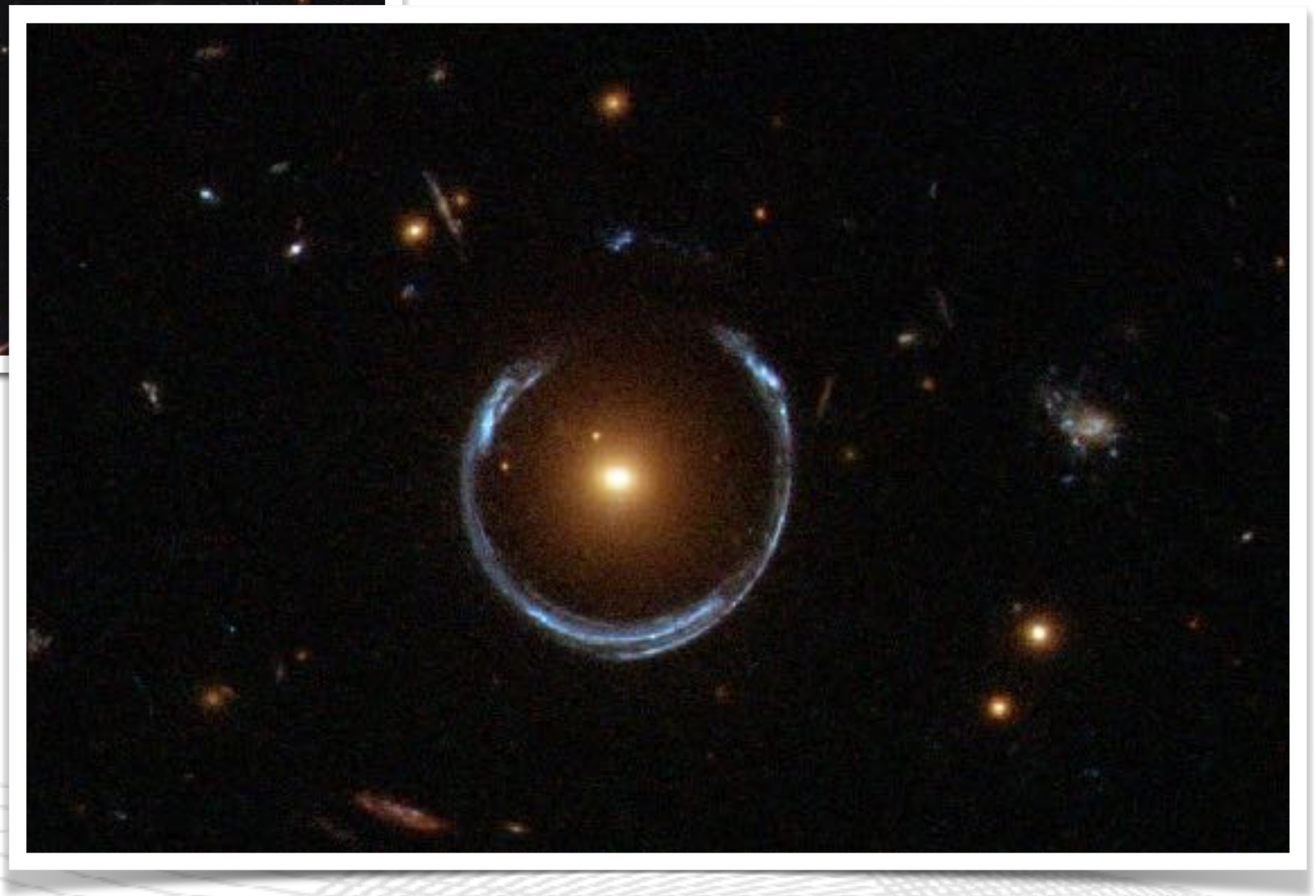
Table-top gravitational lensing of lights



[credit: Perimeter Institute for Theoretical Physics / source: YouTube]

Examples of gravitational lensing of lights

- **Lensing by galaxies/galaxy clusters**
  - A major sub-discipline of gravitational lensing
  - Cosmic telescope effect of gravitational lenses enables us to study faint and distant galaxies which happen to be strongly magnified by galaxy clusters.
  - The statistics of gravitational lensing events lensed by galaxies offers one of the promising ways of inferring cosmological parameters.



Gravitational lensing for cosmology

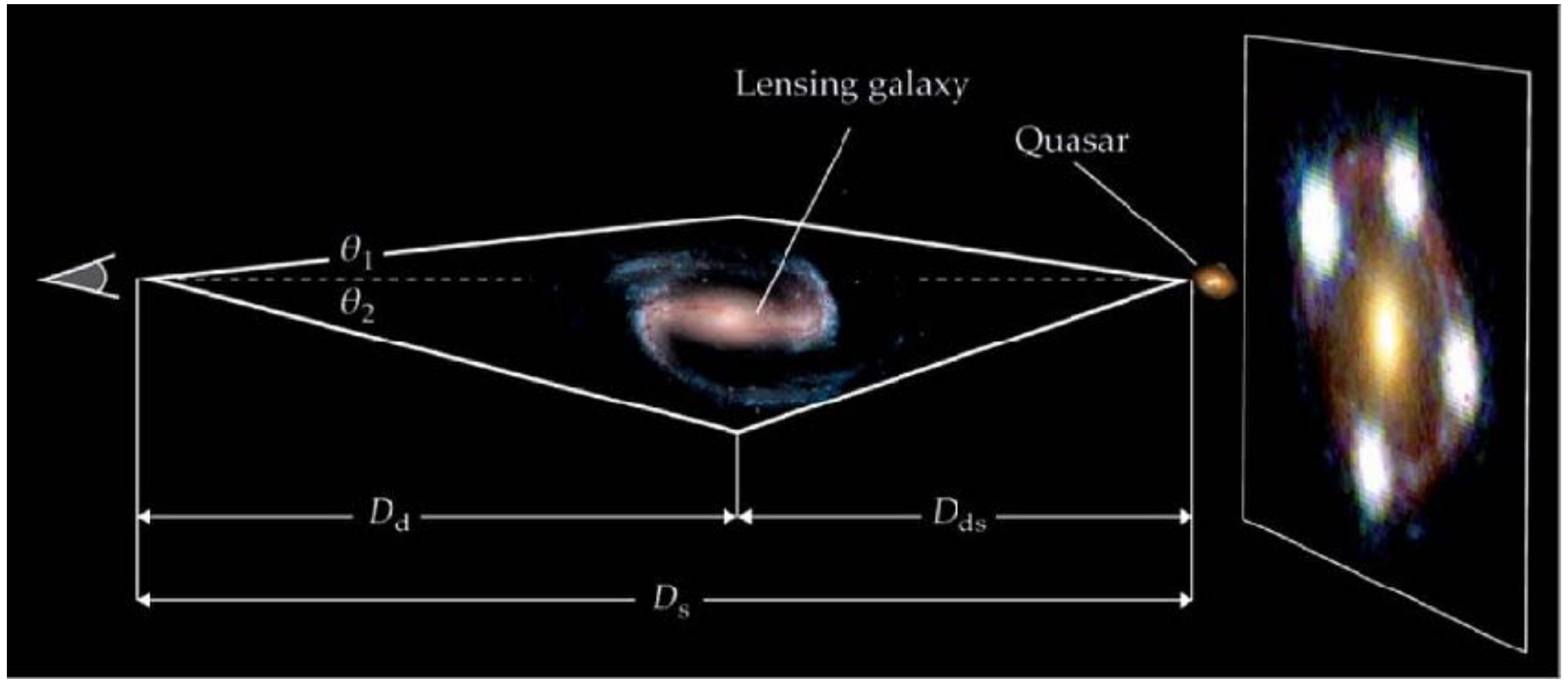
- **Measurement of the Hubble constant H_0 w/ gravitational lensing** (Refsdal 1964)

  - Hubble—Lemaître law: $v = H_0 D$ (v: recessional velocity; D: proper distnance)
    → Unit of H_0: $[\mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{Mpc}^{-1}]$

  - Key: time delay between lensed images

  - Time delay ∝ the difference in the absolute lengths of the light paths ∝ H_0^{-1}

  - If the time delay is measured and
    if an accurate model of a lensed source
    is provided, H_0 could be measured.



[credit: EO Portal]

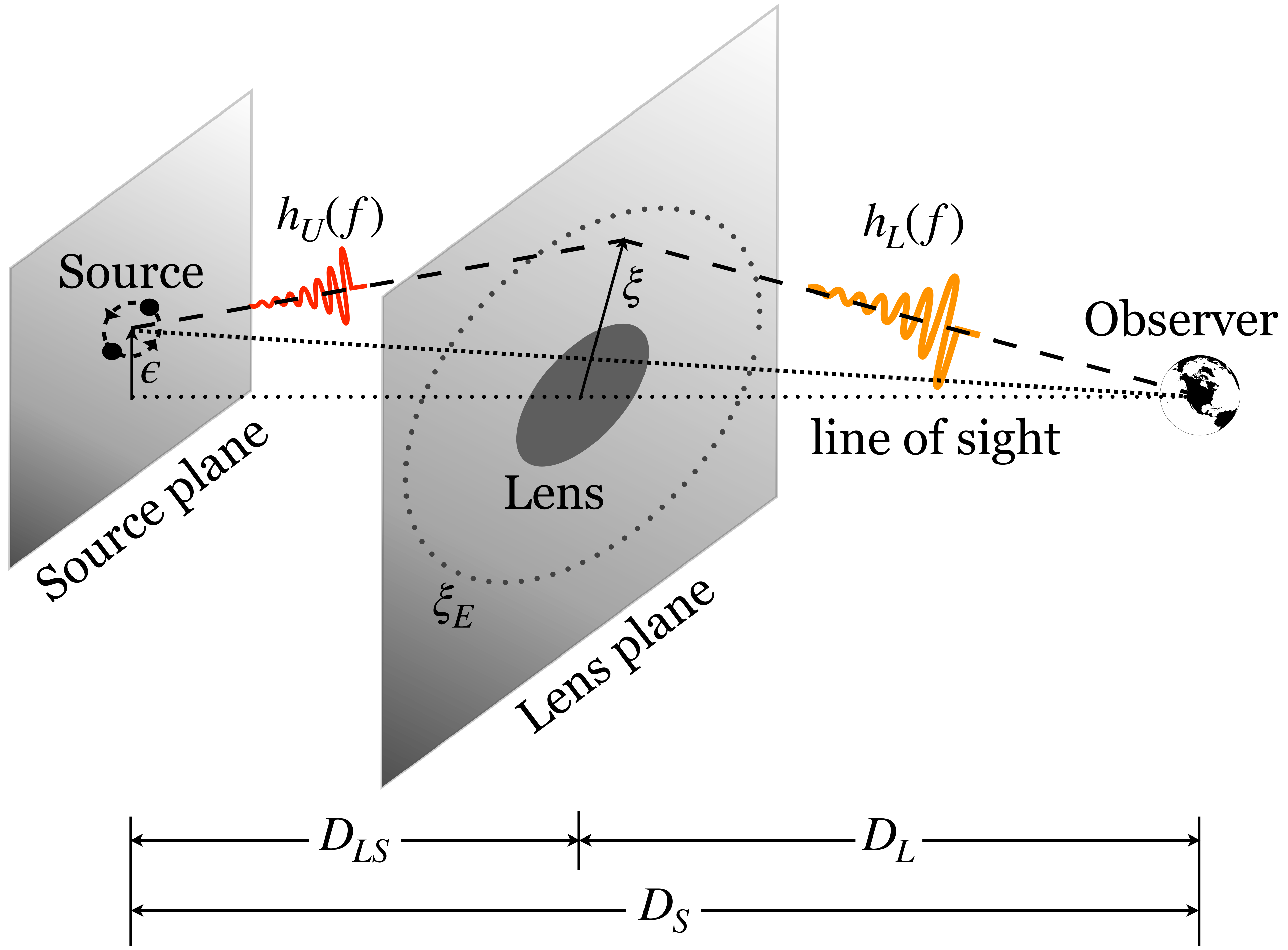
3. 중력파의 중력렌징

Gravitational lensing of gravitational waves

Lensing configuration

- Define the amplification factor as $F(f) \equiv \dfrac{h_L(f)}{h_U(f)}$ ($\rightarrow h_L(f) = F(f)h_U(f)$)

  where $h_L(f)$ and $h_U(f)$ are the lensed and unlensed GWs in the frequency domain.



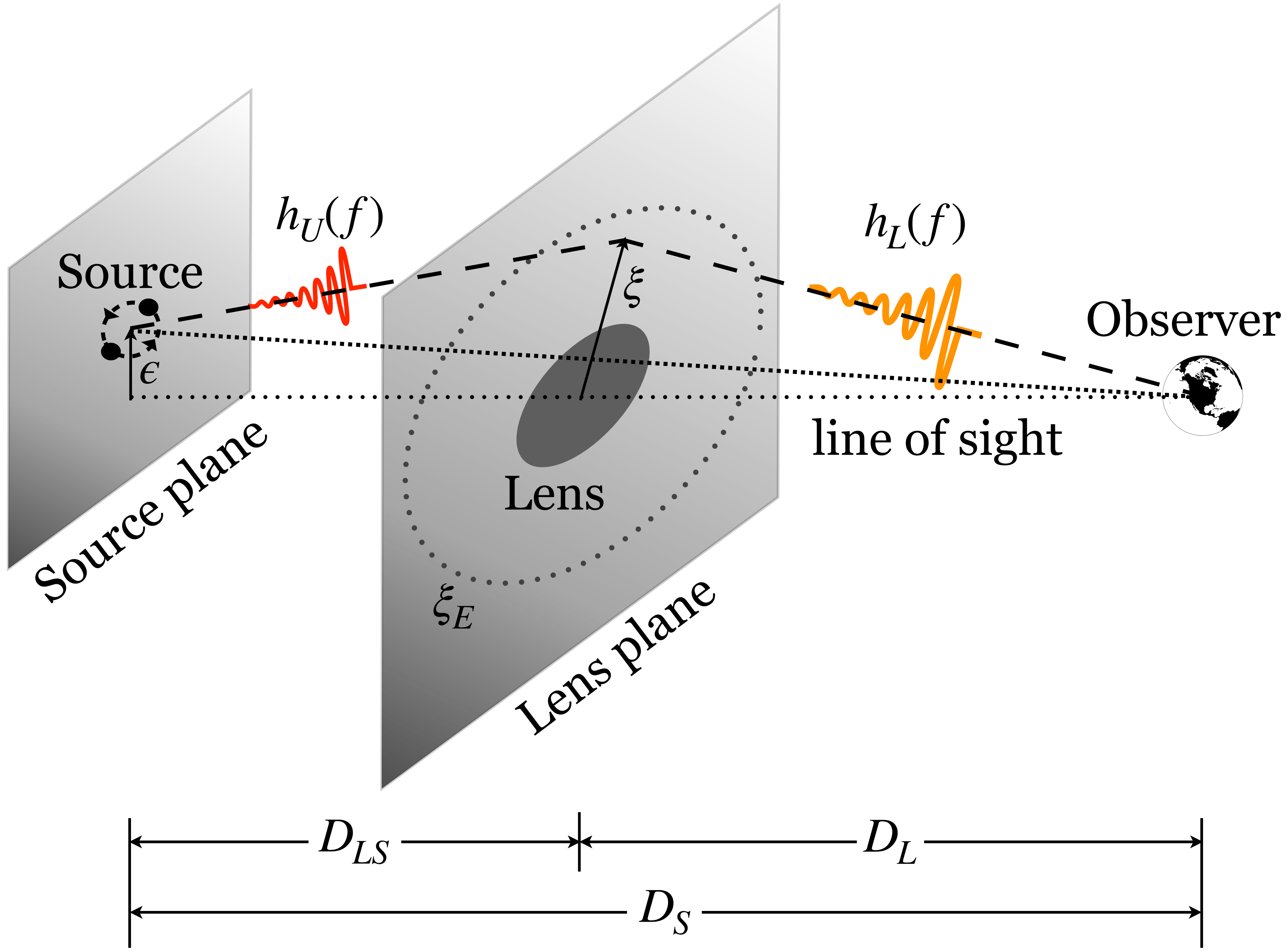
Lensing configuration (cont'd)

- With adopting the thin-lens approximation, we get

$$F(f) = \frac{D_S \xi_0^2 (1+z_L)}{D_L D_{LS}} \frac{f}{i} \int d^2\mathbf{x} \exp[2\pi i f t_d(\mathbf{x}, \mathbf{y})],$$

where

- $\mathbf{x} = \xi/\xi_0$, $\mathbf{y} = \epsilon D_L/\xi_0 D_S$: the source position
  - ξ is the impact parameter in the lens plane
  - ϵ is the position vector of the source in the source plane
- ξ_0 is an arbitrary normalization constant of the length
- t_d is the arrival time at the observer from the source
- F is normalized such that $|F| = 1$ in the no-lens limit



Arrival time

- The arrival time t_d at the observer from the source position ϵ through ξ is given by

$$t_d(\mathbf{x}, \mathbf{y}) = \frac{D_S \xi_0^2 (1 + z_L)}{D_L D_{LS}} \left[\frac{1}{2} |\mathbf{x} - \mathbf{y}|^2 - \psi(\mathbf{x}) + \phi_m(\mathbf{y}) \right],$$

  where

  - $\psi(\mathbf{x})$ is the dimensionless deflection potential
  - $\phi_m(\mathbf{y})$ is an arbitrary constant to be chosen to make the minimum value of t_d is 0

- For convenience, we set ξ_0 is equal to the Einstein radius of the lens such as

$$\xi_0 = \xi_E = \sqrt{(4GM_L/c^2) D_{LS} D_L / D_S}$$

Lens models in geometrical optics limit

Takahashi & Nakamura
(ApJ, 2003)
with $c = G = 1$

- **Point-mass lens**: describe lens mass as a 2-dim Dirac delta function on lens plane

$$F(f) = |\mu_+|^{1/2} - i|\mu_-|^{1/2} e^{2\pi i f \Delta t_d},$$

where

- $\mu_\pm = \dfrac{1}{2} \pm \dfrac{(y^2+2)}{(2y\sqrt{y^2+4})}$: magnification factor for the 1st image (+) and 2nd image (-), respectively

- $\Delta t_d = 4M_{Lz}\left[\dfrac{y\sqrt{y^2+4}}{2} + \ln\left(\dfrac{\sqrt{y^2+4}+y}{\sqrt{y^2+4}-y}\right)\right]$: time delay between 1st and 2nd images

- $M_{Lz} = M_L(1+z)$: redshifted lens mass

Lens models in geometrical optics limit

Takahashi & Nakamura
(ApJ, 2003)
with $c = G = 1$

- **Singular Isothermal Sphere**: circular and symmetric mass distribution on lens plane

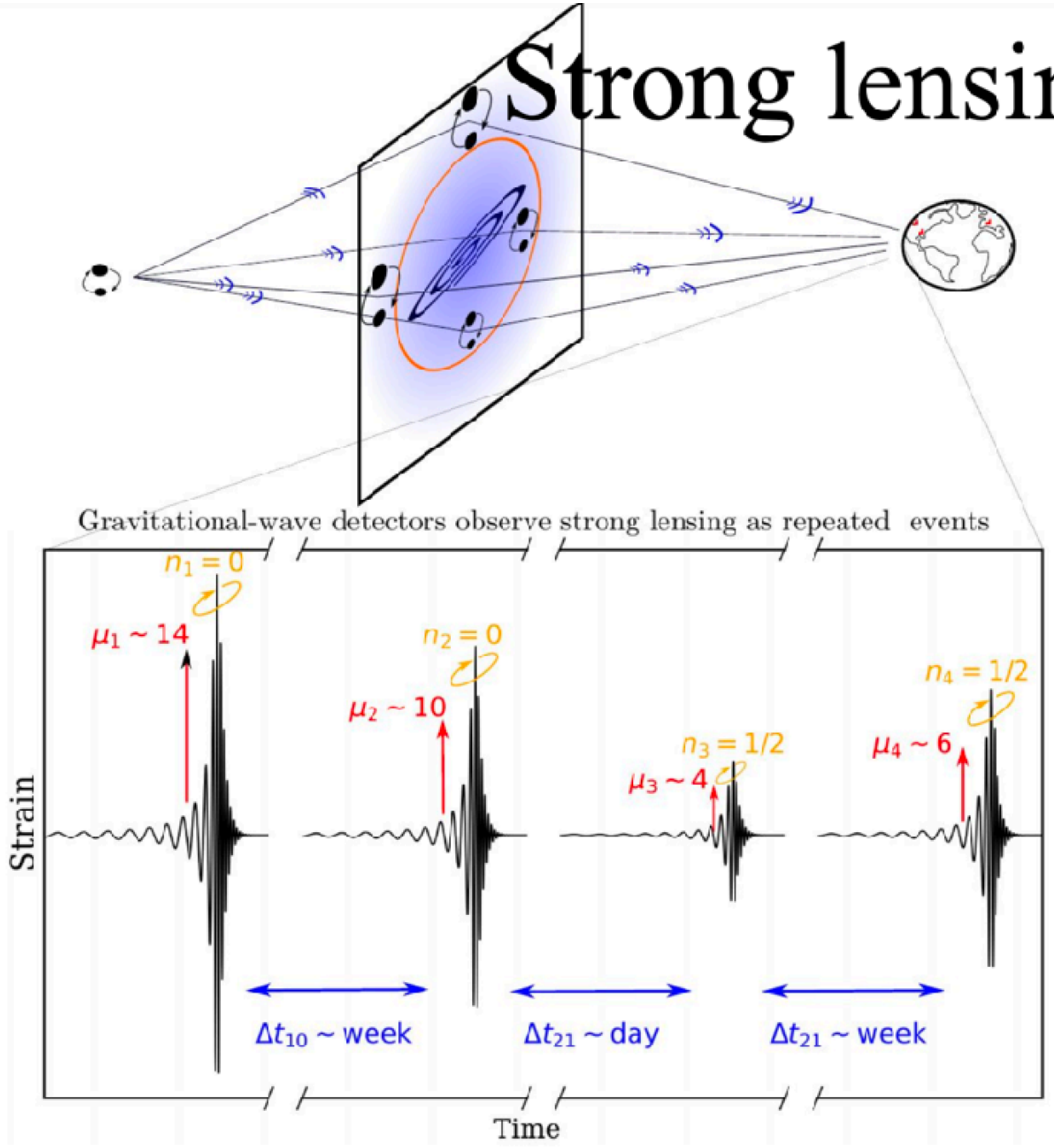
$$F(f) = \begin{cases} |\mu_+|^{1/2} - i|\mu_-|^{1/2} e^{2\pi i f \Delta t_d} & \text{if } y < 1, \\ |\mu_+|^{1/2} & \text{if } y \geq 1 \end{cases},$$

where

- $\mu_\pm = \frac{1}{y} \pm 1$: magnification factor for the 1st image (+) and 2nd image (-), respectively
- $\Delta t_d = 8M_{Lz}y$: time delay between 1st and 2nd images
- $M_{Lz} = M_L(1+z)$: redshifted lens mass

Search for strong lensing of gravitational waves

Strong lensing challenge

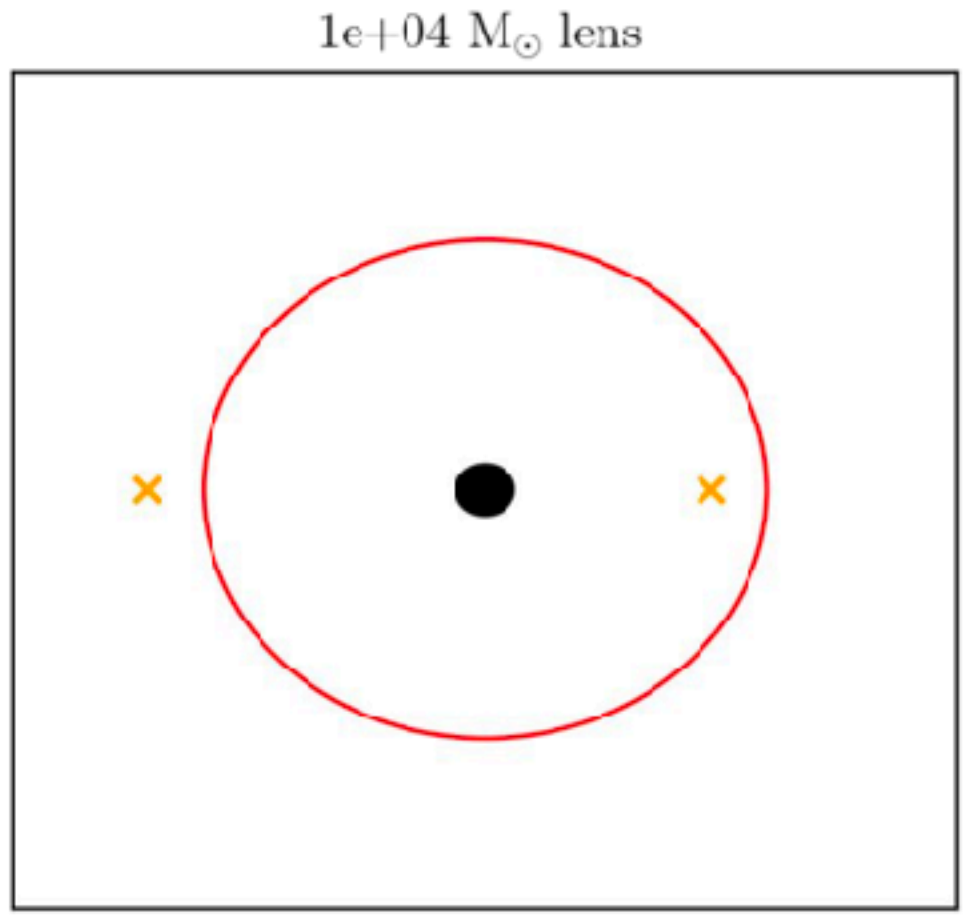


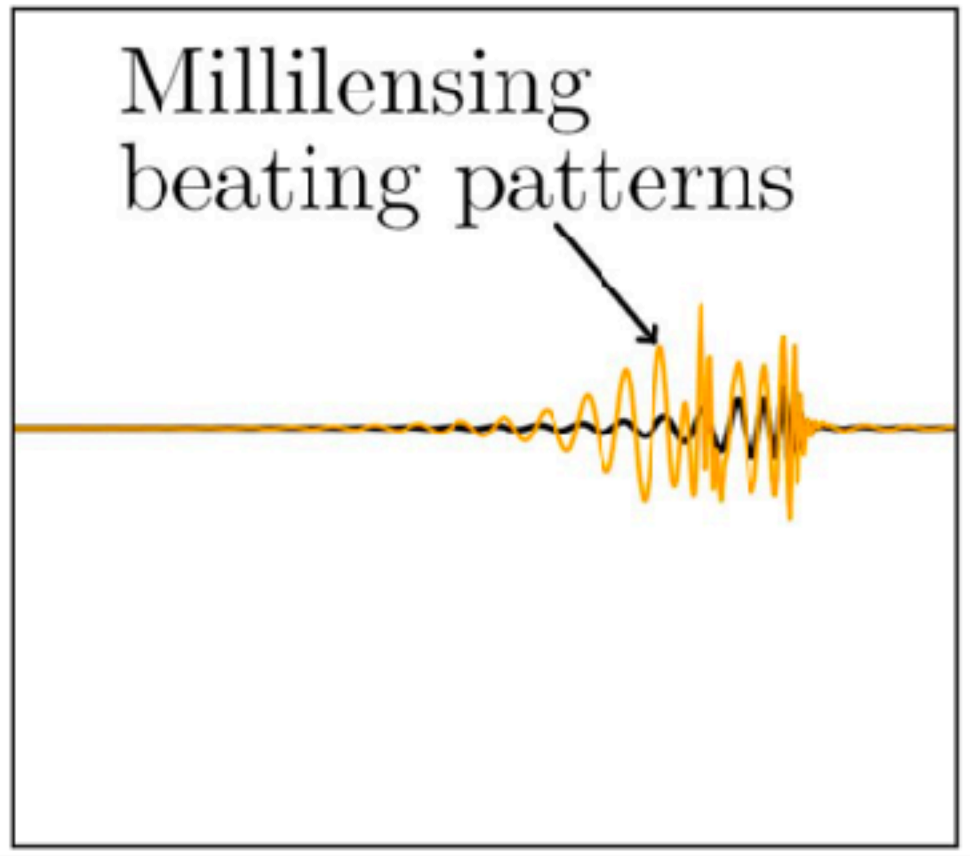
Strong lensing:
- Identify "repeated events" in the data
- Targets: Galaxies, galaxy clusters

Precise measurement of gravitational-wave image properties

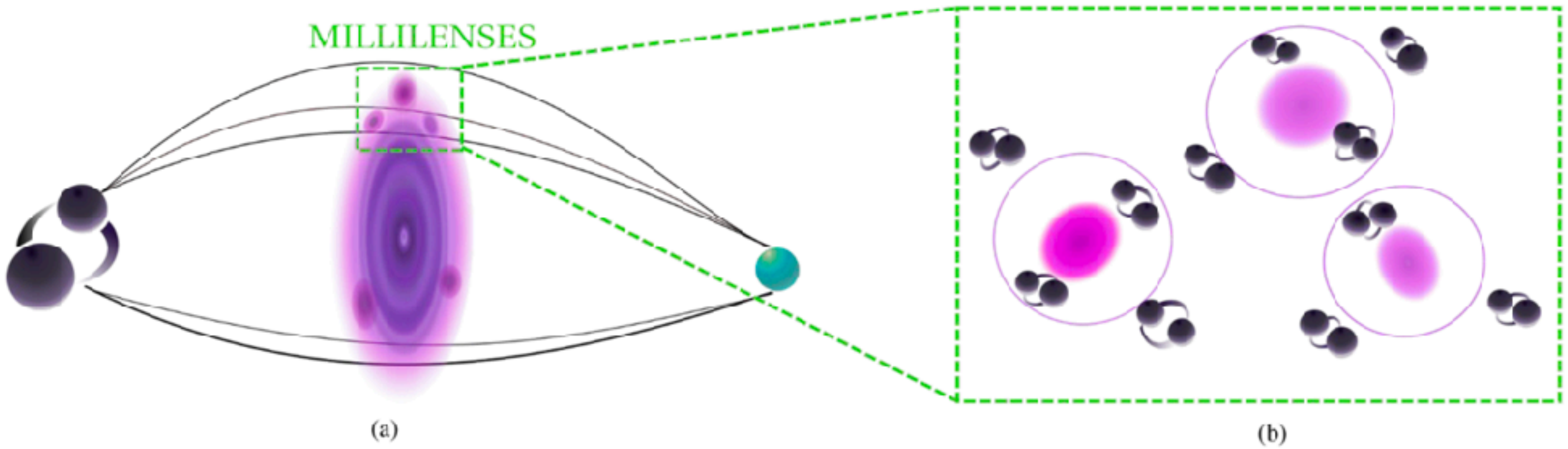
[credit: Otto Hannuksela]

Search for millilensing of gravitational waves





Millilensing challenge

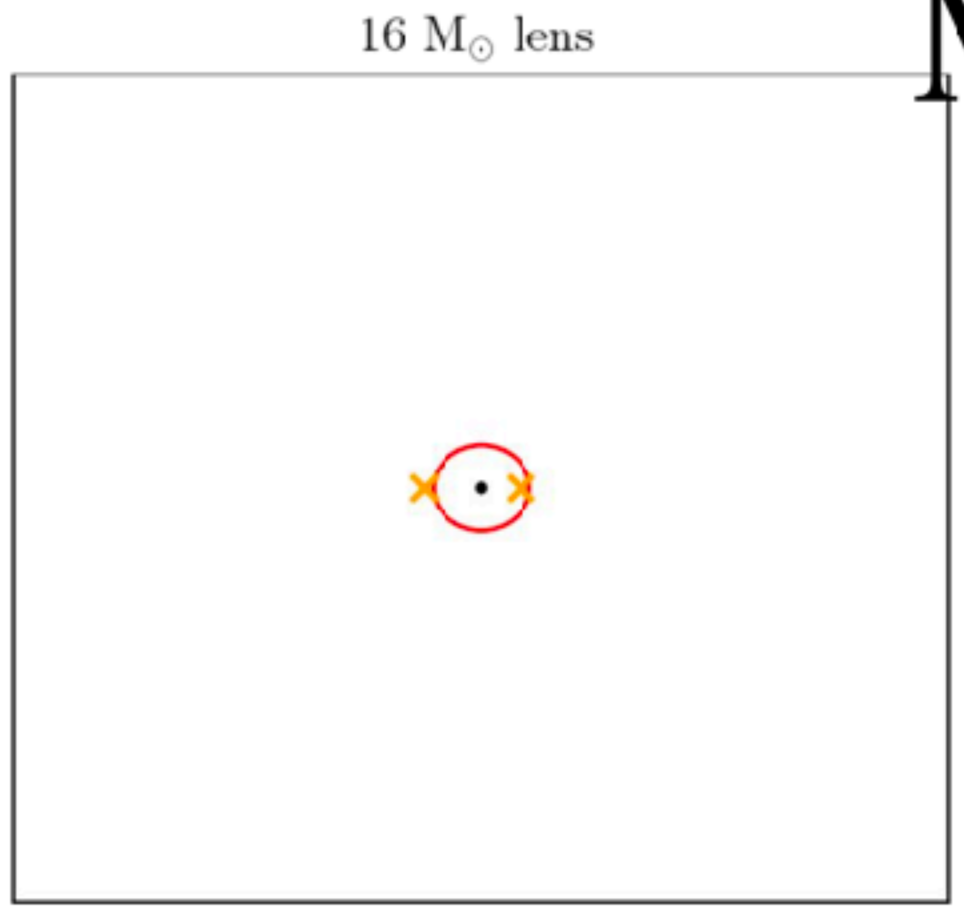


Millilensing:
- Modify gravitational-wave templates to identify millilensing "beating patterns"
- Targets: Subhalos, massive black holes

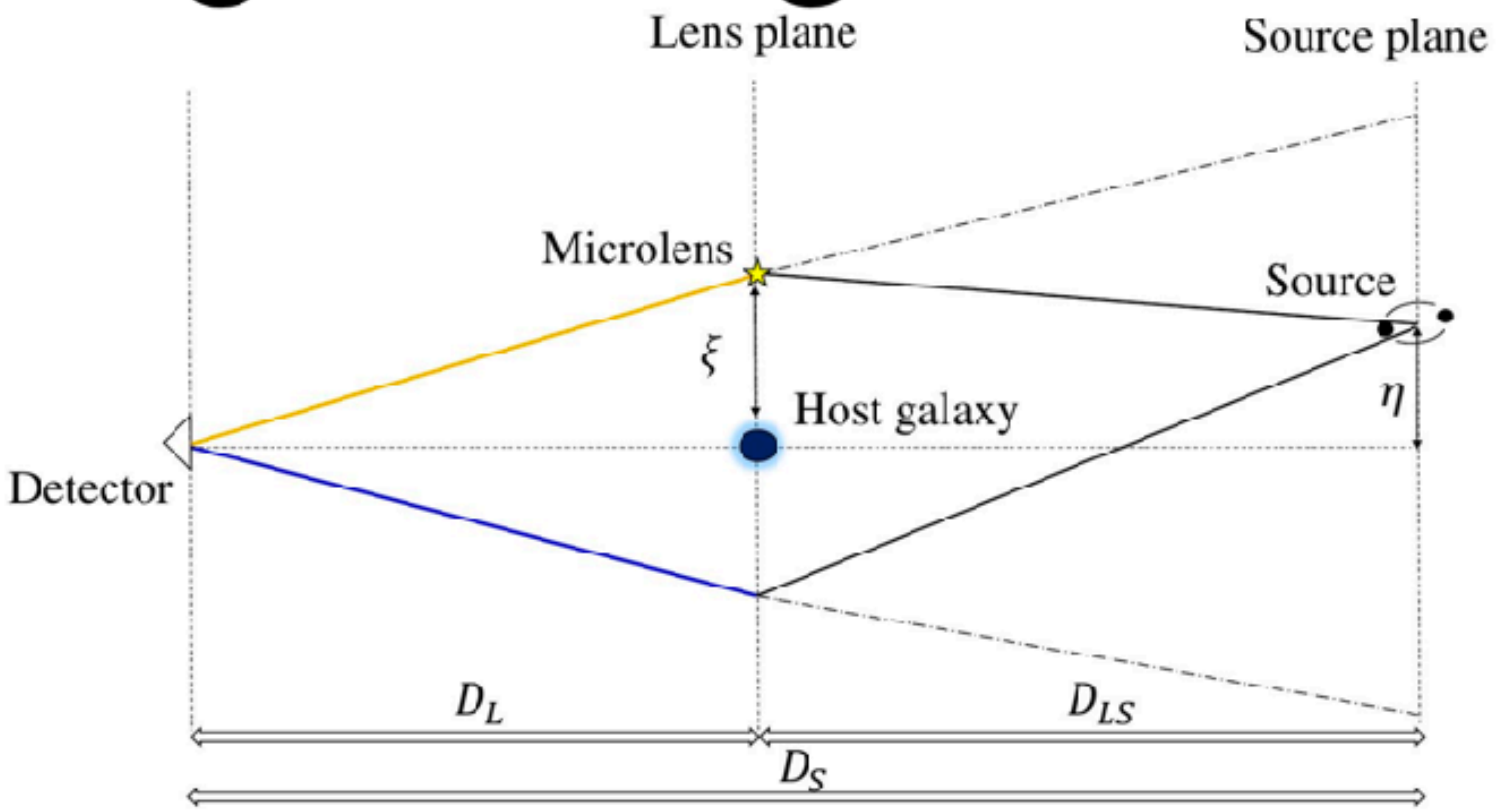
Measurement of the millilens **mass** and source position

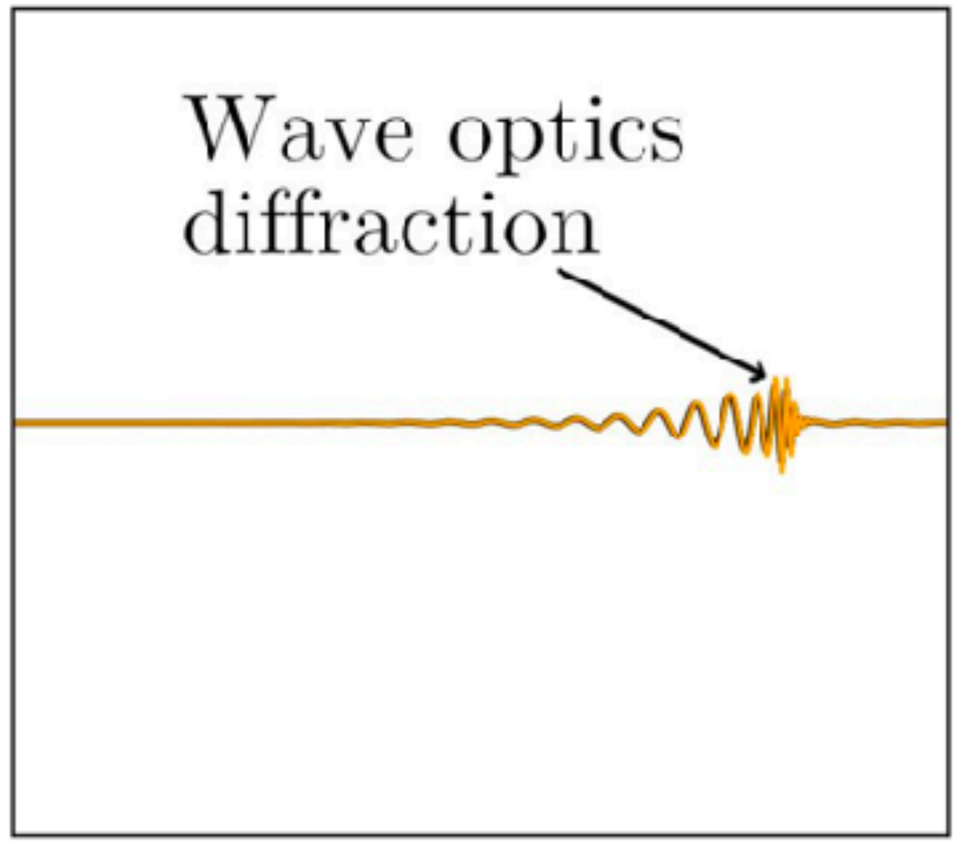
[credit: Otto Hannuksela]

Search for microlensing of gravitational waves



Microlensing challenge





Microlensing:

- Modify gravitational-wave templates to identify microlensing wave optics effects
- Targets: Stars, stellar-mass black holes and primordial black holes

Measurement of the microlensing **mass** and source position

[credit: Otto Hannuksela]

Search results (selected)

THE ASTROPHYSICAL JOURNAL LETTERS (O1 & O2 BBHs)

Search for Gravitational Lensing Signatures in LIGO-Virgo Binary Black Hole Events

O. A. Hannuksela[1], K. Haris[2], K. K. Y. Ng[3,4], S. Kumar[2,5,6], A. K. Mehta[2], D. Keitel[7], T. G. F. Li[1], and P. Ajith[2,8]

Published 2019 March 19 •

The Astrophysical Journal Letters, Volume 874, Number 1

Citation O. A. Hannuksela *et al* 2019 *ApJL* **874** L2

DOI 10.3847/2041-8213/ab0c0f

THE ASTROPHYSICAL JOURNAL (O3a CBCs)

Search for Lensing Signatures in the Gravitational-Wave Observations from the First Half of LIGO–Virgo's Third Observing Run

R. Abbott[1], T. D. Abbott[2], S. Abraham[3], F. Acernese[4,5], K. Ackley[6], A. Adams[7], C. Adams[8], R. X. Adhikari[1], V. B. Adya[9], C. Affeldt[10,11] Show full author list

Published 2021 December 10 •

The Astrophysical Journal, Volume 923, Number 1

Citation R. Abbott *et al* 2021 *ApJ* **923** 14

DOI 10.3847/1538-4357/ac23db

THE ASTROPHYSICAL JOURNAL (O1 & O2 BBHs; microlensing)

OPEN ACCESS

Deep Learning–based Search for Microlensing Signature from Binary Black Hole Events in GWTC-1 and -2

Kyungmin Kim[1,2], Joongoo Lee[3], Otto A. Hannuksela[4], and Tjonnie G. F. Li[4,5,6]

Published 2022 October 24 •

The Astrophysical Journal, Volume 938, Number 2

Citation Kyungmin Kim *et al* 2022 *ApJ* **938** 157

DOI 10.3847/1538-4357/ac92f3

(O3 CBCs)

arXiv > gr-qc > arXiv:2304.08393

General Relativity and Quantum Cosmology

[Submitted on 17 Apr 2023]

Search for gravitational-lensing signatures in the full third observing run of the LIGO-Virgo network

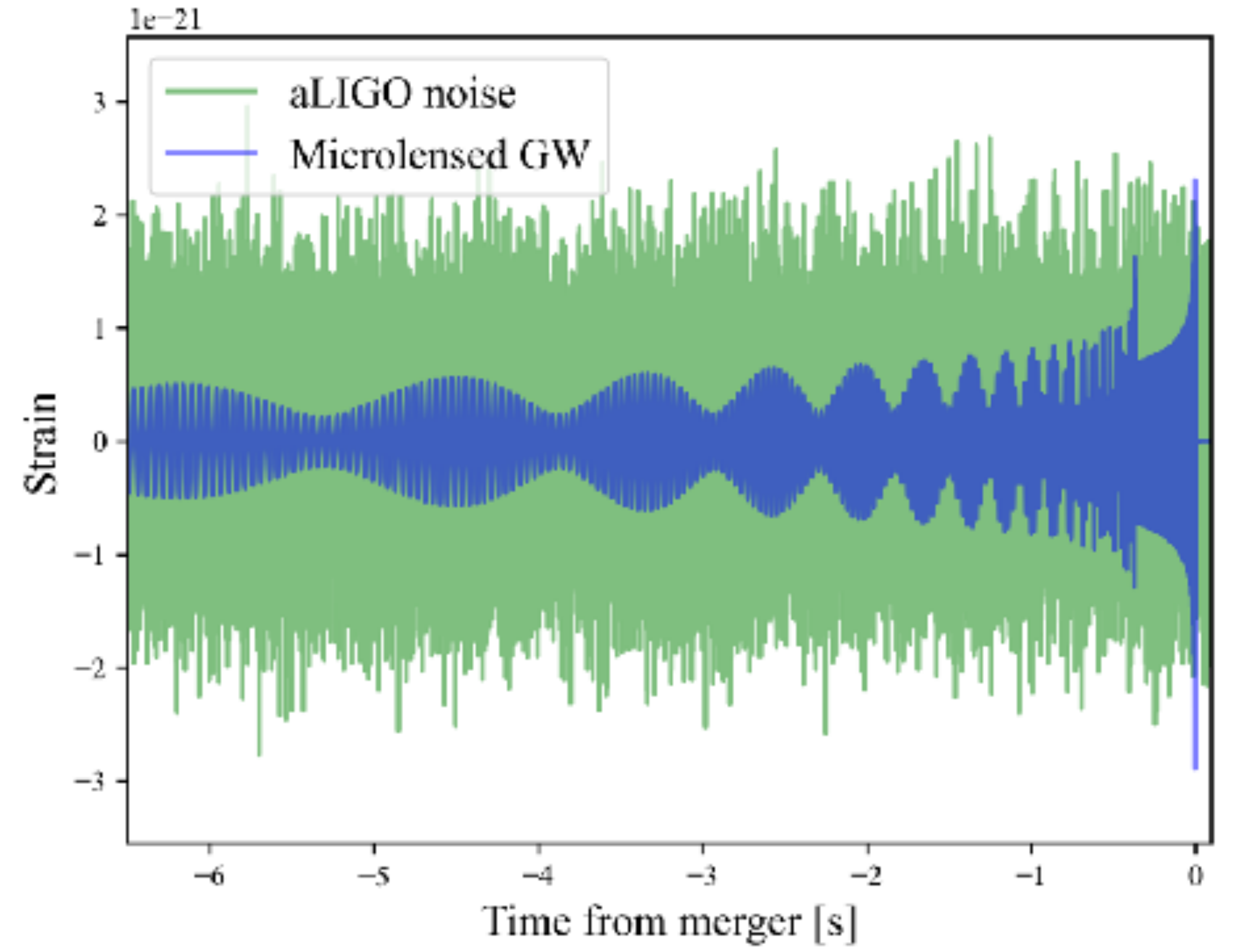
The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration: R. Abbott, H. Abe, F. Acernese, K. Ackley, S. Adhicary, N. Adhikari, R. X. Ad
Agarwal, M. Agathos, O. D. Aguiar, L. Aiello, A. Ain, P. Ajith, T. Akutsu, S. Albanesi, R. A. Alfaidi, C. Alléné, A. Allocca, P. A. Altin, A. Amato, S. Anand, A. Ananye
Ando, T. Andrade, N. Andres, M. Andrés-Carcasona, T. Andrić, S. Ansoldi, J. M. Antelis, S. Antier, T. Apostolatos, E. Z. Appavuravther, S. Appert, S. K. Apple, K.
Arène, N. Aritomi, N. Arnaud, M. Arogeti, S. M. Aronson, H. Asada, G. Ashton, Y. Aso, M. Assiduo, S. Assis de Souza Melo, S. M. Aston, P. Astone, F. Aubin, K. A
S. Bae, Y. Bae, S. Bagnasco, Y. Bai, J. G. Baier, J. Baird, R. Bajpai, T. Baka, M. Ball, G. Ballardin, S. W. Ballmer, G. Baltus, S. Banagiri, B. Banerjee, D. Bankar, J. C. Bai
Barone, B. Barr, L. Barsotti, M. Barsuglia, D. Barta, J. Bartlett, M. A. Barton, I. Bartos, S. Basak, R. Bassiri, A. Basti, M. Bawaj, J. C. Bayley, M. Bazzan, B. Bécsy, V. M
(1595 additional authors not shown)

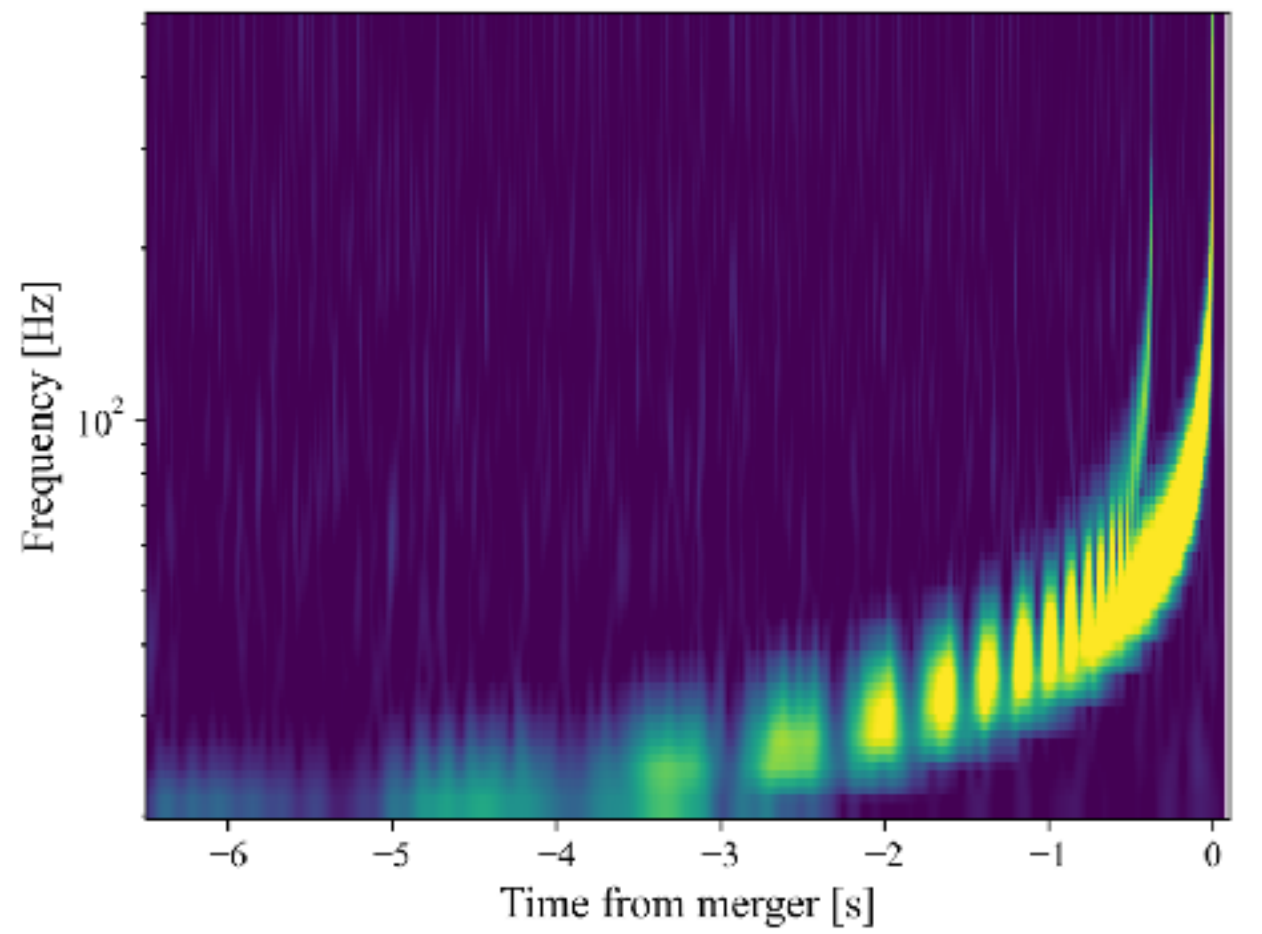
- No strong evidence has been found yet.

Deep learning-based search for microlensing signature from binary black hole events in GWTC-1 and -2

KK et al. (ApJ, 2022)

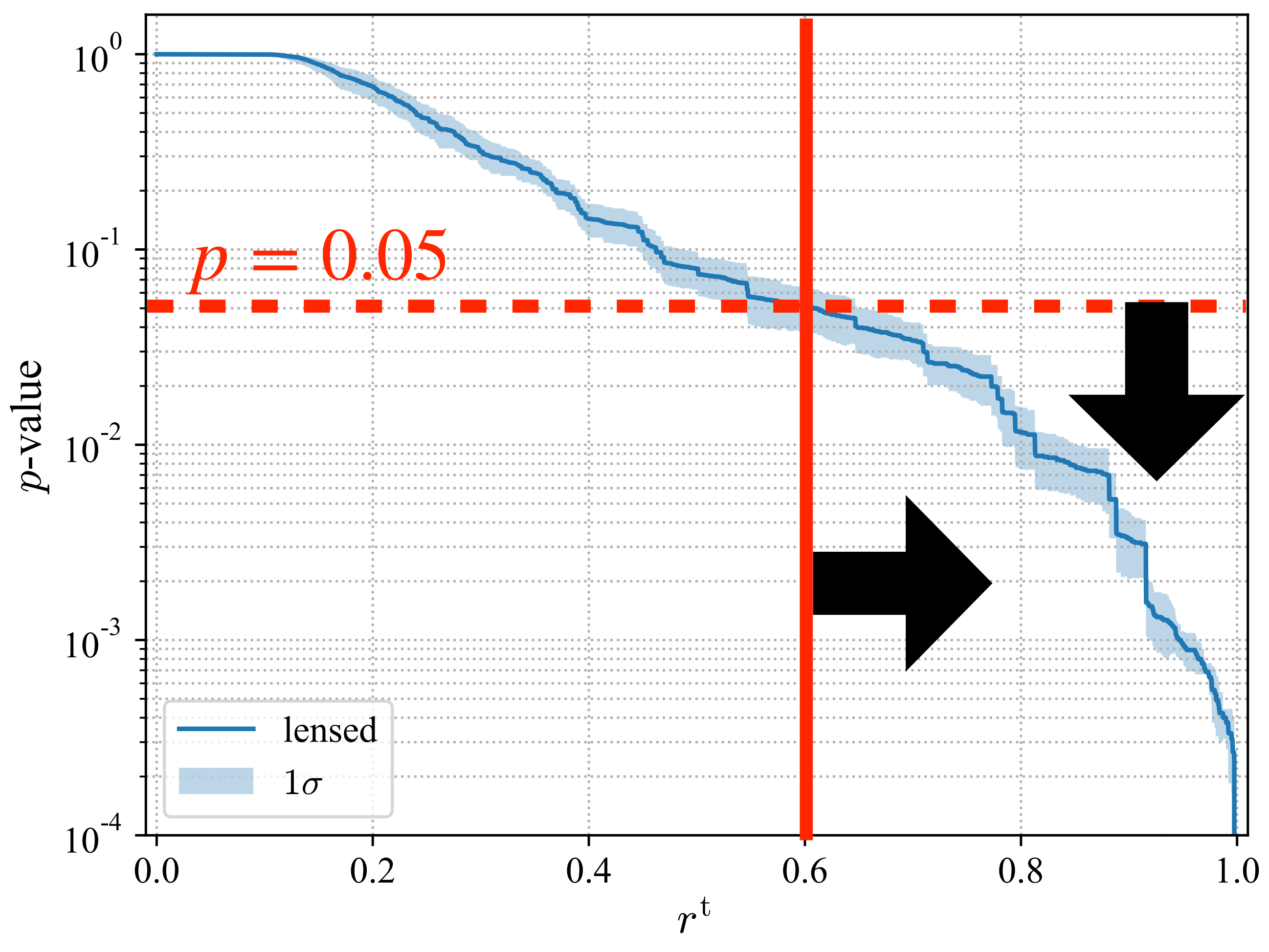
- **Motivation**
  - Microlensing of GWs (GW microlensing)
    - can be caused by stellar objects $\lesssim 10^5 M_\odot$ embedded around macrolenses like galaxies or galaxy clusters.
    - may arrive at detectors with $\mathcal{O}(1) \sim \mathcal{O}(100)$ ms of time delays between multiply lensed signals
      $\Rightarrow$ superposition of those signals
      $\Rightarrow$ interference patterns, a.k.a. *beating patterns*
  - We seek beating patterns from GW signals of binary black hole (BBH) events.
    - The first deep learning (DL)-based search for any lensing signature.
    - Revisit the 46 BBH events in GWTC-1 and -2 already analyzed by LIGO-Virgo-KAGRA collaboration to search lensing signatures in GWs via the Bayes factor-based analysis [Hannuksela+ (2019); Abbott+ (2021)].
    - Search the signature from spectrograms of BBH signals to bring the excellence of state-of-the-art DL models [Kim+ (2021)].

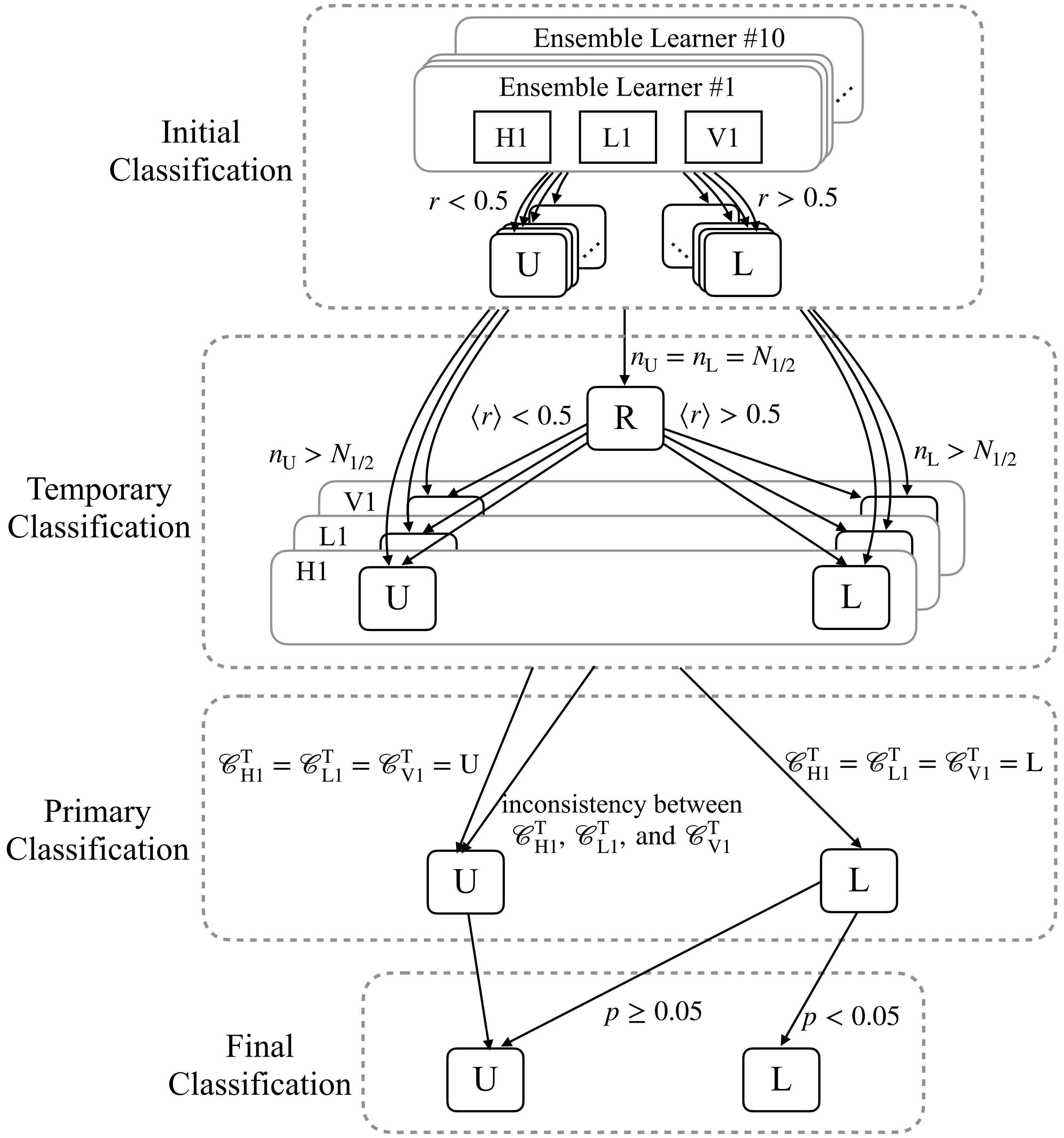




Deep learning-based search for microlensing signature from binary black hole events in GWTC-1 and -2

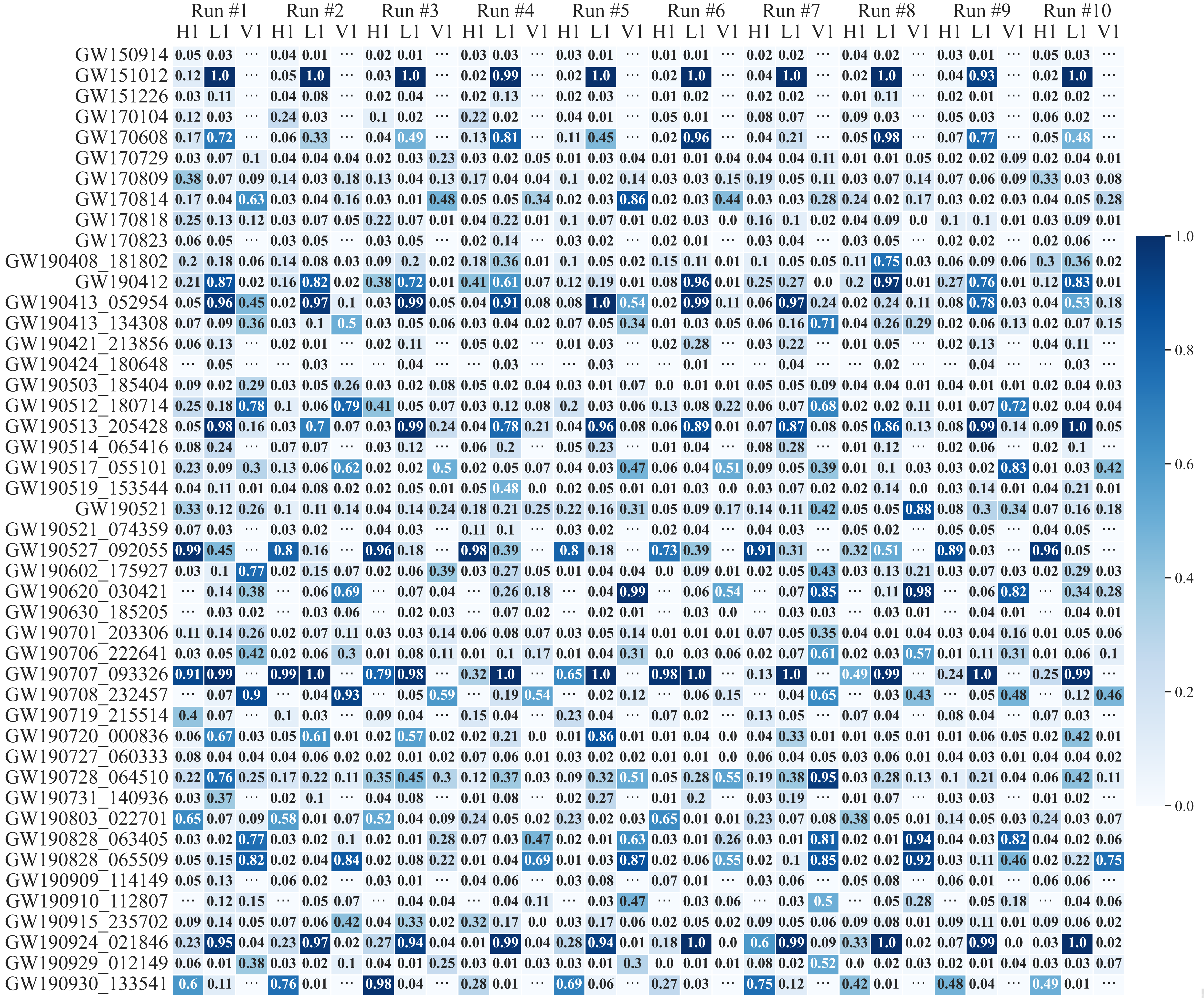
- **Implementation of deep learning**
  - VGG-19 + ensemble learning (to mitigate biased prediction may be caused by a single learner)
    - prepared 10 ensemble learners with altering random seed for the same training data
  - Binary classification scheme with hierarchical criterion
    - ***Initial classification*** on each detector's data based on the prediction, probability r to the lensed class, from ensemble learners
    - Majority voting-based ***temporary classification*** on each detector's data
    - ***Primary classification*** based on consistency between the class of each detector's data of an event
    - ***Final classification*** based on follow-up analyses and a cross-verification
  - Build a p-value model with testing data
    - Use the model as one of verification methods for the final classification.
    - Set the ***empirical criterion*** $r > 0.6$ for claiming the detection of a lensed signal





Deep learning-based search for microlensing signature from binary black hole events in GWTC-1 and -2

- Probability r to the lensed class from all ensemble learners for 46 evaluated BBH events in GWTC-1 and -2.
- Unlensed (U) if $r < 0.5$; Lensed (L) if $r > 0.5$



	Run #1 H1	Run #1 L1	Run #1 V1	Run #2 H1	Run #2 L1	Run #2 V1	Run #3 H1	Run #3 L1	Run #3 V1	Run #4 H1	Run #4 L1	Run #4 V1	Run #5 H1	Run #5 L1	Run #5 V1	Run #6 H1	Run #6 L1	Run #6 V1	Run #7 H1	Run #7 L1	Run #7 V1	Run #8 H1	Run #8 L1	Run #8 V1	Run #9 H1	Run #9 L1	Run #9 V1	Run #10 H1	Run #10 L1	Run #10 V1
GW150914	0.05	0.03	⋯	0.04	0.01	⋯	0.02	0.01	⋯	0.03	0.03	⋯	0.03	0.01	⋯	0.01	0.01	⋯	0.02	0.02	⋯	0.04	0.02	⋯	0.03	0.01	⋯	0.05	0.03	⋯
GW151012	0.12	1.0	⋯	0.05	1.0	⋯	0.03	1.0	⋯	0.02	0.99	⋯	0.02	1.0	⋯	0.02	1.0	⋯	0.04	1.0	⋯	0.02	1.0	⋯	0.04	0.93	⋯	0.02	1.0	⋯
GW151226	0.03	0.11	⋯	0.04	0.08	⋯	0.02	0.04	⋯	0.02	0.13	⋯	0.02	0.03	⋯	0.01	0.02	⋯	0.02	0.02	⋯	0.01	0.11	⋯	0.02	0.01	⋯	0.02	0.02	⋯
GW170104	0.12	0.03	⋯	0.24	0.03	⋯	0.1	0.02	⋯	0.22	0.02	⋯	0.04	0.01	⋯	0.05	0.01	⋯	0.08	0.07	⋯	0.09	0.03	⋯	0.05	0.03	⋯	0.06	0.02	⋯
GW170608	0.17	0.72	⋯	0.06	0.33	⋯	0.04	0.49	⋯	0.13	0.81	⋯	0.11	0.45	⋯	0.02	0.96	⋯	0.04	0.21	⋯	0.05	0.98	⋯	0.07	0.77	⋯	0.05	0.48	⋯
GW170729	0.03	0.07	0.1	0.04	0.04	0.04	0.02	0.03	0.23	0.03	0.02	0.05	0.01	0.03	0.04	0.01	0.01	0.04	0.04	0.04	0.11	0.01	0.01	0.05	0.02	0.02	0.09	0.02	0.04	0.01
GW170809	0.38	0.07	0.09	0.14	0.03	0.18	0.13	0.04	0.13	0.17	0.04	0.04	0.1	0.02	0.14	0.03	0.03	0.15	0.19	0.05	0.11	0.03	0.07	0.14	0.07	0.06	0.09	0.33	0.03	0.08
GW170814	0.17	0.04	0.63	0.03	0.04	0.16	0.03	0.01	0.48	0.05	0.05	0.34	0.02	0.03	0.86	0.02	0.03	0.44	0.03	0.03	0.28	0.24	0.02	0.17	0.03	0.02	0.03	0.04	0.05	0.28
GW170818	0.25	0.13	0.12	0.03	0.07	0.05	0.22	0.07	0.01	0.04	0.22	0.01	0.1	0.07	0.01	0.02	0.03	0.0	0.16	0.1	0.02	0.04	0.09	0.0	0.1	0.1	0.01	0.03	0.09	0.01
GW170823	0.06	0.05	⋯	0.03	0.05	⋯	0.03	0.05	⋯	0.02	0.14	⋯	0.03	0.02	⋯	0.02	0.01	⋯	0.03	0.04	⋯	0.03	0.05	⋯	0.02	0.02	⋯	0.02	0.06	⋯
GW190408_181802	0.2	0.18	0.06	0.14	0.08	0.03	0.09	0.2	0.02	0.18	0.36	0.01	0.1	0.05	0.02	0.15	0.11	0.01	0.1	0.05	0.05	0.11	0.75	0.03	0.06	0.09	0.06	0.3	0.36	0.02
GW190412	0.21	0.87	0.02	0.16	0.82	0.02	0.38	0.72	0.01	0.41	0.61	0.07	0.12	0.19	0.01	0.08	0.96	0.01	0.25	0.27	0.0	0.2	0.97	0.01	0.27	0.76	0.01	0.12	0.83	0.01
GW190413_052954	0.05	0.96	0.45	0.02	0.97	0.1	0.03	0.99	0.05	0.04	0.91	0.08	0.08	1.0	0.54	0.02	0.99	0.11	0.06	0.97	0.24	0.02	0.24	0.11	0.08	0.78	0.03	0.04	0.53	0.18
GW190413_134308	0.07	0.09	0.36	0.03	0.1	0.5	0.03	0.05	0.06	0.03	0.04	0.02	0.07	0.05	0.34	0.01	0.03	0.05	0.06	0.16	0.71	0.04	0.26	0.29	0.02	0.06	0.13	0.02	0.07	0.15
GW190421_213856	0.06	0.13	⋯	0.02	0.01	⋯	0.02	0.11	⋯	0.05	0.02	⋯	0.01	0.03	⋯	0.02	0.28	⋯	0.03	0.22	⋯	0.01	0.05	⋯	0.02	0.13	⋯	0.04	0.11	⋯
GW190424_180648	⋯	0.05	⋯	⋯	0.03	⋯	⋯	0.04	⋯	⋯	0.03	⋯	⋯	0.03	⋯	⋯	0.01	⋯	⋯	0.04	⋯	⋯	0.02	⋯	⋯	0.04	⋯	⋯	0.03	⋯
GW190503_185404	0.09	0.02	0.29	0.03	0.05	0.26	0.03	0.02	0.08	0.05	0.02	0.04	0.03	0.01	0.07	0.0	0.01	0.01	0.05	0.05	0.09	0.04	0.04	0.01	0.04	0.01	0.01	0.02	0.04	0.03
GW190512_180714	0.25	0.18	0.78	0.1	0.06	0.79	0.41	0.05	0.07	0.03	0.12	0.08	0.2	0.03	0.06	0.13	0.08	0.22	0.06	0.07	0.68	0.02	0.02	0.11	0.01	0.07	0.72	0.02	0.04	0.04
GW190513_205428	0.05	0.98	0.16	0.03	0.7	0.07	0.03	0.99	0.24	0.04	0.78	0.21	0.04	0.96	0.08	0.06	0.89	0.01	0.07	0.87	0.08	0.05	0.86	0.13	0.08	0.99	0.14	0.09	1.0	0.05
GW190514_065416	0.08	0.24	⋯	0.07	0.07	⋯	0.03	0.12	⋯	0.06	0.2	⋯	0.05	0.23	⋯	0.01	0.04	⋯	0.08	0.28	⋯	0.01	0.12	⋯	0.02	0.06	⋯	0.02	0.1	⋯
GW190517_055101	0.23	0.09	0.3	0.13	0.06	0.62	0.02	0.02	0.5	0.02	0.05	0.07	0.04	0.03	0.47	0.06	0.04	0.51	0.09	0.05	0.39	0.01	0.1	0.03	0.03	0.02	0.83	0.01	0.03	0.42
GW190519_153544	0.04	0.11	0.01	0.04	0.08	0.02	0.02	0.05	0.01	0.05	0.48	0.0	0.02	0.05	0.01	0.01	0.03	0.0	0.03	0.07	0.02	0.02	0.14	0.0	0.03	0.14	0.01	0.04	0.21	0.01
GW190521	0.33	0.12	0.26	0.1	0.11	0.14	0.04	0.14	0.24	0.18	0.21	0.25	0.22	0.16	0.31	0.05	0.09	0.17	0.14	0.11	0.42	0.05	0.05	0.88	0.08	0.3	0.34	0.07	0.16	0.18
GW190521_074359	0.07	0.03	⋯	0.03	0.02	⋯	0.04	0.03	⋯	0.11	0.1	⋯	0.03	0.02	⋯	0.02	0.04	⋯	0.04	0.03	⋯	0.05	0.02	⋯	0.05	0.05	⋯	0.04	0.05	⋯
GW190527_092055	0.99	0.45	⋯	0.8	0.16	⋯	0.96	0.18	⋯	0.98	0.39	⋯	0.8	0.18	⋯	0.73	0.39	⋯	0.91	0.31	⋯	0.32	0.51	⋯	0.89	0.03	⋯	0.96	0.05	⋯
GW190602_175927	0.03	0.1	0.77	0.02	0.15	0.07	0.02	0.06	0.39	0.03	0.27	0.05	0.01	0.04	0.04	0.0	0.09	0.01	0.02	0.05	0.43	0.03	0.13	0.21	0.03	0.07	0.03	0.02	0.29	0.03
GW190620_030421	⋯	0.14	0.38	⋯	0.06	0.69	⋯	0.07	0.04	⋯	0.26	0.18	⋯	0.04	0.99	⋯	0.06	0.54	⋯	0.07	0.85	⋯	0.11	0.98	⋯	0.06	0.82	⋯	0.34	0.28
GW190630_185205	⋯	0.03	0.02	⋯	0.03	0.06	⋯	0.02	0.03	⋯	0.07	0.02	⋯	0.02	0.01	⋯	0.03	0.0	⋯	0.03	0.03	⋯	0.03	0.01	⋯	0.04	0.01	⋯	0.04	0.01
GW190701_203306	0.11	0.14	0.26	0.02	0.07	0.11	0.03	0.03	0.14	0.06	0.08	0.07	0.03	0.05	0.14	0.01	0.01	0.01	0.07	0.05	0.35	0.04	0.01	0.04	0.03	0.04	0.16	0.01	0.05	0.06
GW190706_222641	0.03	0.05	0.42	0.02	0.06	0.3	0.01	0.08	0.11	0.01	0.1	0.17	0.01	0.04	0.31	0.0	0.03	0.06	0.02	0.07	0.61	0.02	0.03	0.57	0.01	0.11	0.31	0.01	0.06	0.1
GW190707_093326	0.91	0.99	⋯	0.99	1.0	⋯	0.79	0.98	⋯	0.32	1.0	⋯	0.65	1.0	⋯	0.98	1.0	⋯	0.13	1.0	⋯	0.49	0.99	⋯	0.24	1.0	⋯	0.25	0.99	⋯
GW190708_232457	⋯	0.07	0.9	⋯	0.04	0.93	⋯	0.05	0.59	⋯	0.19	0.54	⋯	0.02	0.12	⋯	0.06	0.15	⋯	0.04	0.65	⋯	0.03	0.43	⋯	0.05	0.48	⋯	0.12	0.46
GW190719_215514	0.4	0.07	⋯	0.1	0.03	⋯	0.09	0.04	⋯	0.15	0.04	⋯	0.23	0.04	⋯	0.07	0.02	⋯	0.13	0.05	⋯	0.07	0.04	⋯	0.08	0.04	⋯	0.07	0.03	⋯
GW190720_000836	0.06	0.67	0.03	0.05	0.61	0.01	0.02	0.57	0.02	0.02	0.21	0.0	0.01	0.86	0.01	0.01	0.04	0.0	0.04	0.33	0.01	0.01	0.05	0.01	0.01	0.06	0.05	0.02	0.42	0.01
GW190727_060333	0.08	0.04	0.04	0.04	0.06	0.02	0.02	0.01	0.02	0.07	0.06	0.01	0.03	0.02	0.02	0.01	0.01	0.0	0.06	0.04	0.05	0.03	0.06	0.01	0.04	0.03	0.01	0.04	0.04	0.02
GW190728_064510	0.22	0.76	0.25	0.17	0.22	0.11	0.35	0.45	0.3	0.12	0.37	0.03	0.09	0.32	0.51	0.05	0.28	0.55	0.19	0.38	0.95	0.03	0.28	0.13	0.1	0.21	0.04	0.06	0.42	0.11
GW190731_140936	0.03	0.37	⋯	0.02	0.1	⋯	0.04	0.08	⋯	0.01	0.08	⋯	0.02	0.27	⋯	0.01	0.2	⋯	0.03	0.19	⋯	0.01	0.07	⋯	0.03	0.03	⋯	0.01	0.02	⋯
GW190803_022701	0.65	0.07	0.09	0.58	0.01	0.07	0.52	0.04	0.09	0.24	0.05	0.02	0.23	0.02	0.03	0.65	0.01	0.01	0.23	0.07	0.08	0.38	0.05	0.01	0.14	0.05	0.03	0.24	0.03	0.07
GW190828_063405	0.03	0.02	0.77	0.03	0.02	0.1	0.02	0.01	0.28	0.07	0.03	0.47	0.02	0.01	0.63	0.03	0.01	0.26	0.03	0.01	0.81	0.02	0.01	0.94	0.04	0.03	0.82	0.04	0.02	0.06
GW190828_065509	0.05	0.15	0.82	0.02	0.04	0.84	0.02	0.08	0.22	0.01	0.04	0.69	0.01	0.03	0.87	0.02	0.06	0.55	0.02	0.1	0.85	0.02	0.02	0.92	0.03	0.11	0.46	0.02	0.22	0.75
GW190909_114149	0.05	0.13	⋯	0.06	0.02	⋯	0.03	0.01	⋯	0.04	0.06	⋯	0.03	0.08	⋯	0.07	0.01	⋯	0.03	0.06	⋯	0.05	0.08	⋯	0.06	0.01	⋯	0.06	0.06	⋯
GW190910_112807	⋯	0.12	0.15	⋯	0.05	0.07	⋯	0.04	0.04	⋯	0.04	0.11	⋯	0.03	0.47	⋯	0.03	0.06	⋯	0.03	0.5	⋯	0.05	0.28	⋯	0.05	0.18	⋯	0.04	0.06
GW190915_235702	0.09	0.14	0.05	0.07	0.06	0.42	0.04	0.33	0.02	0.32	0.17	0.0	0.03	0.17	0.06	0.02	0.05	0.02	0.09	0.05	0.06	0.09	0.08	0.01	0.09	0.11	0.01	0.09	0.06	0.02
GW190924_021846	0.23	0.95	0.04	0.23	0.97	0.02	0.27	0.94	0.04	0.01	0.99	0.04	0.28	0.94	0.01	0.18	1.0	0.0	0.6	0.99	0.09	0.33	1.0	0.02	0.07	0.99	0.0	0.03	1.0	0.02
GW190929_012149	0.06	0.01	0.38	0.03	0.02	0.1	0.04	0.01	0.25	0.03	0.01	0.03	0.03	0.01	0.3	0.0	0.01	0.01	0.08	0.02	0.52	0.0	0.02	0.03	0.02	0.01	0.04	0.03	0.03	0.07
GW190930_133541	0.6	0.11	⋯	0.76	0.01	⋯	0.98	0.04	⋯	0.28	0.01	⋯	0.69	0.06	⋯	0.27	0.03	⋯	0.75	0.12	⋯	0.42	0.01	⋯	0.48	0.04	⋯	0.49	0.01	⋯

Events	Run #1 H1	Run #1 L1	Run #1 V1	Run #2 H1	Run #2 L1	Run #2 V1	Run #3 H1	Run #3 L1	Run #3 V1	Run #4 H1	Run #4 L1	Run #4 V1	Run #5 H1	Run #5 L1	Run #5 V1	Run #6 H1	Run #6 L1	Run #6 V1	Run #7 H1	Run #7 L1	Run #7 V1	Run #8 H1	Run #8 L1	Run #8 V1	Run #9 H1	Run #9 L1	Run #9 V1	Run #10 H1	Run #10 L1	Run #10 V1
GW150914	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW151012	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯
GW151226	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW170104	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW170608	U	L	⋯	U	U	⋯	U	U	⋯	U	L	⋯	U	U	⋯	U	L	⋯	U	U	⋯	U	L	⋯	U	L	⋯	U	U	⋯
GW170729	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW170809	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW170814	U	U	L	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW170818	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW170823	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190408_181802	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U
GW190412	U	L	U	U	L	U	U	L	U	U	L	U	U	U	U	U	L	U	U	U	U	U	L	U	U	L	U	U	L	U
GW190413_052954	U	L	U	U	L	U	U	L	U	U	L	U	U	L	L	U	L	U	U	L	U	U	U	U	U	L	U	U	L	U
GW190413_134308	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U	U	U
GW190421_213856	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190424_180648	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯	⋯	U	⋯
GW190503_185404	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190512_180714	U	U	L	U	U	L	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	L	U	U	U
GW190513_205428	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U
GW190514_065416	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190517_055101	U	U	U	U	U	L	U	U	L	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U	U	L	U	U	U
GW190519_153544	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190521	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U
GW190521_074359	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190527_092055	L	U	⋯	L	U	⋯	L	U	⋯	L	U	⋯	L	U	⋯	L	U	⋯	L	U	⋯	U	L	⋯	L	U	⋯	L	U	⋯
GW190602_175927	U	U	L	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190620_030421	⋯	U	U	⋯	U	L	⋯	U	U	⋯	U	U	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	U
GW190630_185205	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U
GW190701_203306	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190706_222641	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	L	U	U	U	U	U	U
GW190707_093326	L	L	⋯	L	L	⋯	L	L	⋯	U	L	⋯	L	L	⋯	L	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯
GW190708_232457	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	L	⋯	U	U	⋯	U	U	⋯	U	L	⋯	U	U	⋯	U	U	⋯	U	U
GW190719_215514	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190720_000836	U	L	U	U	L	U	U	L	U	U	U	U	U	L	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190727_060333	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190728_064510	U	L	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	L	U	U	L	U	U	U	U	U	U	U	U	U
GW190731_140936	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190803_022701	L	U	U	L	U	U	L	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190828_063405	U	U	L	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	L	U	U	L	U	U	L	U	U	U
GW190828_065509	U	U	L	U	U	L	U	U	U	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	U	U	U	L
GW190909_114149	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯
GW190910_112807	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯	U	U
GW190915_235702	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U
GW190924_021846	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	U	L	U	L	L	U	U	L	U	U	L	U	U	L	U
GW190929_012149	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	U	L	U	U	U	U	U	U	U	U	U
GW190930_133541	L	U	⋯	L	U	⋯	L	U	⋯	U	U	⋯	L	U	⋯	U	U	⋯	L	U	⋯	U	U	⋯	U	U	⋯	U	U	⋯

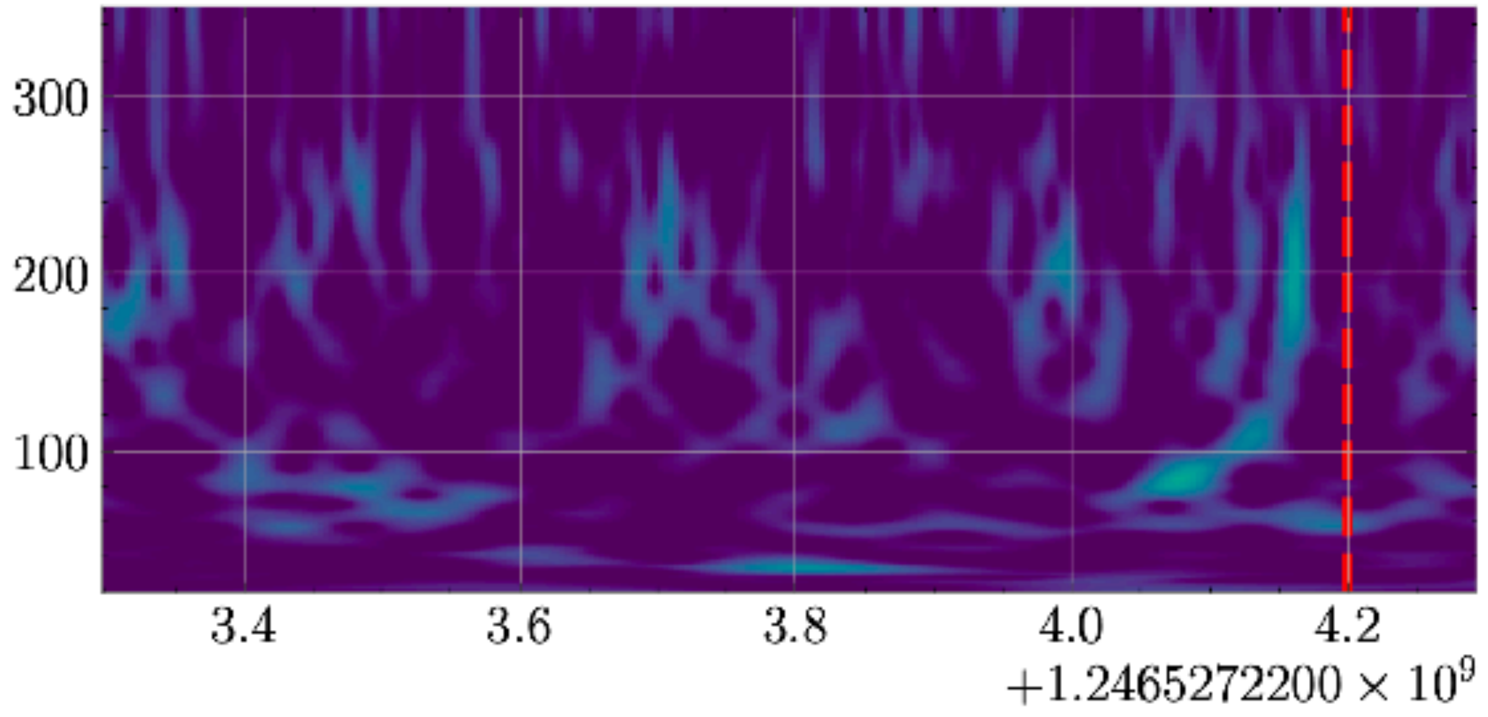
Deep learning-based search for microlensing signature from binary black hole events in GWTC-1 and -2

- **Results: Temporary, Primary, and Final Classes**

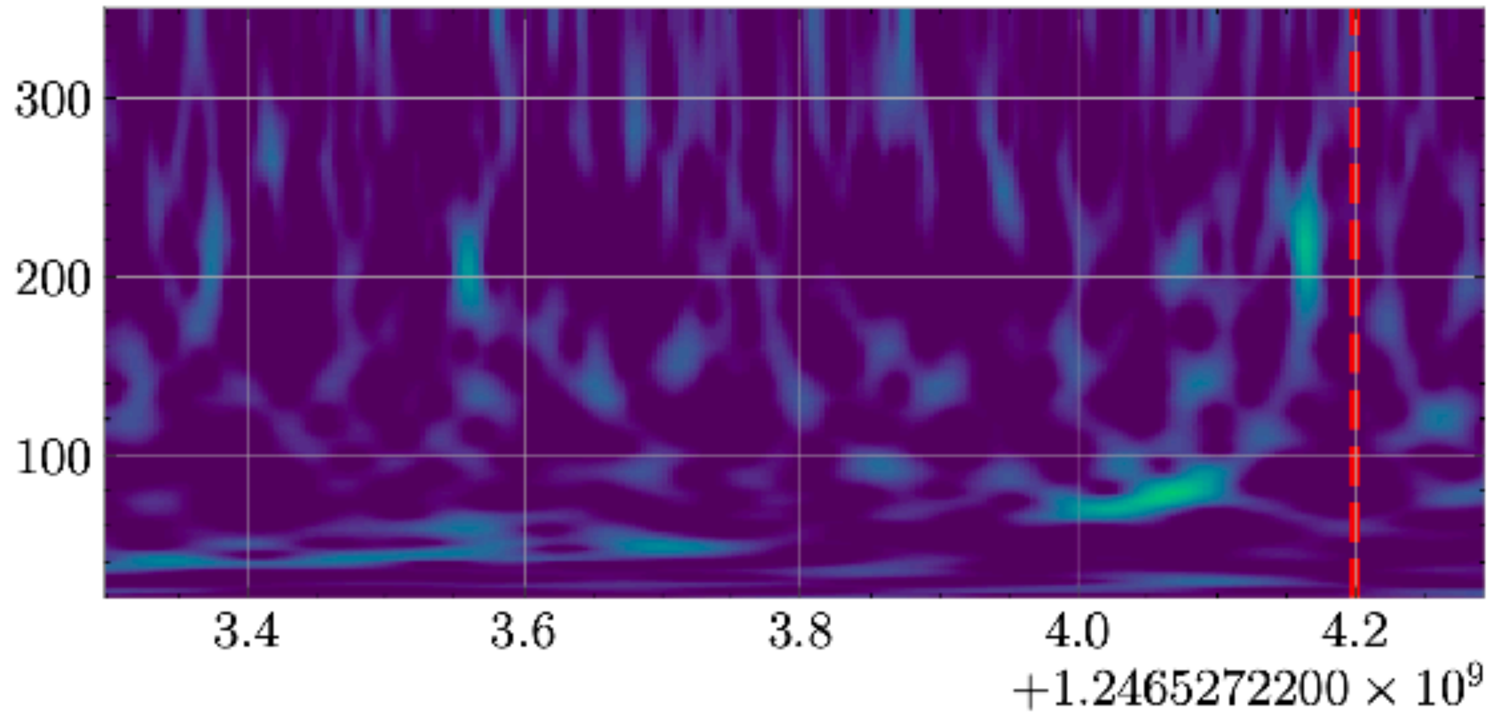
Event	Temporary Class			Primary Class	Final Class
	H1	L1	V1		
GW150914	U	U	···	U	U
GW151012	U	L	···	U	U
GW151226	U	U	···	U	U
GW170104	U	U	···	U	U
GW170608	U	L*	···	U	U
GW170729	U	U	U	U	U
GW170809	U	U	U	U	U
GW170814	U	U	U	U	U
GW170818	U	U	U	U	U
GW170823	U	U	···	U	U
GW190408_181802	U	U	U	U	U
GW190412	U	L	U	U	U
GW190413_052954	U	L	U	U	U
GW190413_134308	U	U	U	U	U
GW190421_213856	U	U	···	U	U
GW190424_180648	···	U	···	U	U
GW190503_185404	U	U	U	U	U
GW190512_180714	U	U	U	U	U
GW190513_205428	U	L	U	U	U
GW190514_065416	U	U	···	U	U
GW190517_055101	U	U	U	U	U
GW190519_153544	U	U	U	U	U
GW190521	U	U	U	U	U
GW190521_074359	U	U	···	U	U
GW190527_092055	L	U	···	U	U
GW190602_175927	U	U	U	U	U
GW190620_030421	···	U	L	U	U
GW190630_185205	···	U	U	U	U
GW190701_203306	U	U	U	U	U
GW190706_222641	U	U	U	U	U
GW190707_093326	L*	L	···	L	U
GW190708_232457	···	U	L*	U	U
GW190719_215514	U	U	···	U	U
GW190720_000836	U	U	U	U	U
GW190727_060333	U	U	U	U	U
GW190728_064510	U	U	U	U	U
GW190731_140936	U	U	···	U	U
GW190803_022701	U	U	U	U	U
GW190828_063405	U	U	L*	U	U
GW190828_065509	U	U	L	U	U
GW190909_114149	U	U	···	U	U
GW190910_112807	···	U	U	U	U
GW190915_235702	U	U	U	U	U
GW190924_021846	U	L	U	U	U
GW190929_012149	U	U	U	U	U
GW190930_133541	L*	U	···	U	U

Deep learning-based search for microlensing signature from binary black hole events in GWTC-1 and -2

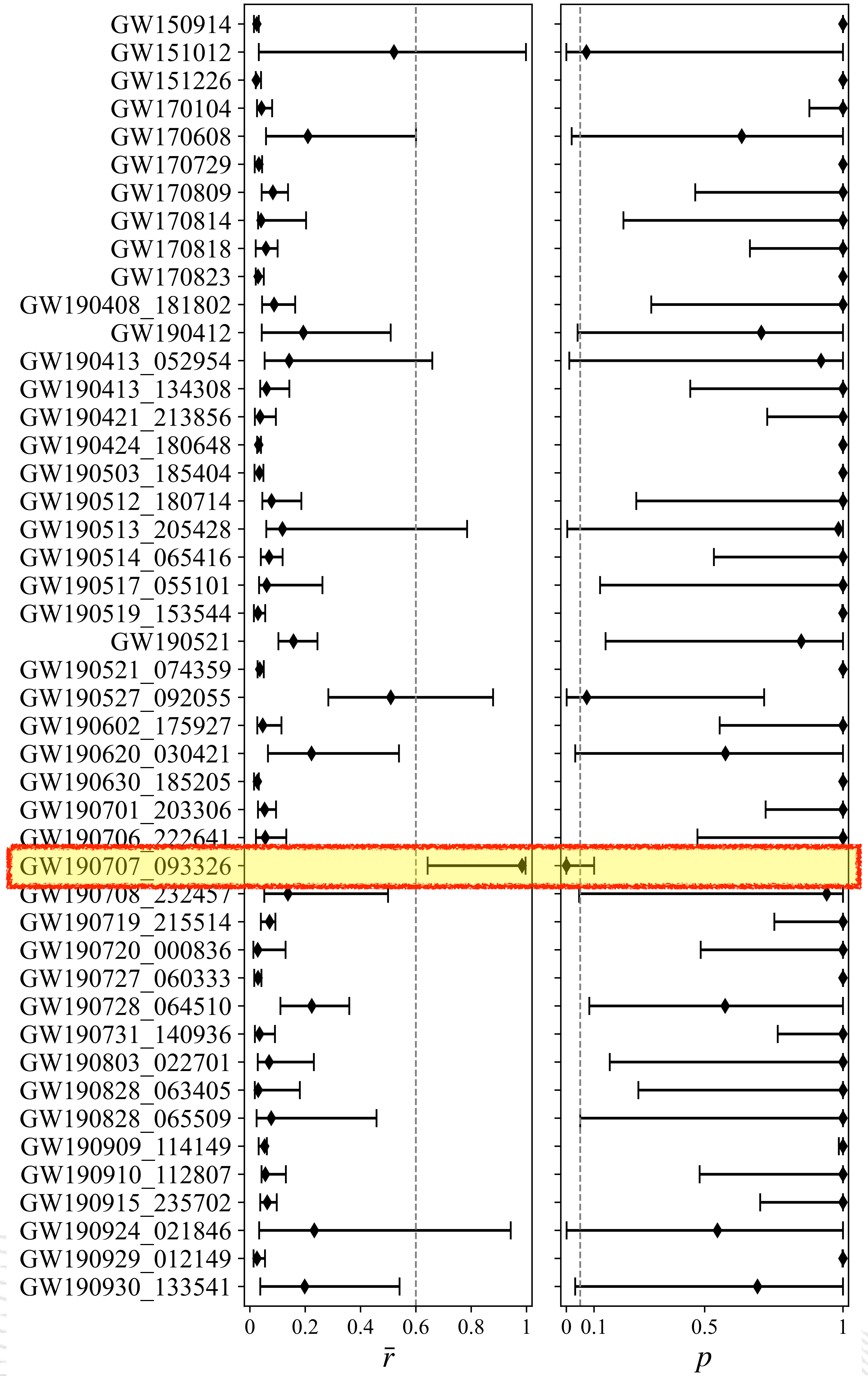
- **GW190707_093326**
  - Primary classification: Lensed (the only event out of 46)
  - $\bar{r} = 0.984^{+0.012}_{-0.342}$ with 90% C.I. (from bootstrapping)
    - $0 \lesssim p \lesssim 0.1$
  - The uncertainty of p includes the possibility of the unlensed hypothesis being true, i.e., $p \geq 0.05$.
    - c.f., $\mathscr{B}^{\mathrm{ML}}_{\mathrm{U}}$ = -0.4 disfavoring lensed hypothesis [Abbott+ (2021)]
  - No visually recognizable signature of beating patterns.

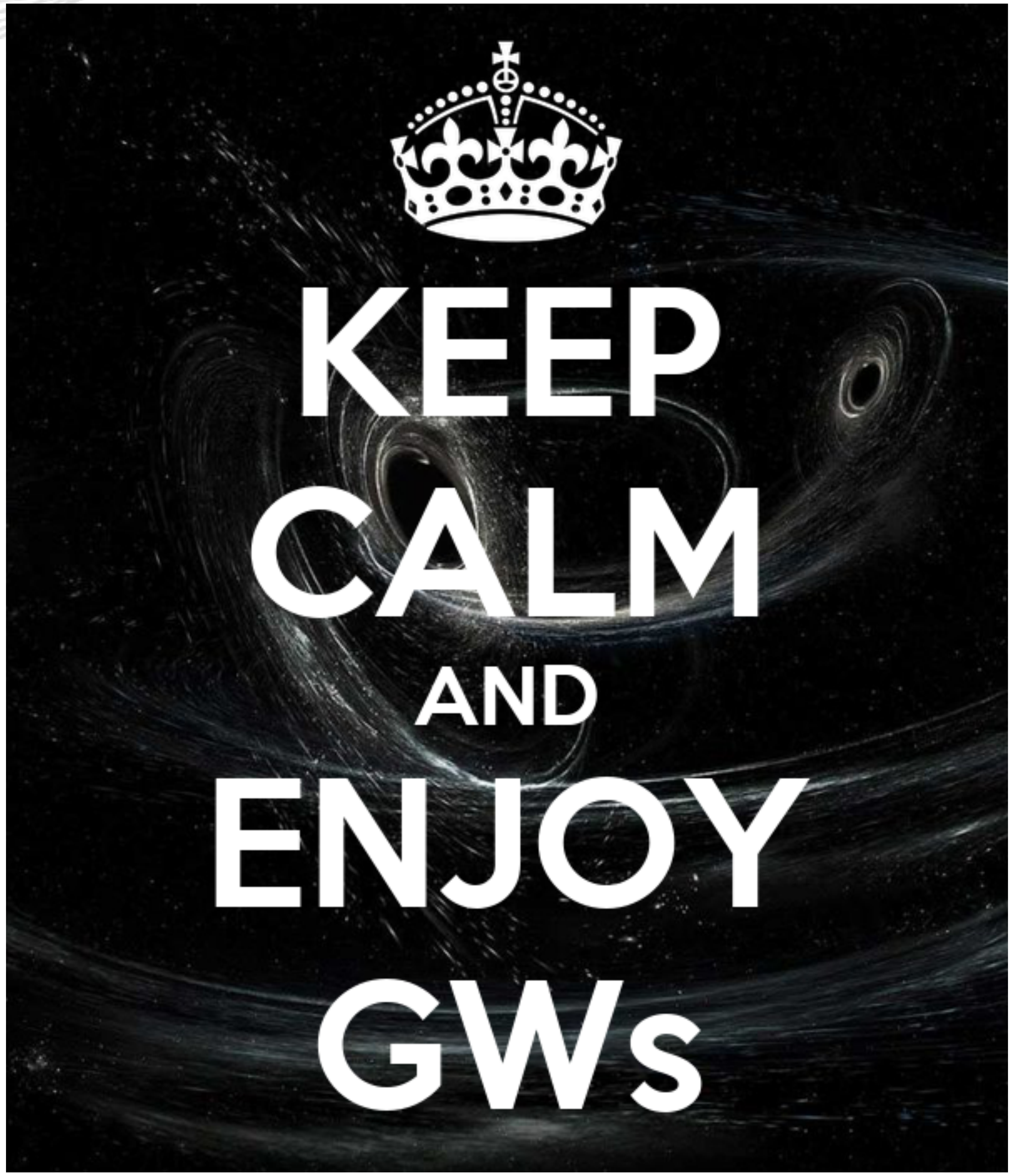


(a) GW190707_093326 (LIGO-Hanford)



(b) GW190707_093326 (LIGO-Livingston)





Thank you!