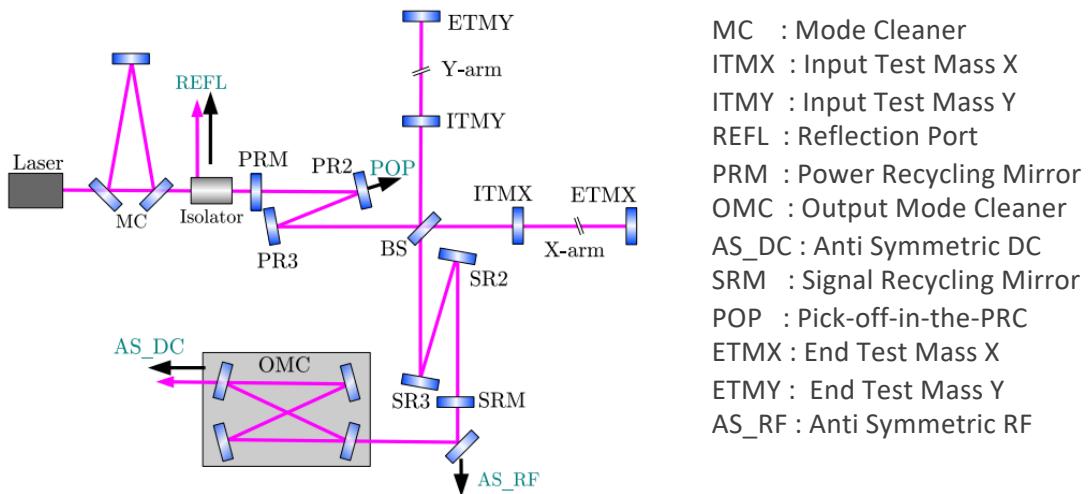
A detailed 3D rendering of a gravitational wave detector, likely the LIGO or Virgo interferometer. It features two long, parallel vertical pipes extending from a central complex of tanks and pumps. A blue horizontal bar highlights a specific section of the lower pipe. The background is a dark, star-filled space with a prominent nebula and a bright star.

# Quantum noise of gravitational wave detector and QND measurement

# Quantum noise of GW detector

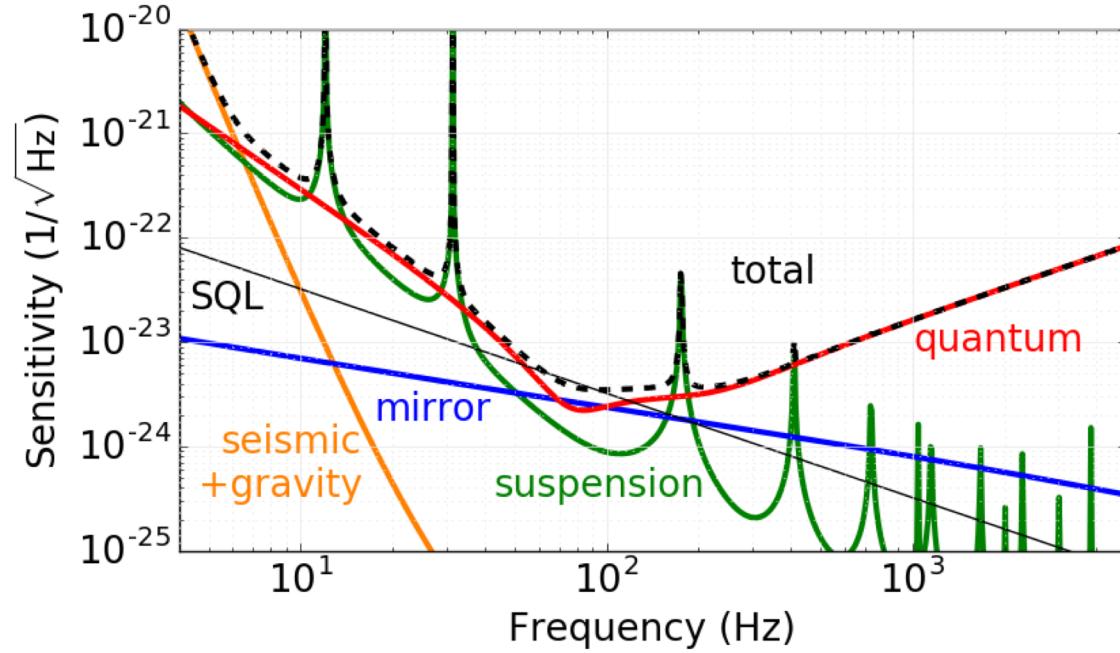
# ■ Interferometer of GW detector



Y. Aso et al. (KAGRA Collaboration), Phys.Rev. D88, 043007 (2013)

Interferometer of KAGRA  
(gravitational wave detector)

# Target sensitivity of KAGRA

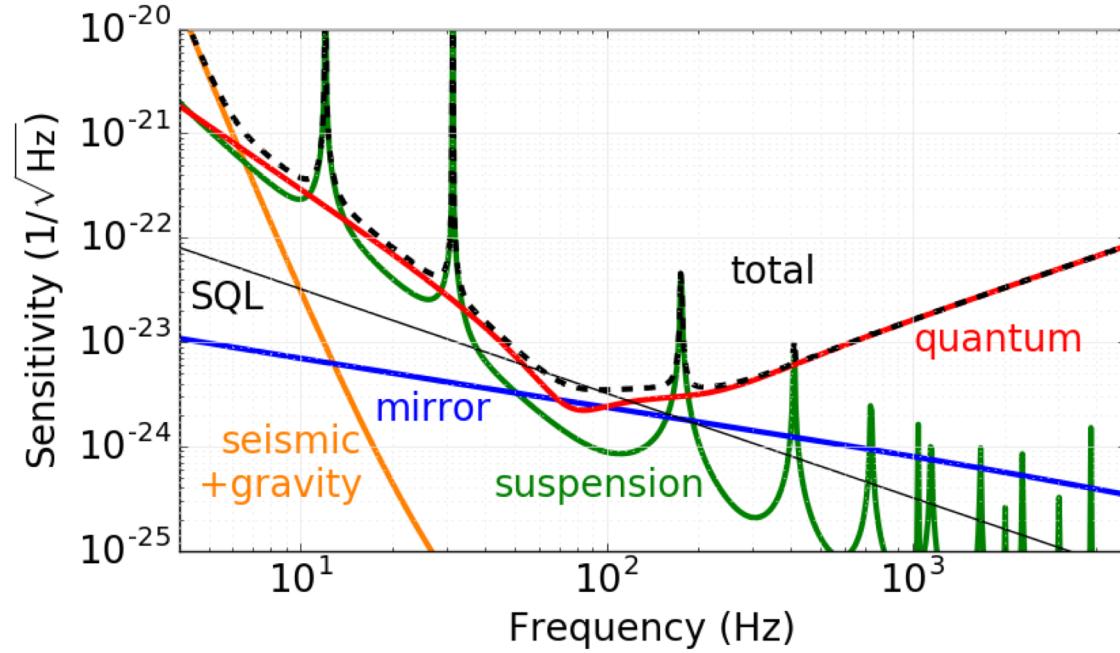


# ■ VIRGO detector

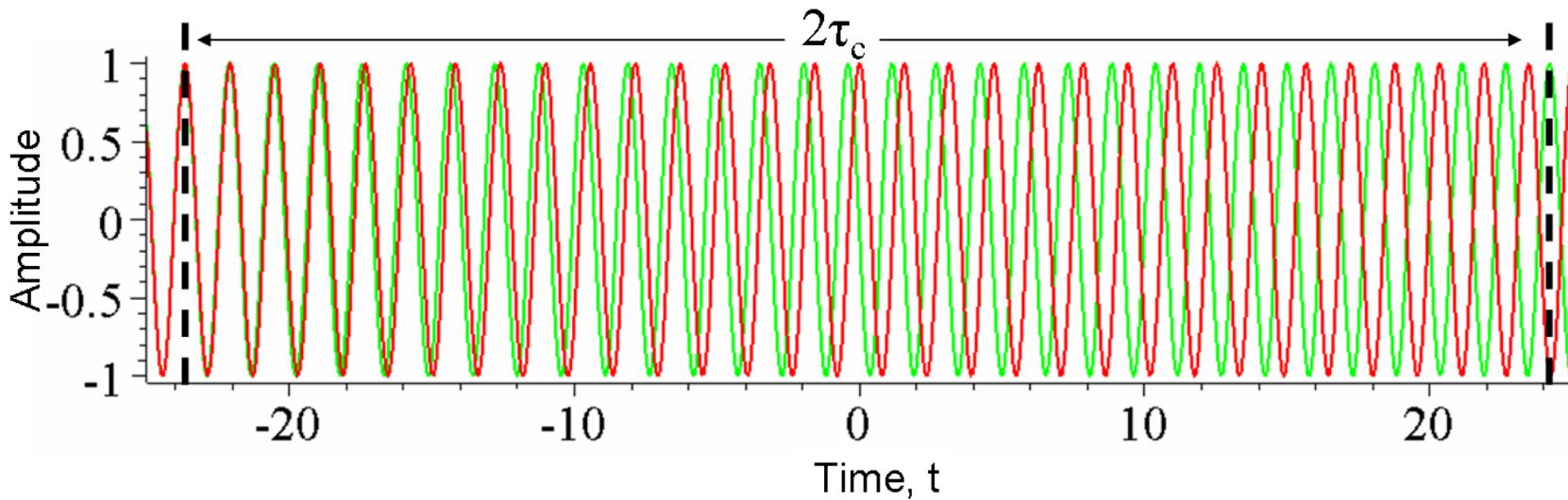
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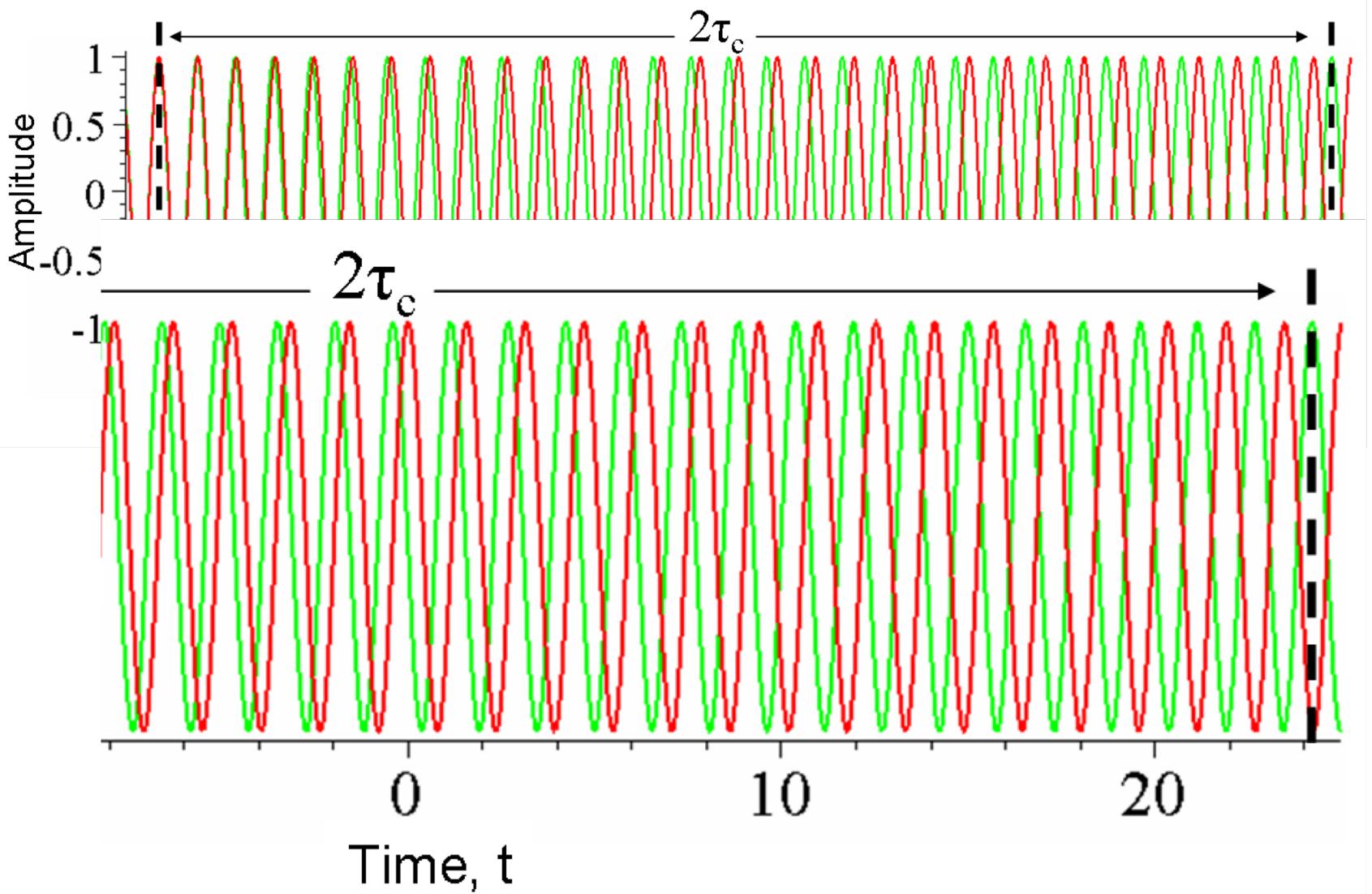
# Target sensitivity of KAGRA



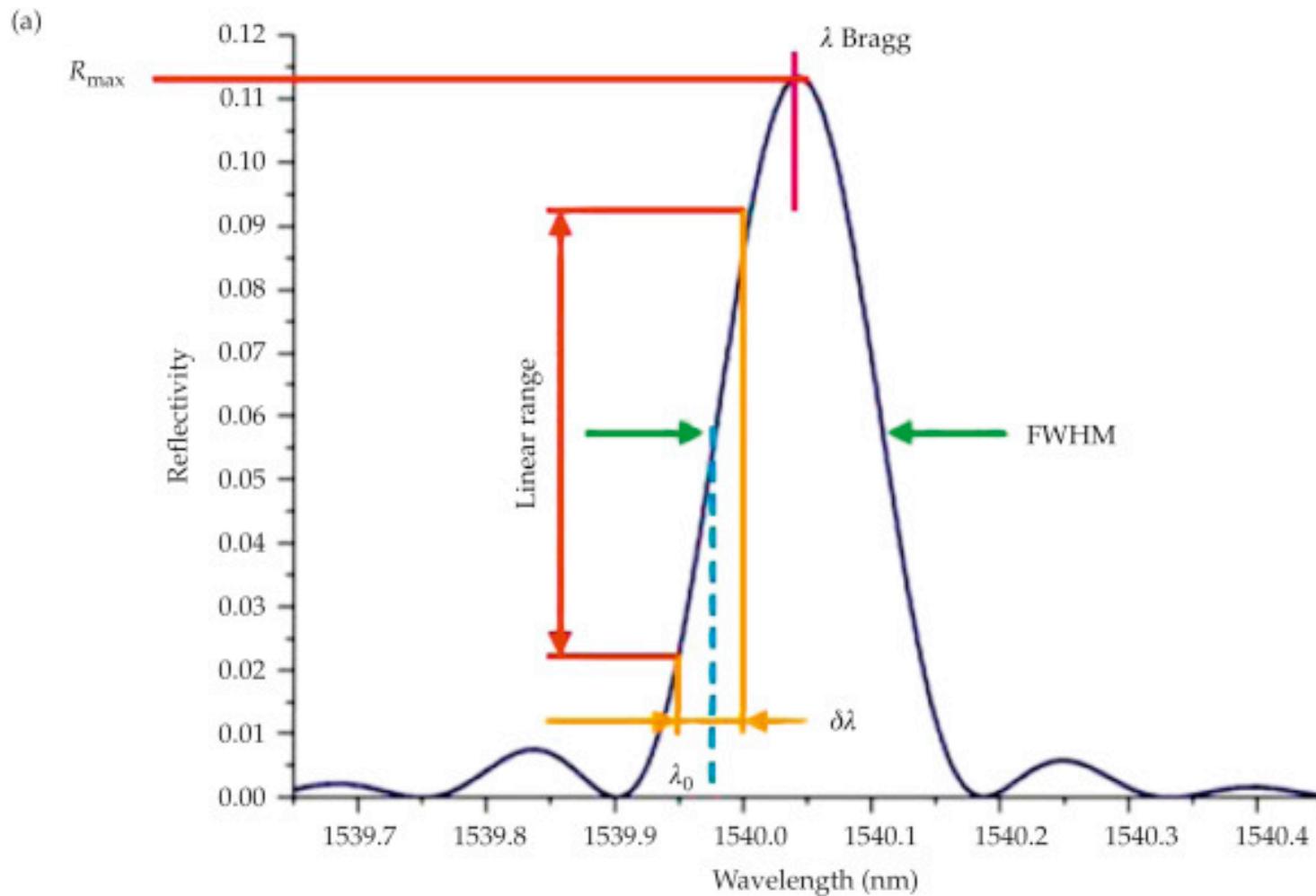
## ■ Coherence



## ■ Coherence



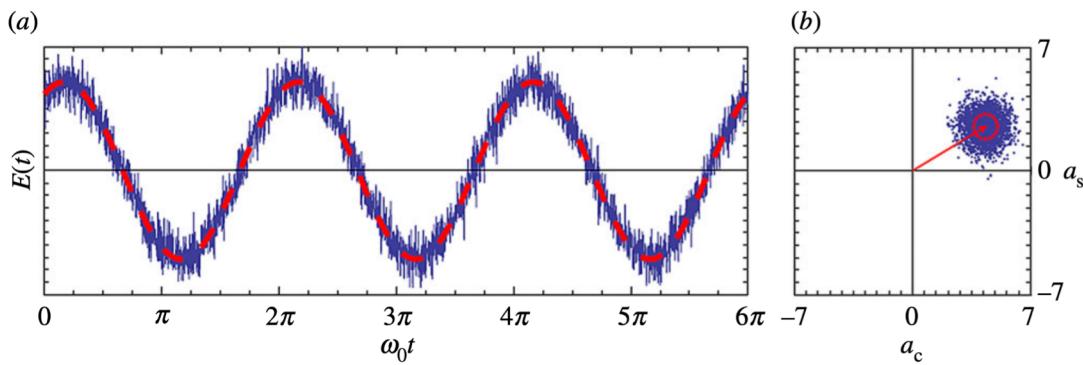
# Linewidth of laser



[Transmitter and Receiver Design for Amplified Lightwave Systems](#)

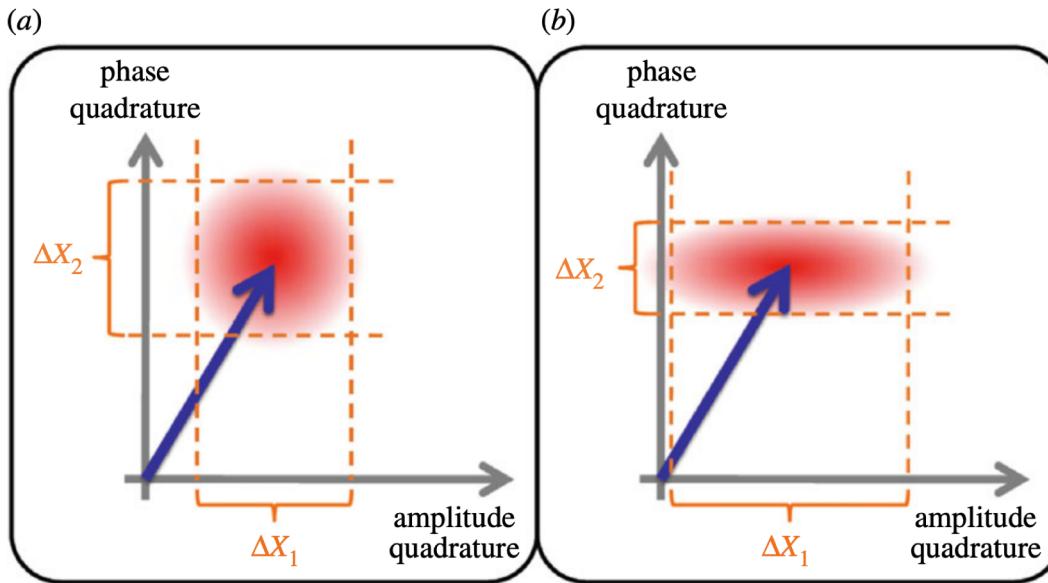
Daniel A. Fishman, B. Scott Jackson, in [Optical Fiber Telecommunications \(Third Edition\), Volume B](#), 1997

# Quantum noise of coherent light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

# Phase and amplitude noise of light

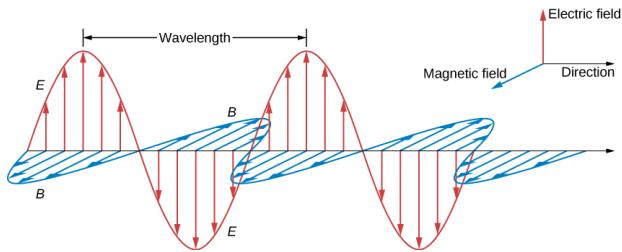


Heurs M. 2018 Gravitational wave detection using  
laser interferometry beyond the standard quantum  
limit. Phil. Trans. R. Soc. A 376: 20170289.

# ■ Amplitude

---

$$\text{Intensity} \rightarrow |\vec{E}|^2 \rightarrow (W/m^2) \propto (\text{Number of photon})$$

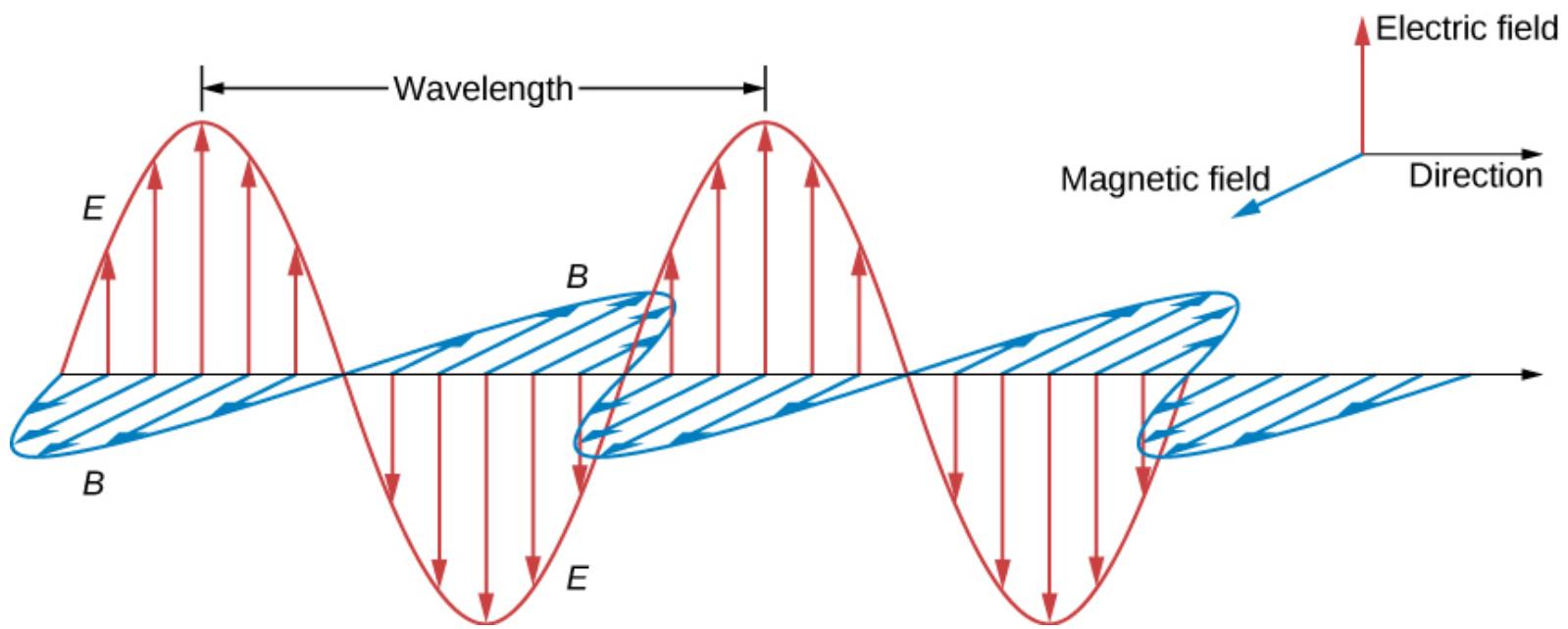


$$\text{Photo diode current } (I) \propto \text{Intensity} \propto (\text{Number of photon})$$

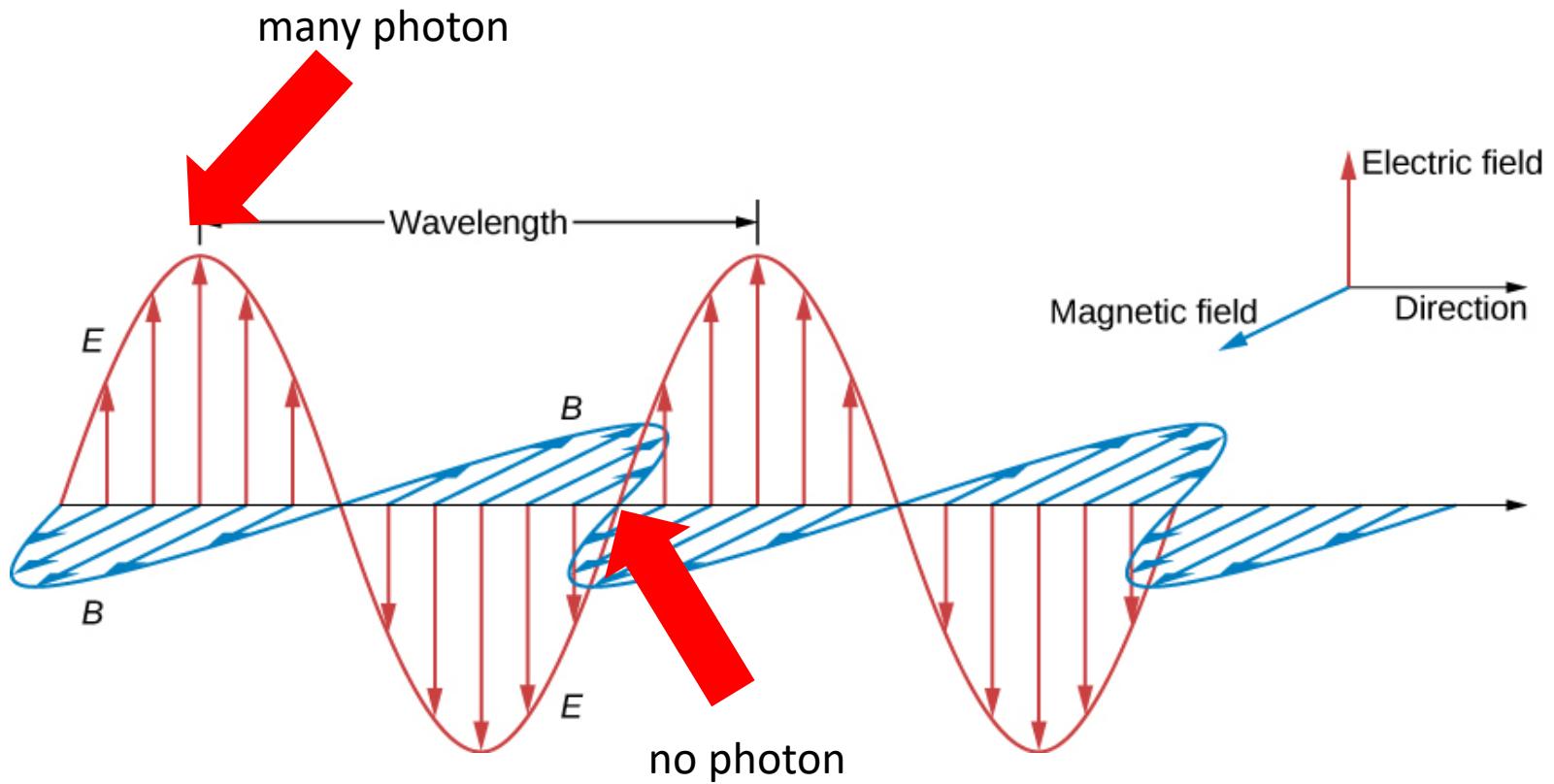
## Momentum

# Phase

---



# Phase



Position

# Classical light

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$$\text{Intensity} \rightarrow |\vec{E}|^2 \rightarrow (W/m^2) \propto (\text{Number of photon})$$

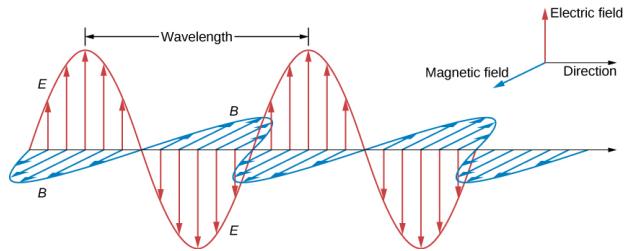
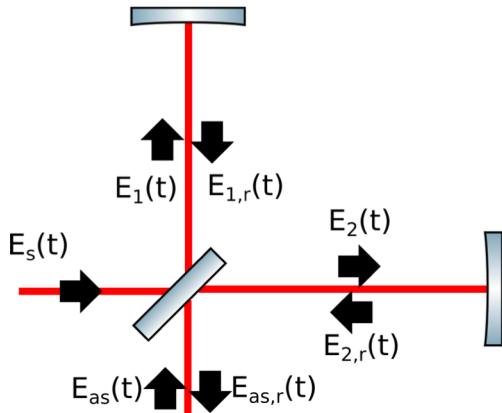


Photo diode current ( $I$ )  $\propto$  Intensity  $\propto$  (Number of photon)

$$E_{1,r}(t) \approx \frac{1}{\sqrt{2}}[E_0 \cos(\omega_0 t) - E_0 \sin(\omega_0 t) \frac{2\omega_0 x_1(t)}{c} + E_{as}(t)]$$
$$E_{2,r}(t) \approx \frac{1}{\sqrt{2}}[E_0 \cos(\omega_0 t) - E_0 \sin(\omega_0 t) \frac{2\omega_0 x_2(t)}{c} - E_{as}(t)]$$

# Quantum noise of interferometer



Assume anti-symmetric port is dark port

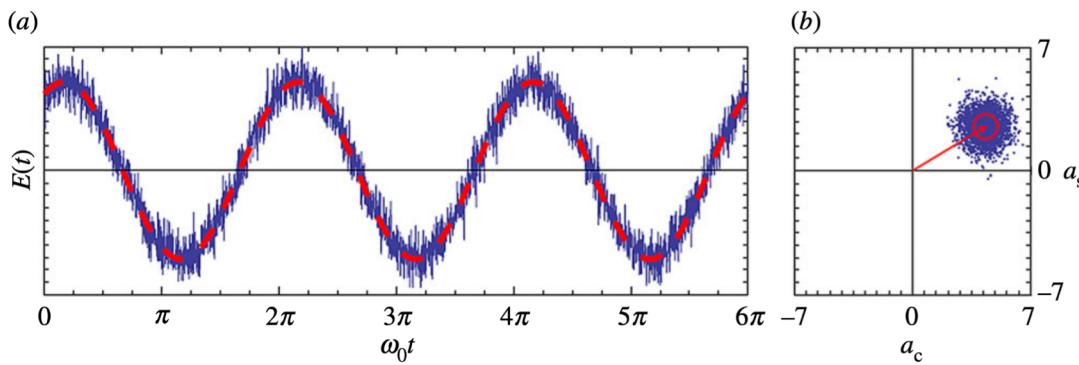
$$E_1(t) = \frac{1}{\sqrt{2}}[E_s(t) + E_{as}(t)]$$

$$E_2(t) = \frac{1}{\sqrt{2}}[E_s(t) - E_{as}(t)]$$

'as' is vacuum field

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

# Quantum noise of coherent light



Heurs M. 2018 Gravitational wave detection using laser interferometry beyond the standard quantum limit. Phil. Trans. R. Soc. A 376: 20170289.

# Quantum shot noise

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Quantum Shot noise

-> Noise induced by photon fluctuation

$$\text{Intensity} \rightarrow |\vec{E}|^2 \rightarrow (W/m^2) \propto (\text{Number of photon})$$

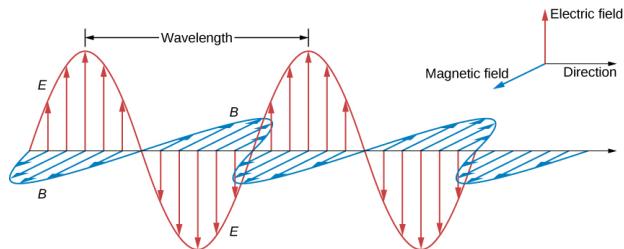
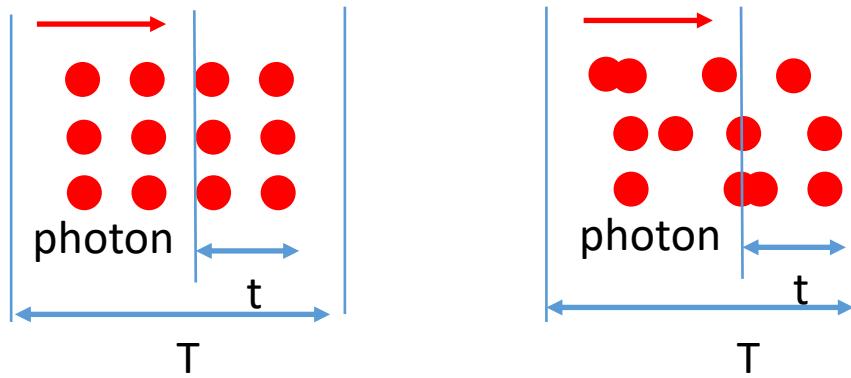


Photo diode current ( $I$ )  $\propto$  Intensity  $\propto$  (Number of photon)

# ■ Shot noise of interferometer

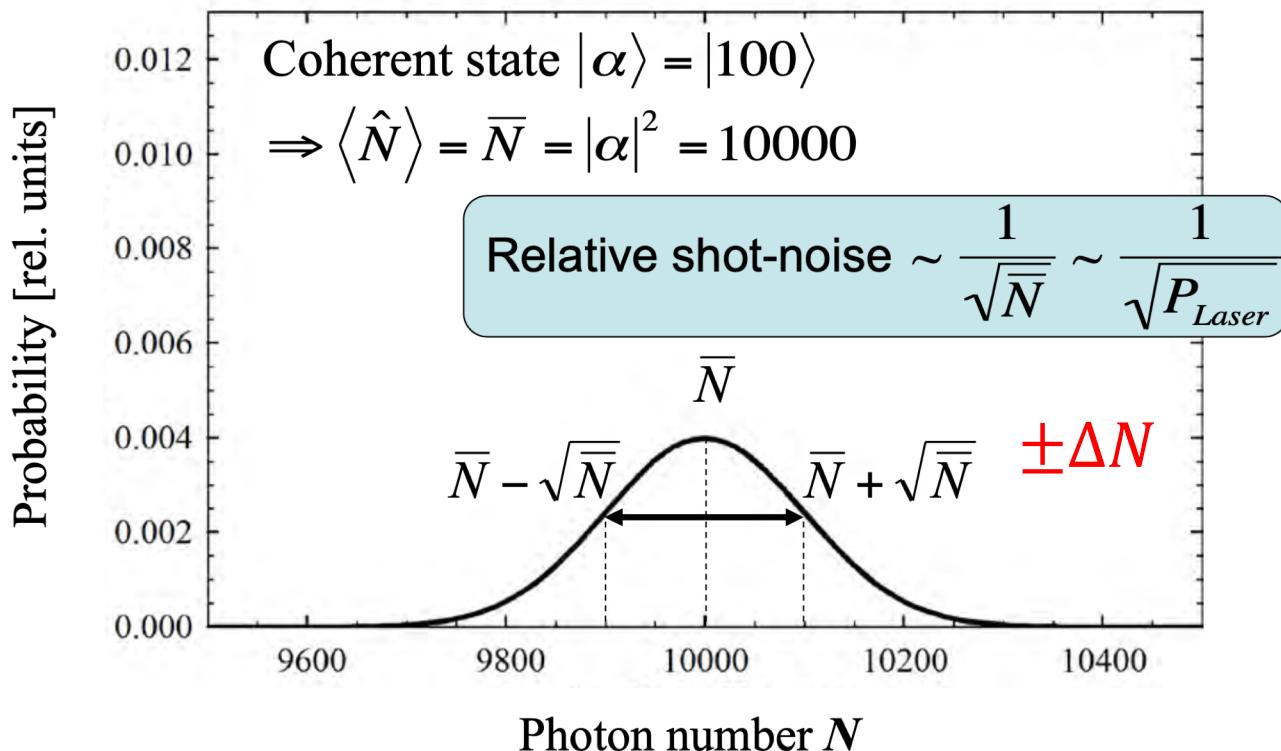
---

Laser power = the number of photon / time



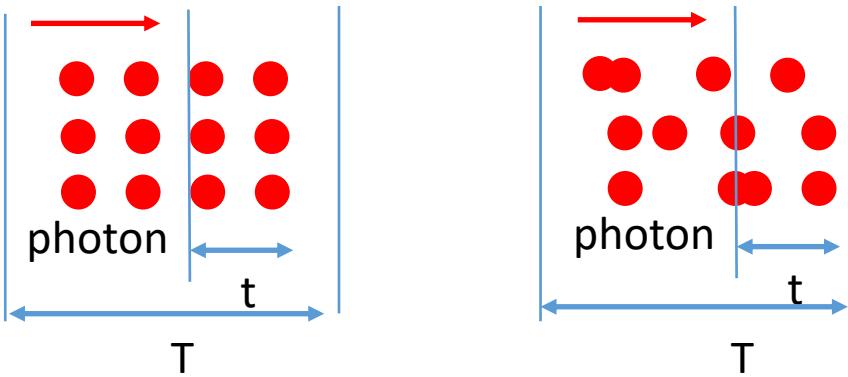
Shot noise

# Photon Counting Statistics



## ■ Shot noise of interferometer

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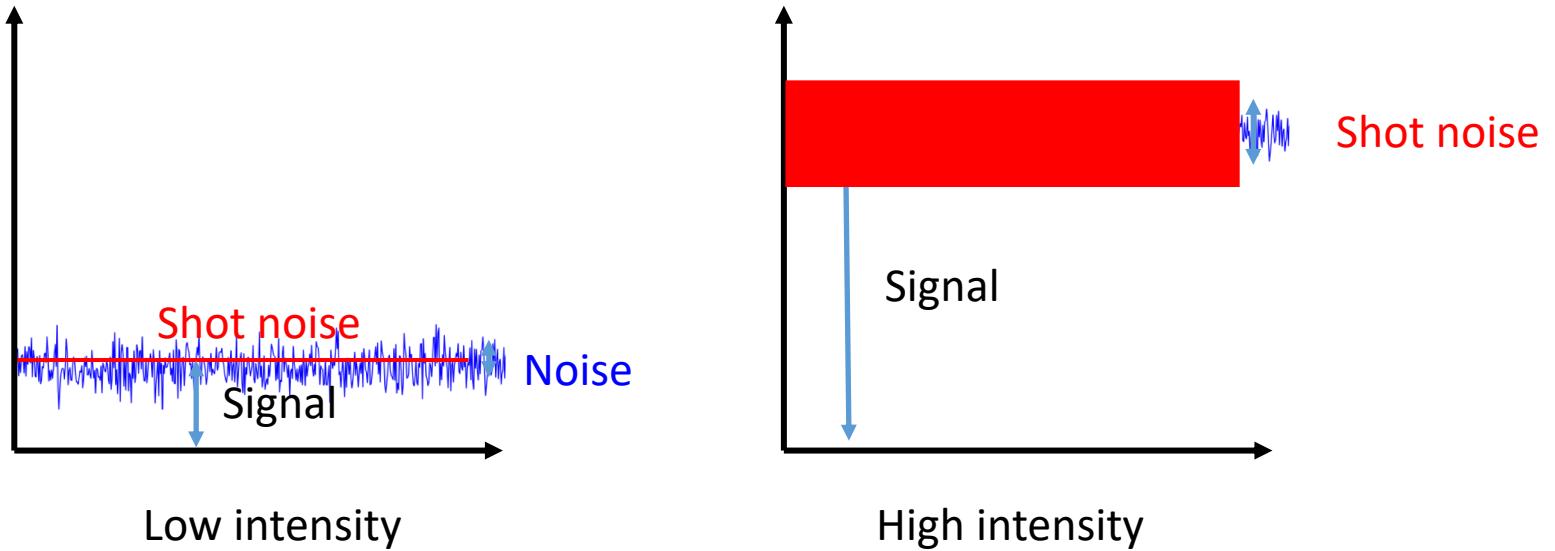
When we have coherent laser source

$$\text{Signal} \propto (\text{Number of photon})$$

$$\text{Shot Noise} \propto \sqrt{\text{Number of Photon}}$$

# ■ Shot noise of interferometer

---

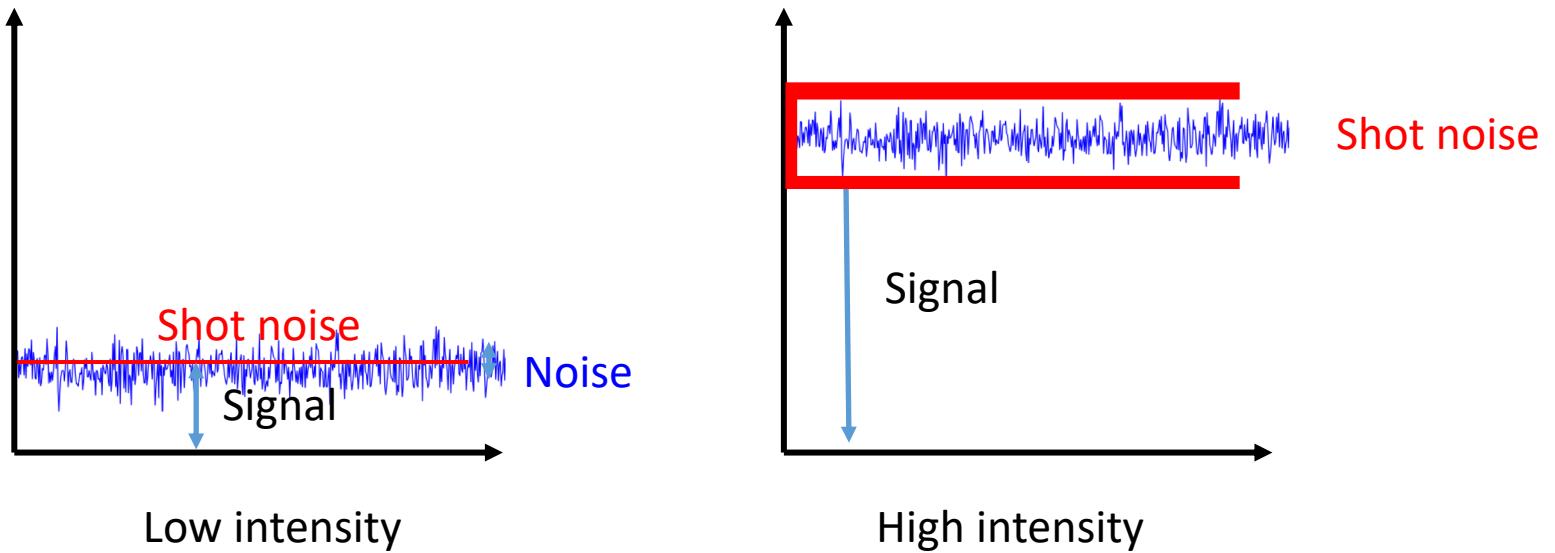


$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)  
We say that it has **shot noise limit sensitivity**

# ■ Shot noise of interferometer

---

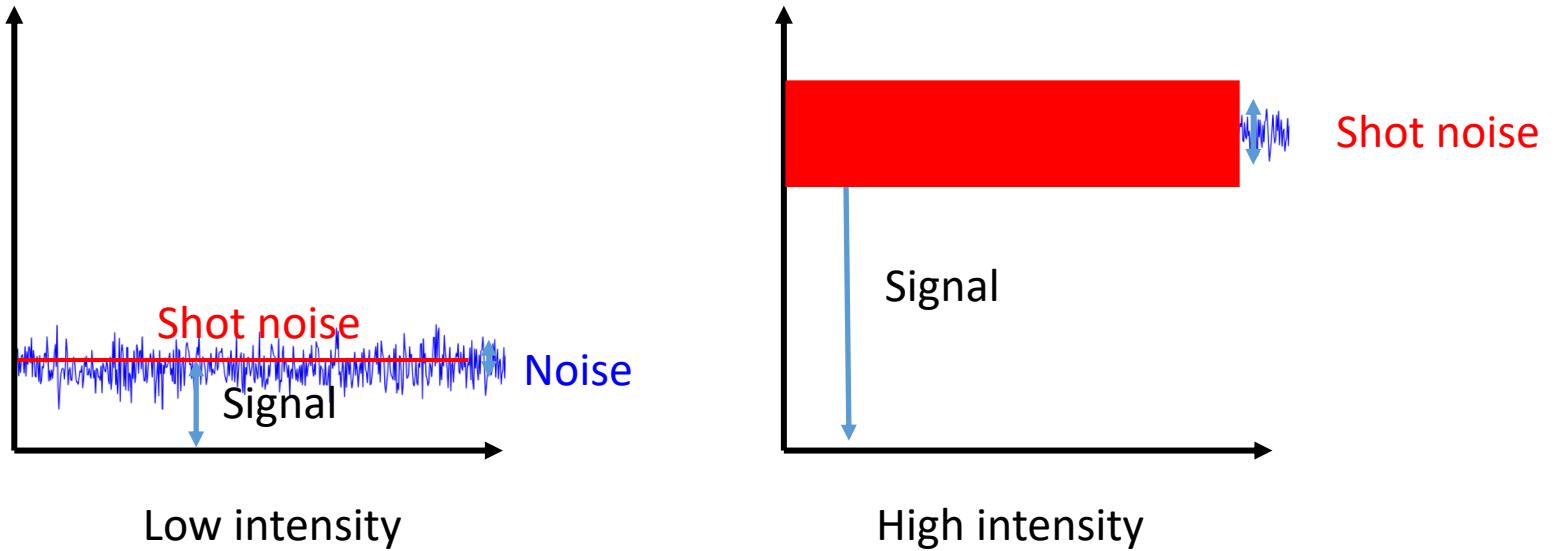


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# ■ Shot noise of interferometer

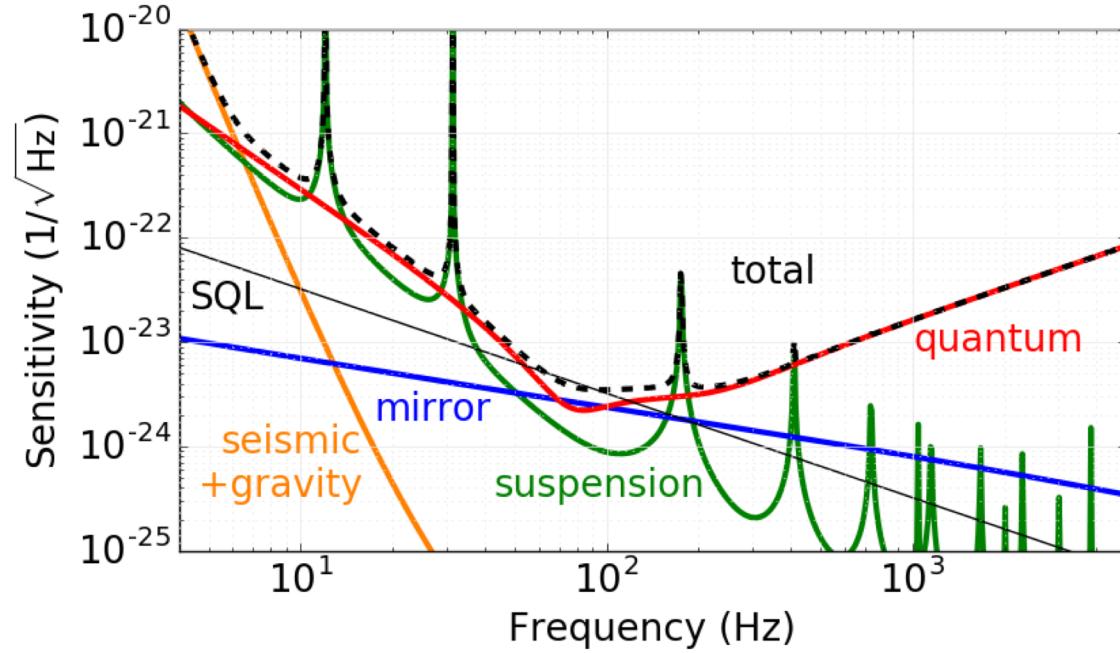
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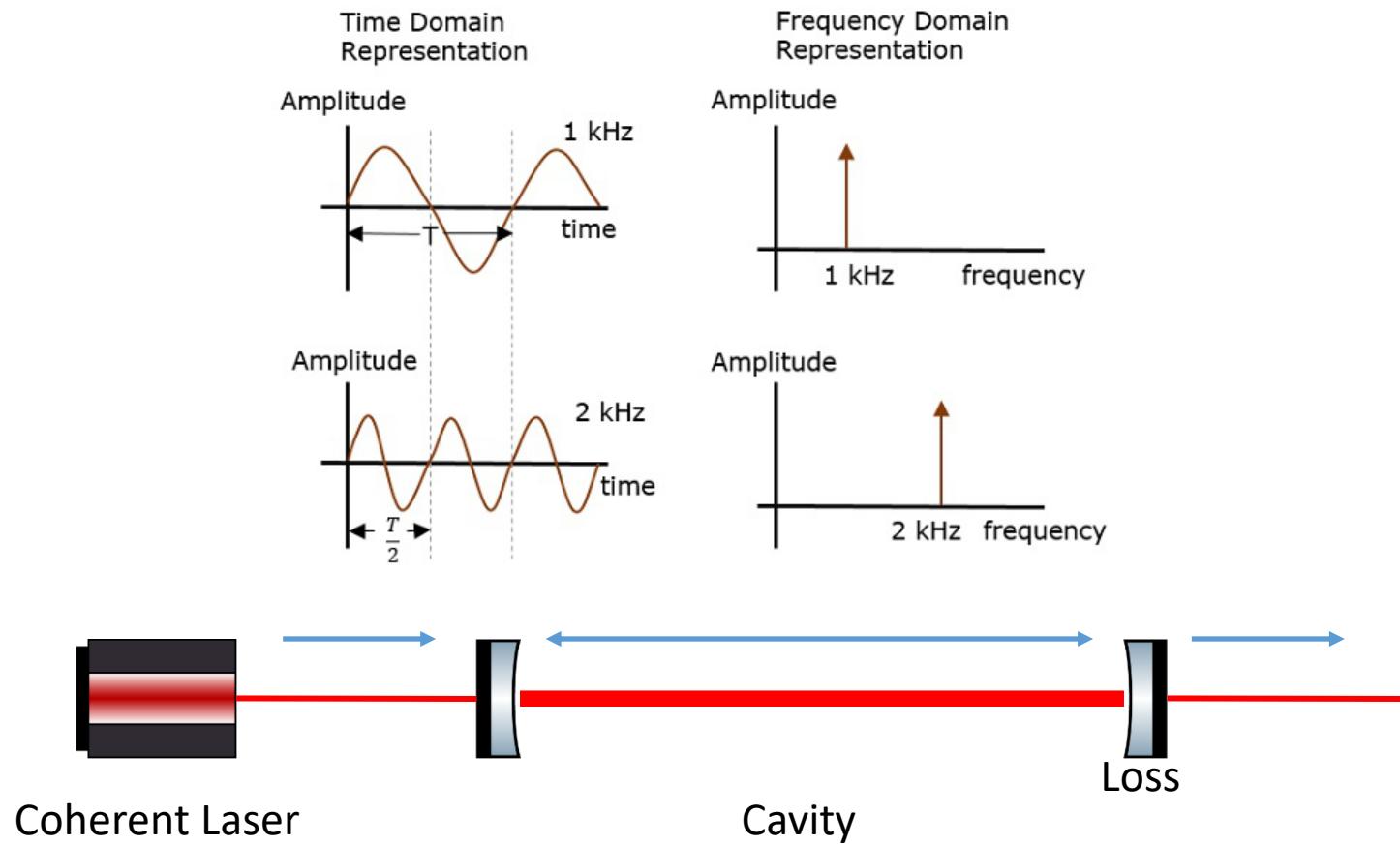
$$\text{Signal to Noise ratio} = \frac{N}{\sqrt{N}}$$

If Shot noise is relatively larger than other noise(Thermal, Electric.. etc)  
We say that it has **shot noise limit sensitivity**

# Target sensitivity of KAGRA

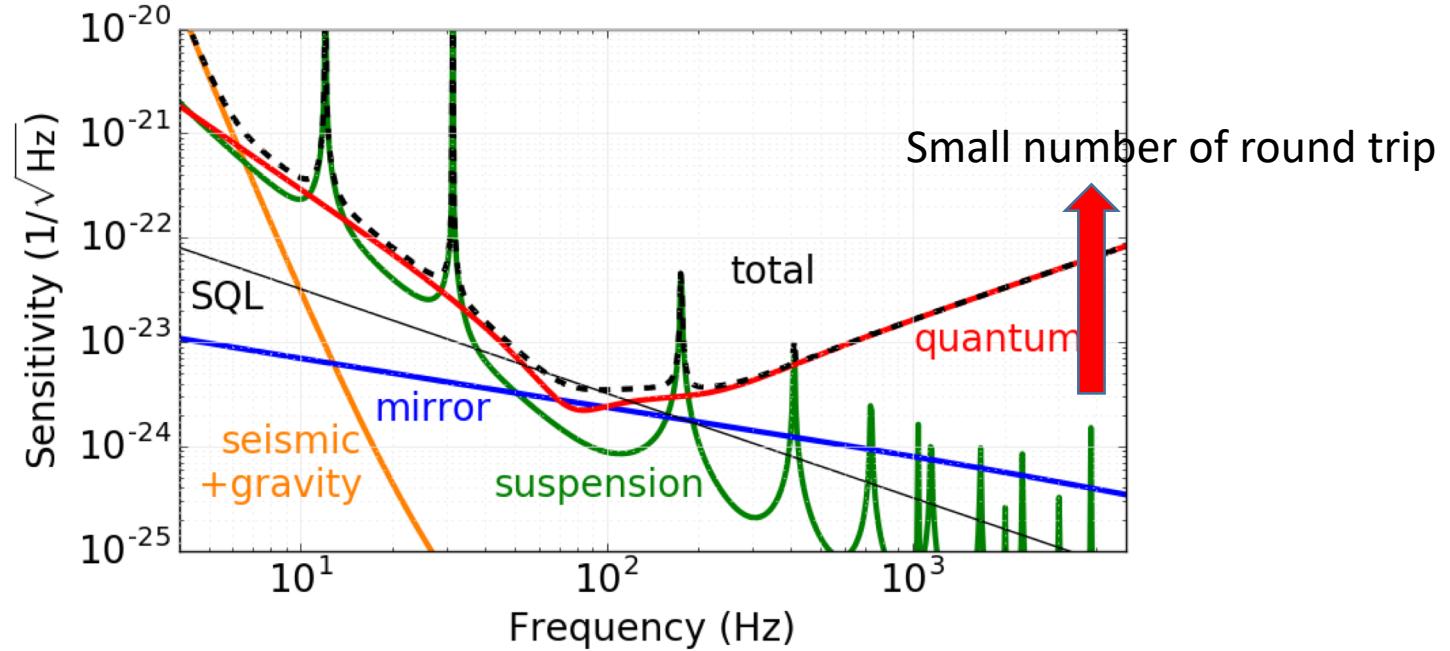


# ■ Shot noise curve of GW detector



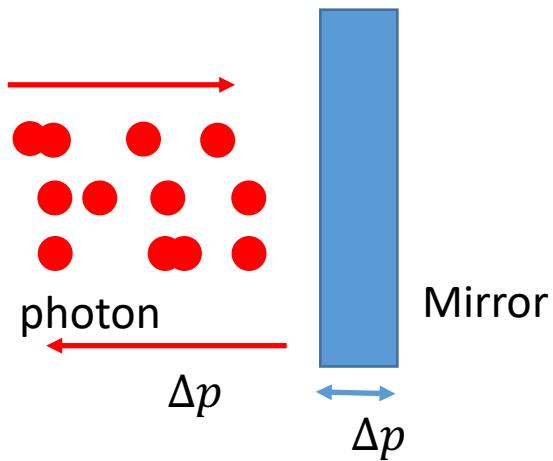
Interaction length  $\sim$  Number of round trip  $\times$  Length

# Target sensitivity of KAGRA



# Radiation pressure noise

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- Stored energy is very high (750 kW)
- Desired sensitivity is very high ( $10^{-21} \sim 10^{-24}$ )

# Radiation pressure noise in GW detector

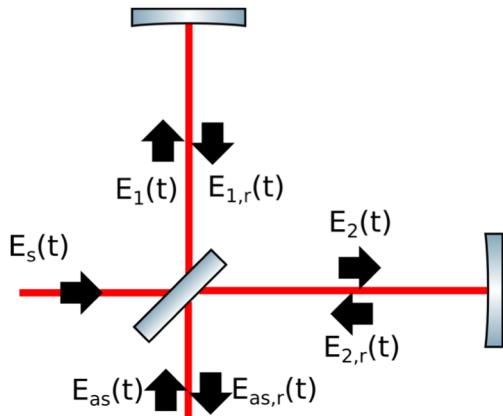


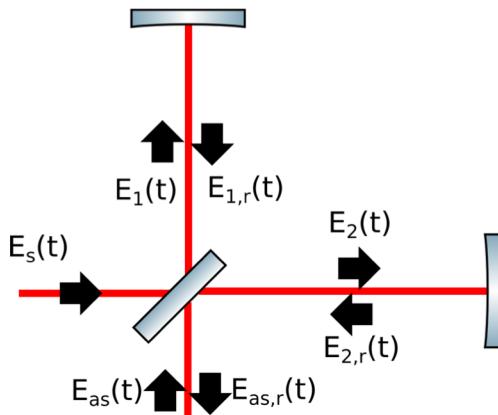
Figure 1-3: Schematic of a Michelson interferometer. A classical carrier field  $E_s(t)$  enters from the interferometer symmetric port while a vacuum fluctuations represented by  $E_{as}(t)$  enter from the anti-symmetric port. The quantum noise level at the readout is contained in the AC component of the field exiting the interferometer  $E_{as,r}(t)$

$$E_s(t) = E_0 \cos(\omega_0 t) + \underline{\delta E_s(t)}$$

Noise term

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

# Radiation pressure noise in GW detector



Assume anti-symmetric port is dark port

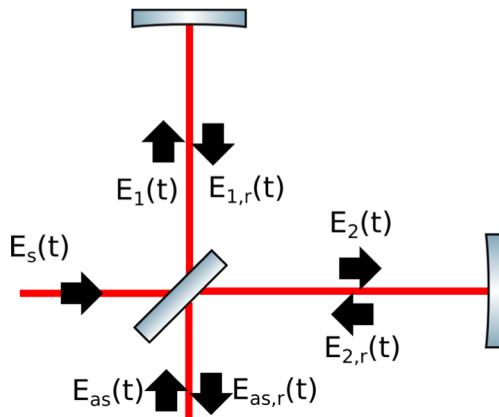
$$E_1(t) = \frac{1}{\sqrt{2}}[E_s(t) + E_{as}(t)]$$

$$E_2(t) = \frac{1}{\sqrt{2}}[E_s(t) - E_{as}(t)]$$

'as' is vacuum field

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

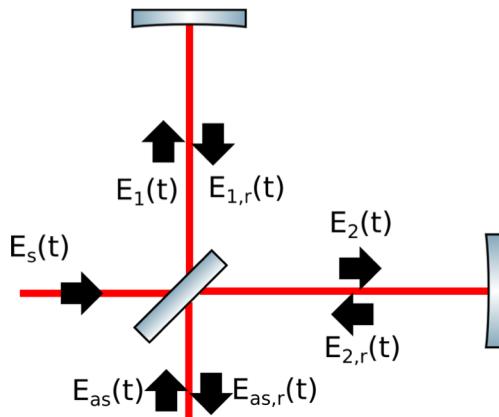
# Radiation pressure noise in GW detector



$$E_{as,r}(t) = E_{as} + E_0 \frac{\omega_0 [x_2(t) - x_1(t)]}{c} \sin(\omega_0 t)$$

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

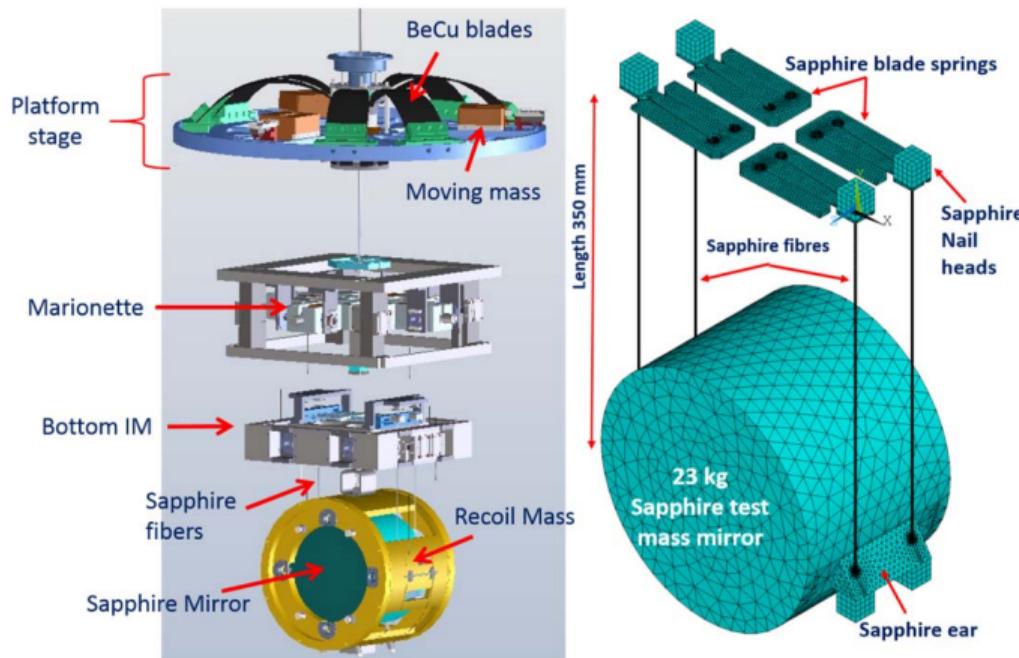
# Radiation pressure noise in GW detector



$$x_1(t) - x_2(t) = \underbrace{x_{cl,1}(t) - x_{cl,2}(t)}_{\text{Thermal, seismic}} + \underbrace{\delta\hat{x}_1(t) - \delta\hat{x}_2(t)}_{\text{Radiation pressure}} + \underbrace{Lh(t)}_{\text{GW source}}$$

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

# Test mass of KAGRA



# I Standard quantum limit of GW detector

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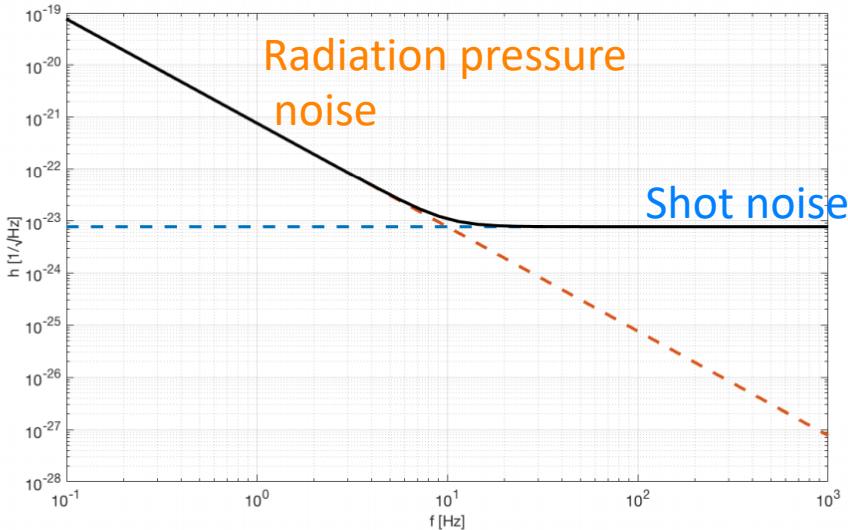
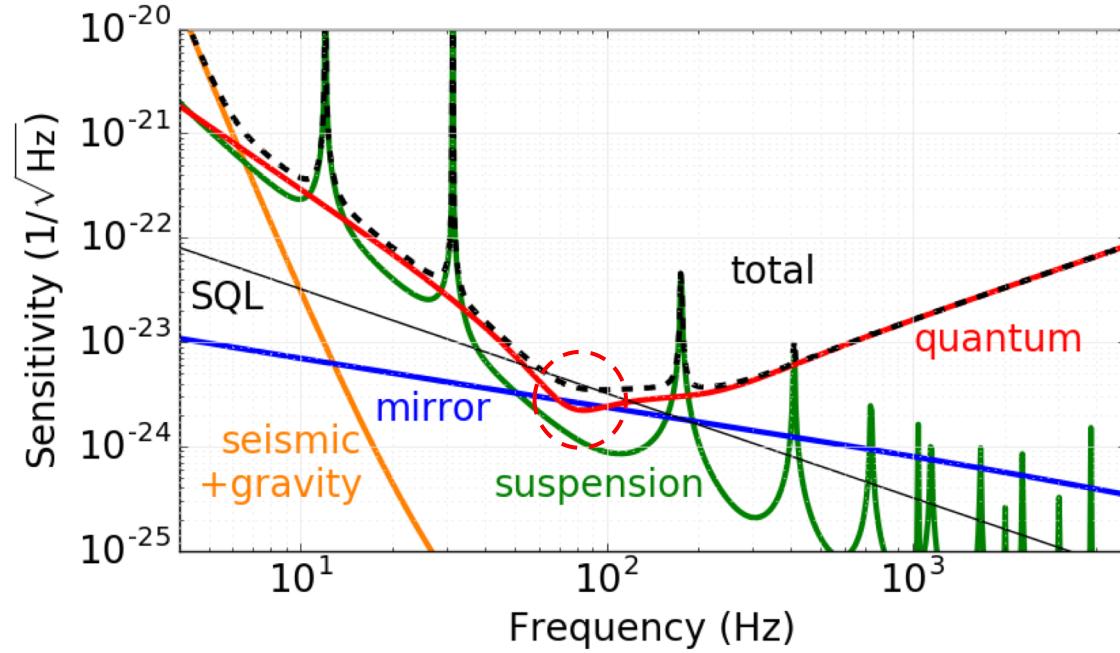


Figure 2.3: The strain equivalent quantum noise is plotted for an interferometer with  $M = 50$  kg,  $L = 3$  km,  $P = 10$  MW. The two contribution of radiation pressure noise and shot noise are shown in blue and red respectively.

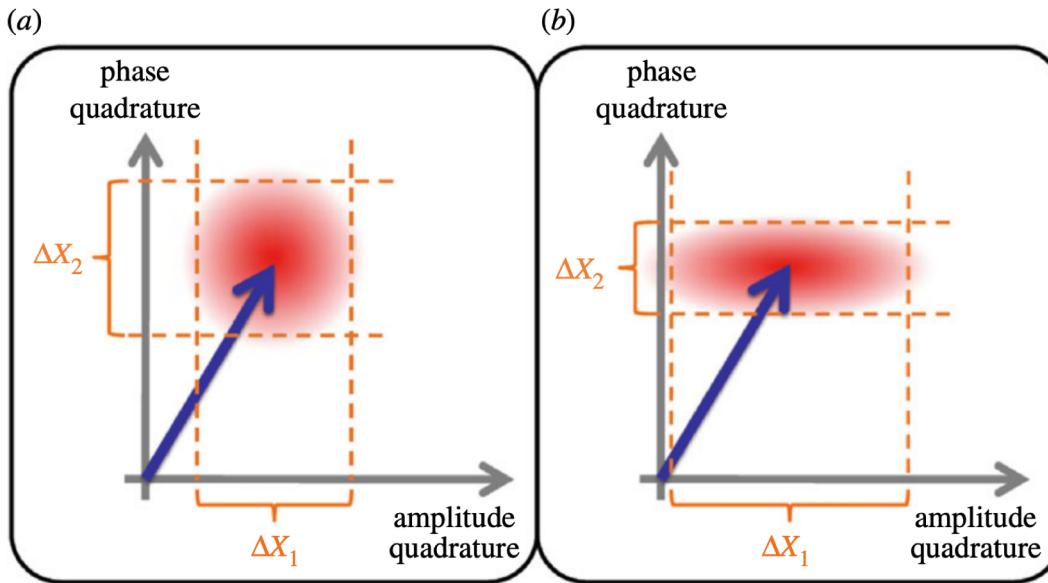
Standard quantum limit of gravitational wave detector  
Shot noise + Radiation pressure noise

# Target sensitivity of KAGRA



# Squeezed vacuum injection in GW detector

# Phase and amplitude noise of light



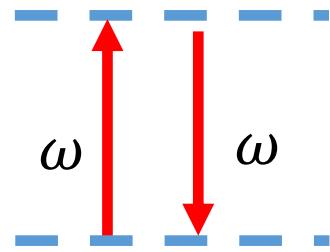
Heurs M. 2018 Gravitational wave detection using  
laser interferometry beyond the standard quantum  
limit. Phil. Trans. R. Soc. A 376: 20170289.

## ■ Non linear crystal

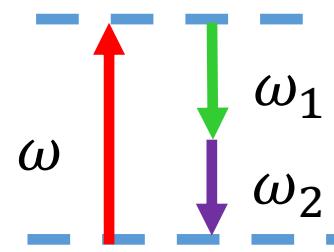
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Non-linear crystal

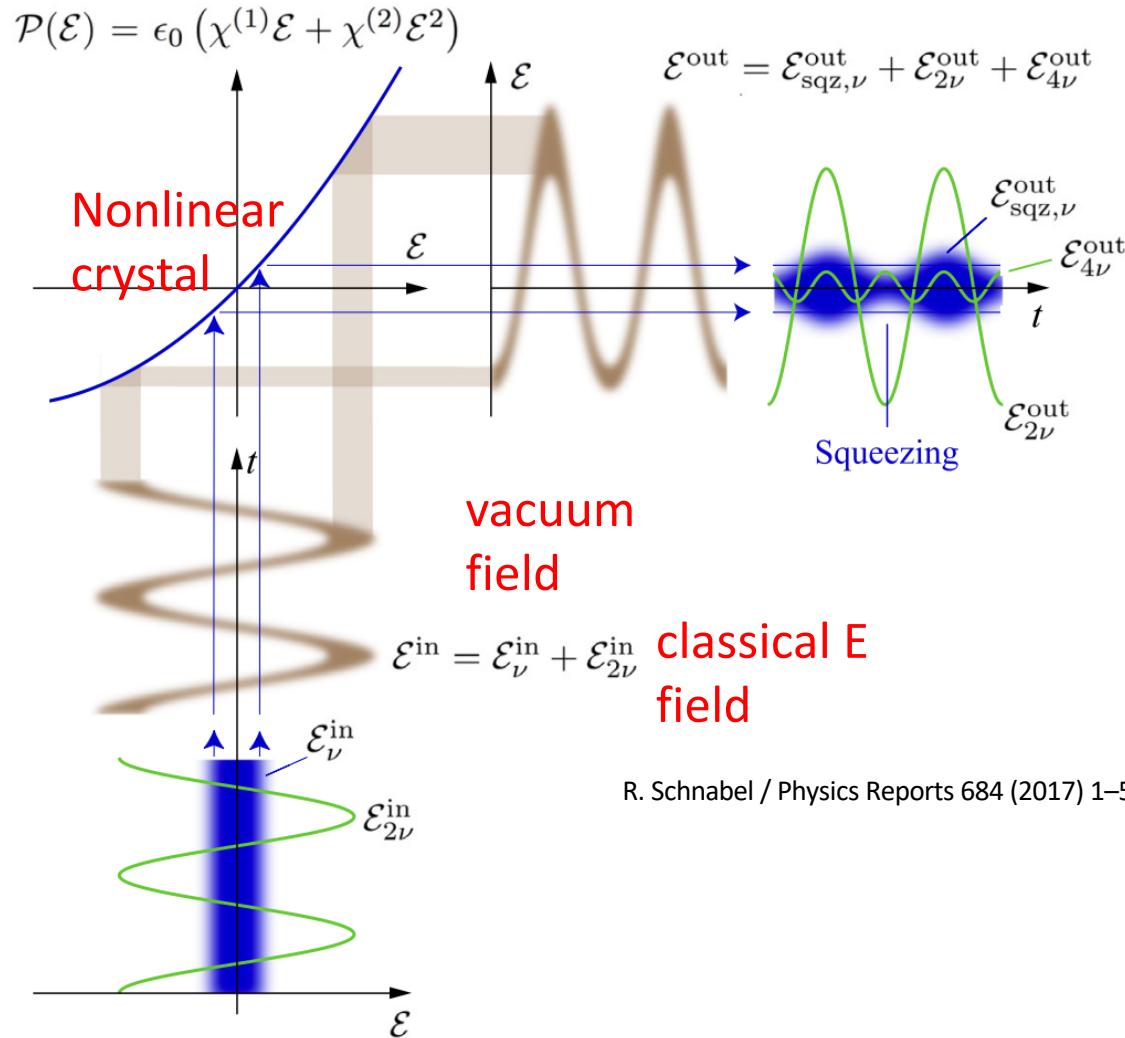


linear optics

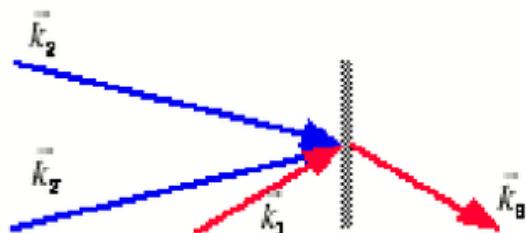


Non linear optics

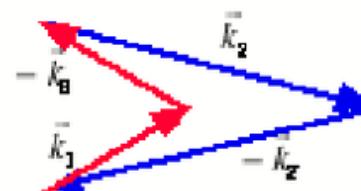
# Parametric down conversion process



# Phase matching condition



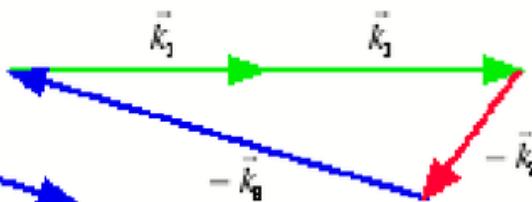
$$\omega_b = \omega_1 + \omega_2 - \omega_3$$



$$\Delta\vec{k} = \vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k}_4 = 0$$

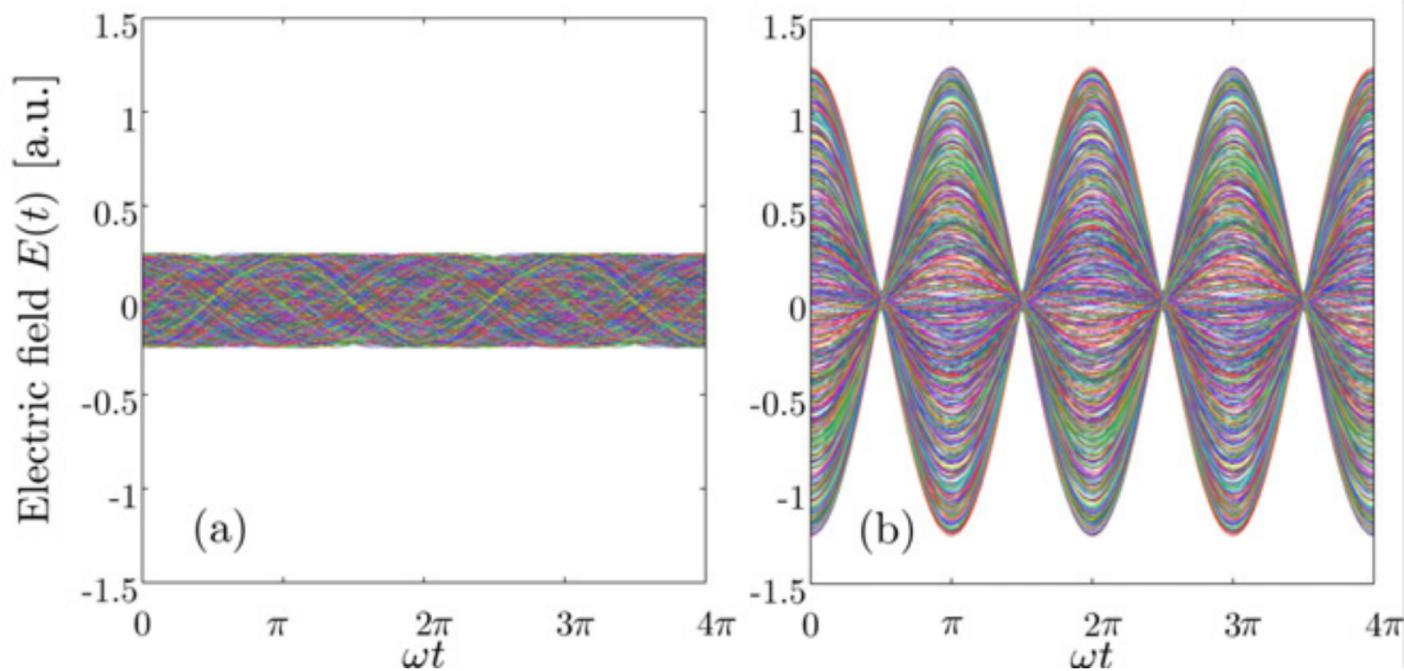


$$\omega_b = 2\omega_1 - \omega_2$$



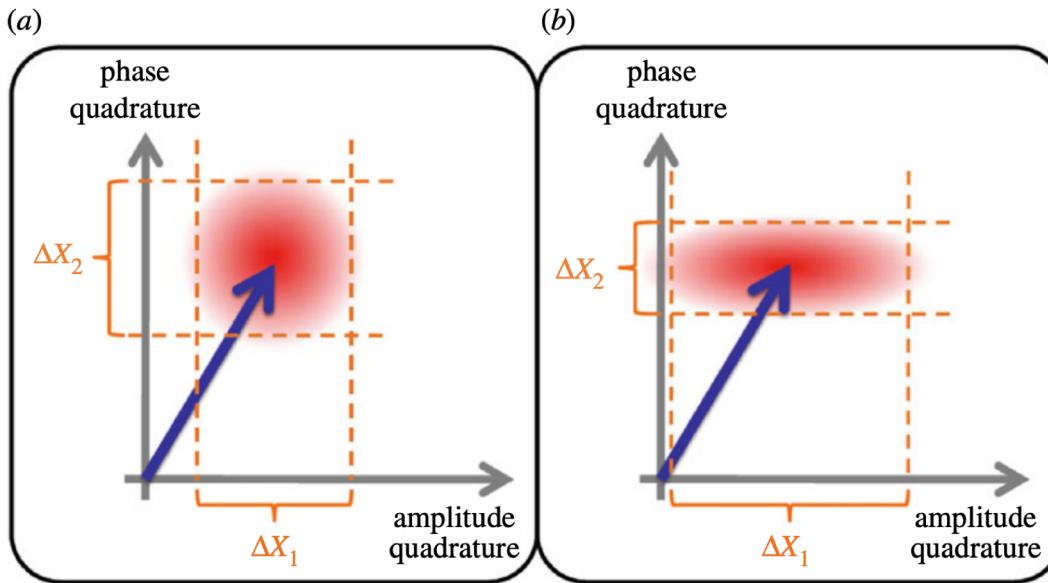
$$\Delta\vec{k} = 2\vec{k}_1 - \vec{k}_2 - \vec{k}_3 - \vec{k}_4 = 0$$

# Parametric down conversion process



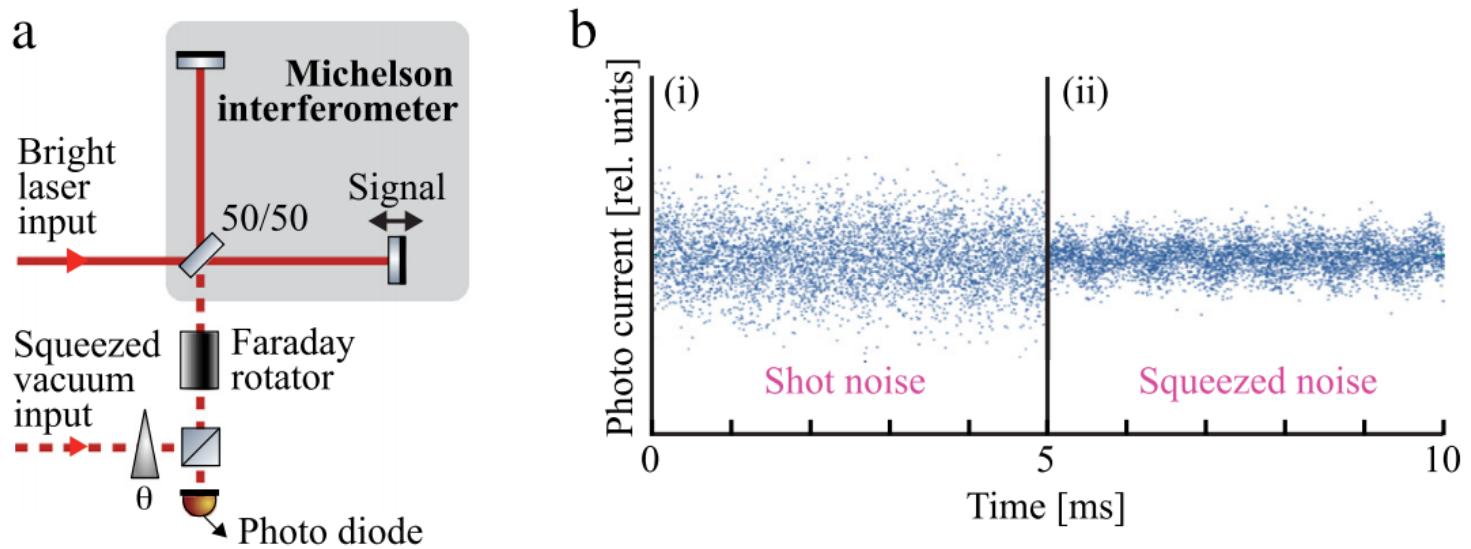
**Figure 1.6:** Simulation of electric field in time for (a) vacuum state and for (b) squeezed vacuum.

# Phase and amplitude noise of light



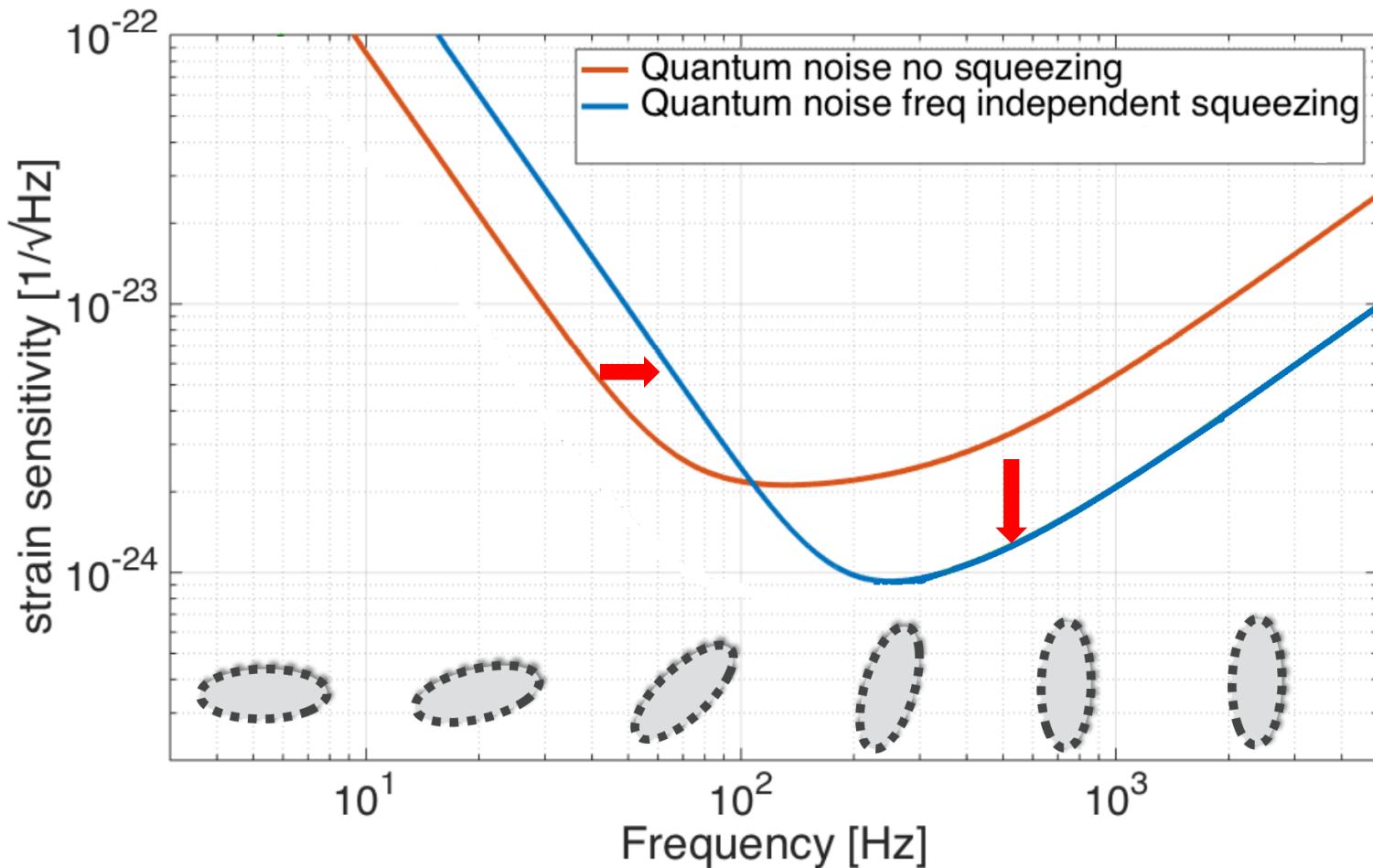
Heurs M. 2018 Gravitational wave detection using  
laser interferometry beyond the standard quantum  
limit. Phil. Trans. R. Soc. A 376: 20170289.

# Squeezed state of light



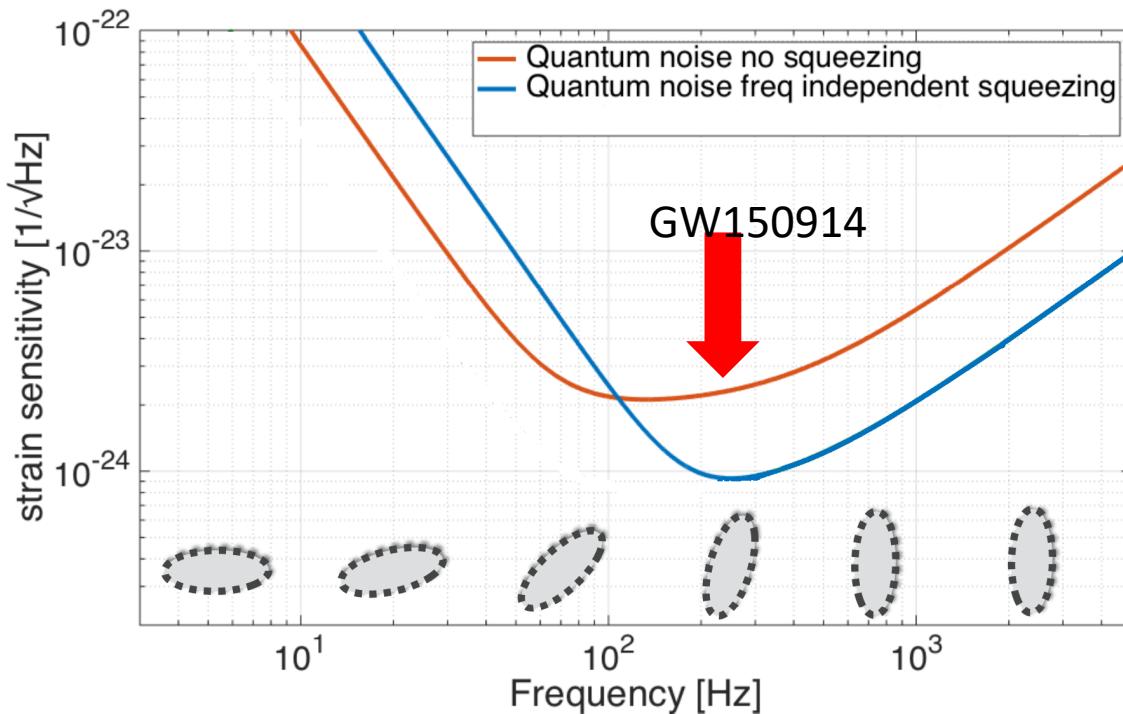
R. Schnabel / Physics Reports 684 (2017) 1–51

# Frequency independent squeezing



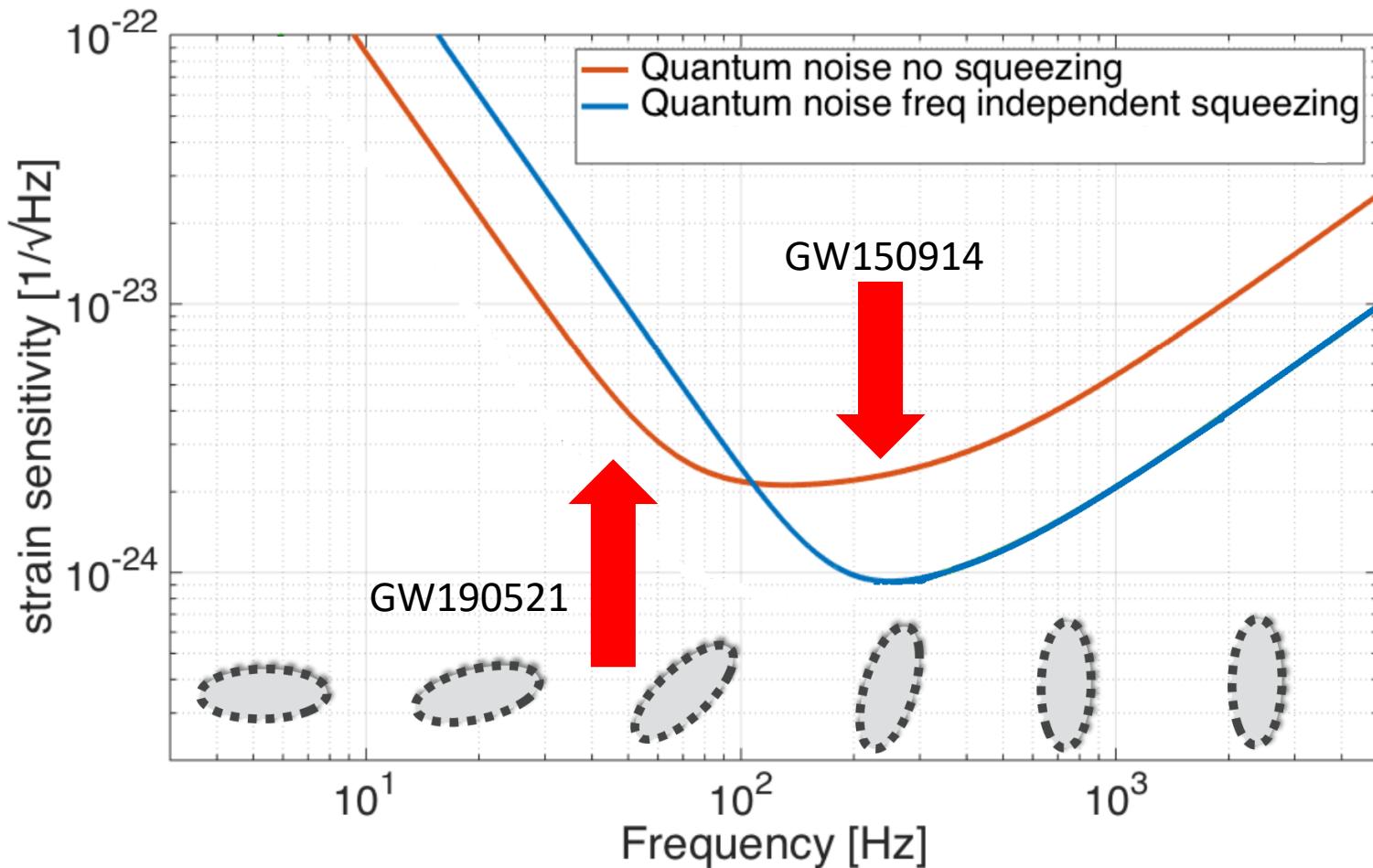
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction  
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

# Frequency independent squeezing



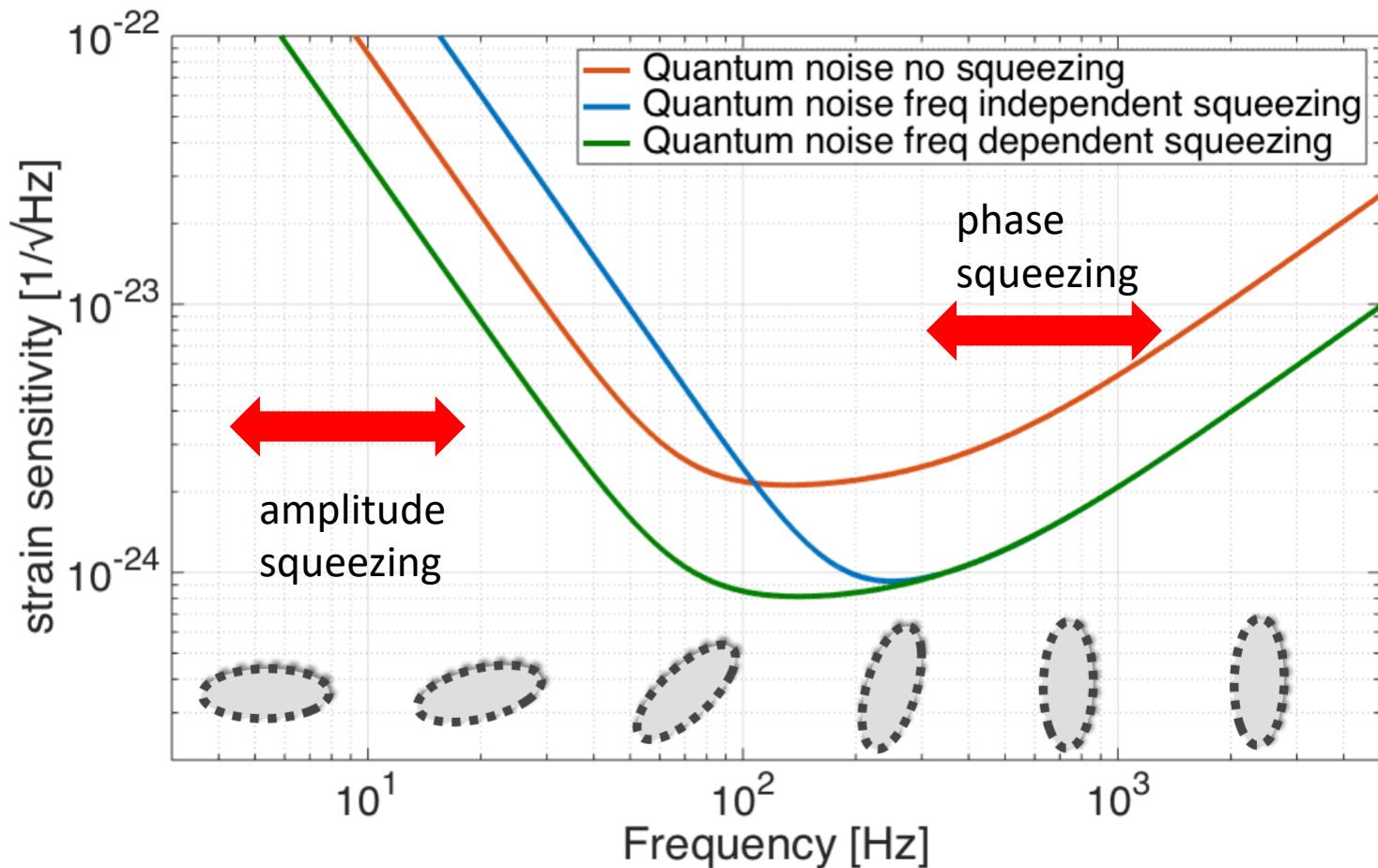
Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction  
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT  
(2017)

# Frequency independent squeezing



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction  
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

# Frequency dependent squeezing(FDS)



Optical and noise studies for Advanced Virgo and filter cavities for quantum noise reduction  
in gravitational-wave interferometric detectors, Eleonora Capocasa, UNIVERSITÉ PARIS DIDEROT (2017)

## 4. Frequency dependent squeezing in GW detector

# First suggestion of filter cavity in FD squeezing

Conversion of conventional gravitational-wave interferometers into quantum nondemolition interferometers by modifying their input and/or output optics

H. J. Kimble,<sup>1</sup> Yuri Levin,<sup>2,\*</sup> Andrey B. Matsko,<sup>3</sup> Kip S. Thorne,<sup>2</sup> and Sergey P. Vyatchanin<sup>4</sup>

<sup>1</sup>*Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125*

<sup>2</sup>*Theoretical Astrophysics, California Institute of Technology, Pasadena, California 91125*

<sup>3</sup>*Department of Physics, Texas A&M University, College Station, Texas 77843-4242*

<sup>4</sup>*Physics Faculty, Moscow State University, Moscow, 119899, Russia*

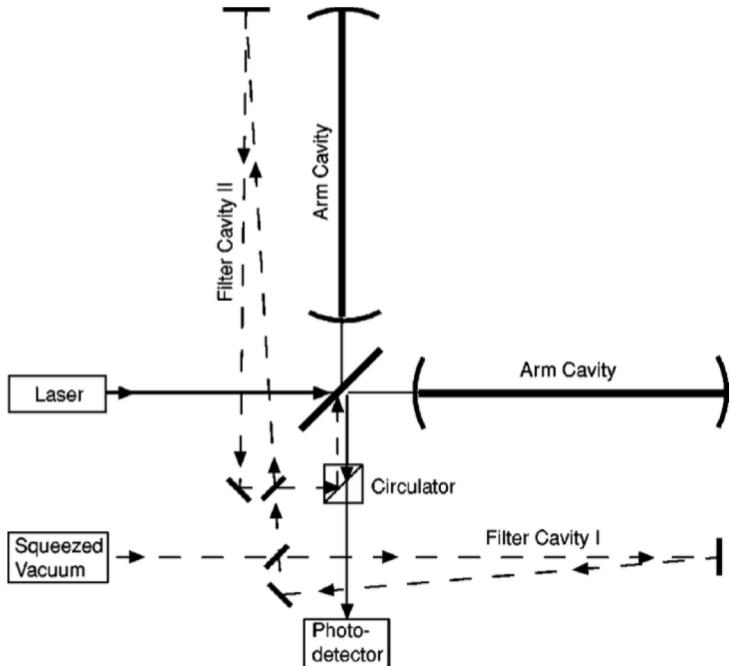
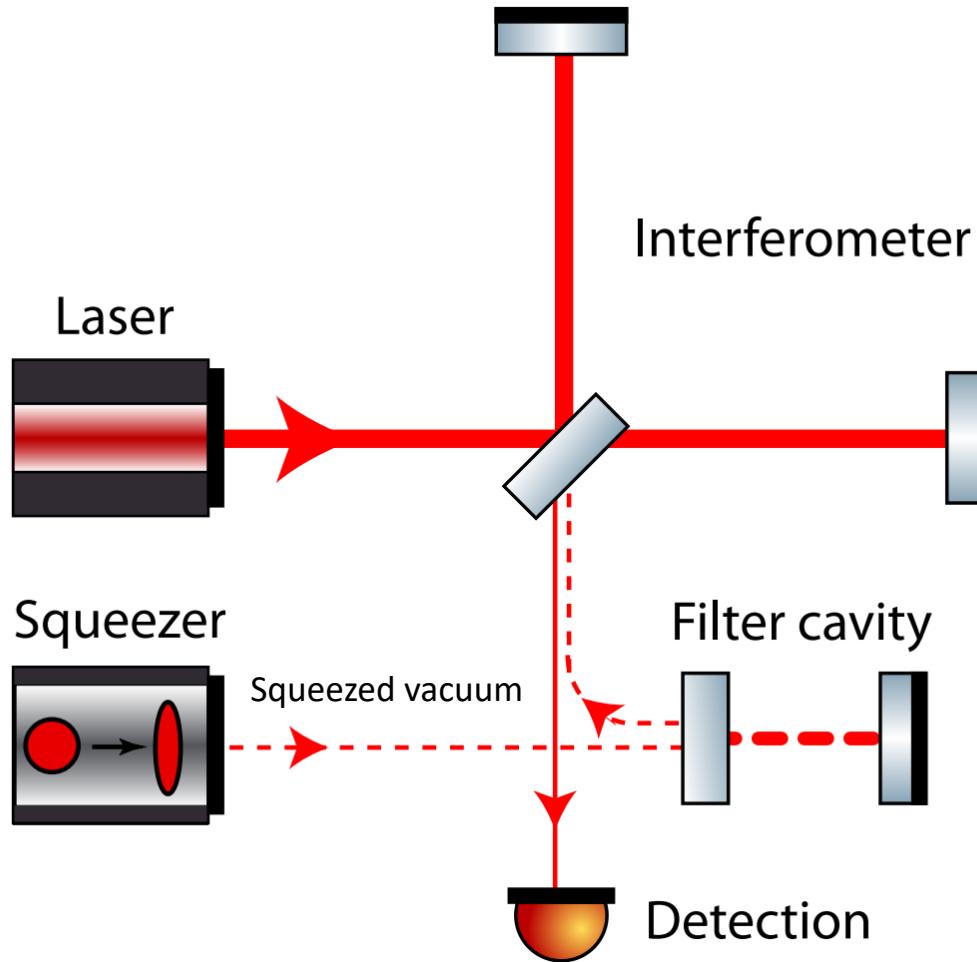


FIG. 1. Schematic diagram of a squeezed-input interferometer.

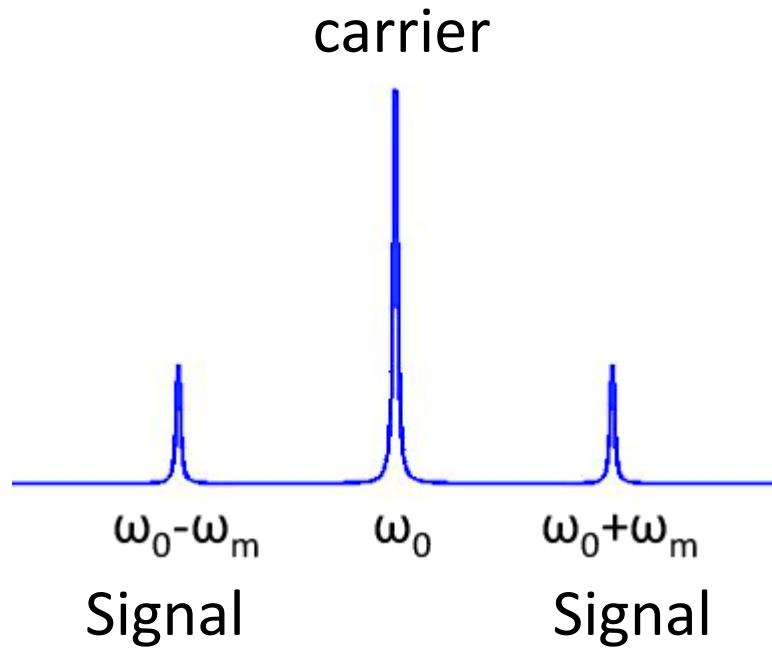
# Squeezed vacuum injection with filter cavity



M. Evans, L. Barsotti, P. Kwee, J. Harms, and H. Miao  
Phys. Rev. D **88**, 022002 – Published 29 July 2013

## ■ Side band figure

---



# Detuned cavity



## Simple picture

Stefan Hild et al, "Detuned arm cavities", 3rd GEO simulation workshop, Hannover, June 2007

**B:**

- less carrier light in cavity => less GW sidebands are produced.
- Since one GW sideband is resonant, it gets enhanced.

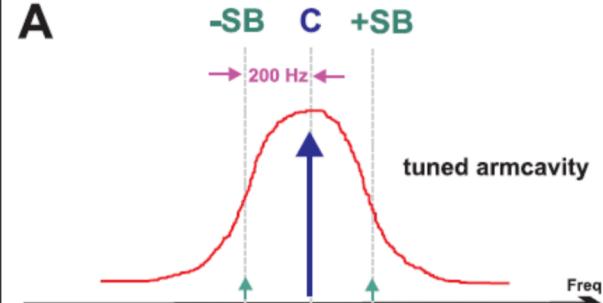
**=> Smaller GW signal**

**C:**

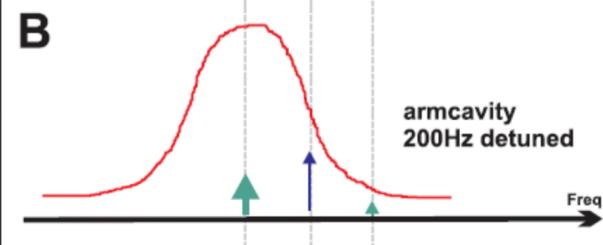
- optical power is restored in the cavity by larger PR-gain.
  - Same amount of GW sidebands are produced.
  - Since one GW sideband is resonant, it gets enhanced.
- Overall we win GW signal.

**=> Larger GW signal**

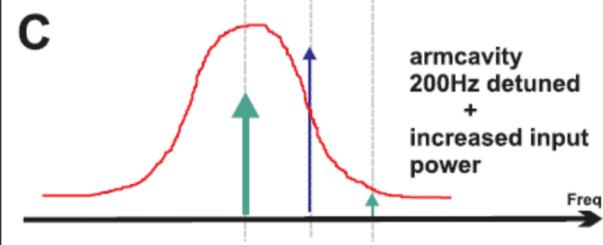
**A**



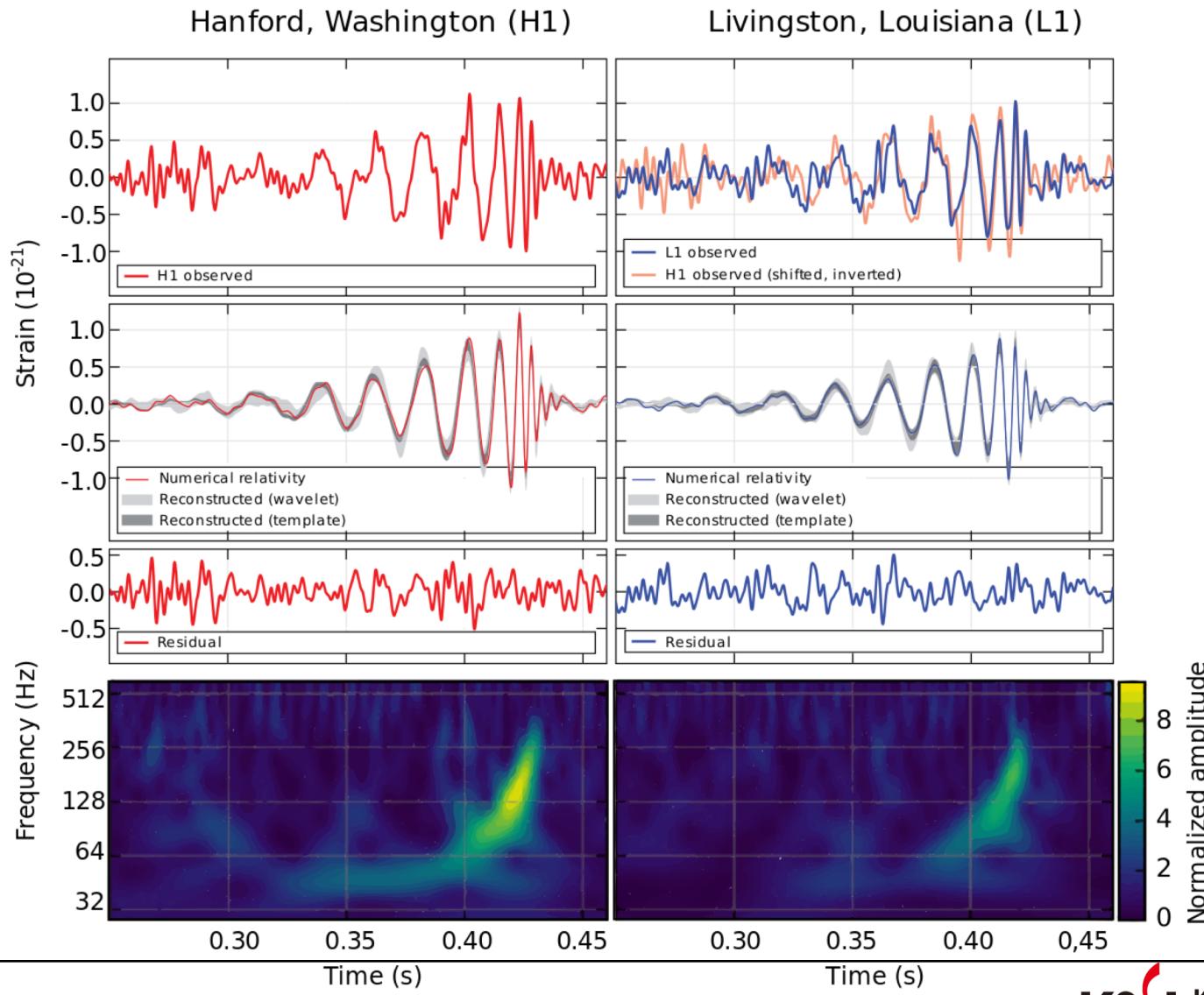
**B**



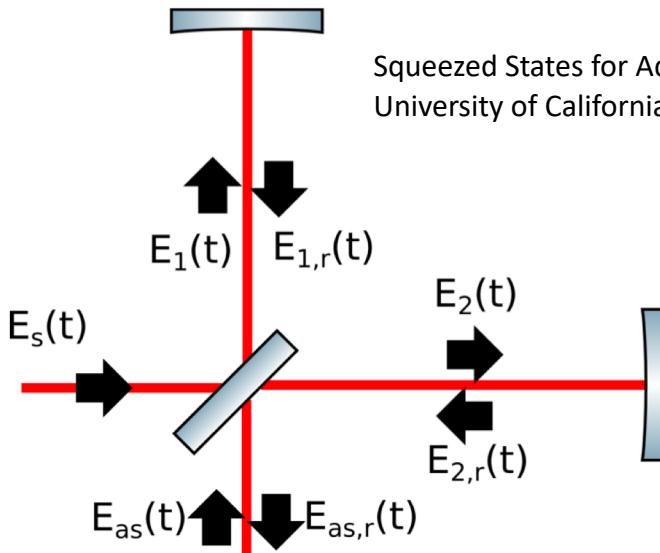
**C**



# Gravitational wave signal



# Electric field in simple Michelson interferometer



Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

Assume anti-symmetric port is dark port

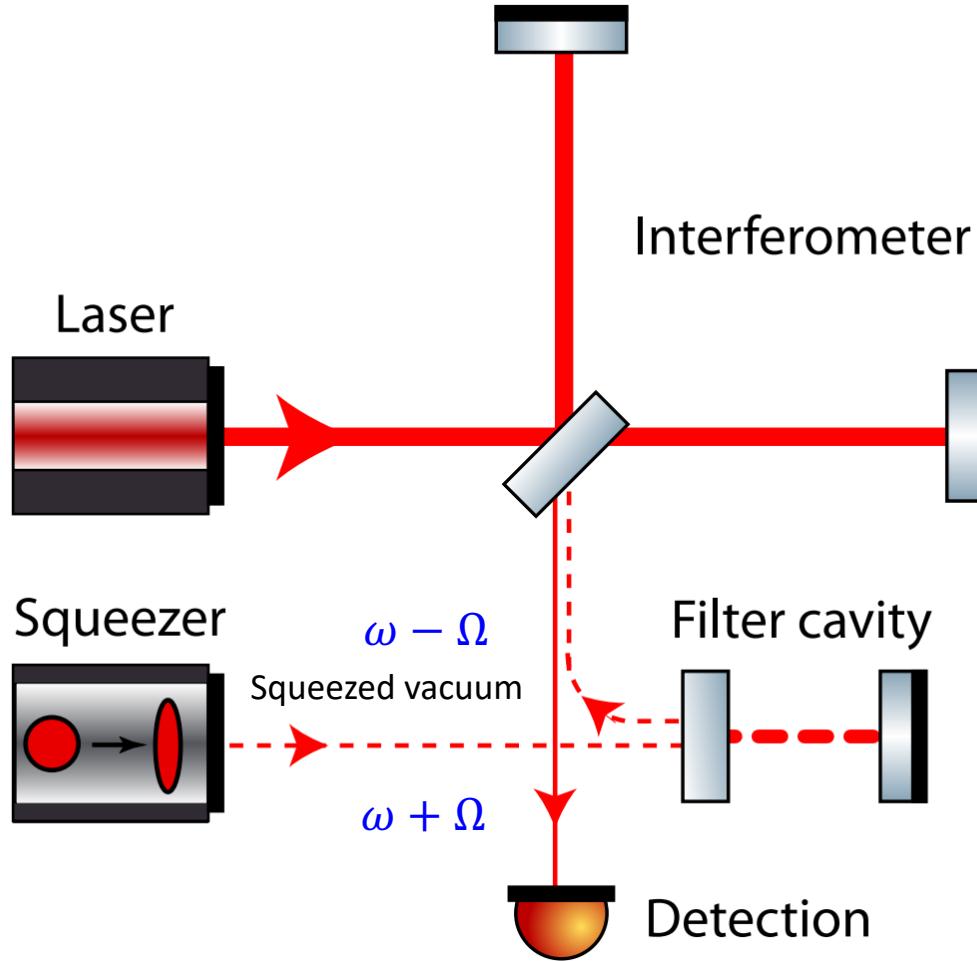
$$E_1(t) = \frac{1}{\sqrt{2}}[E_s(t) + E_{as}(t)]$$

$$E_2(t) = \frac{1}{\sqrt{2}}[E_s(t) - E_{as}(t)]$$

'as' is vacuum field

Squeezed States for Advanced Gravitational Wave Detectors, B.A.,  
University of California Berkeley, Eric Oelker (2009)

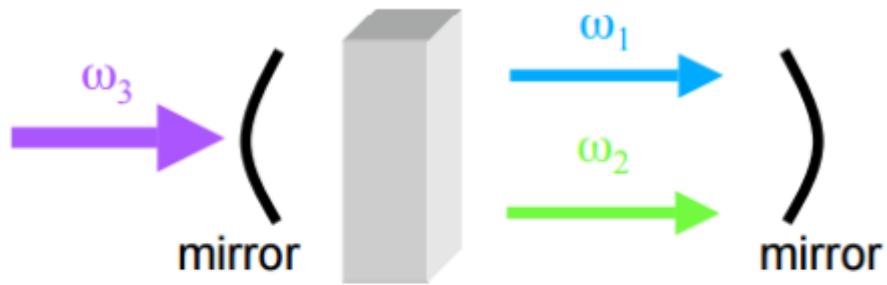
# Squeezed vacuum injection with filter cavity



M. Evans, L. Barsotti, P. Kwee, J. Harms, and H. Miao  
Phys. Rev. D **88**, 022002 – Published 29 July 2013

# Optical parametric oscillator

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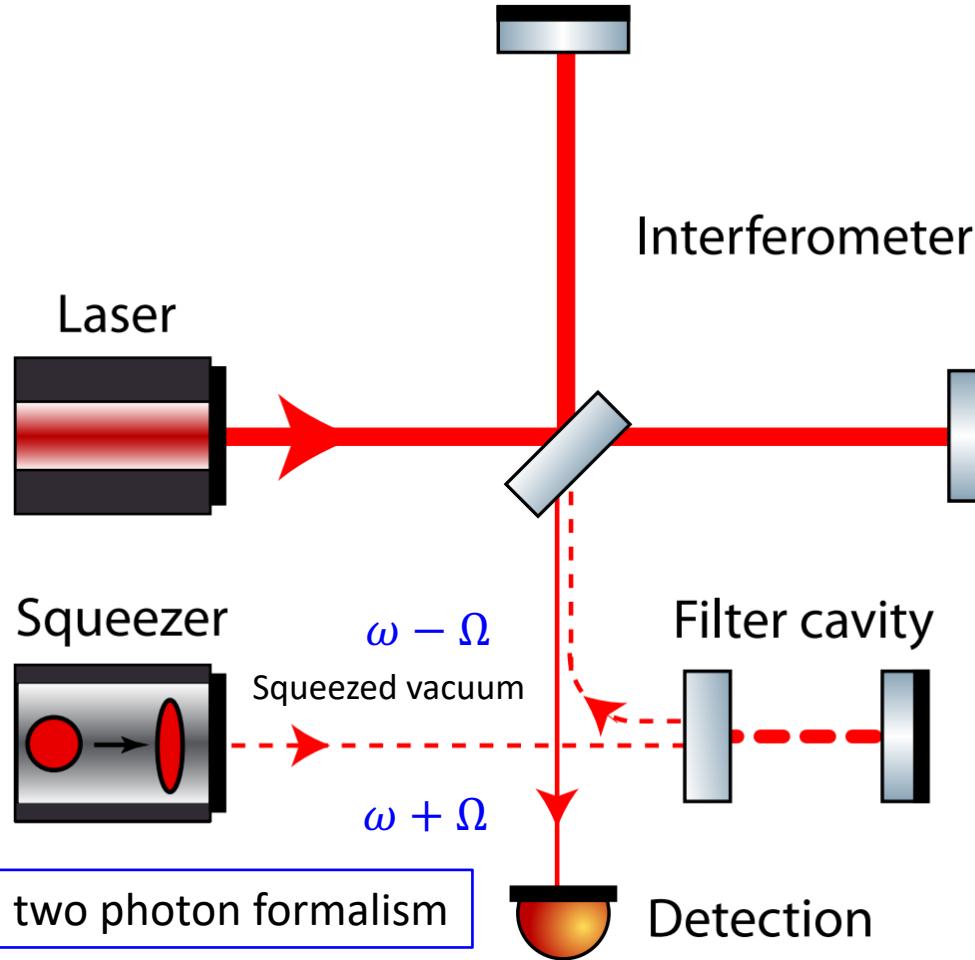


Optical Parametric  
Oscillation (OPO)

$\omega_1 = \omega_2$  : degenerated OPO

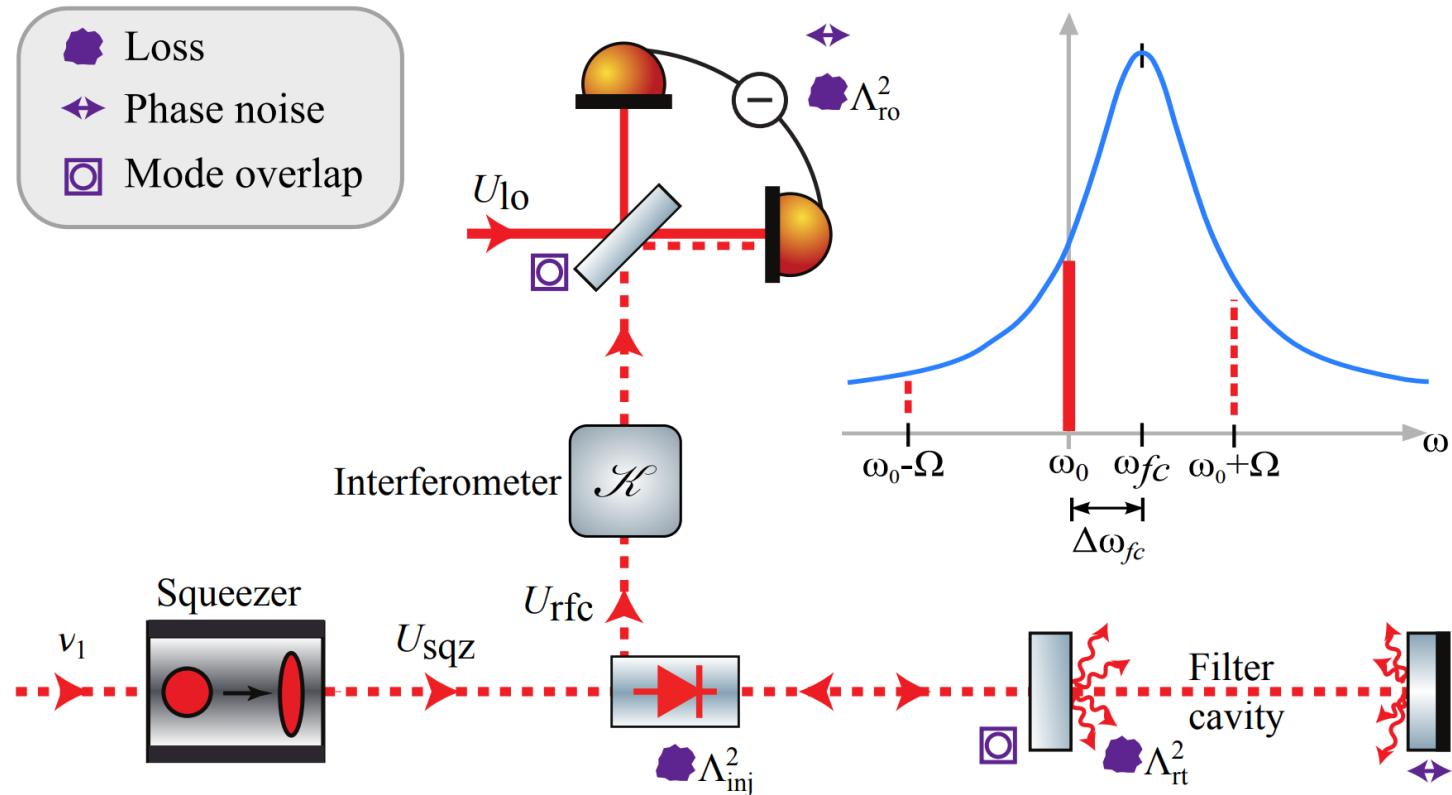
All-Optical Electron Acceleration with Ultrafast THz  
Pulses, Wenqian Ronny Huang, MIT(2017)

# Squeezed vacuum injection with filter cavity



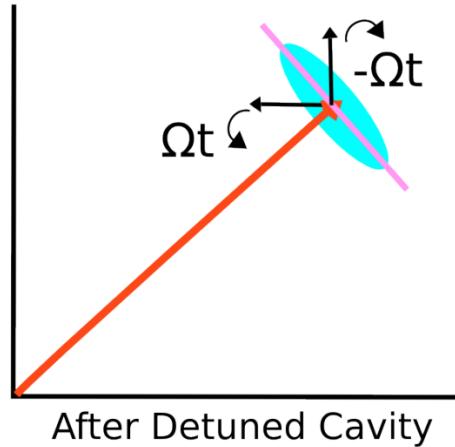
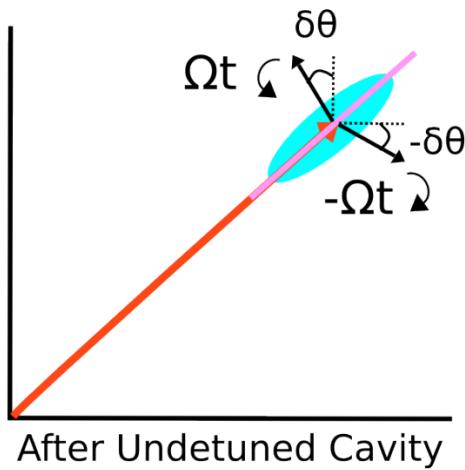
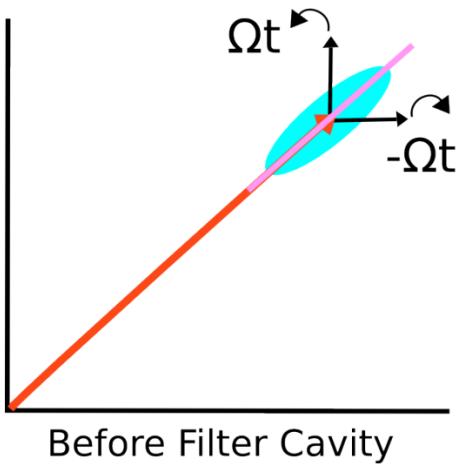
M. Evans, L. Barsotti, P. Kwee, J. Harms, and H. Miao  
Phys. Rev. D **88**, 022002 – Published 29 July 2013

# Detuned filter cavity



P. Kwee, J. Miller, T. Isogai, L. Barsotti, and M. Evans  
Phys. Rev. D 90, 062006 – Published 5 September 2014

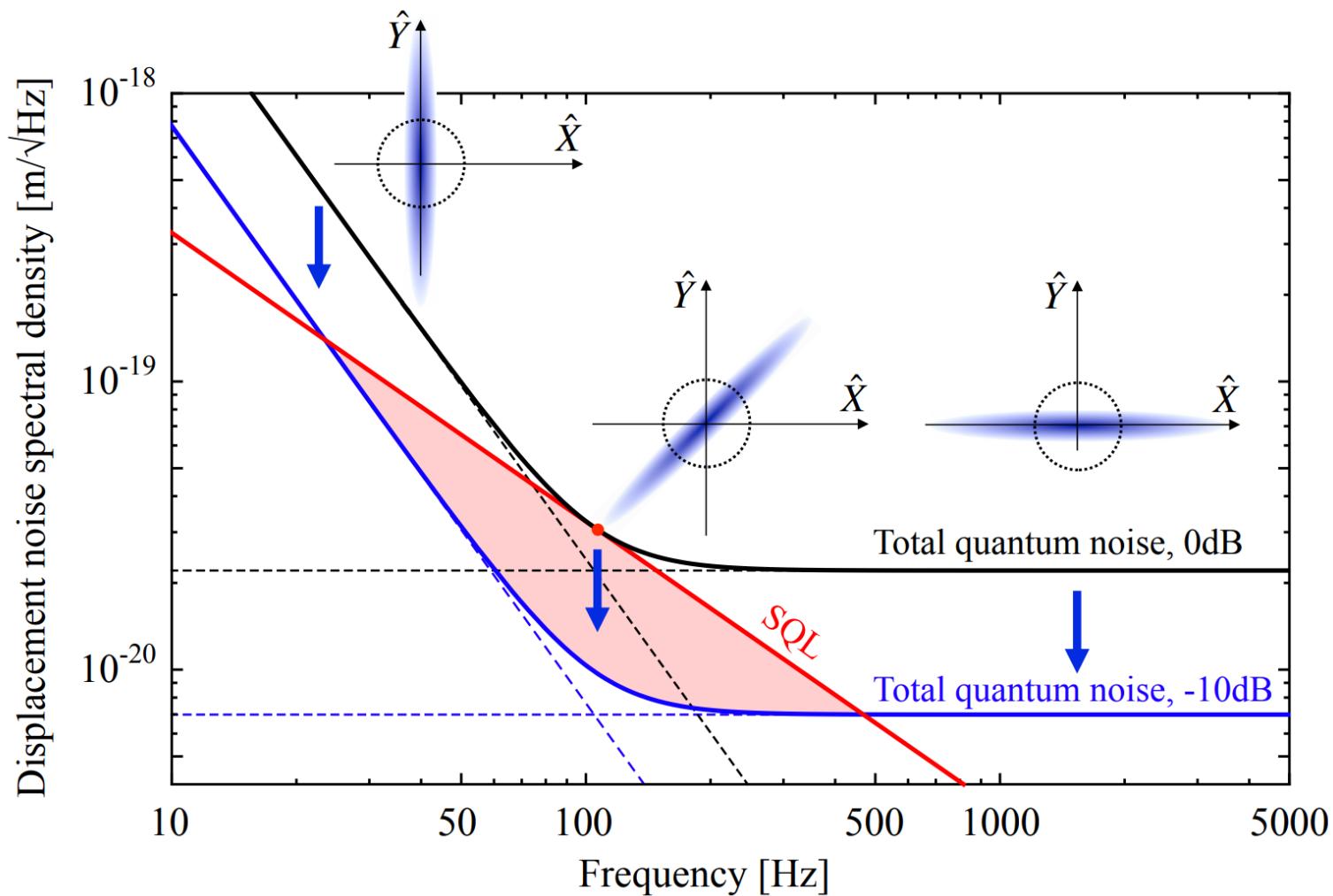
# Squeeze angle



— Squeeze angle

M. Evans, L. Barsotti, P. Kwee, J. Harms, and H. Miao  
Phys. Rev. D **88**, 022002 – Published 29 July 2013

# Squeeze angle rotation



## ▪ Squeeze angle rotation

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$$\alpha_p = \arctan \left( \frac{2\gamma_{\text{fc}}\Delta\omega_{\text{fc}}}{\gamma_{\text{fc}}^2 - \Delta\omega_{\text{fc}}^2 + \Omega^2} \right)$$

$\gamma$  = loss of filter cavity  
 $\omega_{fc}$  = detuned frequency

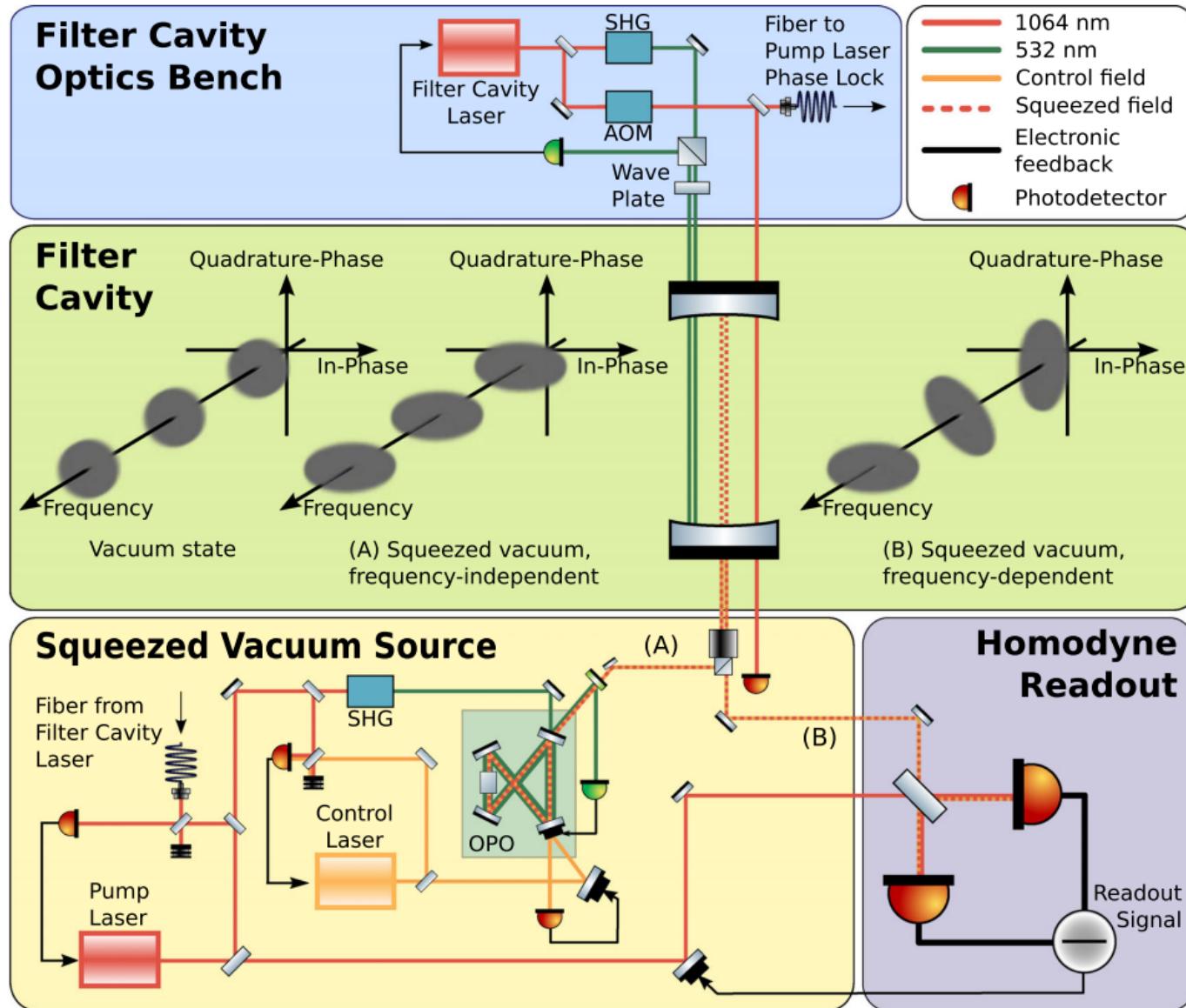
$$t_{\text{st}} = \frac{1}{\gamma_{\text{fc}}} = \frac{\sqrt{2}}{\Omega_{\text{SQL}}} \simeq 3 \text{ ms}$$

# LIGO filter cavity

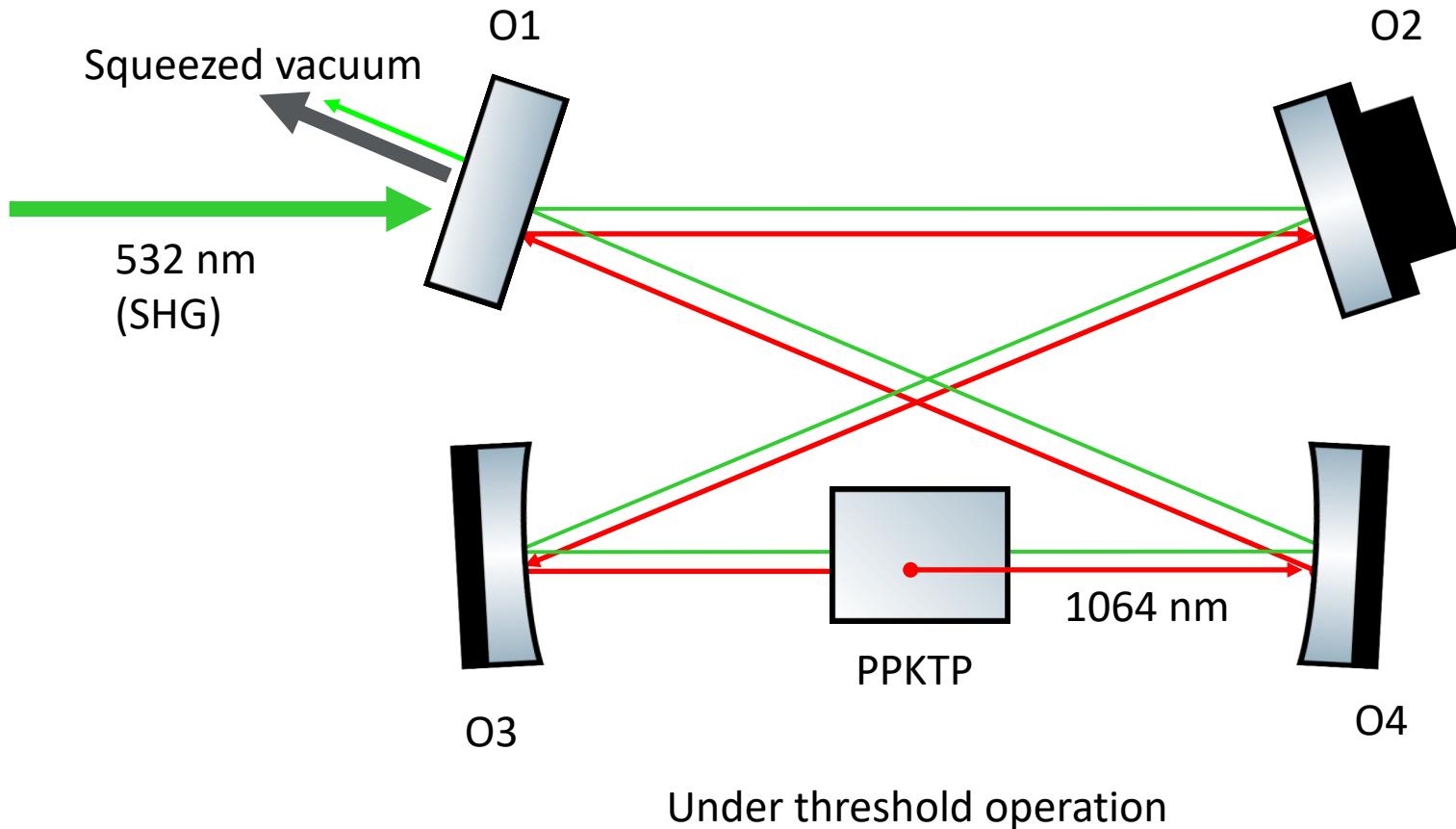
PRL 116, 041102 (2016)

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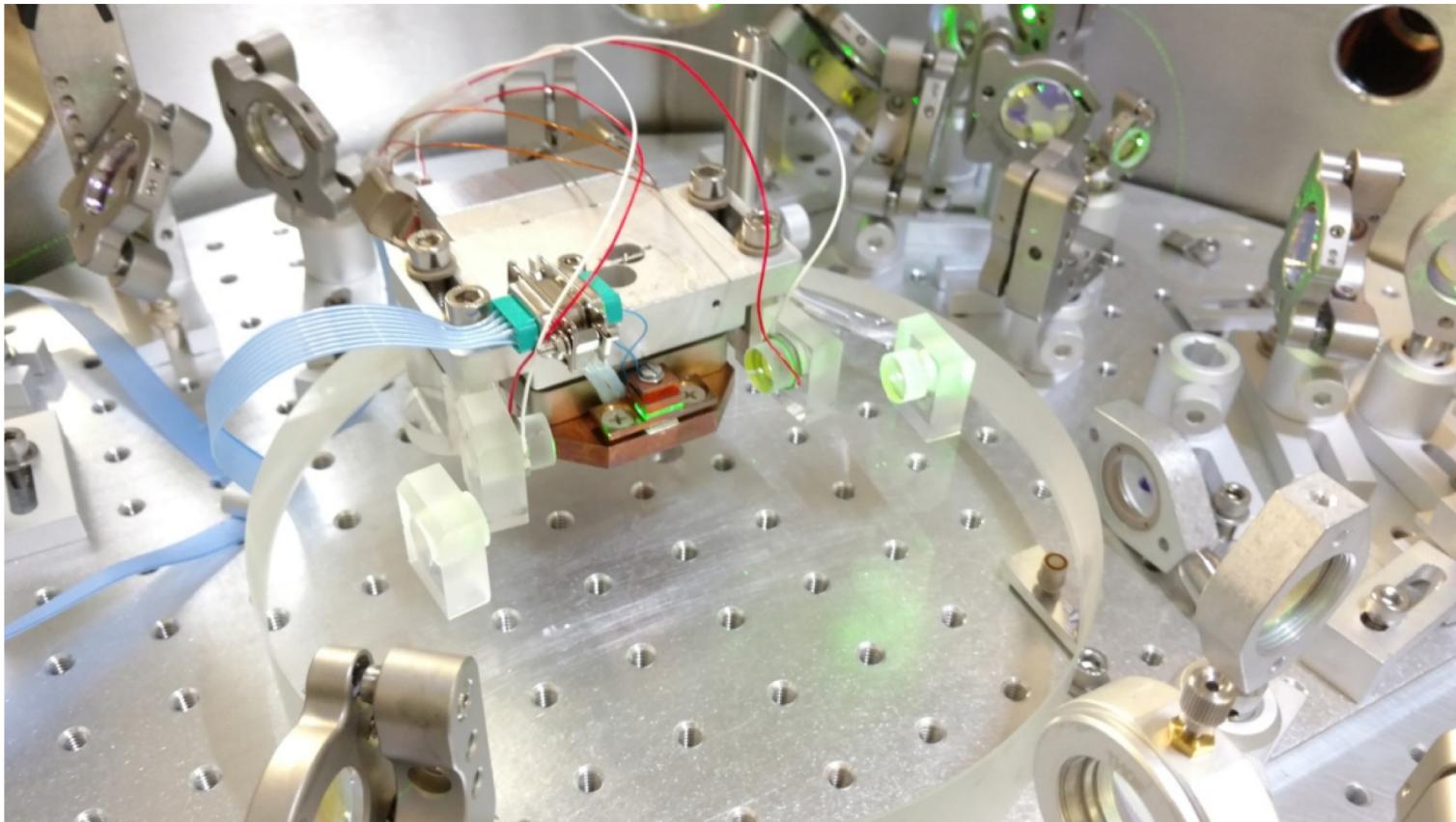


# OPO of LIGO squeezer



# LIGO OPO

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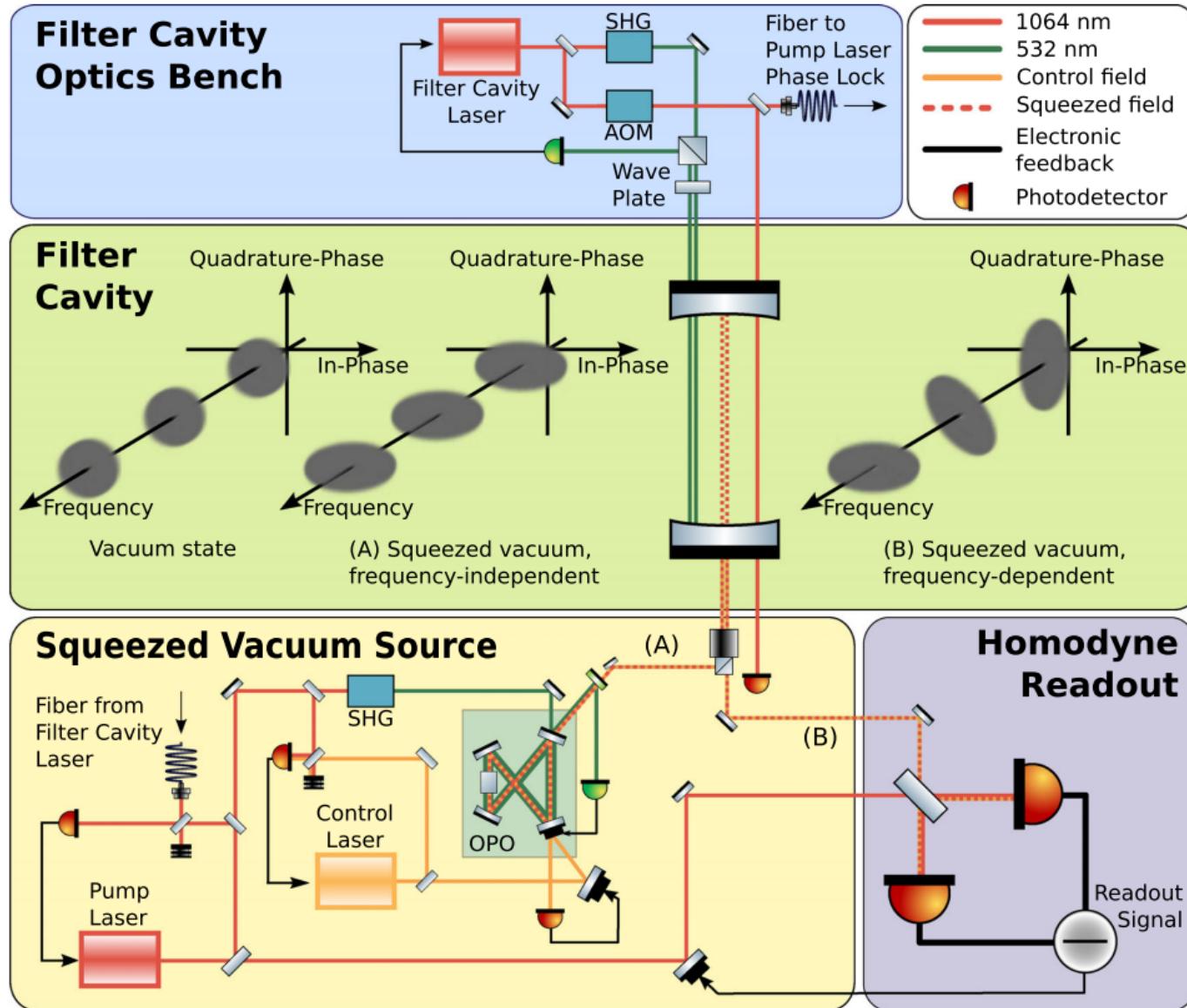


# LIGO filter cavity

PRL 116, 041102 (2016)

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# ALS with green laser

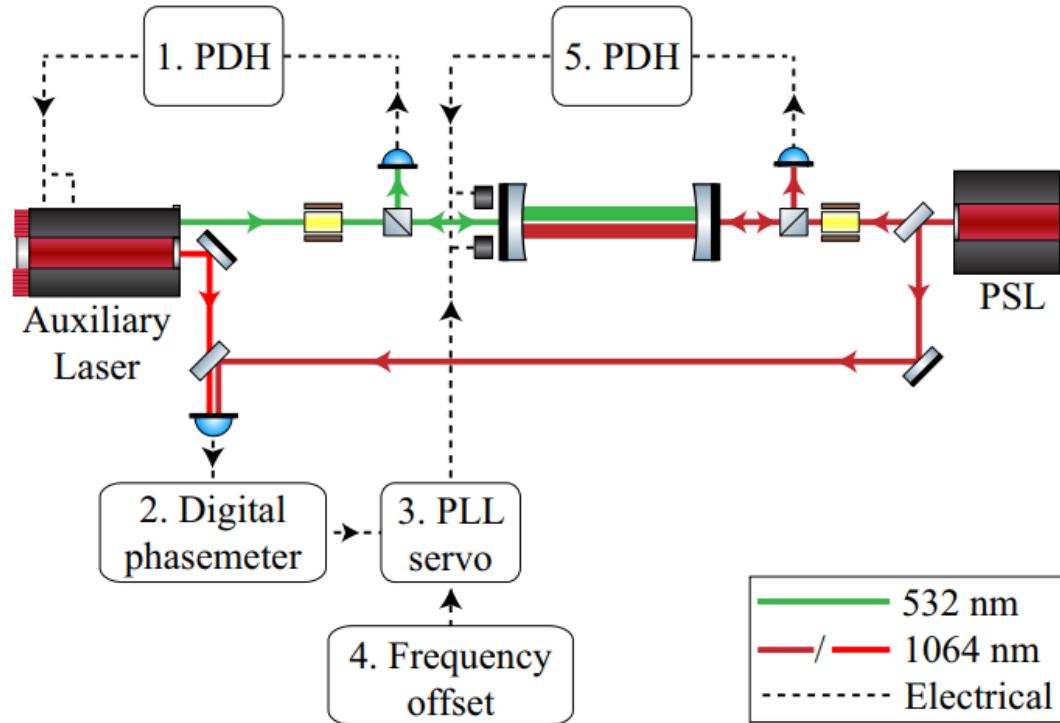


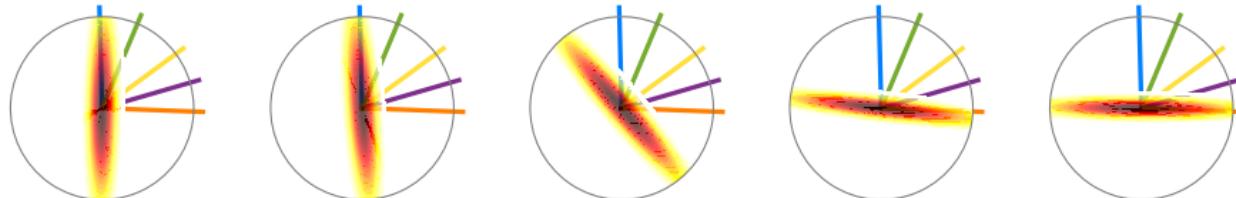
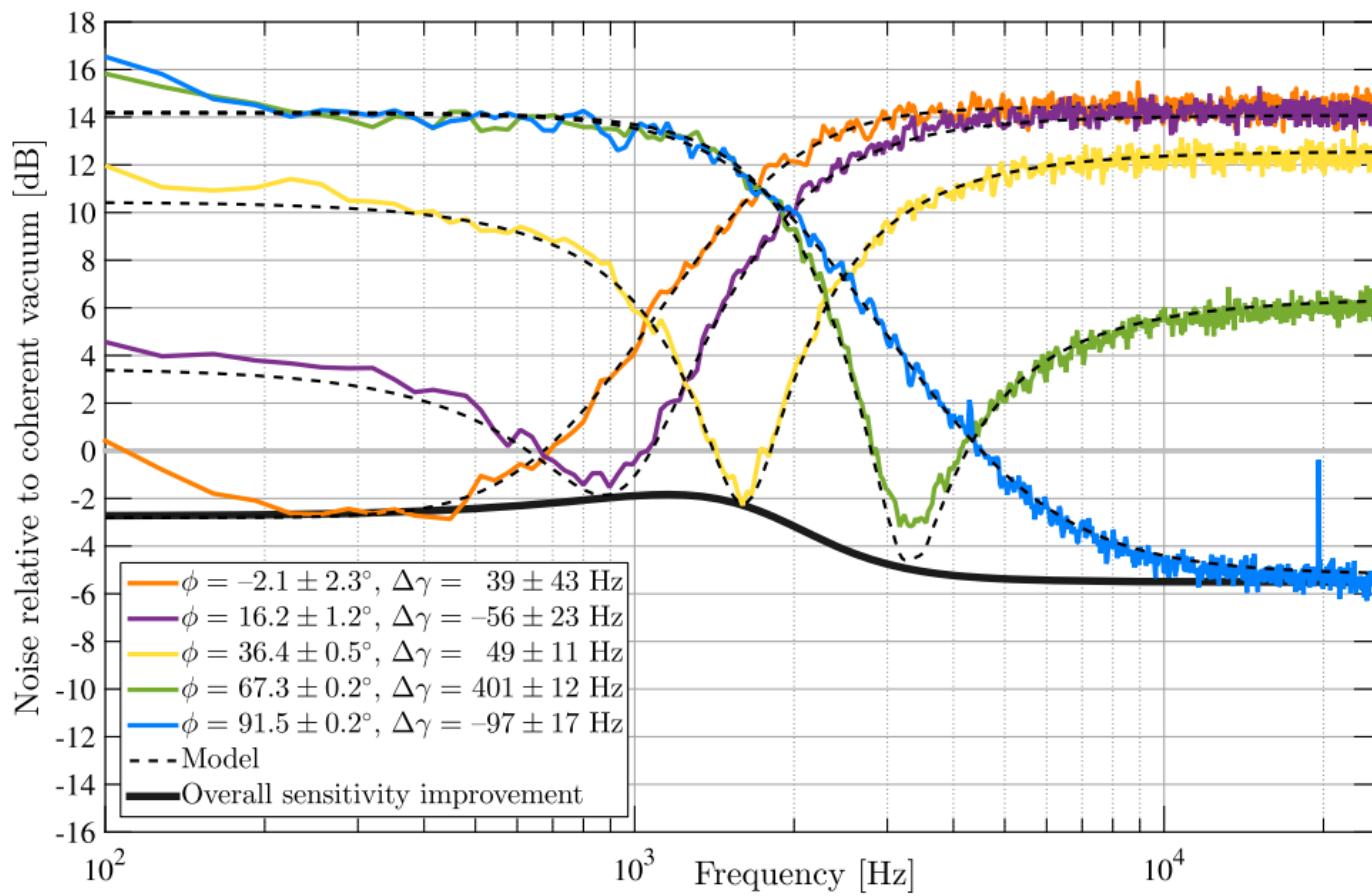
Fig. 2. (Color online) Schematic of the arm-length stabilisation system. The numbering indicates the flow of the lock acquisition process and corresponds to the enumerated list below.

# LIGO filter cavity

PRL 116, 041102 (2016)

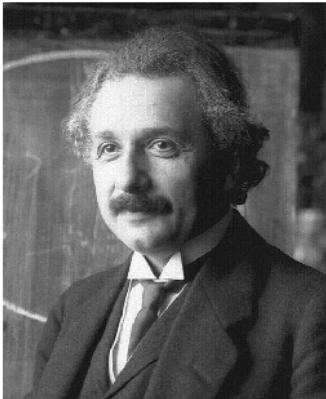
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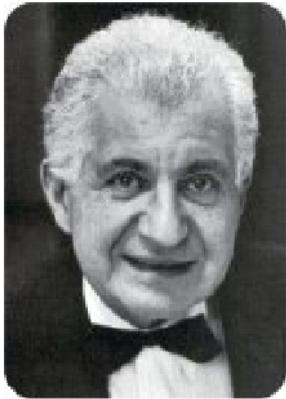


## 5. Frequency dependent squeezing using EPR entanglement

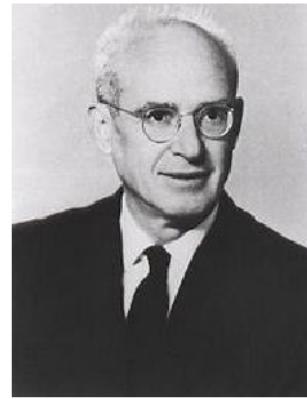
# EPR paradox



Albert Einstein



Boris Podolsky



Nathan Rosen

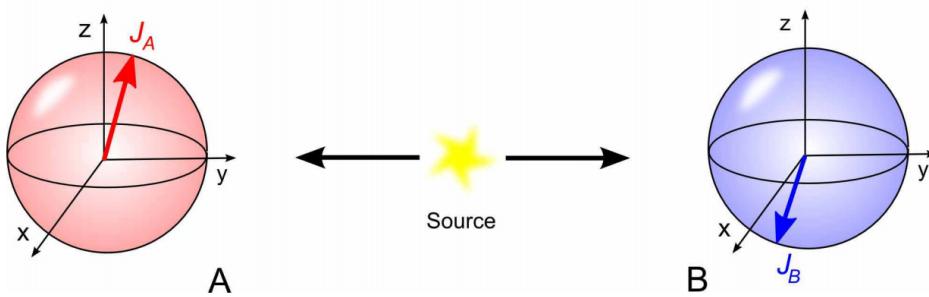
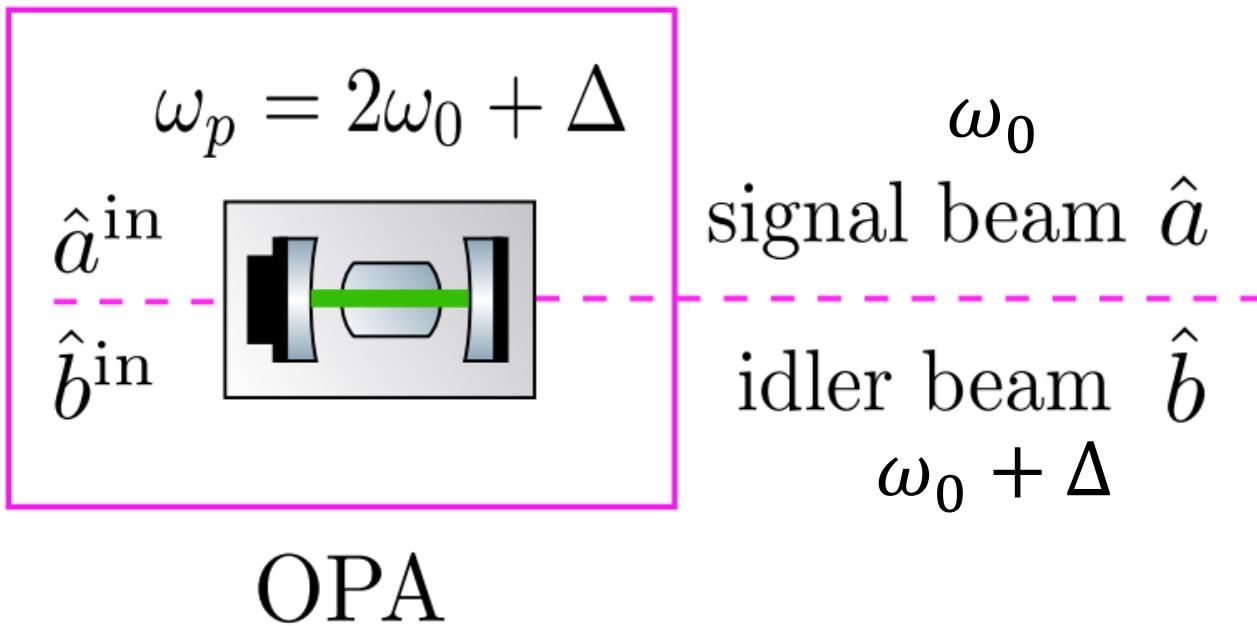


Fig. 1. Schematic diagram of Bohm's EPR experiment with correlated spins at spatially-separated locations A and B.

**E. G. Cavalcanti, P. D. Drummond, H. A. Bachor, and M. D. Reid, "Spin entanglement, decoherence and Bohm's EPR paradox," Opt. Express 17, 18693-18702 (2009)**

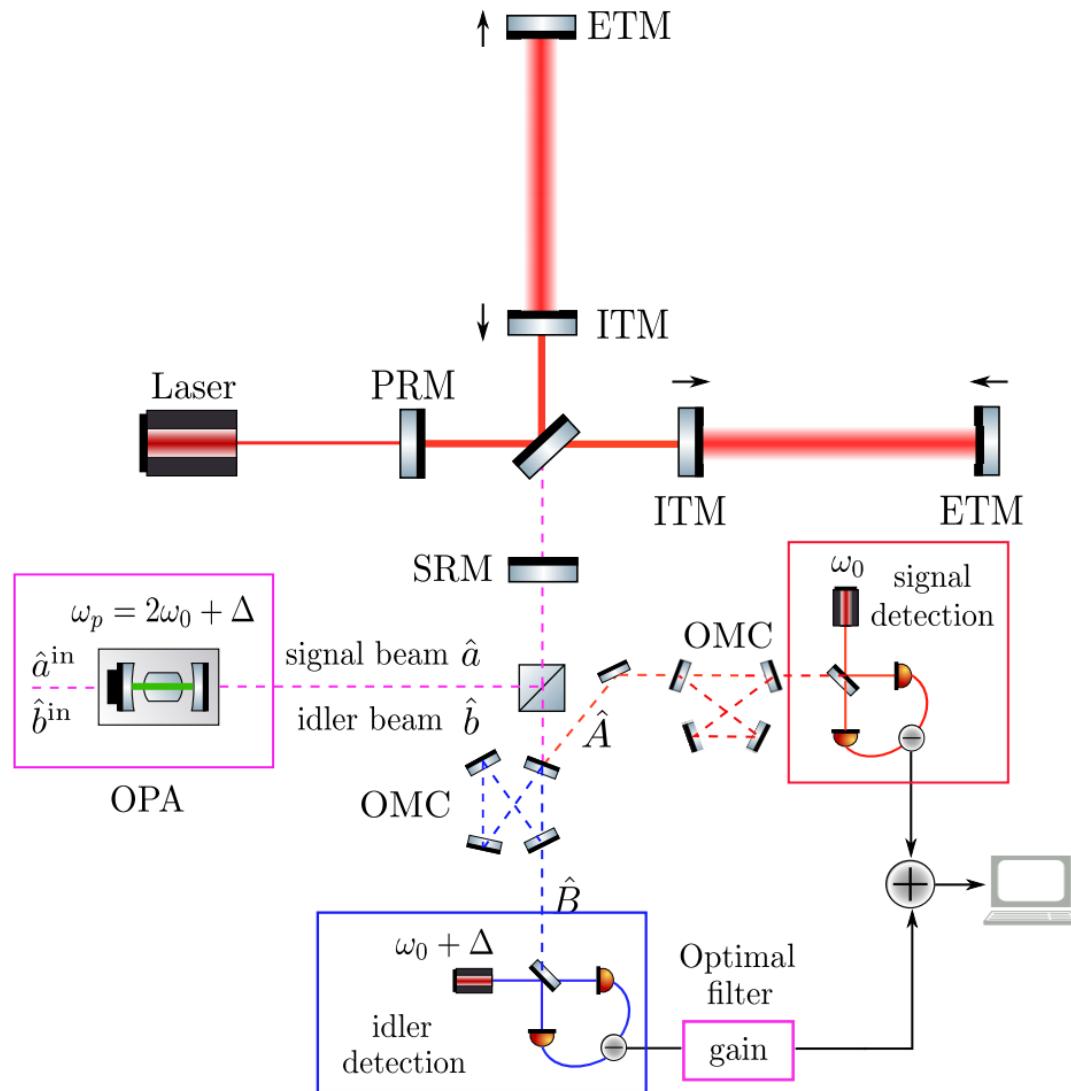
## ■ Non-degenerated PDC

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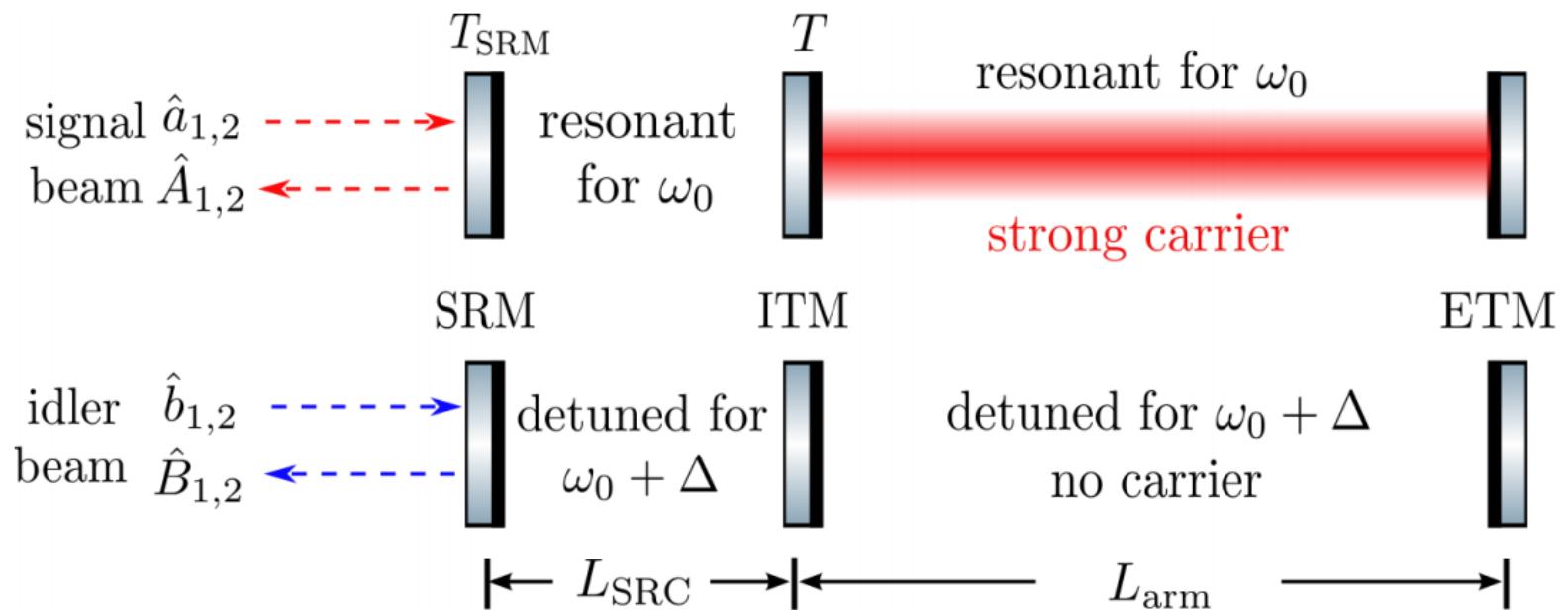


Ma, Y., Miao, H., Pang, B. *et al.* Proposal for gravitational-wave detection beyond the standard quantum limit through EPR entanglement. *Nature Phys* **13**, 776–780 (2017)

# FDS using EPR squeezing

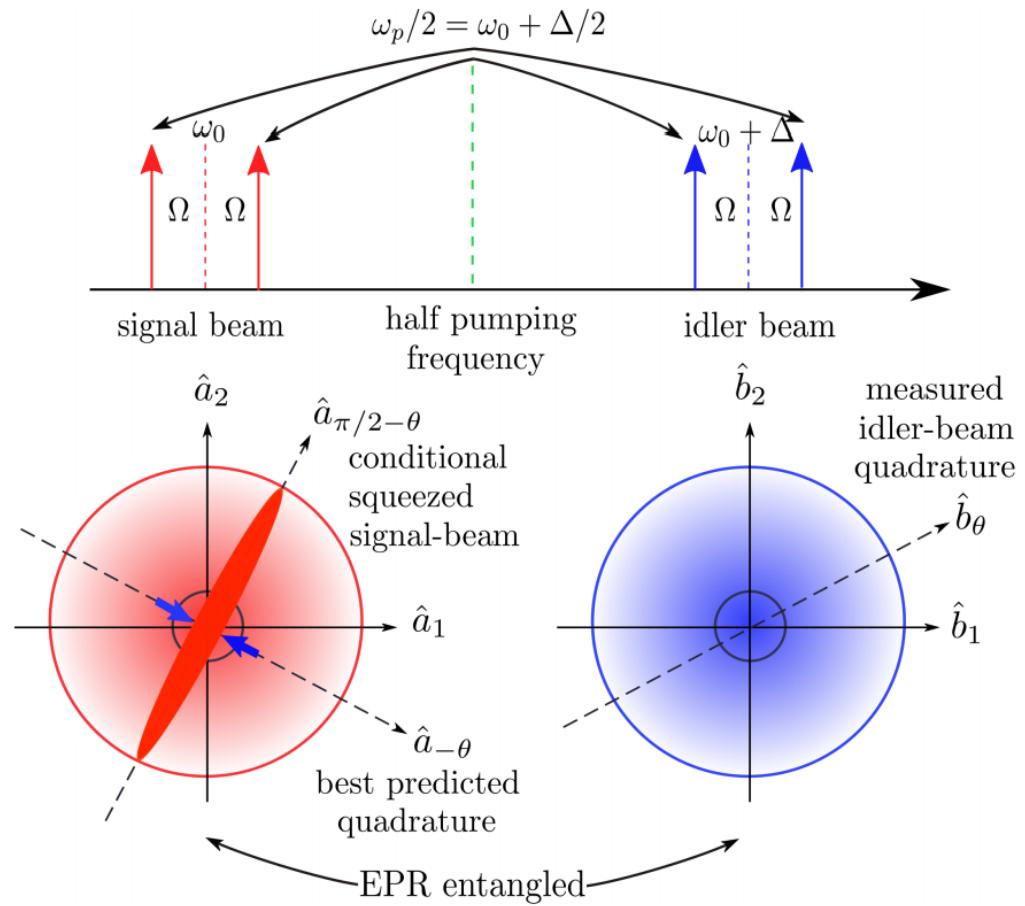


# FDS using EPR squeezing



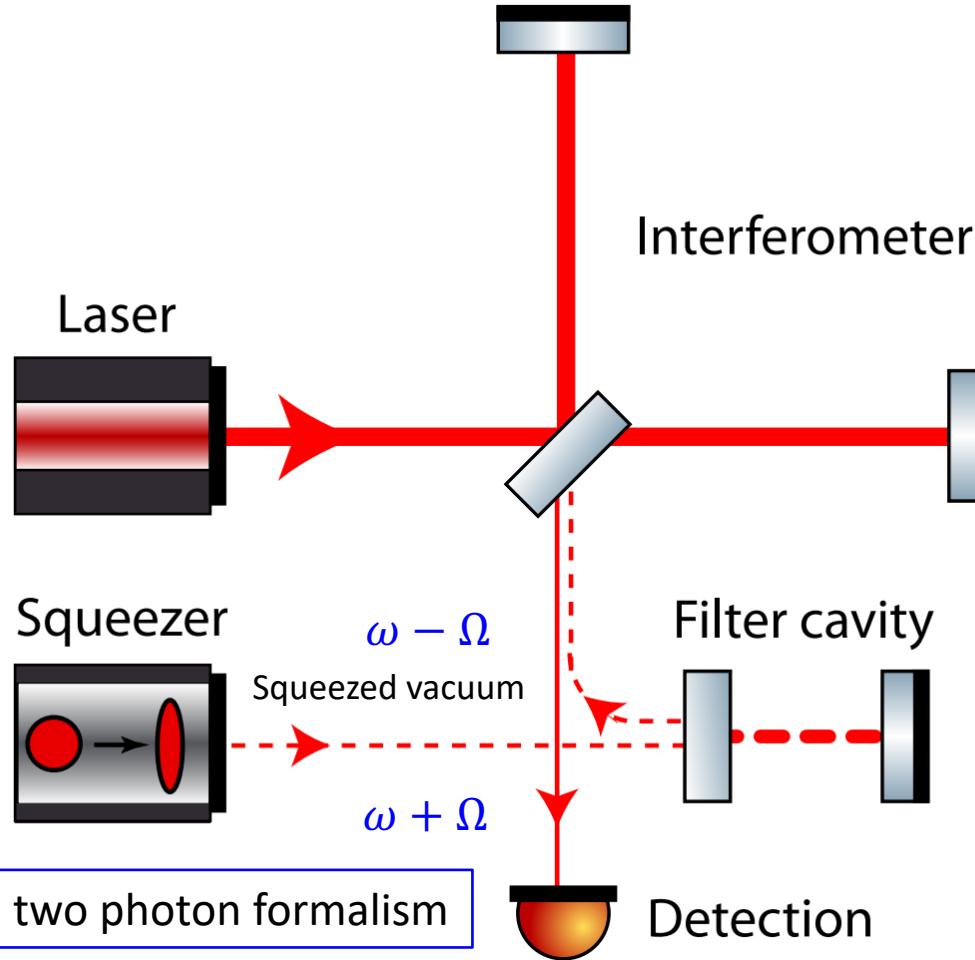
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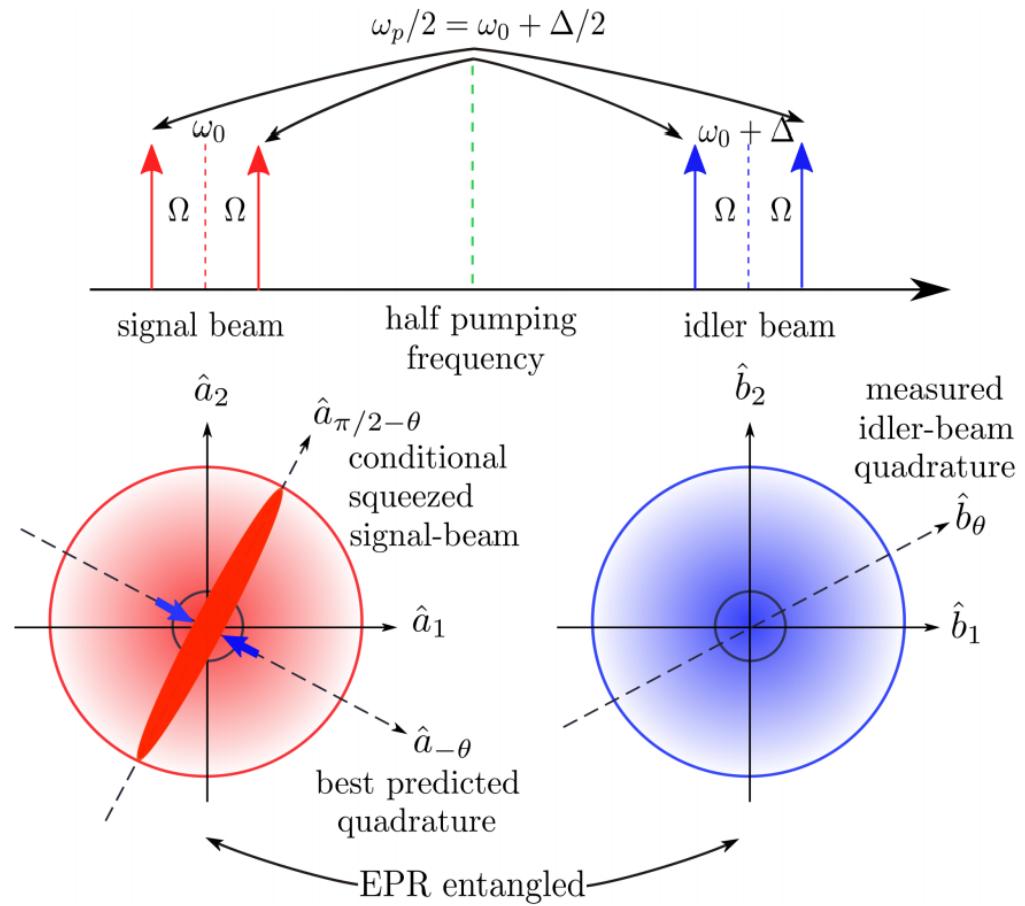
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Thank you