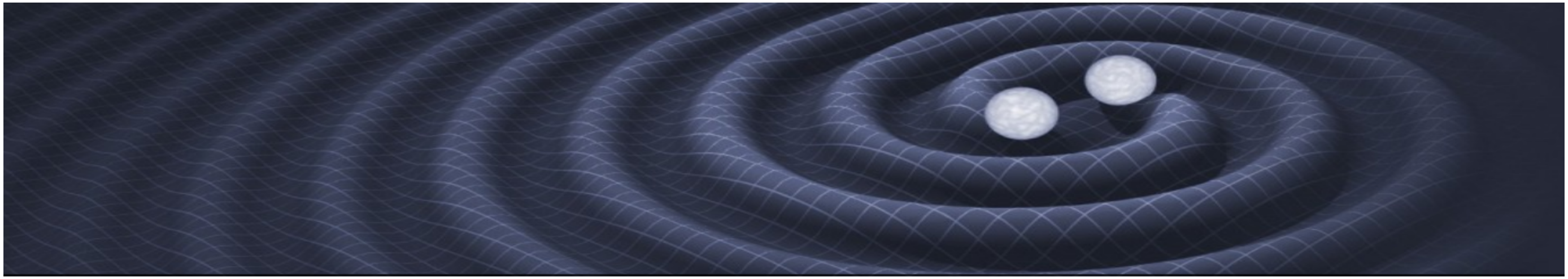
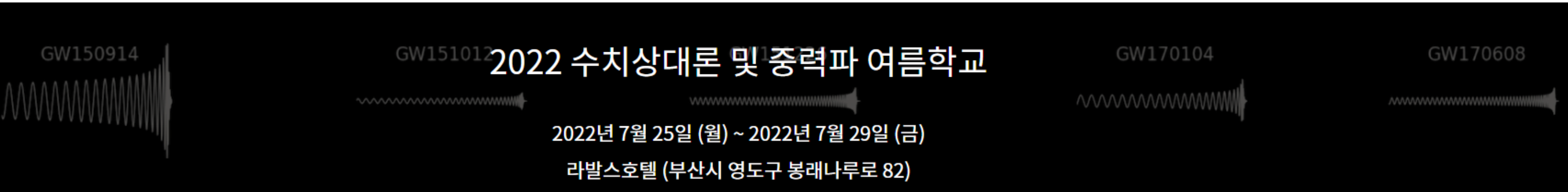


중력파 천체물리학 I

김정리
(이화여자대학교)



강의 개요

- 강의 1: 정의, 개념 소개 + **중력파 천문학**

  “중력파원이란?”
  “중력파 관측 주파수 밴드 vs 중력파원 종류/성질”
  “중력파 천문학 "

- 강의 2: **중력파 천체물리학**

  “중력파천체물리학”
  “밀집 쌍성의 형성과 진화 ”
  “중력파원의 성질과 중력파 파형 모델”
  “관측으로부터 추출 가능한 중력파 천체물리학 관련 정보 ”

- 강의3: **중력파 천체물리학과 중력파 관측**

  “자료 분석과 베이지언 모수 추정“
  “다중신호 천문학” (짧게)
  “저주파 중력파 천체물리학” (매우 짧게)

참고문헌

GW transient catalog -1 https://arxiv.org/abs/1811.12907

GW transient catalog -2" https://arxiv.org/abs/2010.14527

GW transient catalog -3 https://arxiv.org/pdf/2111.03606

및 이 논문에 기재된 참고문헌들

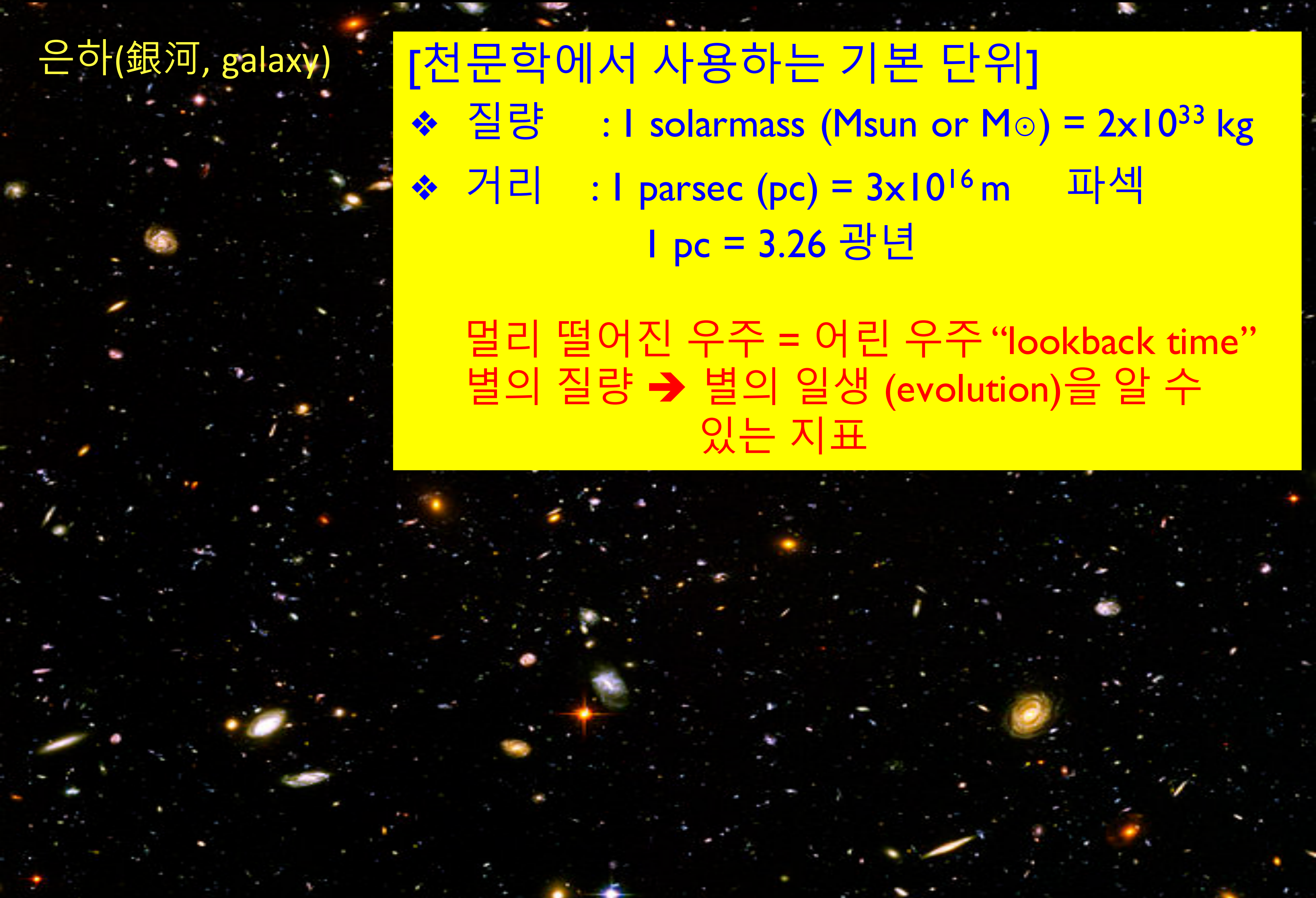
(+) 천문학, 천체물리학 교과서

universe, astrophysics 주제 TED 강연


별(星, star)


성단(星團, star cluster)
Credit:
NASA/HST

빛으로 본 우주 (visible/near IR)



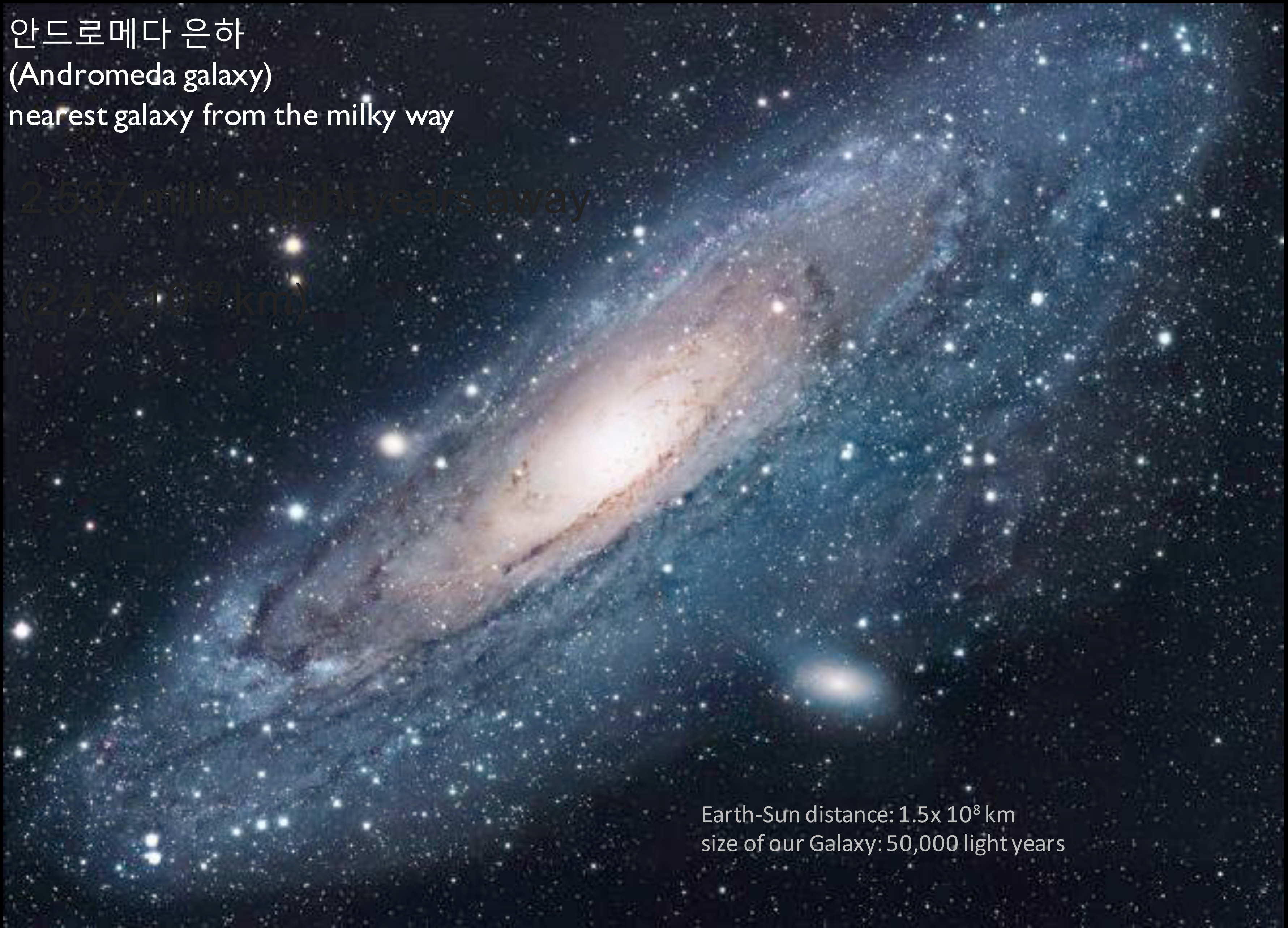
은하(銀河, galaxy)

[천문학에서 사용하는 기본 단위]

- ❖ 질량 : 1 solarmass (Msun or M⊙) = $2x10^{33}$ kg
- ❖ 거리 : 1 parsec (pc) = $3x10^{16}$ m 파섹

  1 pc = 3.26 광년

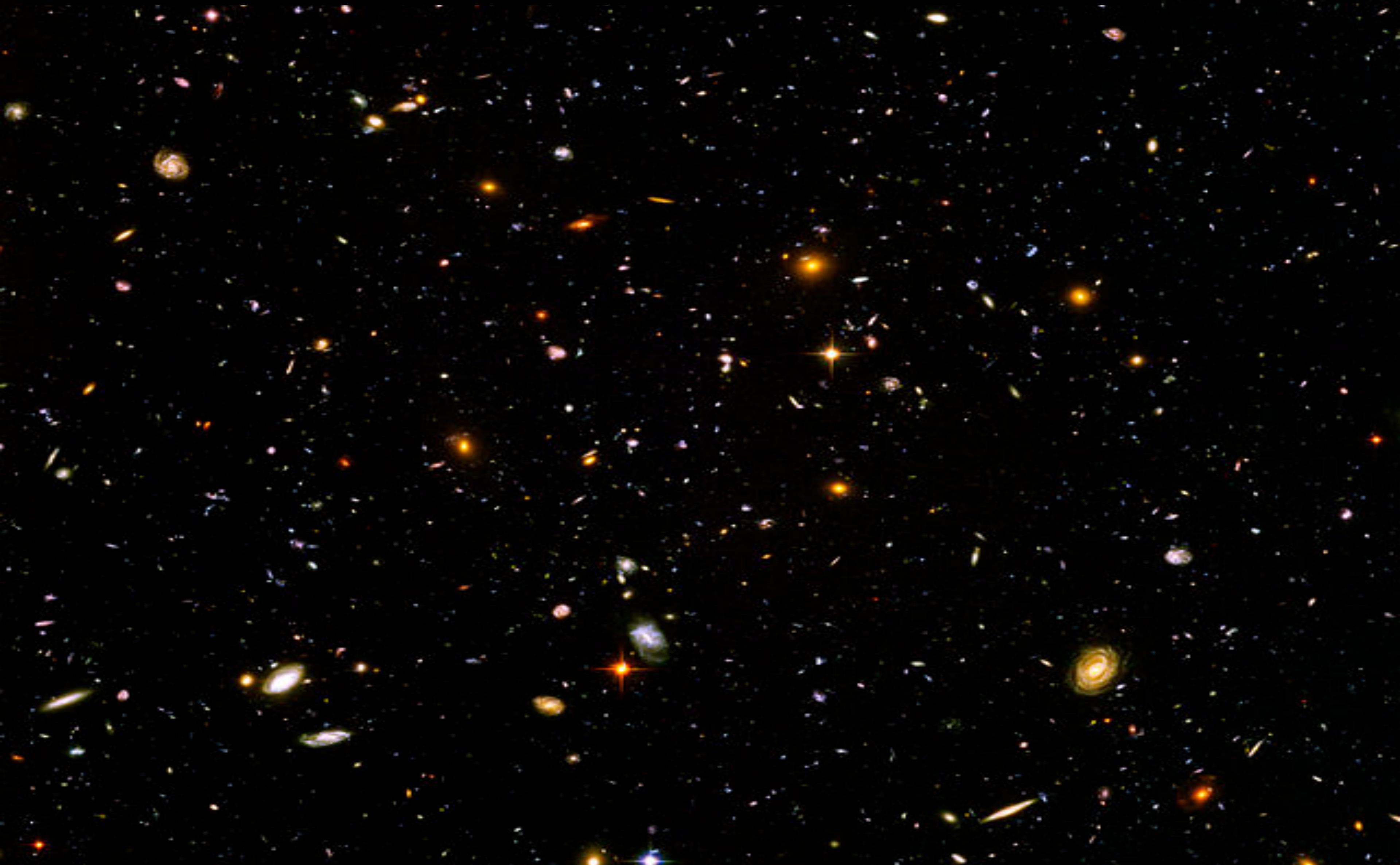
멀리 떨어진 우주 = 어린 우주 “lookback time”
별의 질량 ➔ 별의 일생 (evolution)을 알 수 있는 지표



안드로메다 은하
(Andromeda galaxy)
nearest galaxy from the milky way
2.537 million light years away
(2.4×10^{19} km)
Earth-Sun distance: 1.5×10^{8} km
size of our Galaxy: 50,000 light years

Hubble Ultra Deep field image

우주의 작은 조각 (projected area)에도 수 만 개의 은하가 있다



some heavenly bodies are intrinsically “dark” or too far to “see”

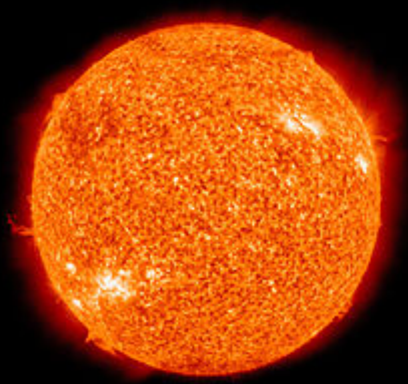
ex) dark matter, black hole

감마선으로 본 우주

("돌발 천체 transients" = one-time strong emission of lights)

현재까지 라이고-비르고로 관측된 중력파 신호는 0.1초 ~ 1분 정도의 1회성 현상에 기인한다. "Gravitational-wave transients"

Some astronomical objects are observable at all times
"continuous sources"



중력파 신호도 "지속해서" 방출되는 것처럼 관측될 수 있다.

예) 두 초거대질량 블랙홀의 공전운동

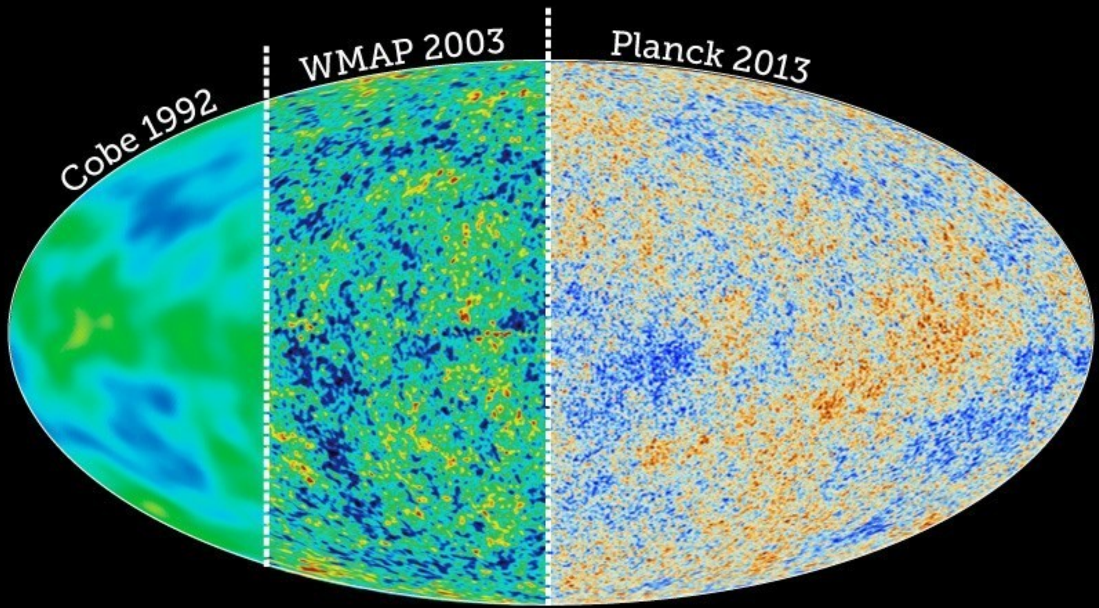
Supermassive BH binaries

➔ emitting GWs at a constant frequency

➔ "continuous sources"

Cosmic microwave background 우주 마이크로파 배경복사

"black body radiation (T=2.7 K)"

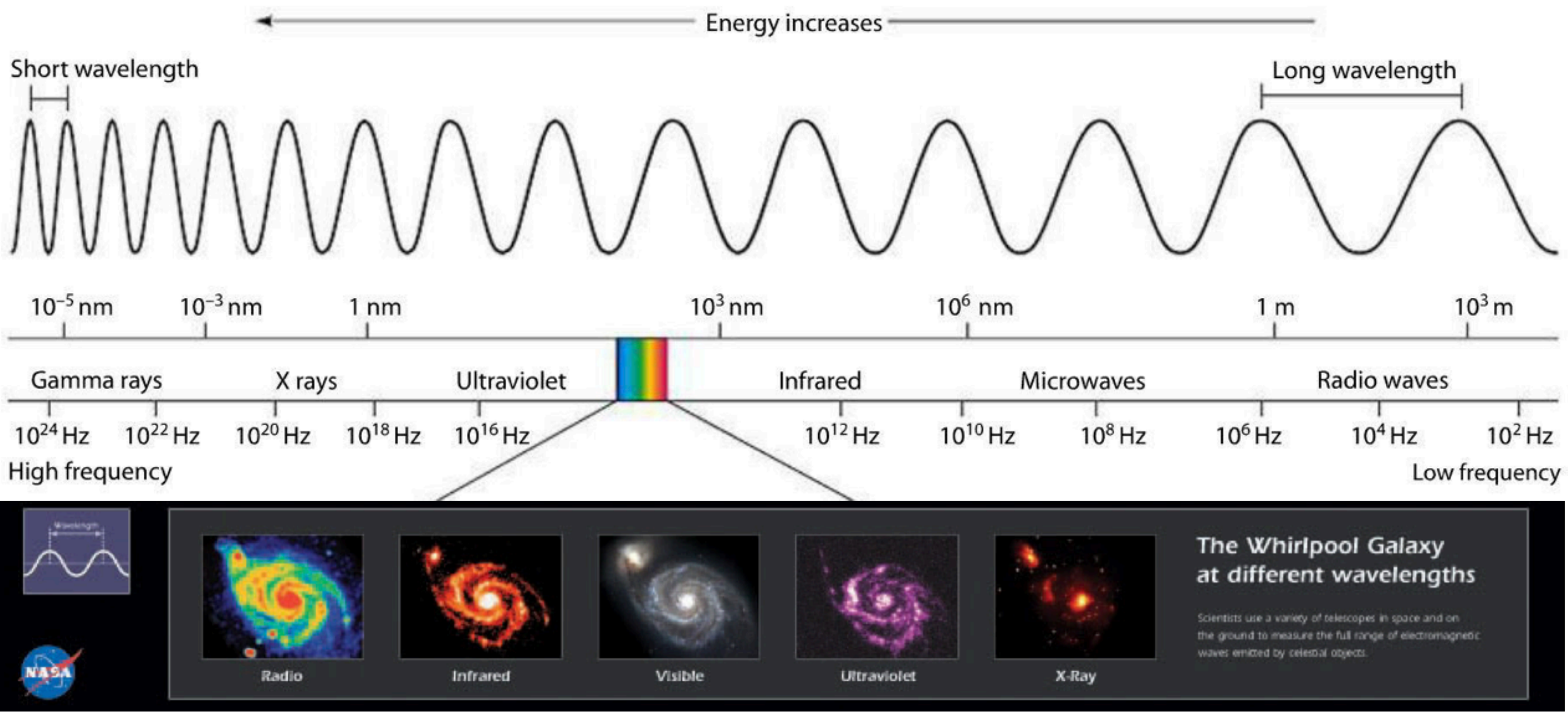


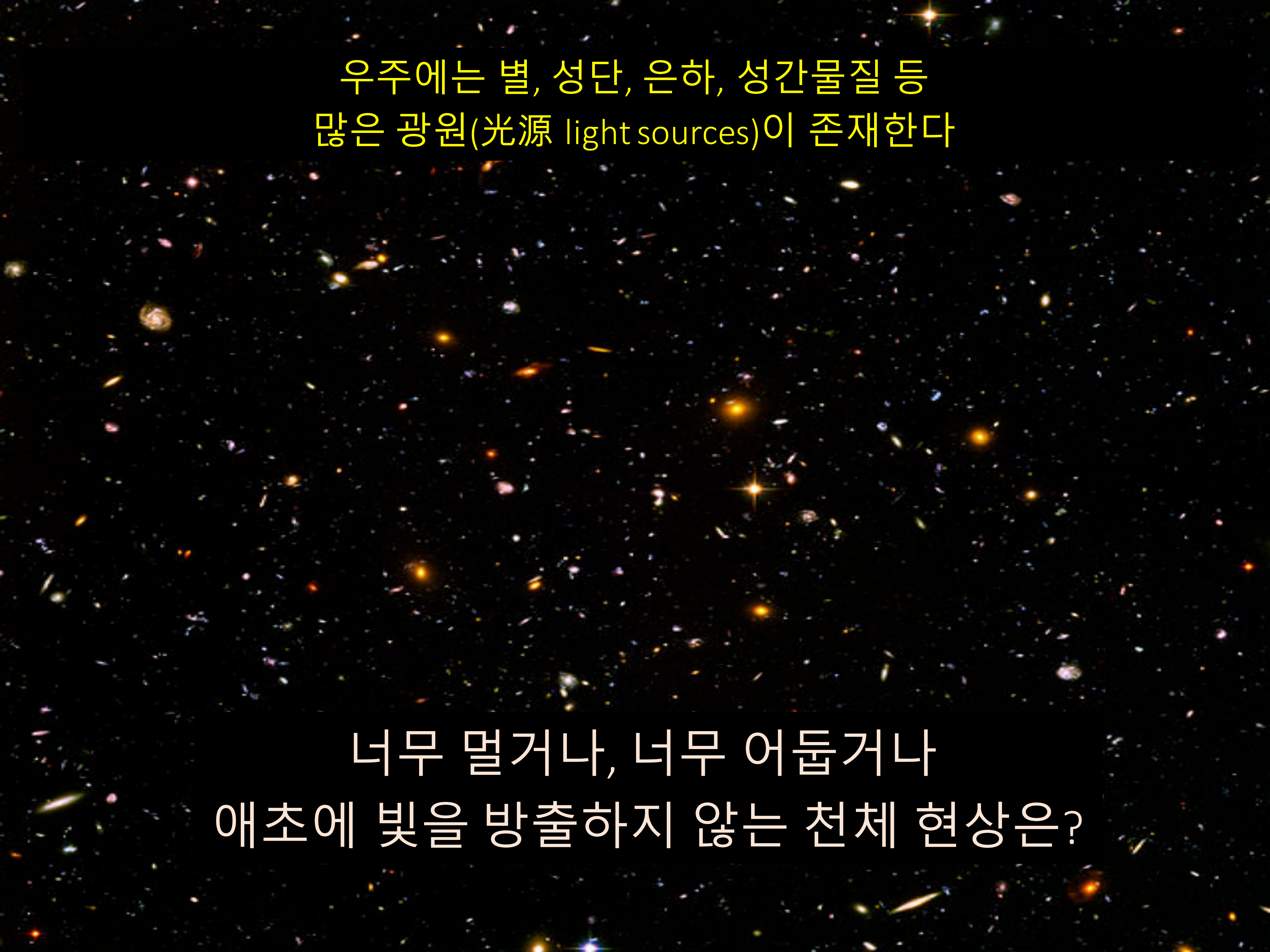
"GW background" are expected in all frequencies

lowest frequencies: early universe "inflation signature"

higher frequencies: astrophysical origin
(superposition of GWs from astrophysical objects)

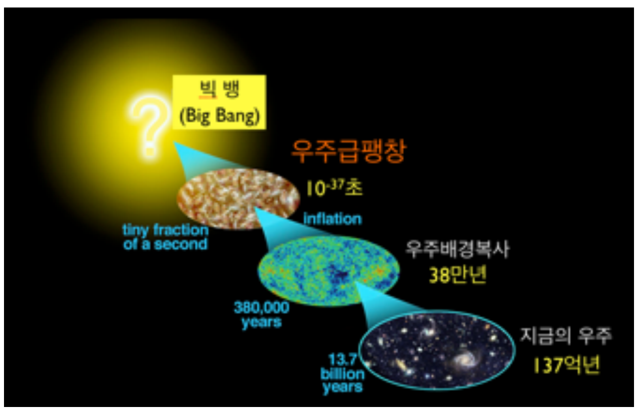
빛 관측과 천문학


Energy increases
Short wavelength
Long wavelength
10⁻⁵ nm
10⁻³ nm
1 nm
10³ nm
10⁶ nm
1 m
10³ m
Gamma rays
X rays
Ultraviolet
Infrared
Microwaves
Radio waves
10²⁴ Hz
10²² Hz
10²⁰ Hz
10¹⁸ Hz
10¹⁶ Hz
10¹² Hz
10¹⁰ Hz
10⁸ Hz
10⁶ Hz
10⁴ Hz
10² Hz
High frequency
Low frequency
Radio
Infrared
Visible
Ultraviolet
X-Ray
The Whirlpool Galaxy at different wavelengths
Scientists use a variety of telescopes in space and on the ground to measure the full range of electromagnetic waves emitted by celestial objects.
NASA

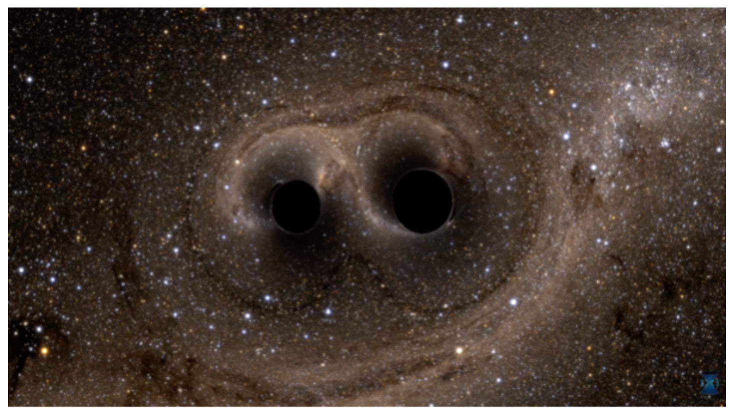

우주에는 별, 성단, 은하, 성간물질 등
많은 광원(光源 light sources)이 존재한다
너무 멀거나, 너무 어둡거나
애초에 빛을 방출하지 않는 천체 현상은?

21세기는 중력파로 우주를 관측할 수 있는 시대!

초기 우주



블랙홀, 중성자별
“쌍성 "



우리 은하내
초신성

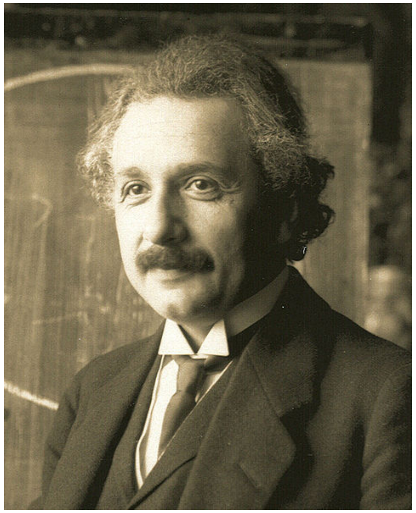
GW frequency: 10^{-16} Hz (early Universe) ~ 10,000 Hz (supernovae)

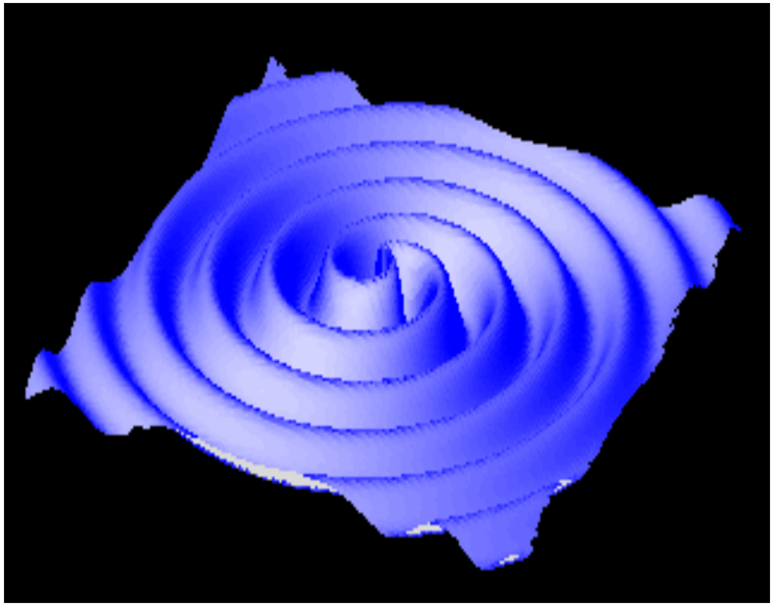
중력파 Gravitational Waves (GWs) are "ripples" in spacetime.

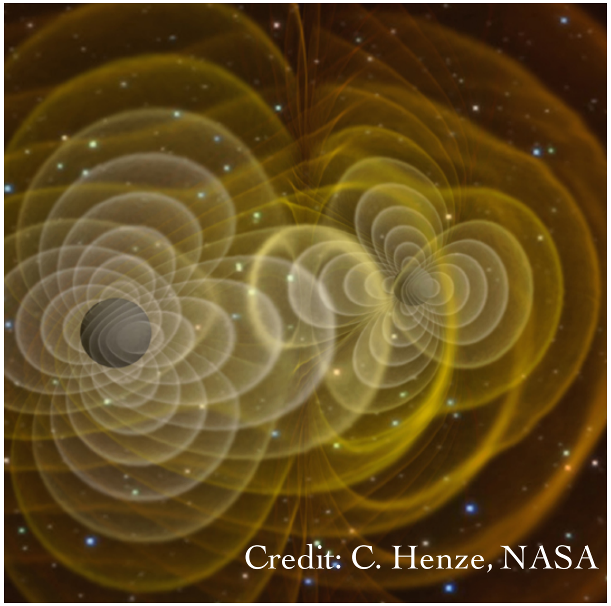
Accelerated mass with non-zero quadruple (or higher) moments perturbs spacetime. The perturbation propagates outward from the "source" at the speed of light : gravitational waves

metric = Minkowski + perturbation *h*

$$h_{\mu\nu} = \frac{2G}{Rc^4}\ddot{I}_{\mu\nu}$$

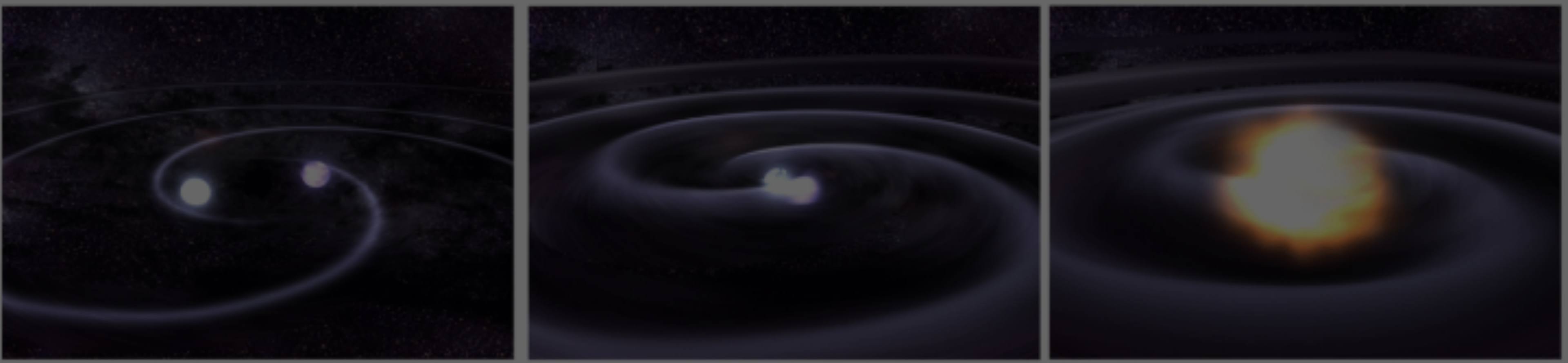






GWs emitted by "inspiral" motion of a binary

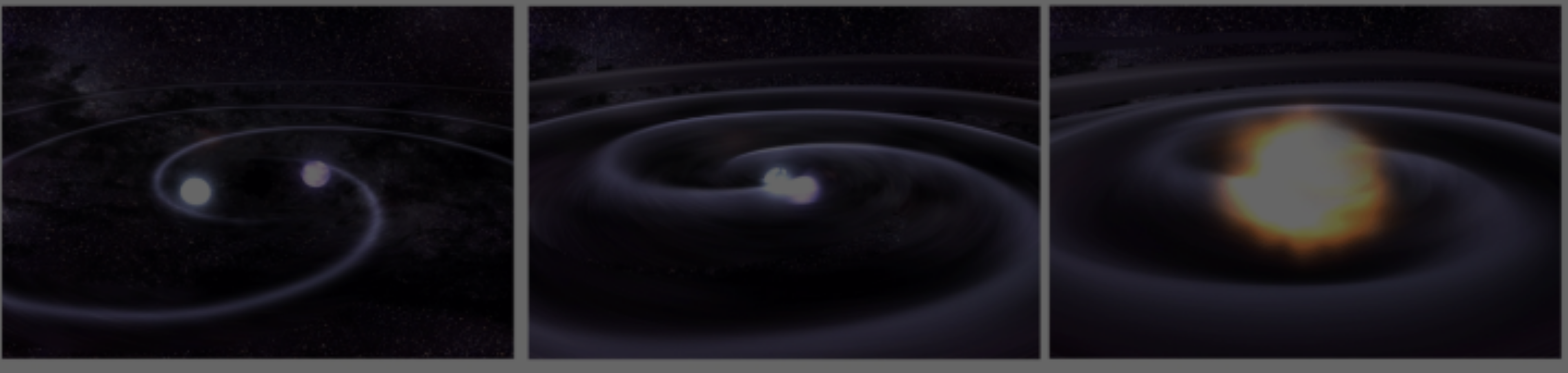
GWs emitted from merging Black hole - Black hole



천체물리학

Astrophysics = Astro + Physics

천문 우주 현상의
근간 원리를 이해하고자 하는 학문

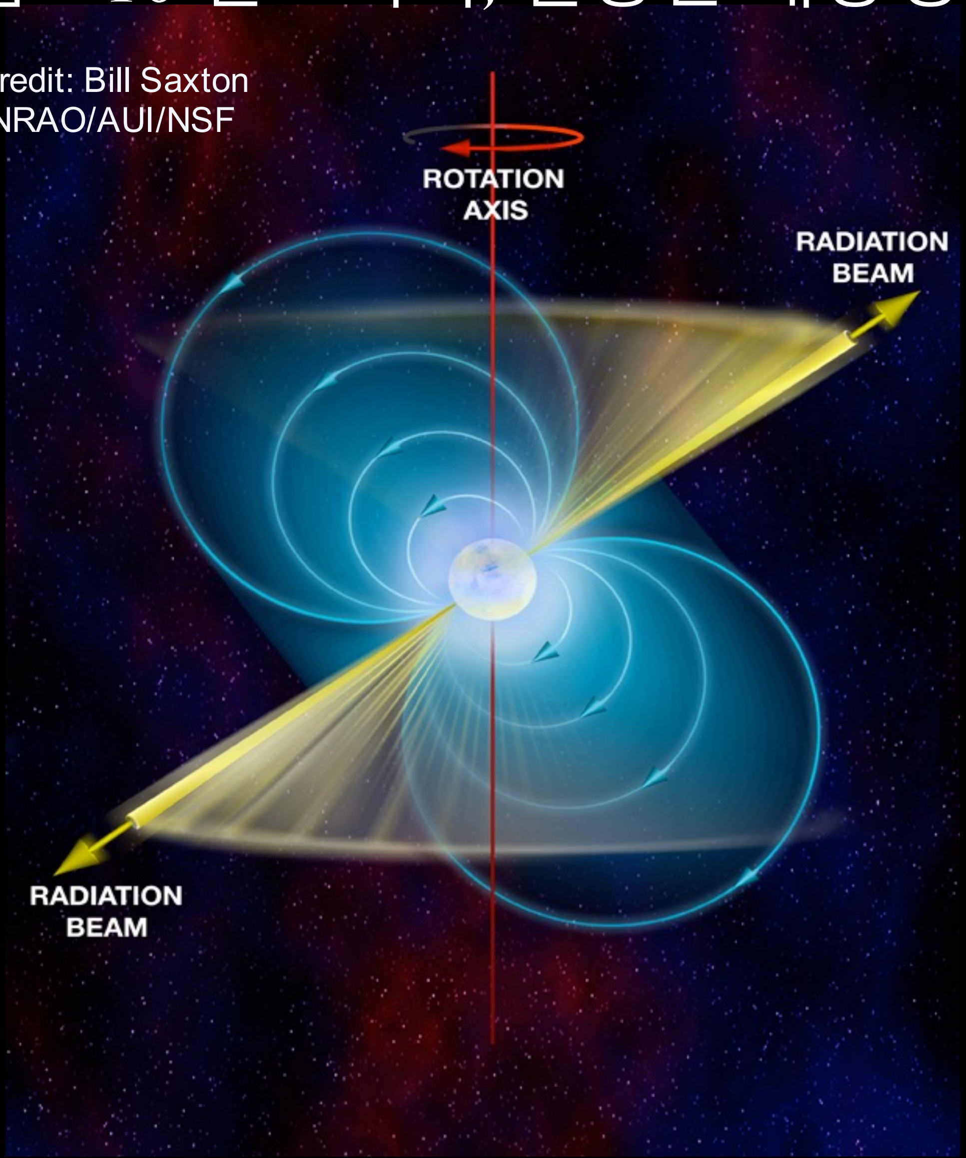


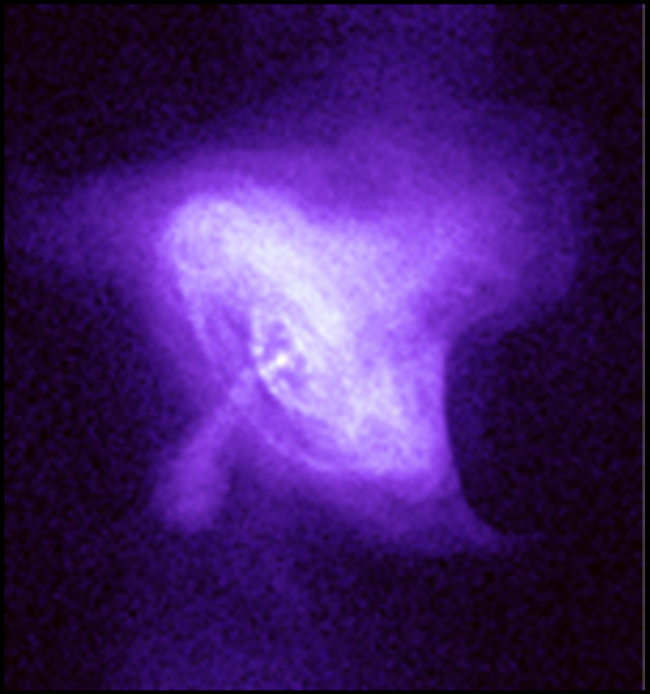
상대성이론 + 천체물리학
= 별의 폭발이나 천체간 충돌로 중력파가 발생한다.

"중력파원(源)"

중성자별 = 중성자로 이루어진 별

지름 ~ 10 킬로미터, 질량은 태양정도

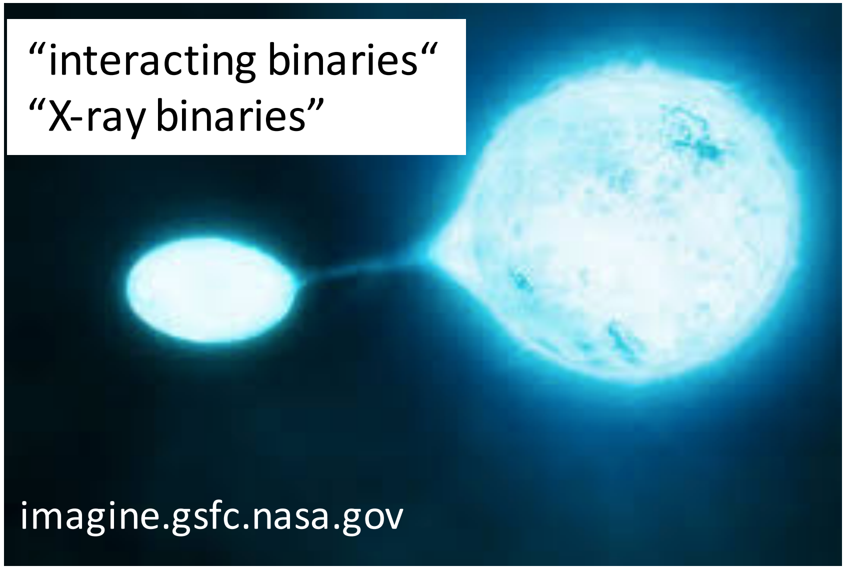




게 펄서
찬드라 위성망원경이
찍은 X-ray 사진 (NASA)

블랙홀 무거운 별들의 최종 진화 단계
“별질량 블랙홀 "

Many stars in the universe are in binaries



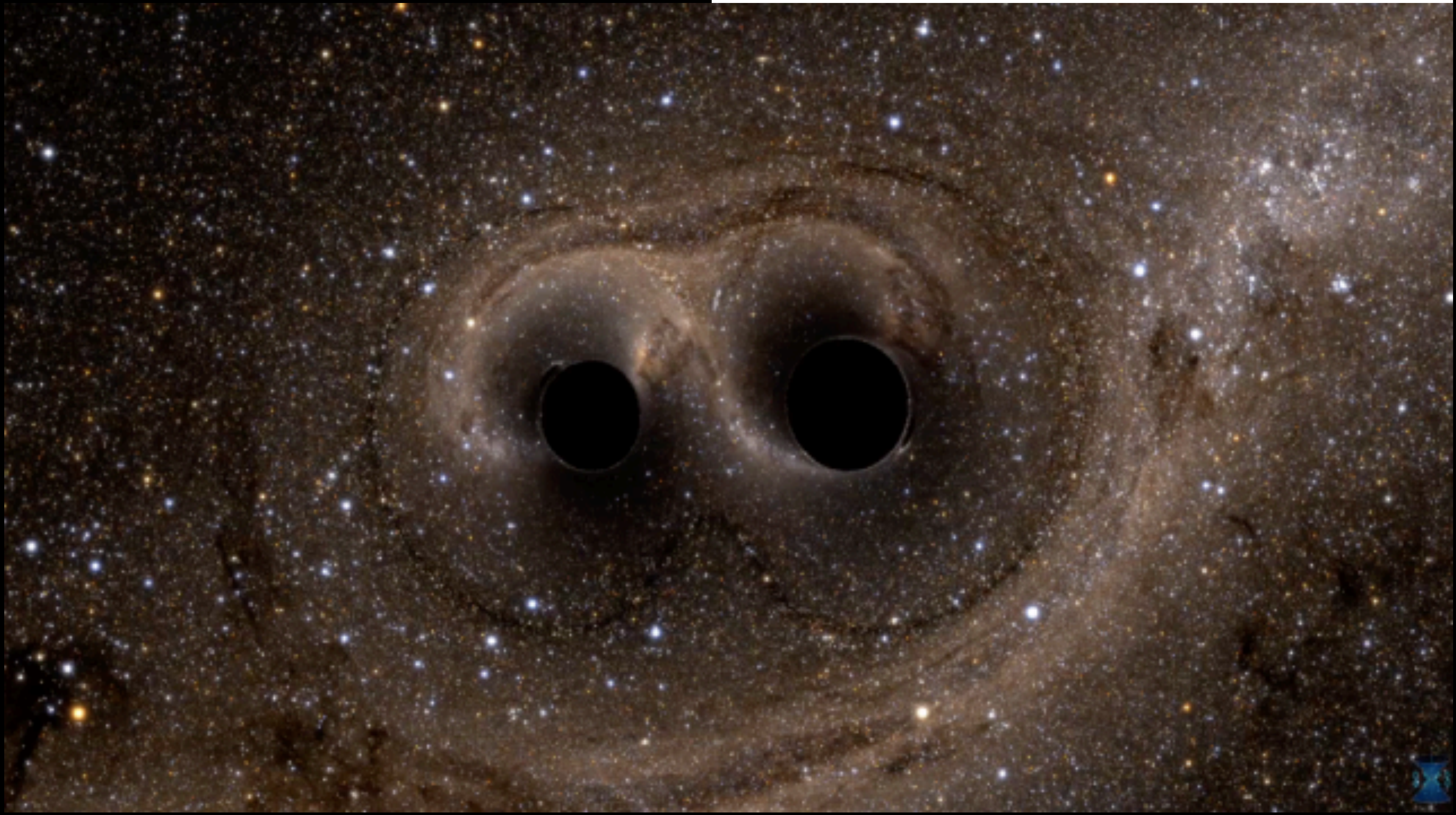
Black Holes (BHs)

not observable with light

BBHs are strong sources of GWs

BBH = binary black holes

$$r_{\mathrm{Schwarz}}(m) = \frac{2Gm}{c^2} = 2.95\left(\frac{m}{\mathrm{M}_\odot}\right)\mathrm{km}$$



compact binary coalescences (CBCs) 밀집 쌍성 병합

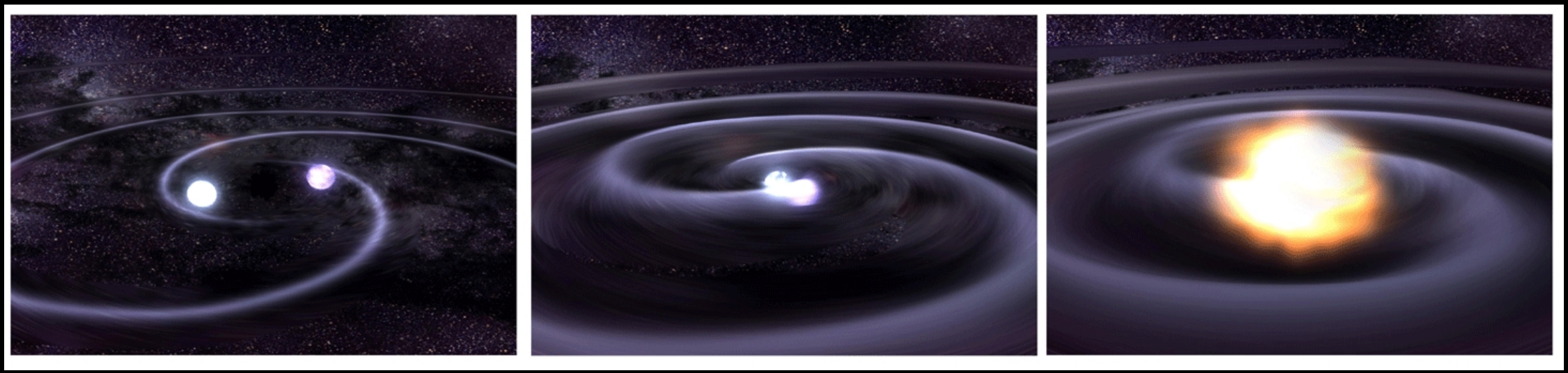
merging binaries consisting of WD/NS/BH

➔ 조건: 중력파 방출로 인한 da/dt (anow ➔ 충돌) 시간이
허블시간 보다 작으면 “병합 쌍성 " 으로 구분

$$\tau_{\rm mrg} = 9.83 \times 10^6\,{\rm yr} \left(\frac{P_b}{\rm hr}\right)^{8/3} \left(\frac{\mu}{\rm M_\odot}\right)^{-1} \left(\frac{m_1+m_2}{\rm M_\odot}\right)^{-2/3}$$

$$\mu = m_1 m_2/(m_1+m_2)$$

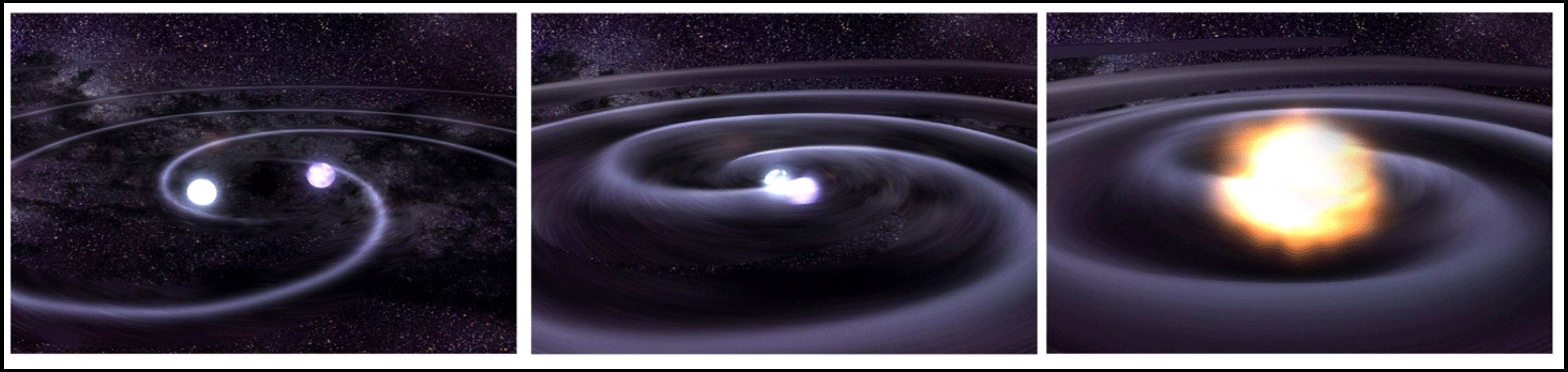
https://arxiv.org/pdf/astro-ph/0402162.pdf (following Peters (1964))



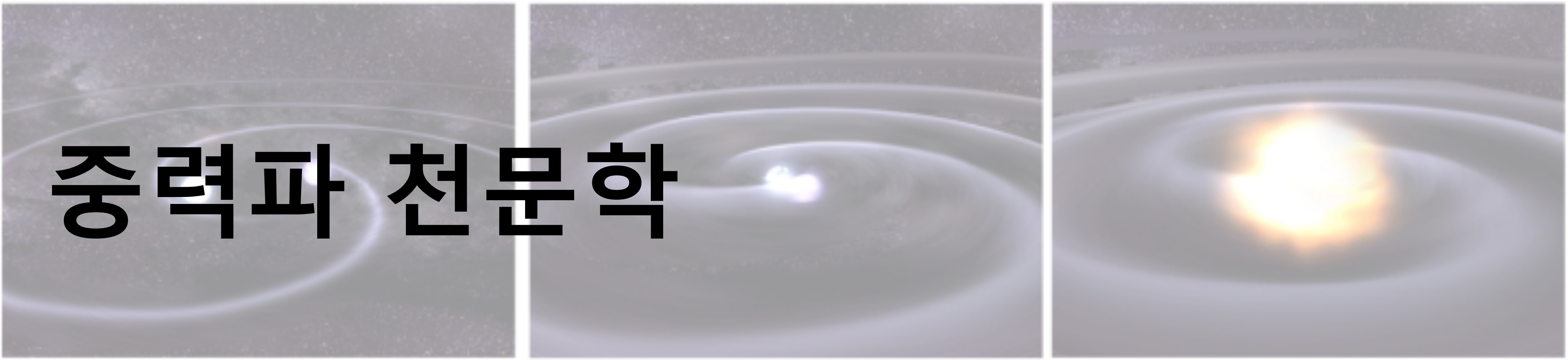
compact binary coalescences (CBCs) 밀집 쌍성 병합

3 phases of a CBC = inspiral + merge + ringdown

➔ 편의상 병합 과정을 3단계로 나누어 중력파 파형을 계산하거나 신호를 분석한다.



중력파 천문학



GW frequency ~ 1/(source mass)$^{\alpha}$

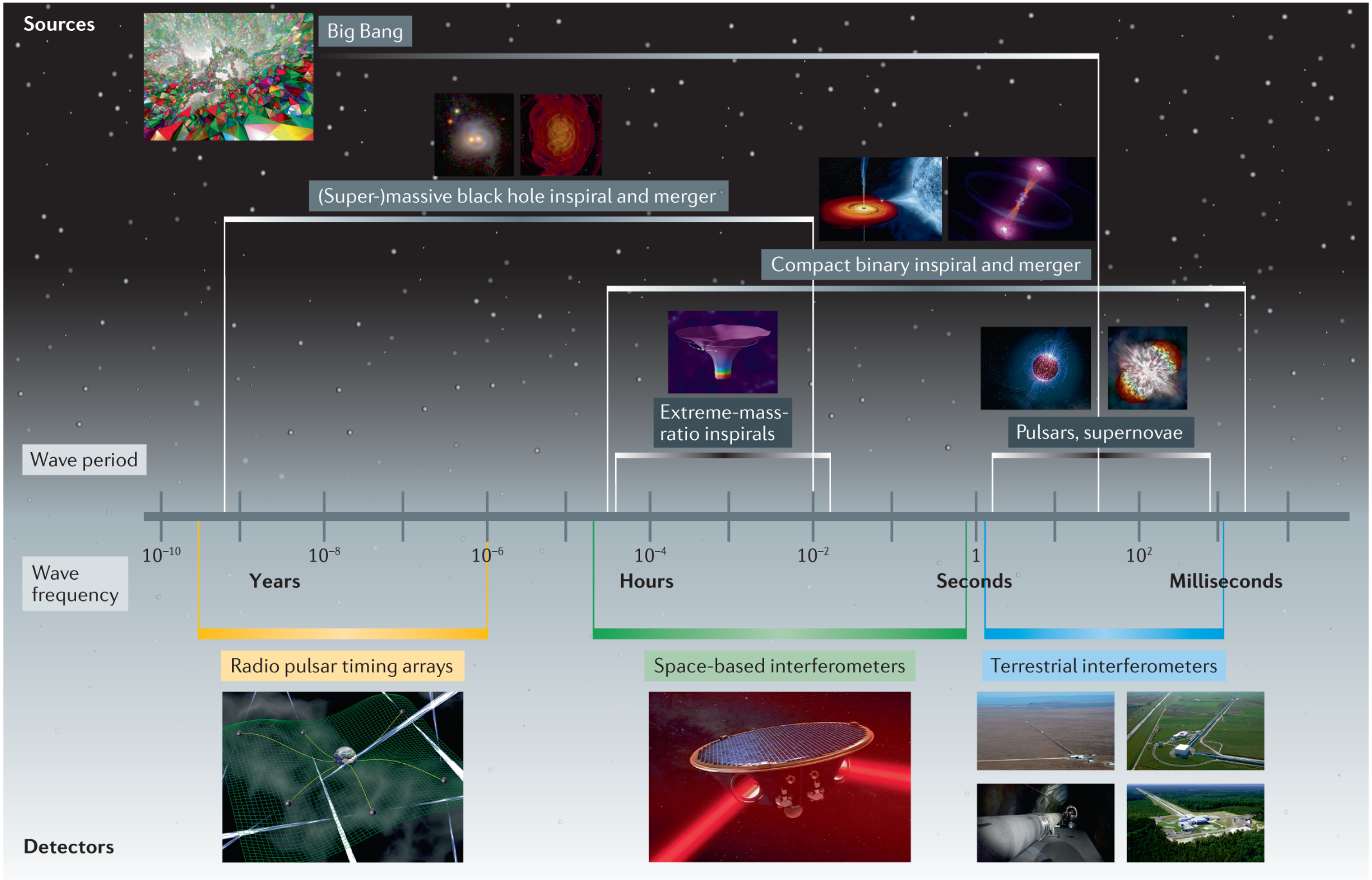
- massive source emits GW signals at lower frequencies
- different frequencies require different observational technique

Successful technique to observe GW signals:

- laser interferometry (on Earth, in space)
- pulsar timing array (radio observation using fast-rotating radio pulsars)

중력파 천문학과 중력파원

관측 주파수(Hz) ← → 중력파원의 “특징적인 시간 " 기준 → 중력파원의 질량



https://www.nature.com/articles/s42254-021-00303-8/figures/2

중력파 관측 ➔ 밀집성의 질량 ➔ 밀집성의 형성과 진화

CBC = binary의 기본 물리량 {m1, m2, a, 천구상의 위치, 거리} 등

- 중력파 관측 ➔ m1,m2 개별 질량 측정 가능
- 빛 관측 ➔ 케플러 방정식 적용 ➔ m1+m2 만 측정 가능

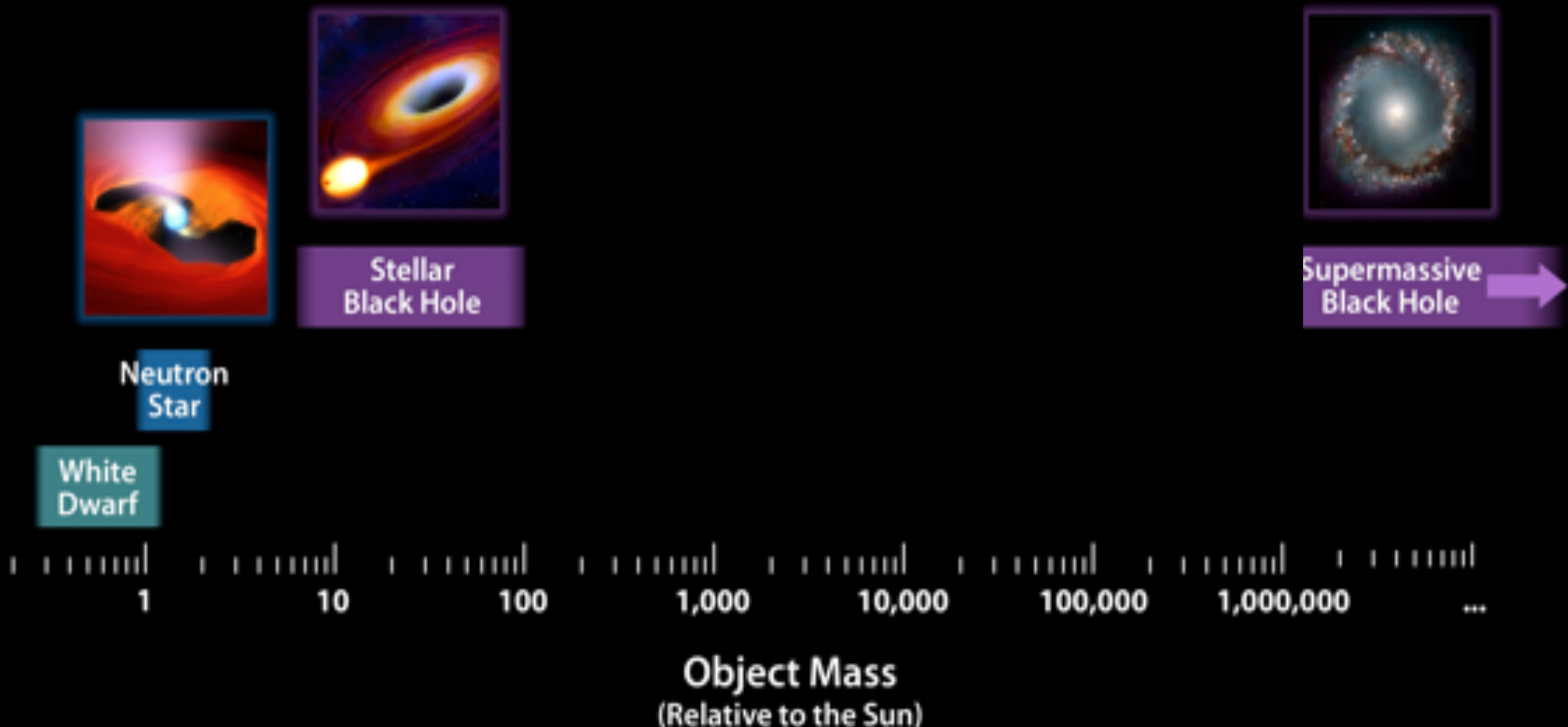


Image credit: NASA/JPL-Caltech

중력파 관측 ➔ 밀집성의 질량 ➔ 밀집성의 형성과 진화

블랙홀의 질량 : 별질량 블랙홀 (태양 질량의 2.5 ~ 1000 배) ➔ 라이고 관측 “2.5~300배“
중간질량 블랙홀, 거대질량 블랙홀, 초거대질량 블랙홀

블랙홀의 형성과 진화 : 밀집성 천체물리학 + 은하 진화 + 우주론 필요

2015~현재까지 관측된 중력파원은 대부분 (별질량) 블랙홀-블랙홀 병합

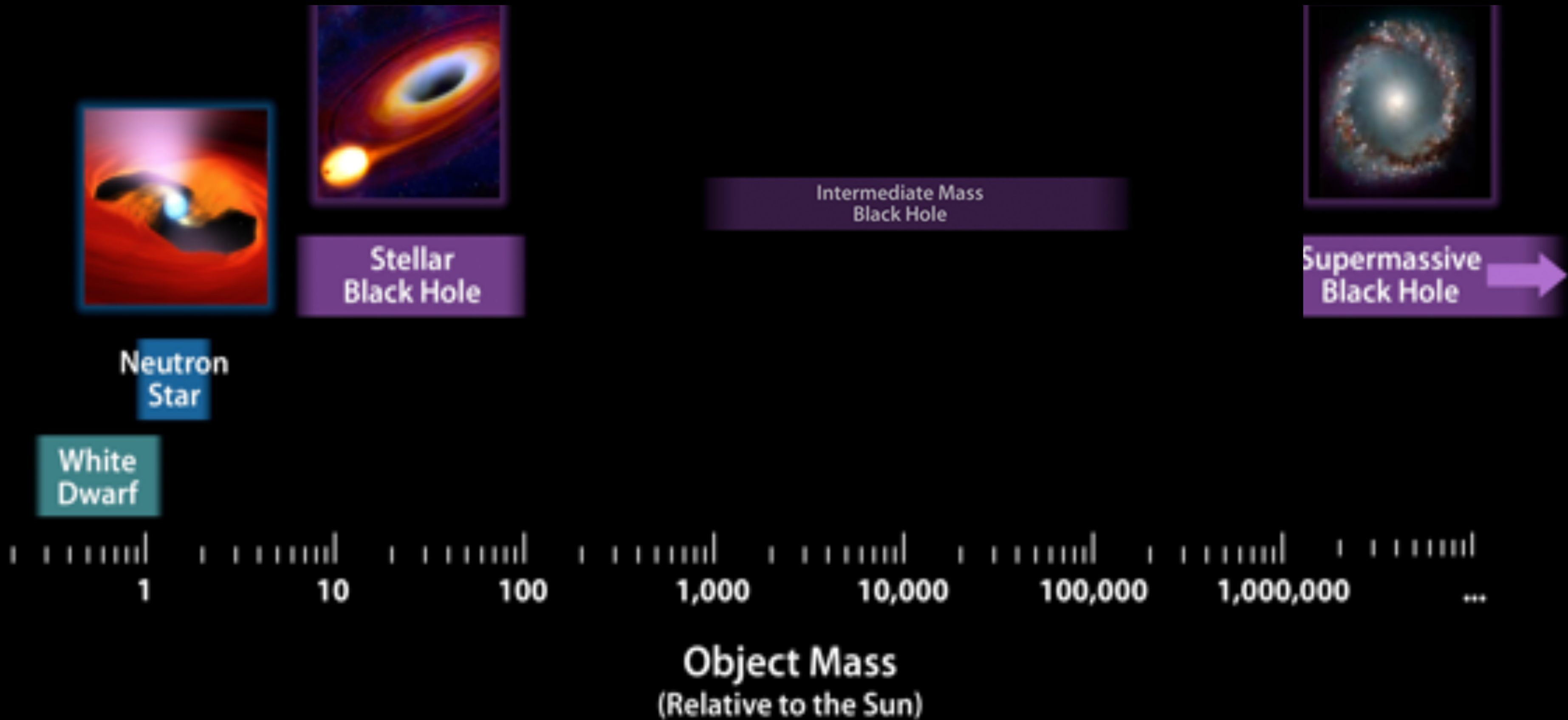
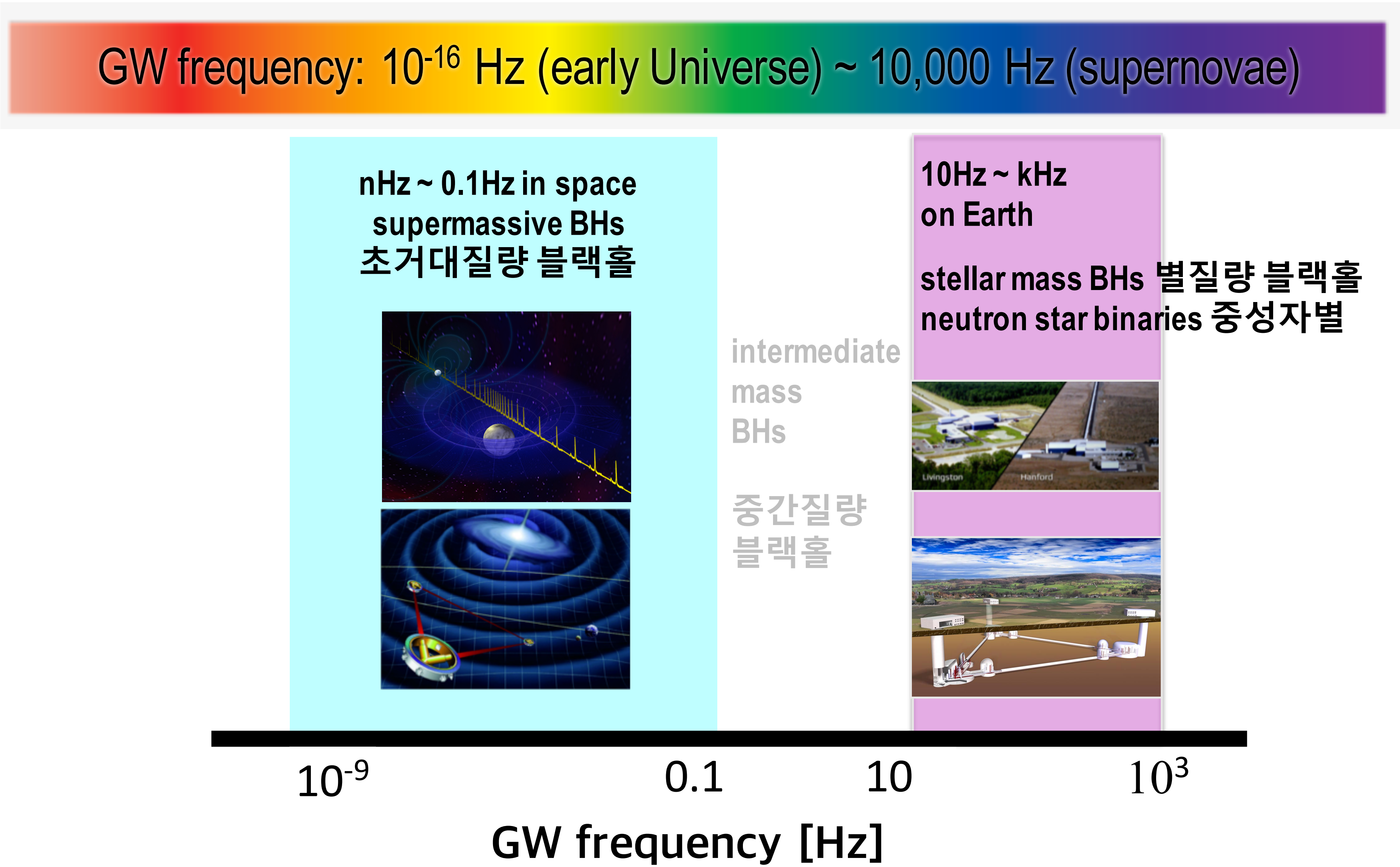


Image credit: NASA/JPL-Caltech

- 중력파원이 존재할 것으로 예측되는 중력파 스펙트럼은 매우 넓다

GW frequency: 10^{-16} Hz (early Universe) ~ 10,000 Hz (supernovae)

- CBC 관측 스펙트럼 : nHz ~ kHz



중력파 관측 → 밀집성의 질량 → 밀집성의 형성과 진화

GW frequency = 2 x orbital frequency

frequency ← Kepler's law

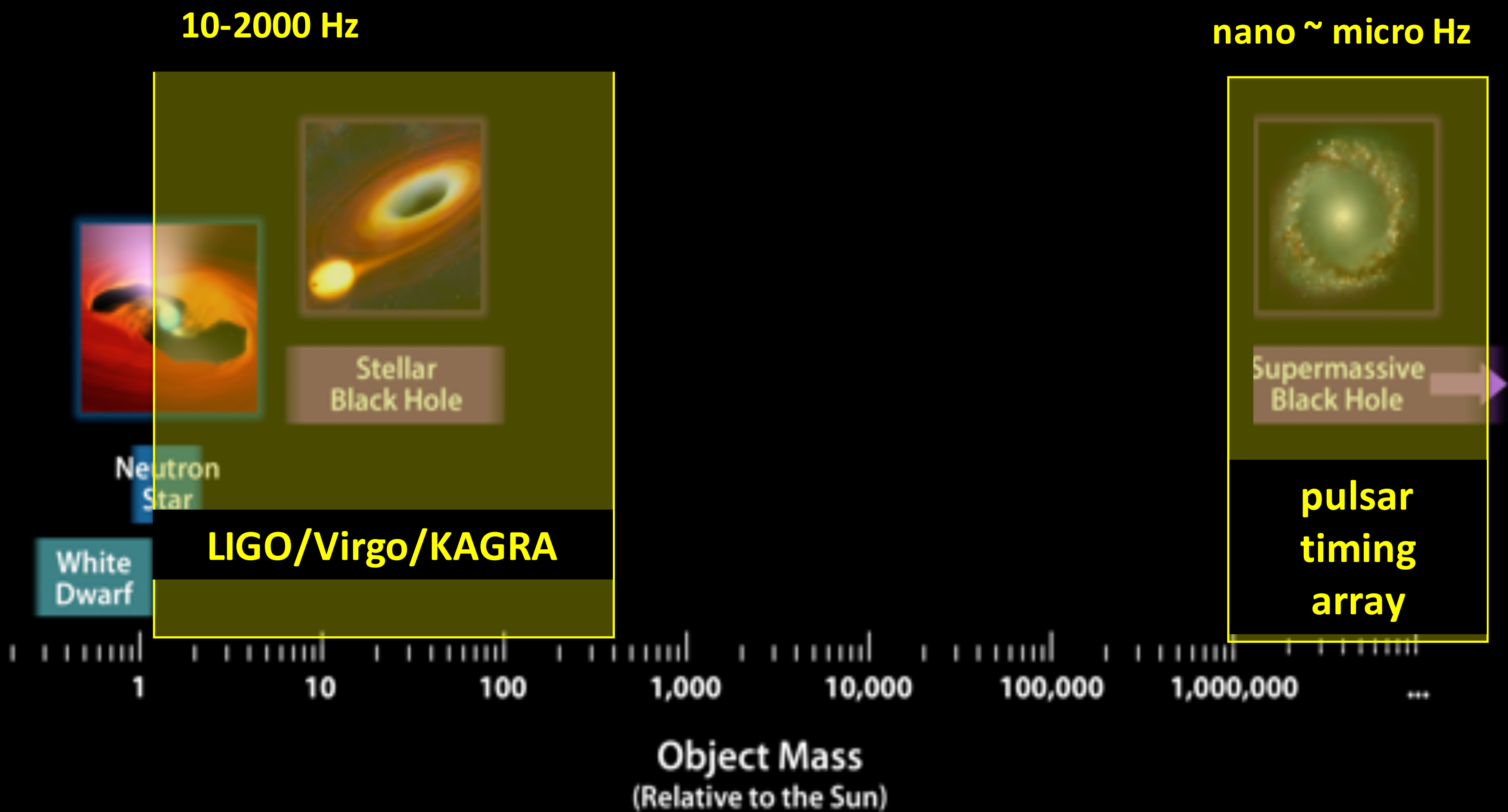


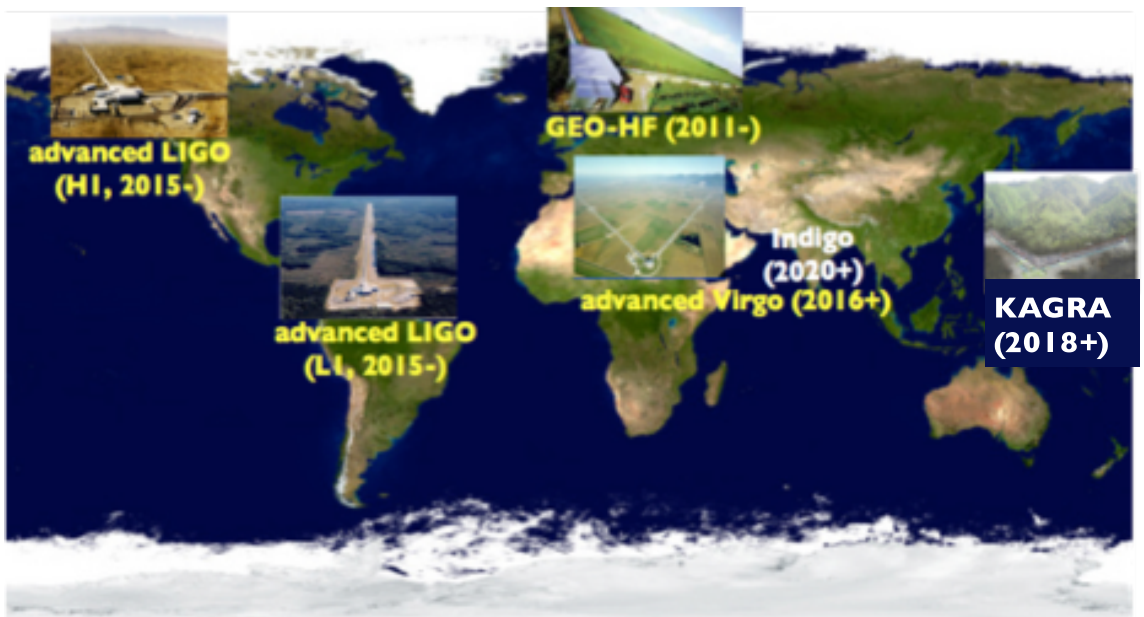
Image credit: NASA/JPL-Caltech

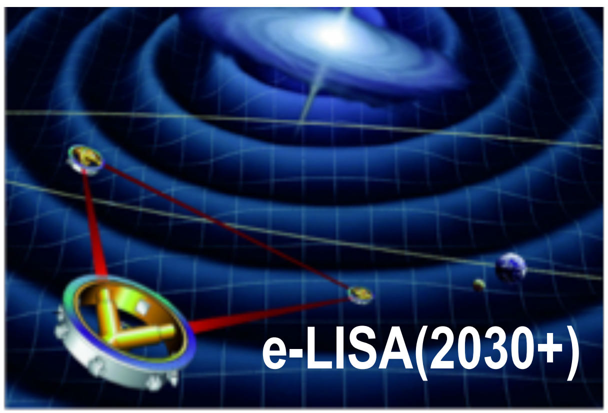
International GW observatory network

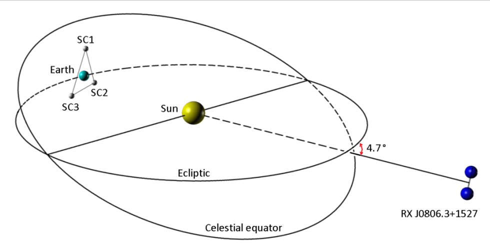
laser interferometry

- 2017 - present   aLIGO(USA), aVirgo(Europe) ← "a" = advanced
- 2020, 2021   aLIGO, aVirgo, KAGRA(Japan)
- 2030+   aLIGO, aVirgo, KAGRA, and LIGO-India(India) : on Earth

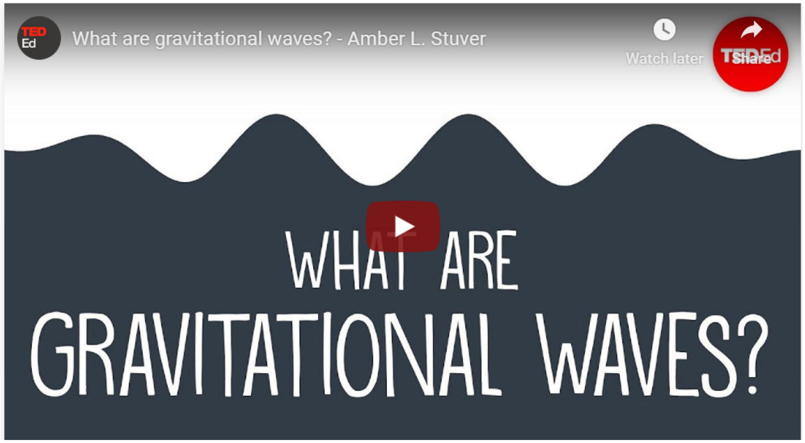
  LISA (Europe/USA) , Chinese concept (2030+)   : in Space



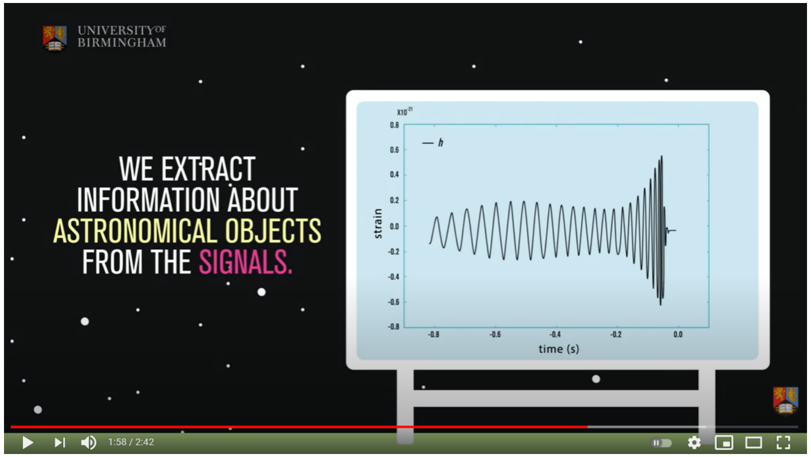




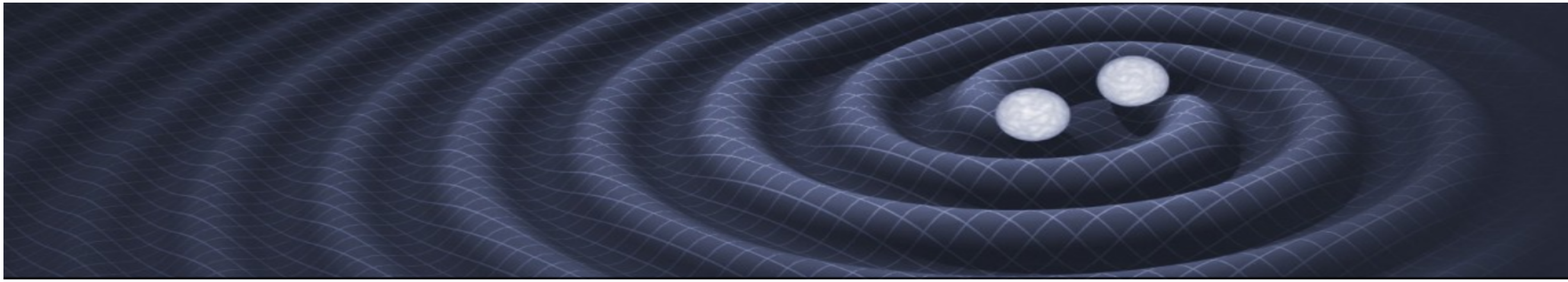
참고 자료



https://ed.ted.com/lessons/what-are-gravitational-waves-amber-l-stuver

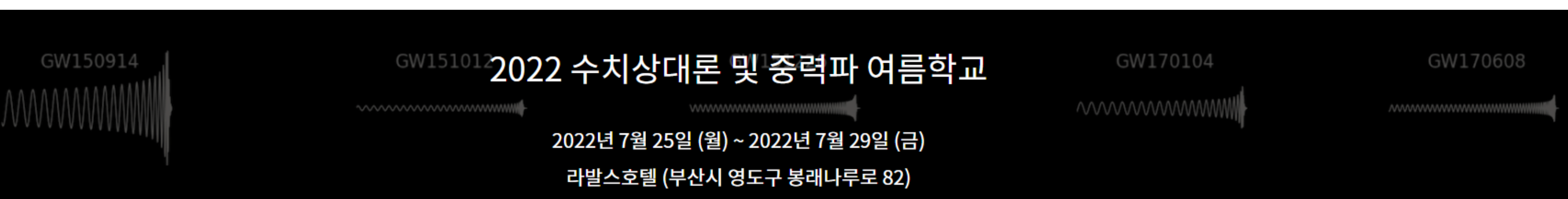


https://youtu.be/L7XqMuIPtrM



중력파 천체물리학 II

김정리
(이화여자대학교)

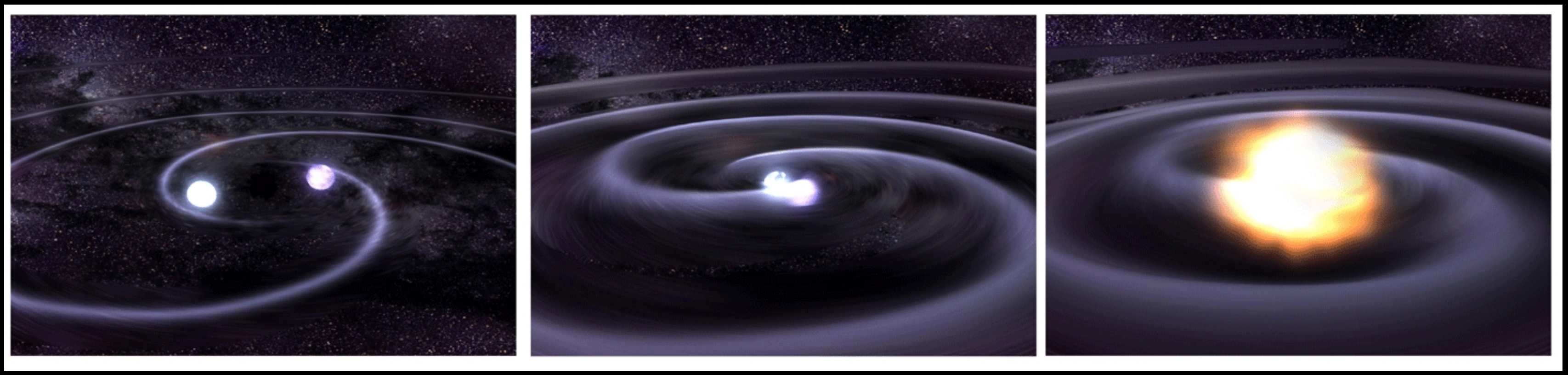


compact binary coalescences (CBCs) 밀집 쌍성 병합

inspiral motion 나선근접궤도 운동

- 점질량 근사
- post-Newtonian formalism으로 중력파 파형 h(t), h(f) 계산

- 보정: 중성자별 모양 변화(NS EOS) 등



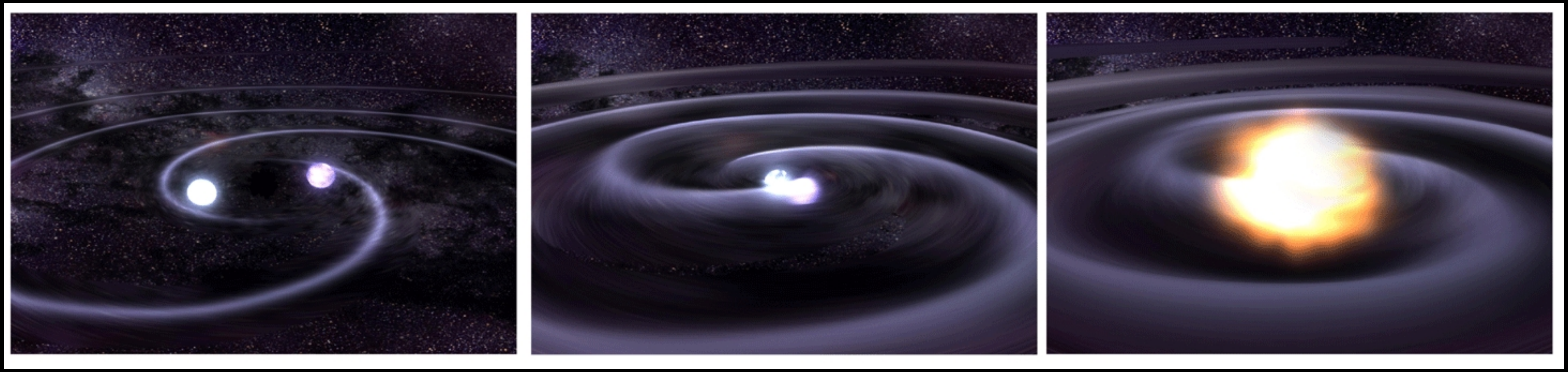
compact binary coalescences (CBCs) 밀집 쌍성 병합

inspiral motion 나선근접궤도 운동

matched filter + 중력파 파형 h(t) ➔ 중력파원의 물리량

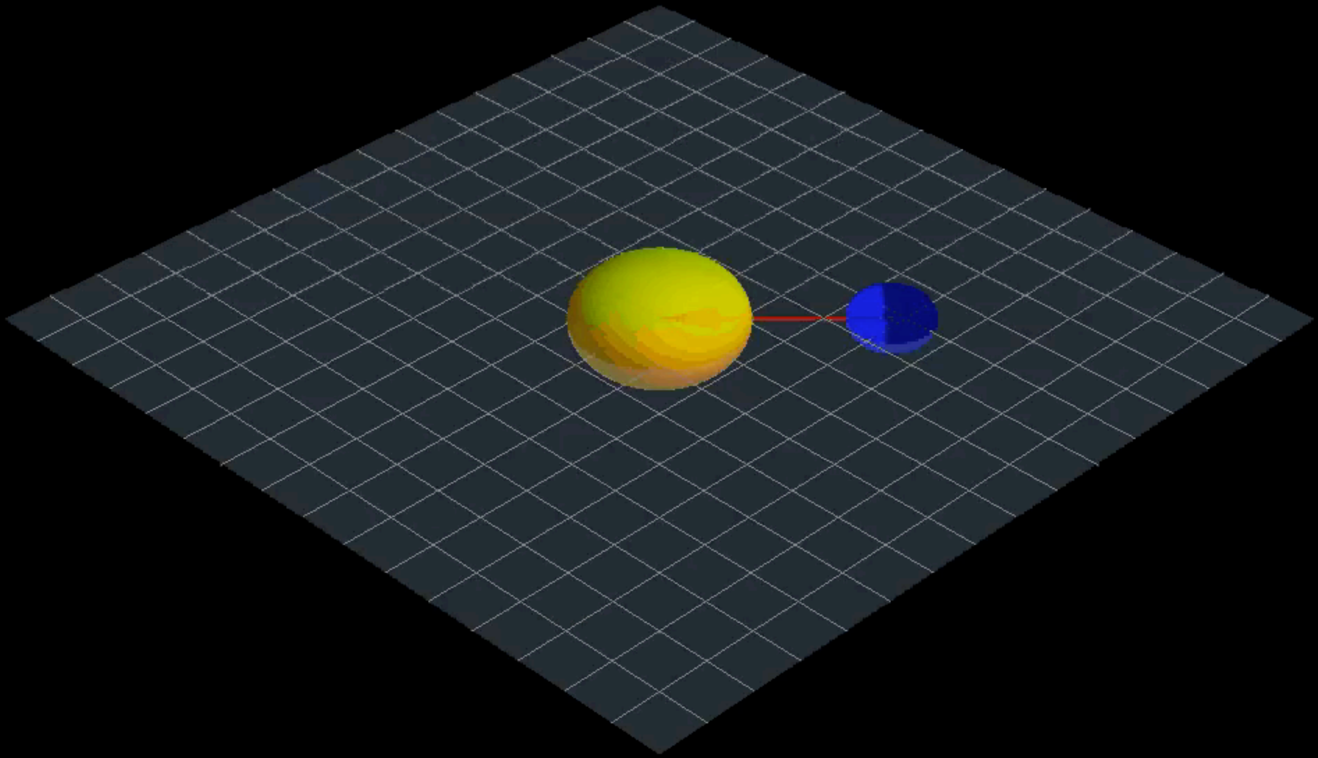
{m1,m2,s1,s2, a, d, RA, dec} ➔ “관측가능한 물리량”
observables

단, 중력파 자료 분석 ➔ “통계적 추정” inferences



Newton's universal gravity and the binary motion

쌍성의 궤도운동이 시간과 공간을 변화시키지 않음



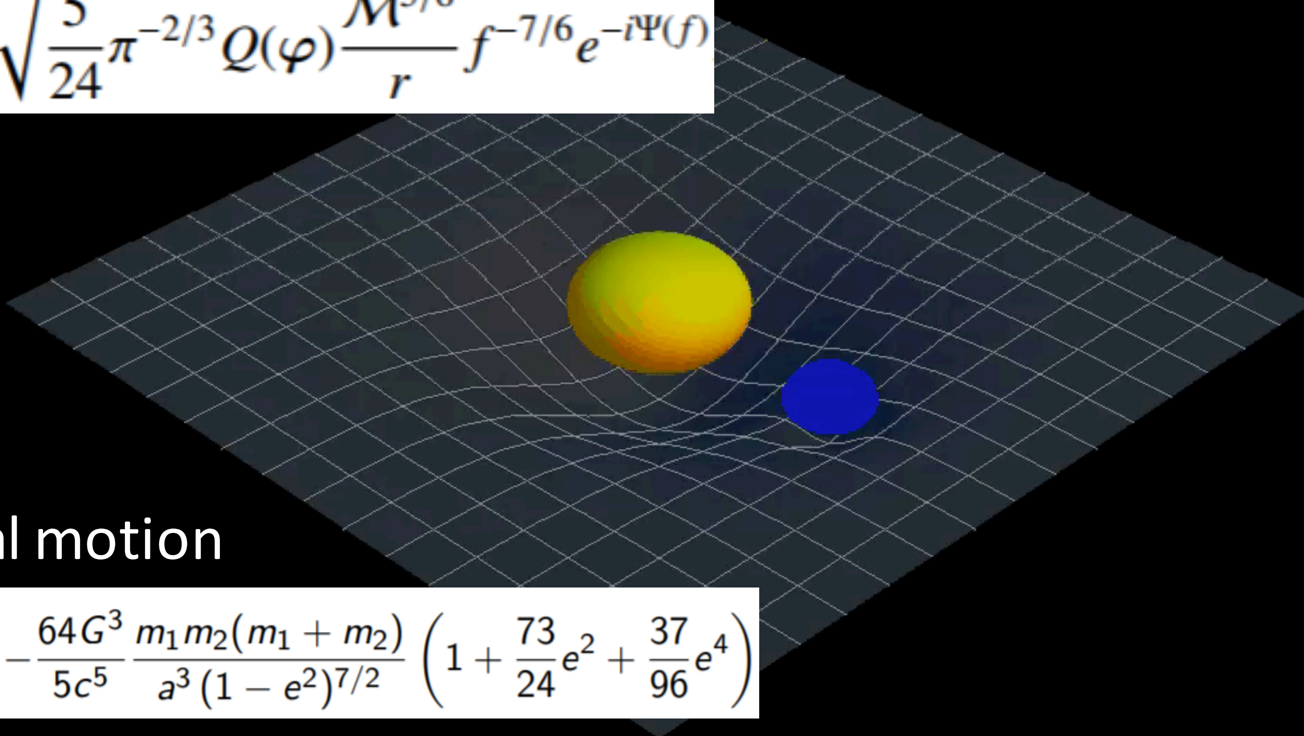
Credit: Carl Rodriguez

Einstein's general relativity and the binary motion

쌍성의 궤도운동이 시간과 공간을 변화시킨다.

➔ 시공간의 변화가 쌍성으로부터 빛의 속력으로 "퍼져 " 나간다 ➔ 중력파

$$\tilde{h}(f) = \sqrt{\frac{5}{24}}\pi^{-2/3} Q(\varphi) \frac{\mathcal{M}^{5/6}}{r} f^{-7/6} e^{-i\Psi(f)}$$



inspiral motion

$$\left\langle \frac{\mathrm{d}a}{\mathrm{d}t} \right\rangle = -\frac{64 G^3}{5c^5} \frac{m_1 m_2 (m_1 + m_2)}{a^3 (1 - e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

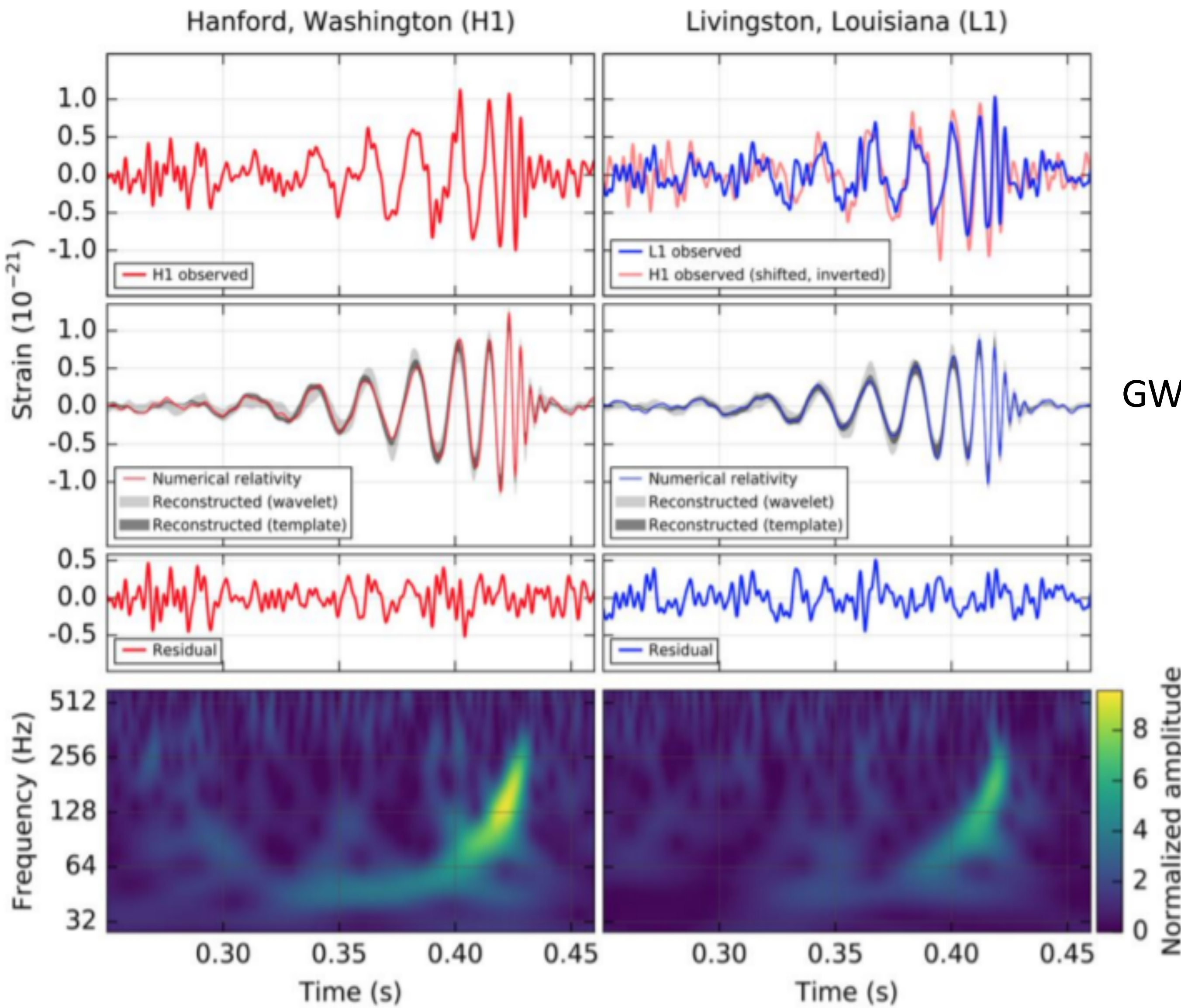
circularization

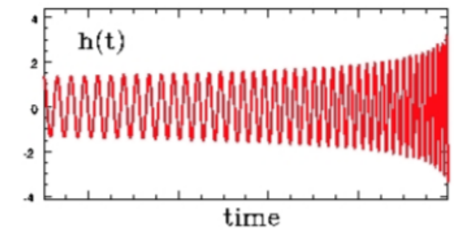
$$\left\langle \frac{\mathrm{d}e}{\mathrm{d}t} \right\rangle = -\frac{304 G^3}{15c^5} \frac{m_1 m_2 (m_1 + m_2)}{a^4 (1 - e^2)^{5/2}} \left(e + \frac{121}{304} e^3 \right)$$

Credit: Carl Rodriguez

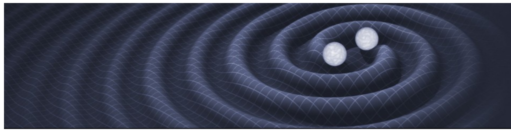
“chirp”

블랙홀-블랙홀 병합 GW150914 중력파





GW waveform of GW150914



https://www.ligo.caltech.edu/page/detection-companion-papers

PHYSICAL REVIEW X **9**, 031040 (2019)

GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing Runs

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 14 December 2018; revised manuscript received 27 March 2019; published 4 September 2019)

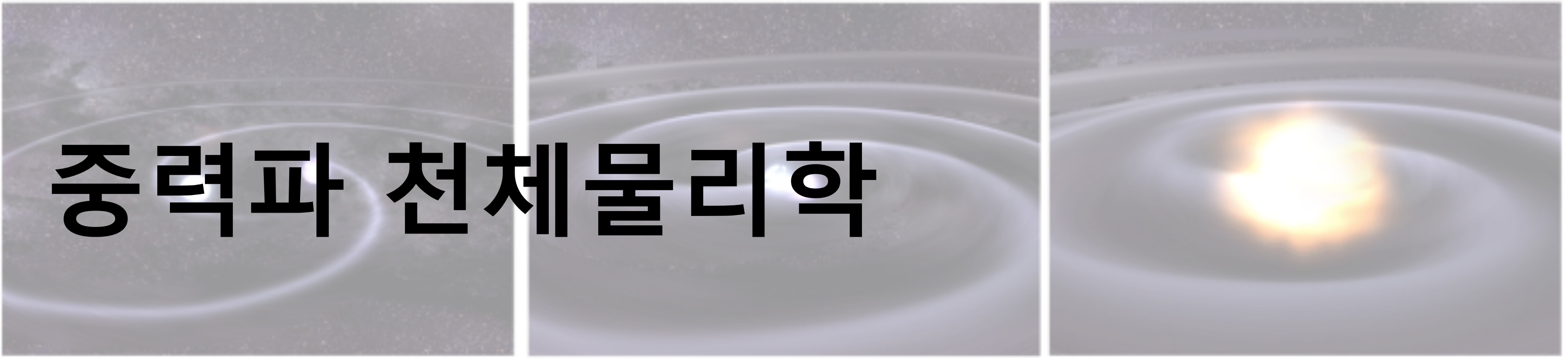
1 NS-NS (GW170817)
10 BH-BH (including GW150914)

GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run

arXiv:2010.14527v1
(posted on Oct 27, 2020)

- **47 compact binary mergers** detected with a false-alarm rate (FAR) < 1 yr^{-1} in GWTC-2
- **Advanced LIGO–Virgo observing runs O3a**
- **multiple detections on the same date**

- **search space : individual mass = [2, 100] M_{sun}, z up to 2.3, also**
- **individual mass = [1, 400] M_{sun}, total mass = [2, 758] M_{sun}**



중력파 천체물리학

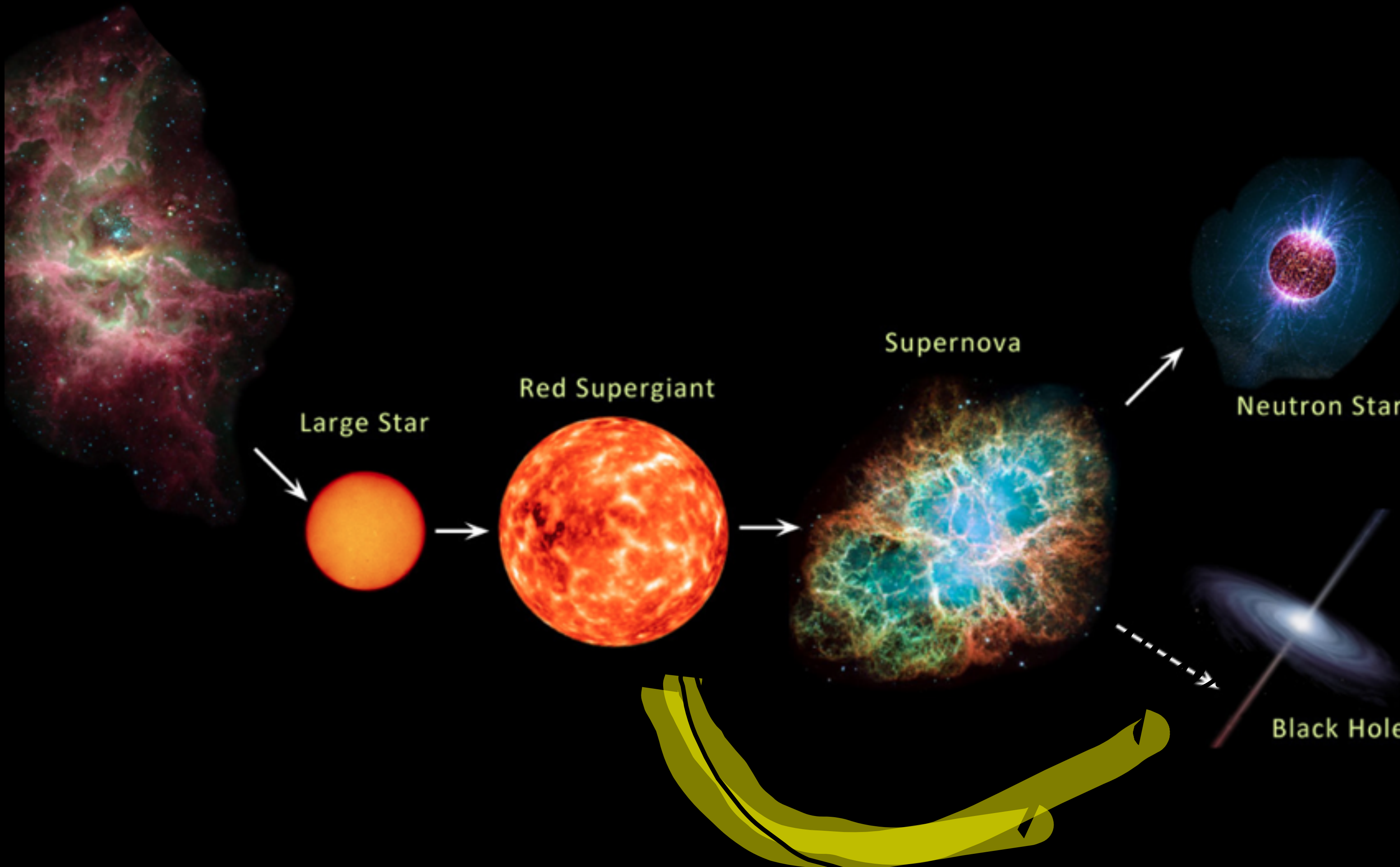
We can trace down or reconstruct the formation and evolutionary "history" of a source by GW observation

requirement: good data (strong signal, less noise or identified noises)
accurate GW waveform (model)

method: matched filtering + Bayesian inference "parameter estimation"
➔ 15 parameters to determine GW signal h(t)

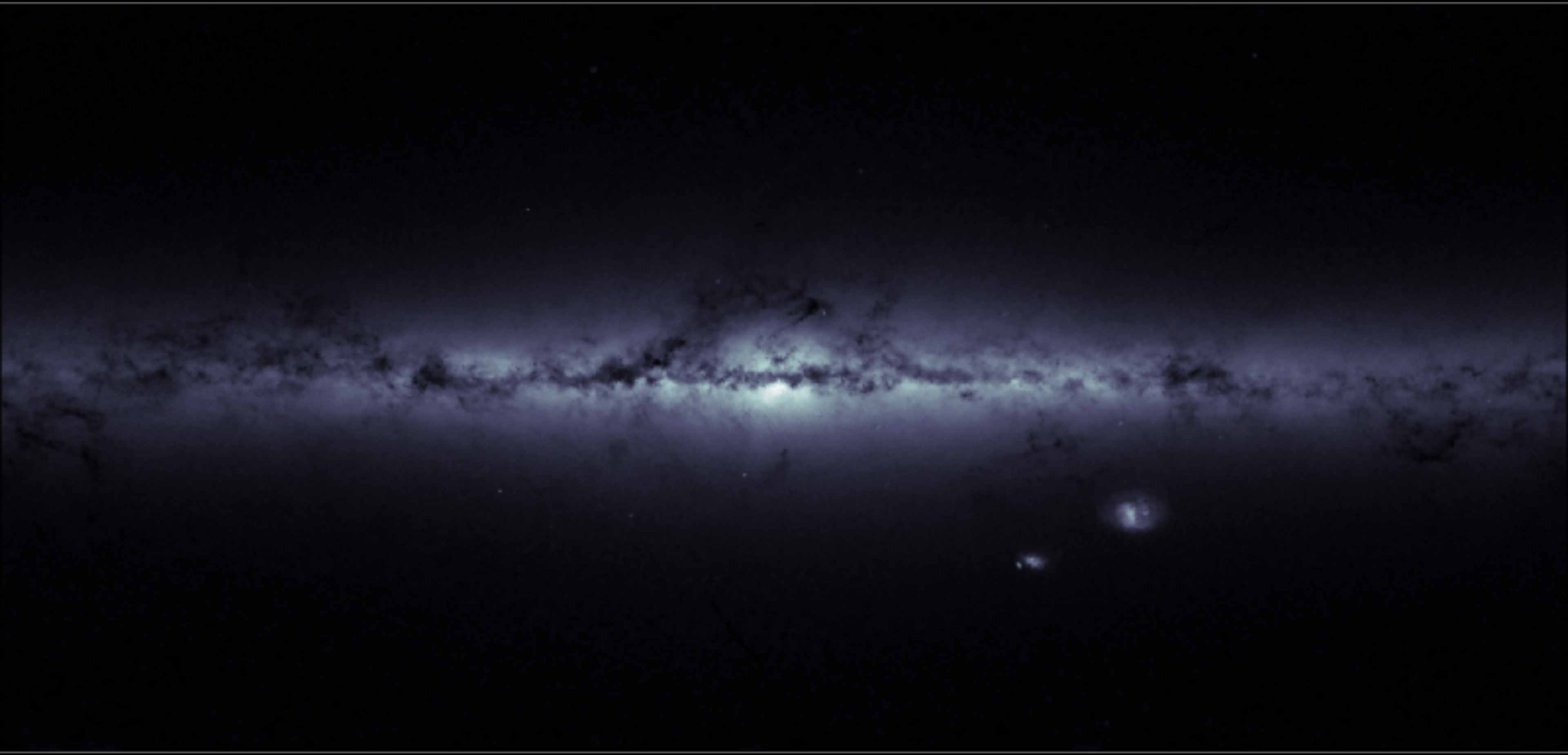
a priori knowledge helps ! (constraints, independent observations)

Evolution of a massive star “single star evolution”



http://essayweb.net/astronomy/images/Stellar_Evolution_large.jpg

typically, BBHs and NS-NS binaries reside in a galactic disk

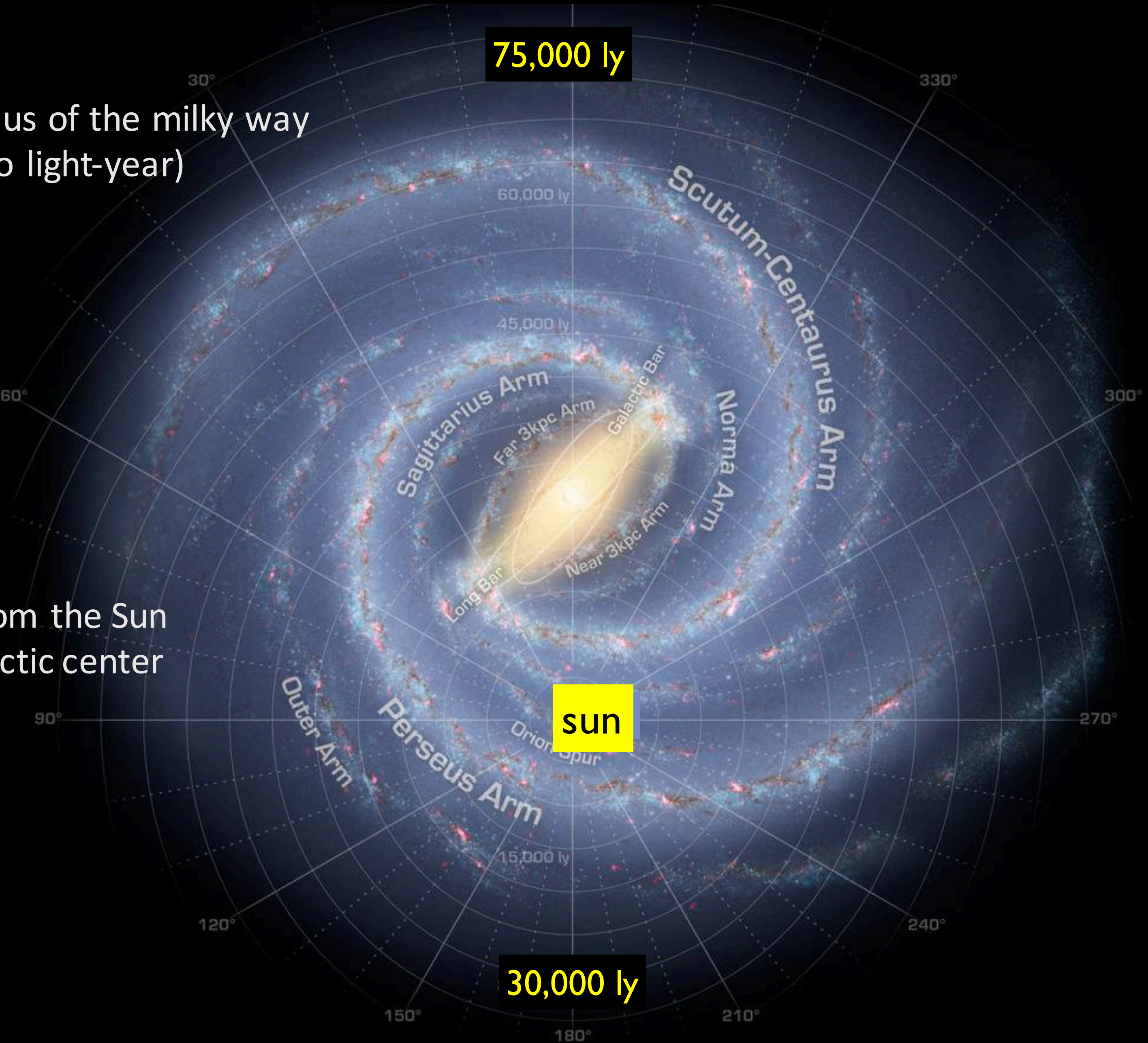


Credit: ESA/Gaia

Our Galaxy (Milky Way). 우리 은하 (은하수)

average radius of the milky way
~ 50 kly (kilo light-year)
~ 16 kpc

distance from the Sun
to the Galactic center
~ 24 kly
~ 8 kpc

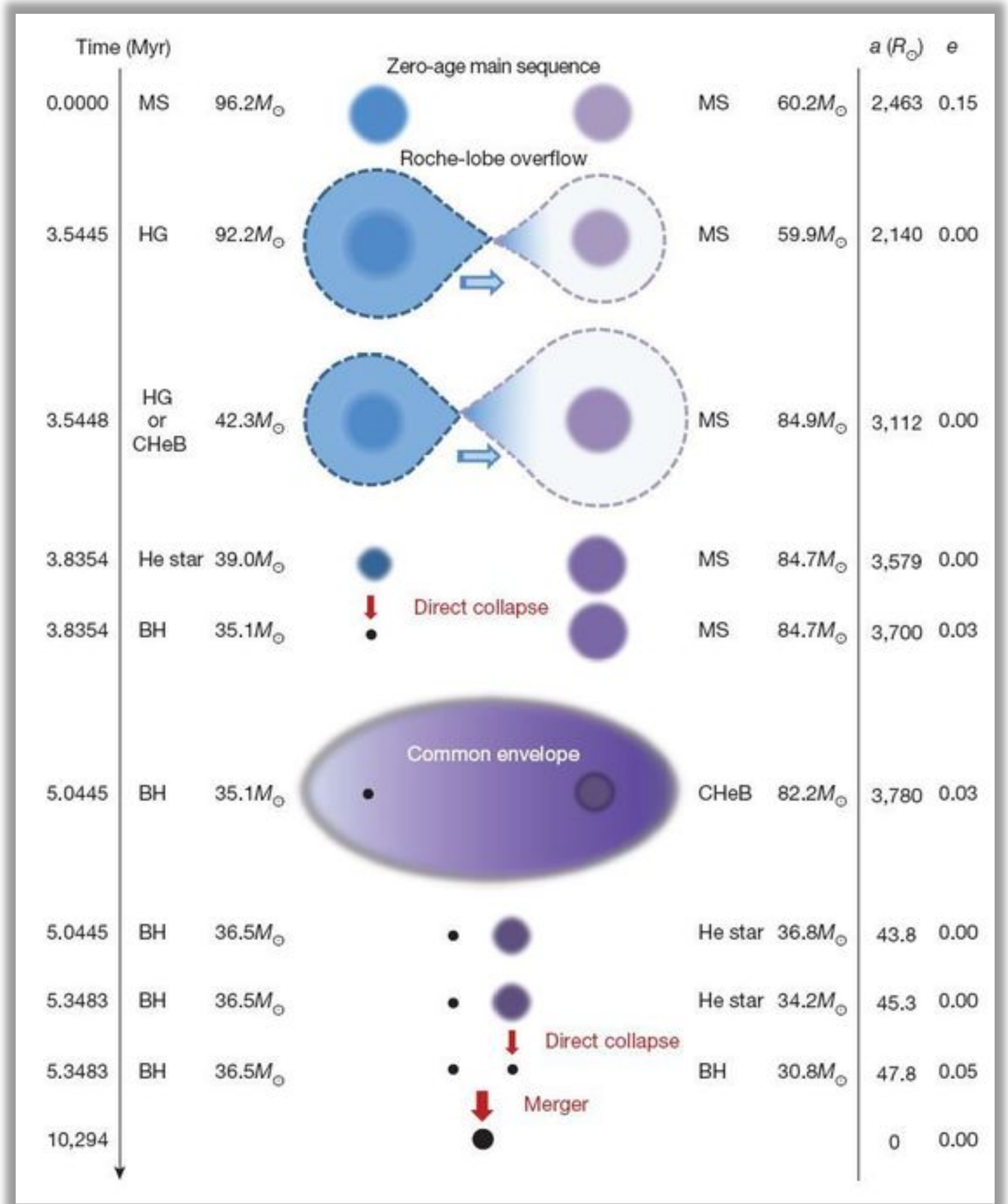


Credits: NASA/JPL-Caltech/R. Hurt (SSC/Caltech)

Evolution of a binary

"standard" binary evolution in the Galactic disk

- initial conditions (mass ratio, separation, metallicity)
- common envelope
- supernovae
- mass/angular momentum transfer

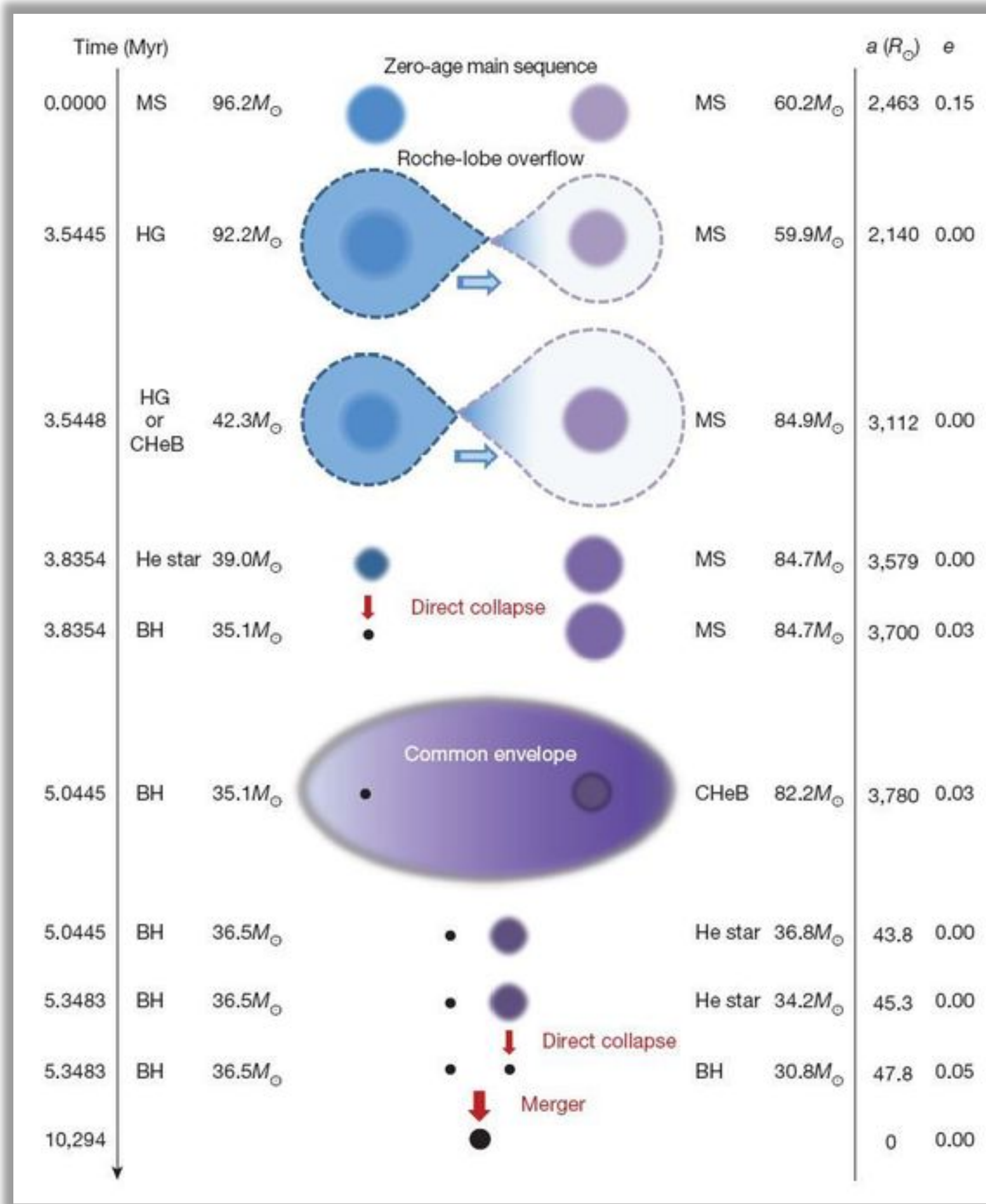


https://www.nature.com/articles/nature18322

Evolution of a binary

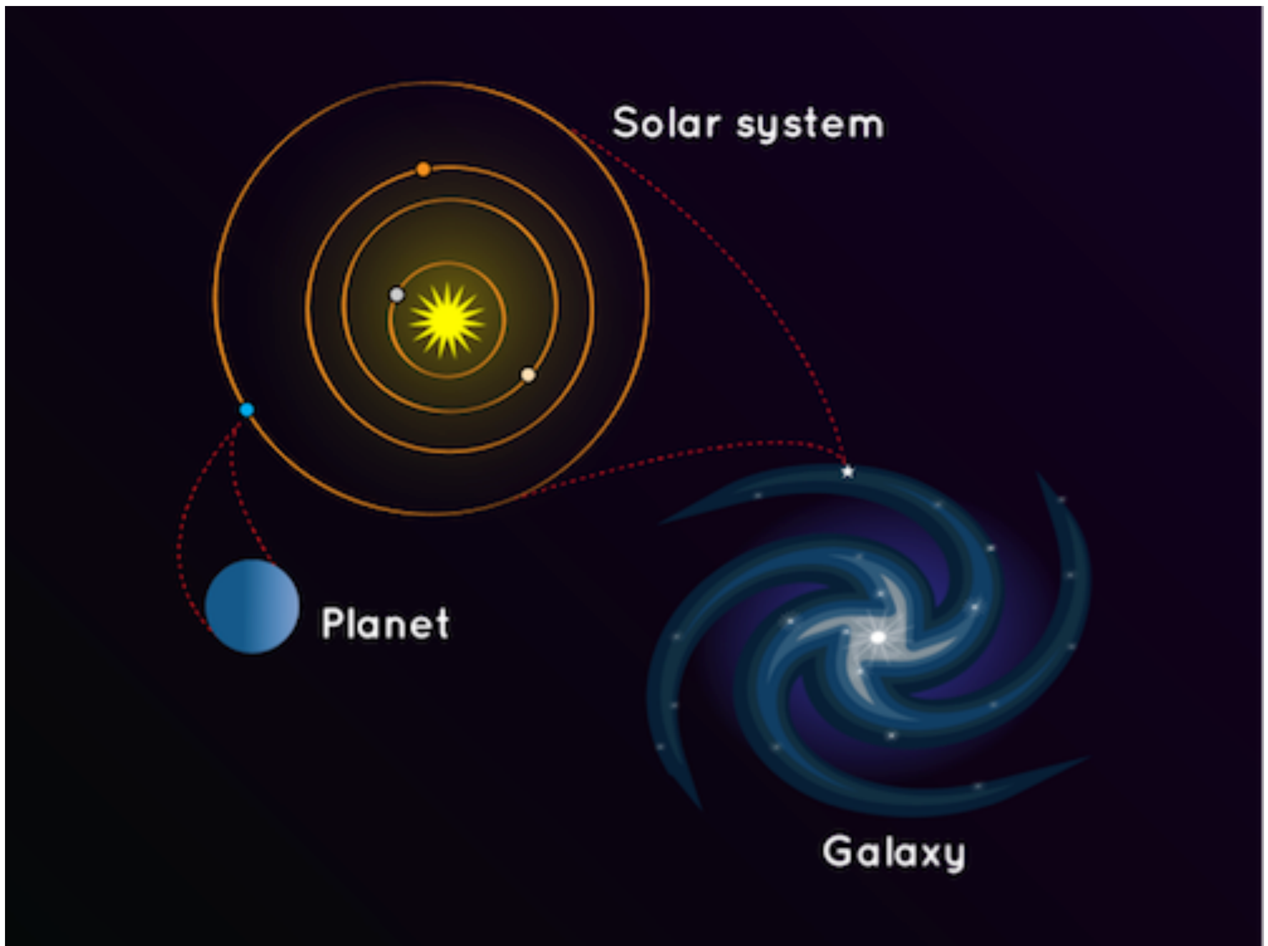
“standard” binary evolution in the Galactic disk

- CBC location ➔ within the host galaxy
- metallicity of a galaxy ➔ stellar population (fraction of massive stars)



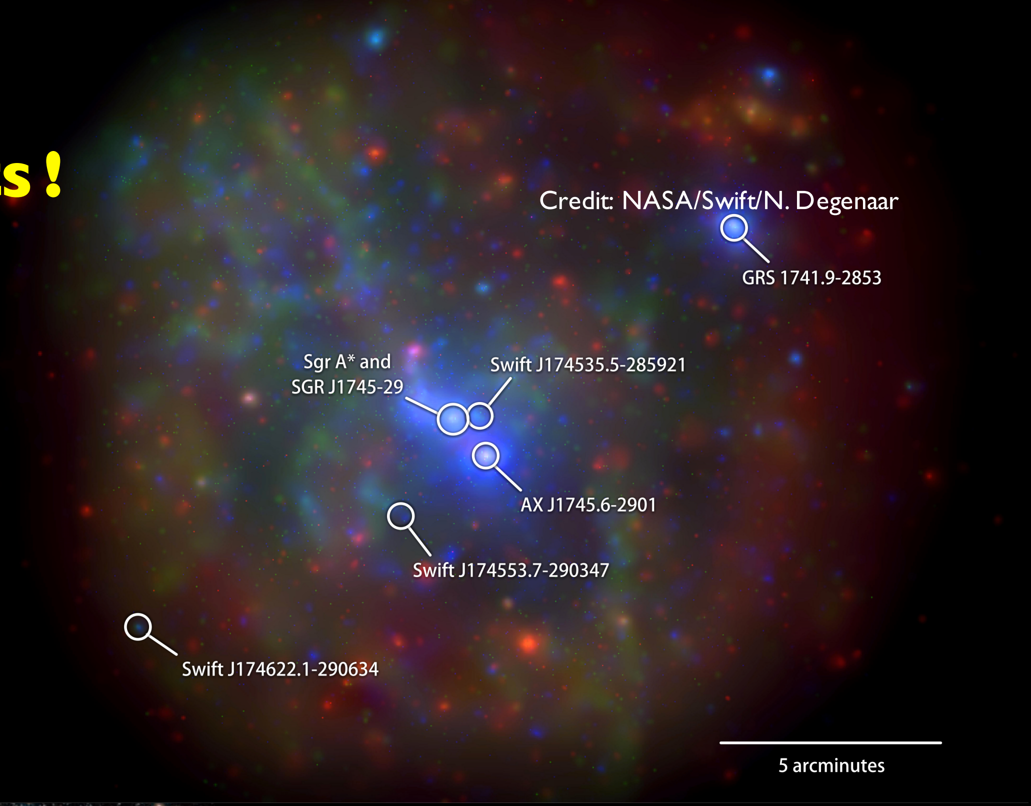
https://www.nature.com/articles/nature18322 building blocks of the universe

star < cluster < galaxy < many galaxies ➔ universe



Some binaries can be formed in dense stellar environments !

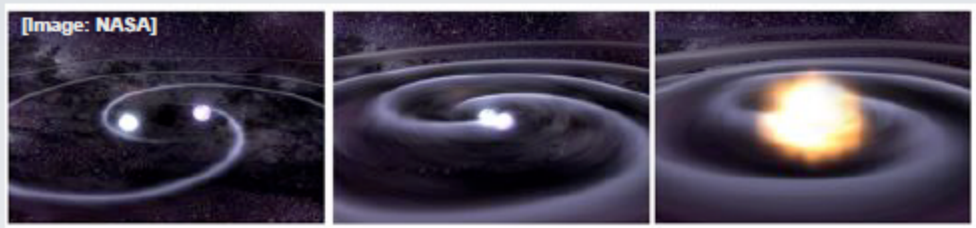
(구상성단, 은하중심)



CBC location

➔ ejected from a cluster
➔ “offset” from the host galaxy

[요약] 중력파 천문학과 중력파 천체물리학



inspiral | merge | ringdown

개별질량
거리
천구상의 위치
궤도 방향
블랙홀 스핀

중성자별 구조

상태방정식(nuclear eq. of state)

중성자별 내/외부 자기장 효과

블랙홀-블랙홀 쌍성계에서
블랙홀간 상호작용

충돌후 생성된 천체의 성질

중력파 최초검출
중력파 천문학

강한 중력장에서의 물리현상

천체물리학
관측천문학

↔

중력파 모수추정
중력파 검출

중력파원의 성질과 중력파 파형 모델

a typical post-Newtonian “inspiral” waveform in the frequency domain

$$\tilde{h}(f) = \mathcal{A} e^{i\Psi} \qquad \tilde{h}(f) \equiv \int_{-\infty}^{\infty} h(t) e^{2\pi i f t}\, dt$$

$$\mathcal{A} = -M\sqrt{\frac{5\pi}{96}}\left(\frac{M}{D}\right)\sqrt{\eta}(\pi M f)^{-7/6} \quad \left[(1+C^2)^2 F_+^2 + 4C^2 F_\times^2\right]^{1/2}$$

$$\Psi(f) = \phi_c + 2\pi f t_c + \frac{3}{128\eta v^5}\left(1 + \Delta\Psi^{\text{circ.}}_{\text{3.5PN}} + \Delta\Psi^{\text{spin, circ.}}_{\text{4PN}} + \Delta\Psi^{\text{ecc.}}_{\text{3PN}}\right)$$

where t_c and ϕ_c are the coalescence time and phase, and $v \equiv (\pi M f)^{1/3}$ is the PN orbital velocity parameter. Note that the angle β is absorbed into a constant shift to ϕ_c.

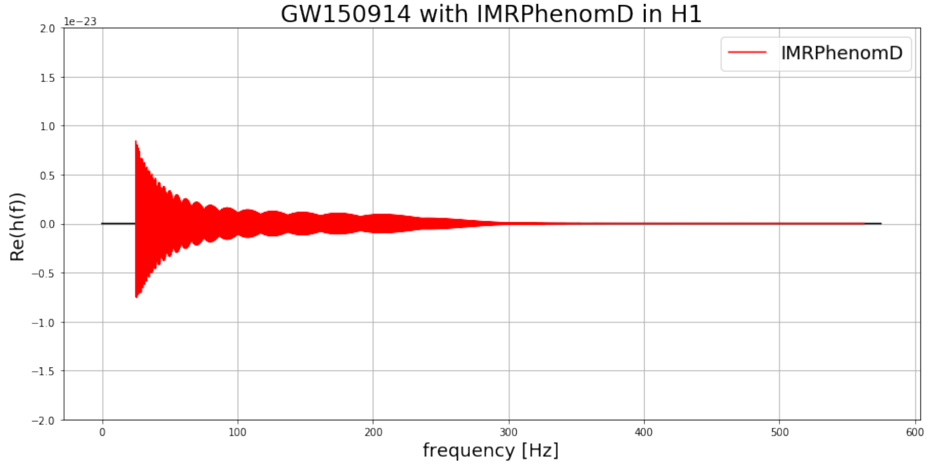
The standard 3.5PN circular contribution is $\Delta\Psi^{\text{circ.}}_{\text{3.5PN}} = \sum_{n=2}^{7} c_n(\eta) v^n$, where the $c_n(\eta)$ can be read off of Eq. (3.18) of [93], and the 2.5PN and 3PN coefficients also depend on $\ln v$.

https://arxiv.org/pdf/2108.05861.pdf

inspiral signal vs inspiral-merge-ringdown signal (ex) GW150914

<Frequency Domain>

IMRPhenomD
25Hz ~ 562.4Hz
(SNR 20.74)



TaylorF2
25Hz ~ 60.9Hz
(SNR 20.00)

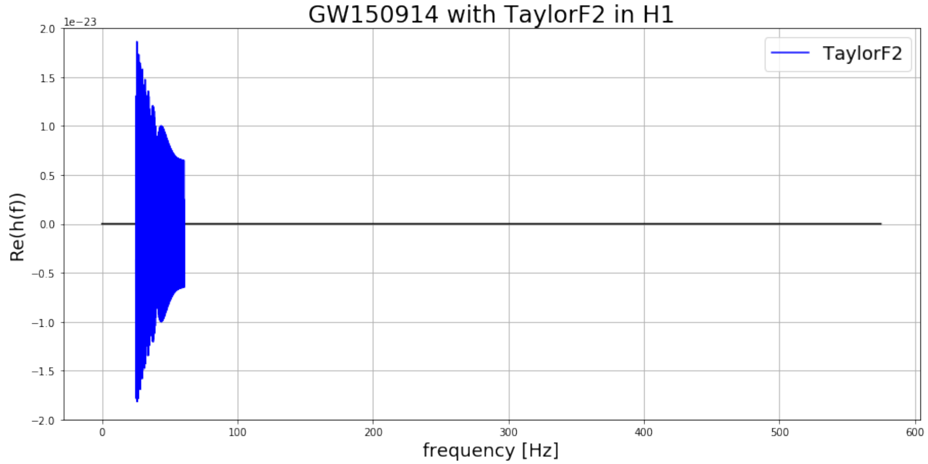
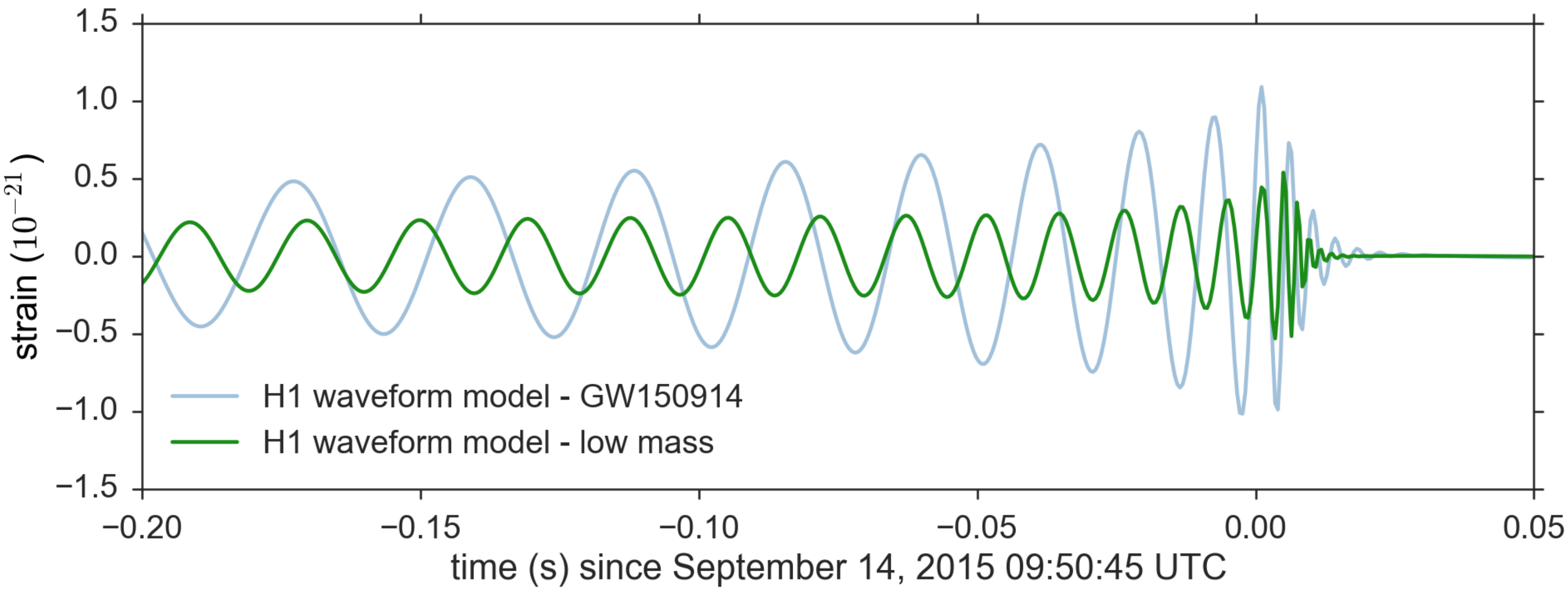


figure credit: Chaeyeon Jeon

중력파 자료 분석에서 중요한 질량 변수

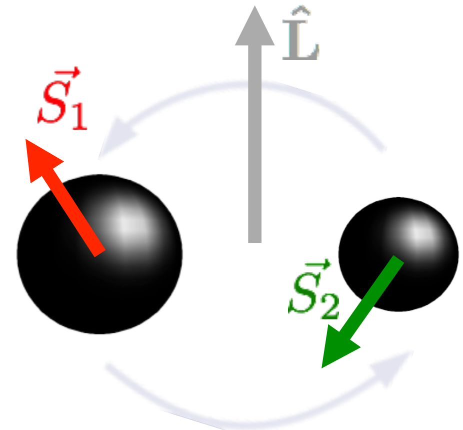
Chirp mass:
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$$

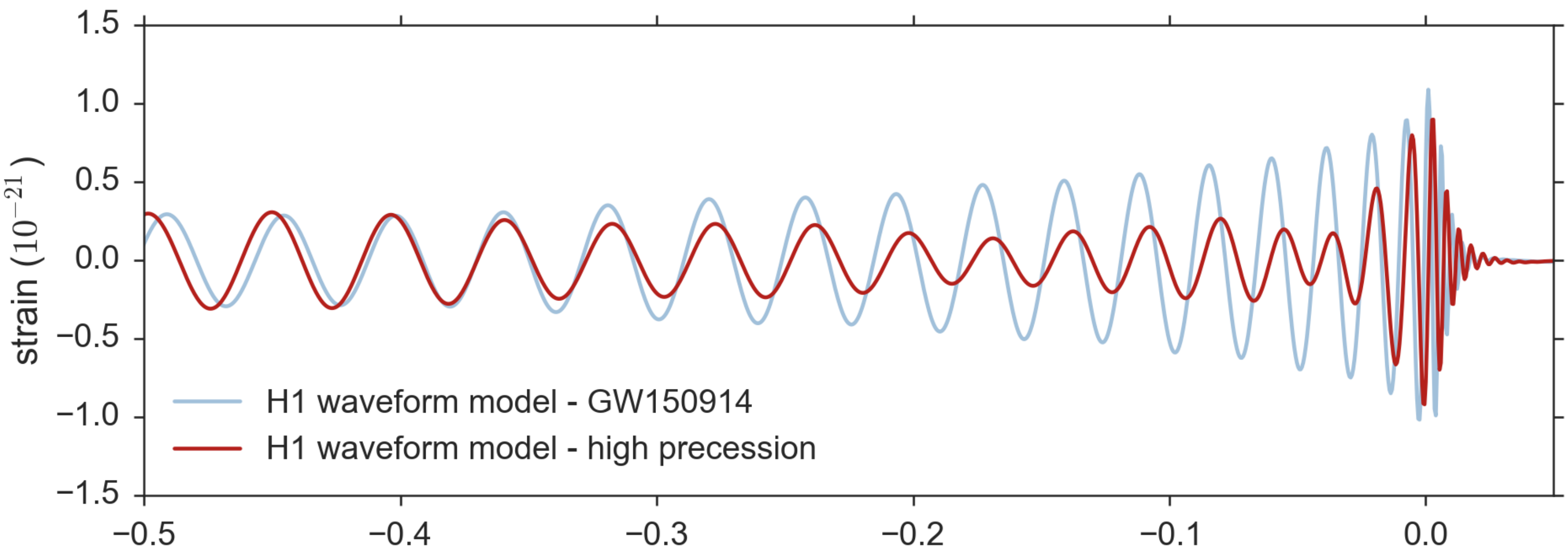
Mass ratio:
$$q = \frac{m_1}{m_2}$$



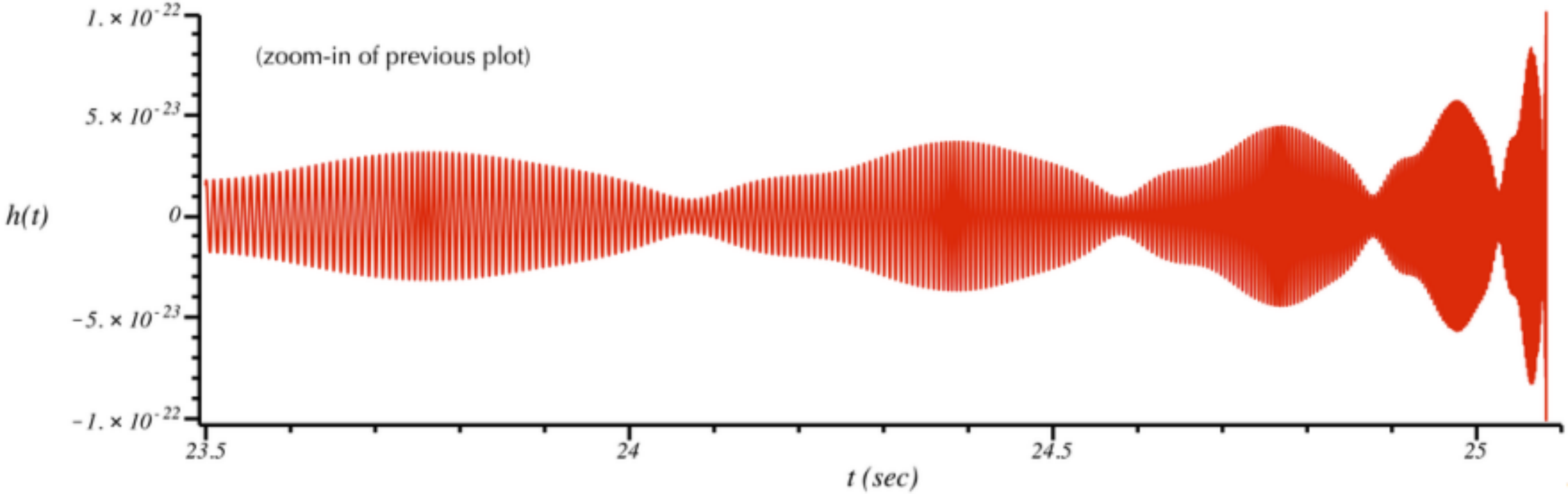
블랙홀 스핀 ➔ 중력파 파형 (세기, 위상)에 영향

➔ 거리 측정 정밀도에 영향





When precessing dominant the GW waveform "inspirals"



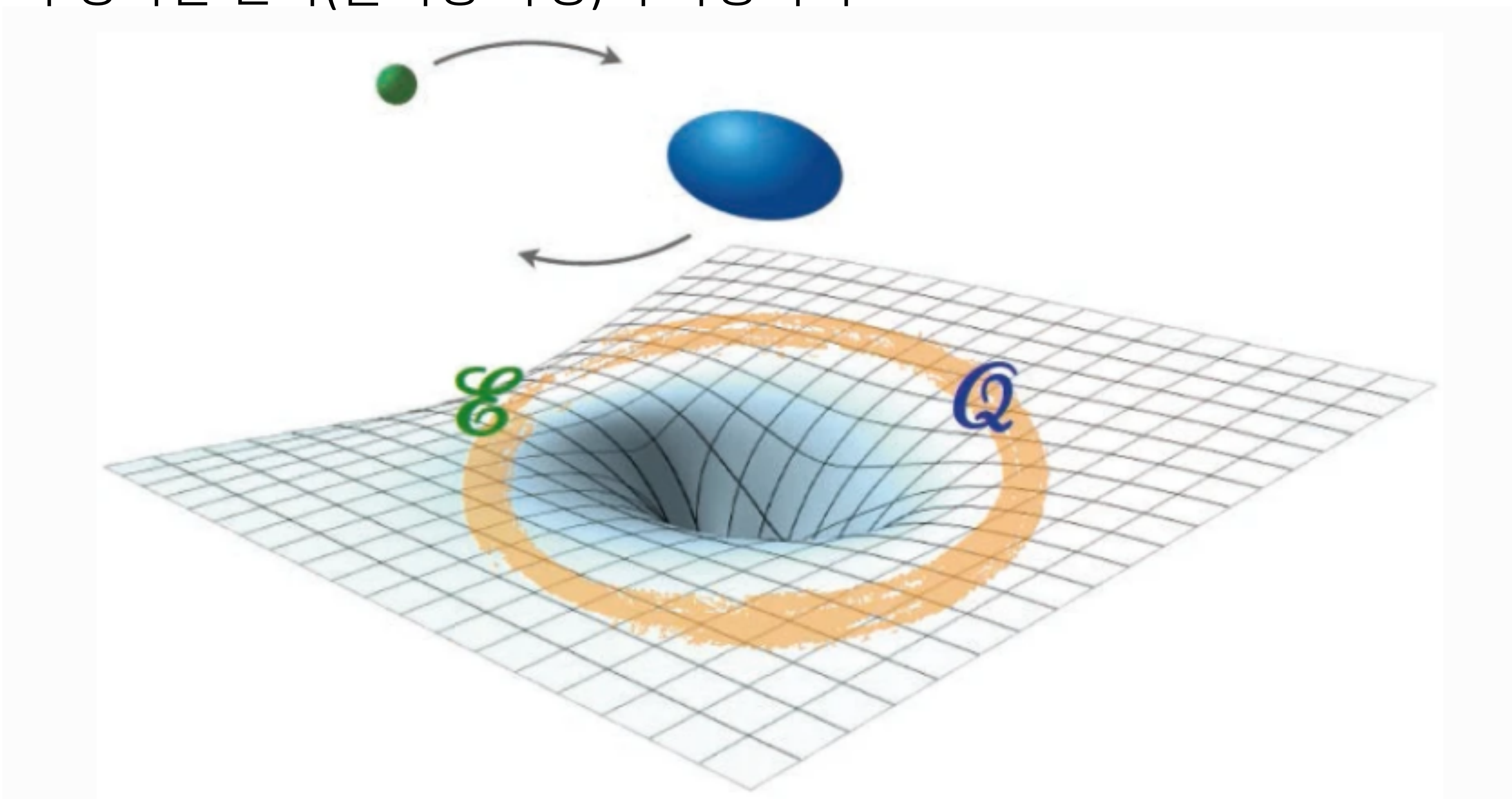
Precessing binary black hole inspiral waveform, m1 = 0.5, m2 = 5 solar masses;
spin1 = 0, spin2 = 0.99 (Kerr dimensionless spin parameter);
spin2 initially misaligned from initial orbital angular momentum by 60 degrees;
detector direction 140 degrees away from initial orbital angular momentum;
initial frequency is 2 f_orb = 40 Hz; ending frequency is 2 f_orb = 4282 Hz.

from Neil Cornish's presentation (2006)
https://indico.cern.ch/event/626266/contributions/2807647/attachments/1593043/2522572/SF6.pdf

중성자별-중성자별 병합 중력파 모델링

- 중성자별의 변형(tidal distortion ➔ NS 상태방정식)
- 나선궤도의 정확한 모양 (원궤도? 타원궤도?)을 고려해야

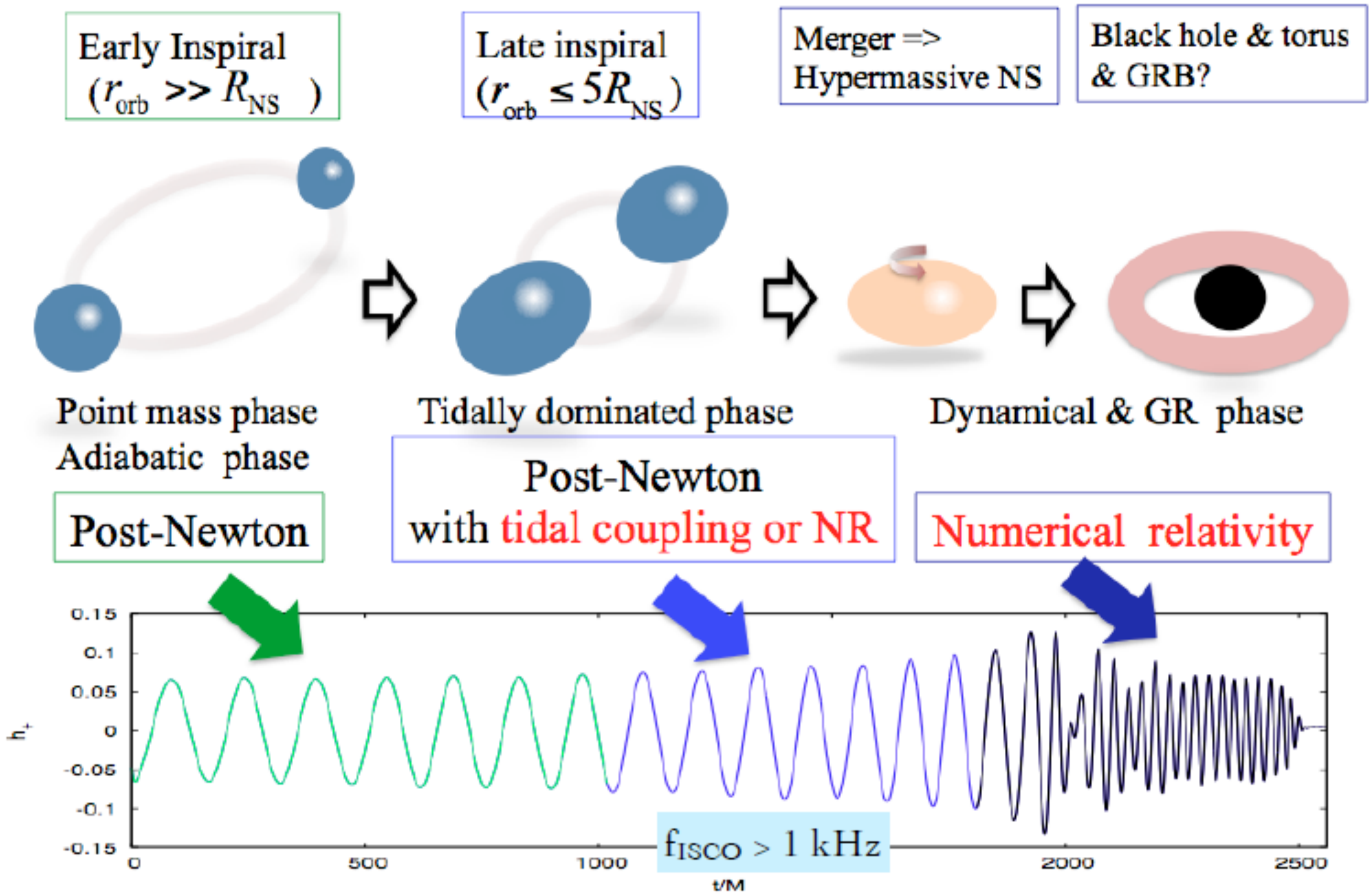
보다 정확한 관측(물리량 측정)이 가능하다



https://link.springer.com/article/10.1007/s10714-020-02751-6/figures/3

GW signals from a NS-NS merger

3 Gravitational waves & EOS

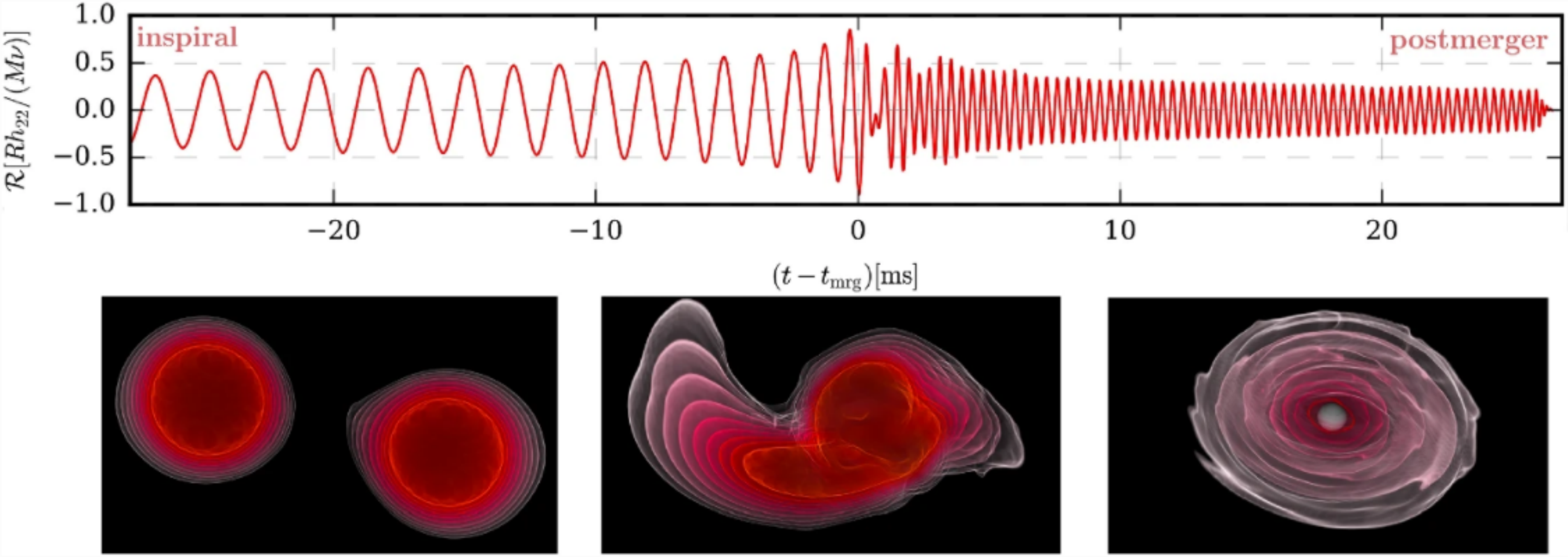


M. Shibata's slide

중성자별-중성자별 병합에서 방출된 중력파 시뮬레이션

Fig. 1

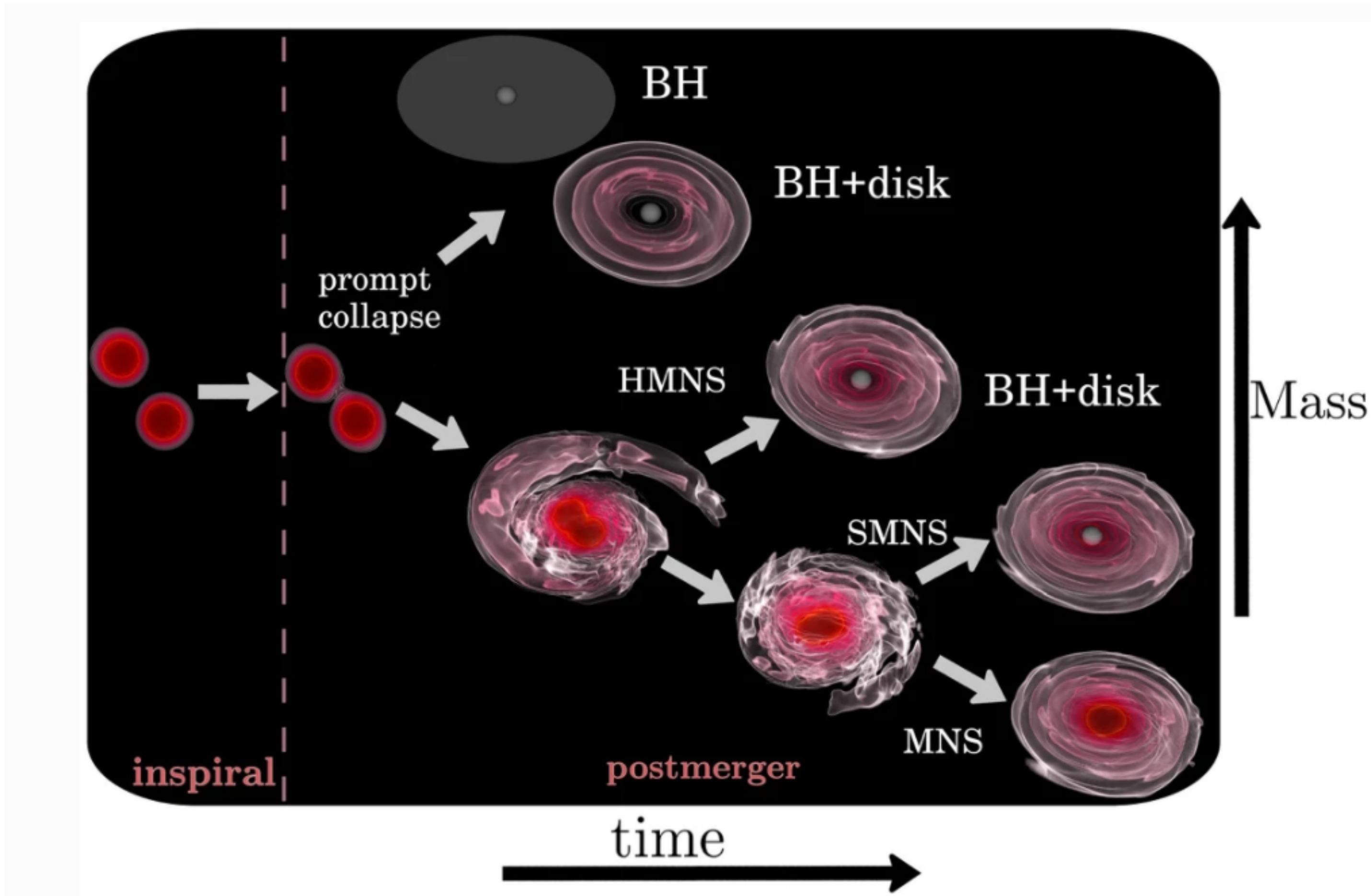
From: Interpreting binary neutron star mergers: describing the binary neutron star dynamics, modelling gravitational waveforms, and analyzing detections



NR simulation of a BNS merger showing the GW signal and the matter evolution. Top panel: GW signal emitted during the last orbits before the merger (late-inspiral phase) and during the postmerger phase of the BNS coalescence. Bottom panel: Rest-mass density evolution for the inspiral (first panel), the merger (second panel) and the postmerger phase after the formation of the black hole (third panel)

https://link.springer.com/article/10.1007/s10714-020-02751-6/figures/1

중성자별-중성자별 병합 (중력파 방출) 이후 빛, 중성미자도 방출 ?
➔ 다중신호 천문학 multimessenger astronomy



https://link.springer.com/article/10.1007/s10714-020-02751-6/figures/4

International GW community and our group

KGWG (Korea GW Group) PI: Hyung Mok Lee

KOREAN GRAVITATIONAL WAVE GROUP

Ewha & Inje collaboration → LSC (Jeongcho Kim, PhD)
KAGRA (Chaeyeon Jeon, master's)

"parameter estimation for stellar-mass BH binaries + NS-NS inspirals"
"systematic biases in GW measurements"

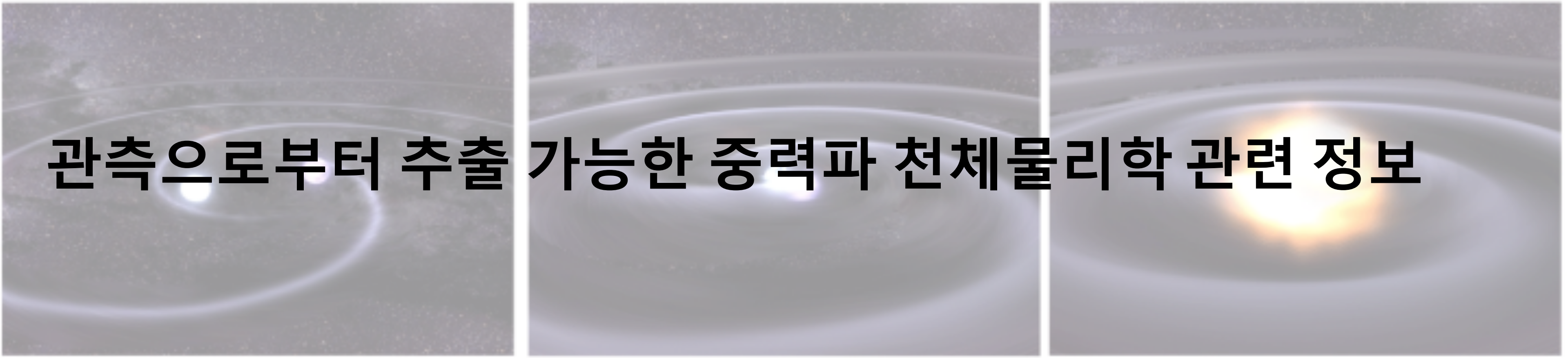
LIGO LSC

VIRGO

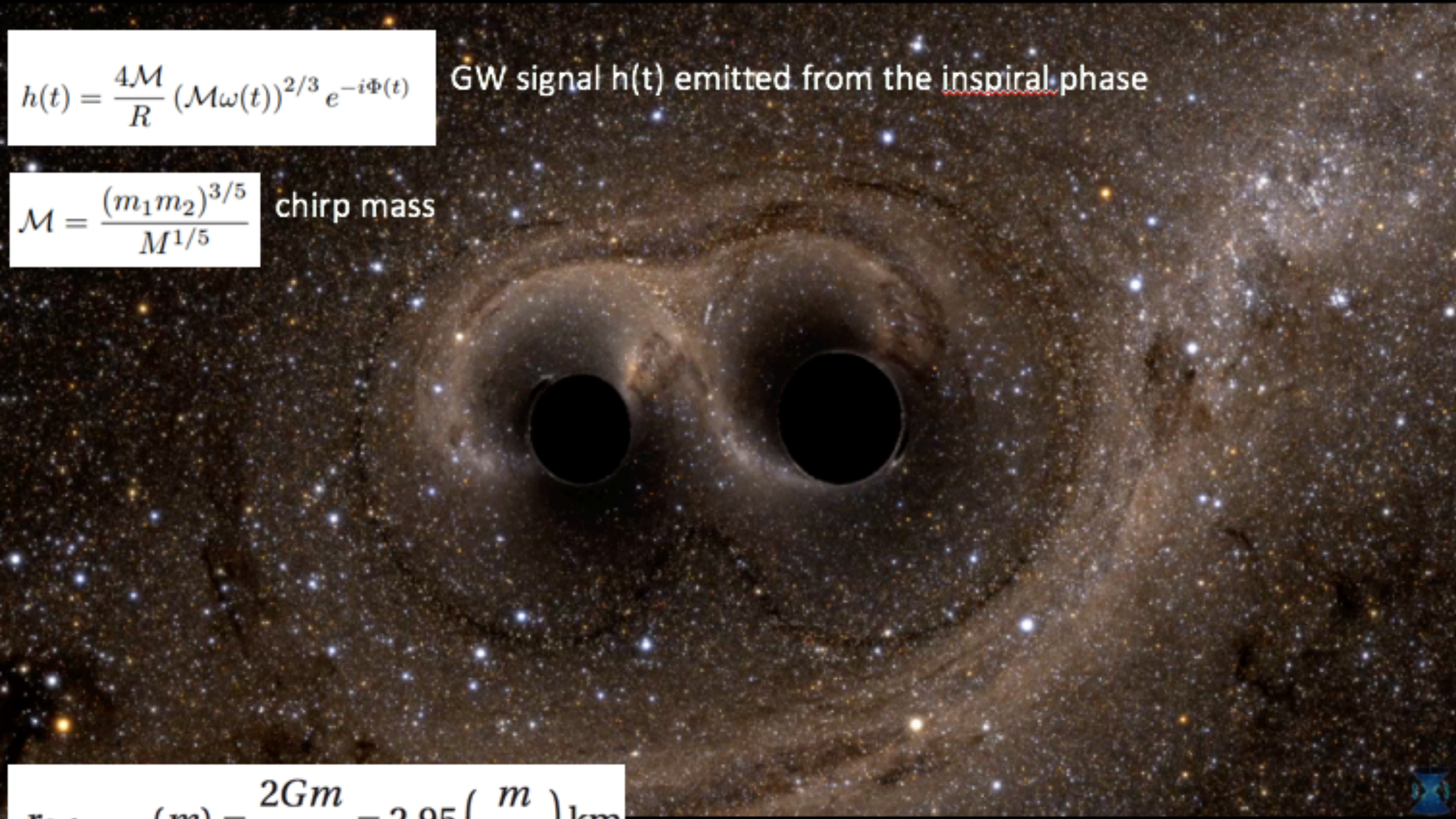
KAGRA

LIGO / LSC	Virgo	KAGRA
1330 members	465 members	360 members
860 authors	360 authors	200 authors
101 groups	96 groups	110 groups
20 countries	8 countries	14 regions

관측으로부터 추출 가능한 중력파 천체물리학 관련 정보



BBH = binary black holes ➔ 2 masses, 2 spins before merge



$$h(t) = \frac{4\mathcal{M}}{R}\left(\mathcal{M}\omega(t)\right)^{2/3} e^{-i\Phi(t)}$$

GW signal h(t) emitted from the inspiral phase

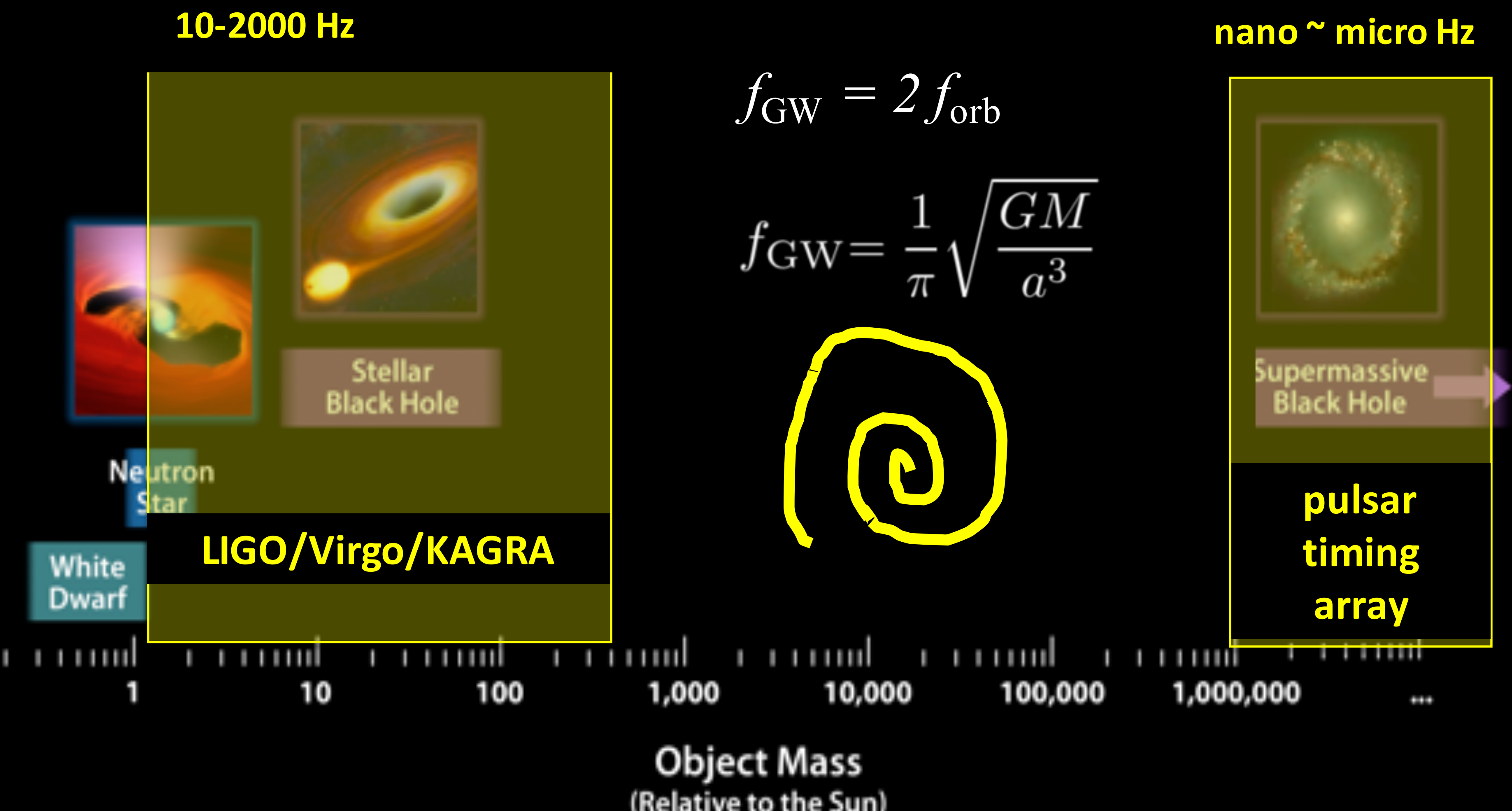
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{M^{1/5}}$$

chirp mass

$$r_{\mathrm{Schwarz}}(m) = \frac{2Gm}{c^2} = 2.95\left(\frac{m}{\mathrm{M}_{\odot}}\right)\mathrm{km}$$

GW150914 = 2 black holes of 30-40 M_{sun}

multi-wavelength GW observations would shed lights on the BH mass distribution

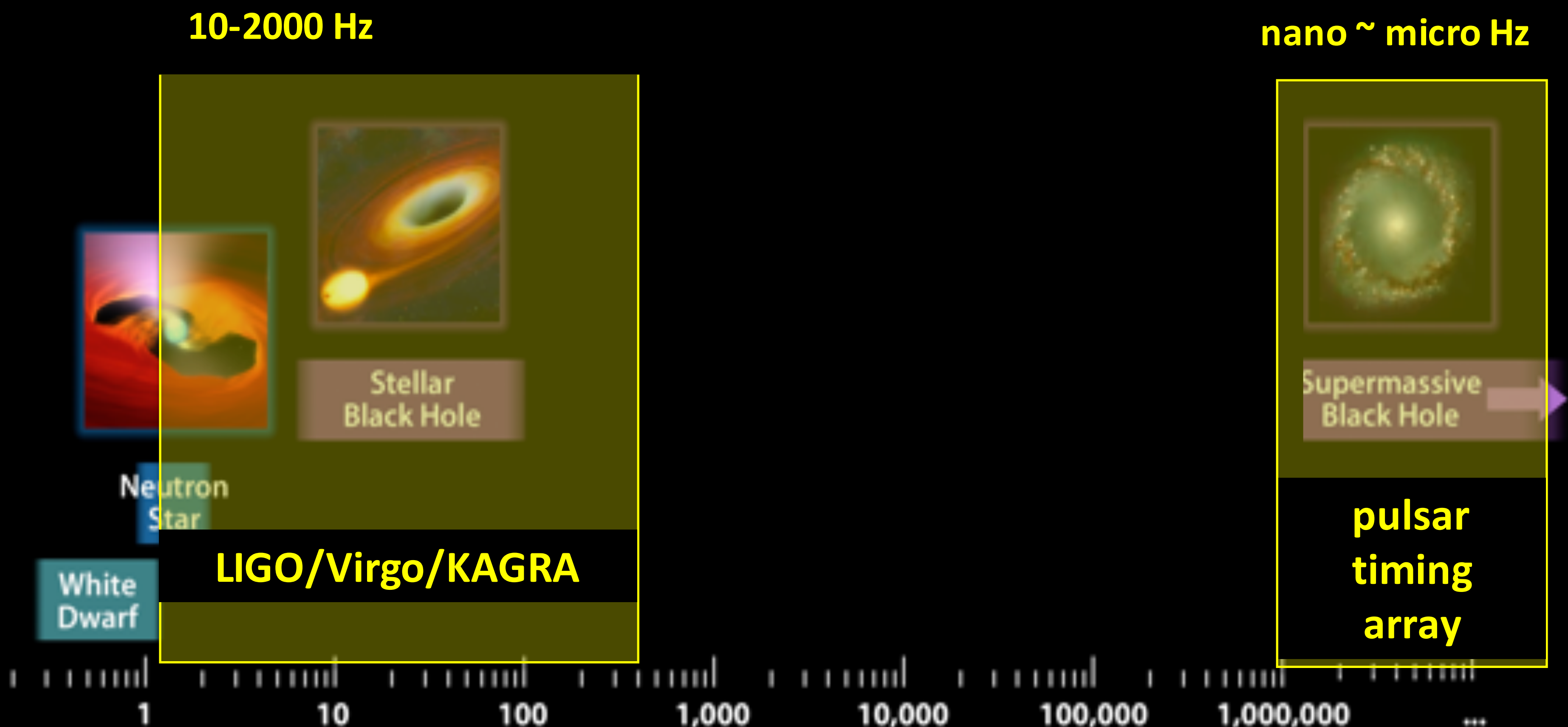


$$f_{\rm GW} = 2\,f_{\rm orb}$$

$$f_{\rm GW} = \frac{1}{\pi}\sqrt{\frac{GM}{a^3}}$$

Image credit: NASA/JPL-Caltech

multi-wavelength GW observations would shed lights on the BH mass distribution

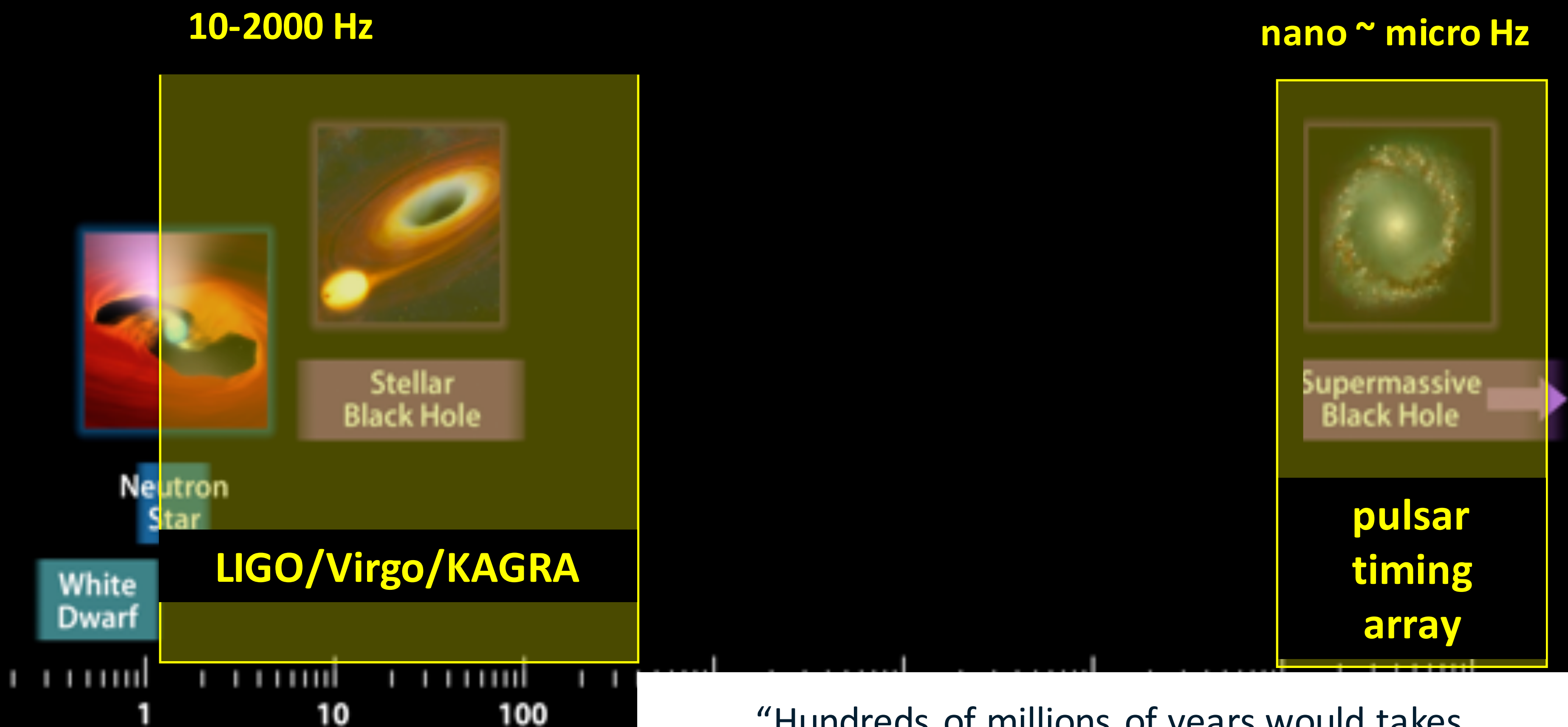


stellar-mass BH **inspirals**, then **mergers**

signal duration < seconds **“transients”**

Image credit: NASA/JPL-Caltech

multi-wavelength GW observations would shed lights on the BH mass distribution



"Hundreds of millions of years would takes for one merger of a SMBH binary to complete"

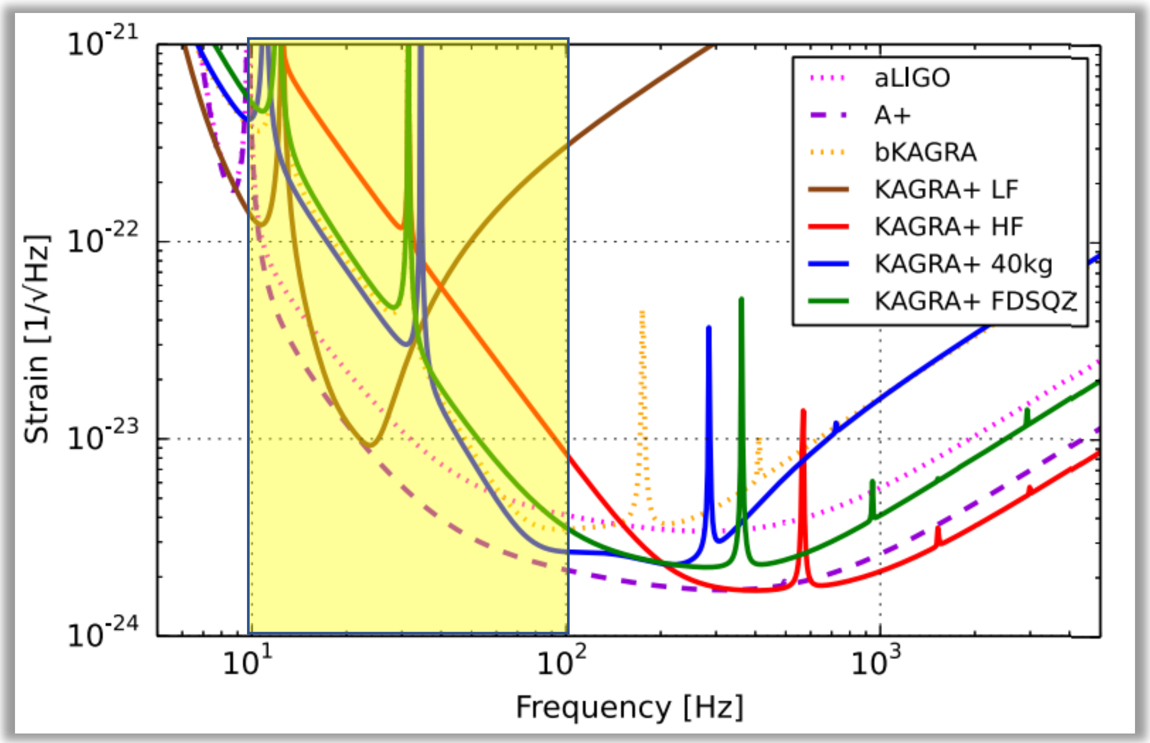
➔ **"continuous" GW signal**

research interests:

stellar-mass black holes (BHs), neutron stars in binaries

$f_{gw} = 2\ f_{orb}$

signal durations ~ sub seconds to minutes



below 30 Hz:

early inspiral ➔ eccentricity

low f ➔ massive binaries

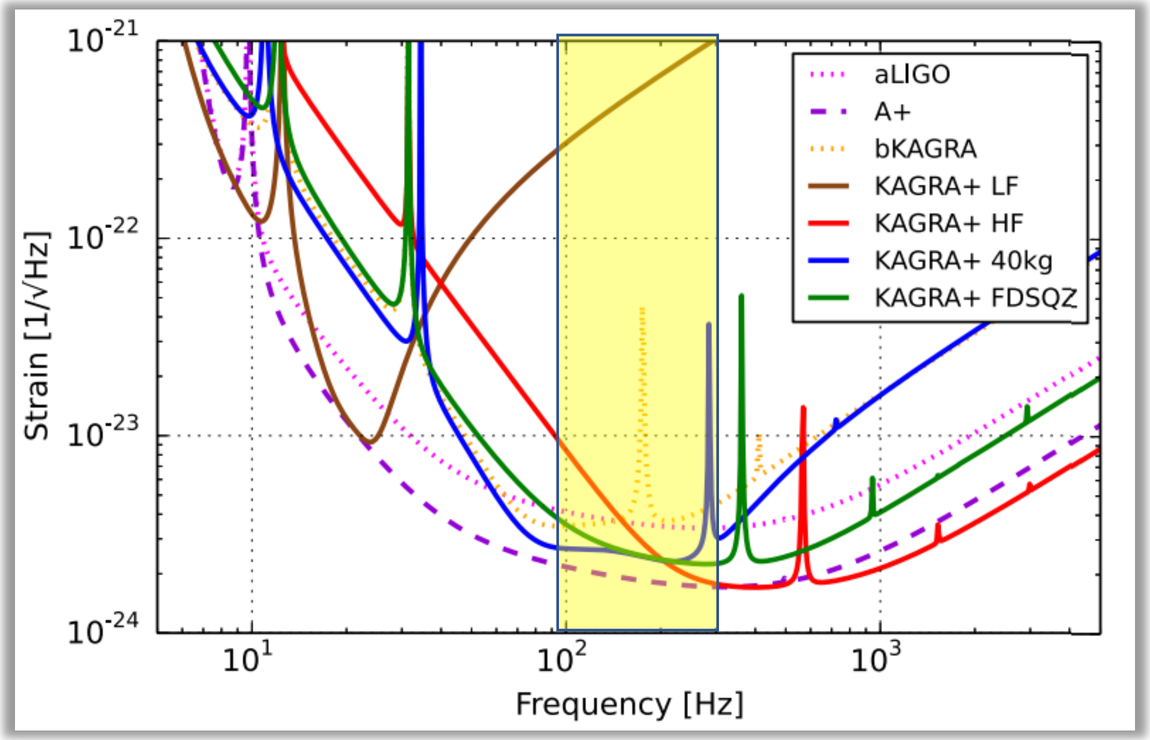
more cycles ➔ far sources

research interests:

stellar-mass black holes (BHs), neutron stars in binaries

$f_{gw} = 2\ f_{orb}$

signal durations ~ sub seconds to minutes



~100-300 Hz:

late inspiral to merge
➔ test of GR (BH-BH)

known pulsars in our Galaxy

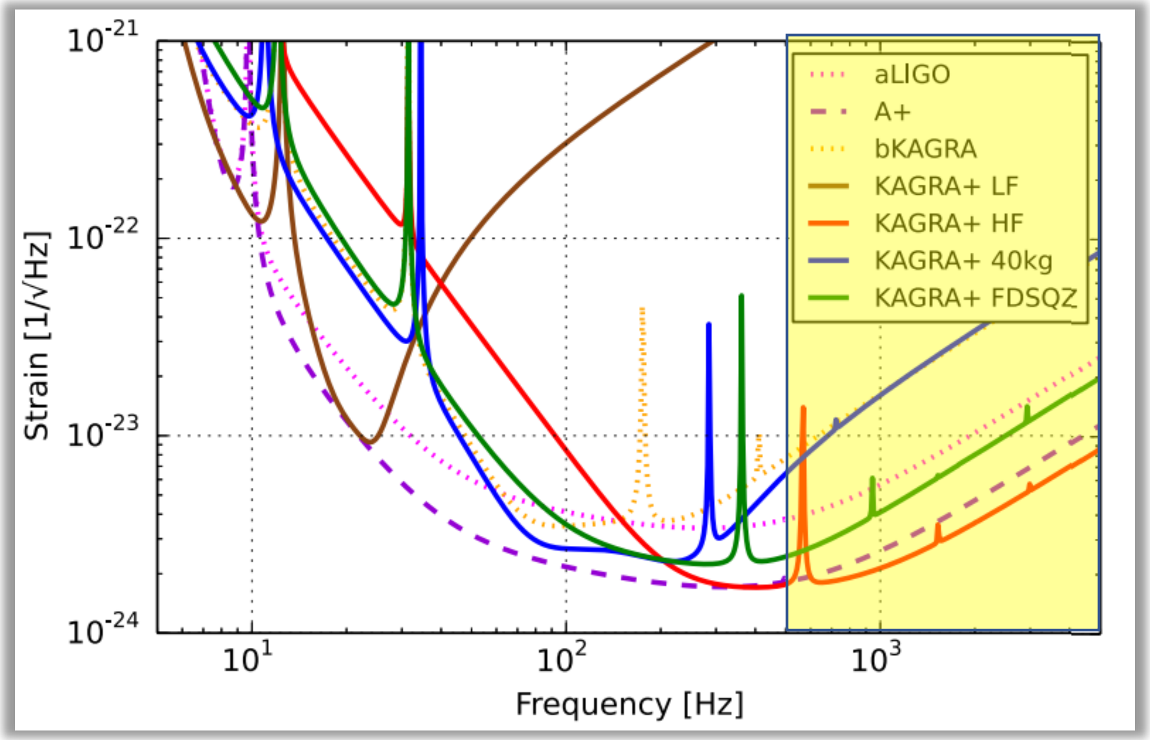
increase in SNR in general

research interests:

stellar-mass black holes (BHs), neutron stars in binaries

$f_{gw} = 2\, f_{orb}$

signal durations ~ sub seconds to minutes



500+ Hz:

supernovae in our Galaxy

merger to ring-down of NS-NS

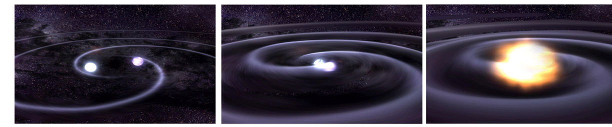
implications of squeezing

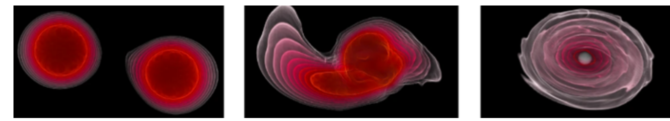
➔ **overall improvement in sensitivity** (f_{gw} ~ O(10) up to 2 kHz)

➔ more BBHs (similar to those already known)

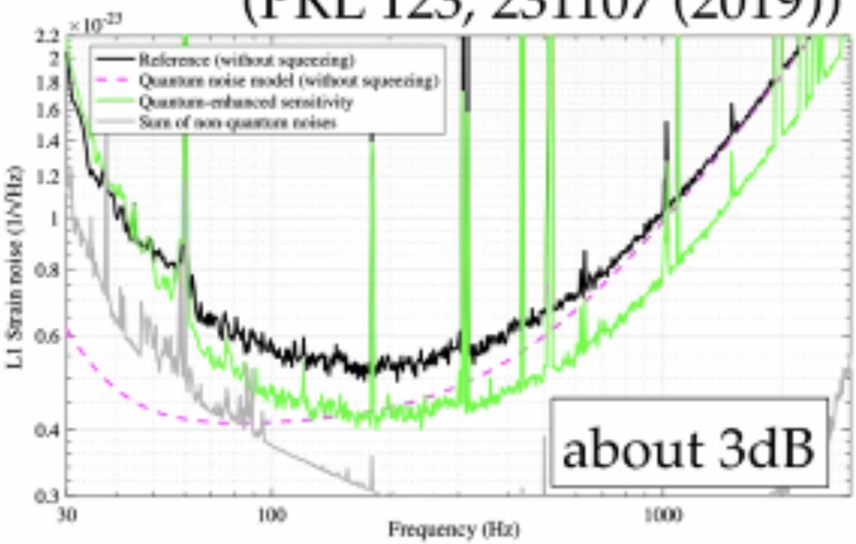
➔ larger SNR for merge-ringdown from BBHs

➔ late-inspiral for NS-NS (helpful to constrain EOS)

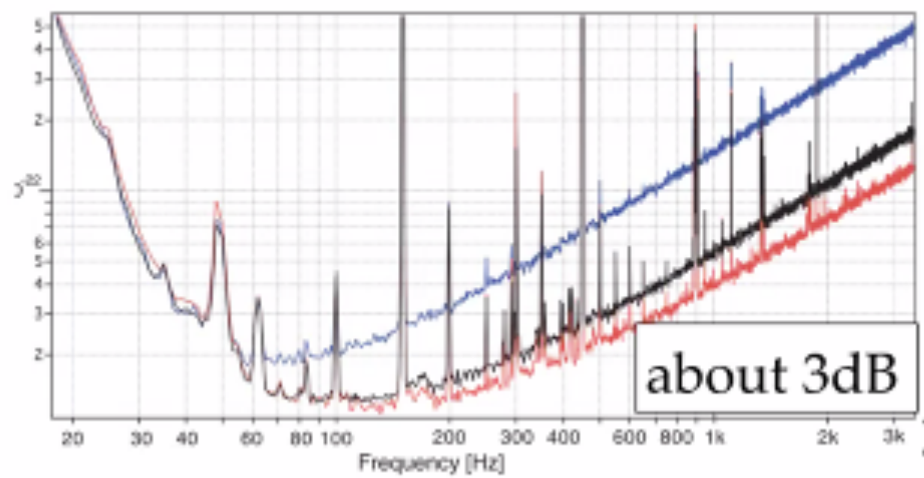




Advanced LIGO
(PRL 123, 231107 (2019))



Advanced Virgo
(PRL 123, 231108 (2019))



Next decades will be a golden era for GW astronomy and astrophysics ! ➔ white dwarfs, neutron stars, black holes

GW astronomy and astrophysics

2020+ : GW obs in 10-2000 Hz

2030+ : GW obs in 10-2000 Hz
in 0.03 mHz – 0.1 Hz
in nHz

Gravitation

strong field gravity

Galaxy formation and evolution

large-scale structure

cosmology

Hubble constant
dark energy, standard model

천체물리학자에게 필요한 교양

역학 “중력의 물리학”

전자기학 “빛의 물리학”

양자역학 “미시세계의 물리학”

열및통계(별, 성간물질, 은하) + 고체물리학/핵물리학 (중성자별) + 중력이론(블랙홀, 우주론)

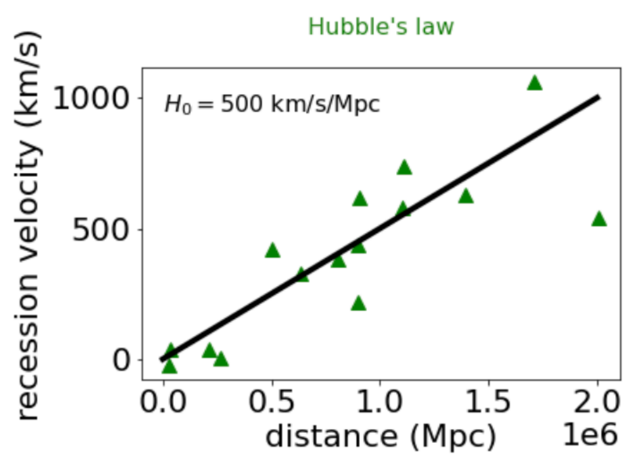
전자공학 “신호처리”

통계/수학 “자료 분석” science with a big data

컴퓨터 프로그래밍, visualization: C, Python, maple, mathematica, matlab

```
int32_t XKGLGetDomain( //begin{proto
    KGLStatus *status,    /**< [in,
    KGLApproximants approximant /**<
    ) //end{proto}
{
  int32_t domain = FREQUENCY_DOMAIN;
  switch(approximant)
  {
    case TaylorF2:
    case TaylorF2Full:
      break;
    case NUMBER_APPROXIMANTS:
    default:
      domain = UNKNOWN_DOMAIN;
      break;
  }
  return domain;
}
```

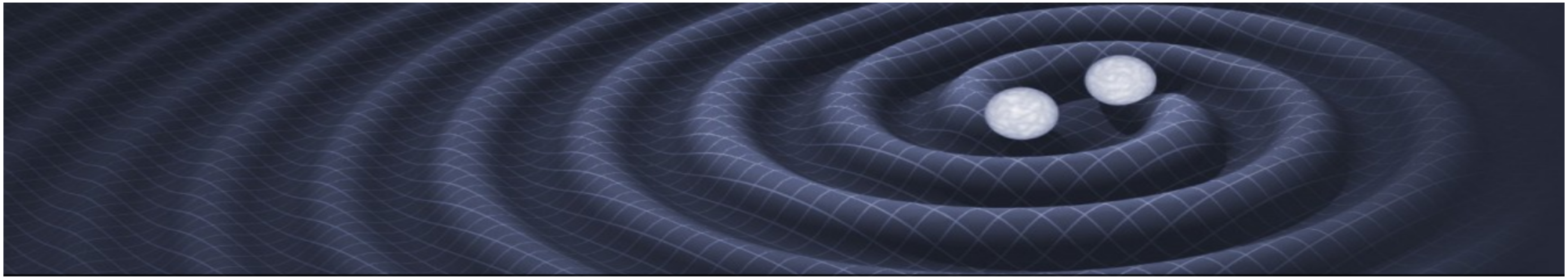
```
 plt.text(x,y) <-- x,y are in data coordinates
 Set figure title
lt.title("Hubble's law", fontsize=16, y=1.08, color='green')

show the figure on the screen
lt.show();
```



물리/천문학
+ 프로그래밍
+ 영어
+ 운전능력
+ 열린 사고와 호기심!

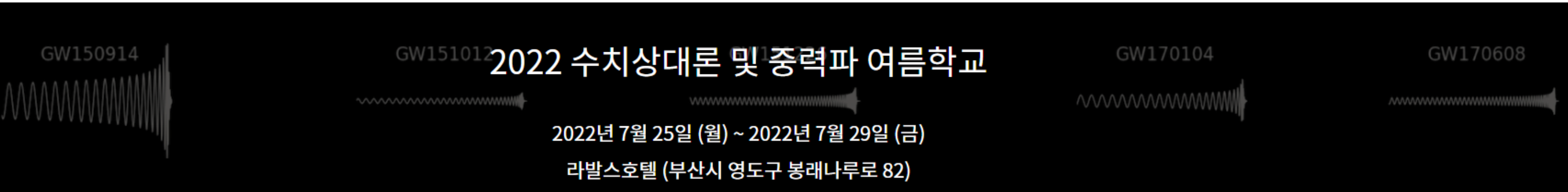
중력파 천체물리학 ➔ 국제 연구 협력 기반

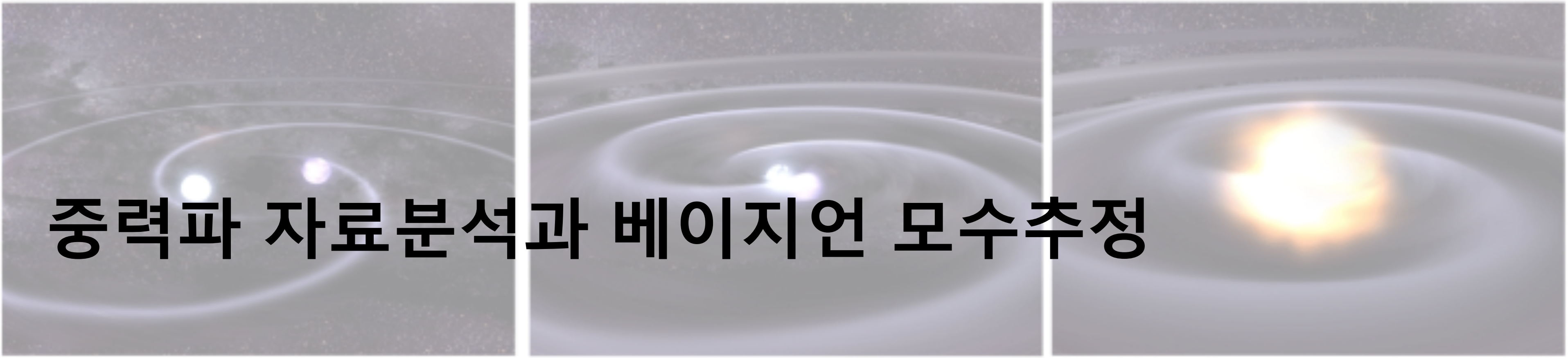
제4회 카그라 국제 워크숍 (2018) @ Ewha



중력파 천체물리학 III

김정리
(이화여자대학교)





중력파 자료분석과 베이지언 모수추정

matched filter

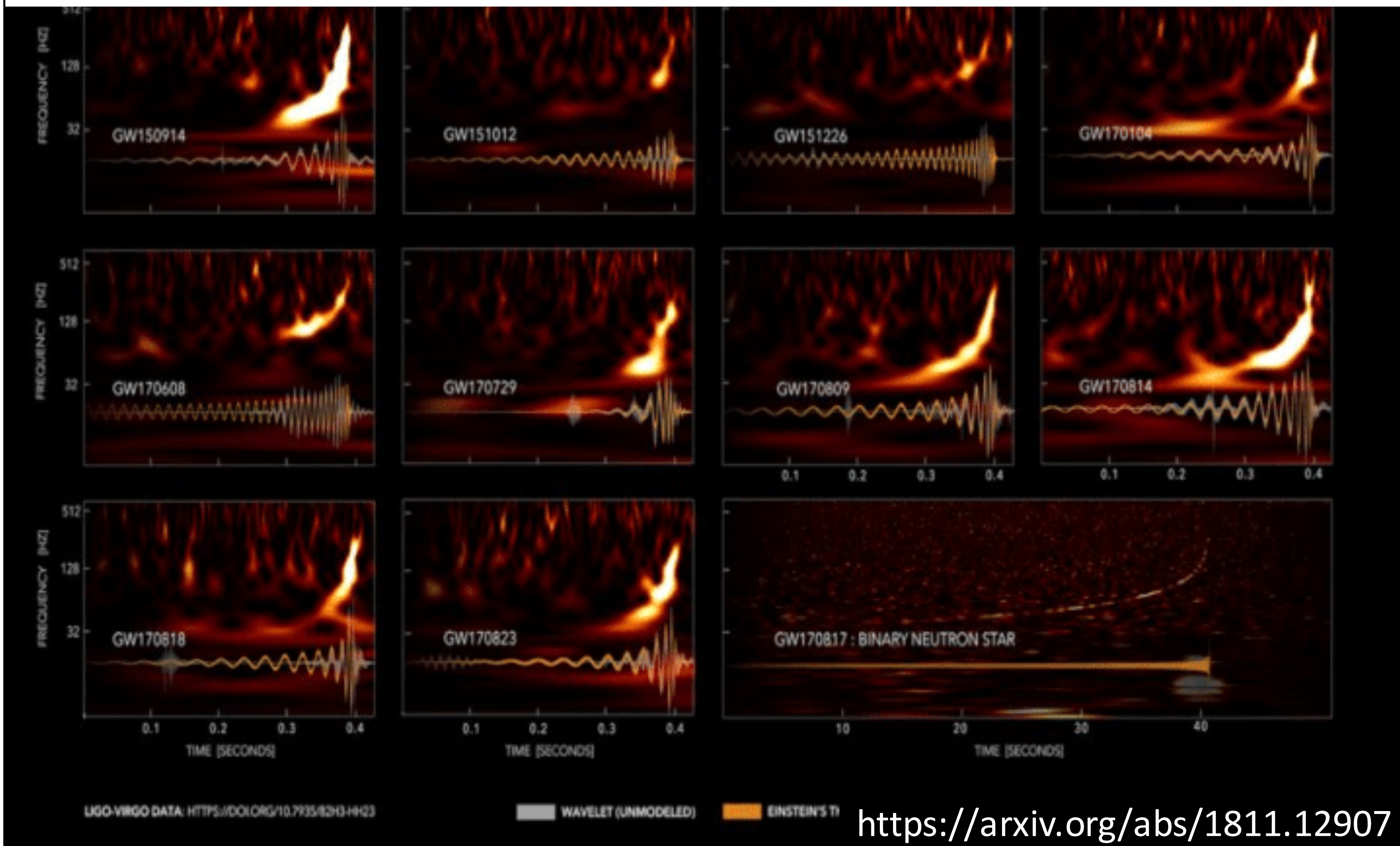
“template h(t), h(f)” ➔ find the best template that matches the GW signal embedded in the data

data = signal + noise   (if signal exists)

www.gw-openscience.org

Gravitational Wave Open Science Center

The **Gravitational Wave Open Science Center** provides data from gravitational-wave observatories, along with access to tutorials and software tools.



https://arxiv.org/abs/1811.12907

GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run

BH data with advanced LIGO-Virgo

+ KAGRA from Y2021

+ LIGO India from Y2030s(?)

1 GW detection → 3 masses + 1 spin

- individual masses (m_1,m_2) → mass ratio
- individual spins (s_1,s_2)
- distance
- sky location (RA, dec)

$$T^2 = \frac{4\pi^2}{G(M+m)} a^3$$

- remnant mass (m_f)
- remnant spin (s_f)

Event	$m_1[M_\odot]$	$m_2[M_\odot]$	FAR [yr^{-1}]
GW150914	$35.7^{+4.8}_{-3.1}$	$30.6^{+3.1}_{-4.4}$	$< 1.0 \times 10^{-7}$
GW151012	$23.3^{+14.7}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	7.9×10^{-3}
GW151226	$13.7^{+8.7}_{-3.2}$	$7.7^{+2.2}_{-2.5}$	$< 1.0 \times 10^{-7}$
GW170104	$31.0^{+7.3}_{-5.7}$	$20.0^{+4.9}_{-4.6}$	$< 1.0 \times 10^{-7}$
GW170608	$11.0^{+5.6}_{-1.7}$	$7.6^{+1.4}_{-2.2}$	$< 1.0 \times 10^{-7}$
GW170729	$50.7^{+16.5}_{-10.5}$	$34.0^{+9.3}_{-10.2}$	2.0×10^{-2}
GW170809	$35.1^{+8.4}_{-5.9}$	$23.8^{+5.2}_{-5.2}$	$< 1.0 \times 10^{-7}$
GW170814	$30.6^{+5.6}_{-3.0}$	$25.3^{+2.8}_{-4.1}$	$< 1.0 \times 10^{-7}$
* GW170817	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	$< 1.0 \times 10^{-7}$
GW170818	$35.4^{+7.5}_{-4.7}$	$26.7^{+4.3}_{-5.2}$	4.2×10^{-5}
GW170823	$39.7^{+11.2}_{-6.7}$	$29.0^{+6.8}_{-7.7}$	$< 1.0 \times 10^{-7}$
GW190408_181802	$24.5^{+5.1}_{-3.4}$	$18.3^{+3.2}_{-3.5}$	1.0×10^{-5}
GW190412	$30.0^{+4.7}_{-5.1}$	$8.3^{+1.6}_{-0.9}$	1.0×10^{-5}
GW190413_052954	$33.4^{+12.4}_{-7.4}$	$23.4^{+6.7}_{-6.3}$	7.2×10^{-2}
GW190413_134308	$45.4^{+13.6}_{-9.6}$	$30.9^{+10.2}_{-9.6}$	4.4×10^{-2}
GW190421_213856	$40.6^{+10.4}_{-6.6}$	$31.4^{+7.5}_{-8.2}$	7.7×10^{-4}
GW190424_180648	$39.5^{+10.9}_{-6.9}$	$31.0^{+7.4}_{-7.3}$	7.8×10^{-1}
* GW190425	$2.0^{+0.6}_{-0.3}$	$1.4^{+0.3}_{-0.3}$	7.5×10^{-4}
GW190503_185404	$42.9^{+9.2}_{-7.8}$	$28.5^{+7.5}_{-7.9}$	1.0×10^{-5}
GW190512_180714	$23.0^{+5.4}_{-5.7}$	$12.5^{+3.5}_{-2.5}$	1.0×10^{-5}
GW190513_205428	$35.3^{+9.6}_{-9.0}$	$18.1^{+7.3}_{-4.2}$	1.0×10^{-5}
GW190514_065416	$36.9^{+13.4}_{-7.3}$	$27.5^{+8.2}_{-7.7}$	5.3×10^{-1}
GW190517_055101	$36.4^{+11.8}_{-7.8}$	$24.8^{+6.9}_{-7.1}$	5.7×10^{-5}
GW190519_153544	$64.5^{+11.3}_{-13.2}$	$39.9^{+11.0}_{-10.6}$	1.0×10^{-5}
GW190521	$91.4^{+29.3}_{-17.5}$	$66.8^{+20.7}_{-20.7}$	2.0×10^{-4}
GW190521_074359	$42.1^{+5.9}_{-4.9}$	$32.7^{+5.4}_{-6.2}$	1.0×10^{-5}

GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run

chirp mass $$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

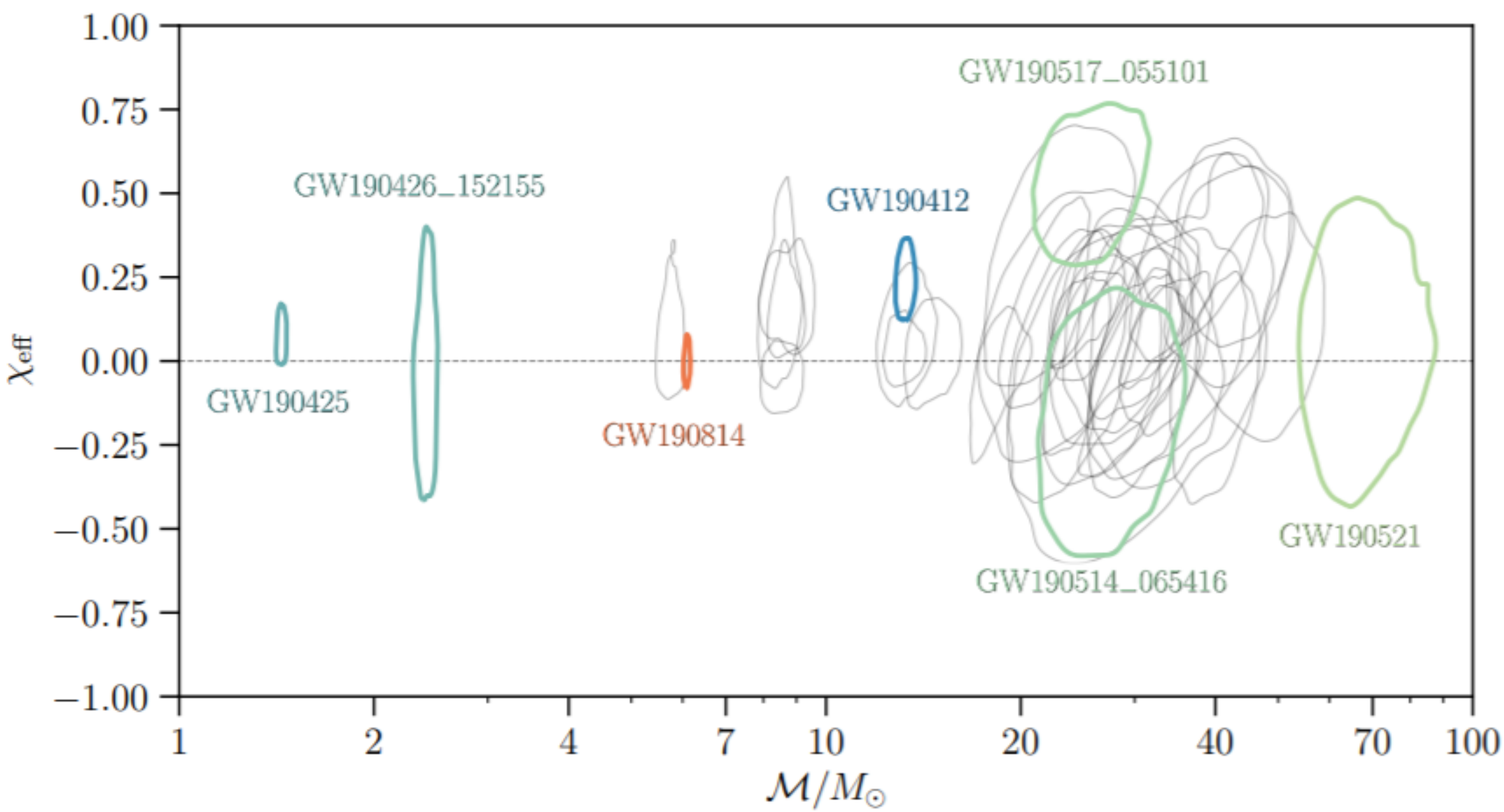
effective spin $$\chi_{\rm eff} = \frac{(m_1 \vec{\chi}_1 + m_2 \vec{\chi}_2) \cdot \hat{L}_{\rm N}}{m_1 + m_2}$$

Event	M ($M_\odot$)	$\mathcal{M}$ ($M_\odot$)	m_1 ($M_\odot$)	m_2 ($M_\odot$)	$\chi_{\rm eff}$	$D_{\rm L}$ (Gpc)	z	$M_{\rm f}$ ($M_\odot$)	$\chi_{\rm f}$	$\Delta\Omega$ (deg^2)	SNR
GW190408_181802	$42.9^{+4.1}_{-2.9}$	$18.3^{+1.8}_{-1.2}$	$24.5^{+5.1}_{-3.4}$	$18.3^{+3.2}_{-3.5}$	$-0.03^{+0.13}_{-0.19}$	$1.58^{+0.40}_{-0.59}$	$0.30^{+0.06}_{-0.10}$	$41.0^{+3.8}_{-2.7}$	$0.67^{+0.06}_{-0.07}$	140	$15.3^{+0.2}_{-0.3}$
GW190412	$38.4^{+3.8}_{-3.7}$	$13.3^{+0.4}_{-0.3}$	$30.0^{+4.7}_{-5.1}$	$8.3^{+1.6}_{-0.9}$	$0.25^{+0.08}_{-0.11}$	$0.74^{+0.14}_{-0.17}$	$0.15^{+0.03}_{-0.03}$	$37.3^{+3.9}_{-3.9}$	$0.67^{+0.05}_{-0.06}$	21	$18.9^{+0.2}_{-0.3}$
GW190413_052954	$56.9^{+13.1}_{-8.9}$	$24.0^{+5.4}_{-3.7}$	$33.4^{+12.4}_{-7.4}$	$23.4^{+6.7}_{-6.3}$	$0.01^{+0.29}_{-0.33}$	$4.10^{+2.41}_{-1.89}$	$0.66^{+0.30}_{-0.27}$	$54.3^{+12.4}_{-8.4}$	$0.69^{+0.12}_{-0.13}$	1400	$8.9^{+0.4}_{-0.8}$
GW190413_134308	$76.1^{+15.9}_{-10.6}$	$31.9^{+7.3}_{-4.6}$	$45.4^{+13.6}_{-9.6}$	$30.9^{+10.2}_{-9.6}$	$-0.01^{+0.24}_{-0.28}$	$5.15^{+2.44}_{-2.34}$	$0.80^{+0.30}_{-0.31}$	$72.8^{+15.2}_{-10.3}$	$0.69^{+0.10}_{-0.12}$	520	$10.0^{+0.4}_{-0.5}$
GW190421_213856	$71.8^{+12.5}_{-8.6}$	$30.7^{+5.5}_{-3.9}$	$40.6^{+10.4}_{-6.6}$	$31.4^{+7.5}_{-8.2}$	$-0.05^{+0.23}_{-0.26}$	$3.15^{+1.37}_{-1.42}$	$0.53^{+0.18}_{-0.21}$	$68.6^{+11.7}_{-8.1}$	$0.68^{+0.10}_{-0.11}$	1000	$10.7^{+0.2}_{-0.4}$
GW190424_180648	$70.7^{+13.4}_{-9.8}$	$30.3^{+5.7}_{-4.2}$	$39.5^{+10.9}_{-6.9}$	$31.0^{+7.4}_{-7.3}$	$0.15^{+0.22}_{-0.22}$	$2.55^{+1.56}_{-1.33}$	$0.45^{+0.22}_{-0.21}$	$67.1^{+12.5}_{-9.2}$	$0.75^{+0.08}_{-0.09}$	26000	$10.4^{+0.2}_{-0.4}$
GW190425	$3.4^{+0.3}_{-0.1}$	$1.44^{+0.02}_{-0.02}$	$2.0^{+0.6}_{-0.3}$	$1.4^{+0.3}_{-0.3}$	$0.06^{+0.11}_{-0.05}$	$0.16^{+0.07}_{-0.07}$	$0.03^{+0.01}_{-0.02}$	–	–	9900	$12.4^{+0.3}_{-0.4}$
GW190426_152155	$7.2^{+3.5}_{-1.5}$	$2.41^{+0.08}_{-0.08}$	$5.7^{+4.0}_{-2.3}$	$1.5^{+0.8}_{-0.5}$	$-0.03^{+0.33}_{-0.30}$	$0.38^{+0.19}_{-0.16}$	$0.08^{+0.04}_{-0.03}$	–	–	1400	$8.7^{+0.5}_{-0.6}$
GW190503_185404	$71.3^{+9.3}_{-8.0}$	$30.1^{+4.2}_{-4.0}$	$42.9^{+9.2}_{-7.8}$	$28.5^{+7.5}_{-7.9}$	$-0.02^{+0.20}_{-0.26}$	$1.52^{+0.71}_{-0.66}$	$0.29^{+0.11}_{-0.11}$	$68.2^{+8.7}_{-7.5}$	$0.67^{+0.09}_{-0.12}$	94	$12.4^{+0.2}_{-0.3}$
GW190512_180714	$35.6^{+3.9}_{-3.4}$	$14.5^{+1.3}_{-1.0}$	$23.0^{+5.4}_{-5.7}$	$12.5^{+3.5}_{-2.5}$	$0.03^{+0.13}_{-0.13}$	$1.49^{+0.53}_{-0.59}$	$0.28^{+0.09}_{-0.10}$	$34.2^{+3.9}_{-3.4}$	$0.65^{+0.07}_{-0.07}$	230	$12.2^{+0.2}_{-0.4}$
GW190513_205428	$53.6^{+8.6}_{-5.9}$	$21.5^{+3.6}_{-1.9}$	$35.3^{+9.6}_{-9.0}$	$18.1^{+7.3}_{-4.2}$	$0.12^{+0.29}_{-0.18}$	$2.16^{+0.94}_{-0.80}$	$0.39^{+0.14}_{-0.13}$	$51.3^{+8.1}_{-5.8}$	$0.69^{+0.14}_{-0.12}$	490	$12.9^{+0.3}_{-0.4}$
GW190514_065416	$64.2^{+16.6}_{-9.6}$	$27.4^{+6.9}_{-4.3}$	$36.9^{+13.4}_{-7.3}$	$27.5^{+8.2}_{-7.7}$	$-0.16^{+0.28}_{-0.32}$	$4.93^{+2.76}_{-2.41}$	$0.77^{+0.34}_{-0.33}$	$61.6^{+16.0}_{-9.2}$	$0.64^{+0.11}_{-0.14}$	2400	$8.2^{+0.3}_{-0.6}$
GW190517_055101	$61.9^{+10.0}_{-9.6}$	$26.0^{+4.2}_{-4.0}$	$36.4^{+11.8}_{-7.8}$	$24.8^{+6.9}_{-7.1}$	$0.53^{+0.20}_{-0.19}$	$2.11^{+1.79}_{-1.00}$	$0.38^{+0.26}_{-0.16}$	$57.8^{+9.4}_{-9.1}$	$0.87^{+0.05}_{-0.07}$	460	$10.7^{+0.4}_{-0.6}$
GW190519_153544	$104.2^{+14.5}_{-14.9}$	$43.5^{+6.8}_{-6.8}$	$64.5^{+11.3}_{-13.2}$	$39.9^{+11.0}_{-10.6}$	$0.33^{+0.19}_{-0.22}$	$2.85^{+2.02}_{-1.14}$	$0.49^{+0.27}_{-0.17}$	$98.7^{+13.5}_{-14.2}$	$0.80^{+0.07}_{-0.12}$	770	$15.6^{+0.2}_{-0.3}$
GW190521	$157.9^{+37.4}_{-20.9}$	$66.9^{+15.5}_{-9.2}$	$91.4^{+29.3}_{-17.5}$	$66.8^{+20.7}_{-20.7}$	$0.06^{+0.31}_{-0.37}$	$4.53^{+2.30}_{-2.13}$	$0.72^{+0.29}_{-0.29}$	$150.3^{+35.8}_{-20.0}$	$0.73^{+0.11}_{-0.14}$	940	$14.2^{+0.3}_{-0.3}$

TABLE VI. Median and 90% symmetric credible intervals on selected source parameters

GWTC-2: Compact Binary Coalescences Observed by LIGO and Virgo During the First Half of the Third Observing Run

BH mass vs effective spin



chirp mass $\mathcal{M} = \dfrac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$

effective spin $\chi_{\rm eff} = \dfrac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot \hat{L}_{\rm N}}{m_1+m_2}$

GWTC-3 data

병합으로 생성된 BH의 스핀

Event	M ($M_\odot$)	$\mathcal{M}$ ($M_\odot$)	m_1 ($M_\odot$)	m_2 ($M_\odot$)	$\chi_{\rm eff}$	$D_{\rm L}$ (Gpc)	z	$M_{\rm f}$ ($M_\odot$)	$\chi_{\rm f}$	$\Delta\Omega$ (deg^2)	SNR
GW191103_012549	$20.0^{+3.7}_{-1.8}$	$8.34^{+0.66}_{-0.57}$	$11.8^{+6.2}_{-2.2}$	$7.9^{+1.7}_{-2.4}$	$0.21^{+0.16}_{-0.10}$	$0.99^{+0.50}_{-0.47}$	$0.20^{+0.09}_{-0.09}$	$19.0^{+3.8}_{-1.7}$	$0.75^{+0.06}_{-0.05}$	2500	$8.9^{+0.3}_{-0.5}$
GW191105_143521	$18.5^{+2.1}_{-1.3}$	$7.82^{+0.61}_{-0.45}$	$10.7^{+3.7}_{-1.6}$	$7.7^{+1.4}_{-1.9}$	$-0.02^{+0.13}_{-0.09}$	$1.15^{+0.43}_{-0.48}$	$0.23^{+0.07}_{-0.09}$	$17.6^{+2.1}_{-1.2}$	$0.67^{+0.04}_{-0.05}$	640	$9.7^{+0.3}_{-0.5}$
GW191109_010717	112^{+20}_{-16}	$47.5^{+9.6}_{-7.5}$	65^{+11}_{-11}	47^{+15}_{-13}	$-0.29^{+0.42}_{-0.31}$	$1.29^{+1.13}_{-0.65}$	$0.25^{+0.18}_{-0.12}$	107^{+18}_{-15}	$0.61^{+0.18}_{-0.19}$	1600	$17.3^{+0.5}_{-0.5}$
GW191113_071753	$34.5^{+10.5}_{-9.8}$	$10.7^{+1.1}_{-1.0}$	29^{+12}_{-14}	$5.9^{+4.4}_{-1.3}$	$0.00^{+0.37}_{-0.29}$	$1.37^{+1.15}_{-0.62}$	$0.26^{+0.18}_{-0.11}$	34^{+11}_{-10}	$0.45^{+0.33}_{-0.11}$	3600	$7.9^{+0.5}_{-1.1}$
GW191126_115259	$20.7^{+3.4}_{-2.0}$	$8.65^{+0.95}_{-0.71}$	$12.1^{+5.5}_{-2.2}$	$8.3^{+1.9}_{-2.4}$	$0.21^{+0.15}_{-0.11}$	$1.62^{+0.74}_{-0.74}$	$0.30^{+0.12}_{-0.13}$	$19.6^{+3.5}_{-2.0}$	$0.75^{+0.06}_{-0.05}$	1400	$8.3^{+0.2}_{-0.5}$
GW191127_050227	80^{+39}_{-22}	$29.9^{+11.7}_{-9.1}$	53^{+47}_{-20}	24^{+17}_{-14}	$0.18^{+0.34}_{-0.36}$	$3.4^{+3.1}_{-1.9}$	$0.57^{+0.40}_{-0.29}$	76^{+39}_{-21}	$0.75^{+0.13}_{-0.29}$	980	$9.2^{+0.7}_{-0.6}$
GW191129_134029	$17.5^{+2.4}_{-1.2}$	$7.31^{+0.43}_{-0.28}$	$10.7^{+4.1}_{-2.1}$	$6.7^{+1.5}_{-1.7}$	$0.06^{+0.16}_{-0.08}$	$0.79^{+0.26}_{-0.33}$	$0.16^{+0.05}_{-0.06}$	$16.8^{+2.5}_{-1.2}$	$0.69^{+0.03}_{-0.05}$	850	$13.1^{+0.2}_{-0.3}$
GW191204_110529	$47.2^{+9.2}_{-8.0}$	$19.8^{+3.6}_{-3.3}$	$27.3^{+11.0}_{-6.0}$	$19.3^{+5.6}_{-6.0}$	$0.05^{+0.26}_{-0.27}$	$1.8^{+1.7}_{-1.1}$	$0.34^{+0.25}_{-0.18}$	$45.0^{+8.6}_{-7.6}$	$0.71^{+0.12}_{-0.11}$	3700	$8.8^{+0.4}_{-0.6}$
GW191204_171526	$20.21^{+1.70}_{-0.96}$	$8.55^{+0.38}_{-0.27}$	$11.9^{+3.3}_{-1.8}$	$8.2^{+1.4}_{-1.6}$	$0.16^{+0.08}_{-0.05}$	$0.65^{+0.19}_{-0.25}$	$0.13^{+0.04}_{-0.05}$	$19.21^{+1.79}_{-0.95}$	$0.73^{+0.03}_{-0.03}$	350	$17.5^{+0.2}_{-0.2}$
GW191215_223052	$43.3^{+5.3}_{-4.3}$	$18.4^{+2.2}_{-1.7}$	$24.9^{+7.1}_{-4.1}$	$18.1^{+3.8}_{-4.1}$	$-0.04^{+0.17}_{-0.21}$	$1.93^{+0.89}_{-0.86}$	$0.35^{+0.13}_{-0.14}$	$41.4^{+5.1}_{-4.1}$	$0.68^{+0.07}_{-0.07}$	530	$11.2^{+0.3}_{-0.4}$
GW191216_213338	$19.81^{+2.69}_{-0.94}$	$8.33^{+0.22}_{-0.19}$	$12.1^{+4.6}_{-2.3}$	$7.7^{+1.6}_{-1.9}$	$0.11^{+0.13}_{-0.06}$	$0.34^{+0.12}_{-0.13}$	$0.07^{+0.02}_{-0.03}$	$18.87^{+2.80}_{-0.94}$	$0.70^{+0.03}_{-0.04}$	490	$18.6^{+0.2}_{-0.2}$
GW191219_163120	$32.3^{+2.2}_{-2.7}$	$4.32^{+0.12}_{-0.17}$	$31.1^{+2.2}_{-2.8}$	$1.17^{+0.07}_{-0.06}$	$0.00^{+0.07}_{-0.09}$	$0.55^{+0.25}_{-0.16}$	$0.11^{+0.05}_{-0.03}$	$32.2^{+2.2}_{-2.7}$	$0.14^{+0.06}_{-0.06}$	1500	$9.1^{+0.5}_{-0.8}$
GW191222_033537	79^{+16}_{-11}	$33.8^{+7.1}_{-5.0}$	$45.1^{+10.9}_{-8.0}$	$34.7^{+9.3}_{-10.5}$	$-0.04^{+0.20}_{-0.25}$	$3.0^{+1.7}_{-1.7}$	$0.51^{+0.23}_{-0.26}$	$75.5^{+15.3}_{-9.9}$	$0.67^{+0.08}_{-0.11}$	2000	$12.5^{+0.2}_{-0.3}$
GW191230_180458	86^{+19}_{-12}	$36.5^{+8.2}_{-5.6}$	$49.4^{+14.0}_{-9.6}$	37^{+11}_{-12}	$-0.05^{+0.26}_{-0.31}$	$4.3^{+2.1}_{-1.9}$	$0.69^{+0.26}_{-0.27}$	82^{+17}_{-11}	$0.68^{+0.11}_{-0.13}$	1100	$10.4^{+0.3}_{-0.4}$
GW200105_162426	$11.0^{+1.5}_{-1.4}$	$3.42^{+0.08}_{-0.08}$	$9.0^{+1.7}_{-1.7}$	$1.91^{+0.33}_{-0.24}$	$0.00^{+0.13}_{-0.18}$	$0.27^{+0.12}_{-0.11}$	$0.06^{+0.02}_{-0.02}$	$10.7^{+1.5}_{-1.4}$	$0.43^{+0.05}_{-0.02}$	7900	$13.7^{+0.2}_{-0.4}$

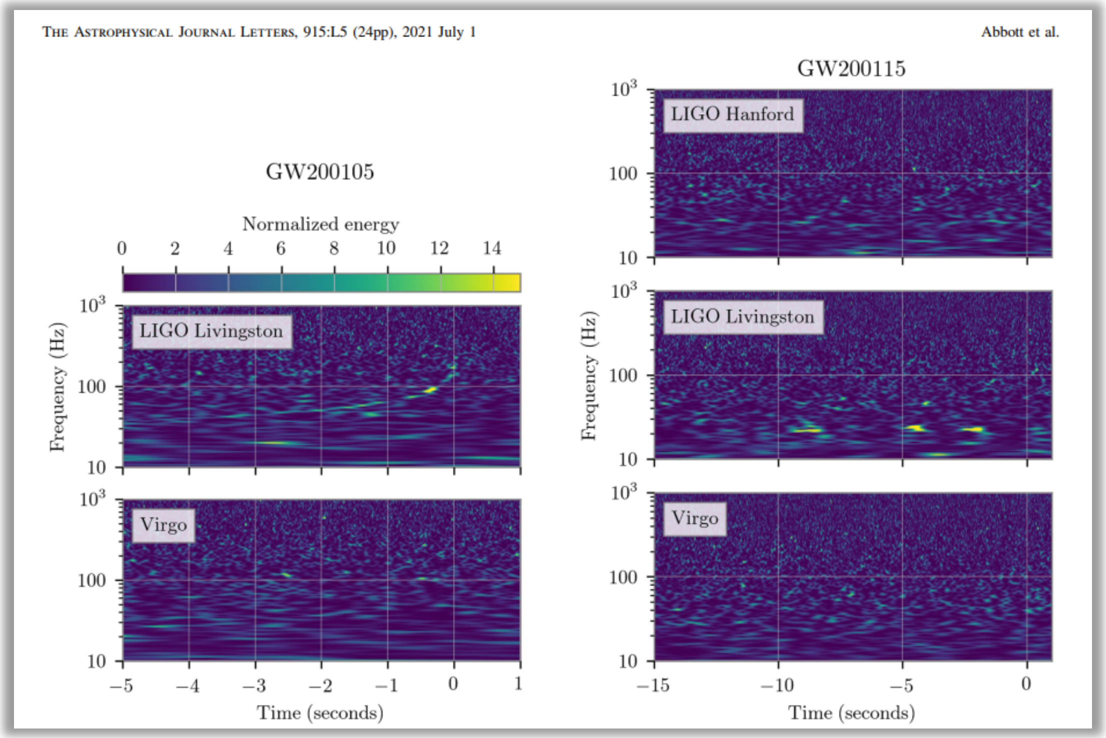
BH

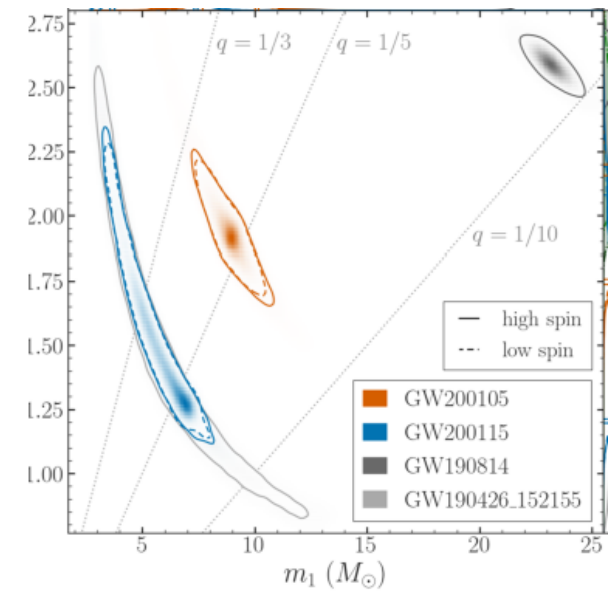
NS

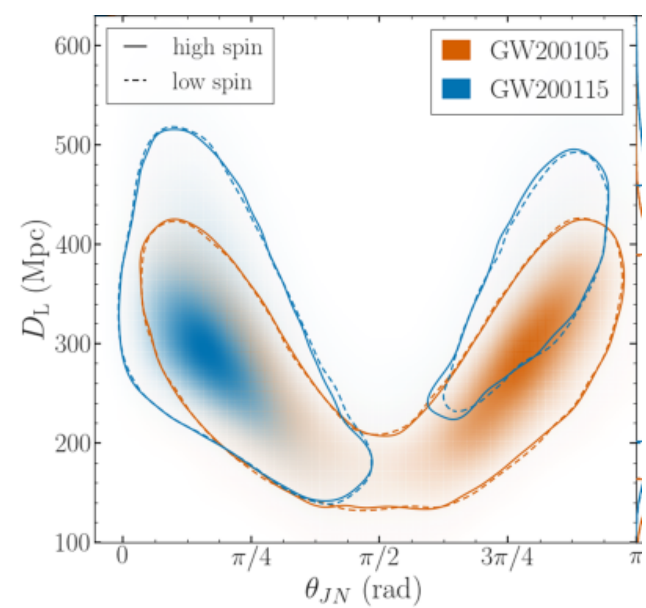
거리

병합 후에 생성된 BH 질량

Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences







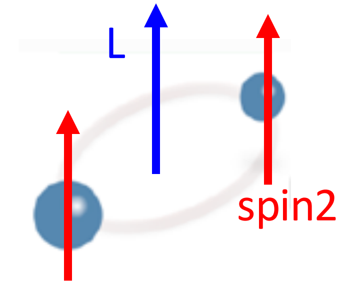
SNR ~ 13.9
(mostly LIGO Livingston only)

SNR ~ 11.6
(coincident detection)

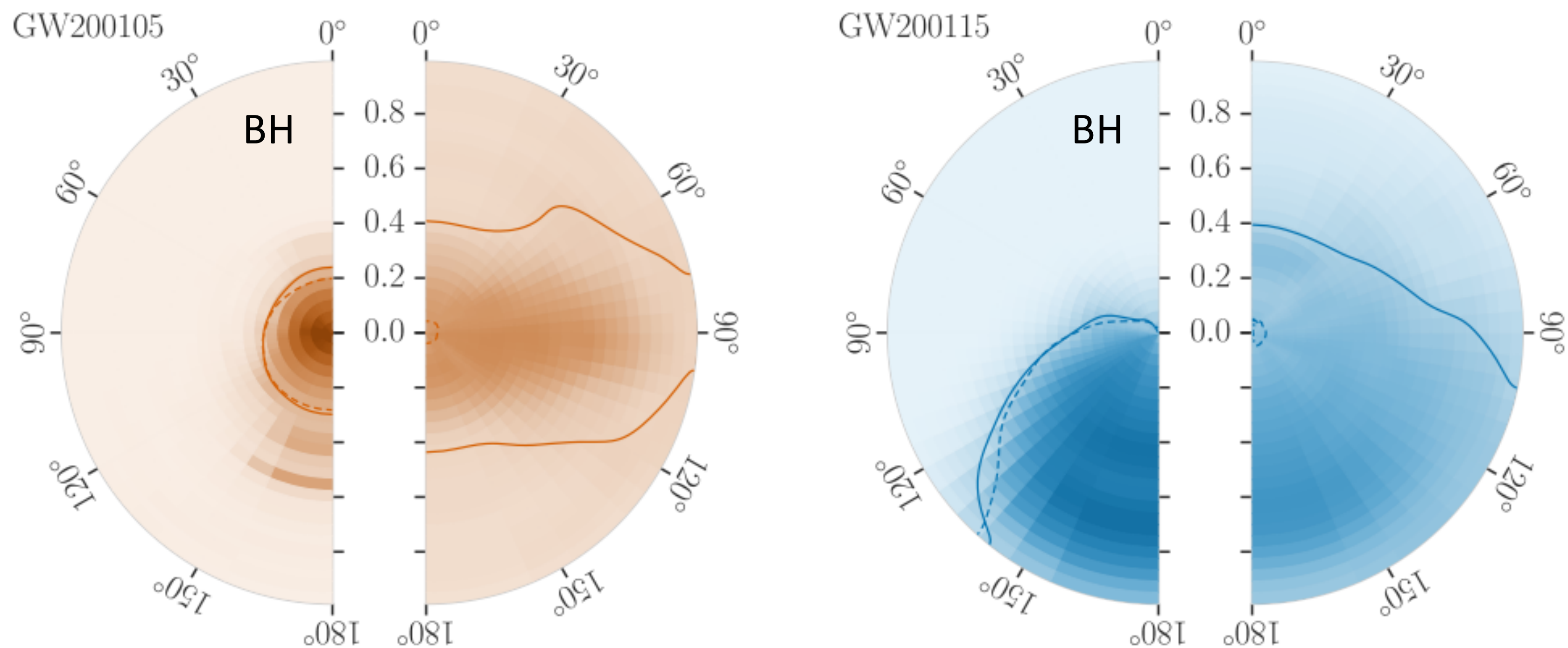
https://iopscience.iop.org/article/10.3847/2041-8213/ac082e/pdf

GW200105 ➔ BH spin < 0.2 for all directions

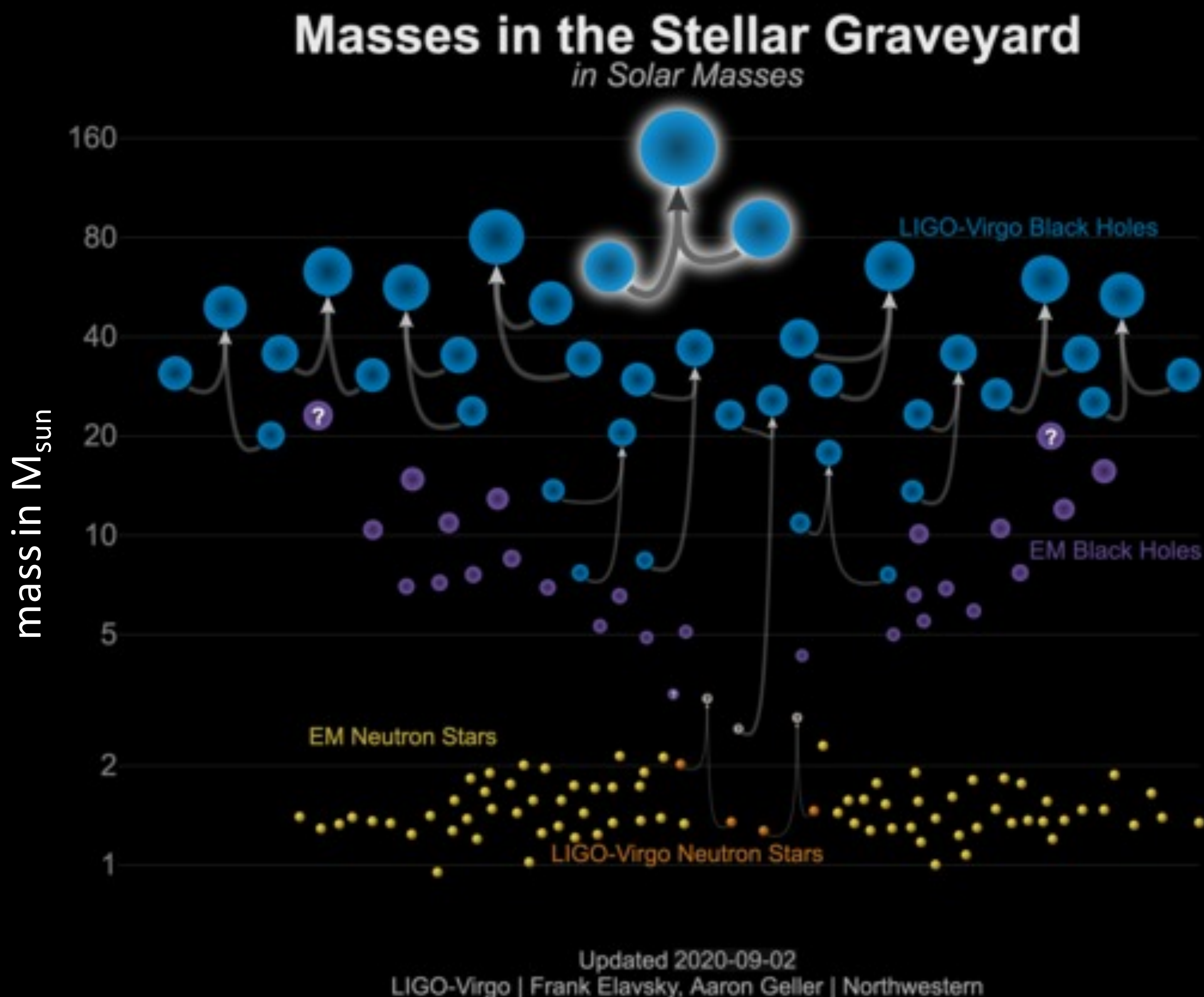
GW200115 ➔ BH spin magnitude (0, 1), negatively aligned from L ?



The Astrophysical Journal Letters, 915:L5 (24pp), 2021 July 1 Abbott et al.

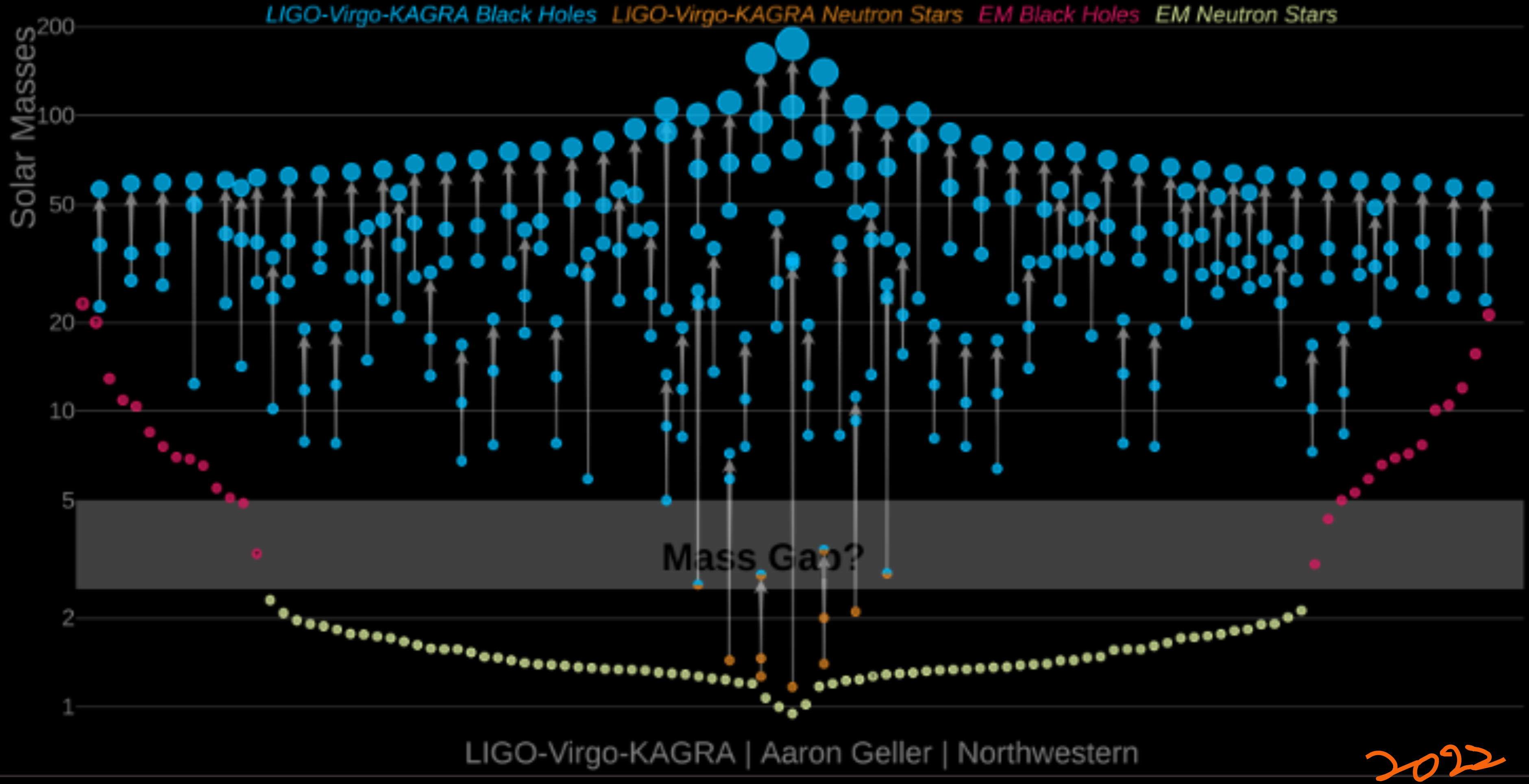


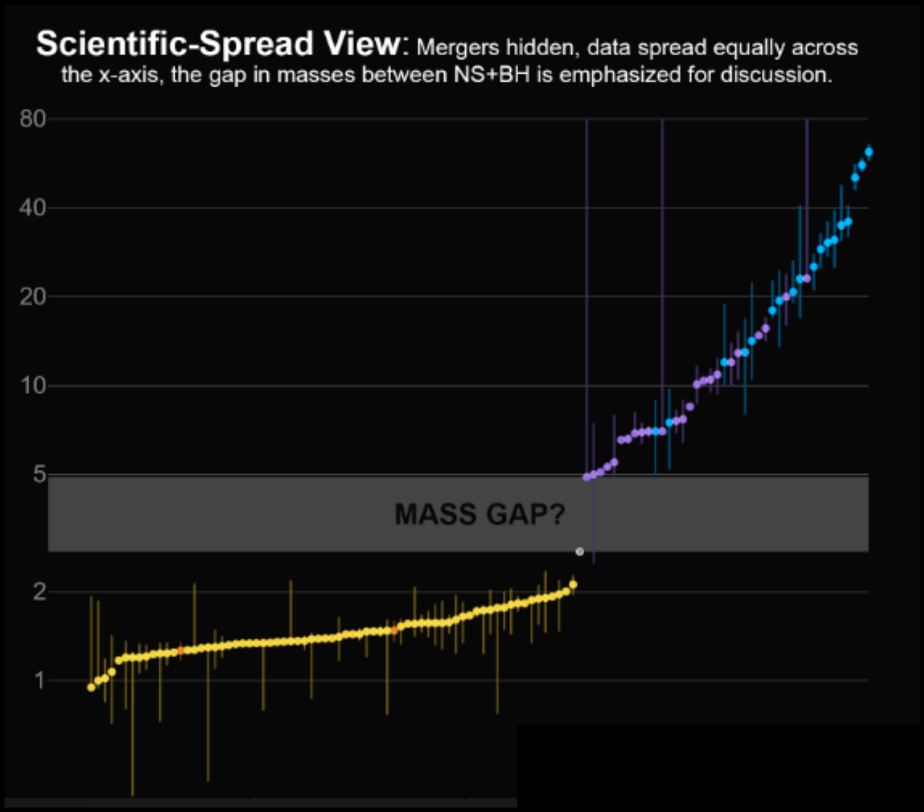
$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right) \cdot \hat{L}$$

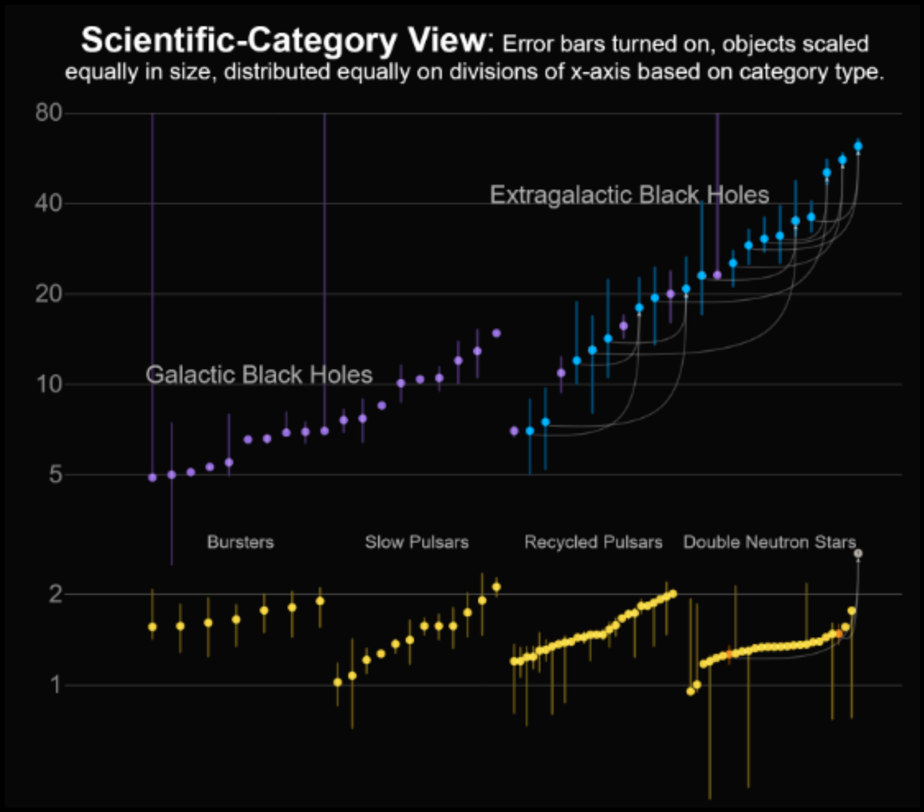


Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
mass in M_sun
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
Updated 2020-09-02
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Masses in the Stellar Graveyard







LIGO-Virgo | Frank Elavsky | Northwestern

https://www.informationisbeautifulawards.com/showcase/2642-masses-in-the-stellar-graveyard

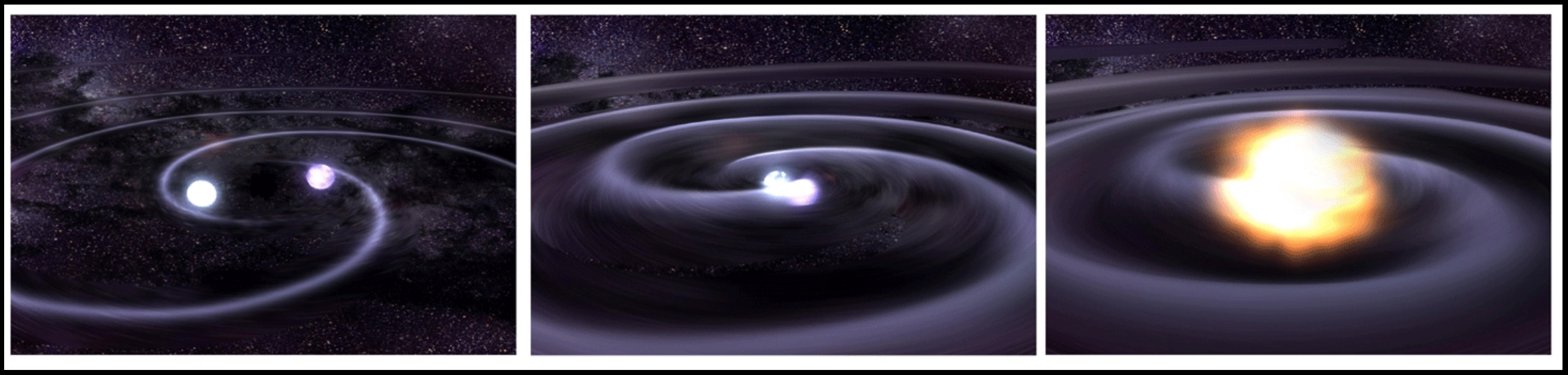
compact binary coalescences (CBCs) 밀집 쌍성 병합

inspiral motion 나선근접궤도 운동

matched filter + 중력파 파형 h(t) ➔ 중력파원의 물리량

{m1,m2,s1,s2, a, d, RA, dec} ➔ "관측가능한 물리량"
+ eccentricity observables

단, 중력파 자료 분석 ➔ "통계적 추정" inferences



How to measure physical quantities of GW sources?

→ **Parameter estimation** for compact binary coalescences

모수 추정

Parameter estimation (모수 추정)

베이지언 추론에 기반.
현상(관측 자료)로부터 모델을 "미루어 짐작"(추정)

재료:
1. 중력파 파형 모델, 잡음 모델
2. likelihood (관측 자료 혹은 시뮬레이션 사용)
3. prior (중력파원 천체물리학)

결과:
현상을 설명할 수 있는 모델(posterior)

방법: 수치 통계적으로 (numerically, sampling, marginalization)

Bayes' theorem

$$p(H|d,I) = \frac{p(d|H,I)p(H|I)}{p(d|I)}$$

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

참고문헌

BAYESIAN REASONING IN HIGH-ENERGY PHYSICS: PRINCIPLES AND APPLICATIONS G. D'Agostini

CERN 99–03
19 July 1999

http://dx.doi.org/10.5170/CERN-1999-003

What is probability?

a measure of the degree of belief that an event will occur

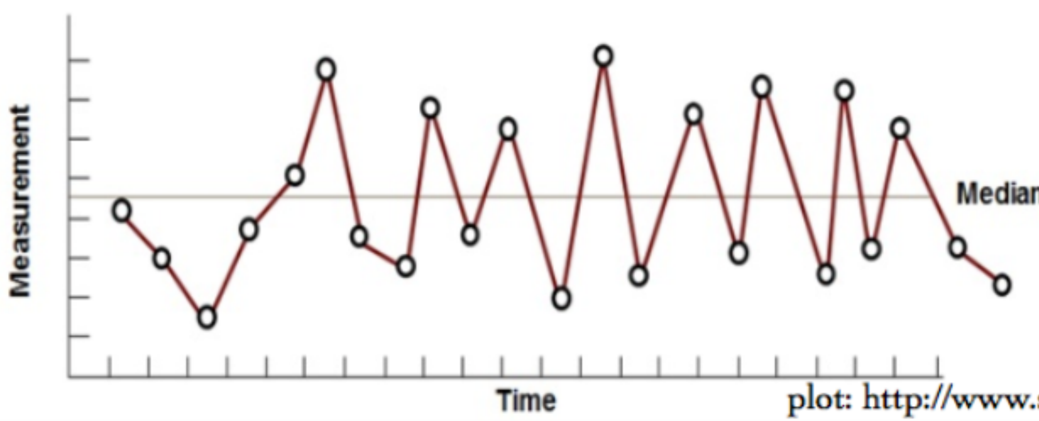
What is data?

많은 경우, 천문 관측으로 얻은 데이터에는 우리가 관심있는 신호뿐 아니라 자연적,인공적 원인으로 발생한 다양한 잡음이 섞여있다

$$\text{data} = \text{noise} + \text{signal}$$

이들은 보통 시간의 함수이다

$$\text{data}(t) = \text{noise}(t) + \text{signal}(t)$$



plot: http://www.sixsigma-institute.org

확률밀도함수(probability density function)

presents a "relative likelihood for this random variable to take on a given value"

신뢰구간(confidence interval):

확률밀도함수(PDF) $f_X(x)$를 따르는
a set of randome variables $\{x\}$에 대해
특정 상수값 X가 구간 [a,b]사이에 존재할 확률

$$\Pr[a \le X \le b] = \int_a^b f_X(x)\,dx.$$

Central Limit Theorem (CLT)

The central limit theorem and the law of large numbers are the two *fundamental theorems* of probability.

Roughly, the central limit theorem states that the distribution of the sum (or average) of **a large number of independent, identically distributed variables** will be approximately normal, regardless of the underlying distribution.

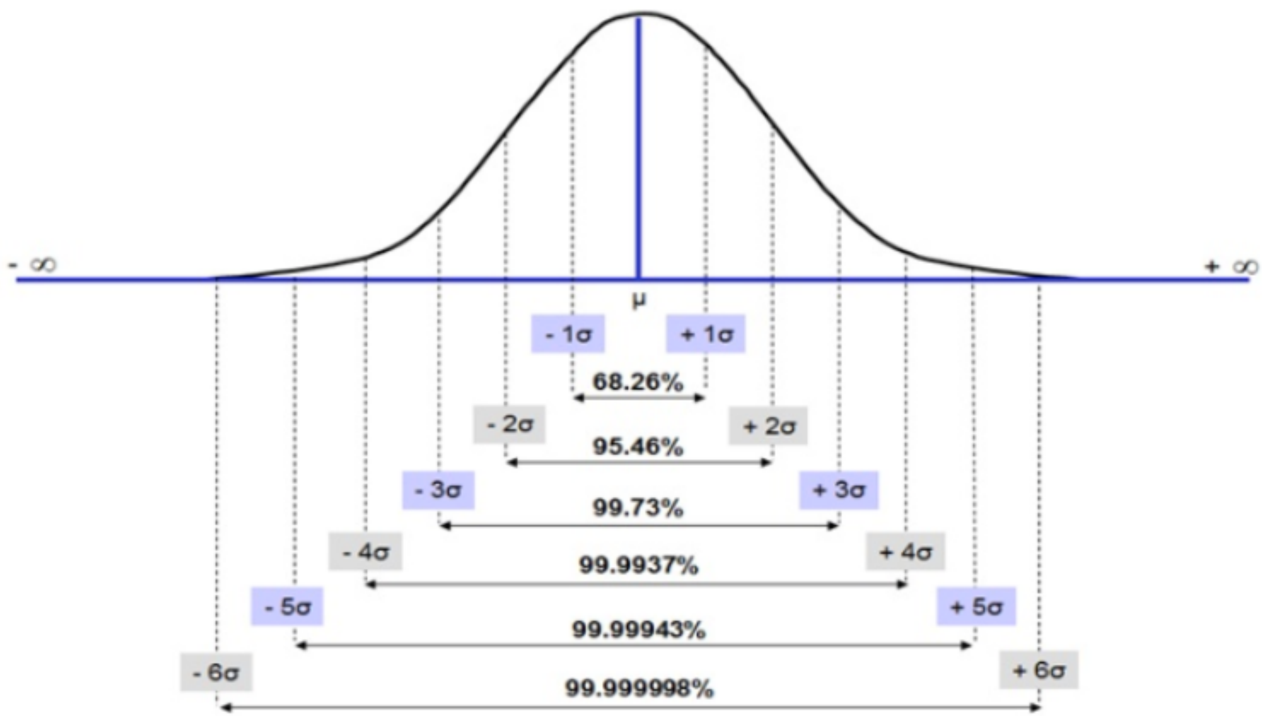
The importance of the central limit theorem is hard to overstate; indeed it is the reason that many statistical procedures work.

http://www.math.uah.edu/stat/sample/CLT.html

수많은 반복 측정 —> 측정값이 가우시안 함수를 따른다

Example of a PDF: standard, normal distribution "Gaussian distribution"

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$



plot is taken from http://www.sixsigma-institute.org/

Many phenomena we observe in nature have
MANY occurences that can be described by **Gaussian distribution**

(ex) human (in age, size, etc),
mass measurement of Higgs from N observations

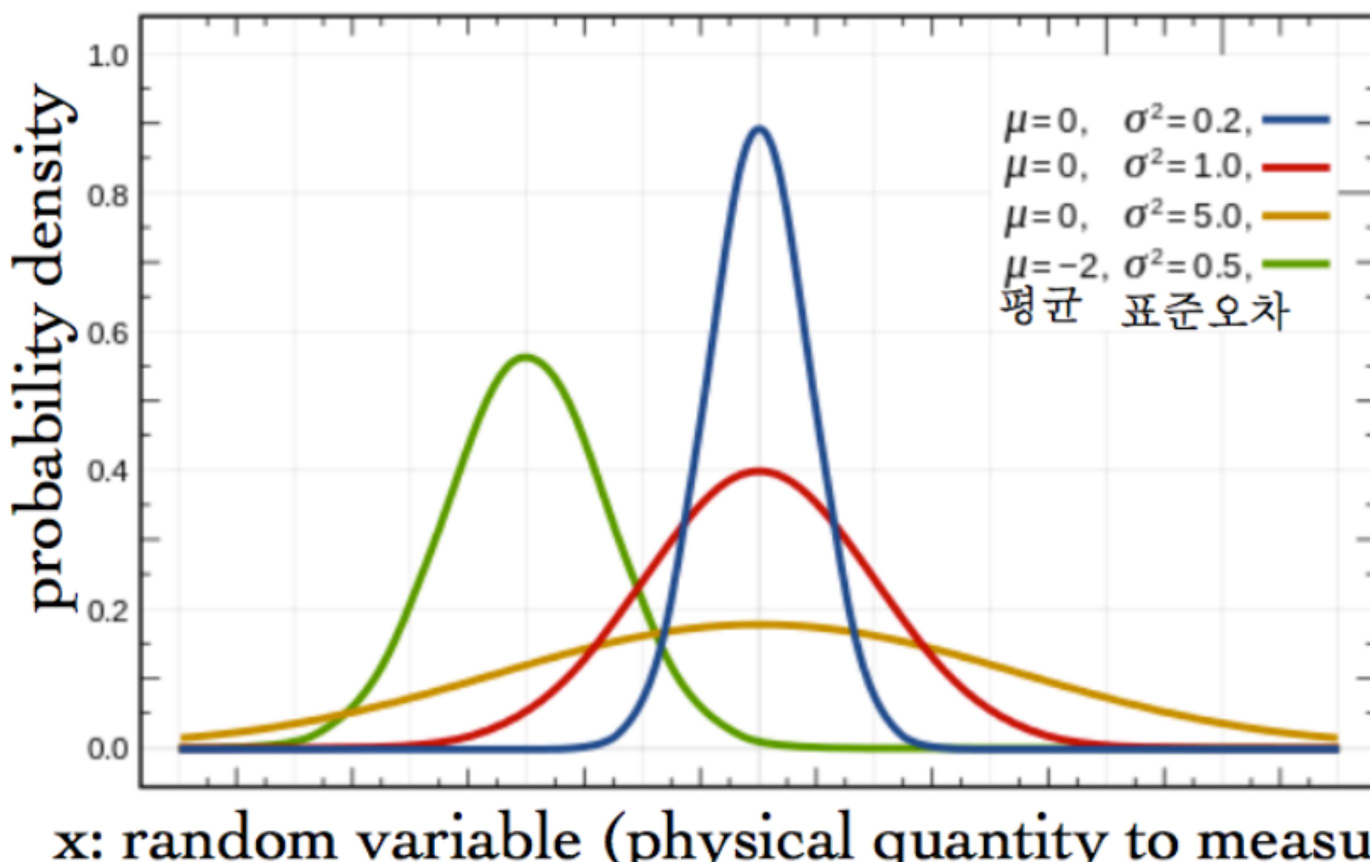


figure from Wikipedia

통계추론(statistical inference)

가설의 검증 (hypothesis test)

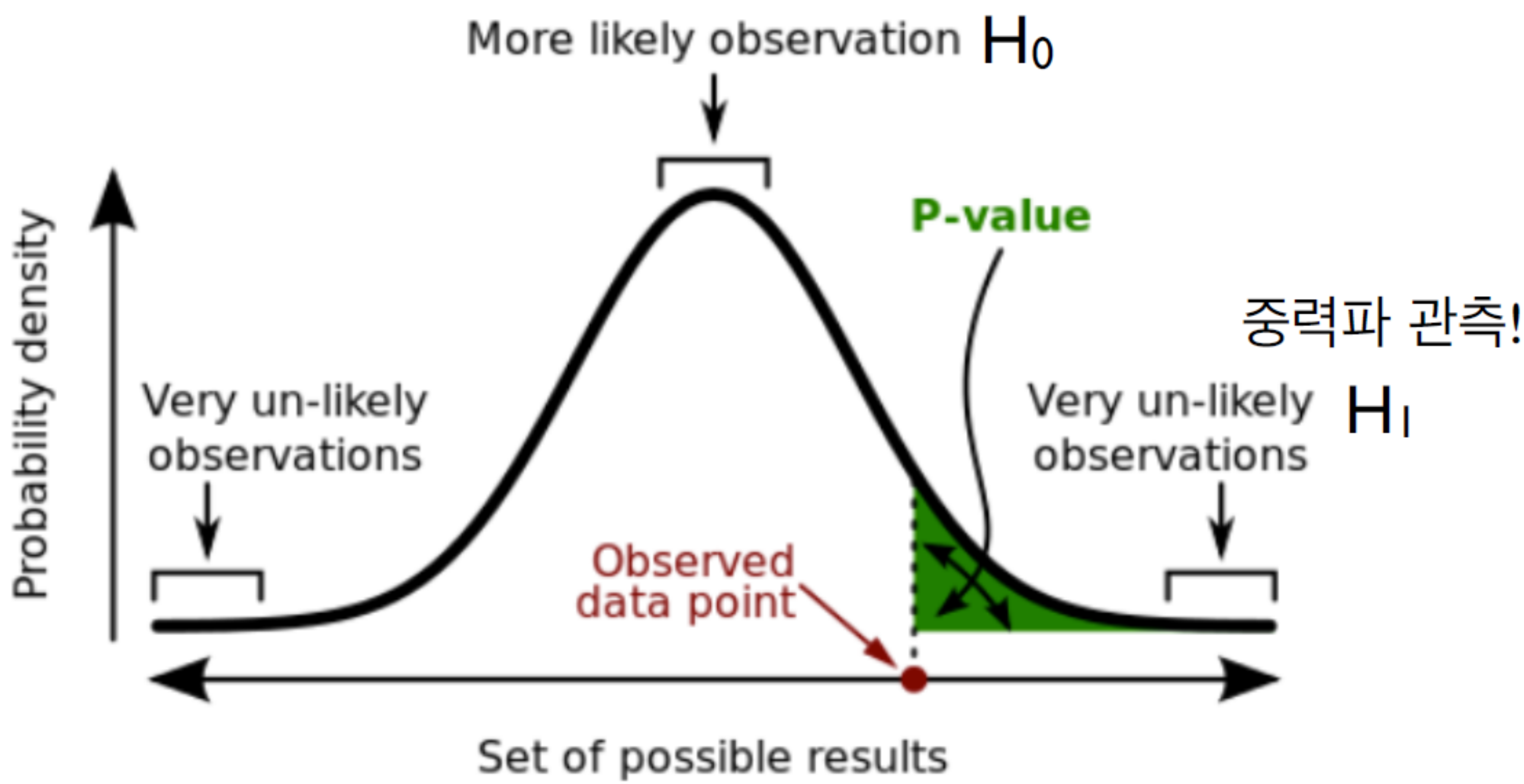
가설(hypothesis) : 관찰을 통해 증명/부정할 사실

H_0 null hypothesis
(예: 중력파 신호가 데이터에 존재하지 않음)

H_1 alternative hypothesis
(예: 중력파 신호가 데이터에 존재)

p-value : 검출된 중력파 신호가
"중력파 신호가 데이터에 존재하지 않음"이라는 가설에서
얼마나 벗어나 있는가를 나타내준다.

관측 자료 대부분은 사실 "잡음"이다



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

(wikipedia)

Bayesian inference in a nutshell

different data shows different **likelihood**
Bayesian inference deals with a **probability density function (PDF)**

observation (experiment, or simulation)
➔ obtain the data "time series"
➔ a data follows a likelihood (distribution or PDF)

GOAL of the **Bayesian inference** is to calculate
a joint probability of likelihood x prior
this is the posterior PDF, *given a hypothesis*

example of a hypothesis:

"there is a GW signal embedded in noise"

Parameter estimation in a nutshell

In the context of GW waveform modeling,
we need **more than 10 physical parameters** in order to determine the **GW waveform h(t)**

joint posterior PDF

P(m1, m2, s1, s2, d, sky position, orbital shape and angles, polarizations, etc)

➔ compute a single posterior for each parameters
by *marginalizing* the joint posterior PDF

$$P(X) = \int_y P(X, Y = y) \mathrm{d}y$$

examples: P(chirp mass), P(distance), P(eccentricity)

$$\mathcal{M}_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$$

➔ Then we can discuss the general shape, width of the PDF

example of ideal PE results

all physical parameters are random and independent
detector noise follows Gaussian distribution
➔ single marginalized posteriors would look like this

In reality, there is strong correlations (degeneracy) between parameters in the waveform eq.

example: mass vs distance,
mass ratio vs spin,
orientation vs distance

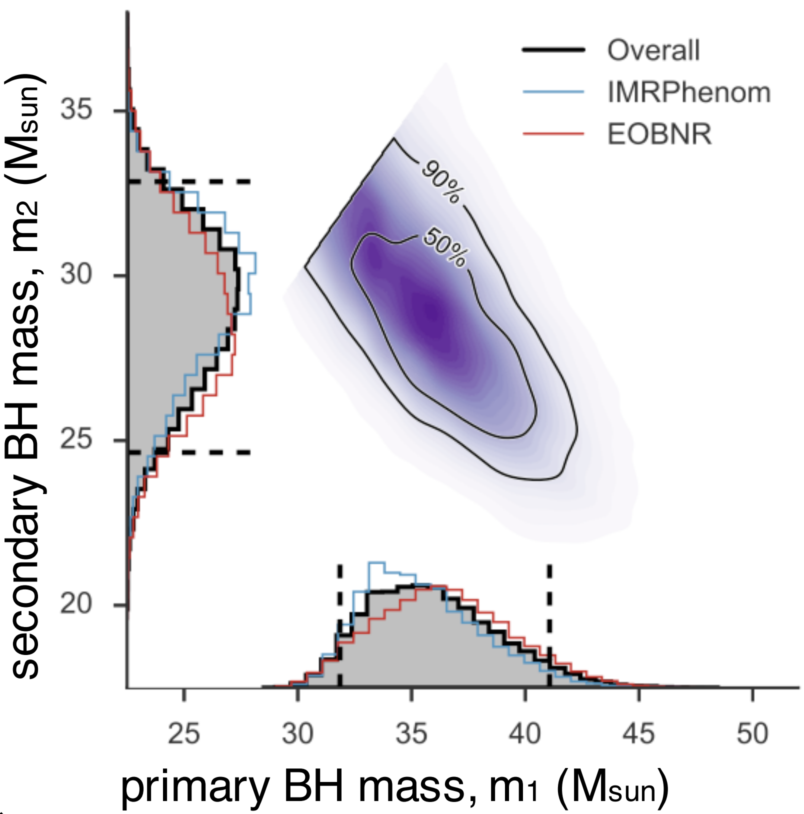
PE requires

- **understanding about the detector characteristics**
- **GW waveform**
- **physics/astrophysics/astronomy/cosmology/gravitation...**

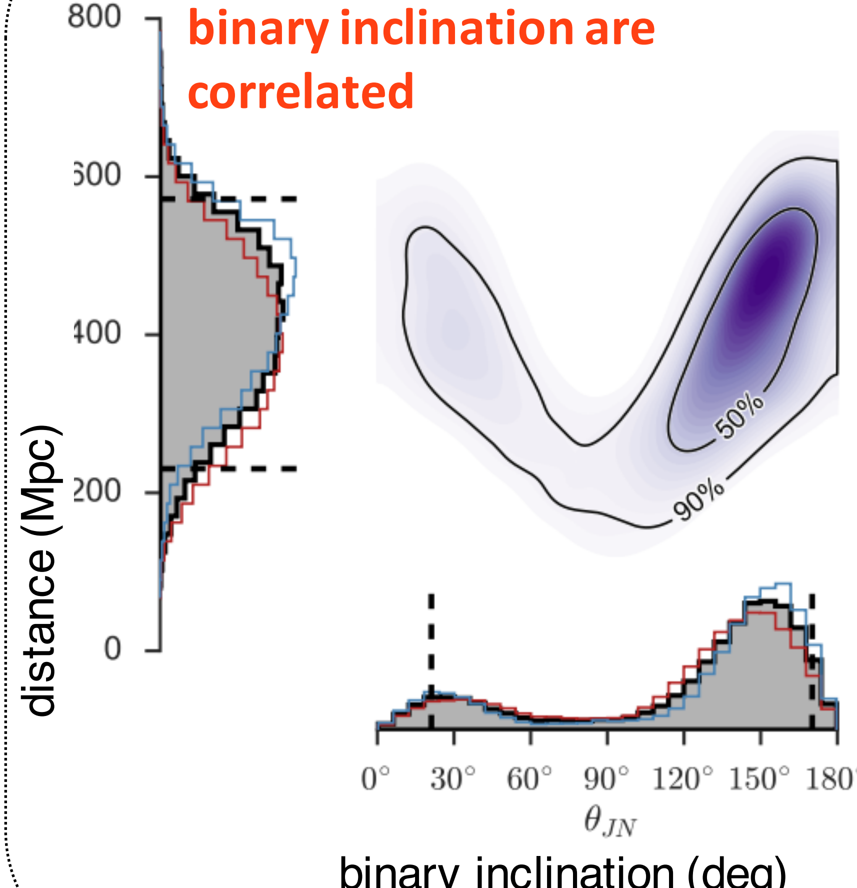
example of parameter estimation: GW150914

two GW waveform models for BH-BH binaries (IMRPhenom and EOBNR)

individual masses are measured



distance and binary inclination are correlated



mass and spin of the final, single BH remnant formed after the merger of two BHs (GW150914)

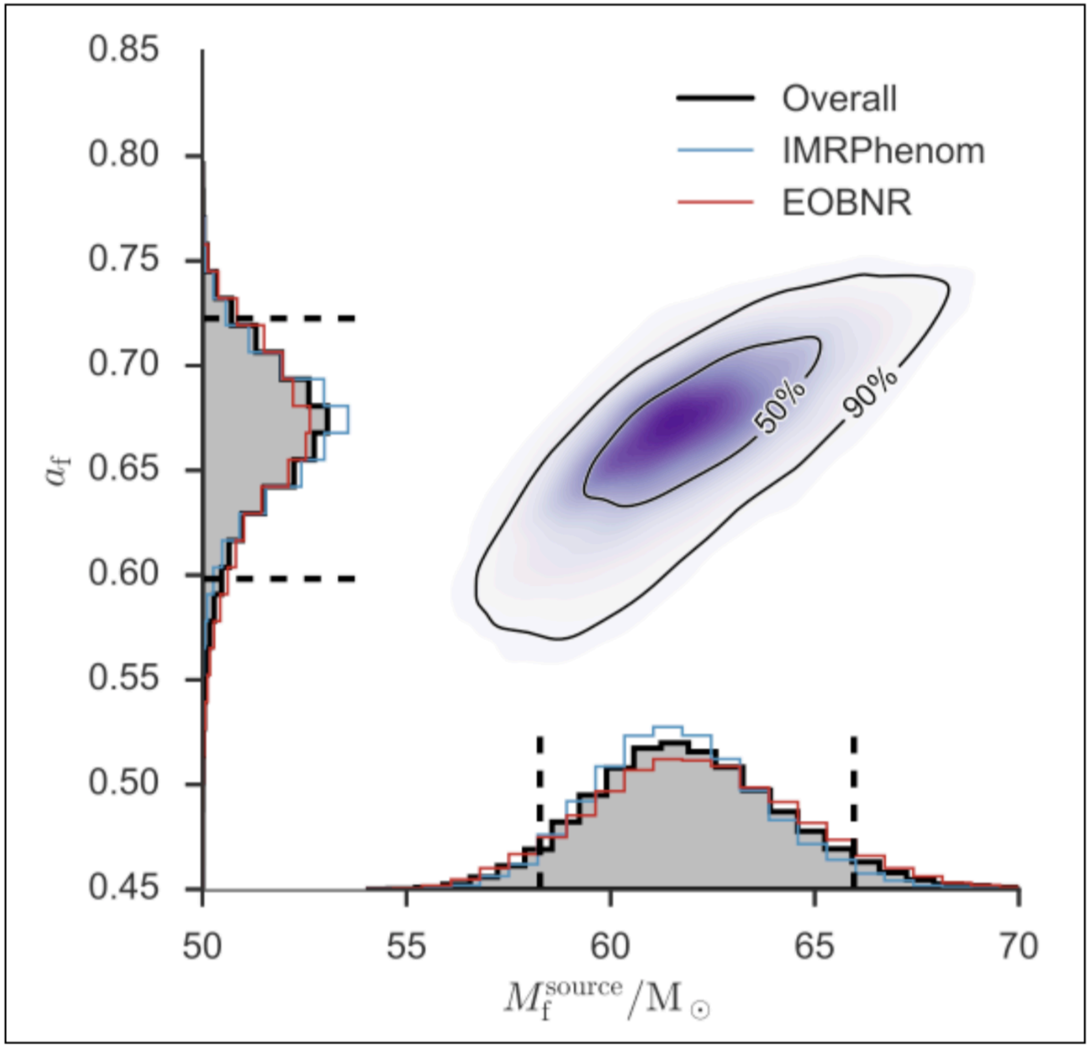
GW energy ~ three Suns ($E=mc^2$)

$m_1 + m_2 = m_{final}$ + GW energy

First concrete measurement of a BH spin ~ 0.65

dimensionless spin parameter
(0 =< a =< 1)

$$a = \frac{cJ}{GM^2}$$

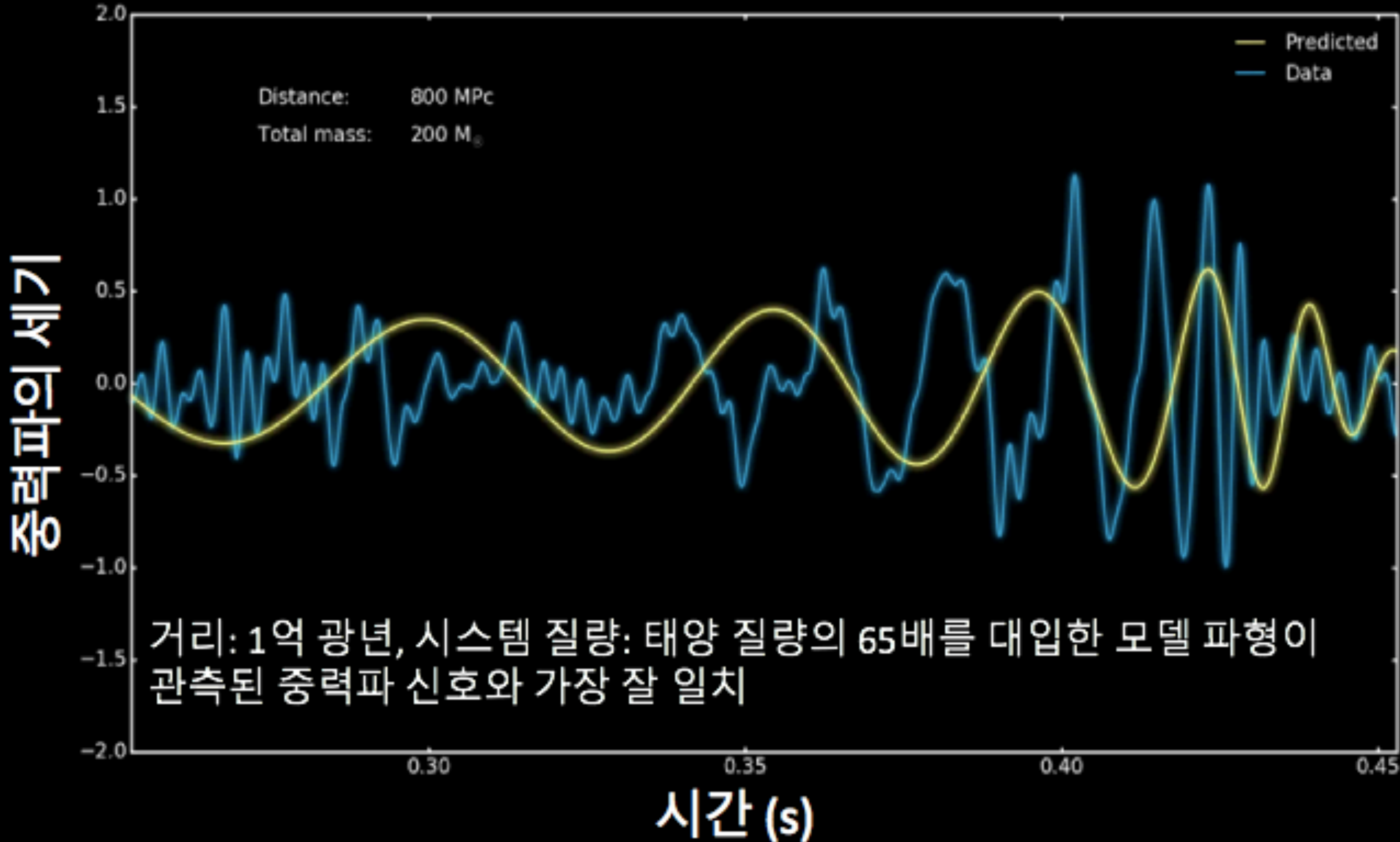


중력파로 거리 측정하기 ➔ Bayesian parameter estimation

베이지언 추론 기반 모수 추정

중력파는 "wave" = 진폭 * 위상

GW150914 : 35 M_{sun} - 30 M_{sun} 블랙홀 쌍성 병합에서 방출된 중력파

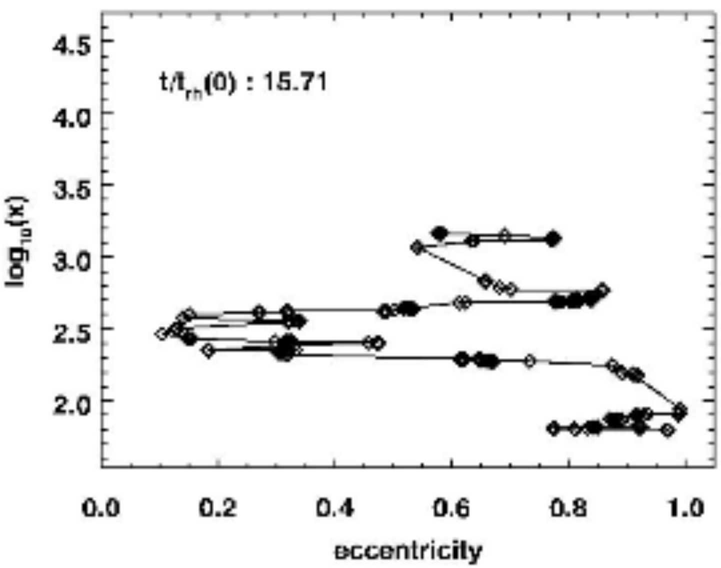


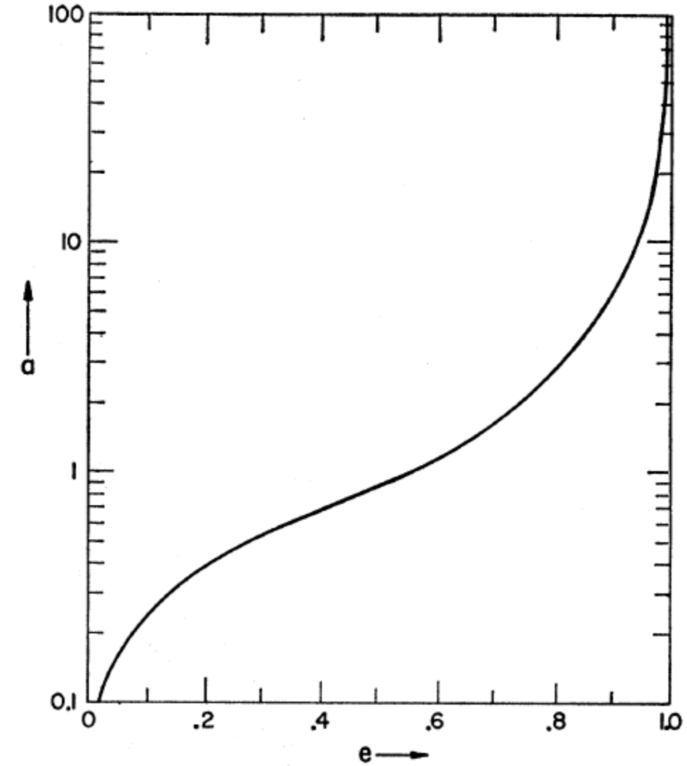
measuring eccentricity is challenging but interesting!

Peters (1964)

$e \ll 10^{-6}$ of known NS-NS binaries @ fgw=20 Hz

stellar interactions can produce eccentric binaries in clusters

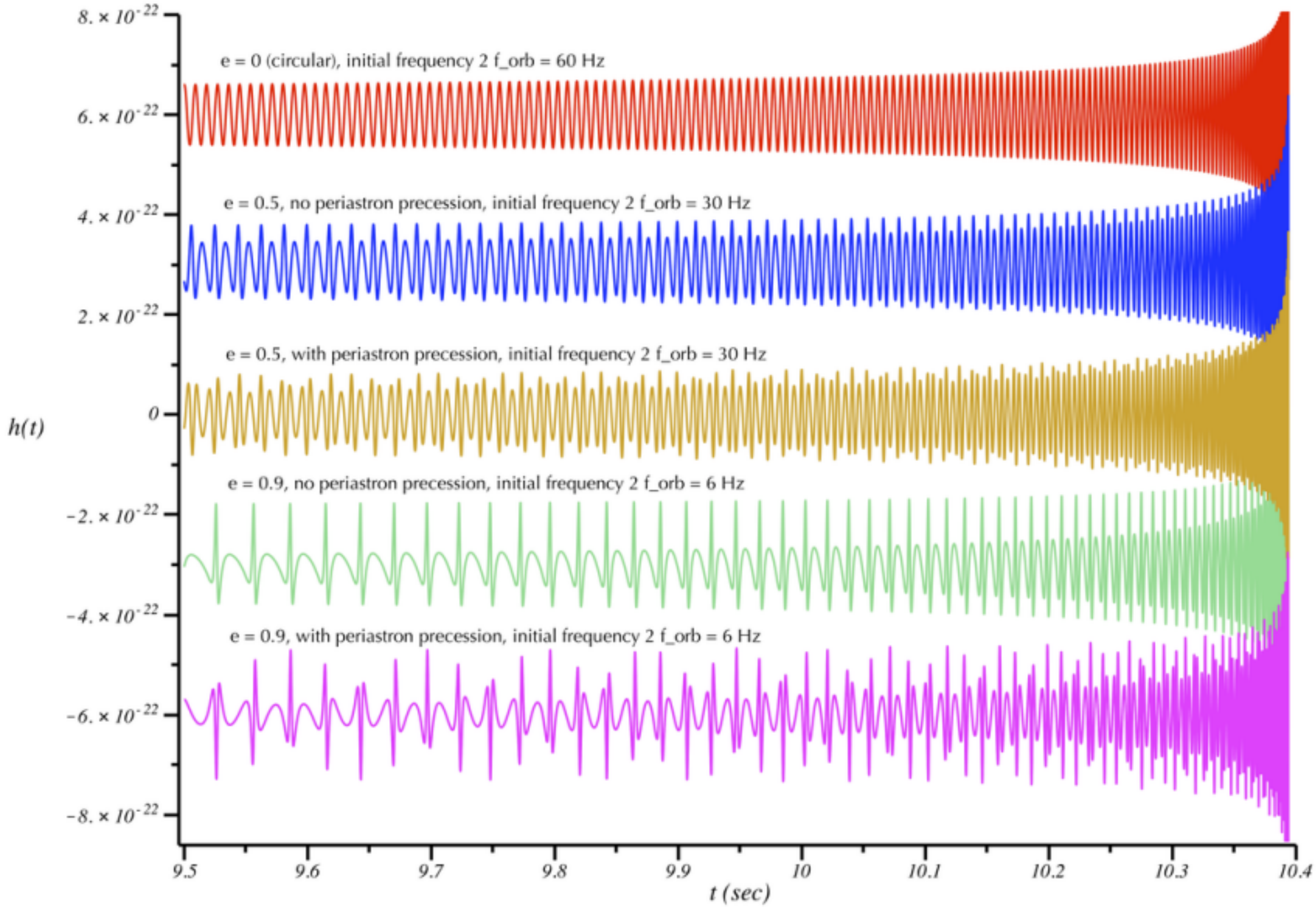




$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304}{15} e \frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^4 (1-e^2)^{5/2}} \left(1 + \frac{121}{304} e^2\right)$$

$$\left\langle \frac{da}{de} \right\rangle = \frac{12}{19} \frac{a}{e} \frac{[1+(73/24)e^2+(37/96)e^4]}{(1-e^2)[1+(121/304)e^2]}$$

eccentricity and GW waveform “inspirals”



circular chirp

e=0.5

e=0.9

Population study meets GW data analysis **example**

dynamical binaries in clusters

- possibility to measure physical parameter(s)
- measurement accuracy

➔ possibly more "eccentric" This would be more important in O4+

Constraining the orbital eccentricity of inspiralling compact binary systems with Advanced LIGO

Marc Favata, Chunglee Kim, K. G. Arun, JeongCho Kim, and Hyung Won Lee

Phys. Rev. D **105**, 023003 – Published 3 January 2022 https://arxiv.org/pdf/2108.05861.pdf

A. Gravitational waveform model

In terms of the GW polarizations $h_{+,\times}$ and the corresponding antenna pattern functions $F_{+,\times}$, the GW signal readout from the detector is

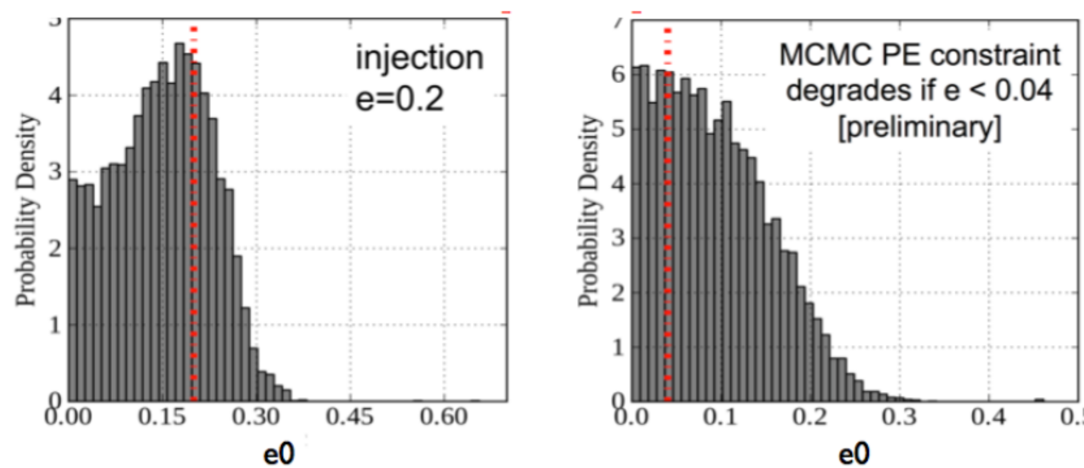
$$h(t) = F_+ h_+(t) + F_\times h_\times(t) = A(t)\cos[2\phi(t) - 2\beta - 2\Phi_0], \tag{2.1a}$$

where

$$A(t) = -\frac{2\eta M}{D}[v(t)]^2\left[(1+C^2)^2F_+^2 + 4C^2F_\times^2\right]^{1/2} \tag{2.1b}$$

and $$\Phi_0 = \frac{1}{2}\arctan\left[\frac{2F_\times C}{F_+(1+C^2)}\right]. \tag{2.1c}$$

Here $C \equiv \cos\iota$, with ι the binary inclination angle (the angle of the Newtonian orbital angular momentum direction relative to the line from source to detector), β spec-



aLIGO can constrain eccentricity of a CBC inspiral if e0 > 0.04 (if low cur-off f > 25 Hz) http://webzine.kps.or.kr/contents/data/webzine/webzine/14762086606.pdf

물리학과 첨단기술

중력파

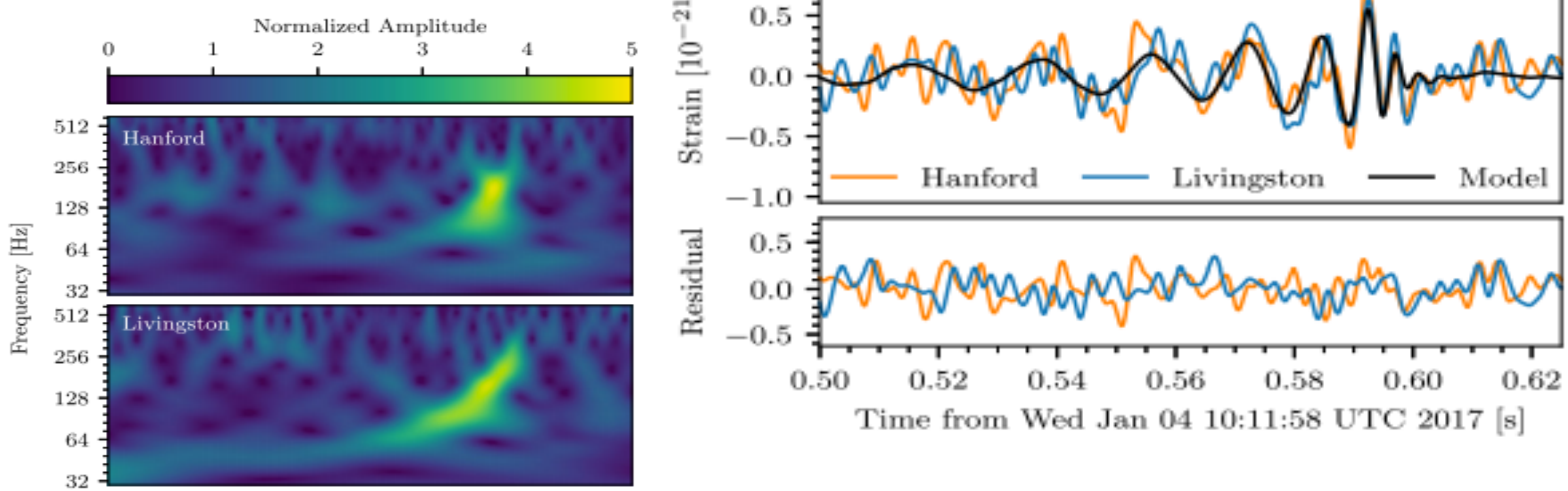
특집
特輯

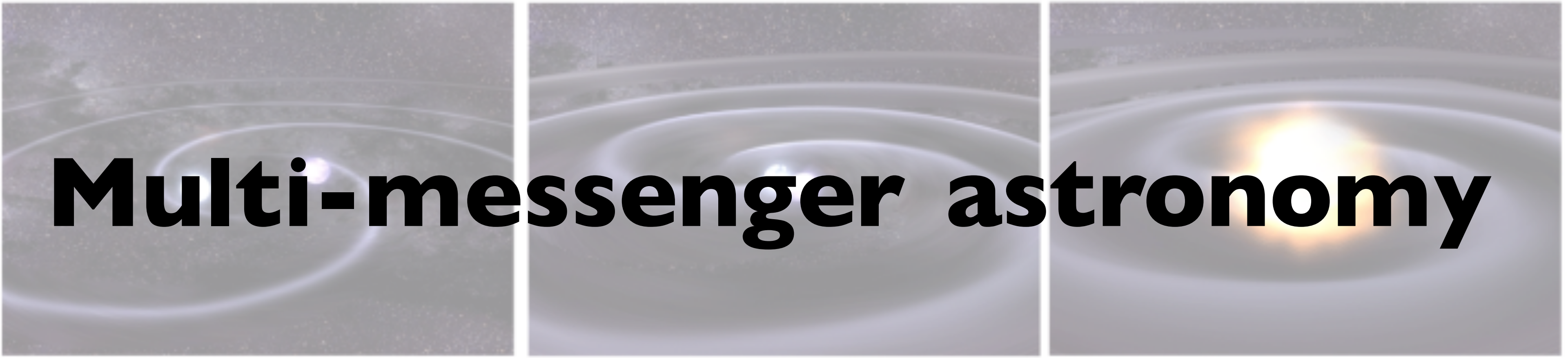
중력파 검출 100년을 기다린 시공간의 속삭임

중력파 천문학 및 천체물리학 시대를 맞이하여[1)]

DOI: 10.3938/PhiT.25.011

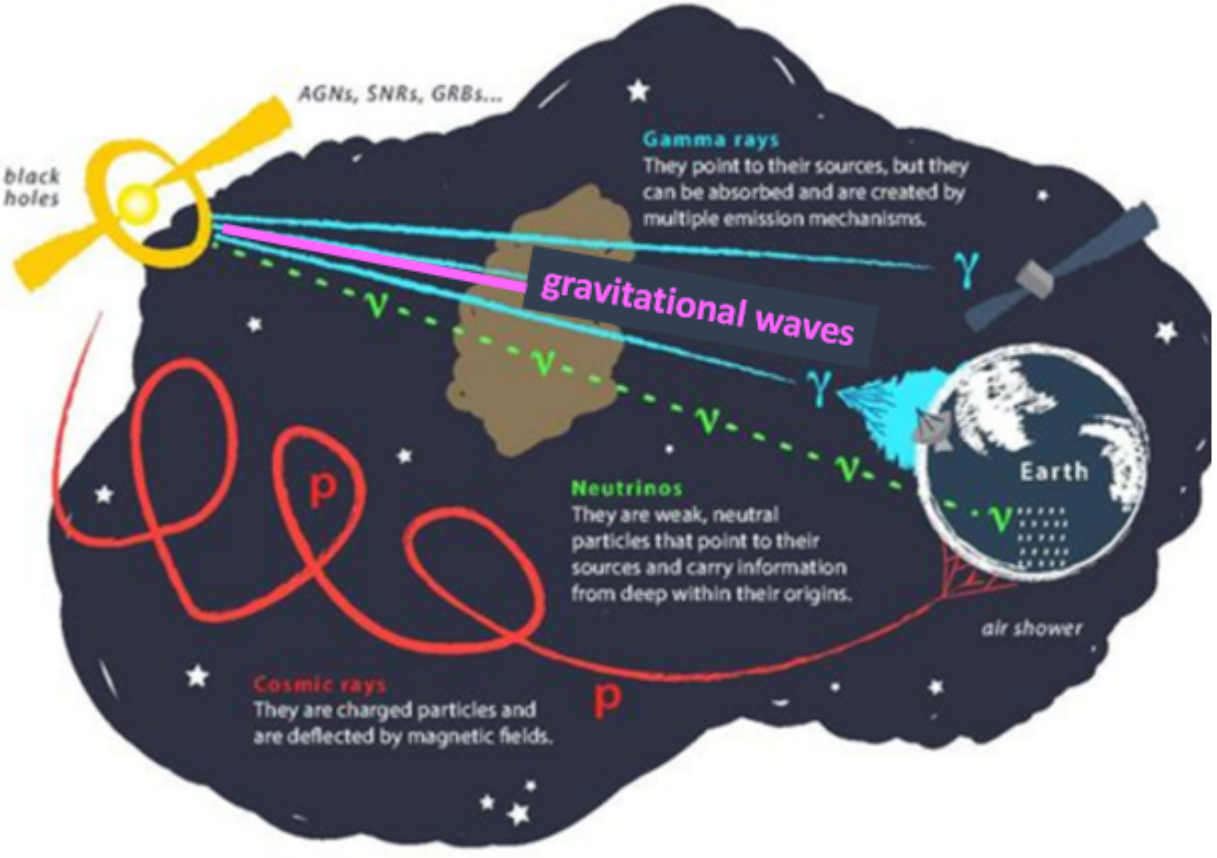
김정리·조희석·이형원·이창환·이현규·강궁원





Multi-messenger astronomy

light + particles (neutrinos, cosmic rays) + gravitational waves



CBC 중력파 관측 자료 처리와 중력파 천문학

CBC 중력파 탐색 [가급적 실시간 + 오프라인도 탐색]

중력파원 질량 및 스핀을 고려한 템플릿 은행 구축(m1,m2,a)
각종 잡음 보정/제거(detector characterization)
관측 자료에 기본 중력파형 모델 대조(matched-filtering) [질량,스핀만 고려]

CBC 모수추정 [오프라인 우선 + 실시간 R&D]

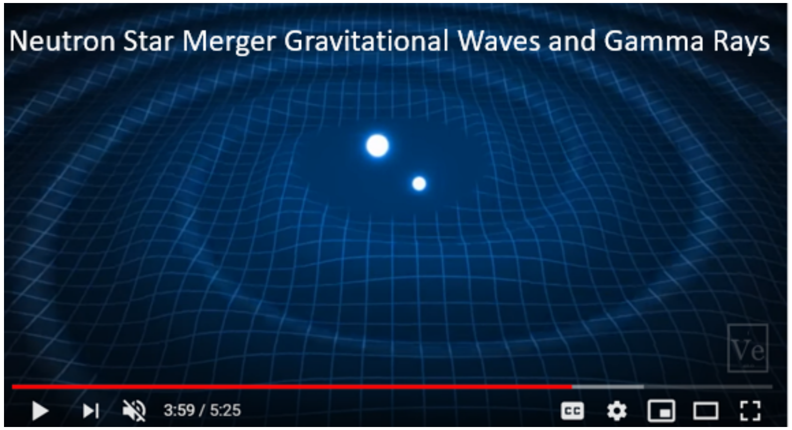
검출된 신호에 가장 적합한 중력파형 모델을 사용(*모든* 파라미터 고려)
기기 감도 곡선 세밀 보정
중력파원의 물리량을 확률밀도함수로 계산

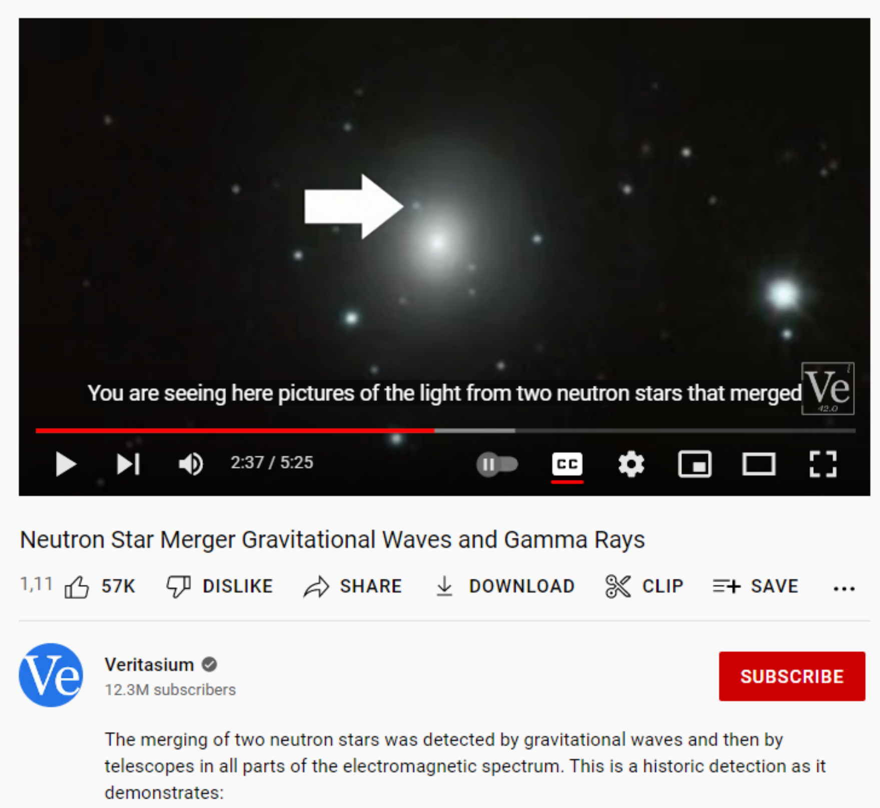
CBC 다중신호 천문학(multi-messenger astronomy) [가급적 실시간]

중력파원의 천구상의 위치 지도 작성
중력파원 질량,거리,검출시간(GPS time)을 천문 커뮤니티와 공유

다중신호 천문관측의 성공 예

- GW170817 (중력파) 외부은하에서 일어난 중성자별-중성자별 병합 최초 관측 NS-NS 병합 ➔ 감마선폭발 ➔ 킬로노바 발생 (이론 예측)이 관측으로 확인





original URL: https://www.youtube.com/watch?v=EAyk2OsKvtU

다중신호 천문관측의 성공 예

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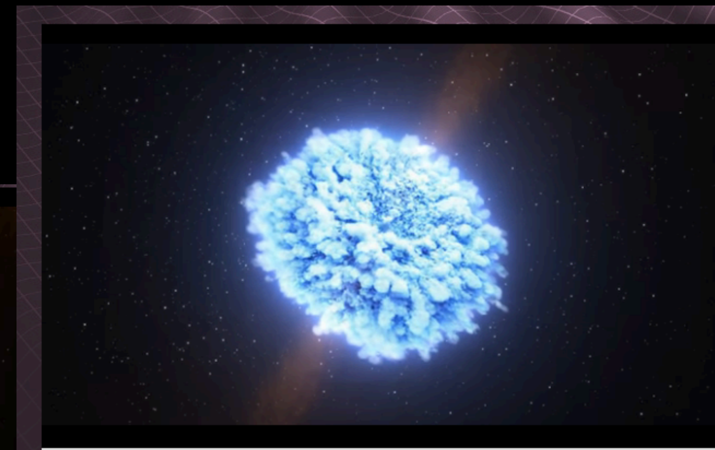
https://kmtnet.kasi.re.kr/kmtnet-eng/

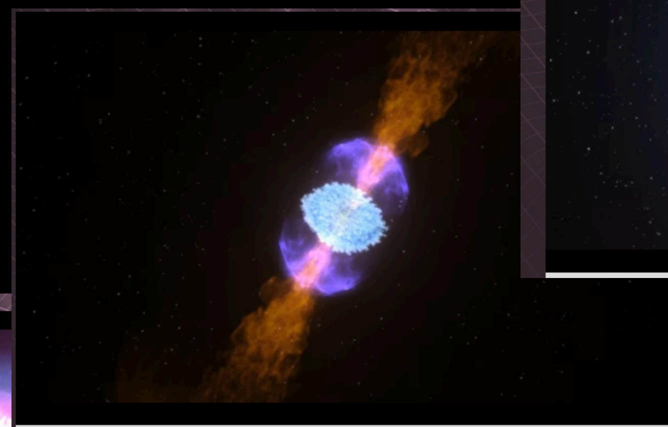
KMTNet 24시간이내 가시광선 관측

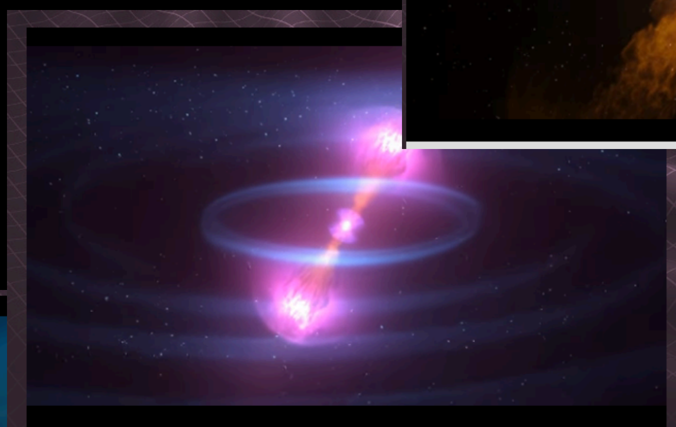
Best targets for MMA:

mergers of two neutron stars (ex: GW170817)

➔ gravitational waves, neutrinos, and light







중력파 관측과 허블상수

$$v = H_0 d$$ (*v* : 후퇴속도, *d* : 거리, H_0 : 허블 상수)

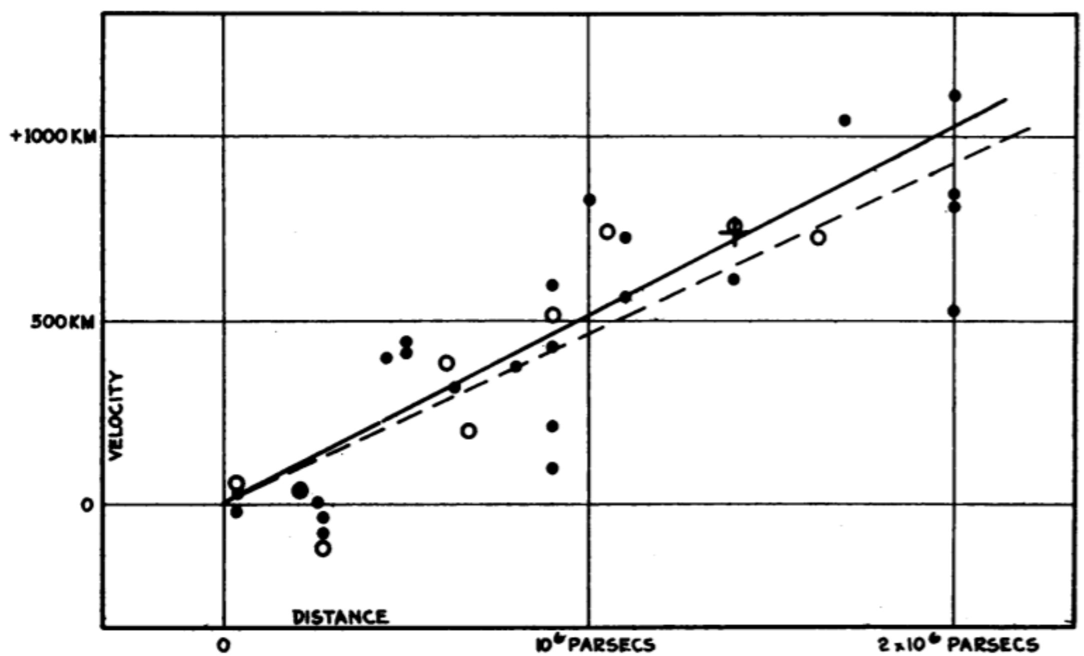


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

(**Hubble,** 1929)

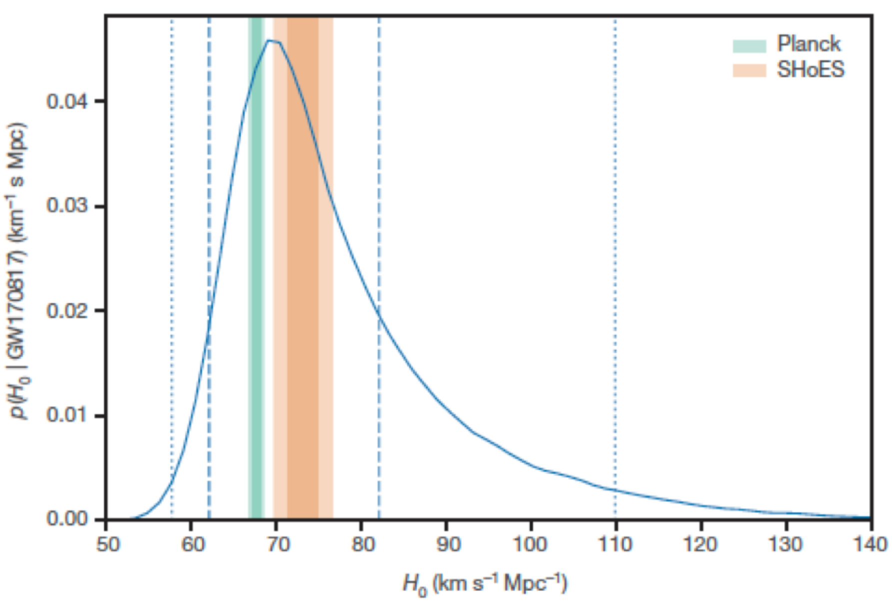
중력파 관측과 허블상수

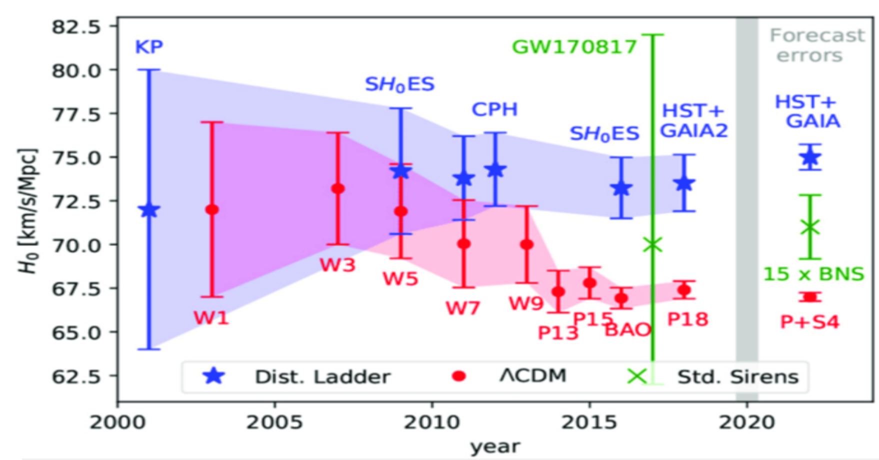
$$v = H_0 d \qquad (v: \text{후퇴속도}, d: \text{거리}, H_0: \text{허블 상수})$$

- **근거리 관측을 통한 허블 상수 측정 (기존 방법): ' 표준 광원 ' 을 활용**

- **원거리 관측을 통한 허블 상수 측정: 우주배경복사 관측**

최신 정밀 관측으로 근거리 허블 상수와 원거리 허블 상수 측정값 사이에 오차 범위를 훨씬 벗어나는 차이가 있다는 것이 발견됨 → 허블상수 갈등 "Hubble tension"

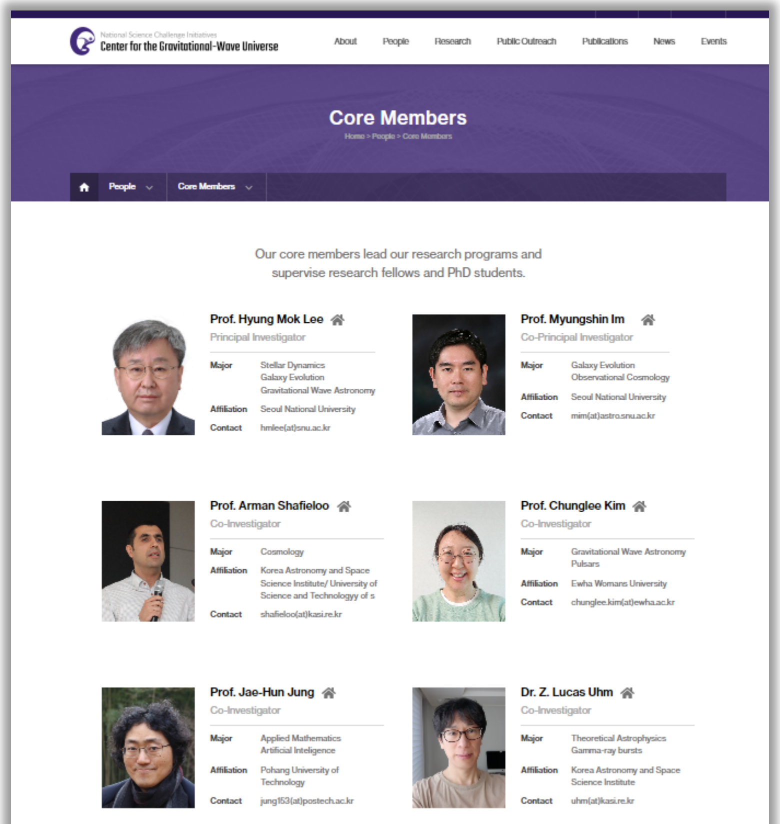


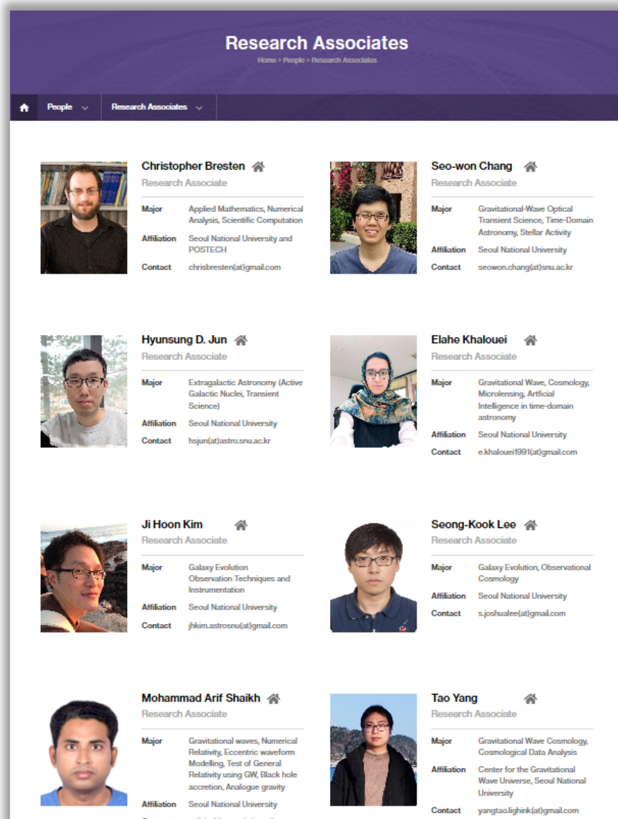


2001년-현재 H_0 의 원거리 측정(붉은색) vs 근거리 측정(보라색). 두 측정간 오차 범위를 훨씬 벗어나는 차이가 보임.
(Freedman 2017, Nature Astronomy)

https://www.youtube.com/watch?v=2ShB8oDQQHY

중력파 우주 연구단
Center for GW Universe





[요약] 중력파원으로서의 밀집쌍성

- compact binary coalescences (CBCs 밀집쌍성병합)
  merging binaries consisting of WD/NS/BH

  3 phases of a CBC = inspiral + merger + ringdown

Types of known CBCs in GW transient catalogues (GWTCs) so far

2 NS-NS "extragalactic population" "EM is not required"

86 BH-BH (binary black holes = BBHs) "stellar-mass" black holes

2 NS-BH (GWTC-3) GW200105, GW200115

[요약] 중력파 천체물리학과 별질량 블랙홀

stellar-mass BBHs (10-2000 Hz) with laser interferometers

- stellar astrophysics/population study based on PE
- precision GW astronomy ➔ GW lensing?
  ➔ Hubble constant "standard siren"

SMBHB mergers (nHz) with radio pulsars

- continuous signals (if resolvable) or GW background
- outcome of a galaxy-galaxy merger?
- stellar dynamics + gas accretion ➔ GW signals of SMBHBs can be more complicated even during the inspiral phase than stellar-mass BBHs

GW science goes on !

LIGO Hanford Observatory control room. Credit: Caltech/MIT/LIGO Lab.

LIGO-Virgo-KAGRA Webinar to Discuss New Results on the Gravitational-wave Background

News Release • February 2, 2021

On Thursday 4 February, at 10:00 Eastern US (other time zones below), the LVK will host an online webinar entitled "Constraining astrophysical and cosmological gravitational-wave backgrounds with Advanced LIGO and Virgo's third observing run." We will present results from our recent papers: **arxiv.org/abs/2101.12248** and **arxiv.org/abs/2101.12130**. The webinar is open to all.

Register for the webinar.

LIGO-Virgo-KAGRA Webinar
Thu, 4 Feb at 10:00 Eastern Time (US and Canada)
Thu, 4 Feb at 07:00 (US PST/Los Angeles)
Thu, 4 Feb at 09:00 (US CST/Chicago)
Thu, 4 Feb at 16:00 (CET/Pisa, Italy)
Thu, 4 Feb at 20:30 (IST/Pune, India)
Fri, 5 Feb at 00:00 (JST/Tokyo)
Fri, 5 Feb at 02:00 (AEDT/Sydney)

A recording will be posted after the seminar for those who cannot attend the live event.

Update: View the webinar:

GW detections so far

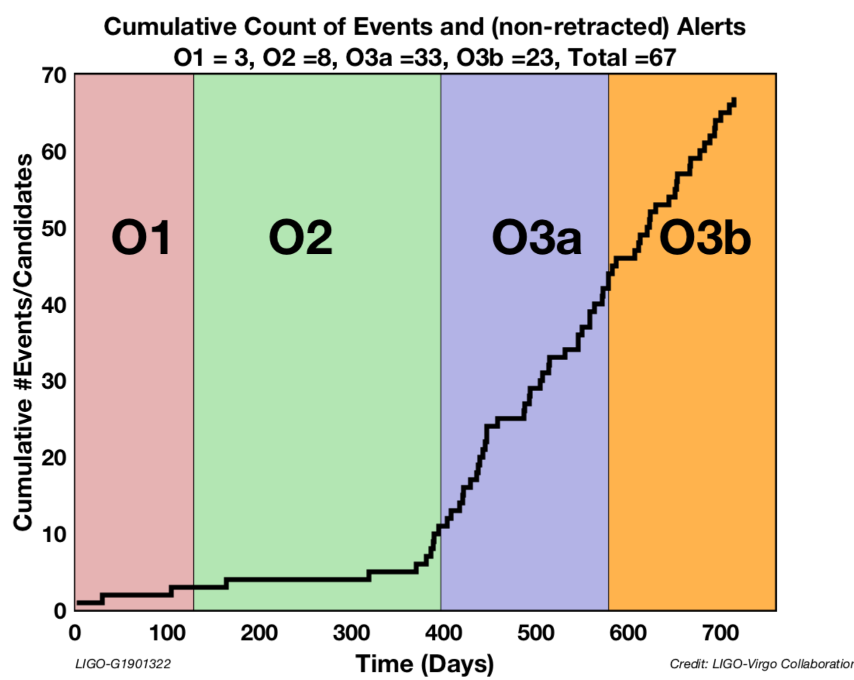
better sensitivities ➔ more detections, many surprises!

O1 : GW150914
O2: GW170817 (NS-NS binary merger)
O3: GW190814 (NS-BH candidate)
GW190521 (Intermediate-mass BH)

GWTC-3 contains 90 confirmed sources

5-times more O1+O2 (total 11 detections)

Image credit: LIGO-Virgo Collaboration.



GW observation will be resumed hopefully soon!

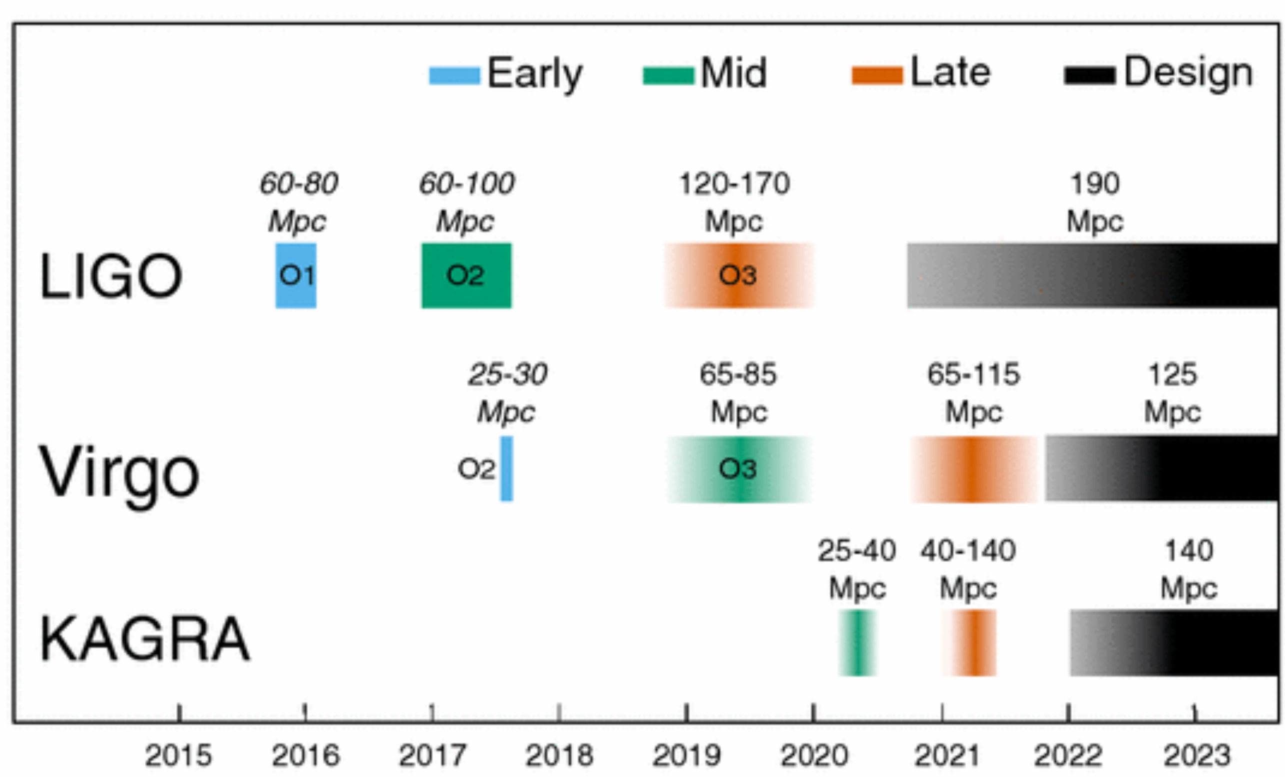
Review Article | Open Access | Published: 26 April 2018

Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA

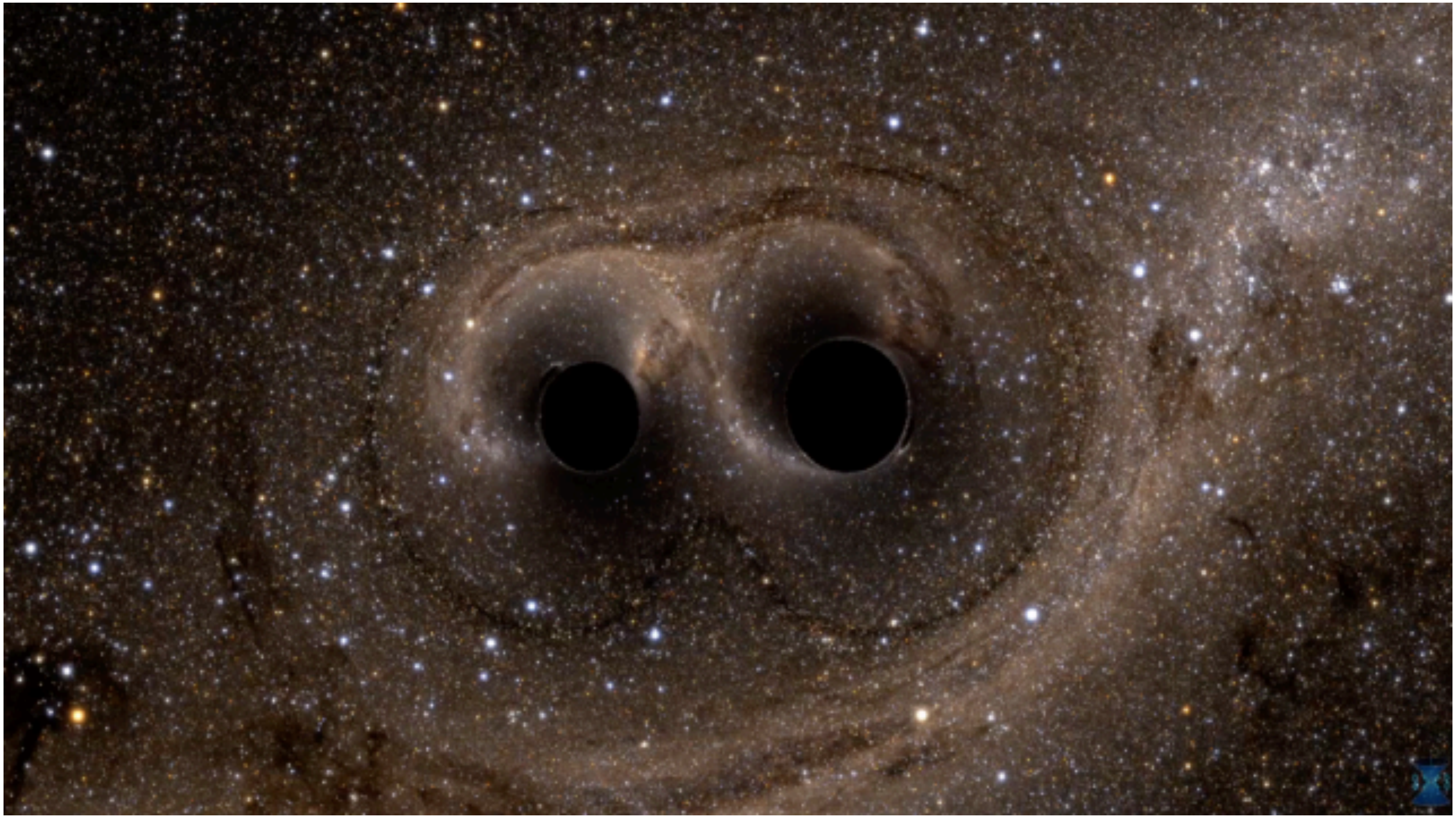
B. P. Abbott, R. Abbott, [...] KAGRA Collaboration, LIGO Scientific Collaboration and Virgo Collaboration

Living Reviews in Relativity **21**, Article number: 3 (2018) | Cite this article

https://link.springer.com/article/10.1007/s41114-018-0012-9



Would more GW observations bring answers or more questions ?



감사합니다

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