Ultralight Dark Matter

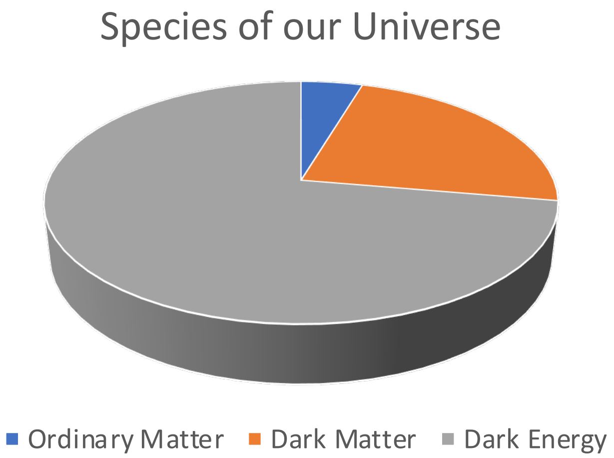
Hyeonmo Koo

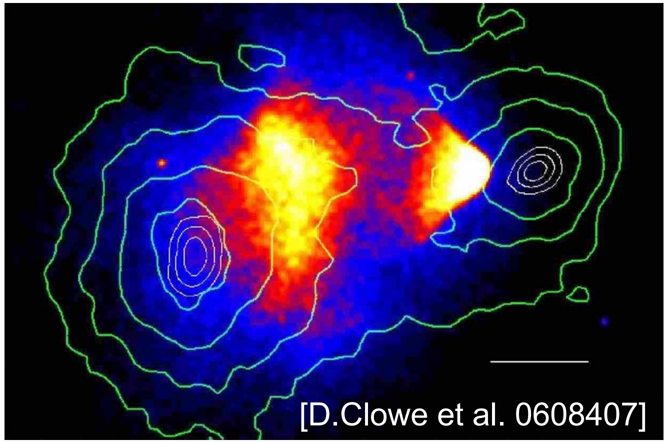
With Jae-Weon Lee (JWU) and Dongsu Bak (UOS)

Contents

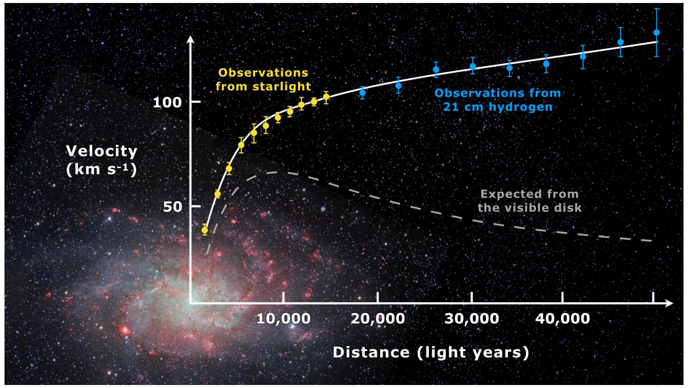
Prologue : Cold Dark Matter (CDM) and its Limitation

Cosmological Evidences for DM

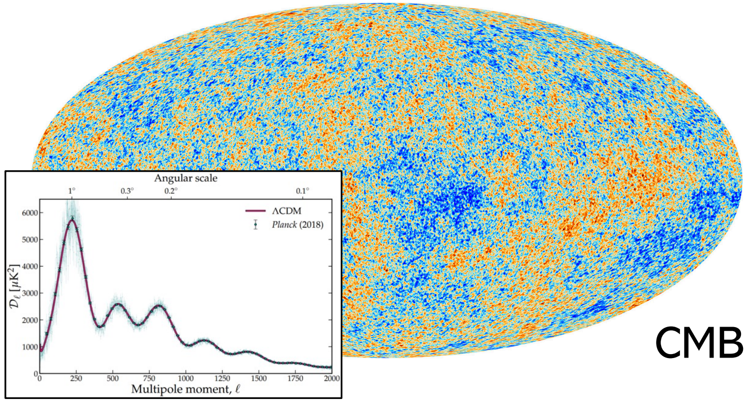



[D.Clowe et al. 0608407]

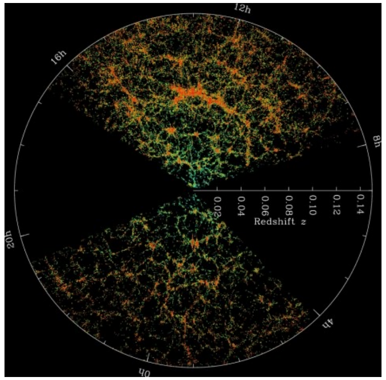
Bullet Cluster Collision



Galaxy Rotation Curve



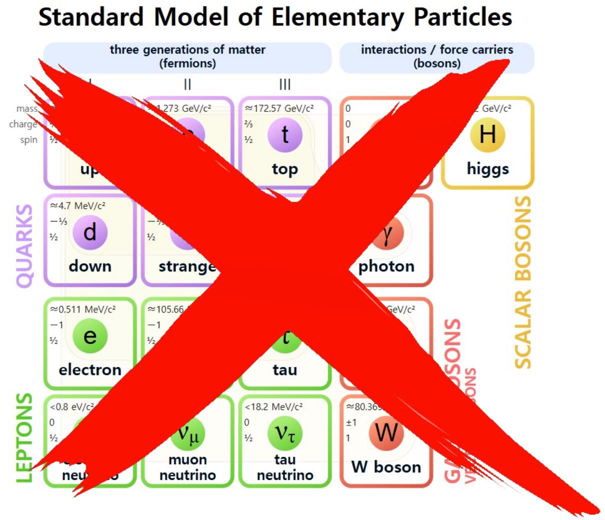
CMB



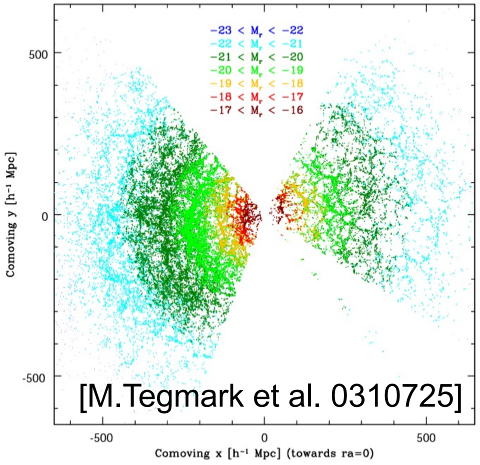
Large-scale Structure

Weakly Interacting Massive Particle (WIMP)

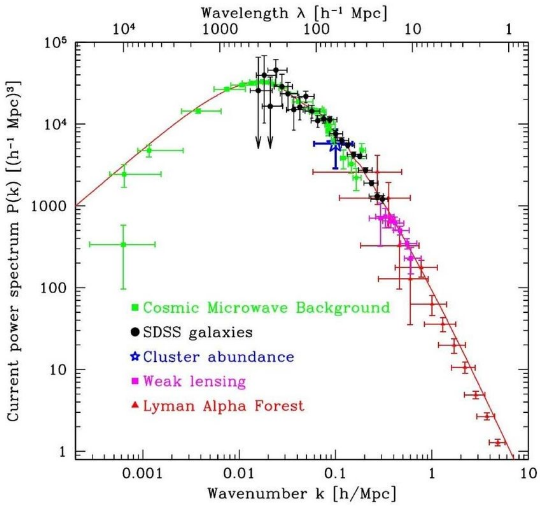
- Proposed by Benjamin W. Lee (1970)
- Only interact via Gravity (& Weak Interaction)
- $m = O(1\mathrm{GeV})\sim O(100\mathrm{TeV})$



"Cold Dark Matter (CDM)"



[M.Tegmark et al. 0310725]



- CDM-based cosmology successfully explain the large-scale structure of our universe.
- How about on the **small-scale**?

Problems of Galactic Scale in CDM Model

[J.S.Bullock et al. 1707.04256]

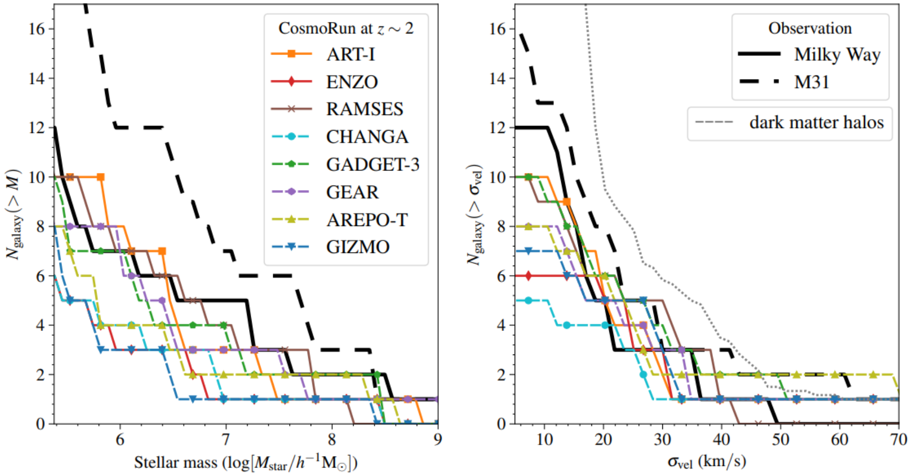
- **Missing Satellite Problem** : Too much # of Satellites
  [A.A.Klypin et al. 9901240 ; B.Moore et al. 9907411]
- **Cusp-Core Problem** : Too sharp of DM halo central density
  [R.A.Flores et al. 9402004 ; B.Moore, Nature 370 (1994)]
- **Galaxy Cluster Collision** : Offsets between DM-Stars
  [A.Mahdavi et al. 0706.3048 ; D.Harvey et al. 1503.07675]
- **Planar Structure of Satellites** [M.S.Pawlowski et al. 1505.07465]
- **Angular Momentum Catastrophe** : Too slow galactic bar speed
  [M.Roshan et al. 2106.10304 ; M.Steinmetz et al. ApJ 513 (1999)]
- **Final Parsec Problem :** SMBH binary does not merge within $\sim H_0^{-1}$
  [Begelman, Blandford, Rees, 1980]
- **…**

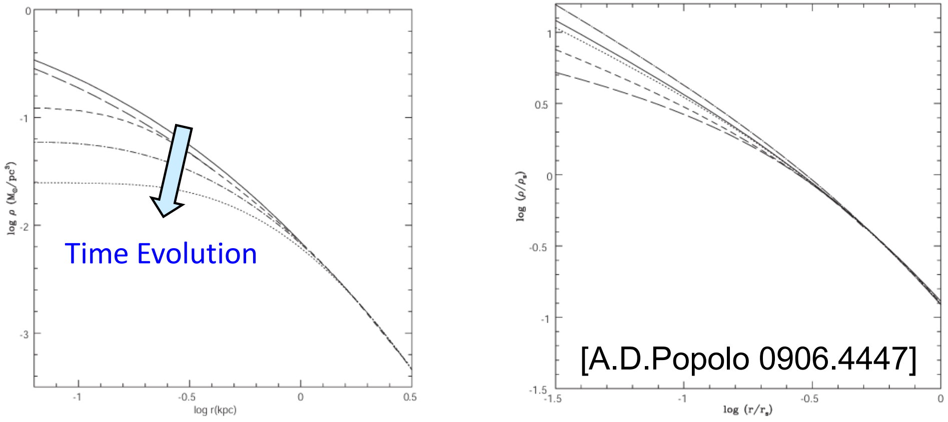
How about Considering Baryonic Matter?

Considering baryon ($p^+, n^0, e^-, \mathrm{He}$ ...) inside galactic/cosmological simulations ⇒ Additional gas dynamics (Atomic/Chemical) is expected!

- Tidally stripping of stellar structure
- Supernova Feedback
- Energy transfer from baryon to DM halo?

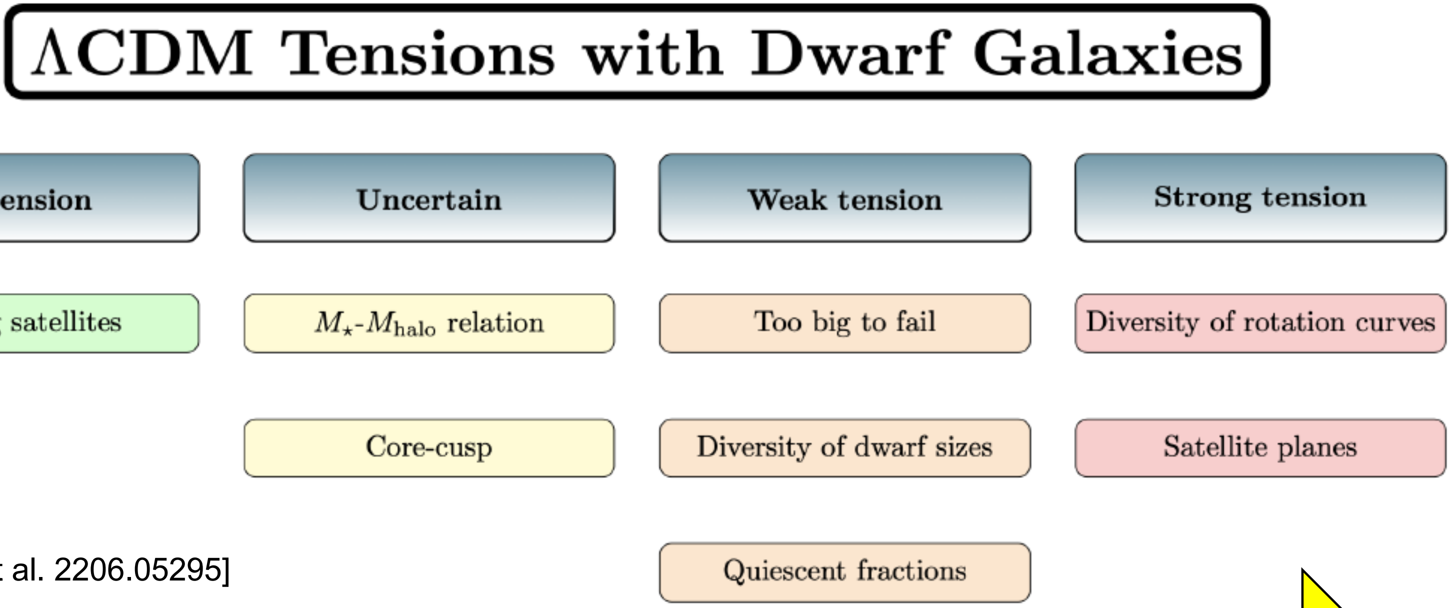
[M.Jung et al. 2402.05392]





Time Evolution

[A.D.Popolo 0906.4447]

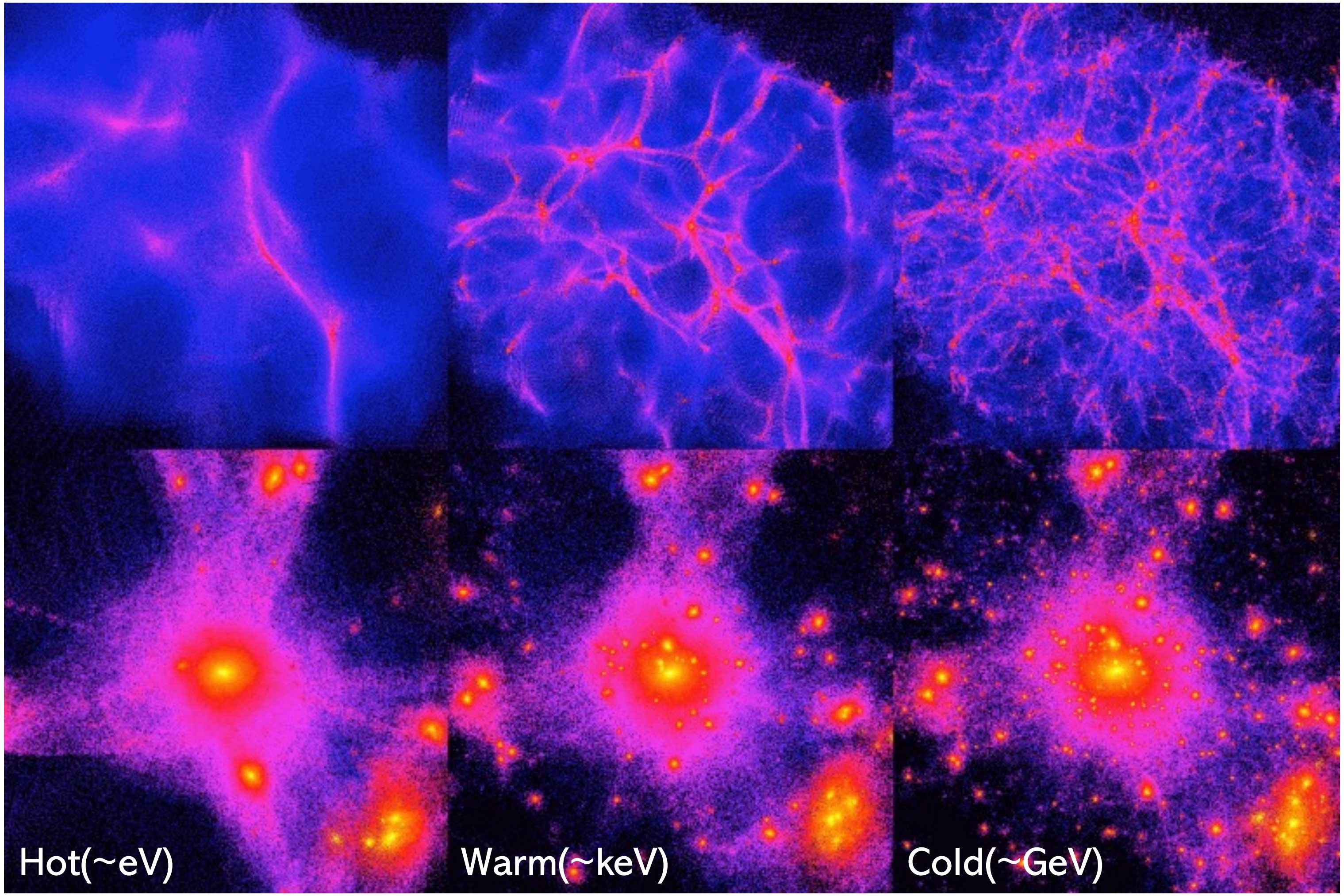


ΛCDM Tensions with Dwarf Galaxies

No tension	Uncertain	Weak tension	Strong tension
Missing satellites	$M_\star$-$M_{\rm halo}$ relation	Too big to fail	Diversity of rotation curves
	Core-cusp	Diversity of dwarf sizes	Satellite planes
		Quiescent fractions	

[L.V.Sales et al. 2206.05295]

Harder to resolve… Alternative DM model required?



Or Else?

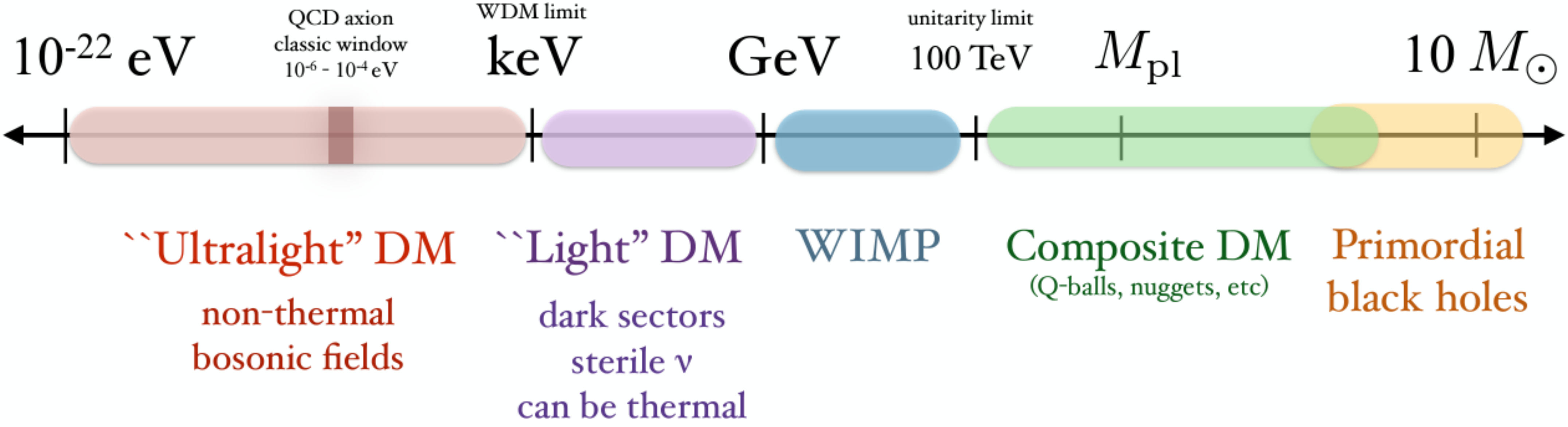
Ultralight Dark Matter (ULDM)

Motivation (as briefly as possible)

Mass scale of dark matter

(not to scale)

[T.Lin 1904.07915]



10^{-22} eV

QCD axion
classic window
10^{-6} - 10^{-4} eV

WDM limit
keV

GeV

unitarity limit
100 TeV

M_{pl}

10 $M_\odot$

``Ultralight" DM
non-thermal
bosonic fields

``Light" DM
dark sectors
sterile ν
can be thermal

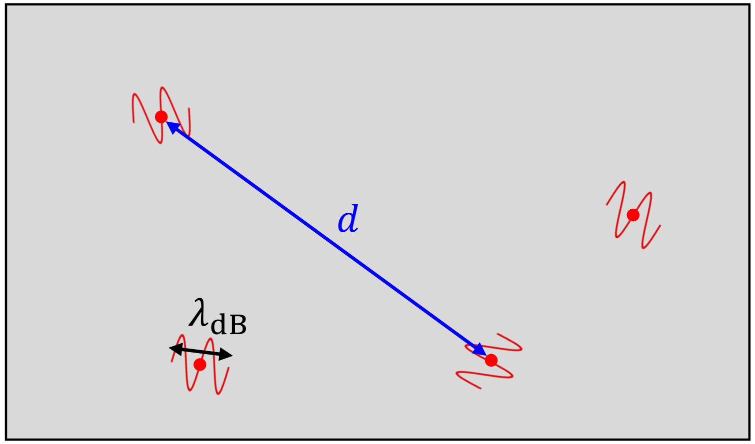
WIMP

Composite DM
(Q-balls, nuggets, etc)

Primordial
black holes

CDM (WIMP)

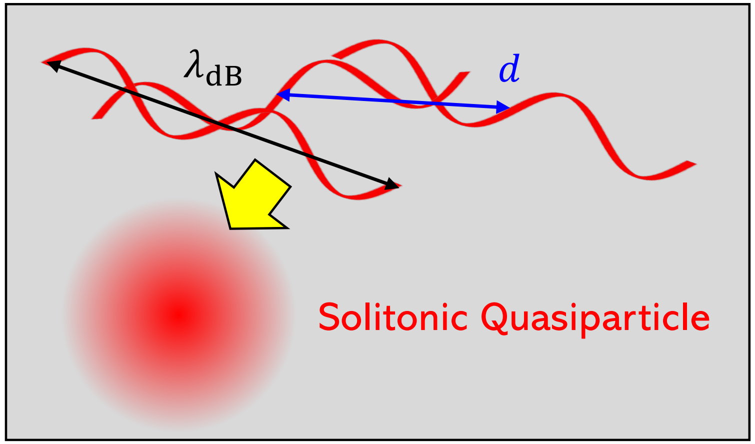
- Heavy : $m \sim O(1\mathrm{GeV})$
- $d \gg \lambda_{\mathrm{dB}} \sim O(10^{-13}\mathrm{m})$ → Low number density
- Particle-Like
- Newton's EoM
- Random Motion

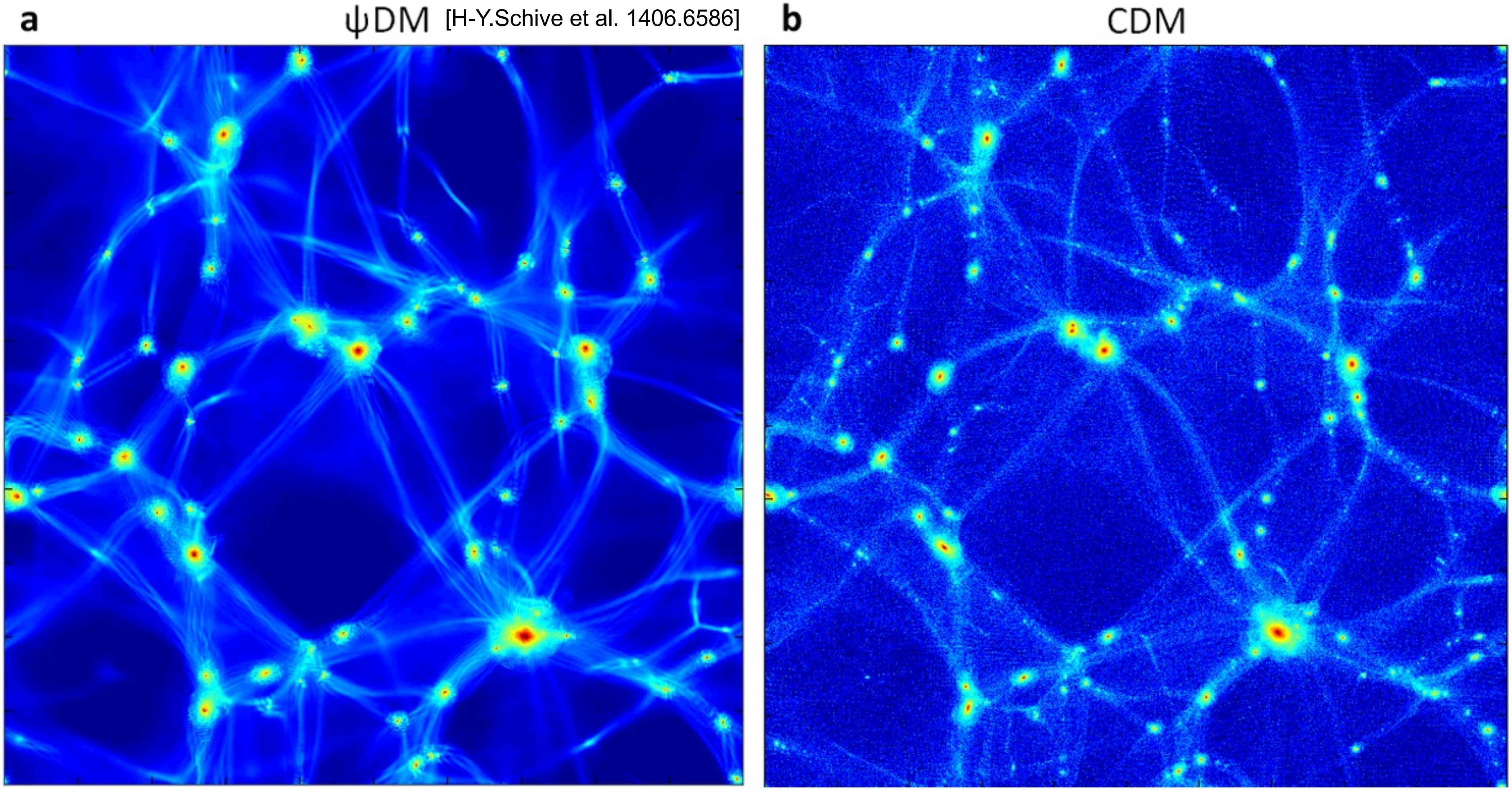


ULDM

- Ultra-Light : $m \sim O(10^{-22}\mathrm{eV})$
- $d \ll \lambda_{\mathrm{dB}} \sim O(1\mathrm{kpc})$ → High number density
- Wave-Like (or Superfluid-Like)
- Schrodinger-like EoM
- Coherent Motion

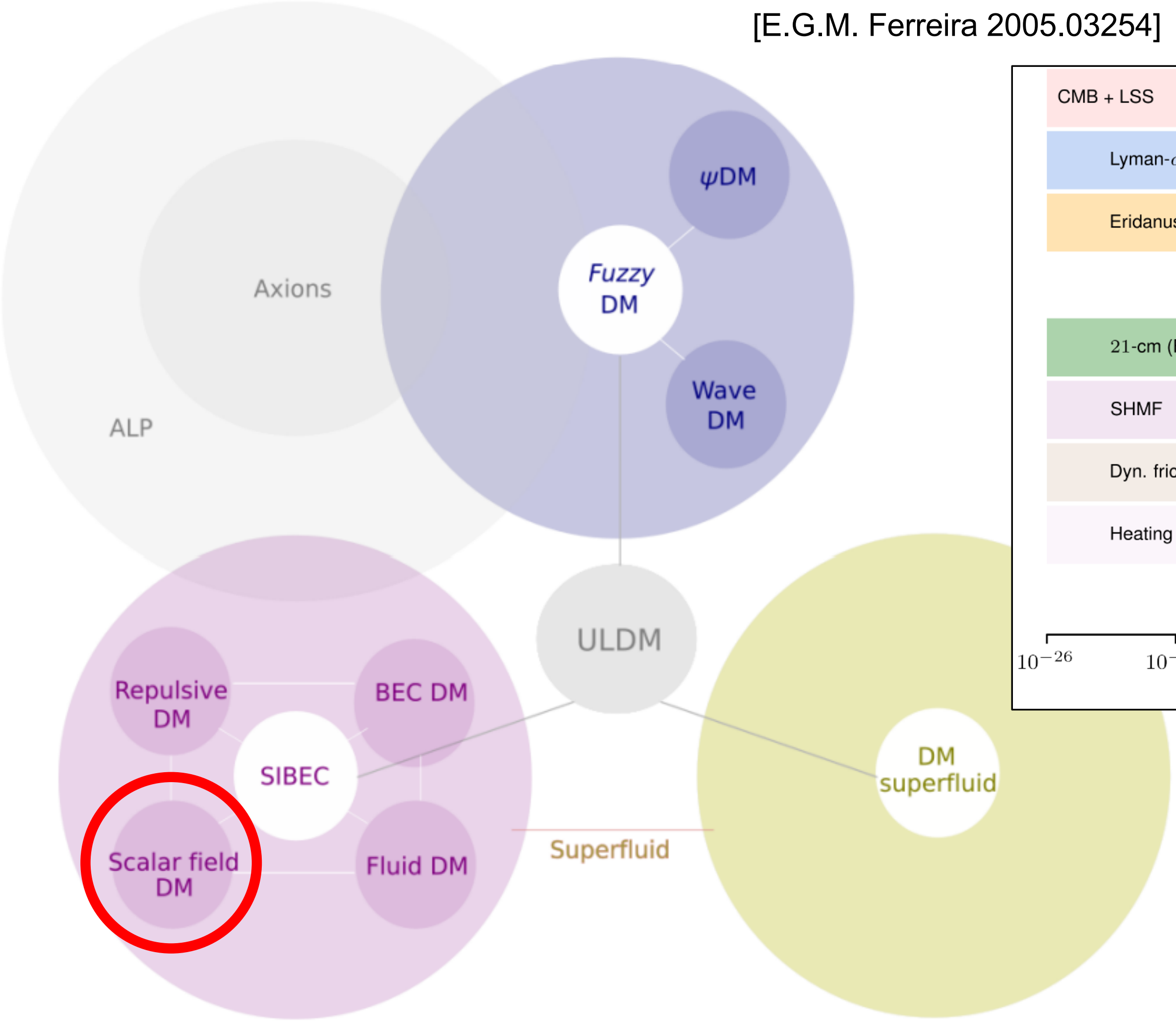
Well summarized in [J-W.Lee 1704.05057]

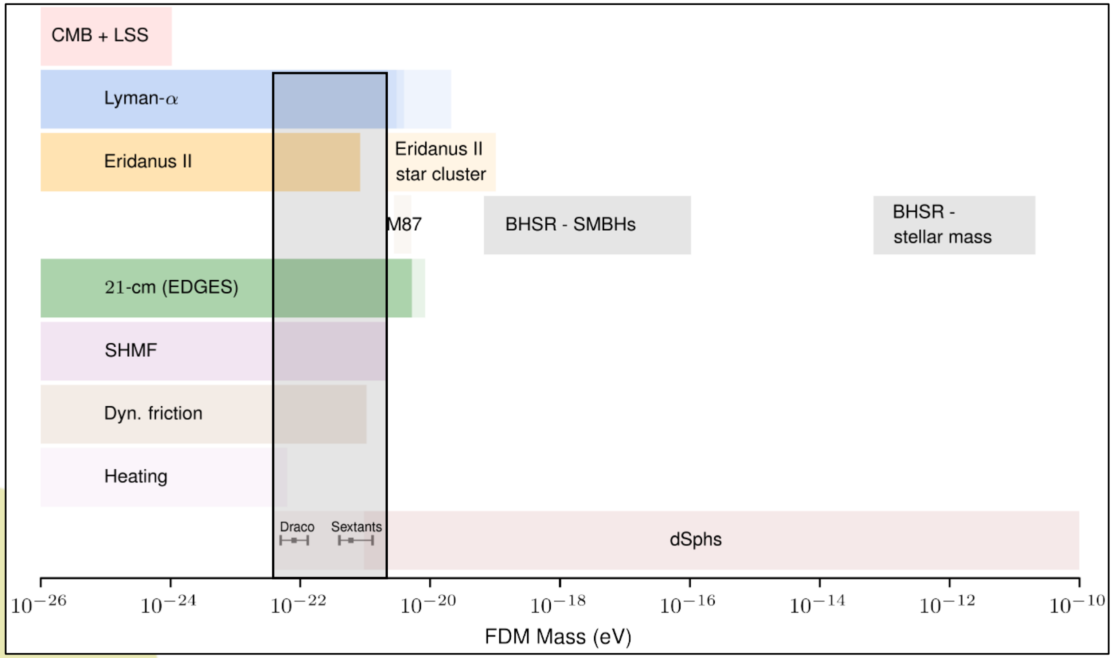




[H-Y.Schive et al. 1406.6586]

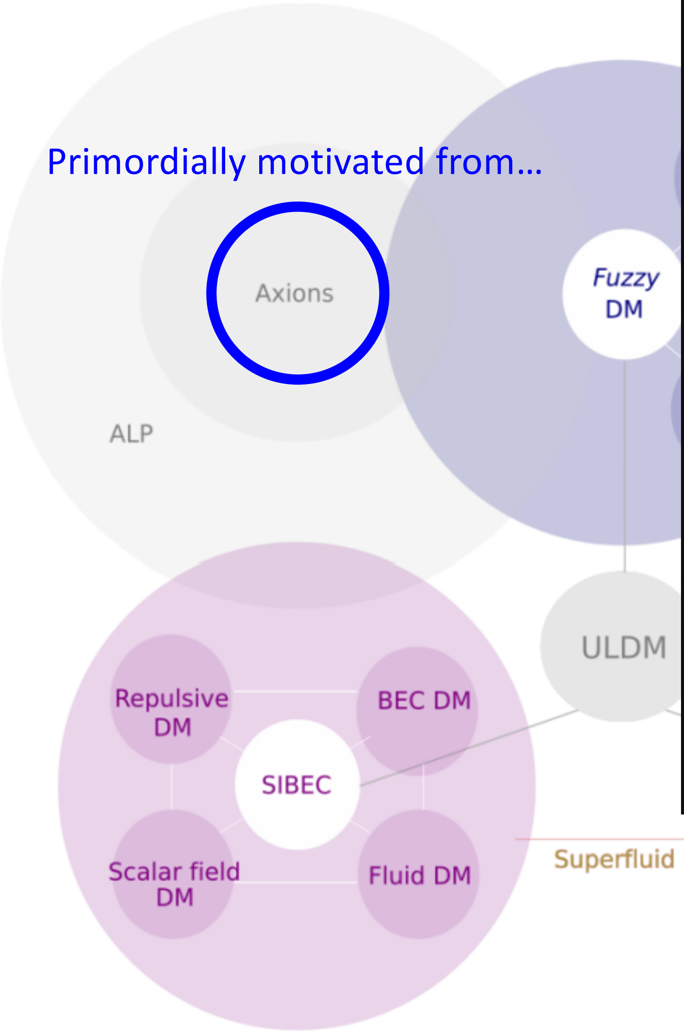
[E.G.M. Ferreira 2005.03254]

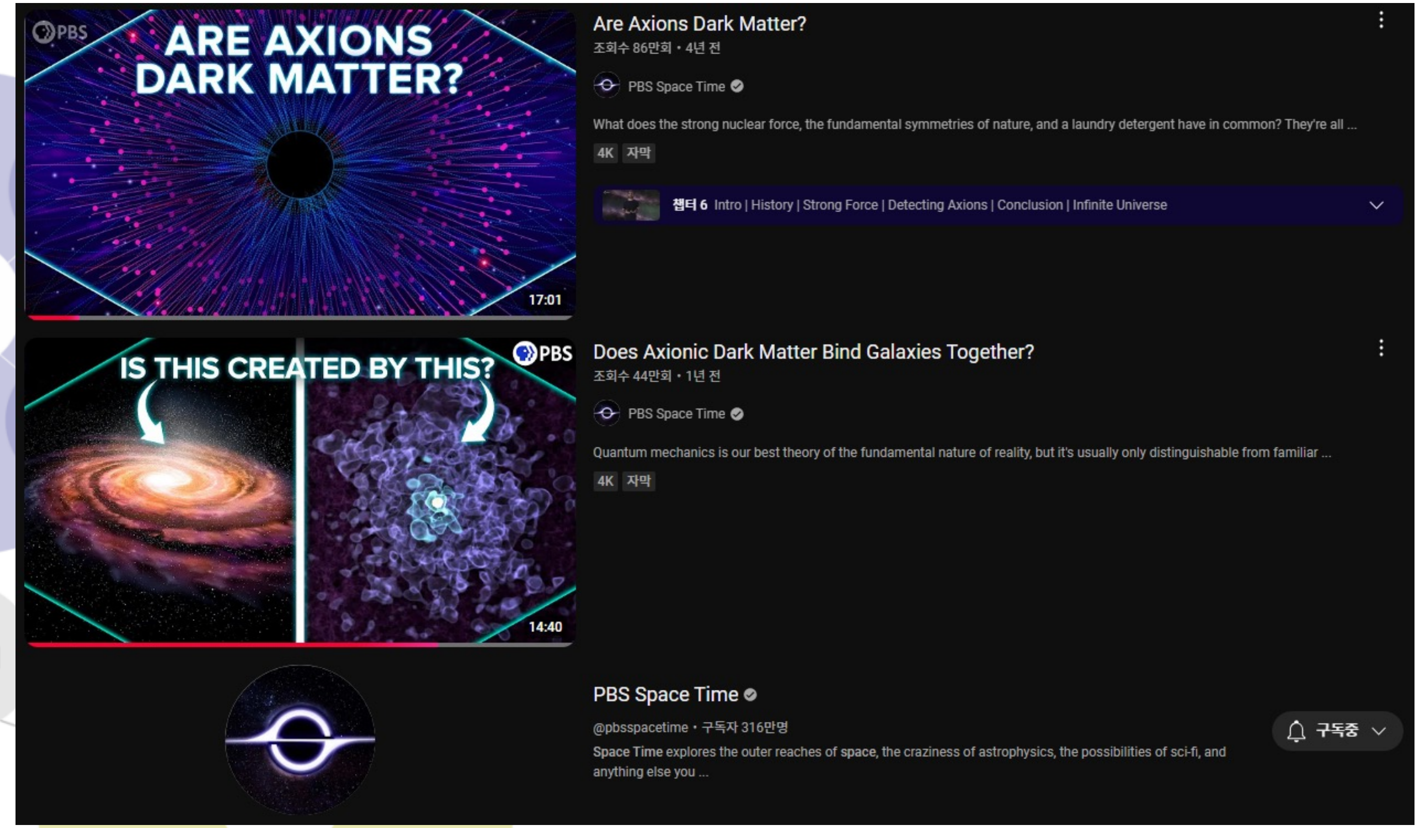




Some tensions in ULDM constraint ⇒
Better to consider “another” interaction?

Primordially motivated from…





Are Axions Dark Matter?
조회수 86만회 • 4년 전
PBS Space Time

What does the strong nuclear force, the fundamental symmetries of nature, and a laundry detergent have in common? They're all ...

챕터 6 Intro | History | Strong Force | Detecting Axions | Conclusion | Infinite Universe

Does Axionic Dark Matter Bind Galaxies Together?
조회수 44만회 • 1년 전
PBS Space Time

Quantum mechanics is our best theory of the fundamental nature of reality, but it's usually only distinguishable from familiar ...

PBS Space Time
@pbsspacetime • 구독자 316만명
Space Time explores the outer reaches of space, the craziness of astrophysics, the possibilities of sci-fi, and anything else you ...

Ultralight Dark Matter (ULDM)

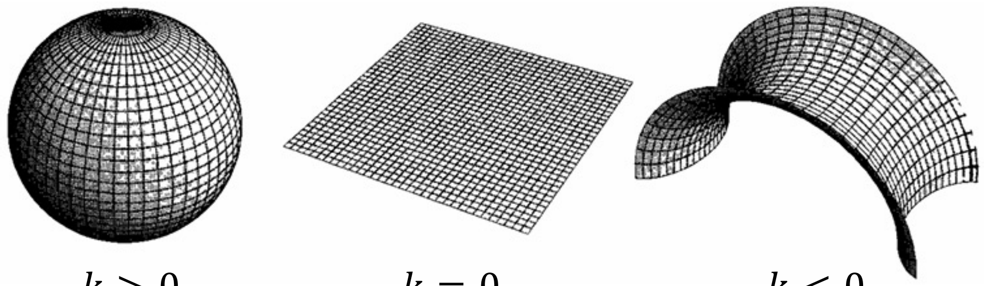
Cosmological Evolution

Robertson-Walker (RW) Metric

- Homogeneous & Isotropic Universe

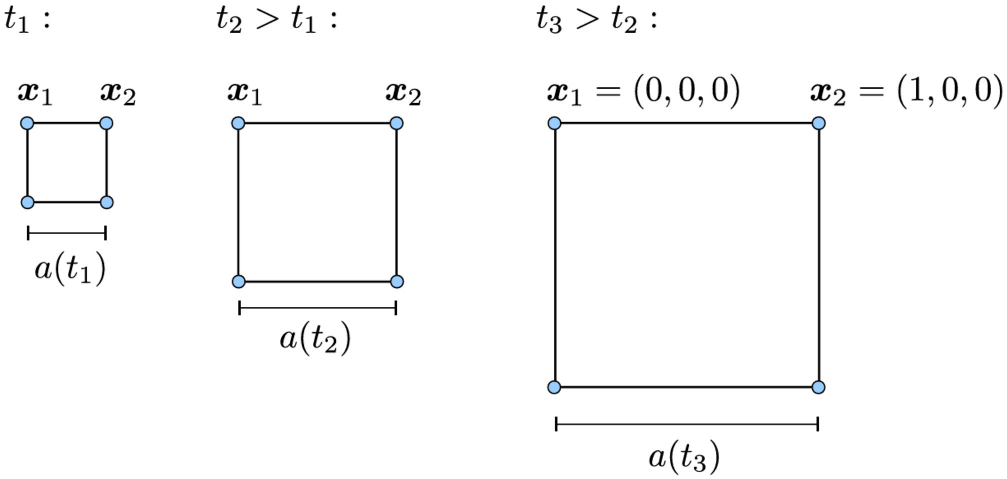
$$ds^2 = -(cdt)^2 + a^2(t)\left[\frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

  - Curvature of our spacetime k
  - Scale Factor $a(t)$: Expansion of our universe


$k > 0$ "Closed"

$k = 0$ "Flat"

$k < 0$ "Opened"



Friedmann Equations

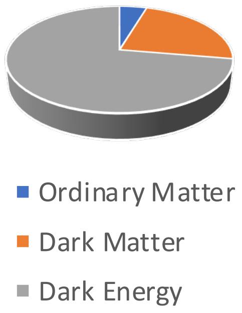
- For RW Metric…

$$\underbrace{H^2+\frac{kc^2}{a^2}=\frac{8\pi G}{3}\rho, \qquad \frac{\ddot{a}}{a}+\frac{4\pi G}{3}(\rho+3P)=0,}_{\text{Einstein Field eq.} \Rightarrow \text{Friedmann eq. (00), } (ij)} \qquad \underbrace{\dot{\rho}+3H(\rho+P)=0}_{\text{Energy Conservation}}$$

- Assuming “flat” universe as a “Perfect Fluid”: $P = w\rho$

Composition	Matter Only	Radiation Only	Dark Energy Only
Pressure	$P = 0$	$P = \frac{1}{3}\rho$	$P = -\rho$
Density	$\rho(t) \propto a^{-3}(t)$	$\rho(t) \propto a^{-4}(t)$	$\rho(t) = \rho_\Lambda(\text{Const.})$
Scale Factor	$a(t) \propto t^{2/3}$	$a(t) \propto t^{1/2}$	$a(t) \propto e^{Ht}$
Hubble Parameter $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$	$H(t) = \frac{2}{3t}$	$H(t) = \frac{1}{2t}$	$H(t) = \sqrt{\frac{8\pi G}{3}\rho_\Lambda}$

- Entropy conservation leads: $T(t) \propto a^{-1}(t)$
- Current Hubble parameter $H_0 \cong 70\mathrm{kms^{-1}Mpc^{-1}} \;\Rightarrow\; h = 0.7$
- Critical Density $\rho_c = \frac{3H_0^2}{8\pi G}$
- Density fraction $\Omega_i = \frac{\rho_i}{\rho_c}$



$$\left.\right\} \quad \begin{aligned} \Omega_b h^2 &= 0.022 \\ \Omega_c h^2 &= 0.122 \\ &\ldots \end{aligned}$$

Evolution of Scalar Field

- Massive, but ultra-light scalar field

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} m_\phi^2 \phi^2$$

- EOM in Flat RW metric ⇒ $\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$
- For radiation era: $H(t) = (2t)^{-1}$, EOM has an exact solution.

$$\phi(t) = \phi_i \left(\frac{2}{m_\phi t}\right)^{1/4} \Gamma\left(\frac{5}{4}\right) J_{1/4}(m_\phi t) \cong \phi_i \times \begin{cases} 1 & (t \ll m_\phi^{-1}) \\ \frac{\Gamma(5/4)}{\sqrt{\pi}} \left(\frac{2}{m_\phi t}\right)^{3/4} \cos(m_\phi t - \cdots) & (t \gg m_\phi^{-1}) \end{cases}$$

$\sim O(100\mathrm{nHz})$

"ULDM Oscillation on Cosmological Scale"

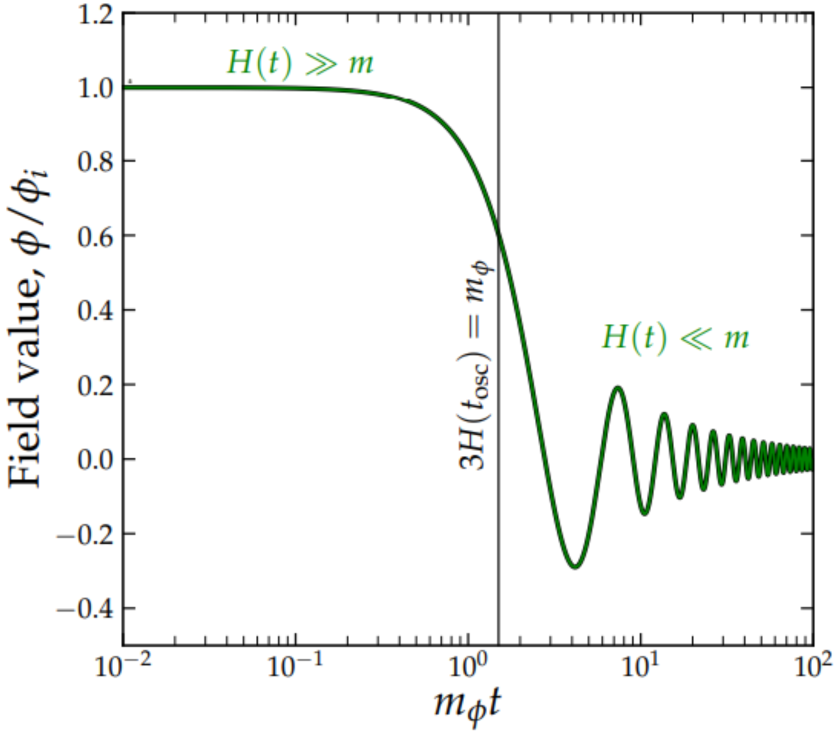
- Energy-momentum of scalar field $T_{\mu\nu} = \mathrm{diag}(-\rho, P, P, P)$

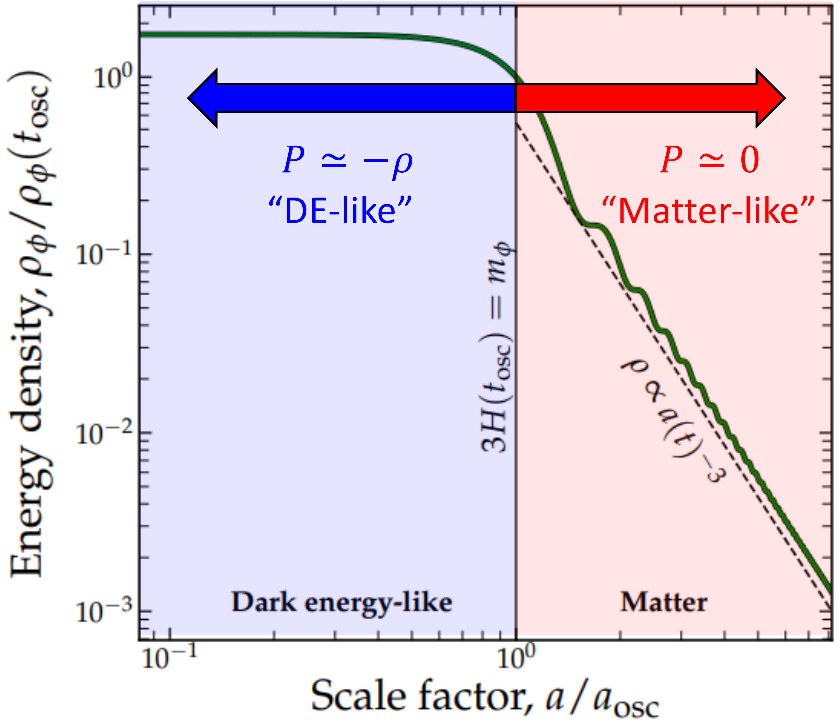
$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} m_\phi^2 \phi^2, \qquad P = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2} m_\phi^2 \phi^2$$

- Starting point of oscillation satisfies $3H(t_{\mathrm{osc}}) = m_\phi$

$$t_{\mathrm{osc}} = \frac{3}{2} m_\phi^{-1} \cong 0.313\mathrm{yr}\left(\frac{10^{-22}\mathrm{eV}}{m_\phi}\right)$$

[Ciaran.A.J.O'Hare 2403.17697]





$$\Omega_\phi = 0.12\left(\frac{\phi_i}{8.36\times10^{16}\mathrm{GeV}}\right)^2\left(\frac{m_\phi}{10^{-22}\mathrm{eV}}\right)^{1/2}$$

⇒ ULDM behaves same as CDM for cosmological scale! How about galactic scale?

Ultralight Dark Matter (ULDM)

Wave Dynamics & Phenomenology

Motion Equations for Newtonian Limit

- Repulsively self-interacting scalar field

$$S_{\mathrm{tot}} = S_{\mathrm{EH}} + S_{\phi} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right], \qquad V(\phi) = \frac{1}{2}m_a^2\phi^2 + \frac{\lambda}{4}\phi^4$$

$$ds^2 = -(1+2\Phi)(cdt)^2 + a^2(t)(1-2\Phi)\delta_{ij}dx^i dx^j$$

$$\phi(\vec{x},t) = \frac{\hbar}{\sqrt{2m}}\left[\psi(\vec{x},t)e^{-im_ac^2t/\hbar} + \mathrm{c.c.}\right]$$

Klein-Gordon eq. $\frac{\delta S_{\mathrm{tot}}}{\delta\phi} = 0$

Einstein eq. $\frac{\delta S_{\mathrm{tot}}}{\delta g^{\mu\nu}} = 0$

$$-\frac{\hbar^2}{2m_a a^2}\nabla^2\psi + m_a\Phi\psi + \frac{4\pi a_s\hbar^2}{m_a}|\psi|^2\psi = i\hbar\left(\frac{\partial\psi}{\partial t} + \frac{3}{2}H\psi\right) \quad + \quad \frac{\nabla^2\Phi}{4\pi G a^2} = m_a|\psi|^2 - \frac{3H^2}{8\pi G}$$

$$\lambda = \frac{8\pi a_s m_a c}{\hbar}$$

“Gross-Pitaevskii-Poisson (GPP) System”

Madelung Formalism

- Also called “Quantum Hydrodynamics” [E.Madelung, Die Naturwissenschaften 14 (45) 1004, 1926]

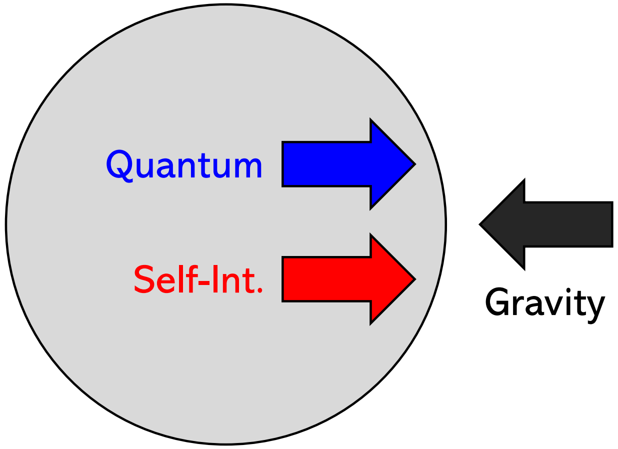
$$\begin{cases} \psi(\vec{x},t) = \sqrt{\dfrac{\rho(\vec{x},t)}{m_a}}\, e^{i\theta(\vec{x},t)} \\ \vec{v}(\vec{x},t) = \left(\dfrac{\hbar}{m_a}\right)\nabla\theta(\vec{x},t) \end{cases} \Longrightarrow \begin{cases} \dfrac{\partial\rho}{\partial t} + 3H\rho + \dfrac{1}{a}\nabla\cdot(\rho\vec{v}) = 0 \\ \dfrac{\partial\vec{v}}{\partial t} + H\vec{v} + \dfrac{1}{a}(\vec{v}\cdot\nabla)\vec{v} = -\dfrac{1}{a}\nabla\Phi - \dfrac{1}{a}\nabla Q - \dfrac{1}{\rho a}\nabla P \end{cases}$$

Quantum Pressure

$$Q = -\frac{\hbar^2}{2m_a^2a^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

Self-Interacting Pressure

$$P = \frac{2\pi a_s\hbar^2}{m_a^3}\rho^2 \Rightarrow c_s^2 = \frac{4\pi a_s\hbar^2}{m_a^3}\rho$$



Conserved Energy & Soliton Solution

$$E_{\text{tot}} = \int d^3\vec{x}\left[\frac{\partial\mathcal{L}}{\partial\dot{\psi}}\dot{\psi} + \frac{\partial\mathcal{L}}{\partial\dot{\psi}^*}\dot{\psi}^* - \mathcal{L}\right] = \int d^3\vec{x}\left(\frac{\hbar^2}{2m_a}|\nabla\psi|^2 + \boxed{\frac{1}{2}m_a\Phi|\psi|^2} + \boxed{\frac{2\pi a_s\hbar^2}{m_a}|\psi|^4}\right)$$

Potential Energy W

Internal Energy U

$$= \int d^3\vec{x}\left(\boxed{\frac{1}{2}\rho|\vec{v}|^2} + \boxed{\frac{\hbar^2}{8m_a^2}\frac{(\nabla\rho)^2}{\rho}} + \frac{1}{2}\rho\Phi + \frac{2\pi a_s\hbar^2}{m_a^3}\rho^2\right)$$

Classical Kinetic Energy Θ_C

Quantum Kinetic Energy Θ_Q

- Static, spherically-symmetric solution ⇒ **“Soliton” or “Boson Star”**
- Letting $\rho(r) = \rho_0 \cdot f\left(\frac{r}{R}, a_s\right)$ introduces five parameters

$$M = \boldsymbol{\eta}(4\pi\rho_0 R^3), \qquad \Theta_Q = \boldsymbol{\sigma}\left(\frac{M\hbar^2}{m_a^2 R^2}\right), \qquad W = -\boldsymbol{\nu}\left(\frac{GM^2}{R}\right)$$

$$U = \boldsymbol{\zeta}\left(\frac{2\pi a_s\hbar^2 M^2}{m_a^3 R^3}\right), \qquad \Theta_C = \boldsymbol{\alpha}\left(\frac{1}{2}M\dot{R}^2\right)$$

- And two length scales $R_Q = \frac{\hbar^2}{GMm_a^2}$, $R_a = \sqrt{\frac{a_s\hbar^2}{Gm_a^3}}$

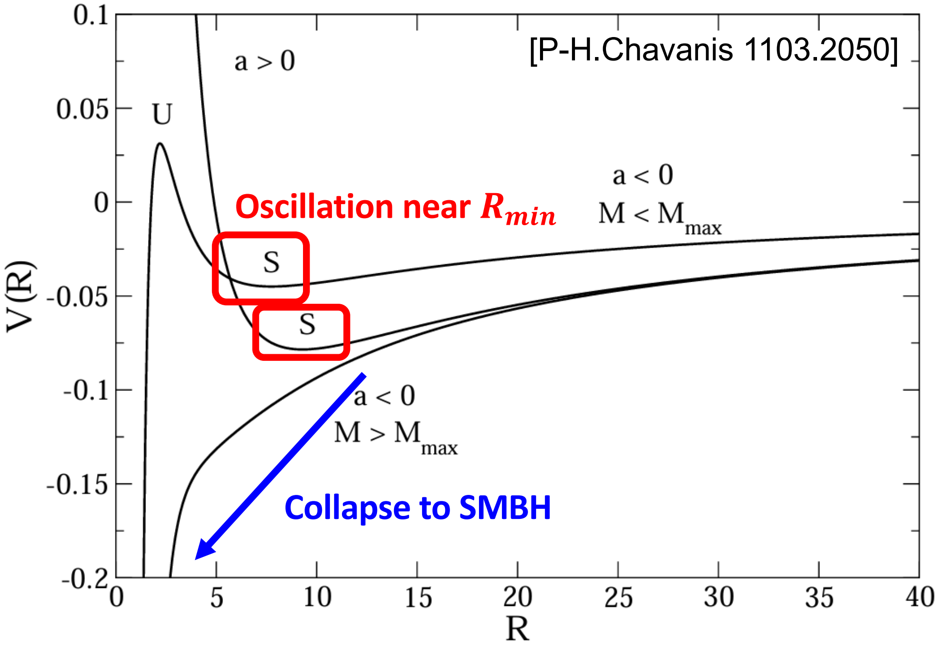
$$E_{\rm tot} = \frac{1}{2}\alpha M \left(\frac{dR}{dt}\right)^2 + \sigma\frac{\hbar^2 M}{m_a^2 R^2} + \zeta\frac{2\pi a_s \hbar^2 M^2}{m_a^3 R^3} - \nu\frac{GM^2}{R} \equiv \frac{1}{2}\alpha M \left(\frac{dR}{dt}\right)^2 + V(R)$$

- Equilibrium Radius scale : $V'(R_0) = 0$

$$R_0 = \frac{\sigma}{\nu}\frac{\hbar^2}{GMm_a^2}\left(1 + \sqrt{1 + \frac{6\pi\zeta\nu}{\sigma^2}\frac{GM^2 m_a a_s}{\hbar^2}}\right)$$

- Oscillation frequency : $\alpha M\ddot{R} + V'(R) = 0$

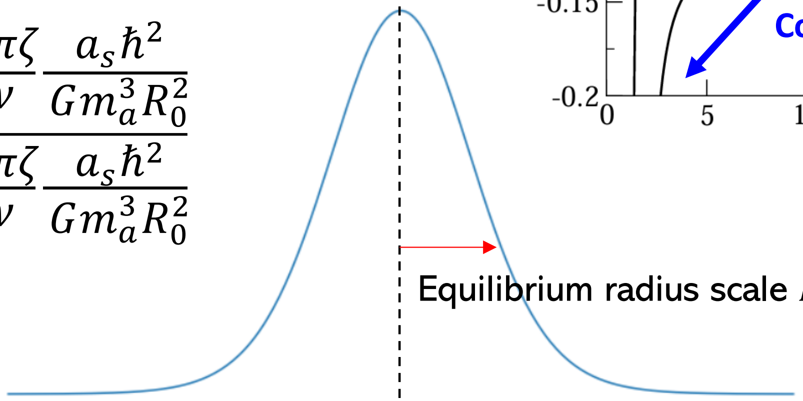
$$\omega^2 = \frac{2\sigma}{\alpha}\frac{\hbar^2}{m_a^2 R_0^4}\frac{1 + \frac{6\pi\zeta}{\nu}\frac{a_s\hbar^2}{Gm_a^3 R_0^2}}{1 - \frac{6\pi\zeta}{\nu}\frac{a_s\hbar^2}{Gm_a^3 R_0^2}}$$



[P-H.Chavanis 1103.2050]

Oscillation near R_{min}

Collapse to SMBH



Equilibrium radius scale R_0

Evolution of the Density Perturbation

$$\rho(\vec{x},t) = \rho_0(t)[1+\delta(\vec{x},t)]$$

Fourier space: $\nabla \rightarrow -i\vec{k}$

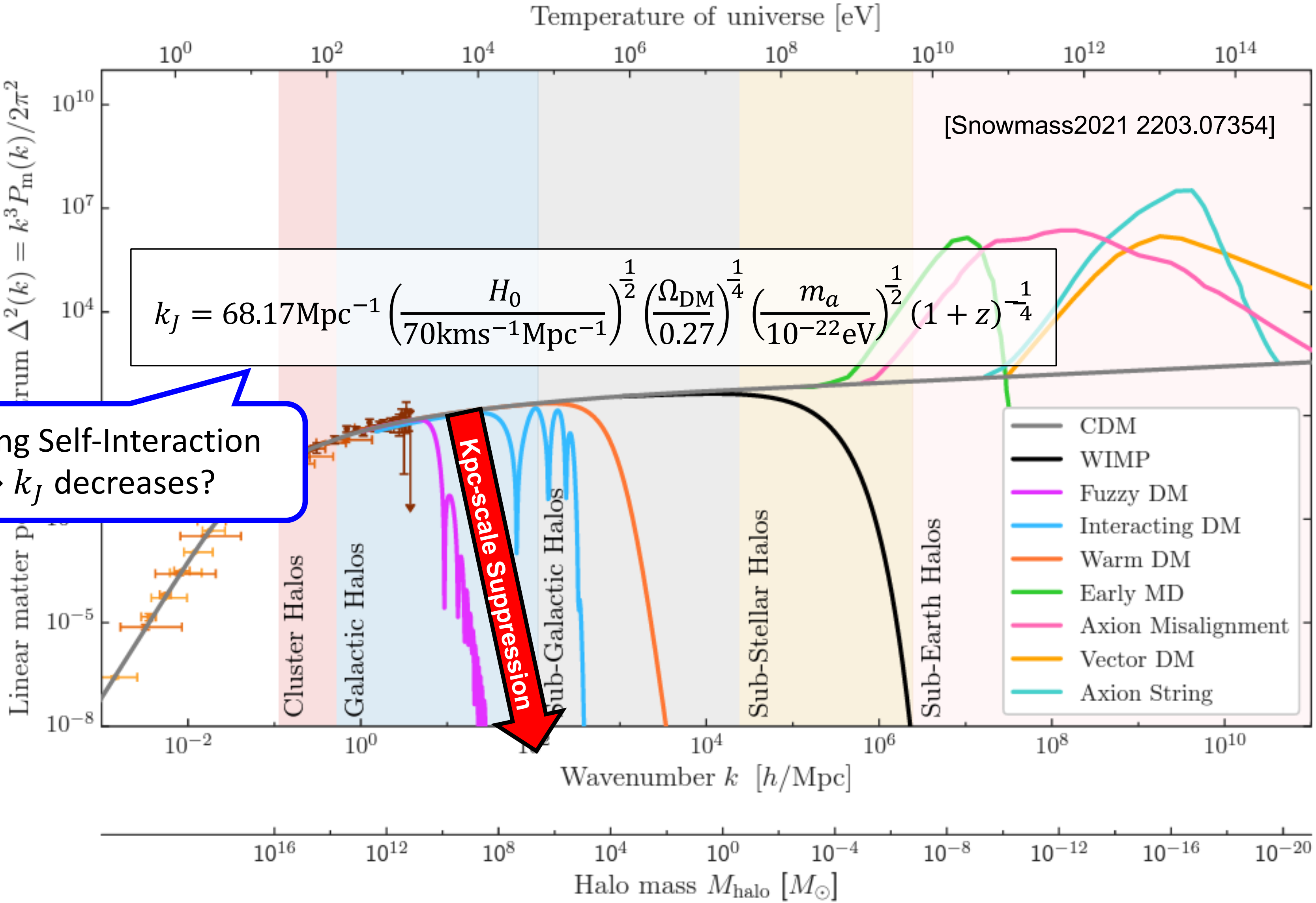
0th order equation $$\frac{\partial \rho_0}{\partial t} + 3H\rho_0 = 0 \quad \Rightarrow \quad \rho_0(t) \propto a^{-3}(t)$$

1st order equation $$\frac{\partial^2 \delta}{\partial t^2} + 2H\frac{\partial \delta}{\partial t} + \left(\frac{\hbar^2 k^4}{4m_a^2 a^4} + \frac{c_s^2}{a^2}k^2 - 4\pi G\rho_0\right)\delta = 0$$

- $k < k_J$ (Super-Galactic) : CDM-like behavior
- $k < k_J$ (Sub-Galactic) : Structure is Suppressed!

“Jeans Instability”

	Self-Interaction X	Self-Interaction O
CDM	X	$k_J^2 = \frac{4\pi G\rho_0 a^2}{c_s^2}$
ULDM	$k_J^4 = \frac{16\pi G\rho_0 m_a^2 a^4}{\hbar^2}$	$k_J^4 = \frac{16\pi G\rho_0 m_a^2 a^2}{\hbar^2}\left[\sqrt{\frac{m_a^2 c_s^4}{4\pi G\rho_0\hbar^2}+1} - \sqrt{\frac{m_a^2 c_s^4}{4\pi G\rho_0\hbar^2}}\right]^2$



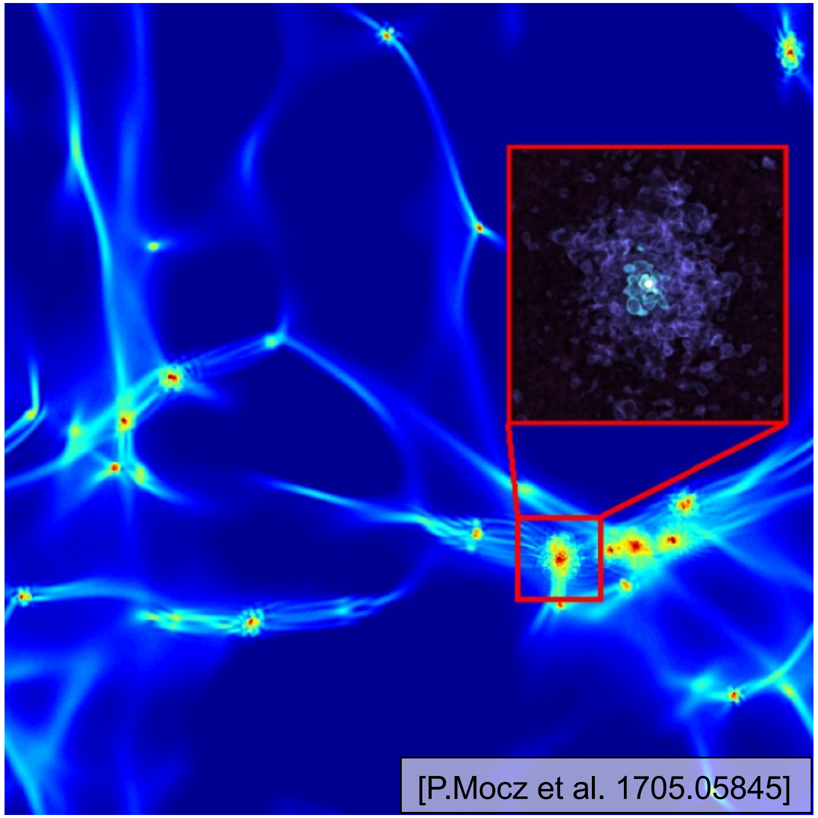
[Snowmass2021 2203.07354]

$$k_J = 68.17\text{Mpc}^{-1}\left(\frac{H_0}{70\text{kms}^{-1}\text{Mpc}^{-1}}\right)^{\frac{1}{2}}\left(\frac{\Omega_{\text{DM}}}{0.27}\right)^{\frac{1}{4}}\left(\frac{m_a}{10^{-22}\text{eV}}\right)^{\frac{1}{2}}(1+z)^{-\frac{1}{4}}$$

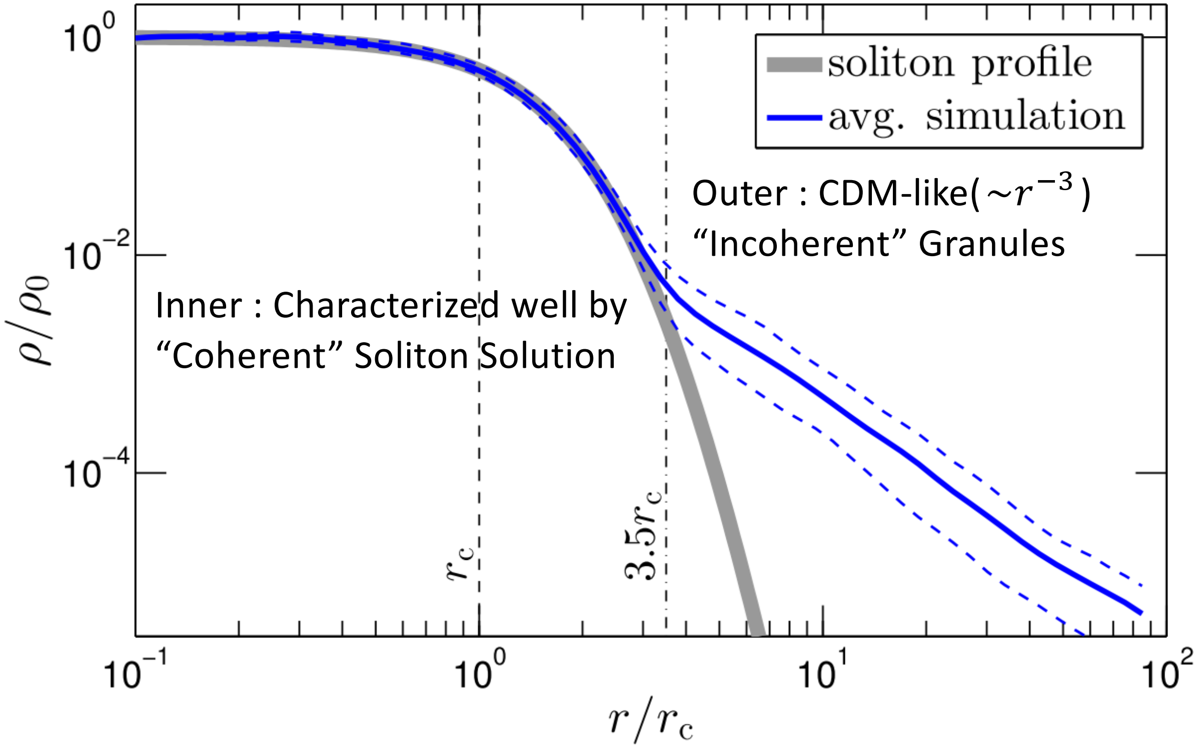
Adding Self-Interaction
⇒ k_J decreases?

ULDM Halo and Soliton

- Spherically-symmetric, time-independent solution



[P.Mocz et al. 1705.05845]



Outer : CDM-like($\sim r^{-3}$)
"Incoherent" Granules

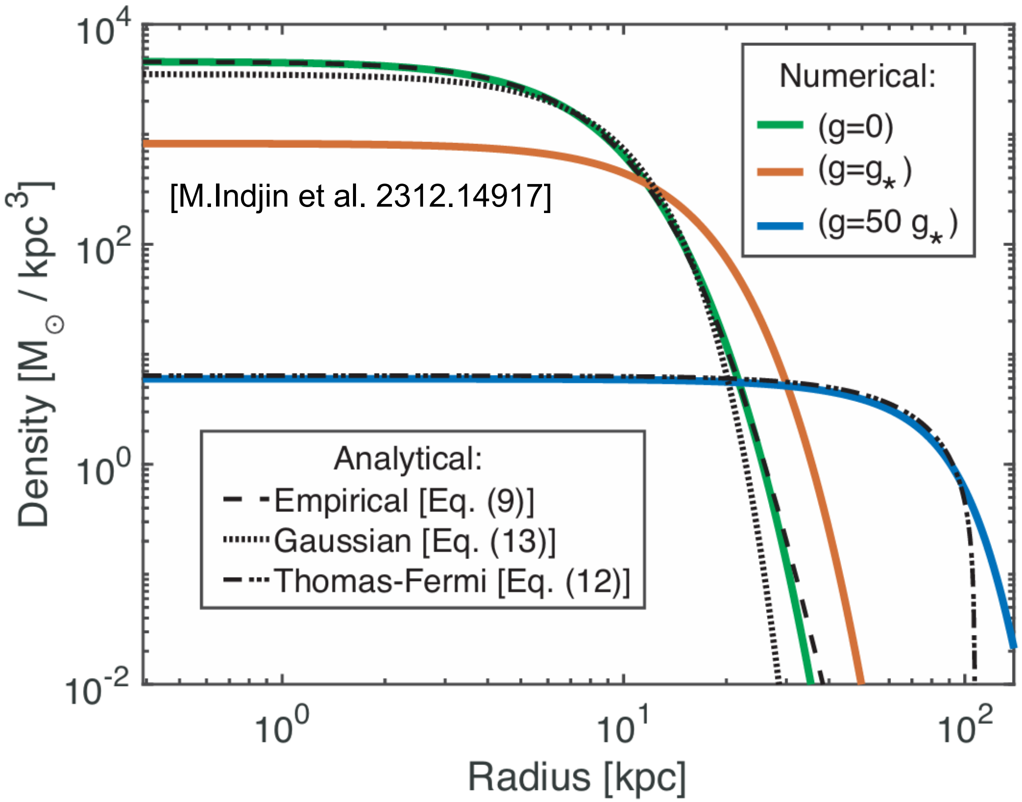
Inner : Characterized well by
"Coherent" Soliton Solution

Common ~~Exact~~ Soliton Profiles

1. Empirical Profile : $\rho(r) \propto [1 + 0.091(r/r_c)^2]^{-8}$
   - Absence of Self-Coupling ($\lambda = 0$)
   - Estimated by [H.Schive et al. 1406.6586]

2. Thomas-Fermi Limit : $\rho(r) \propto \frac{\sin(\pi r/R_{\mathrm{TF}})}{(\pi r/R_{\mathrm{TF}})}$
   - Strongly-interacting limit
   - Exact solution for neglecting quantum pressure
   $$\rho \nabla \Phi = -\nabla P$$
   - valid for $R_a \gg R_Q$ [P-H.Chavanis 1103.2050]
   $$a_s \gg a_Q \equiv \frac{\hbar^2}{GM^2 m_a}$$

3. Gaussian Profile : $\rho(r) \propto e^{-r^2/r_c^2}$
   - Poor description for both $\lambda = 0$ and $\lambda \to \infty$ limit



[M.Indjin et al. 2312.14917]

Numerical Codes for ULDM [J.Zhang et al. 1809.09848]

Direct Solver

$$-\frac{\hbar^2}{2m_a}\nabla^2\psi + m\Phi\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\nabla^2\Phi = 4\pi G m_a|\psi|^2$$

VS

Madelung Solver

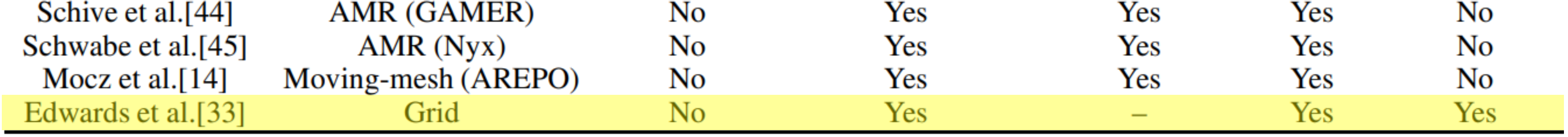
$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\Phi - \nabla Q$$

Table 1. Summary of the Lagriangian based simulation codes for FDM model (Madelung Solvers).

Author	Method (Code Base)	Cosmo-Sim	Granular structure	Solitonic Core	Activity	Open Source
Veltmaat et al.[30]	PIC (NyX)	Yes	No	Yes	Yes	No
Mocz et al.[28]	SPH	No	No	–	No	No
Nori et al.[32]	SPH (P-Gadget3)	Yes	–	–	Yes	No
Zhang et al.[31]	PP (Gadget2)	Yes	No	Yes	Yes	Yes

Table 2. Summary of the Eulerain based simulation codes for FDM model (Schrödinger-Poisson Solvers).



Author	Method (Code Base)	Cosmo-Sim	Granular structure	Solitonic Core	Activity	Open Source
Schive et al.[44]	AMR (GAMER)	No	Yes	Yes	Yes	No
Schwabe et al.[45]	AMR (Nyx)	No	Yes	Yes	Yes	No
Mocz et al.[14]	Moving-mesh (AREPO)	No	Yes	Yes	Yes	No
Edwards et al.[33]	Grid	No	Yes	–	Yes	Yes

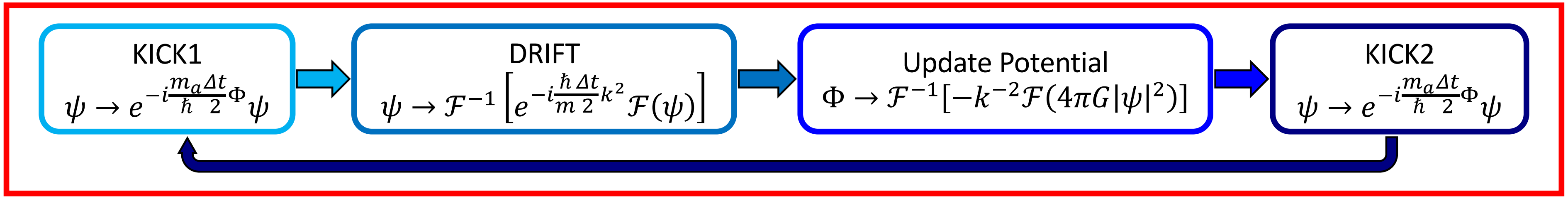
PyUltraLight 2

- Pseudo-Spectral solver for ULDM soliton dynamics with PBC

  https://github.com/auckland-cosmo/PyUltraLight [F.Edwards et al. 1807.04037]

  + N-Body Code https://github.com/Sifyrena/PyUL_NBody [Y.Wang et al. 2110.03428]

- “Python Code”



KICK1

$\psi \rightarrow e^{-i\frac{m_a}{\hbar}\frac{\Delta t}{2}\Phi}\psi$

DRIFT

$\psi \rightarrow \mathcal{F}^{-1}\left[e^{-i\frac{\hbar}{m}\frac{\Delta t}{2}k^2}\mathcal{F}(\psi)\right]$

Update Potential

$\Phi \rightarrow \mathcal{F}^{-1}[-k^{-2}\mathcal{F}(4\pi G|\psi|^2)]$

KICK2

$\psi \rightarrow e^{-i\frac{m_a}{\hbar}\frac{\Delta t}{2}\Phi}\psi$

$\Delta t \geq \frac{m_a}{\hbar}\frac{\Delta x^2}{\pi}$ prevents artifact from $\arg(\psi) \geq \pi$

- Initial soliton condition is generated by RK4 method

$$-\frac{1}{2}f''(r) - \frac{1}{r}f'(r) + \tilde{\phi}(r)f(r) = 0, \qquad \tilde{\phi}''(r) + \frac{2}{r}\tilde{\phi}'(r) - 4\pi f^2(r) = 0 \begin{cases} f(0) = 1 \\ f(\infty) = 0 \end{cases}$$

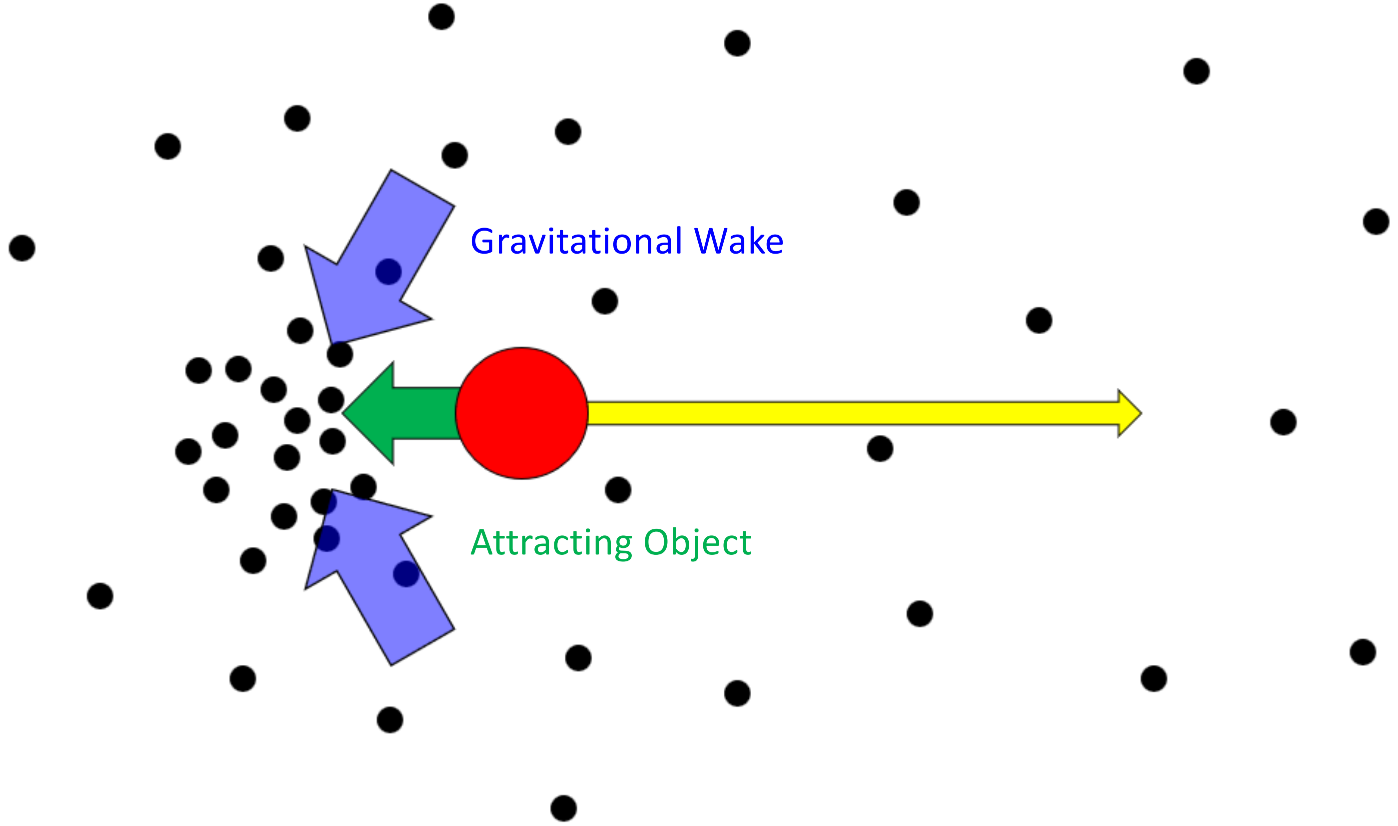
- Adding self-interaction ⇒ Add one term to “KICK” [N.Glennon et al. 2011.09510]

$$\psi \rightarrow e^{-i\frac{m_a}{\hbar}\frac{\Delta t}{2}\left[\Phi + \lambda\frac{\hbar^3}{2m_a^3 c}|\psi(t)|^2\right]}\psi$$

Dynamical Properties

Dynamical Friction (DF)

DF in Classical Particle System



$$F_{\mathrm{DF}} = 4\pi\bar{\rho}\left(\frac{GM}{v}\right)^2 \boxed{C_{\mathrm{DF}}}$$

Effective DF Coefficient

- For sufficiently large speed…

$$\frac{d\vec{v}}{dt} = -4\pi G^2 M m_a n \log\Lambda \frac{\vec{v}}{v^3}$$

$$C_{\mathrm{DF}} = \log\Lambda = \log\frac{b_{\max}}{\max(r_{\mathrm{h}}, GM/v^2)}$$

[Chandrasekhar 1943]

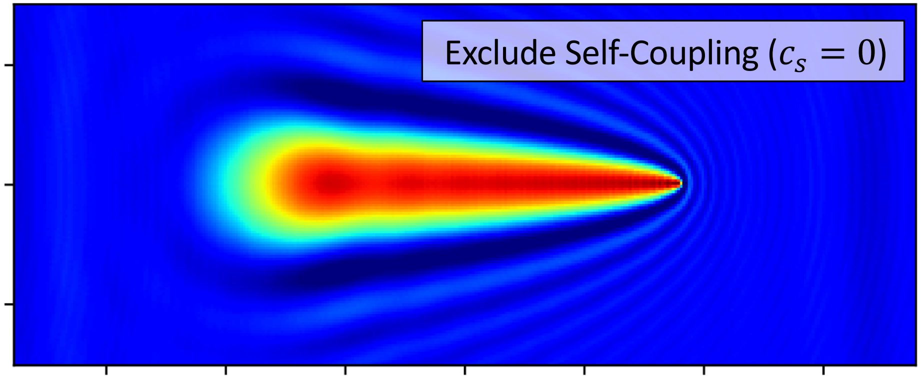
From Linear Perturbation

- Perturbation by a potential $\Phi_{\rm P}$, ULDM self-gravity is negligible?

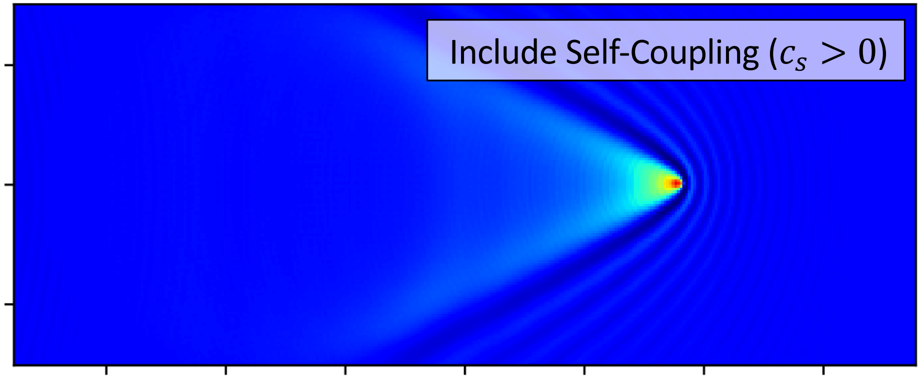
$$\frac{\partial^2 \delta}{\partial t^2} + \frac{\hbar^2}{4m_a^2}\nabla^4\delta - c_s^2\nabla^2\delta = \nabla^2\Phi_{\rm P} \quad \Rightarrow \delta(\vec{x},t) = \int d^3\vec{x}'dt'G(\vec{x}-\vec{x}',t-t')\nabla^2\Phi_{\rm P}(\vec{x}',t')$$

$$G\left(\vec{R},\tau\right) = \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{\exp\left[i\left(\vec{k}\cdot\vec{R}-\omega\tau\right)\right]}{\frac{\hbar^2k^4}{4m_a^2}+c_s^2k^2-\omega^2}$$

- Dynamical friction force woken by density fluctuation $\vec{F}_{\rm DF}(t) = \rho\int d^3\vec{x}(\nabla\Phi_{\rm P})\delta(\vec{x},t)$.



Exclude Self-Coupling ($c_s = 0$)



Include Self-Coupling ($c_s > 0$)

Exact Solution for Zero Coupling

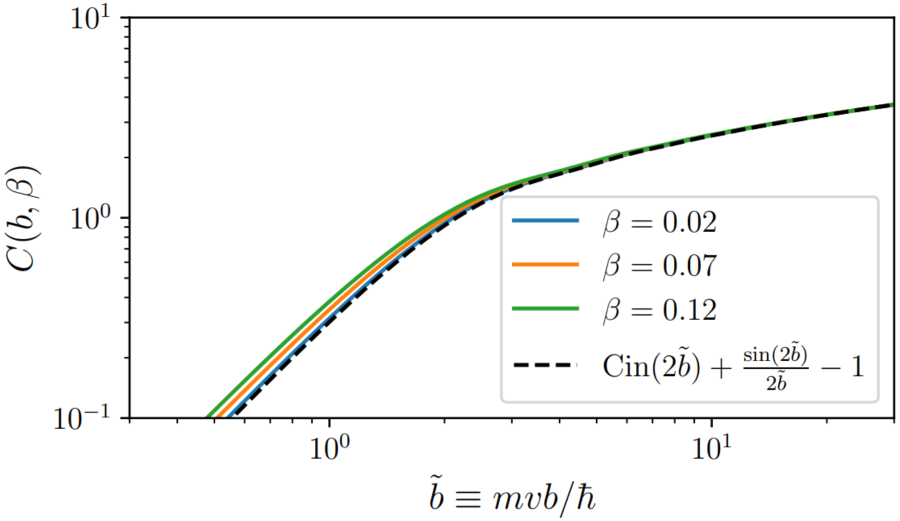
[L.Hul et al. 1610.08297 ; L.Lancaster et al. 1909.06381]

- For $\vec{v} = v_0\hat{z}$, integral of $\vec{F}_{\rm DF}(t) = \rho \int d^3\vec{x}(\nabla\Phi_{\rm P})\delta(\vec{x},t)$ requires a cut-off scale b.

$$\left(\frac{\hbar^2}{2m_a}\nabla^2 + \frac{GM_{\rm P}m_a}{r} + \frac{1}{2}mv_0^2\right)\psi = 0 \;\Rightarrow\; \psi(\tilde{R},\tilde{z}) = \sqrt{\frac{\bar{\rho}}{m_a}}\, e^{i\tilde{z}+\frac{\pi}{2}\beta}|\Gamma(1-i\beta)|\;{}_1F_1\left[i\beta,1;i\left(\sqrt{\tilde{R}^2+\tilde{z}^2}+\tilde{z}\right)\right]$$

$$C_{\rm DF}(\tilde{b},\beta) = \mathrm{Cin}(2\tilde{b}) + \frac{\sin(2\tilde{b})}{2\tilde{b}} - 1 + O(\beta) \simeq \begin{cases} \frac{1}{3}\tilde{b}^2\ (\tilde{b} \ll 1) \\ \log(2\tilde{b}) - 1 + \mathrm{Re}\Psi(1+i\beta)\ (\tilde{b} \gg 1) \end{cases}$$

$$\tilde{b} \equiv \frac{mv_0 b}{\hbar} \qquad \beta = \frac{GMm_a}{\hbar v_0}$$



For Circular Orbit

[L.Berezhiani et al. 2311.07672]

- Circularly orbiting point-like perturber : $\vec{x}_{\mathrm{P}}(t) = r_0(\cos\Omega t\,, \sin\Omega t\,, 0)$

$$\vec{F}_{\mathrm{DF}}(t) = -4\pi\rho\left(\frac{GM_{\mathrm{P}}}{\Omega r_0}\right)^2\left[\Re(I)\hat{r}(t) + \Im(I)\hat{\phi}(t)\right]$$

Effective DF Coefficient

$$I = \boxed{\mathcal{M}^2}\sum_{l=1}^{\infty}\sum_{m=-l}^{l-2}(-1)^{m+1}\frac{(l-m)!}{(l-m-2)!}\frac{S_{l,l-1}^{m} - S_{l,l-1}^{-m-1}}{\Gamma\left(\frac{1-l-m}{2}\right)\Gamma\left(\frac{3-l+m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)}$$

$$\mathcal{M} \equiv \frac{v_\Omega}{c_s}$$

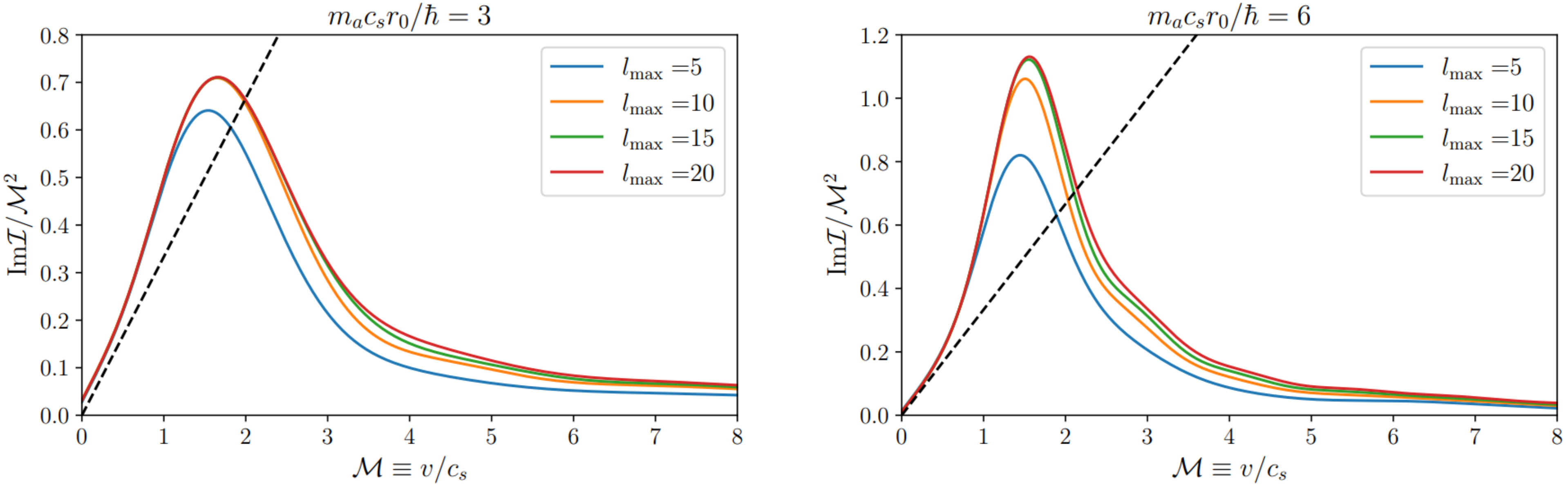
$$S_{l,l-1}^{m} = \begin{cases} \dfrac{\pi i}{2\sqrt{1+m^2/l_q^2}}\left[j_l(l_q\mathcal{M}f_m^-)h_{l-1}^{(1)}(l_q\mathcal{M}f_m^-) - j_l(il_q\mathcal{M}f_m^+)h_{l-1}^{(1)}(il_q\mathcal{M}f_m^+)\right] & (m > 0) \\ \dfrac{-\pi i}{2\sqrt{1+m^2/l_q^2}}\left[j_l(l_q\mathcal{M}f_m^-)h_{l-1}^{(2)}(l_q\mathcal{M}f_m^-) + j_l(il_q\mathcal{M}f_m^+)h_{l-1}^{(1)}(il_q\mathcal{M}f_m^+)\right] & (m < 0) \\ \dfrac{\pi}{2(4l^2-1)} - \dfrac{\pi i}{2}j_l(2il_q\mathcal{M})h_{l-1}^{(1)}(2il_q\mathcal{M}) & (m = 0) \end{cases}$$

$$l_q\mathcal{M} = m_a c_s r_0/\hbar \qquad f_m^{\pm^2} = 2\sqrt{1+m^2/l_q^2} \pm 2$$

- The leading-order term of $\Im(I)$ only arises from $l = 1$.

$$\Im(I)_{l=1} = -\frac{2}{\pi}\mathcal{M}^2\Im\left(S_{1,0}^{-1} - S_{1,0}^{0}\right) = \frac{\mathcal{M}^2 l_q}{\sqrt{l_q^2+1}}\left(\frac{1-\cos 2x}{2x^3} - \frac{\sin 2x}{2x^2}\right)_{x=l_q\mathcal{M}f_1^-} = \frac{1}{3}\mathcal{M}^3 + \mathcal{O}(\mathcal{M}^5)$$

- The larger c_s(self-coupling), the smaller DF coefficient



Circular Orbit without Self-Interaction

[R.Buehler et al. 2207.13740]

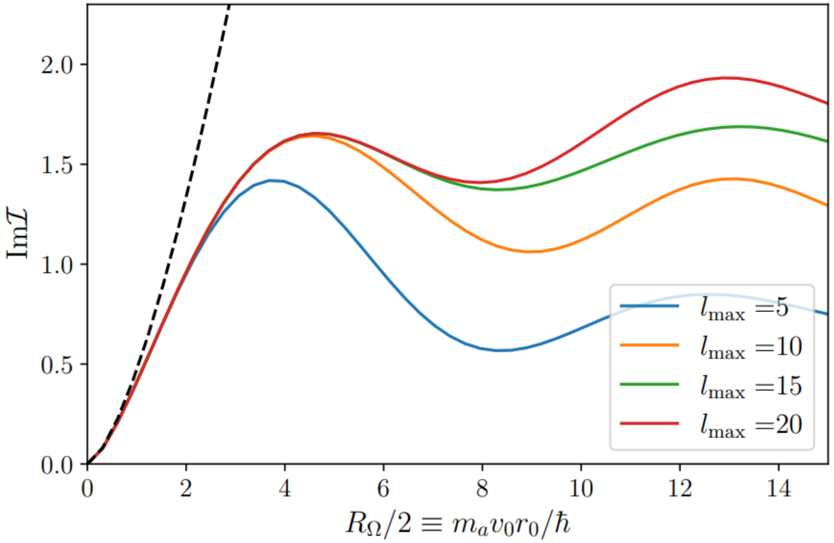
$$I = \mathcal{M}^2 \sum_{l=1}^{\infty} \sum_{m=-l}^{l-2} (-1)^{m+1} \frac{(l-m)!}{(l-m-2)!} \frac{S^{m}_{l,l-1} - S^{-m-1}_{l,l-1}}{\Gamma\left(\frac{1-l-m}{2}\right)\Gamma\left(\frac{3-l+m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)\Gamma\left(\frac{2+l-m}{2}\right)}$$

$$R_\Omega \equiv \frac{2m_a v_0 r_0}{\hbar}$$

$$S^{m}_{l,l-1} = \frac{i\pi \boxed{R_\Omega}}{4m}\left[j_l\left(\sqrt{mR_\Omega}\right)h^{(1)}_{l-1}\left(\sqrt{mR_\Omega}\right) - j_l\left(i\sqrt{mR_\Omega}\right)h^{(1)}_{l-1}\left(i\sqrt{mR_\Omega}\right)\right]$$

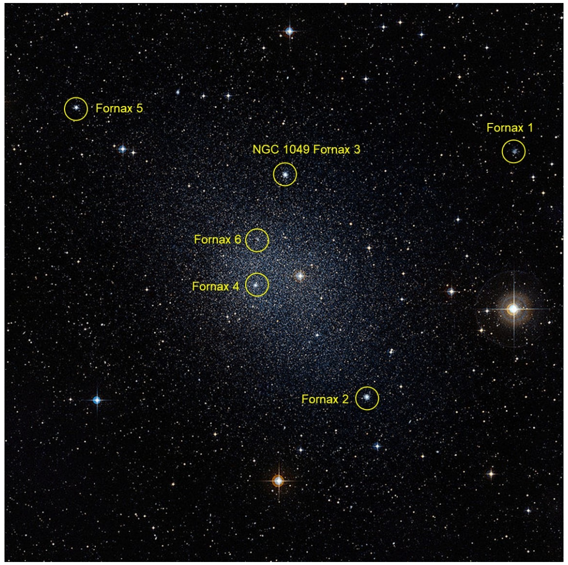
- The leading-order term of $\Im(I)$ only arises from $l = 1$.

$$\Im(I)_{l=1} = \frac{\sqrt{2}}{3}\left(\frac{mv_\Omega r_0}{\hbar}\right)^{3/2} + \mathcal{O}\left(\left(\frac{mv_\Omega r_0}{\hbar}\right)^{5/2}\right)$$



Applying to Fornax dSph & GCs

[H.Koo et al. 25XX.XXXXX]



- The five observed globular clusters (GCs) orbiting Fornax dwarf spheroidal (dSph) has individual "Lifetimes".

$$\tau_{\text{life}} = \frac{M_{\text{P}} v_{\text{P}}}{F_{\text{DF}}} = \frac{v_{\text{P}}^{3}}{4\pi\rho G^{2} M_{\text{P}} C_{\text{DF}}}$$

Object	$M_{\text{P}}[10^5 M_\odot]$	$R_{\text{P}}[\text{kpc}]$
dSph	1420	–
GC1	0.37	1.6
GC2	1.82	1.05
GC3	3.63	0.43
GC4	1.32	0.24
GC5	1.78	1.43

[D.R.Cole et al. 1205.6327]

Observational Result

$\tau_{\text{life}} \sim 10\text{Gyr}$ [M.-Y.Yang et al.1809.07801]

CDM-based Prediction

$\tau_{\text{life}} \sim 1\text{Gyr}$ [K.S.Oh et al. ApJ 531 (2000) 727]

"Timing Problem of Fornax GCs"

[S.D.Tremaine ApJ 203 (1976) 345]

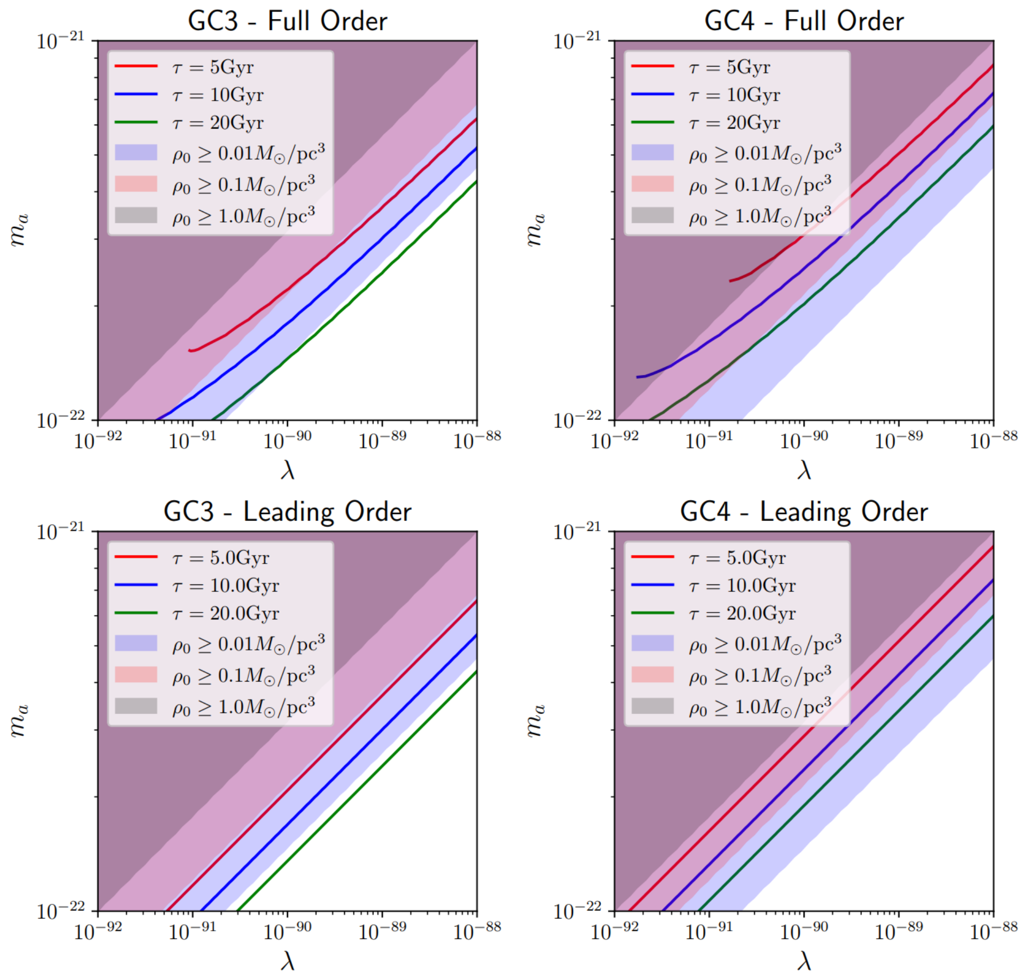
- The ULDM halo for describing Fornax dSph is chosen as

1. Thomas-Fermi Limit Profile (Strong-Interacting)

$$\rho(r) = \frac{\pi M}{4R_{\mathrm{TF}}^3}\frac{\sin(\pi r/R_{\mathrm{TF}})}{(\pi r/R_{\mathrm{TF}})} \quad \Rightarrow R_{\mathrm{TF}} = \pi\sqrt{\frac{a_s\hbar^2}{Gm_a^3}} = 1.957\mathrm{kpc}\times\left(\frac{5\mathrm{eV}}{m_a\lambda^{-1/4}}\right)^2$$

2. Empirical Profile by [H-Y.Schive et al. 1406.6586] (Non-Interacting)

$$\rho(r) \cong \frac{\rho_c}{[1+0.091(r/r_c)^2]^8}\begin{cases} r_c \cong 0.229\mathrm{kpc}\dfrac{10^9 M_\odot}{M}\left(\dfrac{10^{-22}\mathrm{eV}}{m}\right)^2 \\ \rho_c \cong 7.05 M_\odot/\mathrm{pc}^3\left(\dfrac{m}{10^{-22}\mathrm{eV}}\right)^6\left(\dfrac{M}{10^9 M_\odot}\right)^4 \end{cases}$$



- Roughly assuming $10^{-2} \leq \rho_0 [M_\odot/\mathrm{pc}^3] \leq 10^{-1}$…

$$4.680[\mathrm{eV}] \leq \frac{m_a}{\lambda^{1/4}} \leq 6.869[\mathrm{eV}]$$

This constrain is valid in $\lambda \geq 10^{-90}$.

$$\overline{\tau_{\mathrm{life}}} = \frac{v_{\mathrm{P}}^3}{4\pi G^2 \rho M_{\mathrm{P}} \Im(I)_{l=1}} = \frac{3c_s^3}{4\pi G^2 \rho M_{\mathrm{P}}}$$

- Letting $m_a = 3\times10^{-22}$eV leads $\overline{\tau_{\mathrm{life}}}$ to…

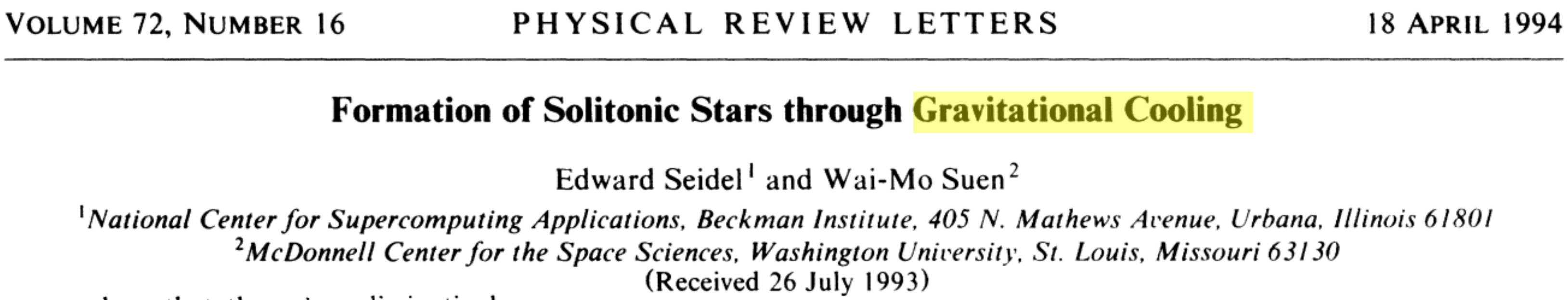
Object	CDM	ULDM, SI X	ULDM, SI O
GC3	0.62	3.99	4.27~15.23
GC4	0.37	1.07	13.06~42.79

CDM Result from [L.Hul et al. 1610.08297]

- The stronger λ, the weaker dynamical friction!

Dynamical Properties

Gravitational Cooling Effect

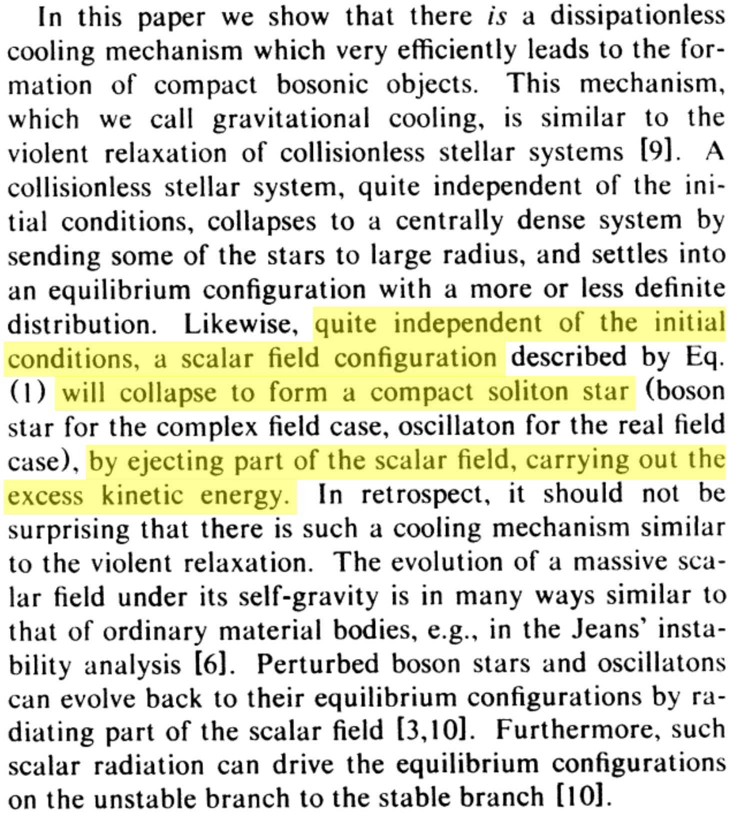


VOLUME 72, NUMBER 16 PHYSICAL REVIEW LETTERS 18 APRIL 1994

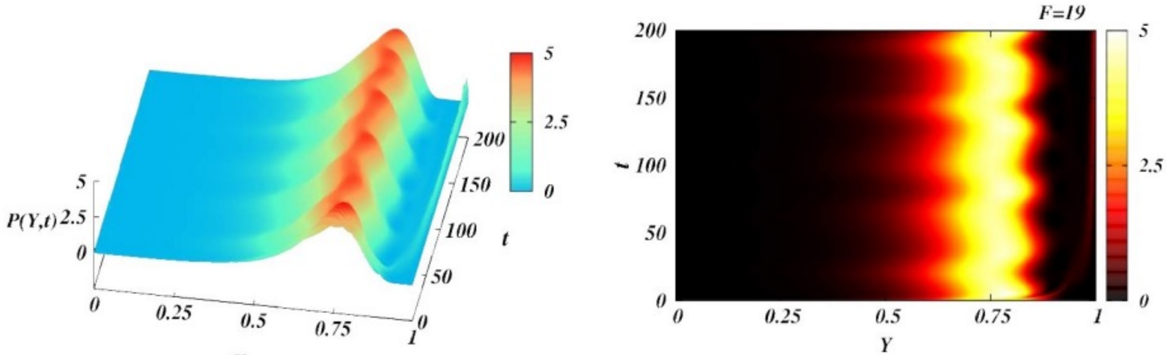
Formation of Solitonic Stars through Gravitational Cooling

Edward Seidel[1] and Wai-Mo Suen[2]
[1]National Center for Supercomputing Applications, Beckman Institute, 405 N. Mathews Avenue, Urbana, Illinois 61801
[2]McDonnell Center for the Space Sciences, Washington University, St. Louis, Missouri 63130

(Received 26 July 1993)

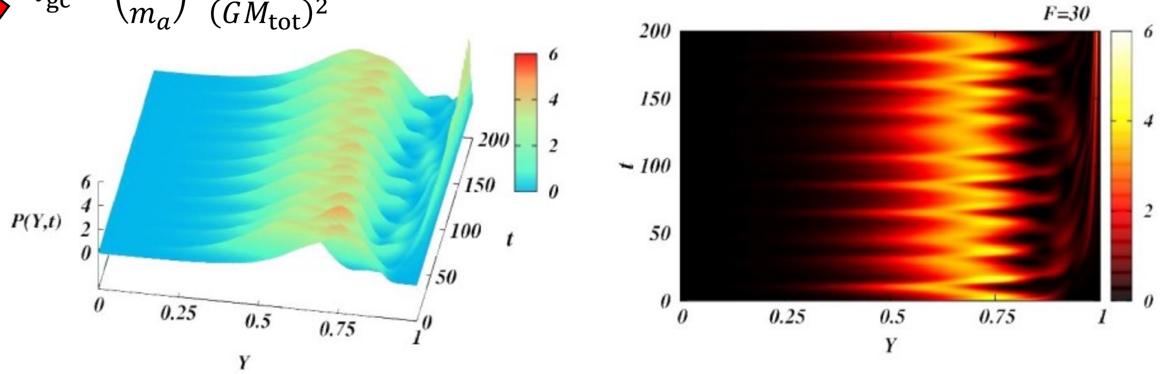


In this paper we show that there *is* a dissipationless cooling mechanism which very efficiently leads to the formation of compact bosonic objects. This mechanism, which we call gravitational cooling, is similar to the violent relaxation of collisionless stellar systems [9]. A collisionless stellar system, quite independent of the initial conditions, collapses to a centrally dense system by sending some of the stars to large radius, and settles into an equilibrium configuration with a more or less definite distribution. Likewise, quite independent of the initial conditions, a scalar field configuration described by Eq. (1) will collapse to form a compact soliton star (boson star for the complex field case, oscillaton for the real field case), by ejecting part of the scalar field, carrying out the excess kinetic energy. In retrospect, it should not be surprising that there is such a cooling mechanism similar to the violent relaxation. The evolution of a massive scalar field under its self-gravity is in many ways similar to that of ordinary material bodies, e.g., in the Jeans' instability analysis [6]. Perturbed boson stars and oscillatons can evolve back to their equilibrium configurations by radiating part of the scalar field [3,10]. Furthermore, such scalar radiation can drive the equilibrium configurations on the unstable branch to the stable branch [10].



$$\tau_{\mathrm{gc}} = \left(\frac{\hbar}{m_a}\right)^3 \frac{1}{(GM_{\mathrm{tot}})^2}$$

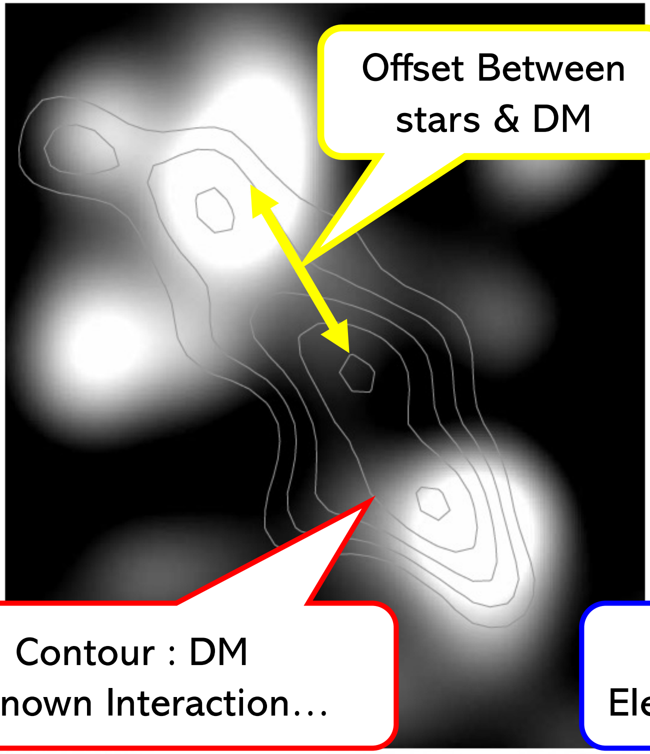
[Dongsu Bak et al. 1811.09694]



Galaxy Cluster Collision

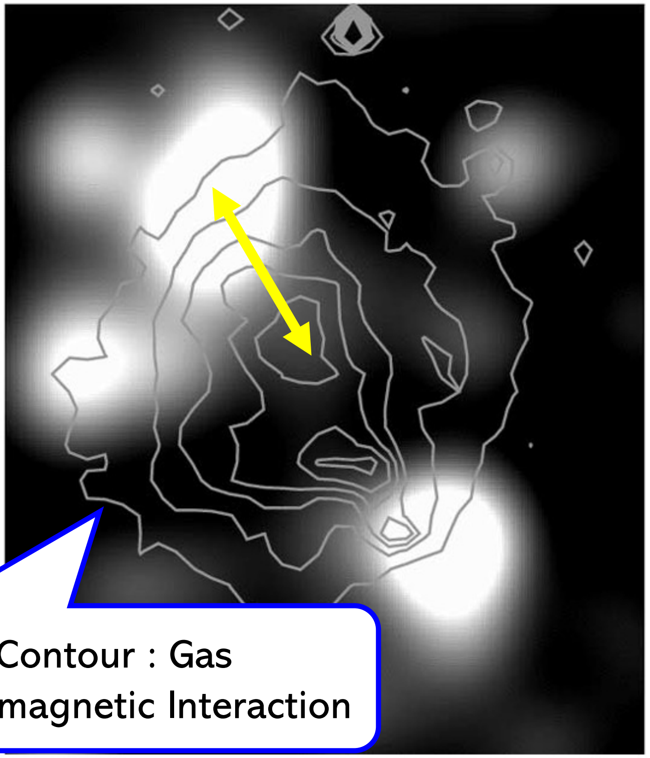
Abell 520 [A.Mahdavi et al. 0706.3048]

$$M = 10^{14} \sim 10^{15} M_{\odot}$$
$$v = 1000 \sim 2000 \mathrm{km/s}$$



Offset Between stars & DM

Contour : DM
Unknown Interaction…



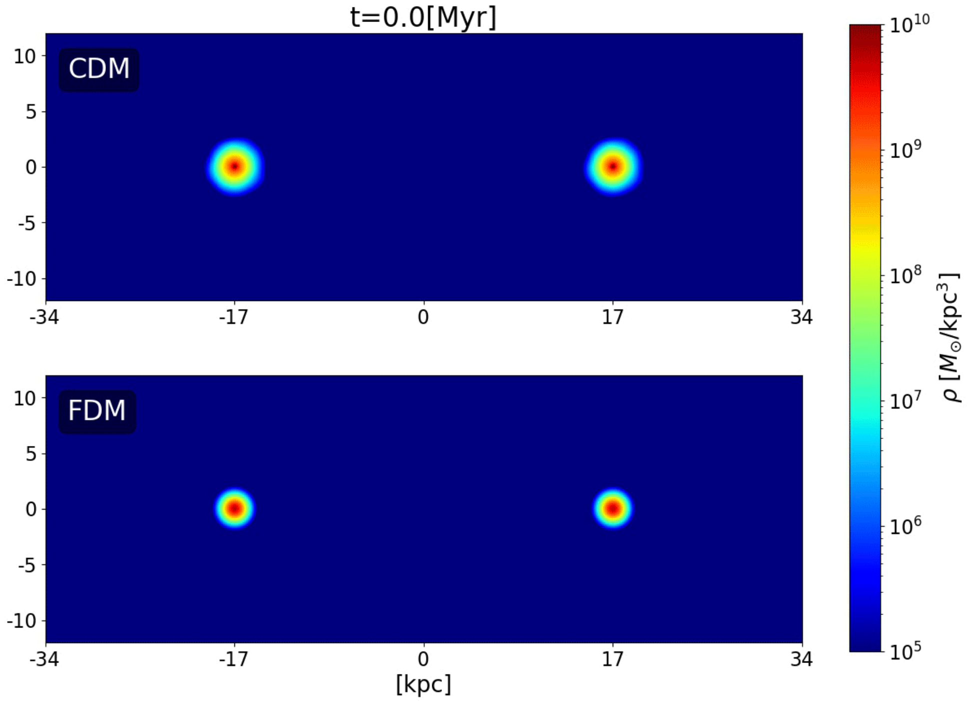
Contour : Gas
Electromagnetic Interaction

Head-on Collision of Identical Subhalos

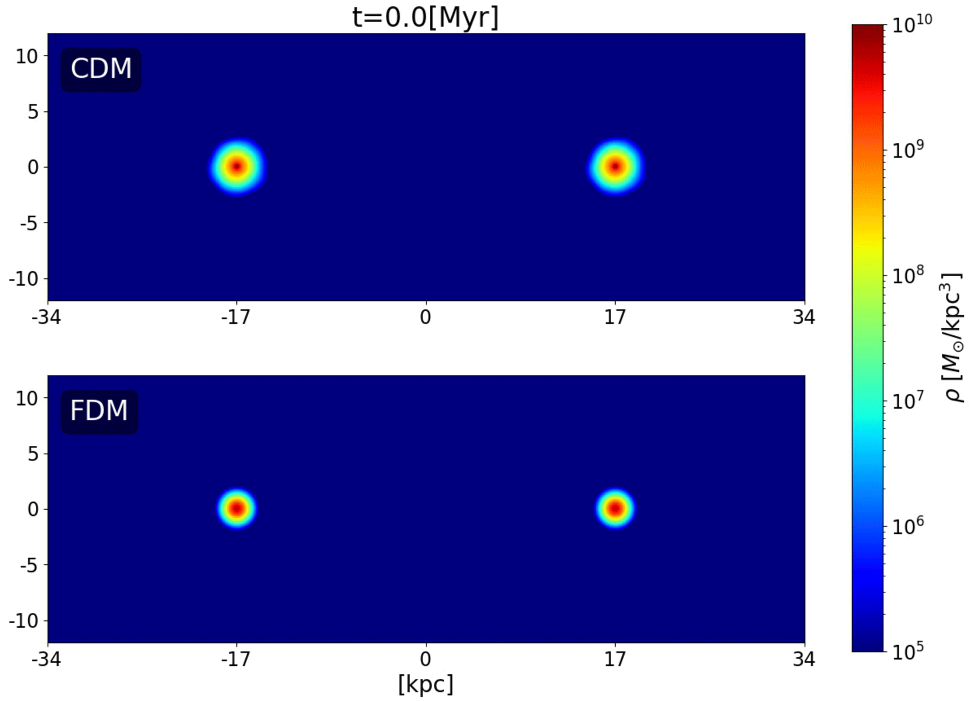
[H.Koo et al. 24XX.XXXXX]

- $M = 2\pi\times10^{8}M_{\odot}, m_a = 10^{-22}\mathrm{eV}/c^2$

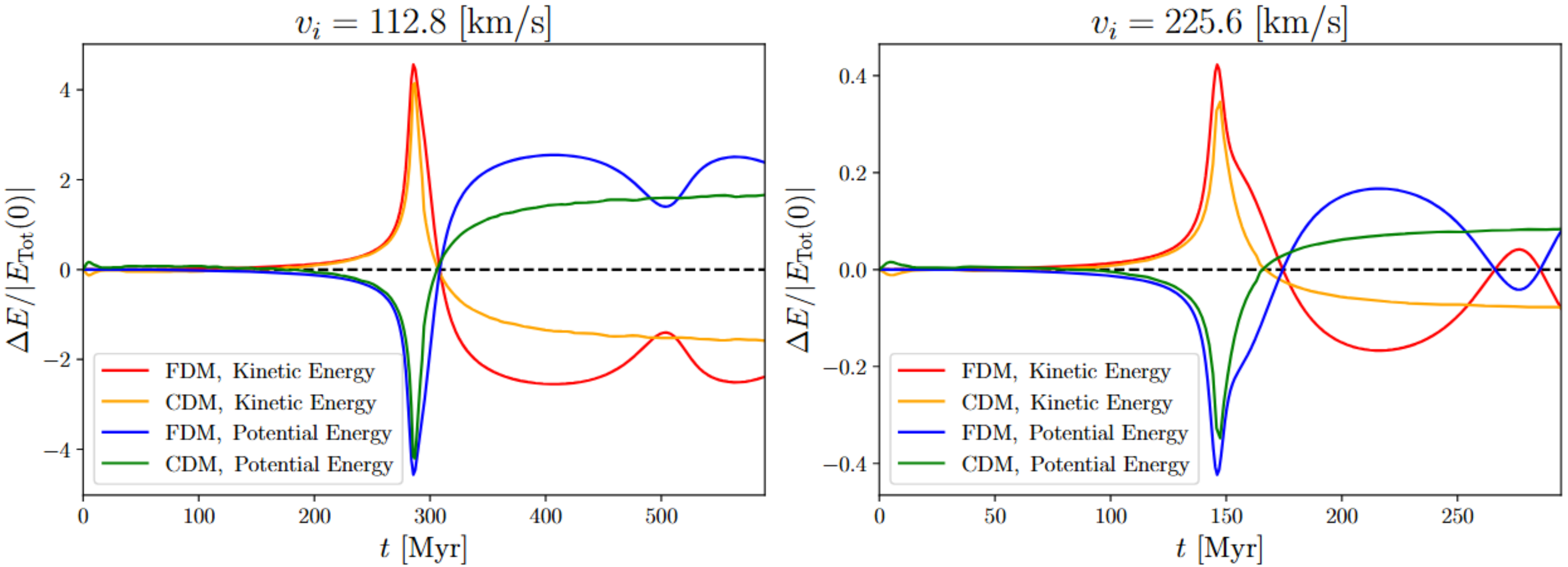
$$v_0^{\mathrm{rel}} = 112.78\mathrm{km/s}$$



$$v_0^{\mathrm{rel}} = 225.56\mathrm{km/s}$$

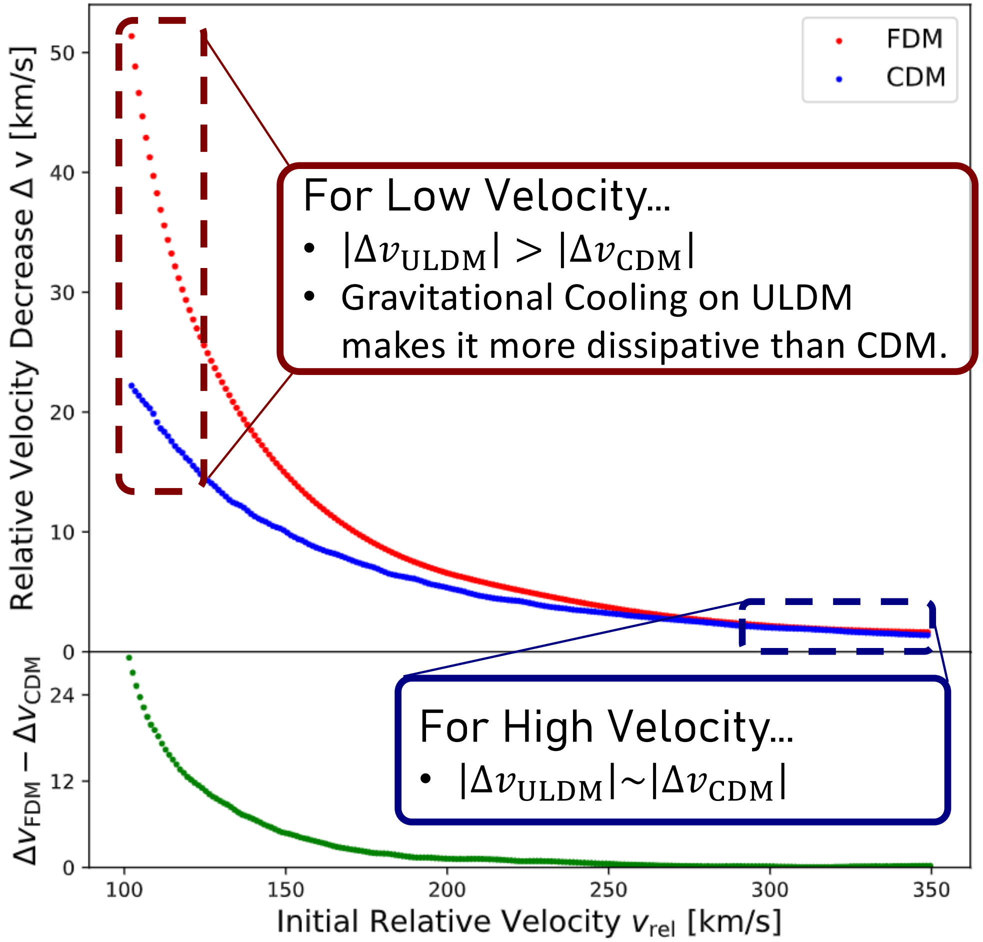


[H.Koo et al. 24XX.XXXXX]



- Kinetic energy dissipation by gravitational cooling is effective at low-speed collision.

[H.Koo et al. 24XX.XXXXX]



For Low Velocity…

- $|\Delta v_{\rm ULDM}| > |\Delta v_{\rm CDM}|$
- Gravitational Cooling on ULDM makes it more dissipative than CDM.

For High Velocity…

- $|\Delta v_{\rm ULDM}| \sim |\Delta v_{\rm CDM}|$

$$\bar{q} \equiv \frac{GM}{v^2 r_{1/2}}$$

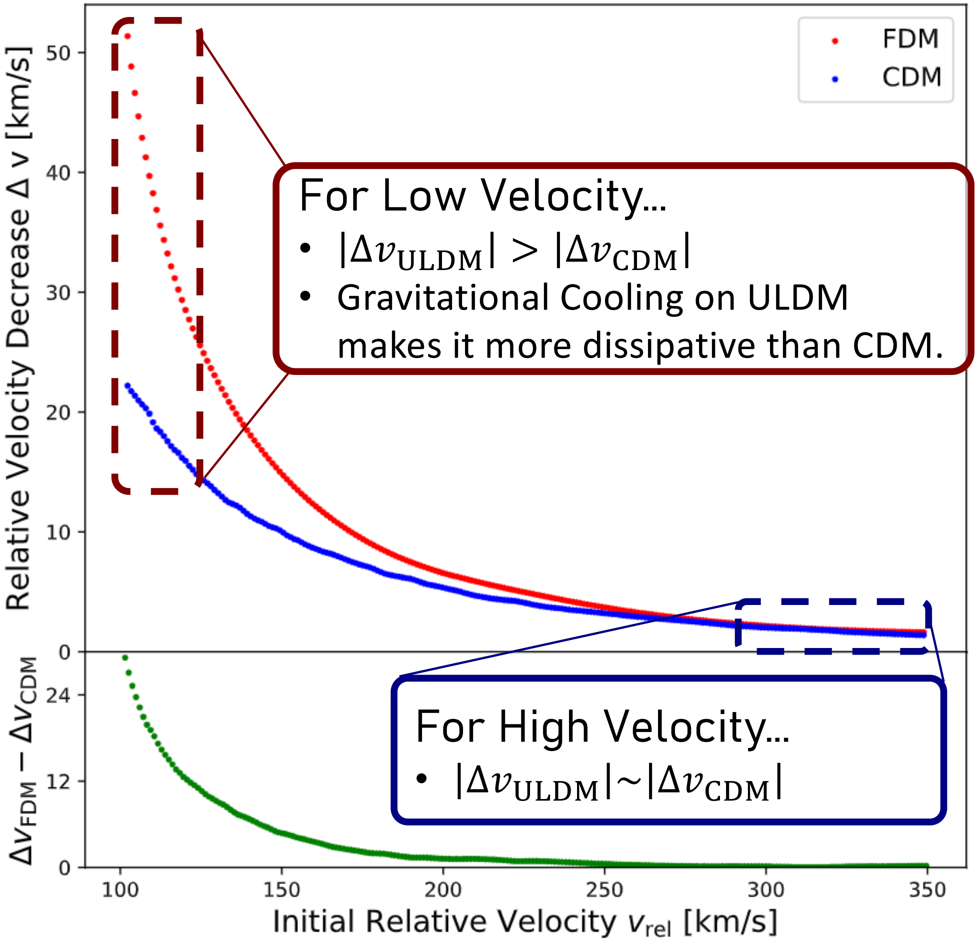
$$v = 100\sim350\mathrm{km/s} \xrightarrow{\qquad} \bar{q} = 0.041\sim0.492$$

$$\frac{\Delta v}{\Delta\tau_{\rm cross}} \propto \frac{1}{v^2}\log\Lambda$$

$$b_{\rm max}\sim r_{1/2} < GM/v^2$$

$$\left|\frac{\Delta v}{v}\right|_{\rm CDM} = A_c\bar{q}^2\left(\log\frac{1}{\bar{q}} + B_c\right) \begin{cases} A_c = 0.516 \pm 0.004 \\ B_c = 1.030 \pm 0.016 \end{cases}$$

[H.Koo et al. 24XX.XXXXX]



For Low Velocity…

- $|\Delta v_{\mathrm{ULDM}}| > |\Delta v_{\mathrm{CDM}}|$
- Gravitational Cooling on ULDM makes it more dissipative than CDM.

For High Velocity…

- $|\Delta v_{\mathrm{ULDM}}| \sim |\Delta v_{\mathrm{CDM}}|$

$$v = 100\sim 350\mathrm{km/s} \xrightarrow{q \equiv \frac{\hbar}{mvr_{1/2}}} q = 0.103\sim 0.360$$

$$\frac{\Delta v}{\Delta \tau_{\mathrm{cross}}} \propto \left[\frac{\Delta \tau_{\mathrm{cross}}}{\tau_{\mathrm{gc}}}\right]$$

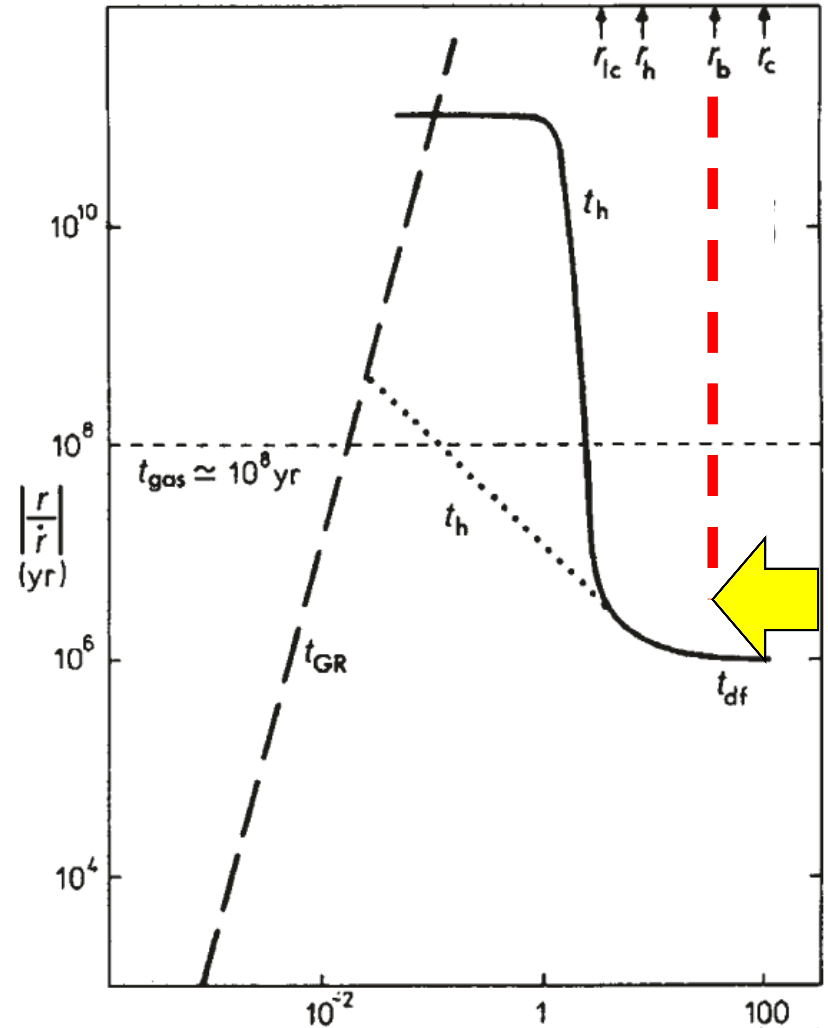
Dissipation Fraction by Gravitational Cooling

- Dissipation effect by GC ≫ DF

$$\left|\frac{\Delta v}{v}\right|_{\mathrm{ULDM}} = A_u q^3 (1 + B_u q^2) \begin{cases} A_u = 3.692 \pm 0.041 \\ B_u = 15.63 \pm 0.294 \end{cases}$$

Fitting Function Derived from [Dongsu Bak et al. 2010.14738]

Can ULDM resolve Final Parsec Problem?

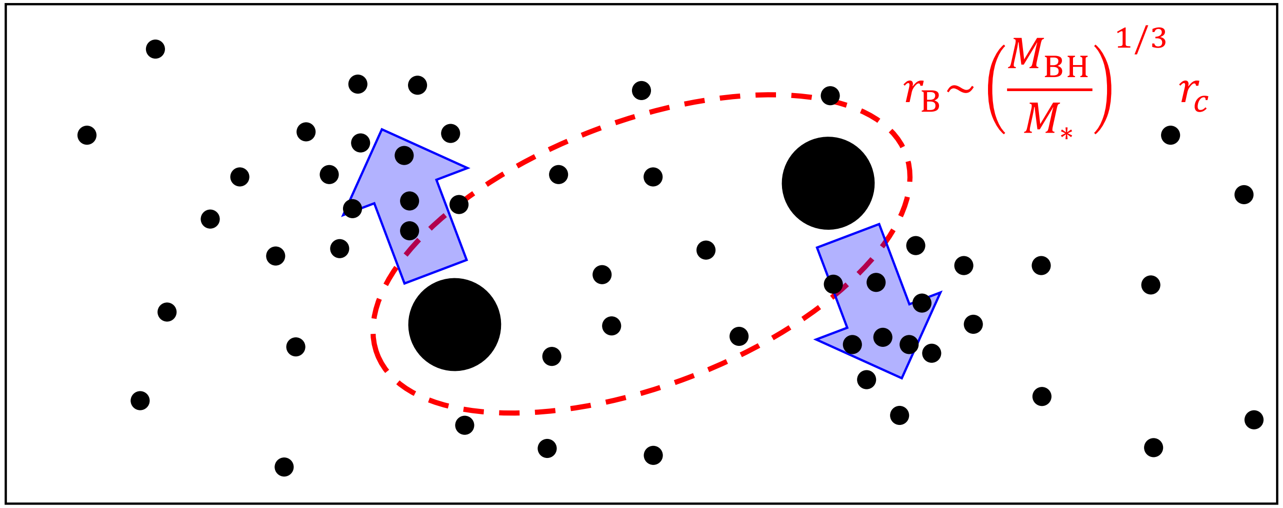


[Begelman, Blandford, Rees, 1980]

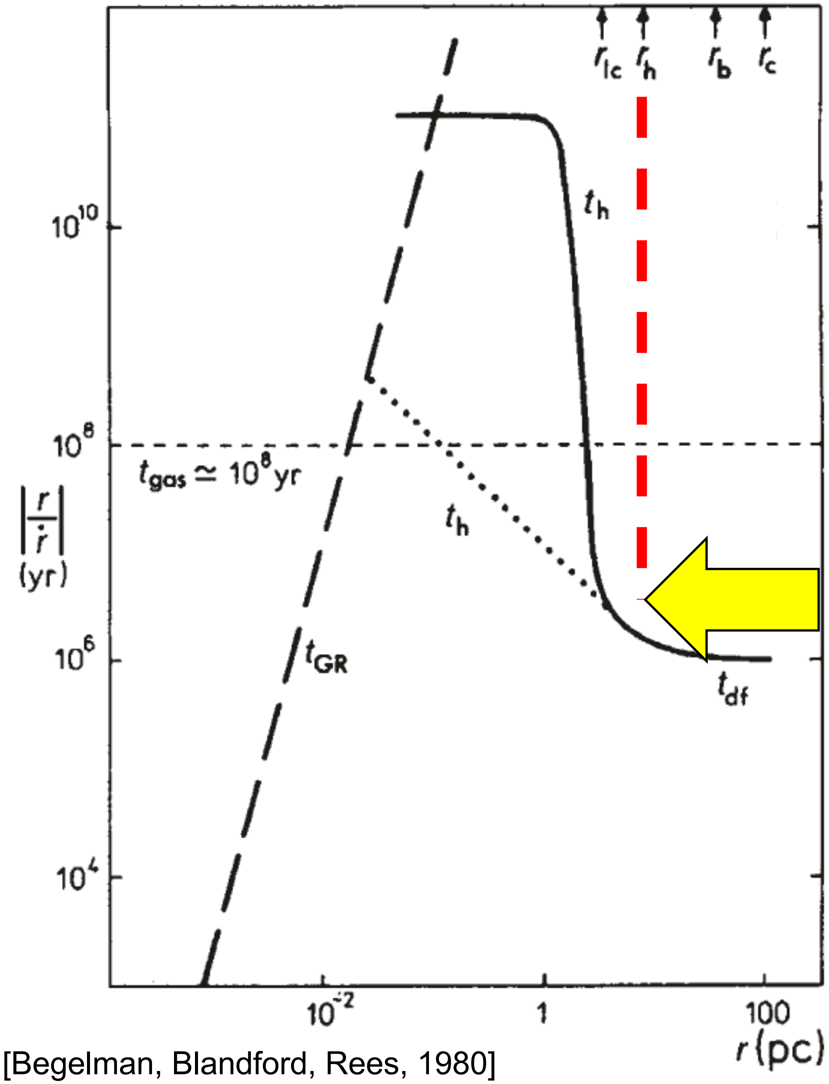
Orbital Decay of SMBHs

1. Surrounding stars near SMBHs (spherical bulge of N_*, σ_*, r_*) cause dynamical friction on them, leads to form SMBH Binary with timescale:

$$t_{\rm df} \simeq \frac{6\times10^6\rm yr}{\ln N_*}\frac{\sigma_*}{300\rm km/s}\left(\frac{r_*}{100\rm pc}\right)^2\frac{10^8 M_\odot}{M_{\rm bh}}$$



Orbital Decay of SMBHs

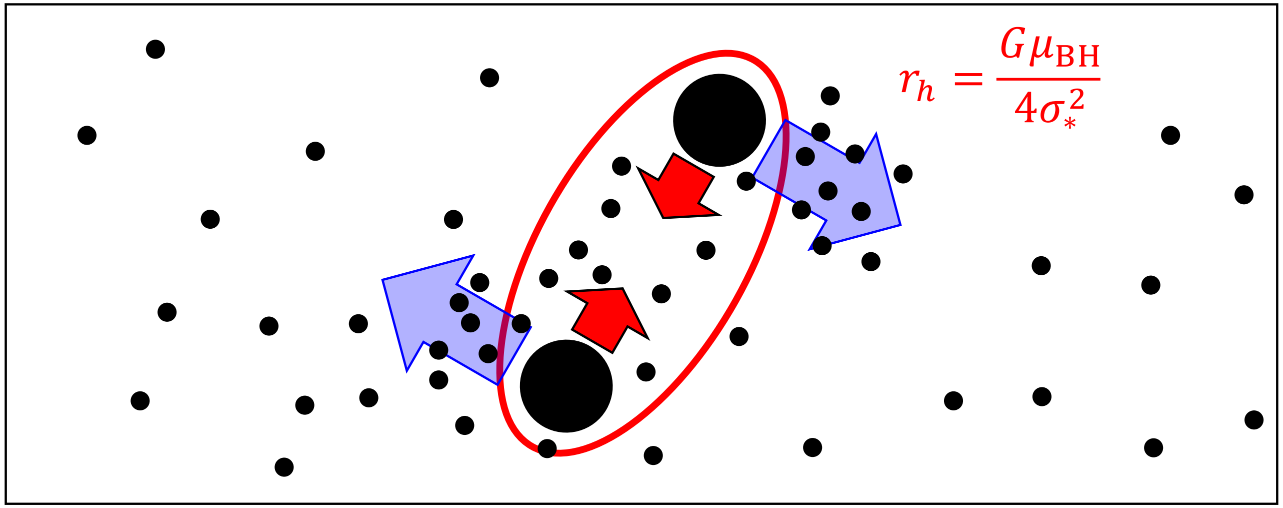


[Begelman, Blandford, Rees, 1980]

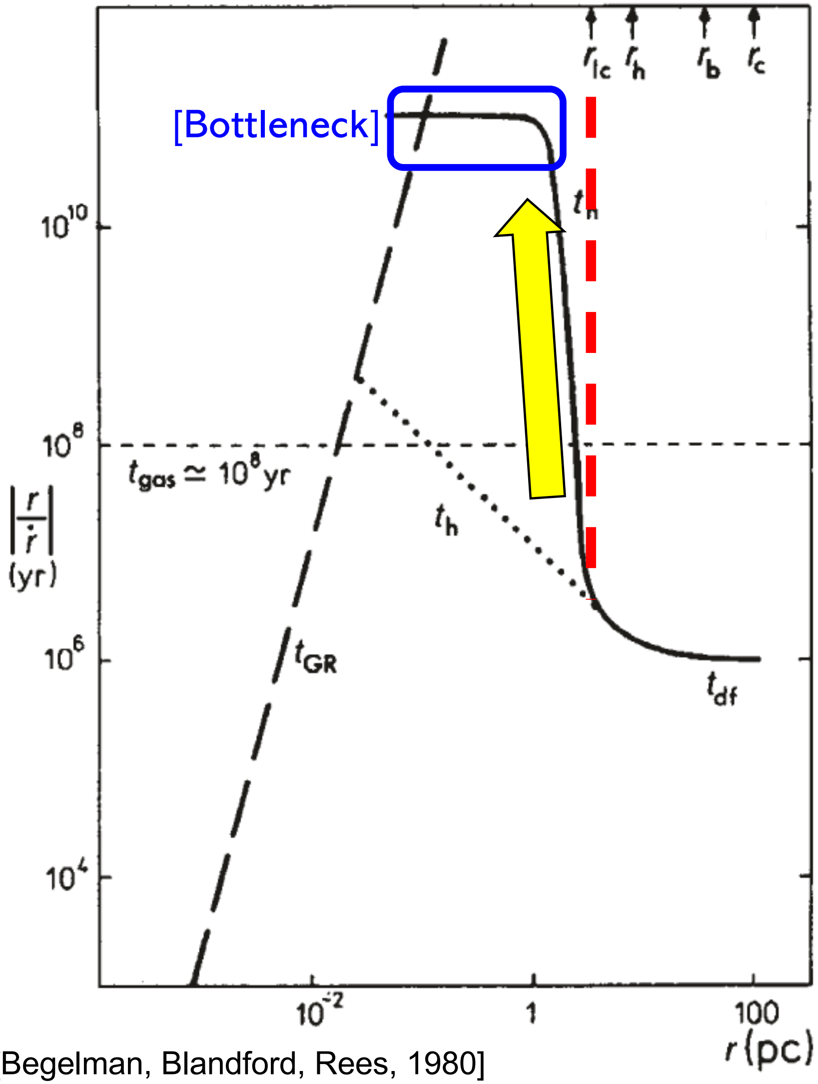
2. When the mass enclosed by the orbit of the SMBH binary is comparable to the mass of the binary itself, binary becomes "<u>Hard</u>".

$$t_h \sim \frac{\sigma_*}{\pi G \rho_* r}$$

[B.C.Bromley et al. 2311.18013]



$$r_h = \frac{G\mu_{\rm BH}}{4\sigma_*^2}$$

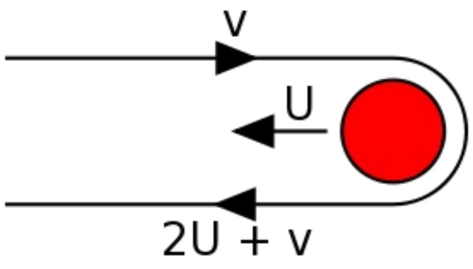


[Begelman, Blandford, Rees, 1980]

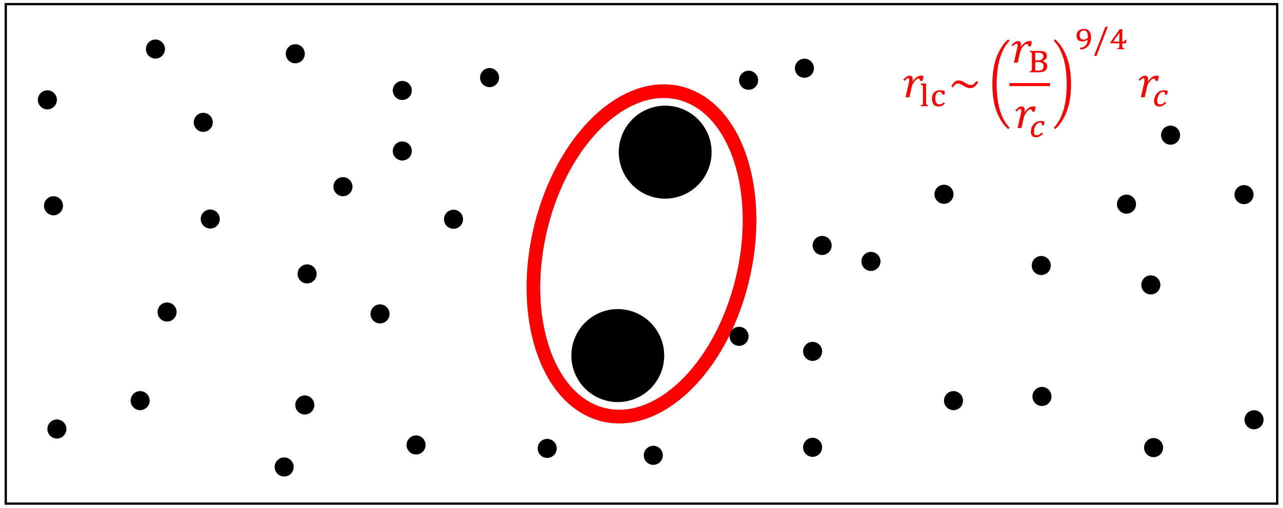
Orbital Decay of SMBHs

3. Most of stars are blown away by slingshot!

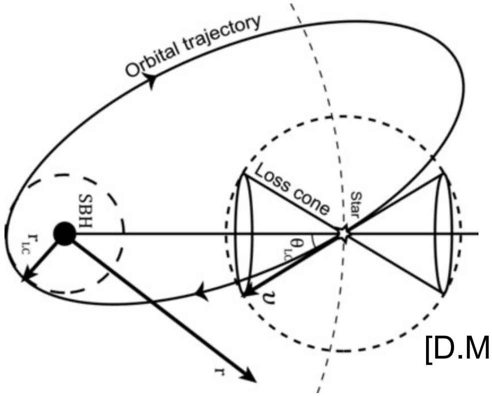
- Not enough stars to decay the SMBHs' orbit…



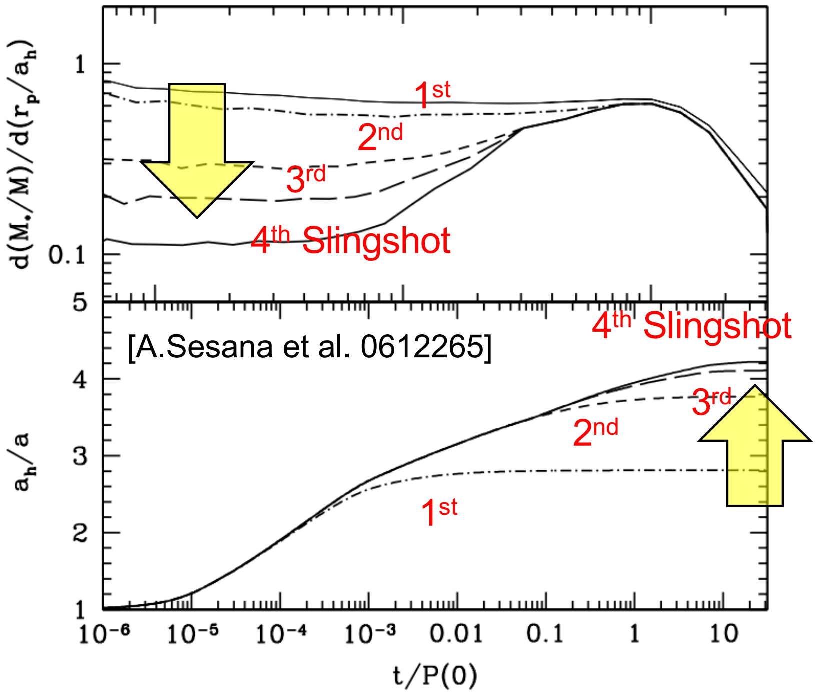
[Diagram of Slingshot]



$$r_{\mathrm{lc}} \sim \left(\frac{r_{\mathrm{B}}}{r_c}\right)^{9/4} r_c$$



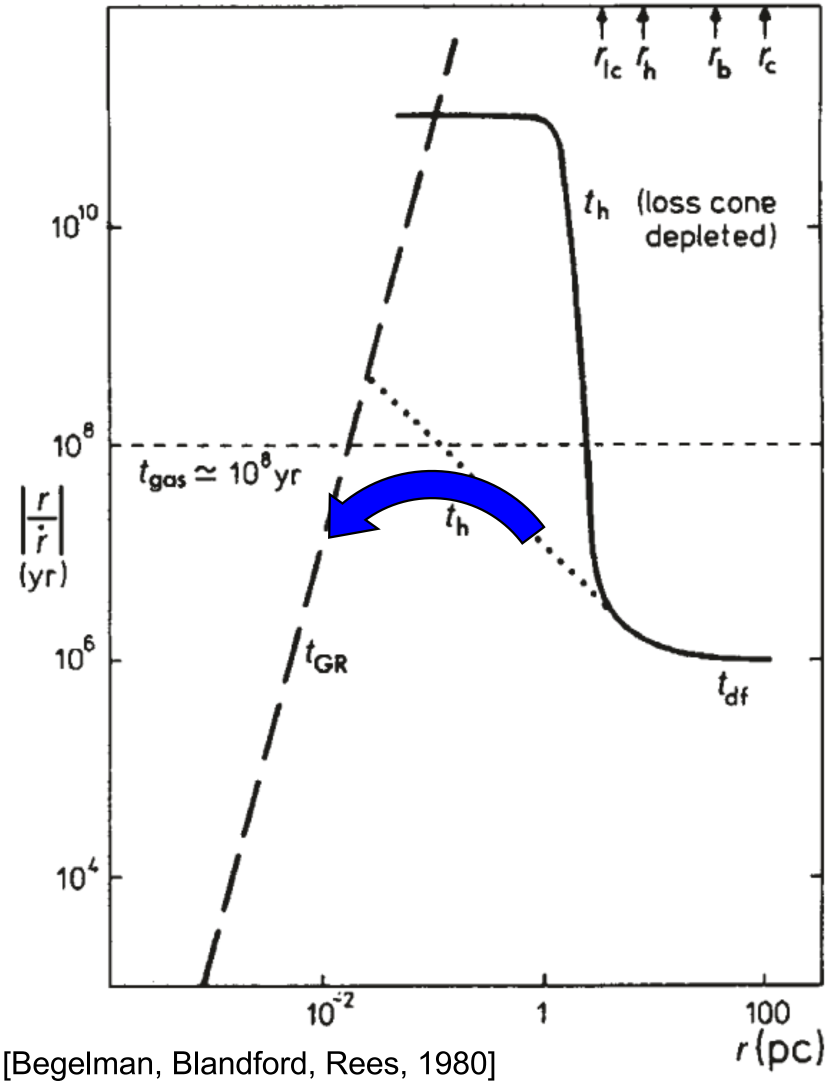
[D.Merritt 1307.3268]



[A.Sesana et al. 0612265]

- Loss cone : Region of Phase space in which stars have close encounters with SMBH binary
- The stellar mass inside loss cone M_* decreases as the average number of slingshots increases, and $\frac{a_h}{a}$ converges to a specific behavior.
- The SMBH binary decay timescale explosively increases beyond $\sim 10\text{Gyr}$.

Loss Cone is Depleted!

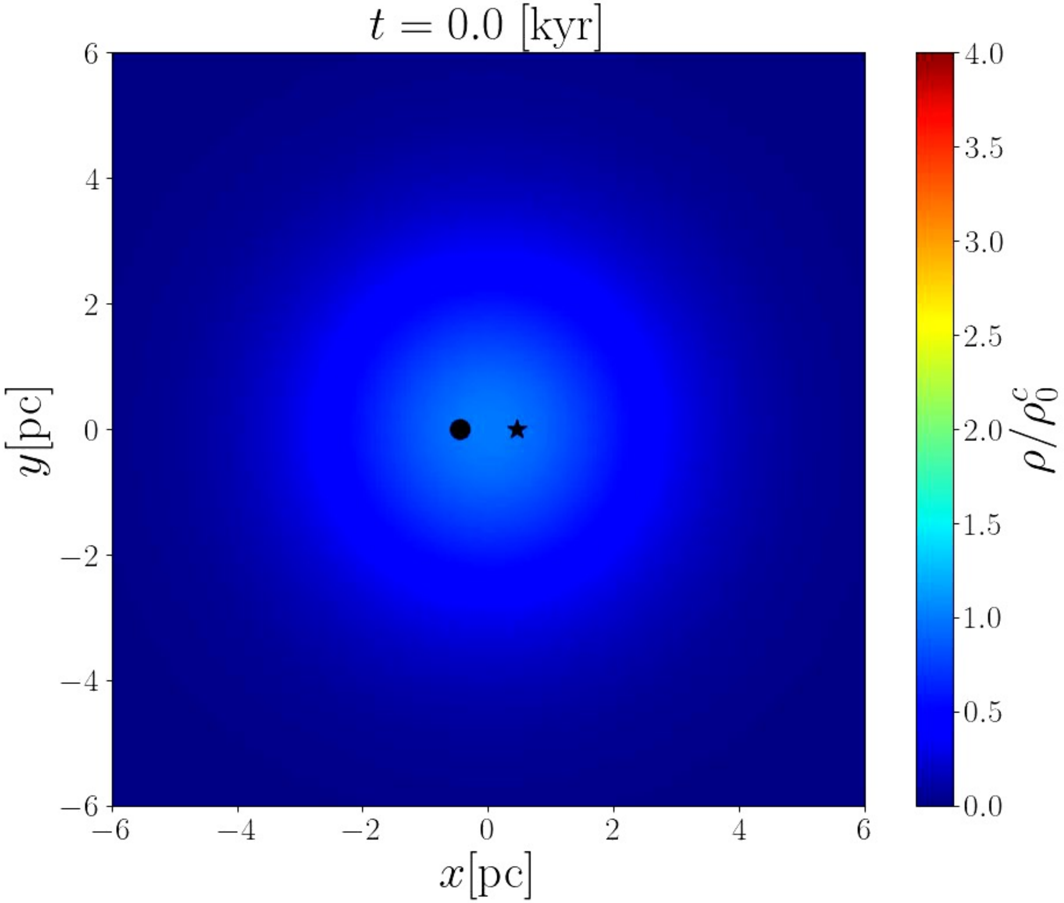


[Begelman, Blandford, Rees, 1980]

- To allow SMBHB to reach an era of gravitational radiation within $\sim H_0^{-1} \sim 10\text{Gyr}$, we need a new mechanism to refill the loss cone.

Supermassive BH Binary in ULDM Halo

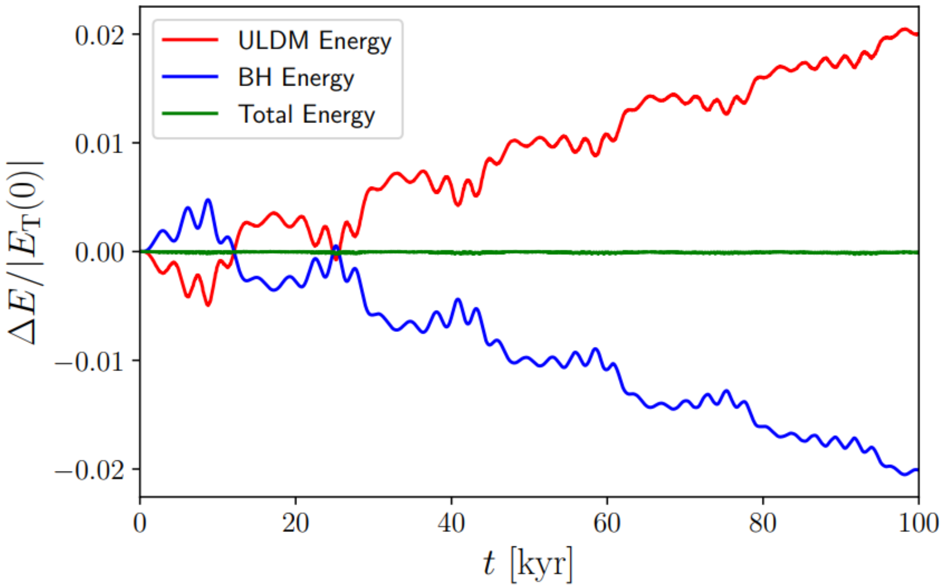
[H.Koo et al. 2311.03412]

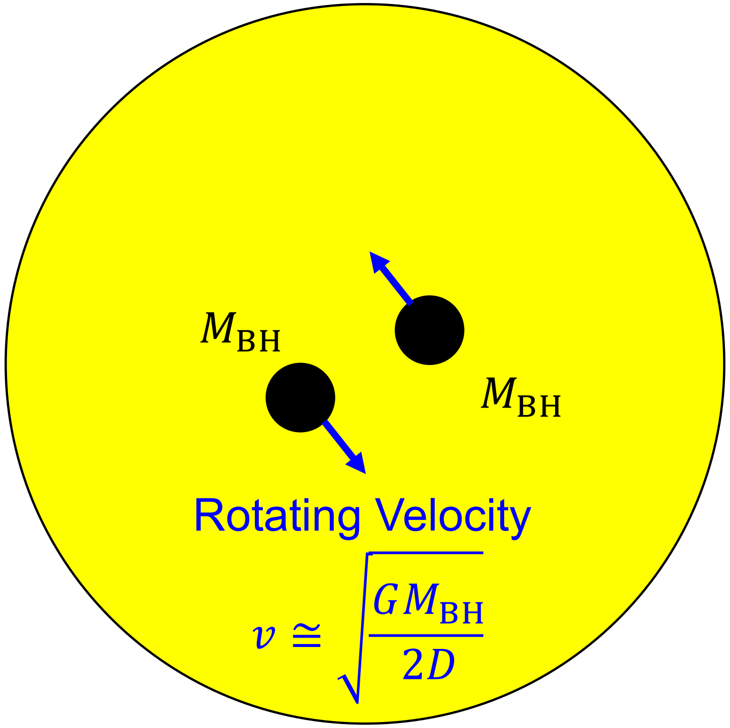


$$m = 10^{-21}\mathrm{eV}\,;\begin{cases} M_\mathrm{s} = 10^9 M_\odot \\ M_\mathrm{bh} = 10^8 M_\odot \end{cases};L = 40\mathrm{pc}$$

Too high ρ_0? ⇒ DM spike near SMBH makes sense

[T.Lacroix 1801.01308]





$$v \cong \sqrt{\frac{GM_{\mathrm{BH}}}{2D}}$$

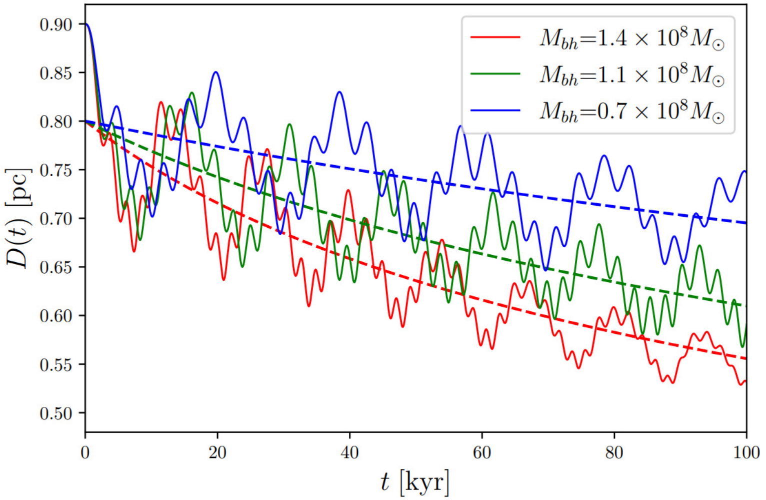
$$M_{\mathrm{bh}}\left|\frac{dv}{dt}\right| = 4\pi\bar{\rho}\left(\frac{GM_{\mathrm{bh}}}{v}\right)^2 C(\tilde{r}) = 4\pi\bar{\rho}\left(\frac{GM_{\mathrm{bh}}}{v}\right)^2 \frac{1}{3}\left(\frac{m_a v r}{\hbar}\right)^2$$

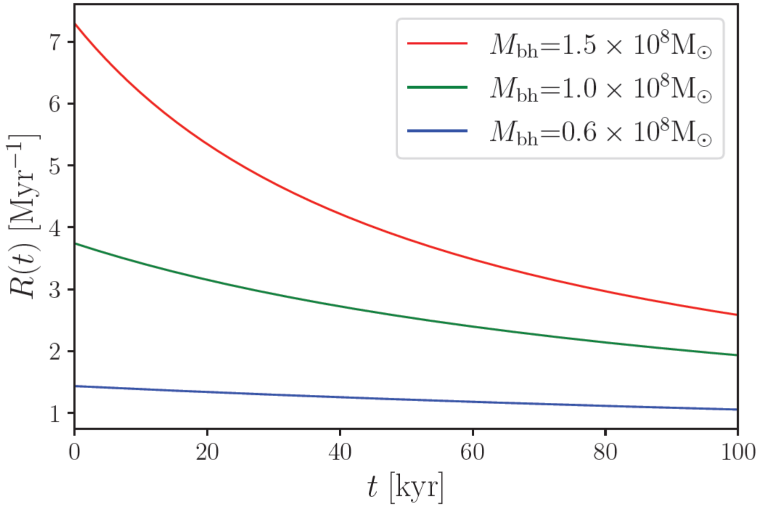
$$\bar{\rho} \simeq \rho_c \equiv 0.0044\left(\frac{Gm_a^2}{\hbar^2}\right)^3 (M_{\mathrm{sol}} + 2\gamma M_{\mathrm{bh}})^4$$

- SMBH binary separation $D(t) = 2r$ has a form:

$$D(t) = D_0\left(1 + \frac{5}{2} Q_0 D_0^{5/2} t\right)^{-2/5}$$

$$Q_0 \cong \frac{7.3524}{[\mathrm{Myr}\cdot\mathrm{pc}^{2.5}]}\left(\frac{\widetilde{M}_{\mathrm{sol}}}{10^9 M_\odot}\right)^4 \left(\frac{M_{\mathrm{bh}}}{10^8 M_\odot}\right)^{1/2} \left(\frac{m}{10^{-21}\mathrm{eV}}\right)^8$$





Decay of binary separation is decomposed to:

1. Global Mode (GC) : Exerting ULDM by breathing with period τ

$$\tau = \tau_0 \left(\frac{10^{-21}\mathrm{eV}}{m_a}\right)^3 \left(\frac{10^9 M_\odot}{M_s}\right)^2 \left(1 + 2\gamma_g \frac{M_{\mathrm{bh}}}{M_s}\right)^{-2}$$

$$\begin{cases} \gamma_g = 1.979 \pm 0.012 \cong 2 \\ \tau_0 = (32.11 \pm 0.115)\mathrm{kyr} \end{cases}$$

2. Local Mode (DF) : Orbital Decay of SMBHB

$$\bar{D}(t) = \bar{D}_0 \left(1 + \frac{5}{2} Q \bar{D}_0^{\frac{5}{2}} t\right)^{-\frac{2}{5}}$$

$$Q = 1.324 \left(\frac{m_a}{10^{-21}\mathrm{eV}}\right)^8 \left(\frac{M_s + 2\gamma_l M_{\mathrm{bh}}}{10^9 M_\odot}\right)^4 \left(\frac{M_{\mathrm{bh}}}{10^8 M_\odot}\right)^{\frac{1}{2}} \mathrm{Myr}^{-1}\mathrm{pc}^{\frac{5}{2}}$$

$$\gamma_l = 2.192 \pm 0.034 \cong 2$$

- Early Times : Gravitational Cooling is dominant

$$F_{\mathrm{GC}} = K_0 \left(\frac{\Delta\tau_c \equiv \pi D / v}{\tau_g} \right) \left(\frac{G \widetilde{M}_s M_{\mathrm{spike}}}{r_h^2} \right)$$

- Late Times : Dynamical Friction is dominant

$$F_{\mathrm{DF}} = 4\pi\rho \left(\frac{G M_{\mathrm{bh}}}{v} \right)^2 \times \frac{1}{3} \left(\frac{m v D}{2\hbar} \right)^2$$

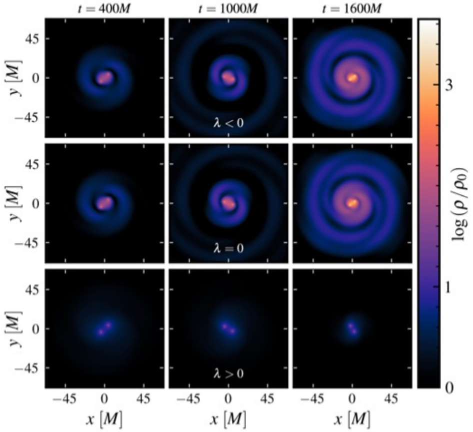
$$\frac{F_{\mathrm{GC}}}{F_{\mathrm{DF}}} \sim D^{\frac{1}{2}}$$

- Initially, gravitational cooling exerts total system’s energy (Wave Mechanics)⇒ Dynamical friction reduces SMBH’s energy & angular momentum (Classical-like Mechanics)

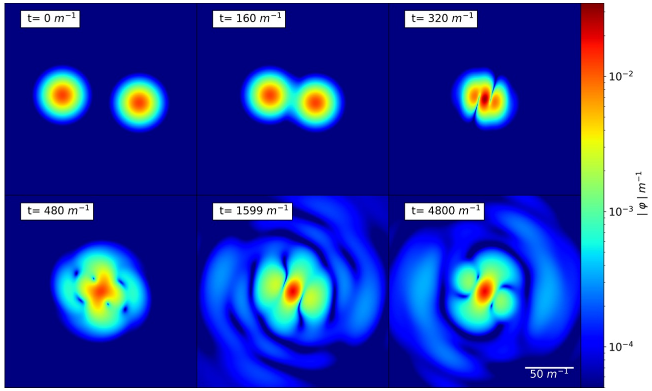
Epilogue : Research Topics of ULDM with GR

(My Private Interest)

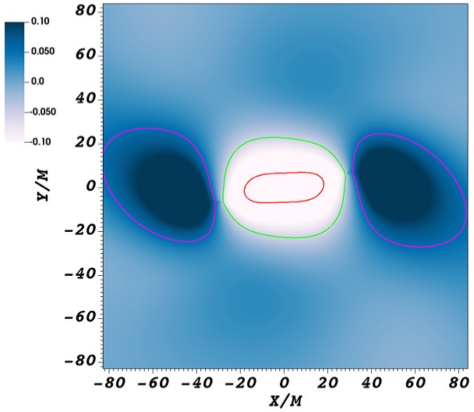
ULDM with Numerical Relativity



BBH Merger inside uniform ULDM [GRChombo 2409.01937]



ULDM Soliton (Boson star) Merger [GRChombo 2207.05690]



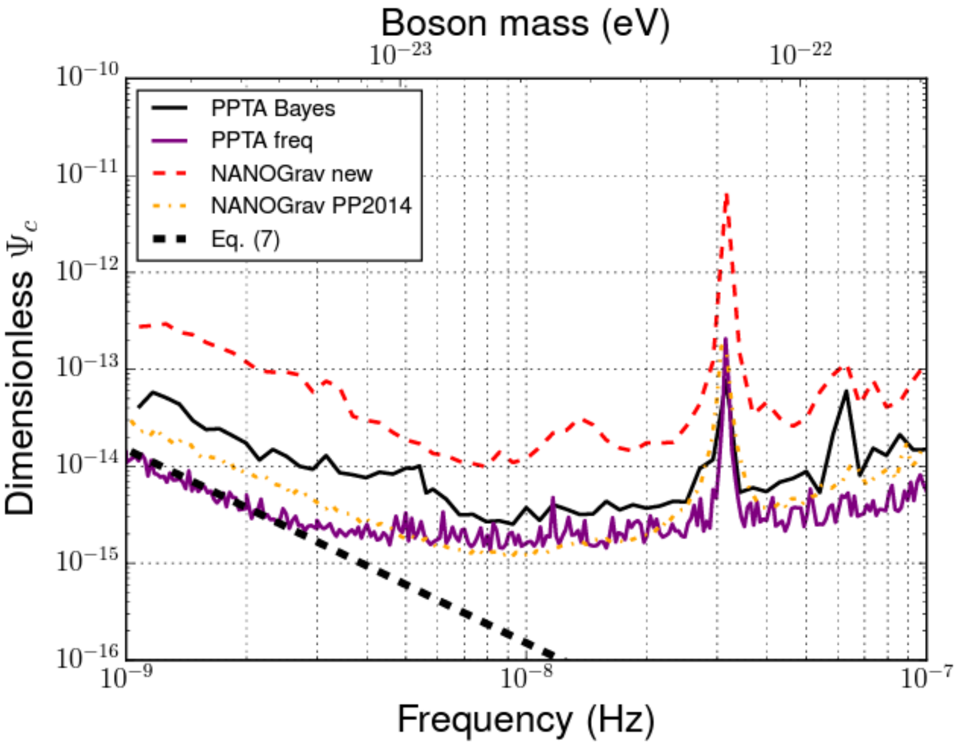
Gravitational Atom formed by ULDM around SMBH Binary [EinsteinToolkit 2010.00008]

- Einstein Toolkit [P.Diener et al. 1111.3344]
- GRChombo [K.Clough et al. 1503.03436]

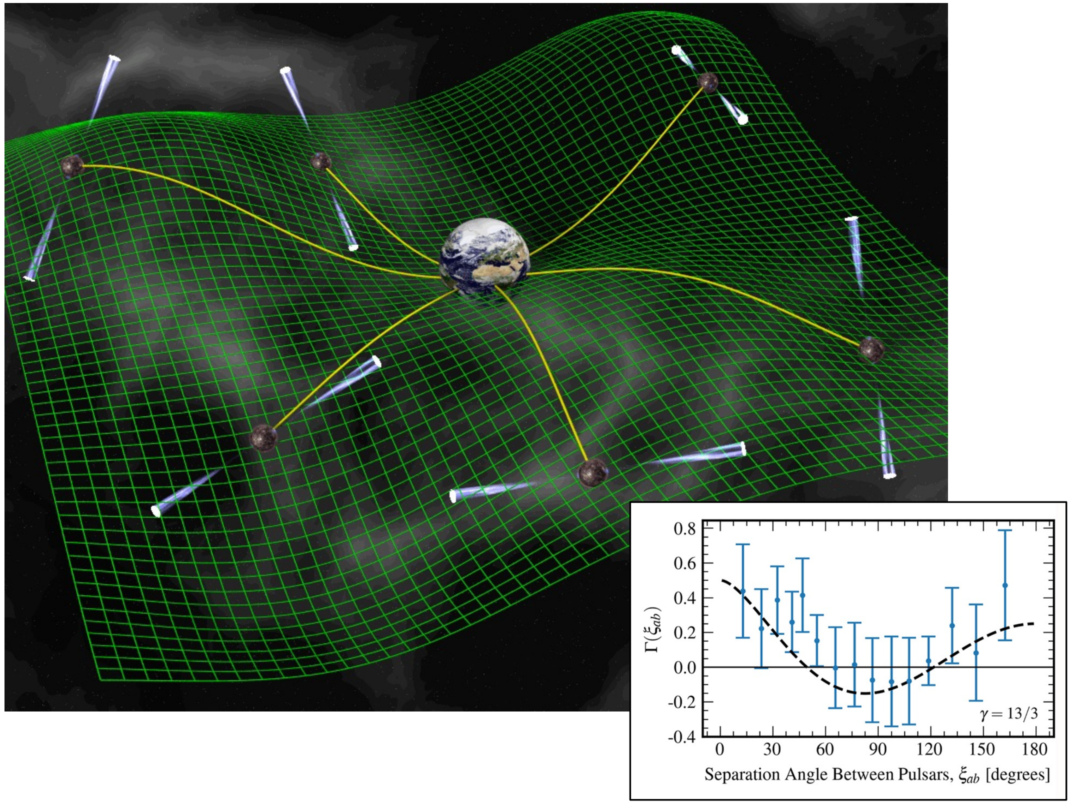
- For $\sim 0.01\text{pc}$, simulating the dynamics of SMBH binary is difficult to use GPP solver

- To develop, numerical relativity is necessary

PTA Signal from ULDM



Parkes PTA Constraints on ULDM [N.K.Porayko et al. 1810.03227]



- ULDM background with oscillation $\omega \sim m$ has a frequency of $O(\mathrm{nHz}) \Rightarrow$ Detectable by PTA

Nonminimally Couples to Gravity?

$$S_{\text{tot}} = S_{\text{EH}} + S_{\phi} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - \frac{1}{2}m_a^2\phi^2 - \frac{1}{2}\xi R\phi^2\right]$$

- Crucial to the renormalizability of a scalar-field theory in curved spacetime
- Motion equations for Newtonian limit

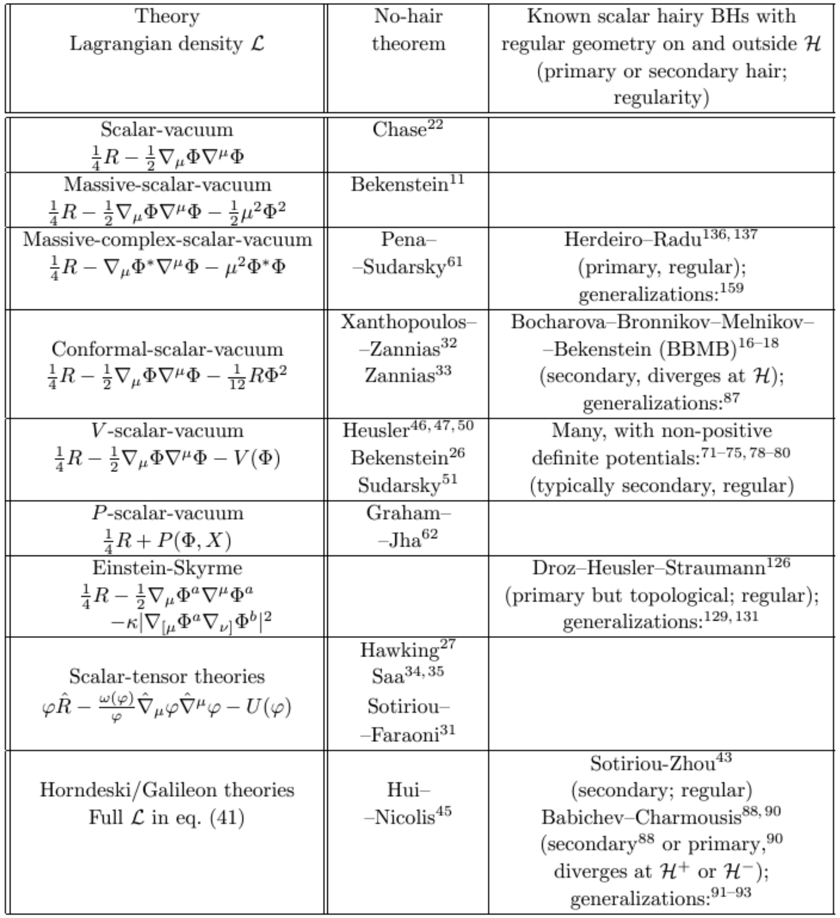
$$\left.\begin{array}{c} -\dfrac{\hbar^2}{2m_a}\nabla^2\psi + m_a\Phi\psi + \xi\mathcal{V}(\psi) = i\hbar\dfrac{\partial\psi}{\partial t} \\ \nabla^2\Phi = m_a\mathcal{V}(\psi) \end{array}\right\} \leftarrow \mathcal{V}(\psi) = \frac{4\pi G|\psi|^2}{1 - 8\pi\xi G|\psi|^2/m_a}$$

[L.Ji 2106.11971]

- For small nonminimal coupling: $\xi \ll \frac{m_a^2}{8\pi G\rho}$, motion equations is similar as GPP system.

Black Hole Hair from ULDM

[C.A.R.Herdeiro et al. 1504.08209]

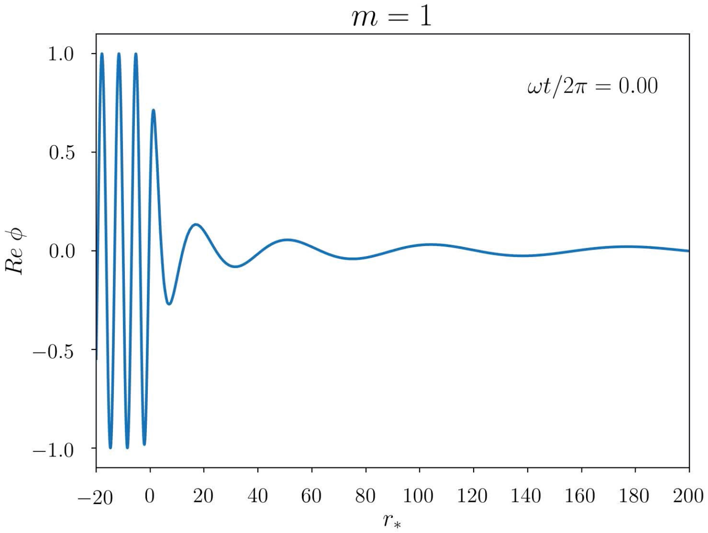


Theory Lagrangian density $\mathcal{L}$	No-hair theorem	Known scalar hairy BHs with regular geometry on and outside $\mathcal{H}$ (primary or secondary hair; regularity)
Scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi$	Chase[22]	
Massive-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{2}\mu^2\Phi^2$	Bekenstein[11]	
Massive-complex-scalar-vacuum $\frac{1}{4}R - \nabla_\mu\Phi^*\nabla^\mu\Phi - \mu^2\Phi^*\Phi$	Pena– –Sudarsky[61]	Herdeiro–Radu[136, 137] (primary, regular); generalizations:[159]
Conformal-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}R\Phi^2$	Xanthopoulos– –Zannias[32] Zannias[33]	Bocharova–Bronnikov–Melnikov– –Bekenstein (BBMB)[16–18] (secondary, diverges at $\mathcal{H}$); generalizations:[87]
V-scalar-vacuum $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi - V(\Phi)$	Heusler[46, 47, 50] Bekenstein[26] Sudarsky[51]	Many, with non-positive definite potentials:[71–75, 78–80] (typically secondary, regular)
P-scalar-vacuum $\frac{1}{4}R + P(\Phi, X)$	Graham– –Jha[62]	
Einstein-Skyrme $\frac{1}{4}R - \frac{1}{2}\nabla_\mu\Phi^a\nabla^\mu\Phi^a$ $-\kappa\lvert\nabla_{[\mu}\Phi^a\nabla_{\nu]}\Phi^b\rvert^2$		Droz–Heusler–Straumann[126] (primary but topological; regular); generalizations:[129, 131]
Scalar-tensor theories $\varphi\hat{R} - \frac{\omega(\varphi)}{\varphi}\hat{\nabla}_\mu\varphi\hat{\nabla}^\mu\varphi - U(\varphi)$	Hawking[27] Saa[34, 35] Sotiriou– –Faraoni[31]	
Horndeski/Galileon theories Full $\mathcal{L}$ in eq. (41)	Hui– –Nicolis[45]	Sotiriou-Zhou[43] (secondary; regular) Babichev–Charmousis[88, 90] (secondary[88] or primary,[90] diverges at $\mathcal{H}^+$ or $\mathcal{H}^-$); generalizations:[91–93]

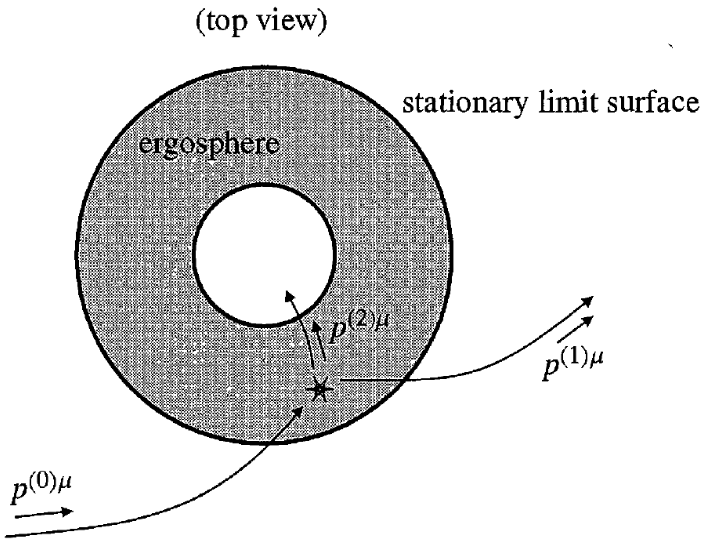
- No Hair Theorem : Only Mass, Angular Momentum, Electric Charge could determine a “Static” profile in the presence of BH

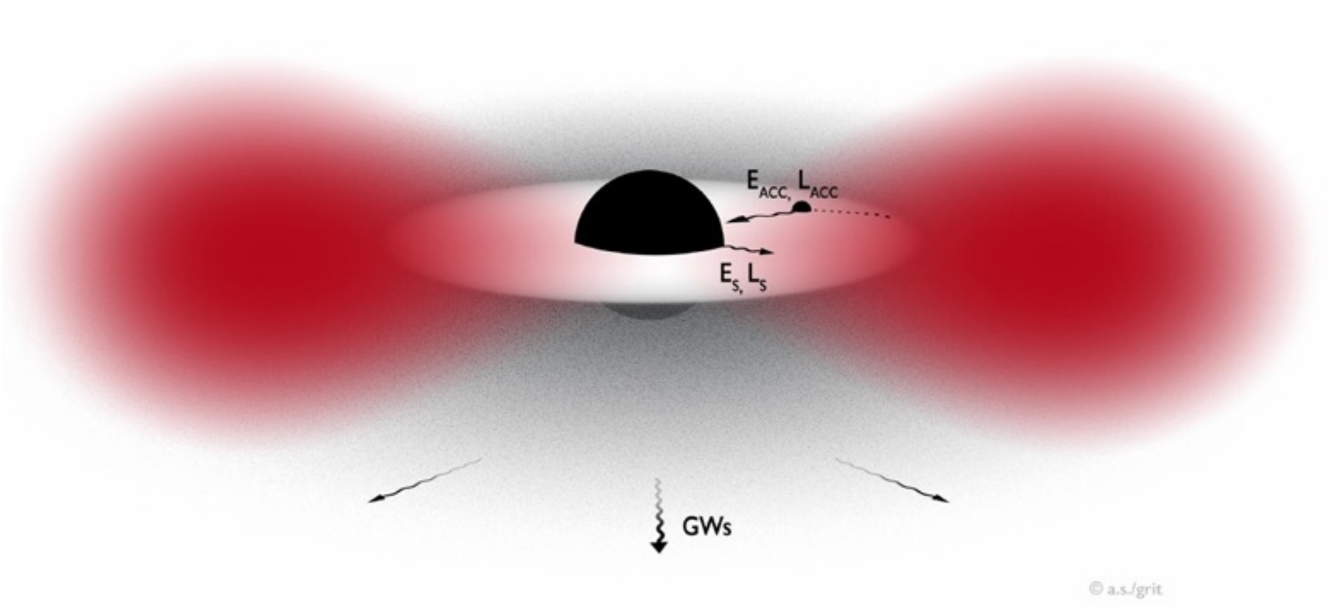
- ULDM Oscillation could give a hair to BH

[L.Hui et al. 1904.12803]



Superradiance





- Scalar field energy is amplified from ergosphere when $\omega < m\Omega_{\mathrm{H}}$
- Expected to change GW patterns from SMBH Binary

Summary

- ULDM of $m_a \sim 10^{-22}\mathrm{eV}/c^2$ leads $\lambda_{\mathrm{dB}} \sim O(\mathrm{kpc})$. This wave-like nature suppresses small-scale structure of our universe, so that solve some problems of CDM.
- The spherically-symmetric solution of GPP equations, corresponds to a “Soliton”, possesses a core of DM halo.
- Dynamical friction of ULDM is less than of CDM, especially for large repulsive self-interaction. This could solve a problem of the lifetime of Fornax dSph.
- Gravitational cooling, which evaporates energy of ULDM system, could explain observational results of galaxy cluster collision of Abell520.
- Both these effects could give a hint to solve final parsec problem.

Appendix

CDM Simulation

- GADGET4 : Parallel cosmological N-body & SPH code https://wwwmpa.mpa-garching.mpg.de/gadget4/#obtaining-the-code [V.Springel et al, 2010.03567]
- TreePM method is applied

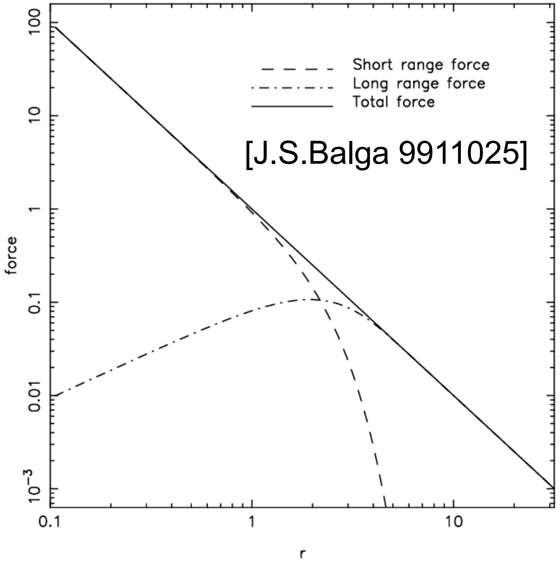
$$\Phi = \Phi_{\text{short-range}} + \Phi_{\text{long-range}}$$

Tree Method

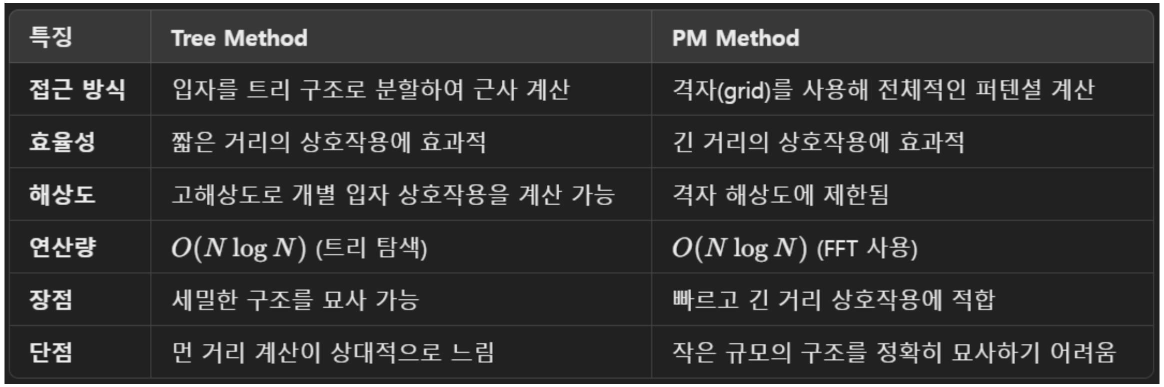
- Computational Cost $\sim O(N \log N)$

Particle-Mesh (PM) Method

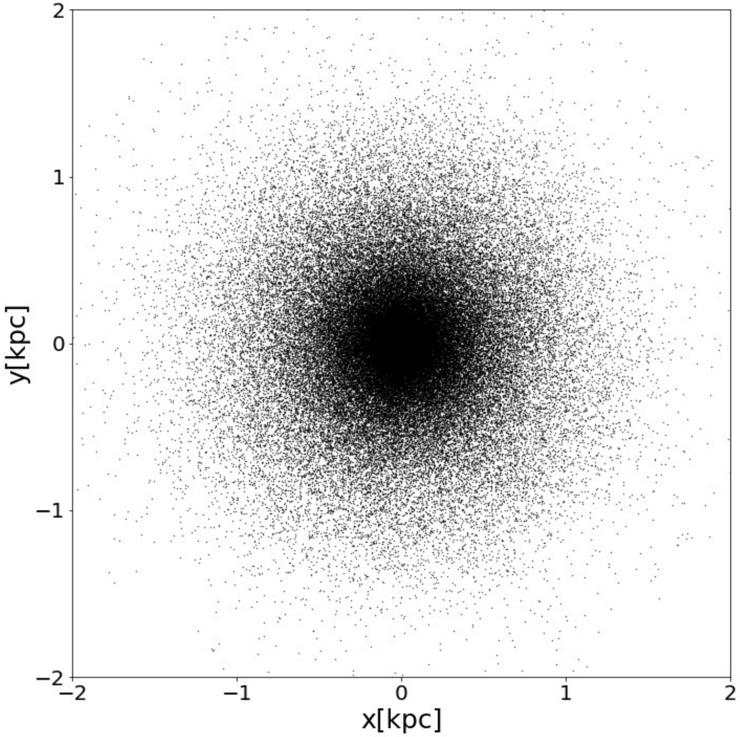
- Useful for paralyzing computation



[J.S.Balga 9911025]



특징	Tree Method	PM Method
접근 방식	입자를 트리 구조로 분할하여 근사 계산	격자(grid)를 사용해 전체적인 퍼텐셜 계산
효율성	짧은 거리의 상호작용에 효과적	긴 거리의 상호작용에 효과적
해상도	고해상도로 개별 입자 상호작용을 계산 가능	격자 해상도에 제한됨
연산량	$O(N \log N)$ (트리 탐색)	$O(N \log N)$ (FFT 사용)
장점	세밀한 구조를 묘사 가능	빠르고 긴 거리 상호작용에 적합
단점	먼 거리 계산이 상대적으로 느림	작은 규모의 구조를 정확히 묘사하기 어려움



Initial Profile

- Hernquist model with $N = 10^5$

$$\rho_{\mathrm{H}}(r) = \frac{\rho_0}{(r/a)(1+r/a)^3} \quad \begin{cases} M = 2\pi\rho_0 a^3 = 2\pi\times10^8 M_\odot \\ r_{1/2} = \left(\sqrt{2}+1\right)a \cong 0.533\mathrm{kpc} \end{cases}$$

- Distribution Function-based Equilibrium Model

$$f\left(\tilde{E}(\vec{x},\vec{v})\right) = \frac{1}{\sqrt{2}(2\pi)^3(GMa)^{3/2}}\frac{\sqrt{\tilde{E}}}{\left(1-\tilde{E}\right)^2}\left[\left(1-2\tilde{E}\right)\left(8\tilde{E}^2-8\tilde{E}-3\right)+\frac{3\sin^{-1}\sqrt{\tilde{E}}}{\sqrt{\tilde{E}\left(1-\tilde{E}\right)}}\right]$$

$$\tilde{E}(\vec{x},\vec{v}) = -\frac{a}{GM}\left(\frac{1}{2}v^2 - \frac{2\pi G\rho_0 a^2}{1+r/a}\right)$$

- Cut-off radius $R_{\mathrm{cut}} = 3r_{1/2} \Rightarrow$ To mimic the compactness of ULDM soliton
- Softening length $SL = 2r_{1/2}/\sqrt{N} \cong 3.377\mathrm{pc}$ [C.Power et al. 0201544]

- Evolution of ULDM energy density after t_{osc} respects $\rho_\phi \propto a^{-3}$

$$\rho_\phi(t_{\mathrm{osc}}) = \frac{1}{2} m_\phi^2 \phi_i^2 \quad \Rightarrow \quad \rho_\phi(t_{\mathrm{now}}) = \rho_\phi(t_{\mathrm{osc}}) \left[\frac{a(t_{\mathrm{osc}})}{a(t_{\mathrm{now}})}\right]^3 = \frac{1}{2} m_\phi^2 \phi_i^2 \left(\frac{T_{\mathrm{now}}}{T_{\mathrm{osc}}}\right)^3$$

- Temperature at t_{osc}…

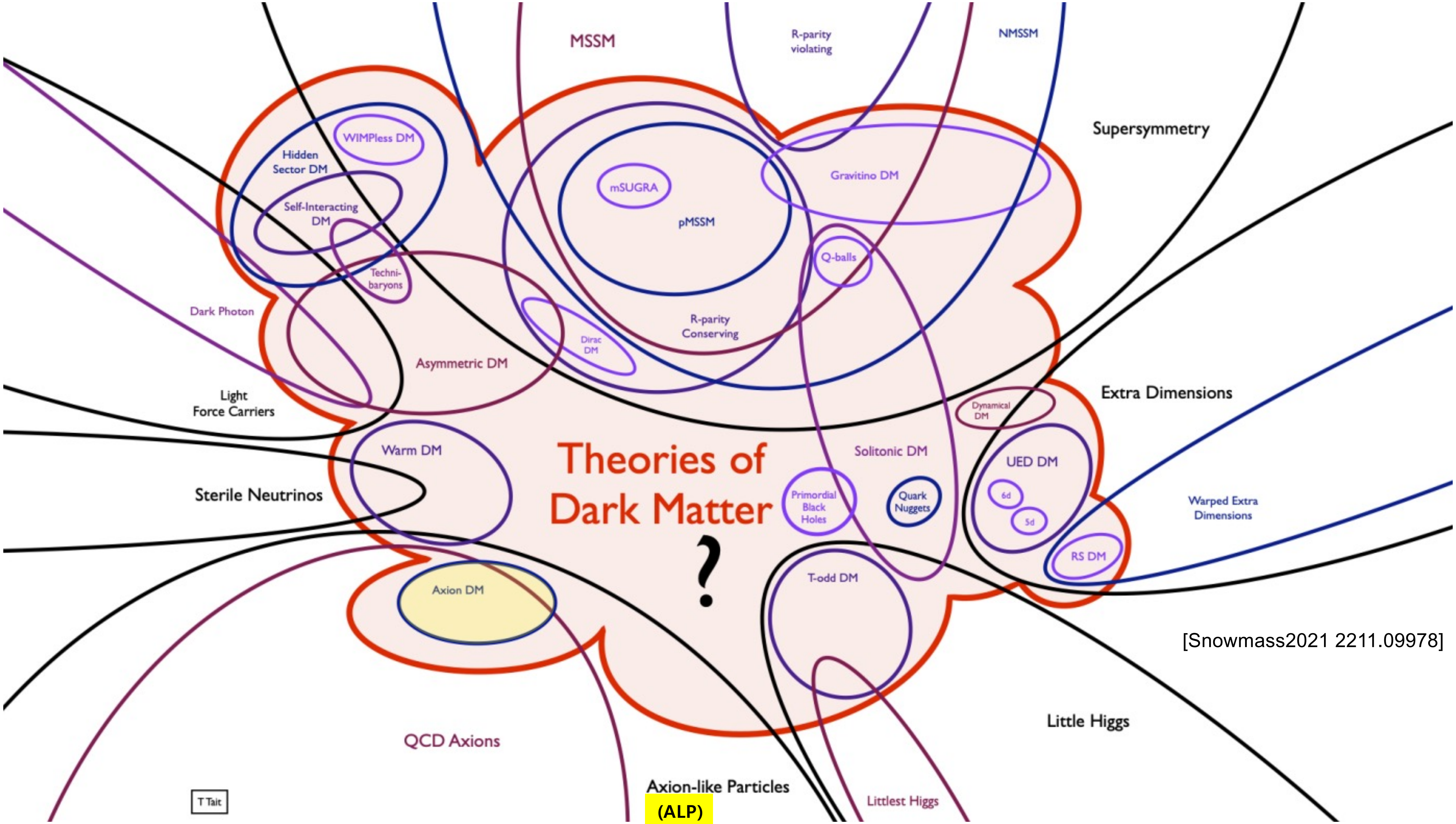
$$H_{\mathrm{osc}}^2 = \left(\frac{m_\phi}{3}\right)^2 = \frac{8\pi G}{3} \times \frac{\pi^2}{30} g_{*,\mathrm{osc}} T_{\mathrm{osc}}^4 \quad \Rightarrow \quad T_{\mathrm{osc}} \cong 4.115\times10^6\mathrm{K} \left(\frac{m_\phi}{10^{-22}\mathrm{eV}}\right)^{\frac{1}{2}}$$

Radiation energy density in t_{osc}

- Amplitude of scalar field should be roughly constrained.

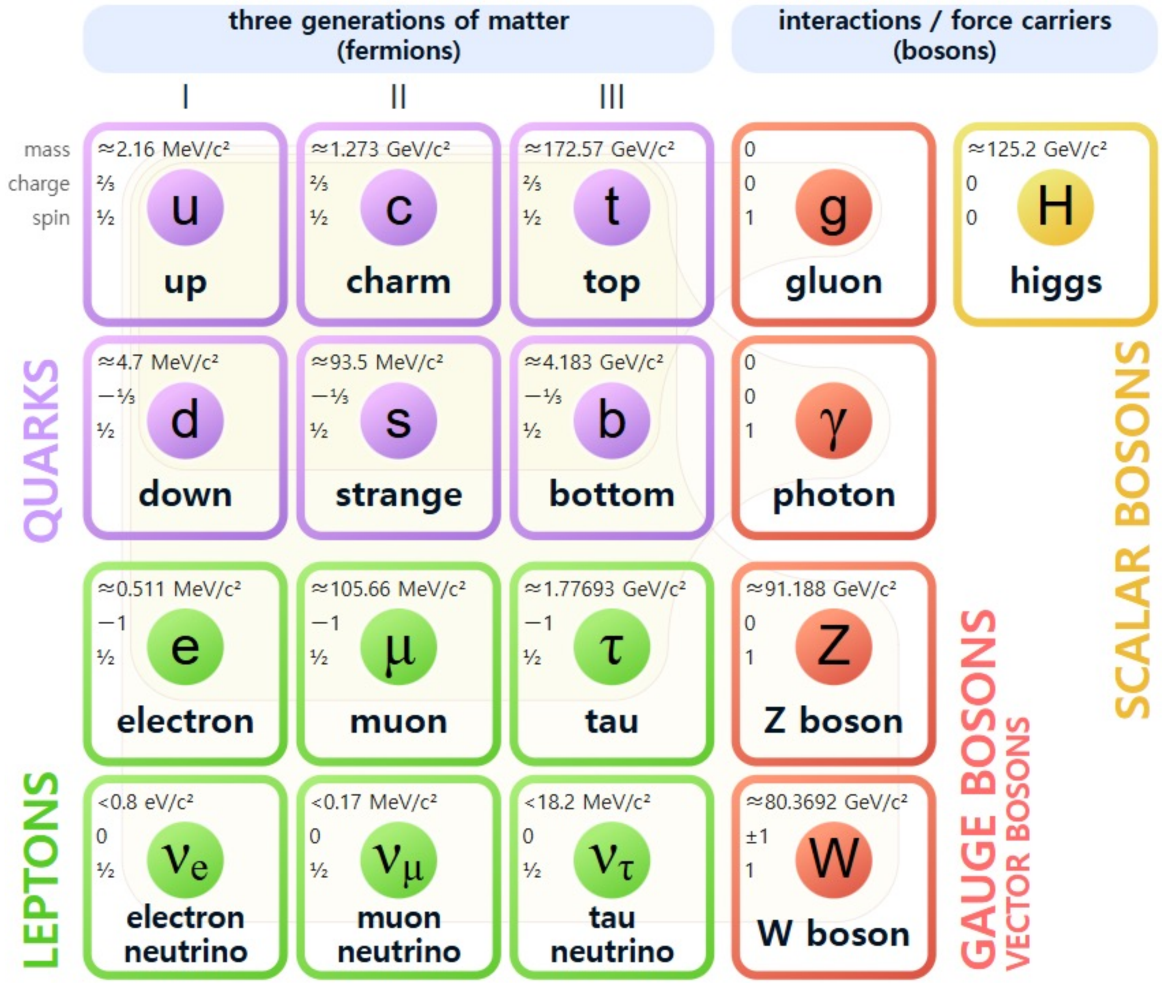
$$10^{16}\mathrm{GeV} \le \phi_i \le 10^{18}\mathrm{GeV}$$

$$\Omega_\phi = 0.12 \left(\frac{\phi_i}{8.36\times10^{16}\mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-22}\mathrm{eV}}\right)^{1/2}$$



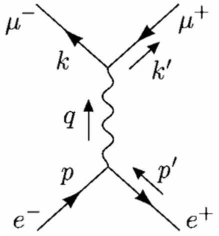
[Snowmass2021 2211.09978]

Standard Model of Elementary Particles

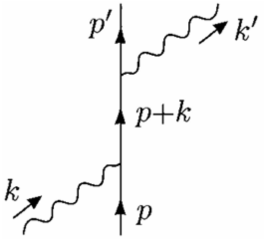


	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass charge spin QUARKS	≈2.16 MeV/c^2 ⅔ ½ u up	≈1.273 GeV/c^2 ⅔ ½ c charm	≈172.57 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≈125.2 GeV/c^2 0 0 H higgs
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d down	≈93.5 MeV/c^2 −⅓ ½ s strange	≈4.183 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
LEPTONS	≈0.511 MeV/c^2 −1 ½ e electron	≈105.66 MeV/c^2 −1 ½ μ muon	≈1.77693 GeV/c^2 −1 ½ τ tau	≈91.188 GeV/c^2 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS
LEPTONS	<0.8 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≈80.3692 GeV/c^2 ±1 1 W W boson	

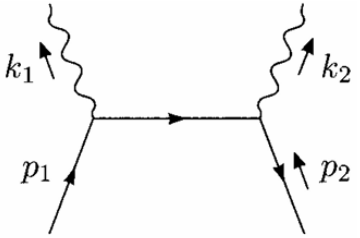
- Electrodynamics : Exchange photon (γ)



Muon Production

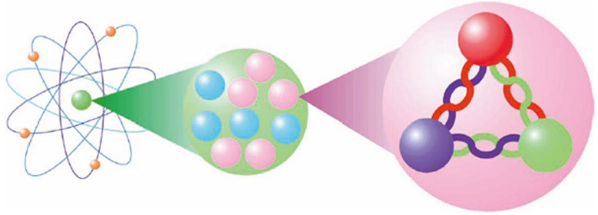


Compton Scattering

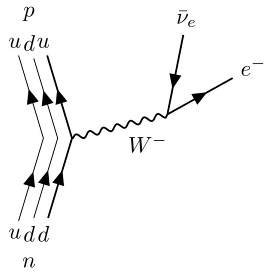


Pair Annihilation

- Strong Interaction : Exchange colors via gluon (g)
  - Binding nucleus : $p^+(uud)$ and $n^0(udd)$

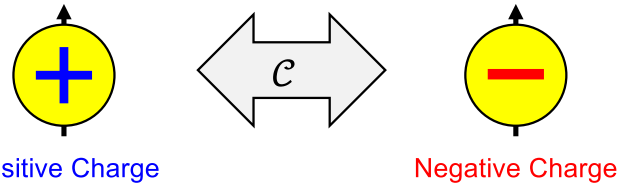


- Weak Interaction : Exchange massive bosons ($W^{\pm}, Z^0$)
  - Beta decay

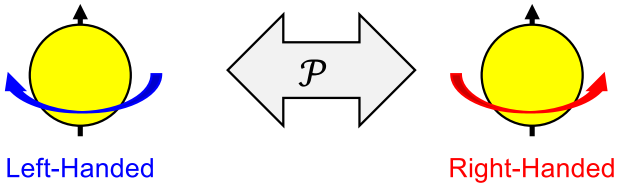


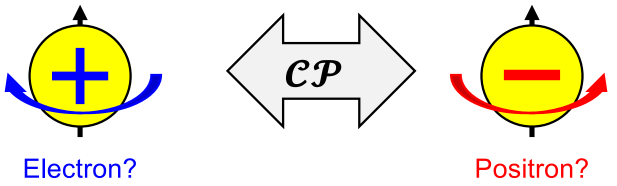
Discrete Symmetries of Standard Model

- Charge Conjugation $\mathcal{C} : \psi(\vec{x},t) \to \bar{\psi}(\vec{x},t)$



- Parity $\mathcal{P} : \psi(\vec{x},t) \to \psi(-\vec{x},t)$





- Time-Reversal
  $\mathcal{T} : \psi(\vec{x},t) \to \psi(\vec{x},-t)$

$$\nabla \cdot \vec{E} = 0$$

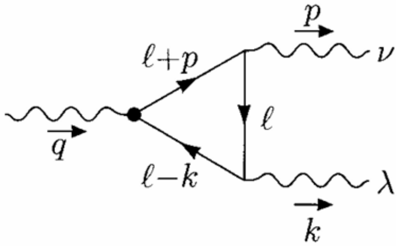
$$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$

	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
$\vec{E}$	$-$	$-$	$+$
$\vec{B}$	$-$	$+$	$-$

$$\boxed{\boldsymbol{\mathcal{CP}} : \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}} \to -\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{B}}}$$

CP-Violation!

- Nature must include an action $S_\theta = \int dt L_\theta \sim \theta \int d^4x \left(\vec{E} \cdot \vec{B}\right)$ (Topological Insulator, Axial Anomaly, etc.)



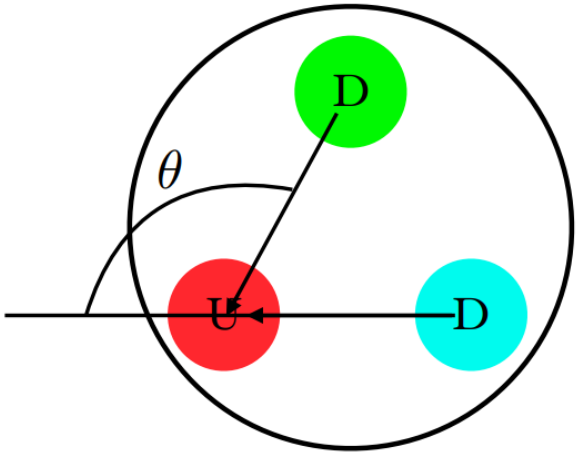
- “CP violation is necessary!”

- Electric Dipole Moment of the neutron d_n is…

$$d_n \sim \begin{cases} 2.4\times10^{-16}\theta[\mathrm{e}\cdot\mathrm{cm}]\ (\mathrm{Predicted}) \\ 1.8\times10^{-26}[\mathrm{e}\cdot\mathrm{cm}]\ (\mathrm{Experiment}) \end{cases} \Rightarrow \boxed{\theta < 10^{-10}}$$

Too Small…
“Strong CP Problem”

[C.A.Baker et al. 0602020 ; C.Abel et al. 2001.11966]



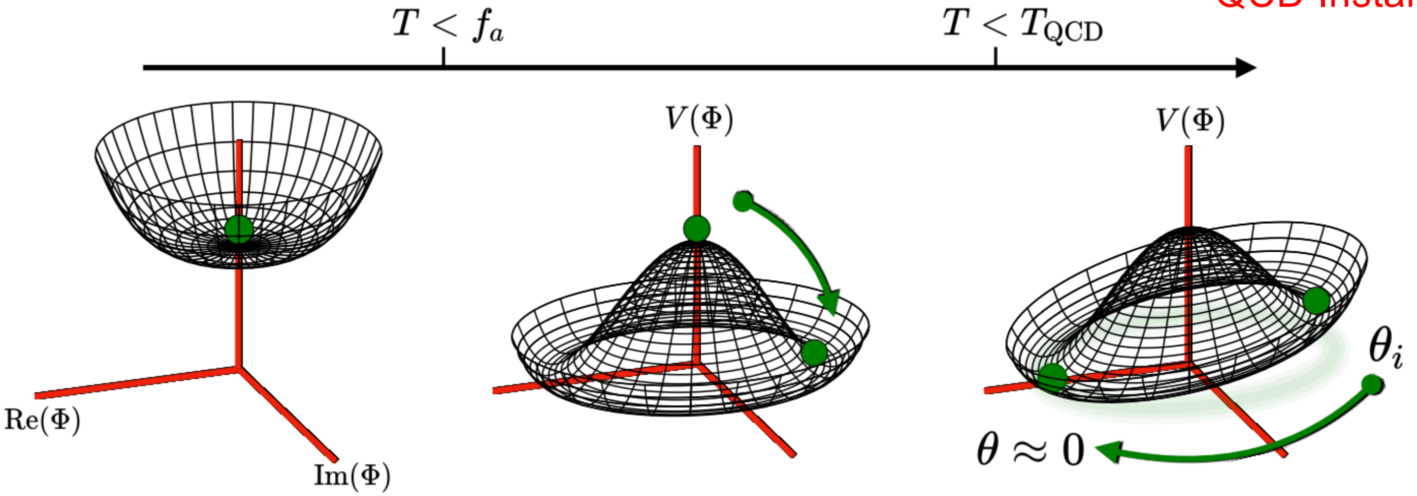
Considering a New Scalar Field?

$$S_\theta \sim \int d^4x \left(\theta + \frac{a(x)}{f_a}\right)(\vec{E}\cdot\vec{B})$$

$$\Omega_{\text{axion}} \sim 0.1 \left(\frac{f_a}{10^{17}\text{GeV}}\right)^2 \left(\frac{m_a}{10^{-22}\text{eV}}\right)^{1/2}$$

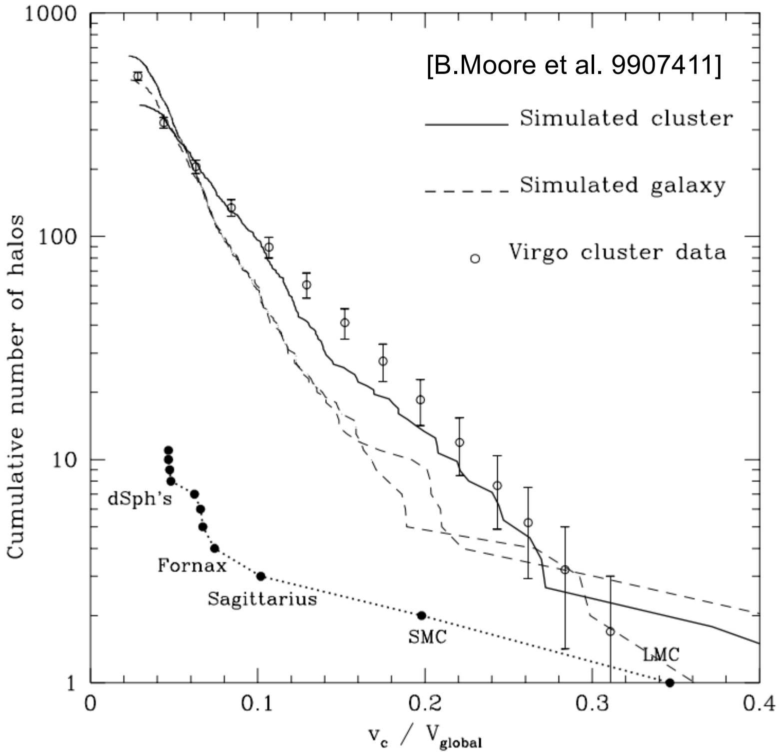
$$V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2 + \underline{\Lambda_{\text{QCD}}^4 \left[1 - \cos\left(\theta + \frac{a}{f_a}\right)\right]}$$

QCD Instanton effect



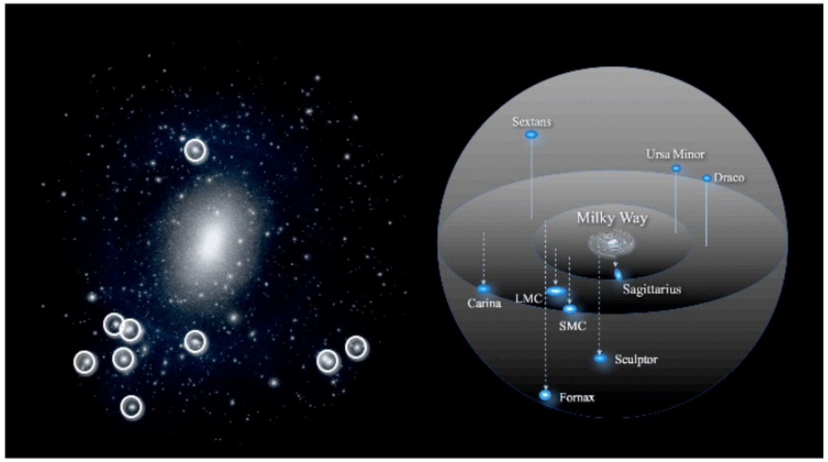
[Ciaran.A.J.O'Hare 2403.17697]

Limitation – Missing Satellite Problem

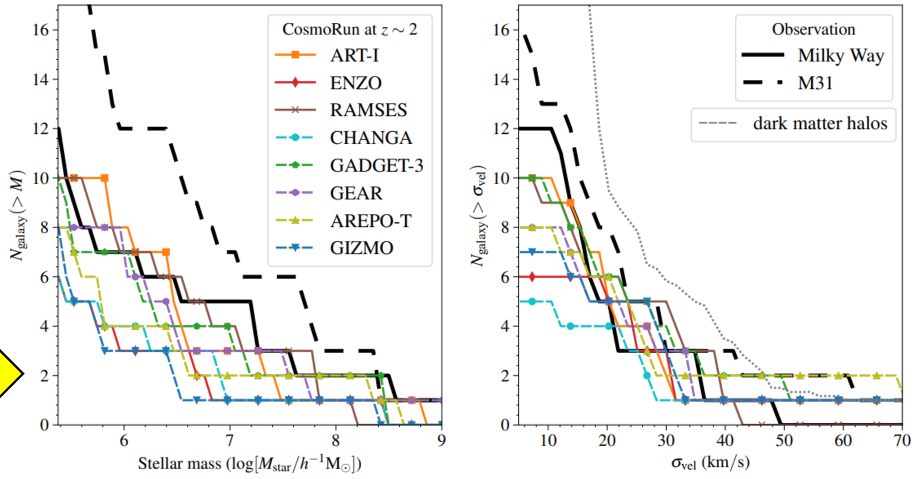


[B.Moore et al. 9907411]

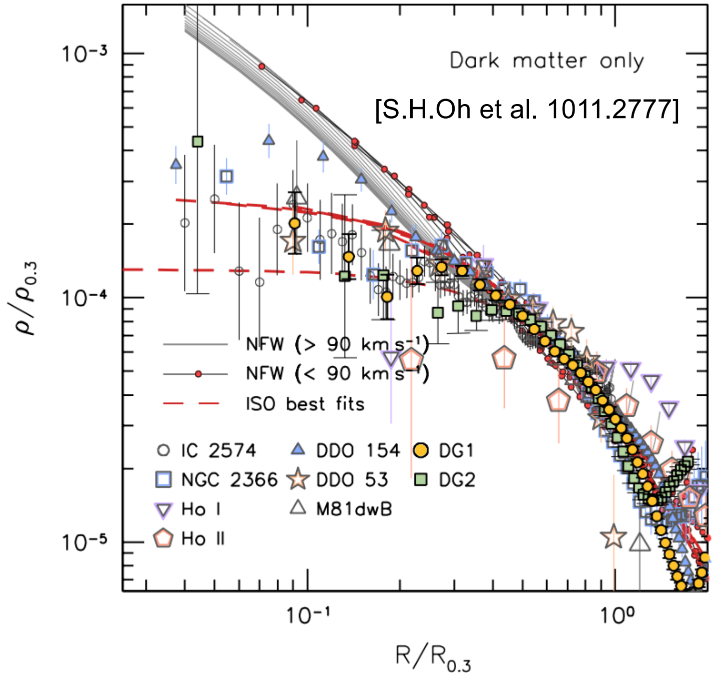
- # of Observed << # of CDM-Simulated
- Potentially solvable by considering baryonic physics in the simulation



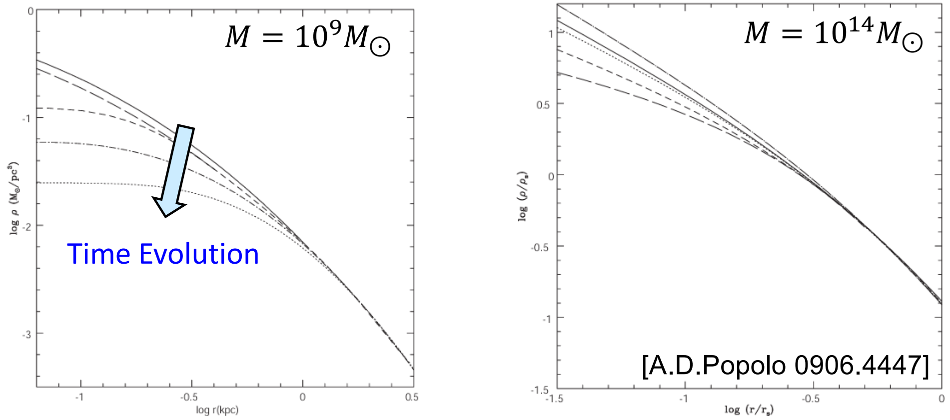
[M.Jung et al. 2402.05392]



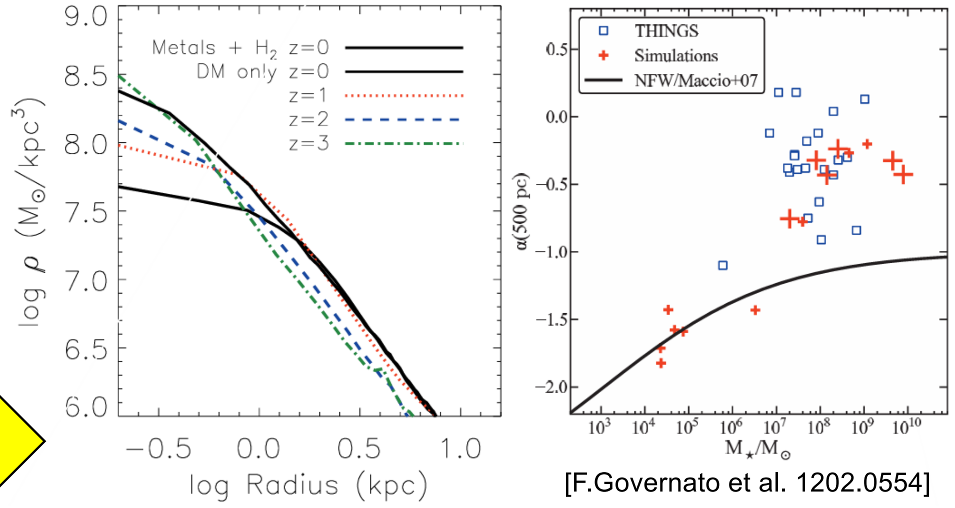
Limitation – Core-Cusp Problem



[S.H.Oh et al. 1011.2777]



[A.D.Popolo 0906.4447]



[F.Governato et al. 1202.0554]

- Observed v_{rot} says "ρ_c of DM halo is not cusp", mismatches with CDM prediction
- Also potentially solvable by considering baryonic physics in the simulation

Mach Number for Thomas-Fermi Limit Profile

$$\rho(r) = \frac{\pi M}{4R_{\mathrm{TF}}^3}\frac{\sin(\pi r/R_{\mathrm{TF}})}{(\pi r/R_{\mathrm{TF}})} \quad \Rightarrow R_{\mathrm{TF}} = \pi\sqrt{\frac{a_s\hbar^2}{Gm_a^3}} = 1.957\mathrm{kpc}\times\left(\frac{5\mathrm{eV}}{m_a\lambda^{-1/4}}\right)^2$$

$$\mathcal{M}^2 = \frac{v^2}{c_s^2} = 1 - \frac{(\pi r/R_{\mathrm{TF}})}{\tan(\pi r/R_{\mathrm{TF}})} \cong \frac{\pi^2}{3}\left(\frac{r}{R_{\mathrm{TF}}}\right)^2$$