Introduction to Radiative Transfer Methods: Applications in Astrophysics

Gas, Dust & Lyα

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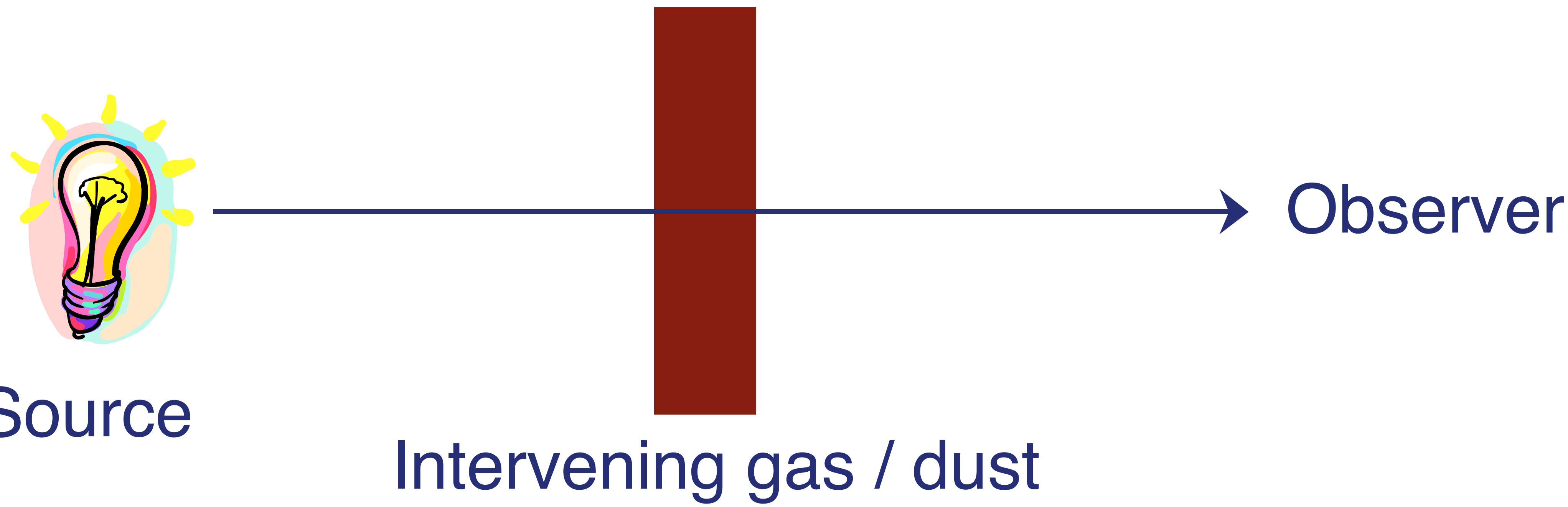
Radiative Transfer (복사전달)

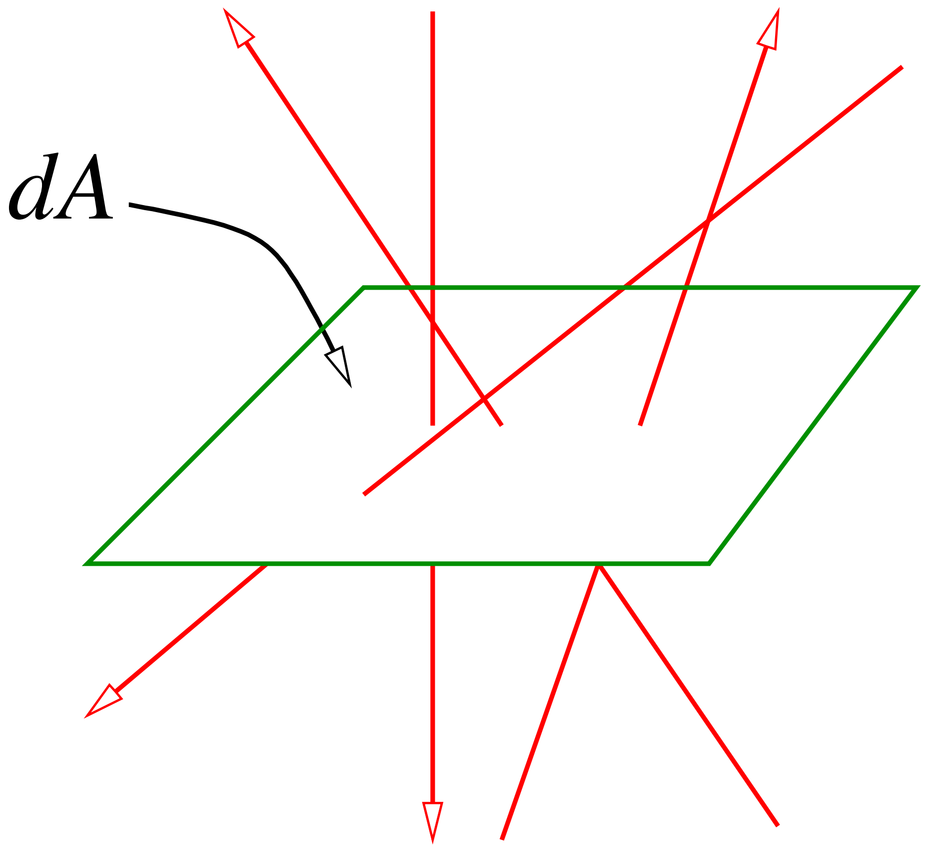
- How is radiation affected as it propagates through intervening gas and dust media to the observer?



- What we can learn?
  - gas/dust temperature, degree of ionization
  - gas kinematics (inflow, outflow)
  - elemental abundances
  - excitation sources

Simplication & Complexity

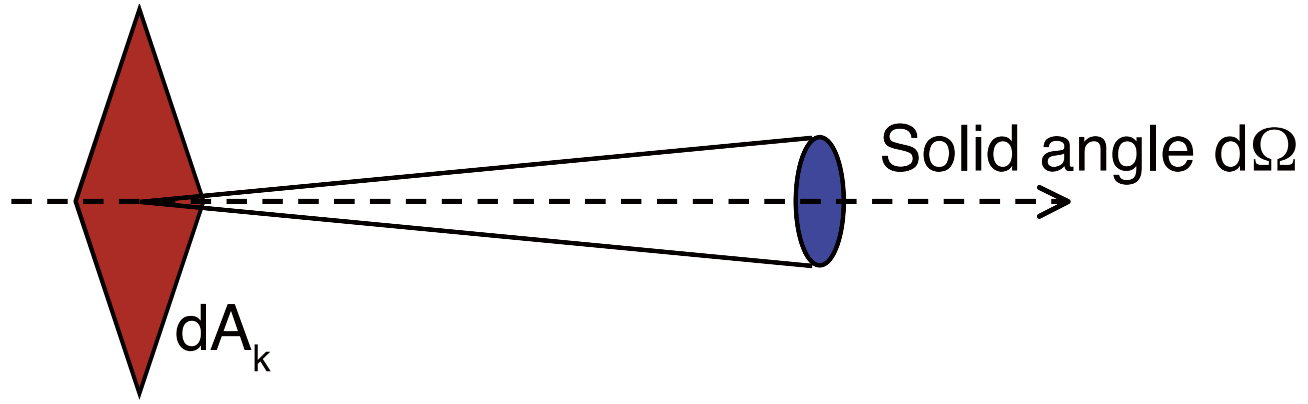
- Simplification:
  - Diffraction can be neglected: Astronomical objects are normally much larger than the wavelength of radiation they emit.
  - Light rays travel to us along straight lines.



- Complexity:
  - At one point, photons can be traveling in several different directions.
  - Full specification of radiation needs to say how much radiation is moving in each direction at every point. Therefore, we are dealing with the five- or six-dimensional problem. ($[x, y, z] + [\theta, \phi] + [t]$)

Intensity (Surface Brightness)

- ***Intensity is the energy carried along by individual rays.***



$\mathbf{k}$ = direction of propagation

- Let dE_ν be the amount of radiant energy which crosses the area dA_k perpendicular to a direction $\mathbf{k}$ within solid angle $d\Omega$ about in a time interval dt with photon frequency between ν and $\nu + d\nu$.
- The monochromatic specific intensity I_ν is then defined by the equation.

$$I_\nu(\mathbf{k}, \mathbf{x}, t) = \frac{dE_\nu}{dA_\mathrm{k} d\Omega d\nu dt}$$

- Unit: $\mathrm{erg\ s^{-1}\ cm^{-2}\ sr^{-1}\ Hz^{-1}}$

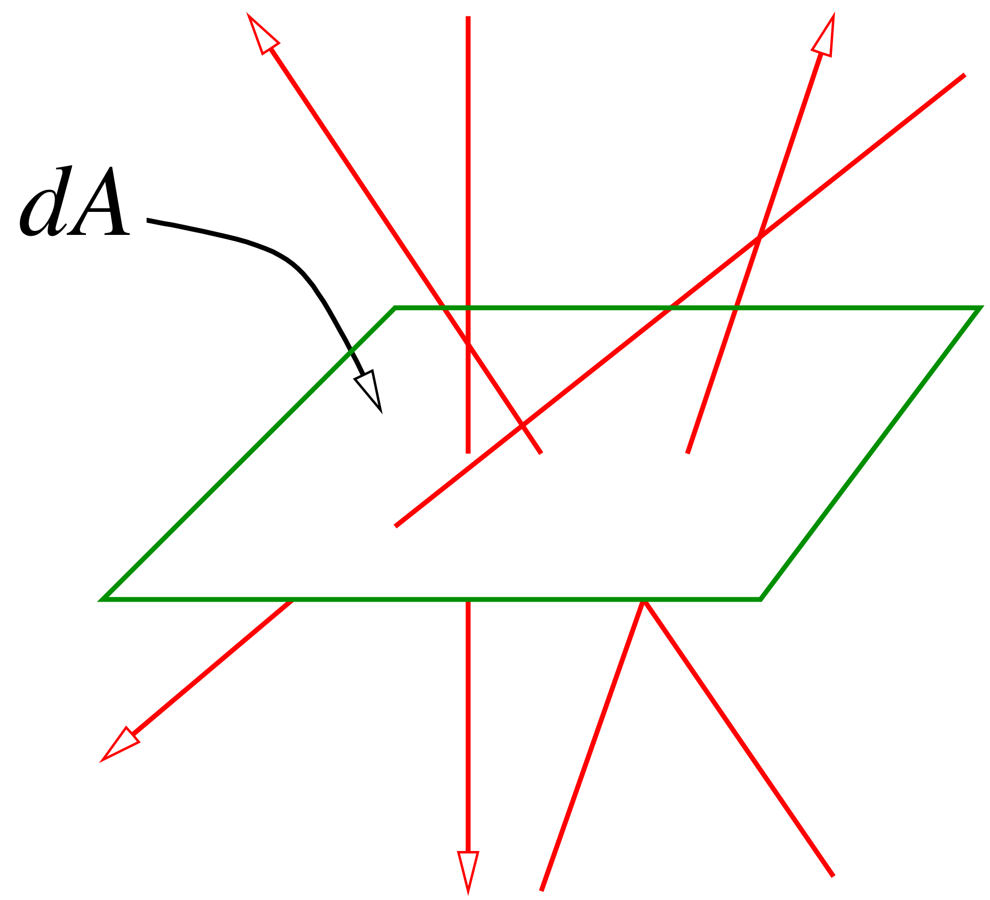
- From the view point of an observer, the specific intensity is called ***surface brightness***.

Flux

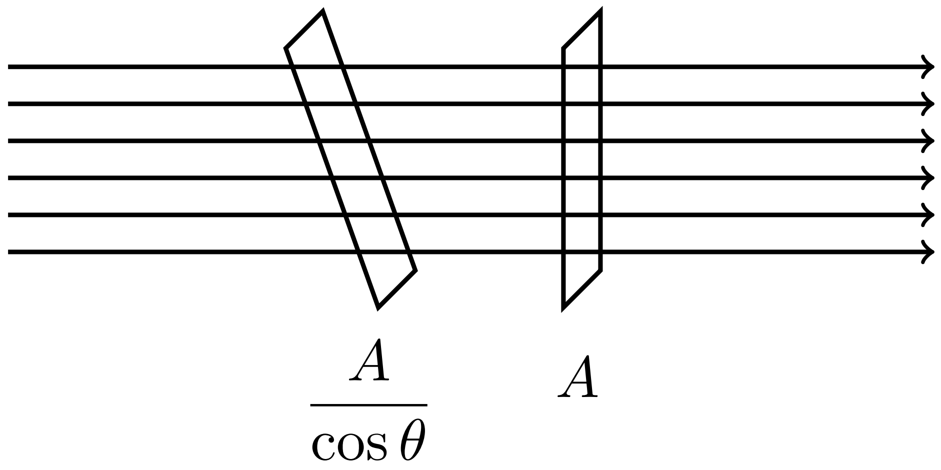
- Definition
  - ***Flux is a measure of the energy carried by all rays passing through a given area***
  - Consider a small area dA, exposed to radiation for a time dt.
  - Flux F_ν is defined as ***the total (net) energy passing through a unit area in all directions within a unit time interval.***

$$F_\nu = \frac{dE_\nu}{dA d\nu dt}$$

  - Note that F_ν ***depends on the orientation of the area element*** dA.
  - Unit: $\mathrm{erg\ cm^{-2}\ s^{-1}}$

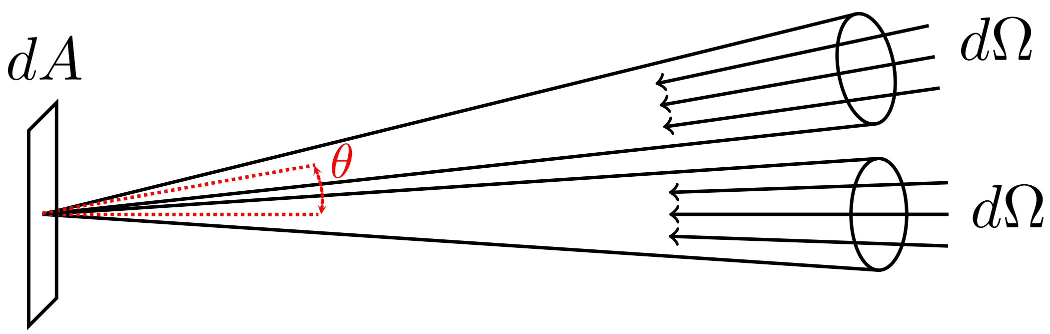


Flux vs. Intensity



The power delivered to the two surfaces are equal though there areas differ.

The flux is **the power per unit area** so the tilted surface gets less flux.



Two intensities are equal.
The upper set or rays delivers less flux.

The rate that energy is delivered to a surface from light traveling around a direction θ is $I\cos\theta d\Omega$.

Relation between the flux and the intensity

- Let's consider a small area dA, with light rays passing through it at all angles to the normal vector $\mathbf{n}$ of the surface.

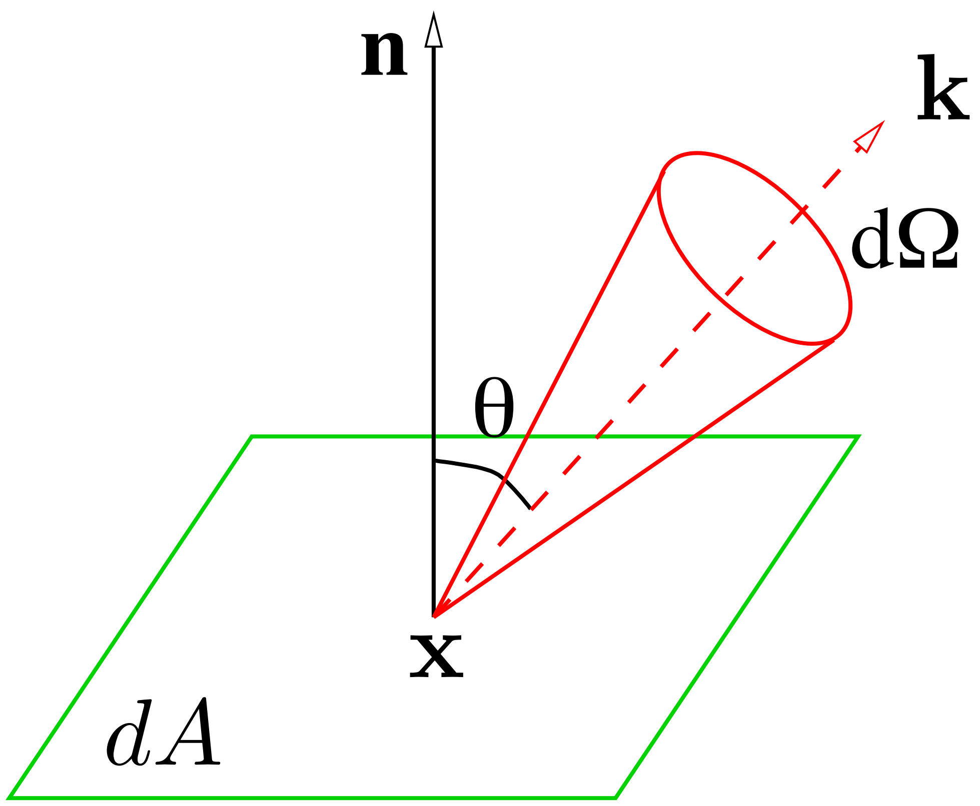
- For a ray centered about $\mathbf{k}$, the area, made by dA, normal to $\mathbf{k}$ is

$$dA_{\mathbf{k}} = dA\cos\theta$$

  - By the definition,

$$F_\nu dA d\nu dt = \int I_\nu(\mathbf{k}, \mathbf{x}, t) dA_{\mathbf{k}} d\Omega d\nu dt$$

- Hence, net flux in the direction of $\mathbf{n}$ is given by integrating over all solid angles:

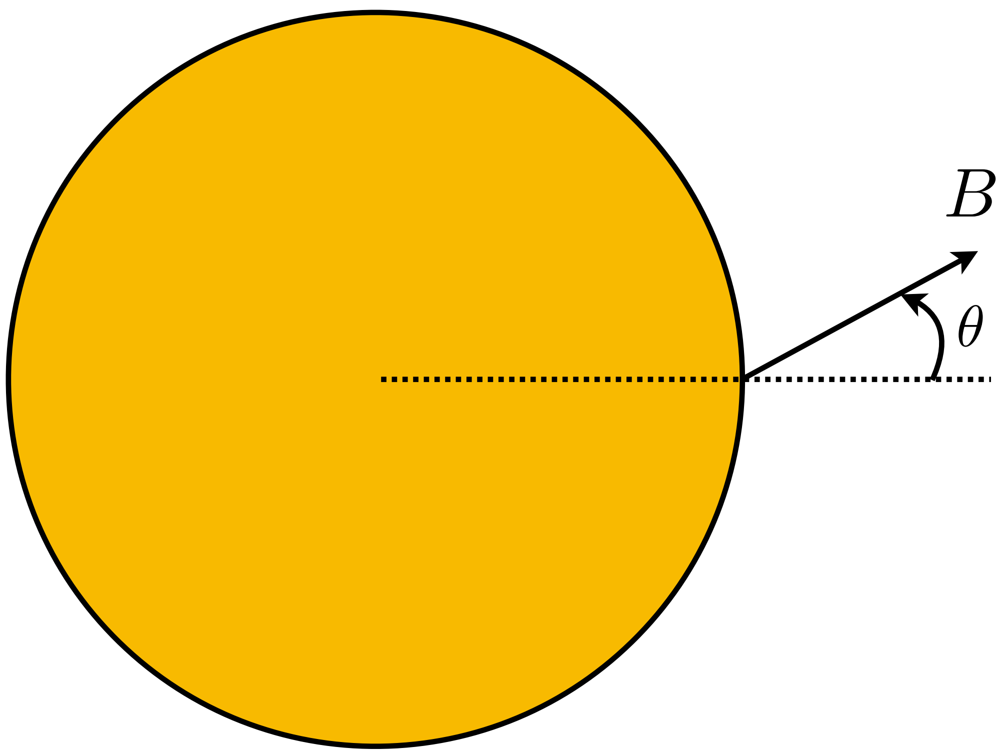


$$F_\nu = \int I_\nu \cos\theta d\Omega = \int_0^{2\pi}\int_0^{\pi} I_\nu(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

[Note] **flux = sum of all ray vectors projected onto a normal vector**
intensity = absolute value of a single ray vector

Flux from the surface of a uniformly bright sphere

- Let's calculate the flux at P on a sphere of uniform brightness B



$$F = \int B \cos\theta d\Omega = \int_0^1 \int_0^{2\pi} B\mu d\mu d\phi$$

$$\boxed{F = \pi B}$$

The total luminosity from the sphere is then

$$\boxed{L = (4\pi R^2)F = (4\pi R^2)\pi B}$$

- In stellar atmosphere, the **astrophysical flux** is defined by F/π .

Constancy of Intensity

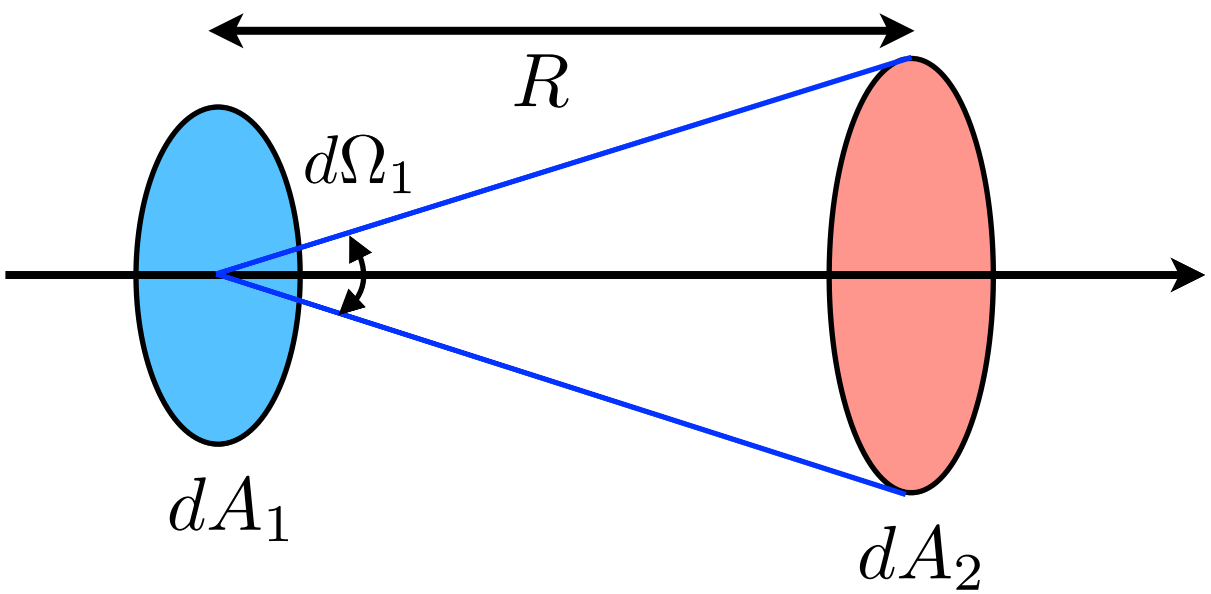
- How does intensity changes along a ray in free space
  - Suppose a bundle of rays and any two points along the rays and construct two “infinitesimal” areas dA_1 and dA_2 normal to the rays at these points.
  - What are the energies carried by the rays passing through both areas?

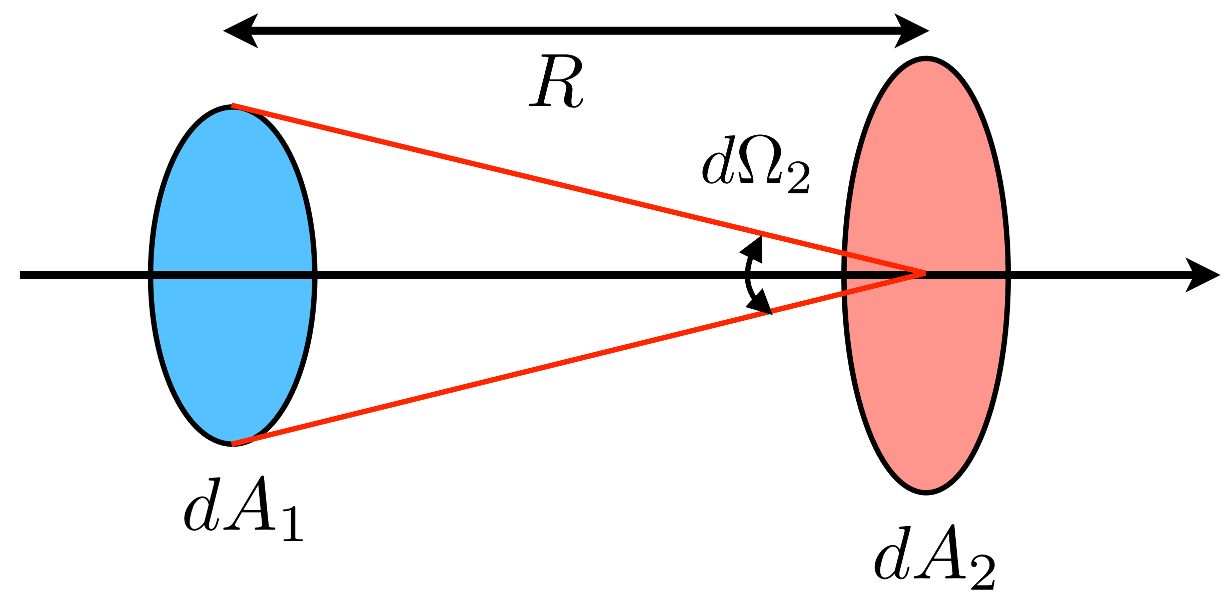
energy passing through 1

$$dE_1 = I_1 dA_1 d\Omega_1 d\nu dt$$

energy passing through 2

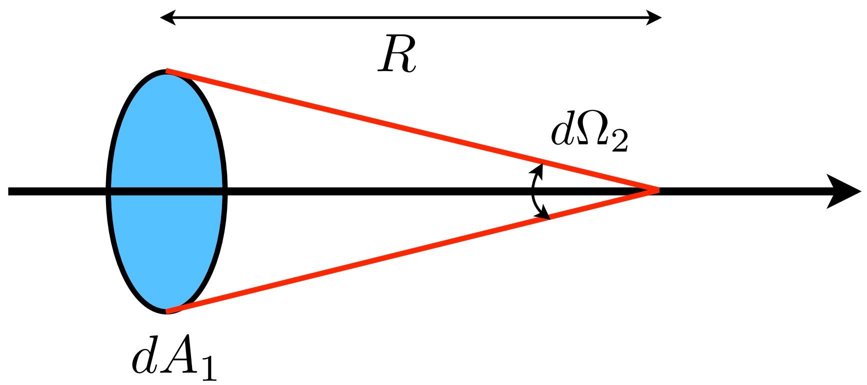
$$dE_2 = I_2 dA_2 d\Omega_2 d\nu dt$$



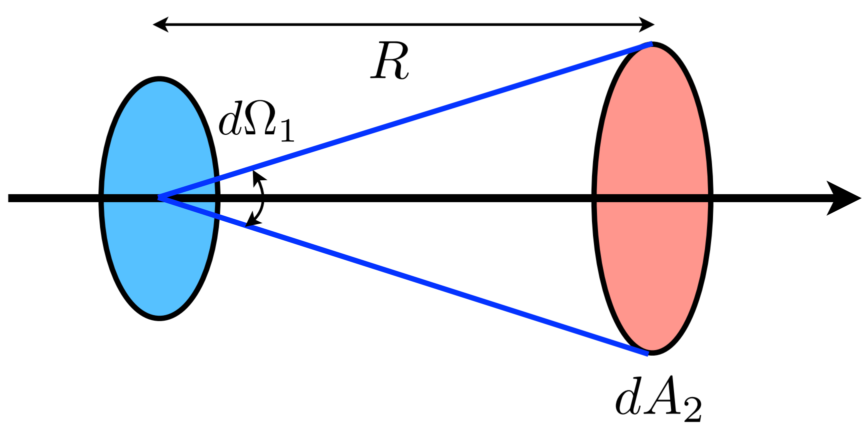


- Here, $d\Omega_1$ is the solid angle subtended by dA_2 at the location 1 and $d\Omega_2$ is the solid angle subtended by dA_1 at the location 2.

: Radiative Transfer Equation in free space



$$d\Omega_2 = \frac{dA_1}{R^2}$$



$$d\Omega_1 = \frac{dA_2}{R^2}$$

Conservation of energy:
Because energy is conserved,

$$\boxed{dE_1 = dE_2 \quad \rightarrow \quad I_1 = I_2}$$

- Conclusion (***the constancy of intensity***): $I_1 = I_2$
  - the specific intensity remains the same as radiation propagates through free space.

- We receive the same specific intensity at the telescope as is emitted at the source.
  - Imagine looking at a uniformly lit wall and walking toward it. As you get closer, a field-of-view with fixed angular size will see a progressively smaller region of the wall, but this is exactly balanced by the inverse square law describing the spreading of the light rays from the wall.

Luminosity

- To determine the energy per unit time, we integrate flux over area.
  - ***Monochromatic luminosity***: Considering a sphere centered on a source with radius *R*, the monochromatic luminosity is

$$L_\nu = R^2 \int d\Omega F_\nu$$
$$= 4\pi R^2 F_\nu \qquad \text{for an isotropic source}$$

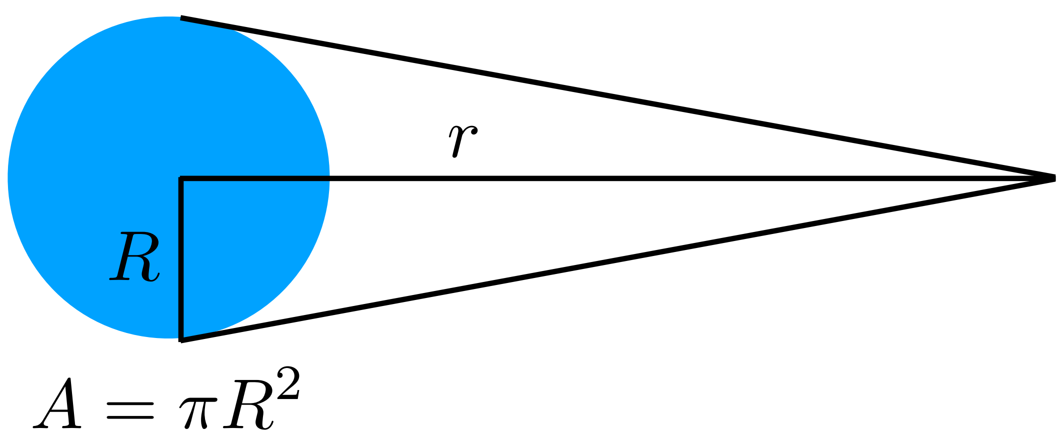
  - The ***bolometric luminosity*** is

$$L_{\rm bol} = \int L_\nu d\nu = \int L_\lambda d\lambda = 4\pi R^2 \int F_\nu d\nu$$

- Flux and Luminosity of an extended source

$$F = \pi I \left(\frac{R}{r}\right)^2 = I\frac{A}{r^2}$$
$$= I\Omega_{\rm source}$$

$$L = (4\pi r^2)F = (4\pi r^2)I\Omega_{\rm source}$$



Energy Density

- Consider a bundle of rays passing through a volume element dV within a time interval dt in a direction Ω.
- Then, the **energy density per unit solid angle** is defined by

$$dE = u_\nu(\Omega) dV d\Omega d\nu$$

- Since radiation travels at velocity c,

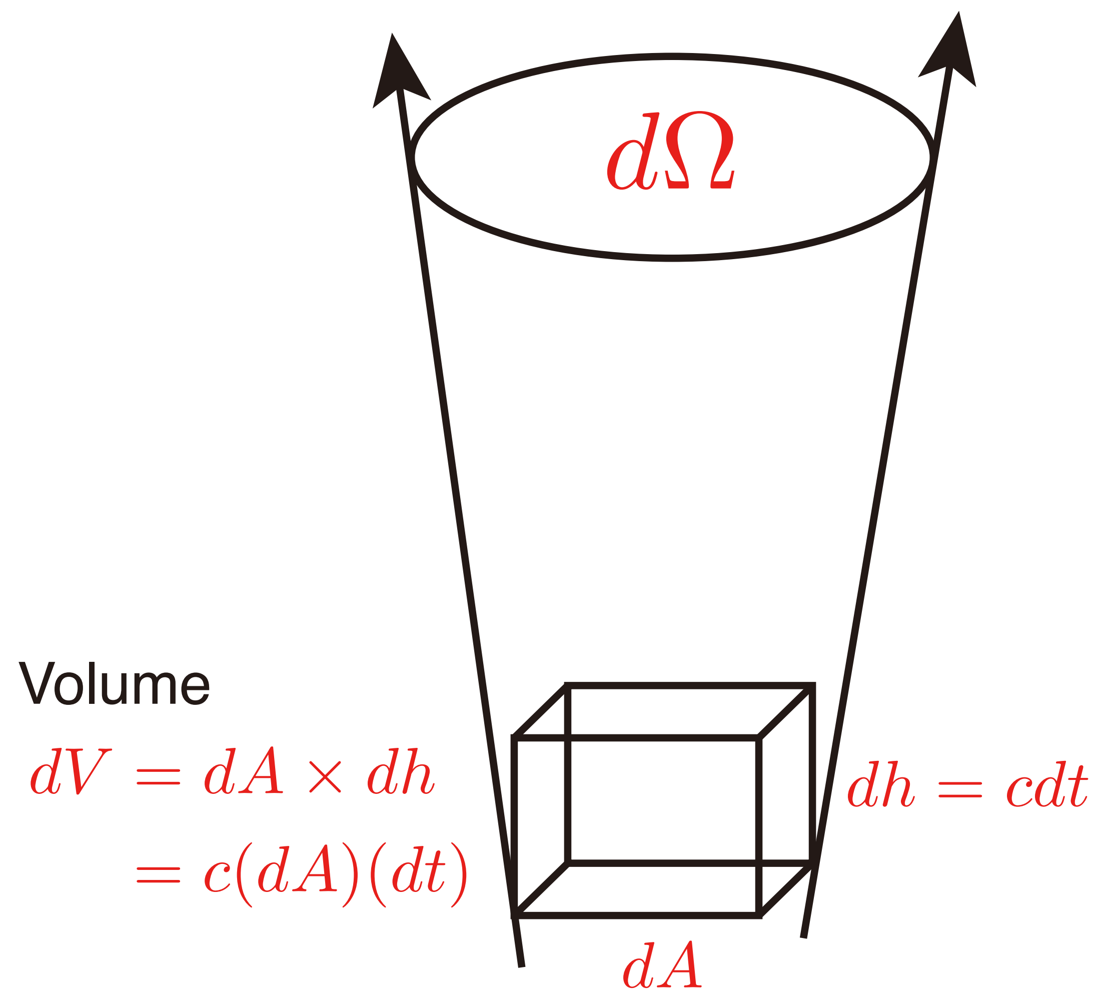
$$dV = dA(cdt)$$

- the definition of the intensity

$$dE = I_\nu dA dt d\Omega d\nu$$

- Therefore,

$$u_\nu(\Omega) = I_\nu(\Omega)/c$$



Energy Density and Mean Intensity

- Integrating over all solid angle, we obtain

$$u_\nu = \int u_\nu(\Omega) d\Omega = \frac{1}{c} \int I_\nu d\Omega$$

- Mean intensity is defined by

$$\boxed{J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega}$$

- Then, the energy density is

$$\boxed{u_\nu = \frac{4\pi}{c} J_\nu}$$

- Total energy density is obtained by integrating over all frequencies.

$$u = \int u_\nu d\nu = \frac{4\pi}{c} \int J_\nu d\nu$$

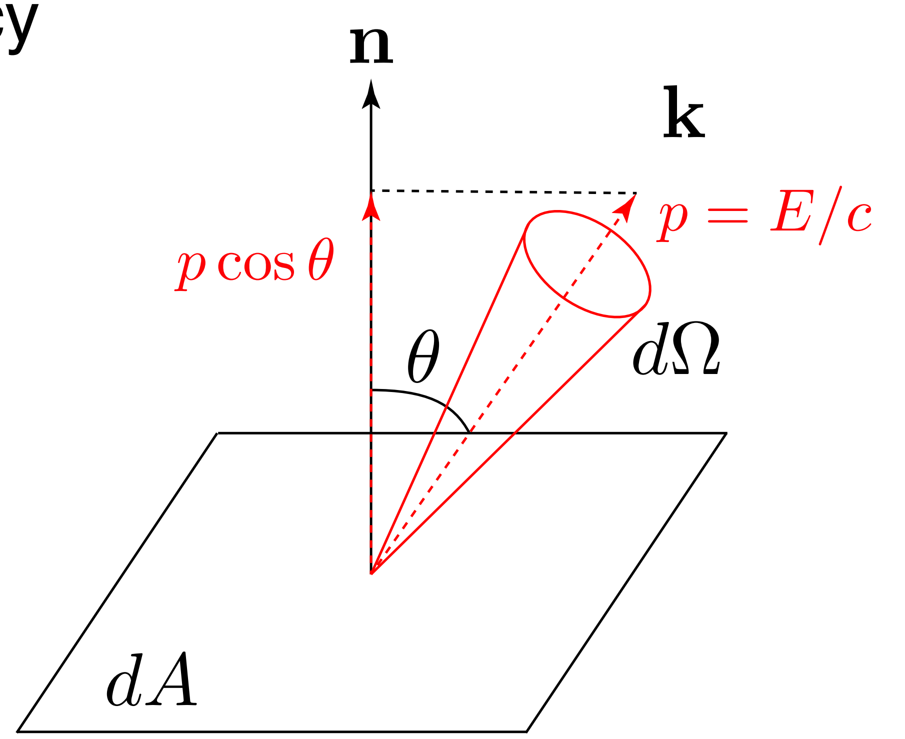
Momentum Flux: Radiation Pressure

- ***Radiation pressure*** due to energy flux propagating along direction $\mathbf{k}$, within solid angle $d\Omega$ and with frequency between $(\nu,\ \nu + d\nu)$, being transported along $\mathbf{n}$:

  - momentum of a photon: $p = E/c$

  - force: $F = \dfrac{\Delta p}{\Delta t} = \dfrac{\Delta E}{c\Delta t}$

  - radiation pressure = force per unit area



$$p_\nu d\Omega d\nu = \frac{\Delta F_\nu}{\Delta A} = \frac{1}{\Delta A}\frac{\Delta E_\nu/c}{\Delta t}\cos\theta$$
$$= \frac{1}{c} I_\nu \cos^2\theta d\Omega d\nu$$

$$\Longleftarrow \quad \Delta E_\nu = I_\nu \Delta A_k \Delta t \Delta\Omega$$
$$\Delta A_k = \Delta A\cos\theta$$

- Integrating over solid angle,

The first cosine factor is due to the reduced area along $\mathbf{k}$.

The second one is due to the projection of the differential flux vector to the normal vector $\mathbf{n}$.

$$P_\nu = \frac{1}{c}\int I_\nu \cos^2\theta d\Omega = \frac{2\pi}{c}\int_{-1}^{1} I_\nu \mu^2 d\mu$$

$$P_\nu = \frac{4\pi}{3c} I_\nu = \frac{1}{3} u_\nu$$

for isotropic radiation field $\left(I_\nu = J_\nu,\ u_\nu = \dfrac{4\pi}{c}J_\nu\right)$

Radiative Transfer Equation: (1) free space

- Recall ***the constancy of intensity***:
  - the specific intensity remains the same as radiation propagates through free space.

$$I_1 = I_2$$

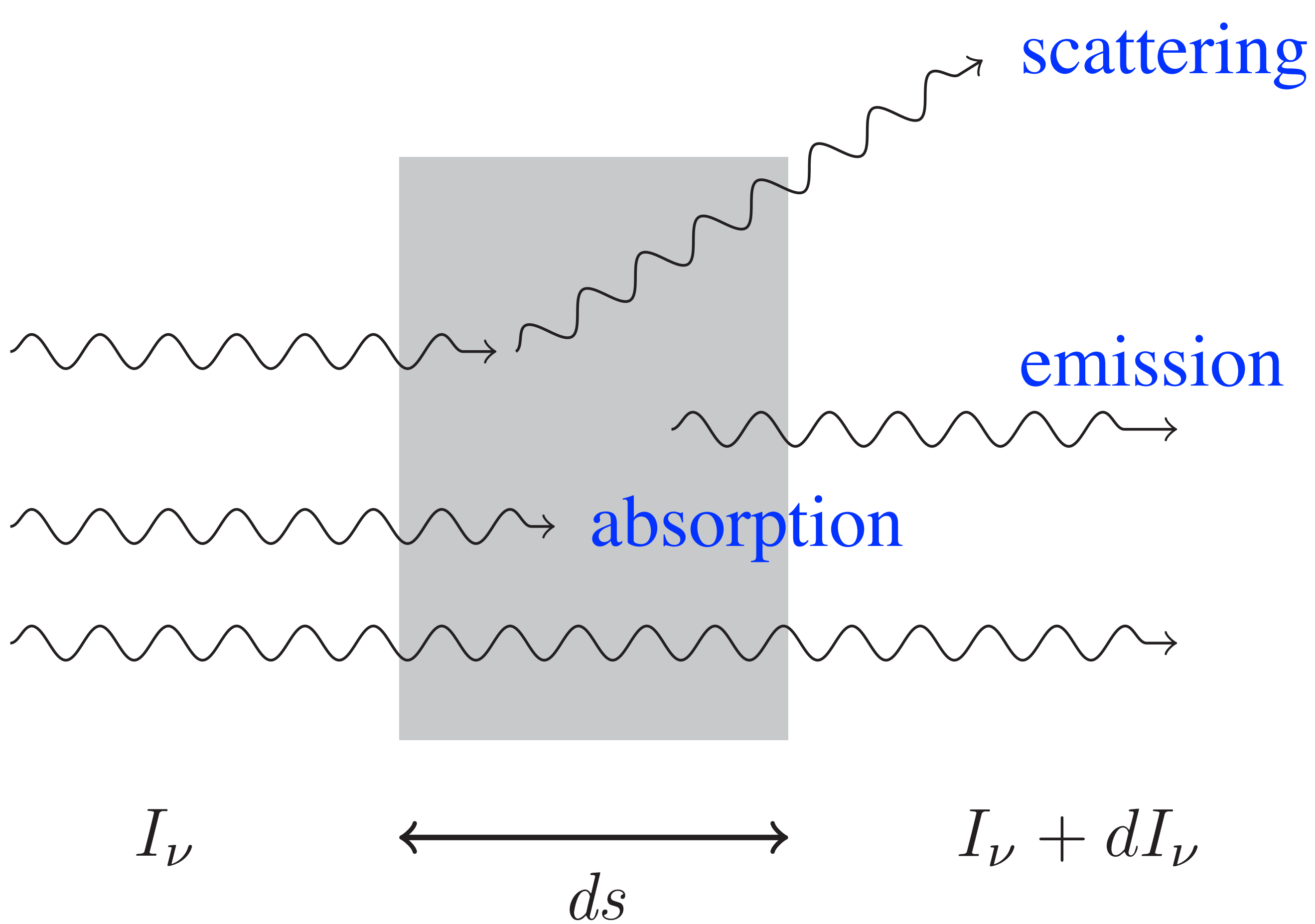
- ***Radiative Transfer Equation in Free Space***: If we measure the distance along a ray by variable s, we can express the result equivalently in differential form:

$$\boxed{\frac{dI}{ds} = 0}$$

radiative transfer equation in free space

Radiative Transfer Equation: (2) in reality

- In reality, as a ray passes through matter, energy may be added, subtracted, or scattered from it by emission, absorption, or scattering.
  - The intensity will not in general remain constant.
  - These interactions are described by the ***radiative transfer equation***.



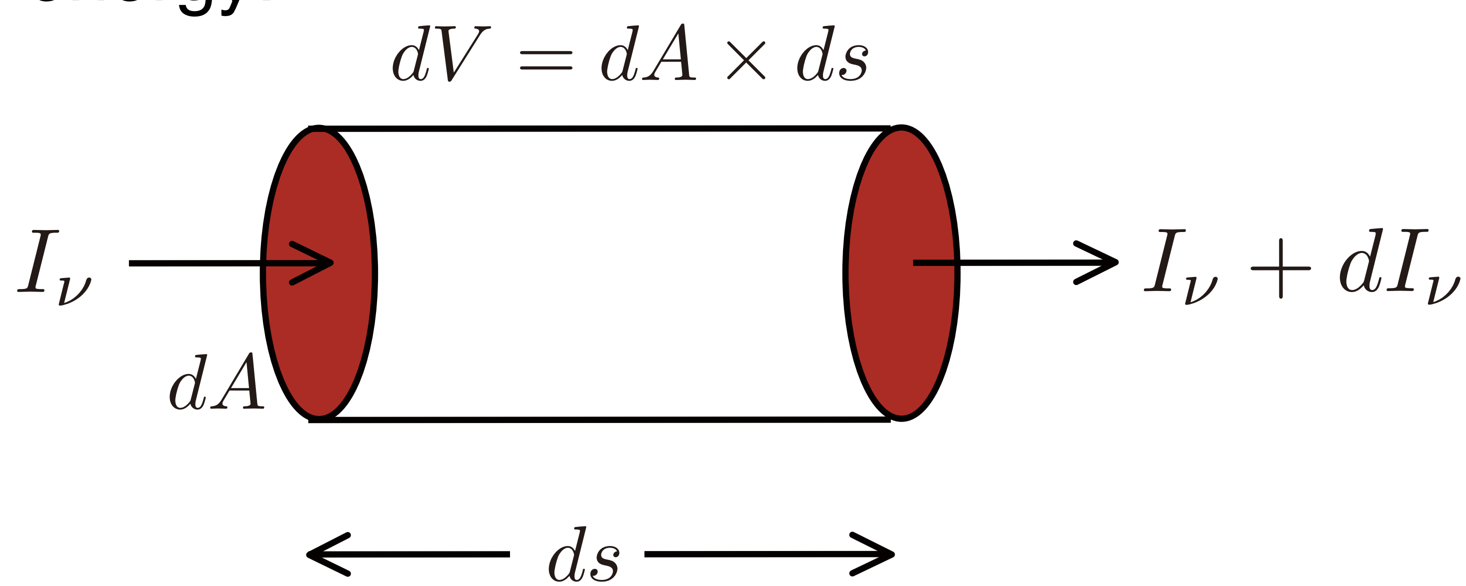
loss: scattering & absorption

gain: emission

[J. P. Williams]
Introduction to the Interstellar Medium

Emission

- If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



- Spontaneous ***"emission coefficient" or "emissivity"*** j_ν is the amount of energy **emitted per unit time, per unit solid angle, per unit frequency, and per unit volume**:

$$dE = j_\nu dV d\Omega dt d\nu \quad (j_\nu : \mathrm{erg\ cm^{-3}\ s^{-1}\ sr^{-1}\ Hz^{-1}})$$

- In going a distance ds, a beam of cross section dA travels through a volume $dV = dAds$. Thus the intensity added to the beam is by ds is

$$dI_\nu = j_\nu ds \quad \longleftarrow \quad dE = (dI_\nu) dA d\Omega dt d\nu$$

- Therefore, the equation of radiative transfer for pure emission becomes:

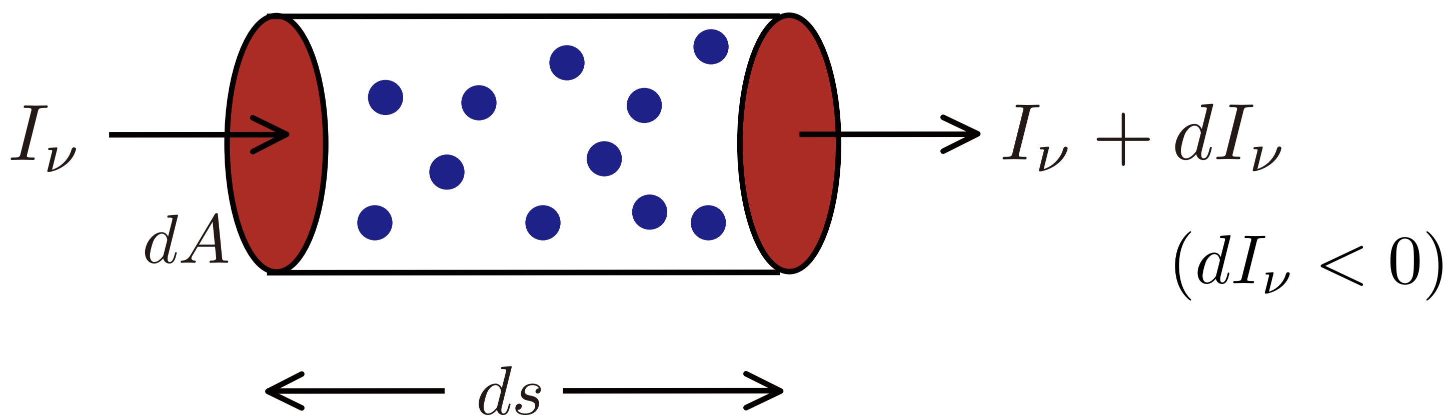
$$\frac{dI_\nu}{ds} = j_\nu$$

- If we know what j_ν is, we can integrate this equation to find the change in specific intensity as radiation propagates through the medium:

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')ds'$$

Absorption

- If the radiation travels through a medium which absorbs radiation, the energy in the beam will be reduced:



  - Let n denote the number density of absorbers (particles per unit volume).
  - Assume that each absorber has a cross-sectional area of σ_ν (in units of cm^2).
  - number of absorbers = $ndAds$

- If a beam travels through ds, total area of absorbers is

$$\text{number of absorbers} \times \text{cross section} = (n \cdot dA \cdot ds) \cdot \sigma_\nu$$

Fraction of radiation absorbed = Fraction of area blocked

$$\frac{dI_\nu}{I_\nu} = -\frac{n dA ds \sigma_\nu}{dA} = -n\sigma_\nu ds \longrightarrow \boxed{\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu}$$

$$dI_\nu = -n\sigma_\nu I_\nu ds \equiv -\alpha_\nu I_\nu ds$$

- ***Absorption coefficient*** is defined as $\alpha_\nu \equiv n\sigma_\nu$ (units: cm^{-1}), meaning the ***total cross-sectional area per unit volume***.

$$\boxed{\begin{aligned} \alpha_\nu &= n\sigma_\nu \quad [\mathrm{cm}^{-1}] \\ &= \rho\kappa_\nu \end{aligned}}$$

where ρ is the mass density and κ_ν is called the **mass absorption coefficient** or the **opacity coefficient**.

The Radiative Transfer Equation: Scattering

- Without scattering term,

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

- Including scattering term, we obtain a general integrodifferential equation.

$$\mathbf{k} \cdot \nabla I_\nu = -\alpha_\nu^{\rm ext} I_\nu + j_\nu + \alpha_\nu^{\rm scatt} \int \phi_\nu(\mathbf{k}, \mathbf{k}') I_\nu(\mathbf{k}') d\Omega'$$

Here, $\alpha_\nu^{\rm ext}$ should be used because the scattering also removes the ray.

extinction coefficient

$$\alpha_\nu^{\rm ext} = \alpha_\nu^{\rm abs} + \alpha_\nu^{\rm scatt}$$

- scattering coefficient $\alpha_\nu^{\rm scatt}\ ({\rm cm}^{-1})$

- scattering phase function $\int \phi_\nu(\mathbf{k}, \mathbf{k}') d\Omega = 1$

  for isotropic scattering

$$\Omega_\nu(\mathbf{k}, \mathbf{k}') = \frac{1}{4\pi}$$

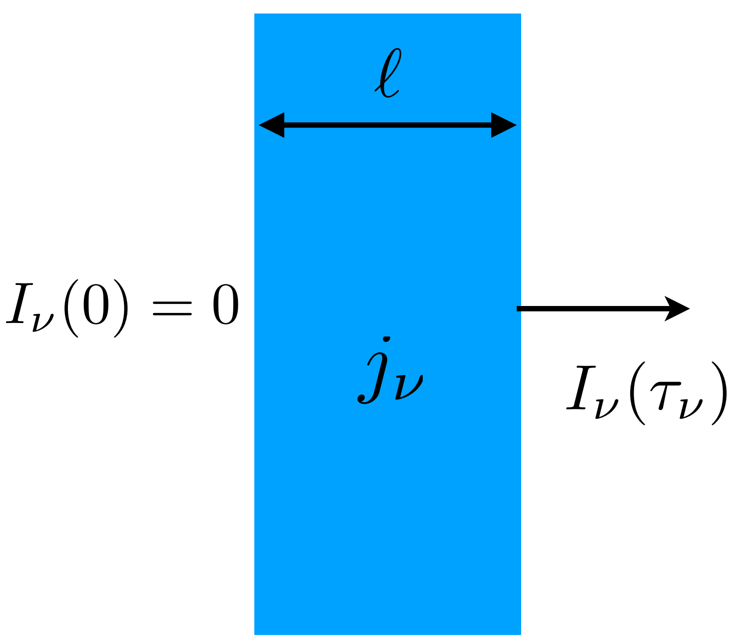
Solution: Emission Only

- For pure emission, $\alpha_\nu = 0$

$$\frac{dI_\nu}{ds} = j_\nu$$

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')ds'$$

  - The brightness increase is equal to the emission coefficient integrated along the line of sight.



$$I_\nu = j_\nu \ell$$
$$\text{if } I_\nu(0) = 0 \text{ and } j_\nu = \text{constant}$$

Solution: Absorption Only

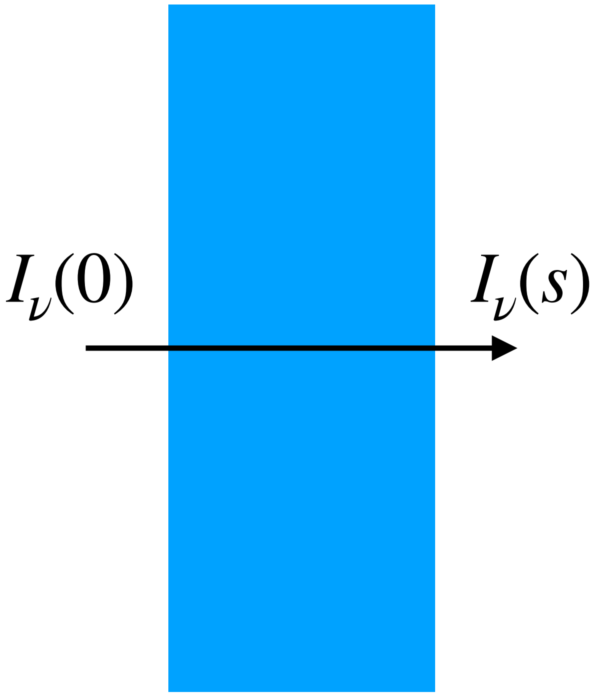
- Pure absorption: $j_\nu = 0$

  Rearranging the previous equation, we obtain the equation of radiative transfer for pure absorption:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

  - The amount of reduced energy depends on how much radiation we already have.
  - Integrate to find how radiation changes along path:

$$I_\nu(s) = I_\nu(0) \exp\left[-\int_0^s \alpha_\nu(s')ds'\right]$$



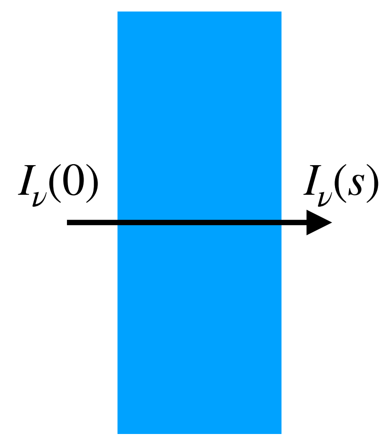
- The brightness decreases along the ray by the exponential of the absorption coefficient integrated along the line of sight.

- ***Optical depth***:

  Imagine radiation traveling into a cloud of absorbing gas, the exponential defines a scale over which radiation is attenuated.

  We define the optical depth τ_ν as:

$$\tau_\nu(s) = \int_0^s \alpha_\nu(s')ds' \quad \text{or} \quad d\tau_\nu = \alpha_\nu ds$$



Transmitted Light

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu}$$

Absorbed Light

$$I_\nu^{\text{abs}}(\tau_\nu) = I_\nu(0)(1 - e^{-\tau_\nu})$$

- A medium is said to be ***optically thick*** at a frequency ν if the optical depth for a typical path through the medium satisfies:

$$\tau_\nu(s) > 1 \qquad I_\nu(\tau_\nu) \rightarrow 0 \qquad I_\nu^{\text{abs}}(\tau_\nu) \rightarrow I_\nu(0)$$

- The medium is ***optically thin*** if, instead:

$$\tau_\nu(s) < 1 \qquad I_\nu(\tau_\nu) \rightarrow I_\nu(0) \qquad I_\nu^{\text{abs}}(\tau_\nu) \rightarrow 0$$

An optically thin medium is one which a typical photon of frequency ν can pass through without being (significantly) absorbed.

Mean Free Path

- From the exponential absorption law, the ***probability of a photon absorbed*** between optical depths τ_ν and $\tau_\nu + d\tau_\nu$ is

$$\int_0^\infty P(\tau_\nu)d\tau_\nu = 1$$

$$|dI_\nu| = \left|\frac{dI_\nu}{d\tau_\nu}\right| d\tau_\nu \quad \Rightarrow \quad \boxed{|dI_\nu| = P(\tau_\nu)d\tau_\nu} \qquad \boxed{P(\tau_\nu) = e^{-\tau_\nu}}$$

$$e^{-\tau_\nu}$$

= **probability density function** for the absorption at an optical depth τ_ν.

- The mean optical depth traveled is thus equal to unity:

$$\langle \tau_\nu \rangle = \int_0^\infty \tau_\nu P(\tau_\nu) d\tau_\nu = \int_0^\infty \tau_\nu e^{-\tau_\nu} d\tau_\nu = 1$$

- ***The mean free path is defined as the average distance a photon can travel through an absorbing material until it is absorbed.*** In a homogeneous medium, the mean free path is determined by

$$\langle \tau_\nu \rangle = \alpha_\nu \ell_{\rm mfp} = 1 \quad \rightarrow \quad \boxed{\ell_{\rm mfp} = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}}$$

- A local mean path at a point in an inhomogeneous material can be also defined.

General Solution (without scattering)

- Source function:

$$\boxed{S_\nu = \frac{j_\nu}{\alpha_\nu}}$$

- The radiative transfer equation can now be written

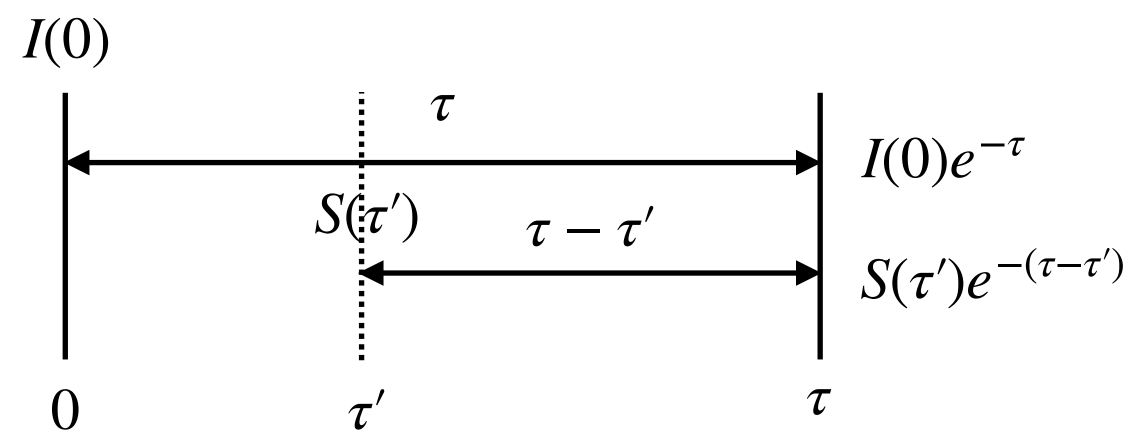
$$\frac{dI_\nu}{\alpha_\nu ds} = -I_\nu + \frac{j_\nu}{\alpha_\nu}$$

$$\boxed{\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu}$$

- This is an alternative and sometimes more convenient way to write the equation.

Formal Solution

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu) d\tau'_\nu$$



- The solution is easily interpreted as the sum of two terms:
  - the initial intensity diminished by absorption
  - the integrated source diminished by absorption.

- For a constant source function, the solution becomes

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right) \\ &= S_\nu + e^{-\tau_\nu}\left(I_\nu(0) - S_\nu\right) \end{aligned}$$

Relaxation

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

- "Relaxation"
  - $I_\nu > S_\nu \quad \rightarrow \quad \dfrac{dI_\nu}{d\tau_\nu} < 0, \quad \text{then } I_\nu \text{ tends to decrease along the ray}$

    $I_\nu < S_\nu \quad \rightarrow \quad \dfrac{dI_\nu}{d\tau_\nu} > 0, \quad \text{then } I_\nu \text{ tends to increase along the ray}$

  - **The source function is the quantity that the specific intensity tries to approach, and does approach if given sufficient optical depth.**

$$\boxed{\text{As } \ \tau_\nu \rightarrow \infty, \quad I_\nu \rightarrow S_\nu}$$

Thermal equilibrium

- In general, equilibrium means a state of balance.
- Thermal Equilibrium
  - ***Thermal equilibrium* refers to steady states of temperature, which defines the average energy of material or photons.**

    (for ideal gas, $E_{\mathrm{avg}} = \frac{3}{2} k_{\mathrm{B}} T$)

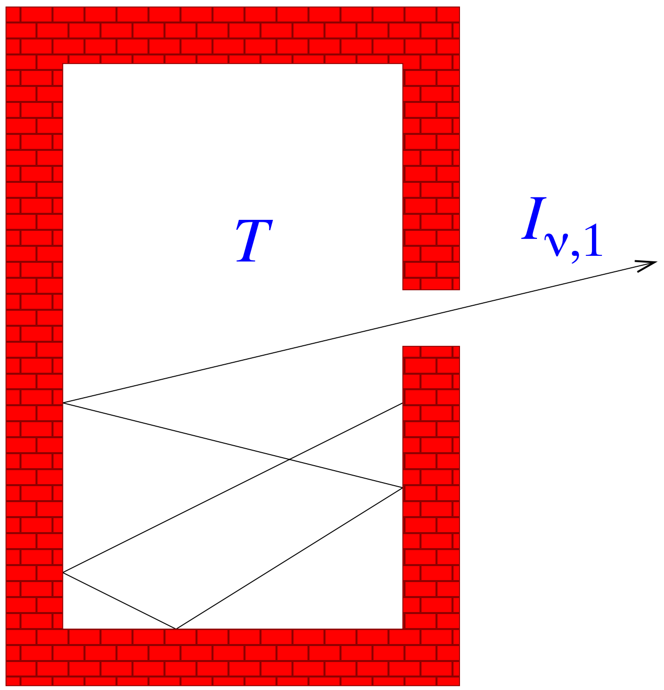
  - In a state of (complete) ***thermodynamic equilibrium (TE)***, no net flows of matter or of energy, no phase changes, and no unbalanced potentials (or driving forces), within the system. ***In TE, matter and radiation are in equilibrium at the same temperature T.***

  - ***When the material is (locally) in thermodynamic equilibrium, and only the radiation field is allowed to depart from its TE***, we refer to the state of the system as being in ***local thermodynamic equilibrium (LTE)***

  - In other words, if the material is (locally) in thermodynamic equilibrium at a well-defined temperature *T*, ***it is said to be in local thermodynamic equilibrium (LTE) even if it is not in equilibrium with the radiation field.***

Blackbody

- Imagine a container bounded by opaque walls with a very small hole.
  - ***Photons will be scattered and absorbed many times, (and eventually trapped and completely absorbed in the box).*** Under such conditions, the particles and photons continually share their kinetic energies. In perfect thermal equilibrium, ***the average particle kinetic energy will equal to the average photon energy, and a unique temperature T can be defined.***



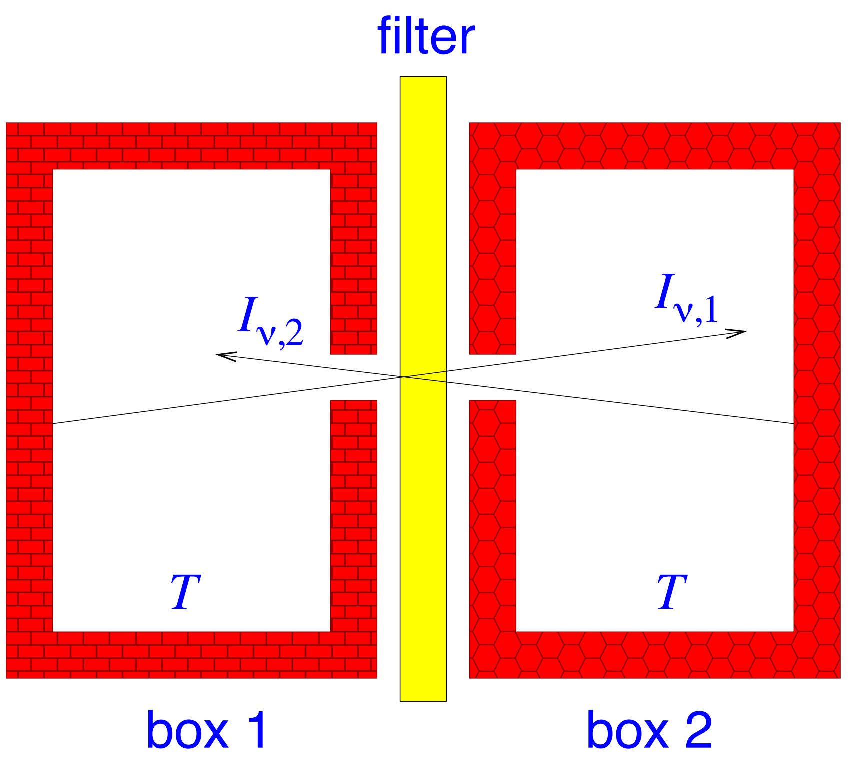
box 1

  - **A blackbody is an idealized physical body that absorbs all incident radiation** regardless of frequency or angle of incidence (i.e., perfect absorber). The above cavity can be regarded to be a blackbody.

  - Radiation from a blackbody in thermal equilibrium is called the **blackbody radiation**.

Blackbody radiation is the universal function.

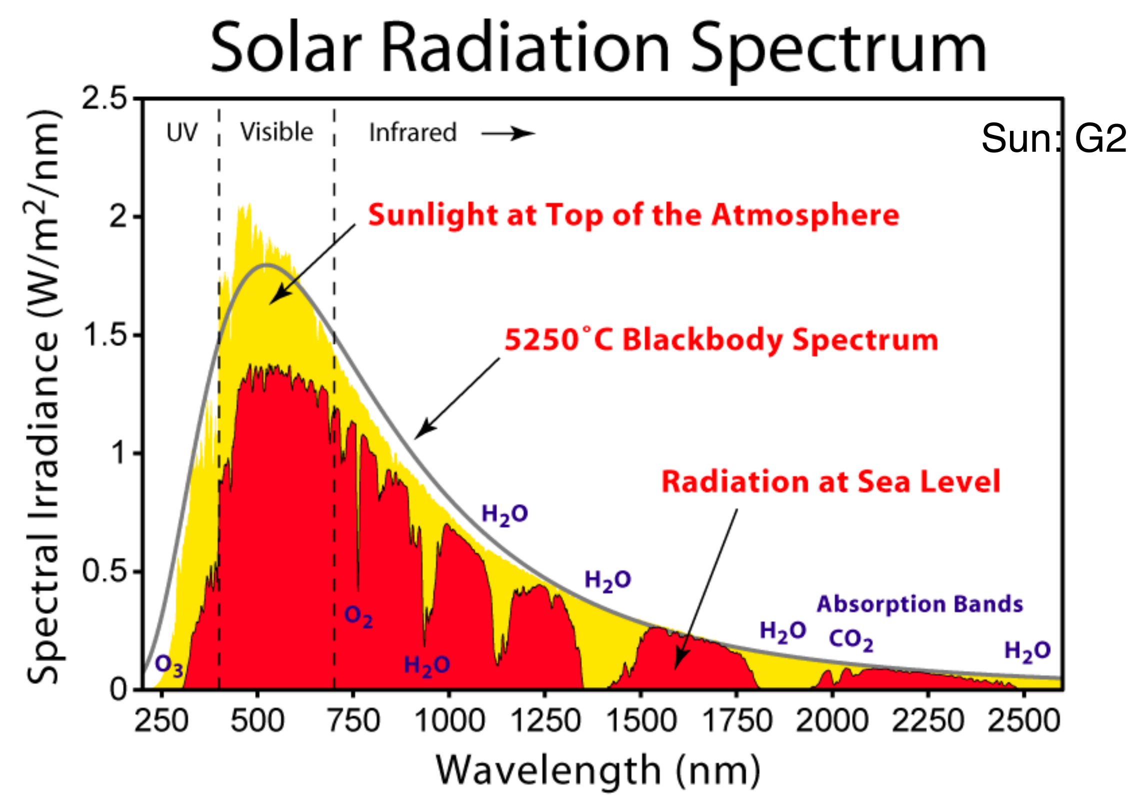
- ***Now, consider another cavity (box 2), also at the same temperature***, but **made of different material or shape** and connect two cavities with a filter transparent only in the narrow frequency range ν and $\nu + d\nu$.
  - ***In equilibrium at T, radiation should transfer no net energy from one cavity to the other.*** Otherwise, one cavity will cool down and the other heats up; this violates the second law of thermodynamics.
  - Therefore, the intensity or spectrum that passes through the holes should be a universal function of T and should be isotropic.
  - ***The intensity and spectrum of the radiation emerging from the hole should be independent of the wall material (e.g., wood, copper, or concrete, etc) and any absorbing material that may be inside the cavity.***



  - The universal function is called the Planck function $B_\nu(T)$.
  - This is the blackbody radiation.

Spectrum of Blackbody Radiation

- There is no perfect blackbody.
  - However, the cosmic microwave background comes quite close; stars can sometimes be usefully approximated as blackbodies.
  - By the end of the 19th century, the blackbody spectrum was fairly well known empirically, from laboratory studies.
  - In 1900, Max Planck, using his idea of quantized energies, derived the blackbody spectrum.



https://pages.uoregon.edu/imamura/321/

- The frequency dependence of blackbody radiation is given by the ***Planck function***:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp\left(h\nu/k_\mathrm{B}T\right) - 1} \quad \text{or} \quad B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp\left(hc/\lambda k_\mathrm{B}T\right) - 1}$$

$$h = 6.63 \times 10^{-27} \text{ erg s (Planck's constant)}$$

$$k_\mathrm{B} = 1.38 \times 10^{-16} \text{ erg K}^{-1} \text{ (Boltzmann's constant)}$$

Rayleigh-Jeans Law & Wien Law

- Rayleigh-Jeans Law (low-energy limit)

$$h\nu \ll k_{\mathrm{B}}T \ (\nu \ll 2 \times 10^{10}\mathrm{Hz}(T/1\mathrm{K})) \quad \rightarrow \quad I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2}k_{\mathrm{B}}T$$

- Originally derived by assuming the classical equipartition of energy

$$\langle E \rangle = 2 \times (1/2)k_{\mathrm{B}}T \qquad u_\nu = \rho_s \langle E \rangle \rightarrow I_\nu = u_\nu c = \rho_s \langle E \rangle c$$

- ultraviolet catastrophe: if the equation is applied to all frequencies, the total amount of energy would diverge.

$$\int \nu^2 d\nu \rightarrow \infty$$

- Wien Law (high-energy limit)

$$h\nu \gg k_{\mathrm{B}}T \quad \rightarrow \quad I_\nu^W(T) = \frac{2h\nu^3}{c^2}\exp\left(-\frac{h\nu}{k_{\mathrm{B}}T}\right)$$

In Local Thermodynamic Equilibrium

- LTE is characterized by the following three equilibrium distribution
  - Maxwellian velocity distribution of particles

$$f(\mathbf{v})d^3\mathbf{v} = \left(\frac{m}{2\pi k_\mathrm{B} T}\right)^{2/3} \exp\left(-\frac{mv^2}{2k_\mathrm{B}T}\right) d^3\mathbf{v}$$

  - Boltzmann distribution: The probability distribution that a system will be in a certain quantum state as a function of that state's energy and the temperature of the system.

$$p_i = \frac{g_i}{U} \exp\left(-\frac{E_i}{k_\mathrm{B}T_\mathrm{ex}}\right) \qquad\qquad U = \sum_k g_k e^{-E_k/k_\mathrm{B}T}$$

where p_i is the population of level i, g_i is its statistical weight, E_i is the level energy, measured from the ground state, and U is the partition function of the ionization stage to which level i belongs.

  - Saha ionization equation:

$$\frac{N_I}{N_{I+1}} = n_e \frac{U_I}{U_{I+1}} \left(\frac{h^2}{2\pi m_e k_\mathrm{B} T}\right)^{3/2} \exp\left(\frac{\chi_I}{k_\mathrm{B}T}\right)$$

where U_I is the partition function of the ionization stage I and χ_I is the ionization potential of ion I.

Breakdown of LTE

- LTE holds if all atomic processes are in detailed balance, i.e., if the number of processes $A \rightarrow B$ is exactly balanced by the number of inverse processes $B \rightarrow A$.

- By the term non-LTE (or NLTE), we describe any state that departs from LTE.

  - **Collisional transitions**: Under many astrophysical circumstances, collisions between particles tend to maintain the local equilibrium (thus, Maxwellian velocity distribution).

  - **Radiative transitions**: However, the fact that the radiation escapes from a system implies that LTE must eventually break down at a certain point. The number of photoexcitations (any atomic transition induced by absorbing a photon) becomes lower than the number of inverse, spontaneous processes.

- Therefore, we expect departures from LTE if the two conditions are met:

  (i) radiative rates in some important atomic transition dominate over the collisional rates

  (ii) radiation is not in equilibrium, i.e., the intensity does not have the Planckian distribution.

It is therefore clear that for low densities departures from LTE will be significant or even crucial.

In the upper layers of the stellar atmosphere, departures from LTE are expected to be largest.

In contrast, deep in the atmosphere, photons do not escape, and so the intensity is close to the equilibrium value. Departures from LTE are therefore small.

- NLTE in most astrophysical systems,
  - The velocity distribution is well represented by a Maxwell distribution at the gas temperature, as the Bremsstrahlung radiation is inefficient, and the velocities are governed by elastic collisions between particles.
  - However, the internal quantum states and ionization stages do not follow the Boltzmann distribution or the Saha equation.

- In NLTE, the excitation temperature $T_{\rm ex}$ for level i is defined as follows:

$$p_i \propto g_i \exp\left(-\frac{E_i}{k_{\rm B} T_{\rm ex}}\right)$$

Here, $T_{\rm ex}$ is the excitation temperature for level i.

$$T_{\rm ex} \neq T_{\rm gas}$$

Kirchhoff's Law in TE

- In (full) thermodynamic equilibrium at temperature *T*, by definition, the following two conditions are satisfied:

$$(1)\ \frac{dI_\nu}{ds} = 0 \quad \text{and} \quad (2)\ I_\nu = B_\nu(T)$$

We also note that $$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

- Then, we obtain ***the Kirchhoff's law for a system in TE***:

$$\boxed{\frac{j_\nu}{\alpha_\nu} = B_\nu(T), \quad j_\nu = \alpha_\nu B_\nu(T)}$$

- This is remarkable because it connects the properties $j_\nu(T)$ and $\alpha_\nu(T)$ of any kind of matter to the single universal spectrum $B_\nu(T)$.

Kirchhoff's Law in LTE

- Recall that Kirchhoff's law was derived for a system in thermodynamic equilibrium.

- ***Kirchhoff's law applies not only in TE but also in LTE***:
  - Recall that $B_\nu(T)$ ***is independent of the properties of the radiating / absorbing material.***
  - In contrast, both $j_\nu(T)$ ***and*** $\alpha_\nu(T)$ ***depend only on the materials in the cavity and on the temperature of that material***; they do not depend on the ambient radiation field or its spectrum.
  - Therefore, the Kirchhoff's law should be true even for the case of LTE.
  - ***In LTE, the ratio of the emission coefficient to the absorption coefficient is a function of temperature and frequency only.***
  - This generalized version of Kirchhoff's law is an exceptionally valuable tool for calculating the emission coefficient from the absorption coefficient or vice versa.

Implications of Kirchhoff's Law

- **A good absorber is a good emitter, and a poor absorber is a poor emitter.** (In other words, a good reflector must be a poor absorber, and thus a poor emitter.)

$$j_\nu = \alpha_\nu B_\nu(T) \quad \rightarrow \quad j_\nu \text{ increases as } \alpha_\nu \text{ increases}$$

  - It is not possible to thermally radiate more energy than a blackbody, at equilibrium.

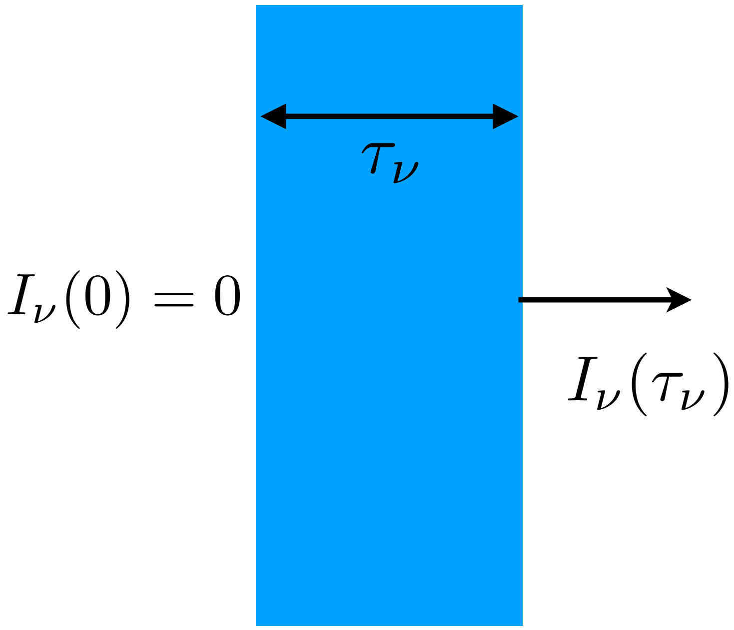
$$\left(dI\right)_{\text{emiss}} = j_\nu ds = \alpha_\nu \frac{j_\nu}{\alpha_\nu} ds \;\leq\; B_\nu(T) \quad \text{because} \quad \alpha_\nu ds = \frac{\left|dI_\nu\right|_{\text{abs}}}{I_\nu} \leq 1$$

  - The radiative transfer equation in LTE can be rewritten: $\boxed{\dfrac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T)}$

- ***Remark***:
  - ***Blackbody radiation*** means $I_\nu = B_\nu(T)$. An object for which the intensity is the Planck function is emitting blackbody radiation.
  - ***Thermal radiation is defined as radiation emitted by "matter" in LTE***. Thermal radiation means $S_\nu = B_\nu(T)$.

- To see the difference between thermal and blackbody radiation,
  - Consider a slab of material with optical depth τ_ν that is producing thermal radiation.
  - If no light is falling on the back side of the slab, the intensity that is measured on the front side of the slab is

$$\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right) \\ &= B_\nu\left(1 - e^{-\tau_\nu}\right) \end{aligned}$$



  - If the slab is optical thick at frequency ν $(\tau_\nu \gg 1)$, then

$$I_\nu \approx B_\nu$$

  - If the slab is optically thin $(\tau_\nu \ll 1)$, then

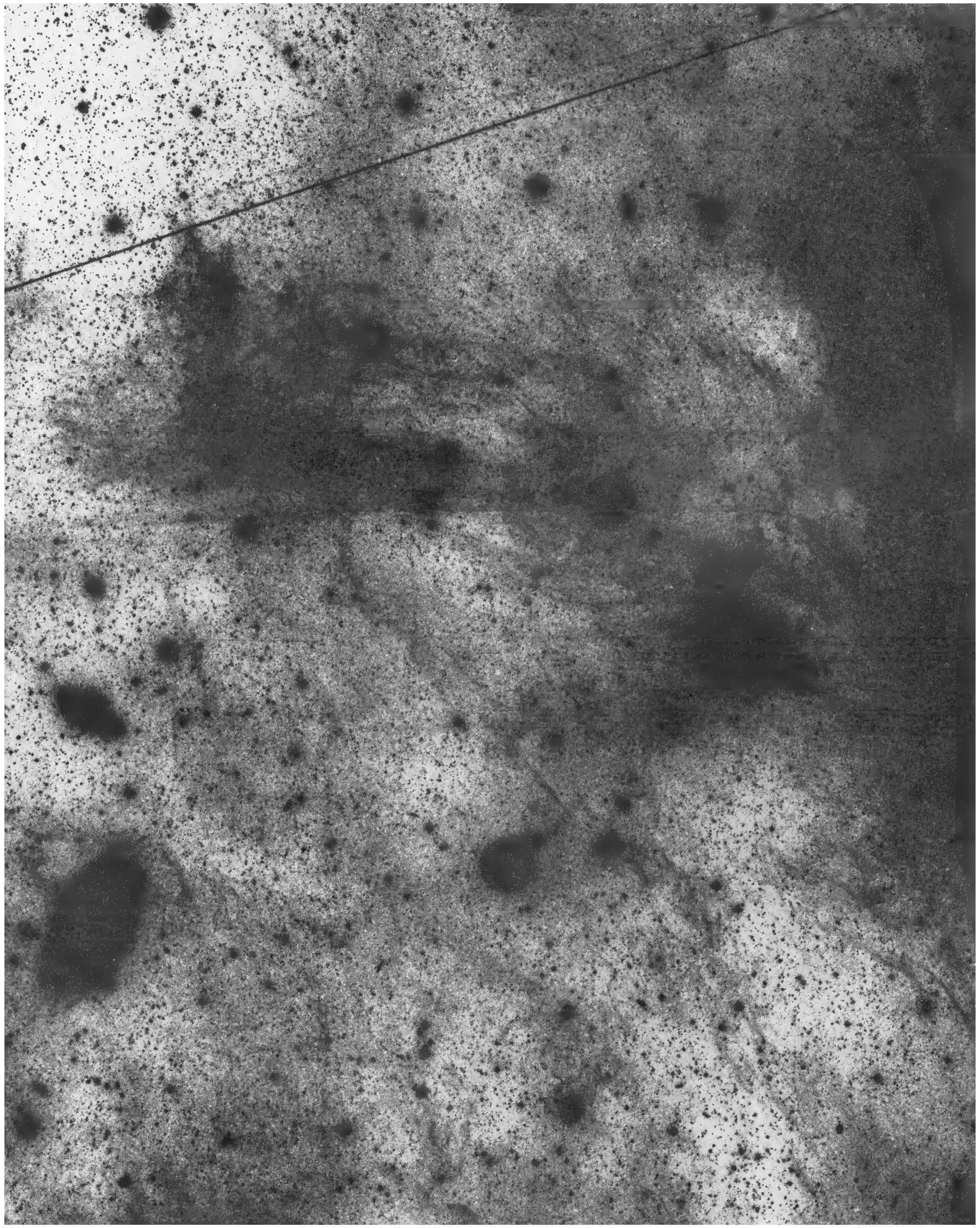
$$I_\nu \approx \tau_\nu B_\nu \ll B_\nu$$

  This indicates that the radiation, although thermal, will not be blackbody.

Thermal radiation becomes blackbody radiation only for optical thick media.

Dust Clouds in our Galaxy

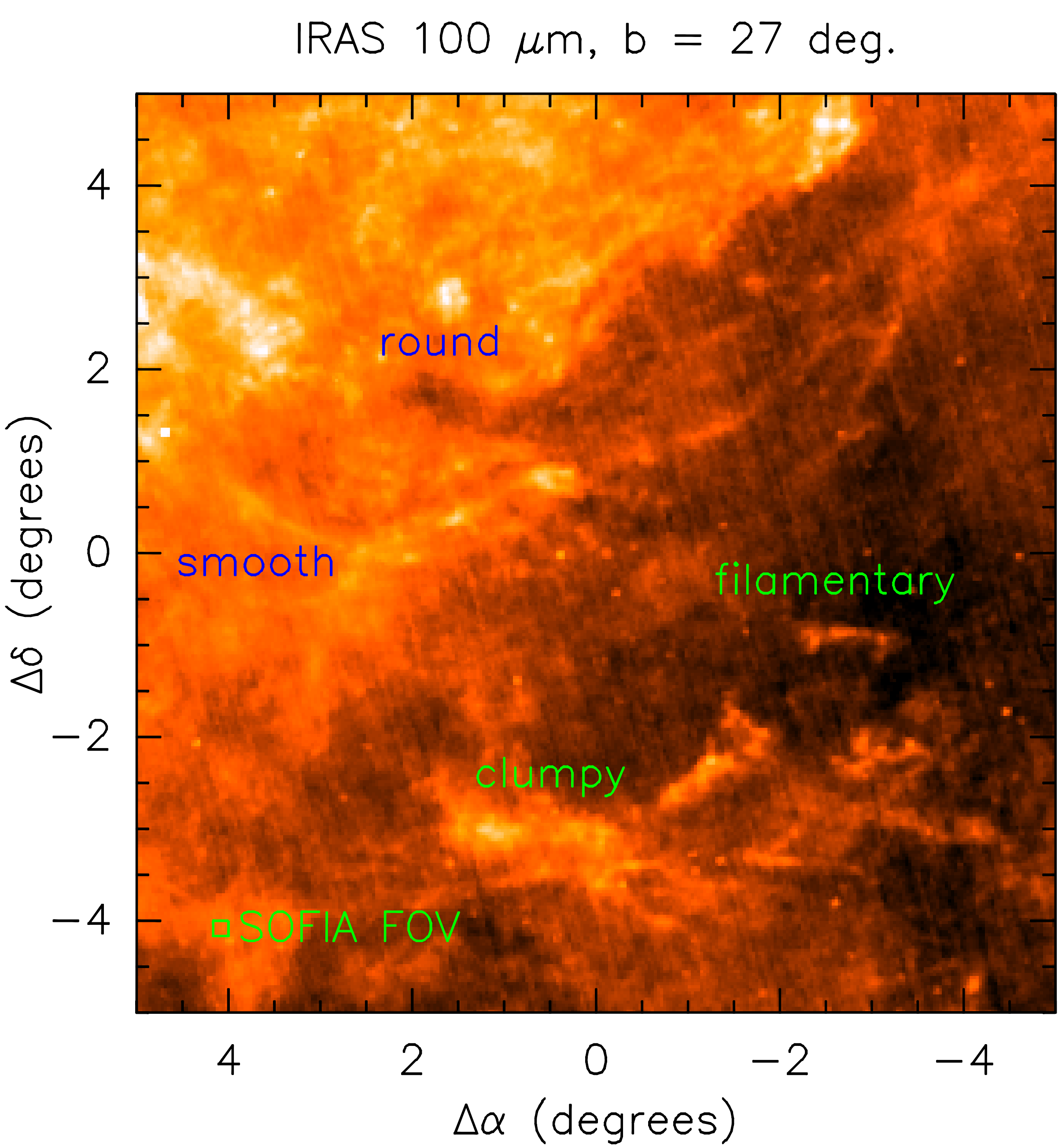
- The ISM consists of **gas** and **dust** (solid particles).
- First Discovery of interstellar dust: de Vaucouleurs (1955)
- First detailed optical images of high latitude clouds in a Galactic polar cap survey - Sandage (1976)
- **The cirrus clouds are observational obstacles in studying extragalactic LSB features.**

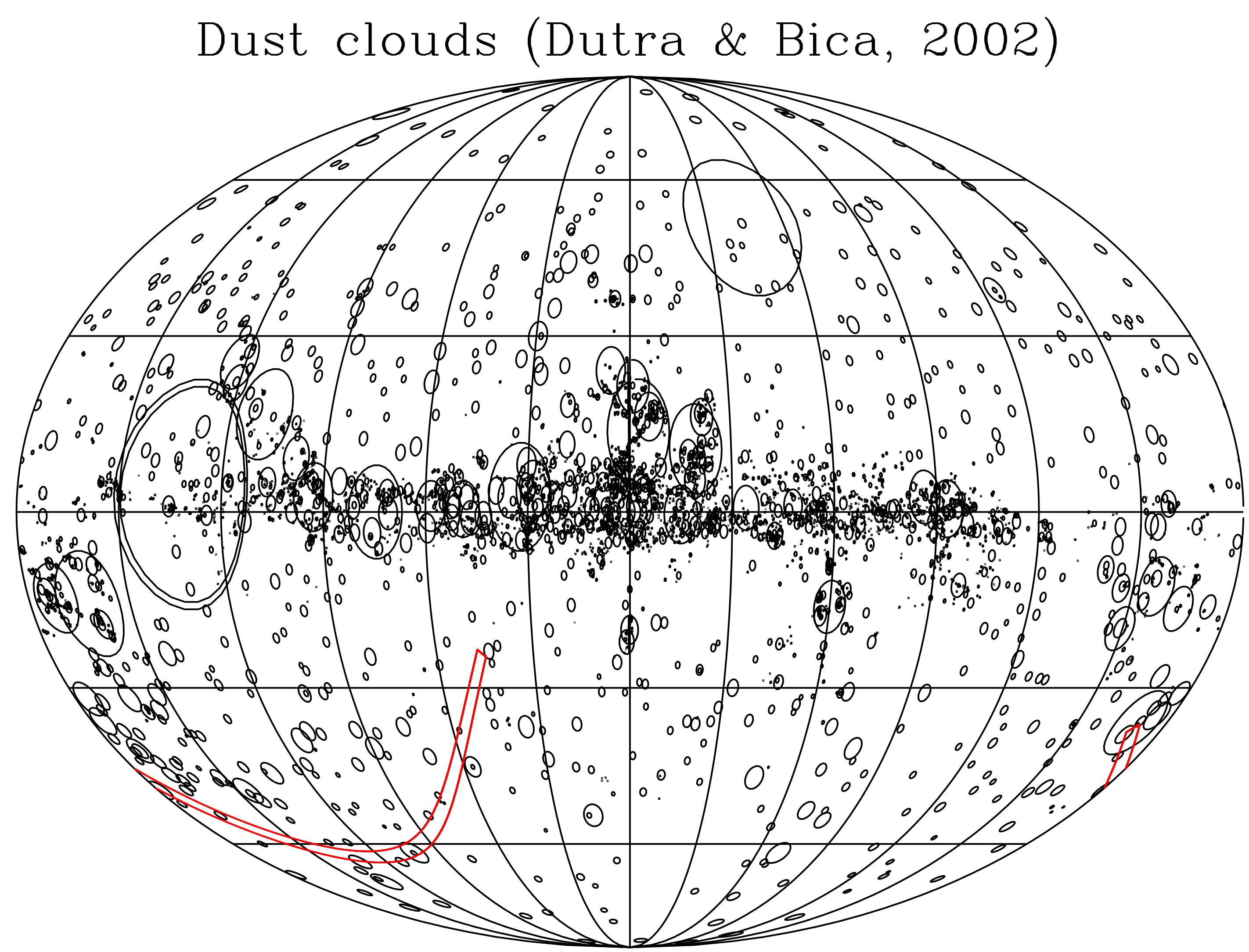


Sandage (1976)

Cirrus Clouds are Ubiquitous!

- Discovery of far-IR (FIR) emitting cirrus clouds over the full sky - Low et al. (1984), IRAS satellite





Radiative Transfer in Astrophysics: Dust

- Dust materials
  - Silicates: The two main types of silicates in dust are pyroxene and olivine.

olivine	$Mg_{2x}Fe_{2-2x}SiO_4$	forsterite $-$ Mg_2SiO_4	fayalite $-$ Fe_2SiO_4
pyroxene	$Mg_xFe_{1-x}SiO_3$	enstatite $-$ $MgSiO_3$	ferrosilite $-$ $FeSiO_3$
	$(x = 0 - 1)$		

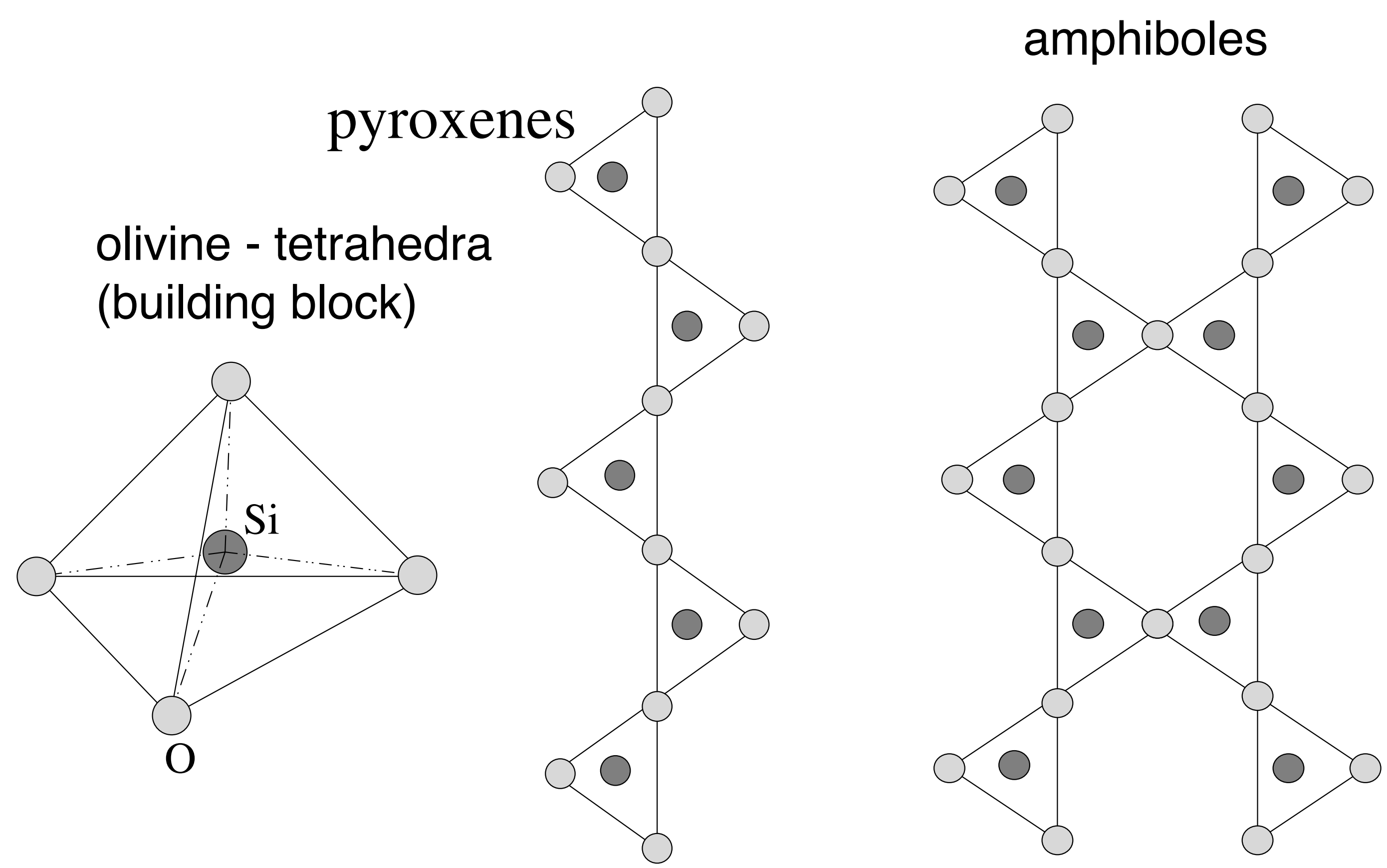
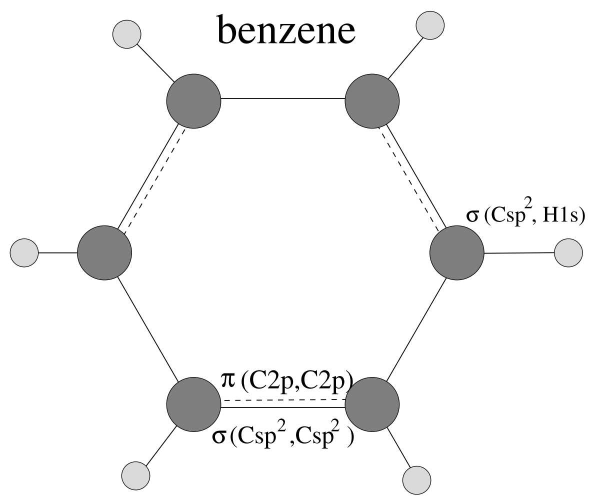


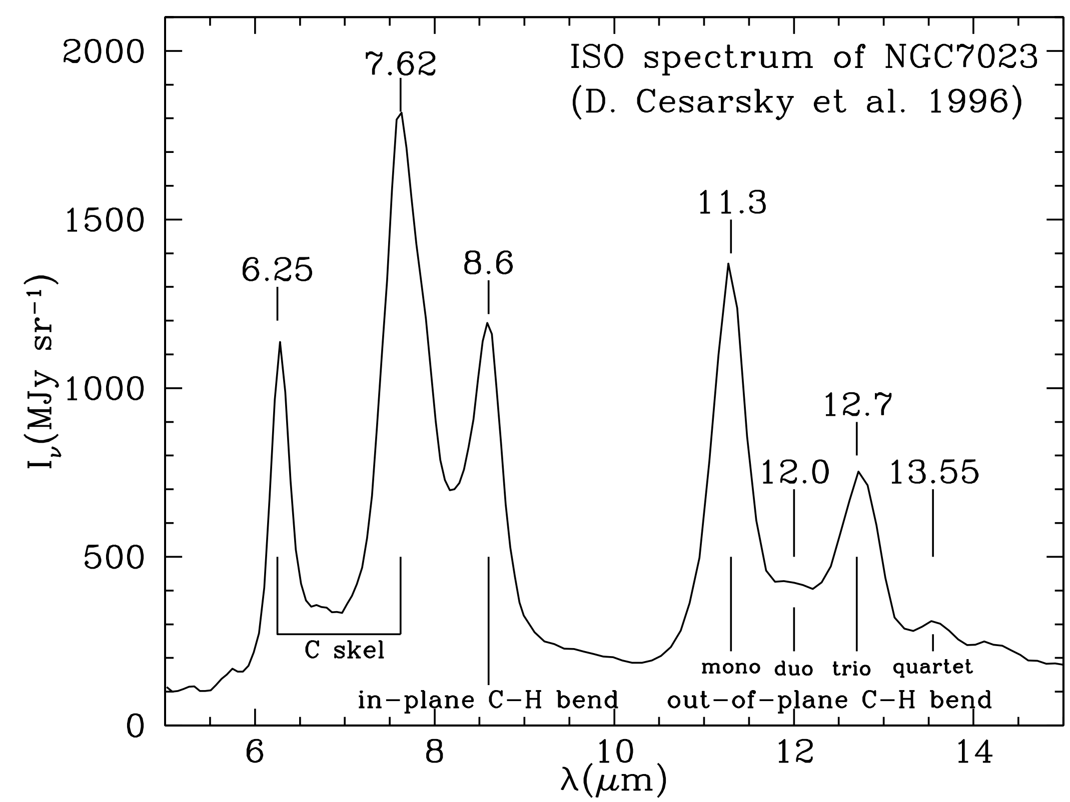
Fig 5.9 Krugel
[An Introduction to the Physics of Interstellar Dust]

- Polycyclic Aromatic Hydrocarbons (PAHs)

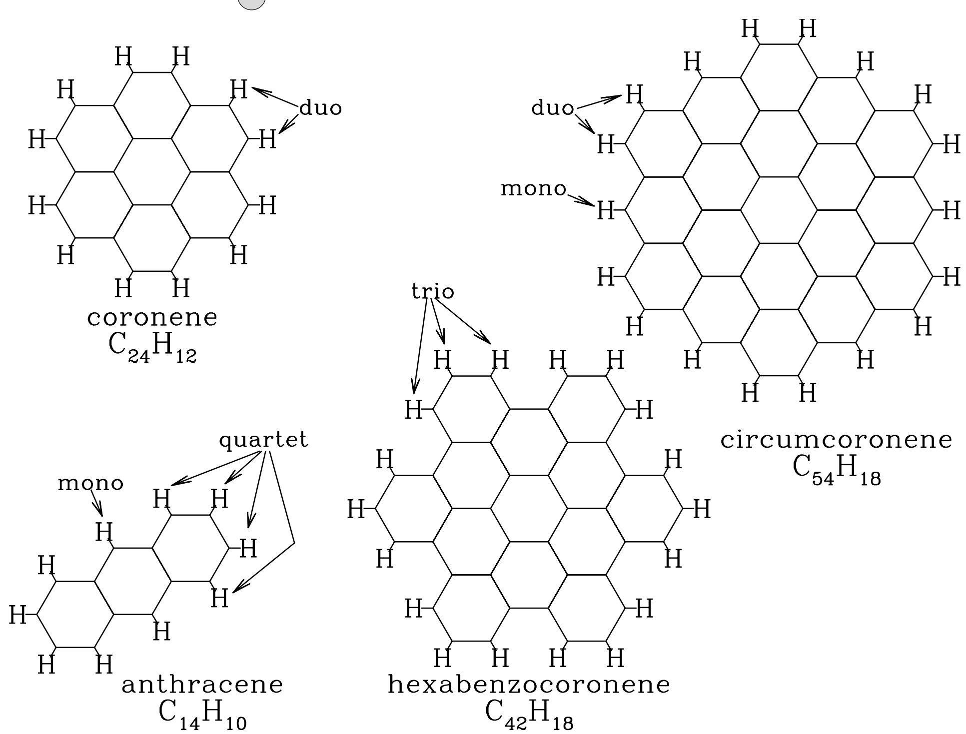


Bezene ring:
The simplest type of PAHs.

[Fig 5.6 in Krugel]



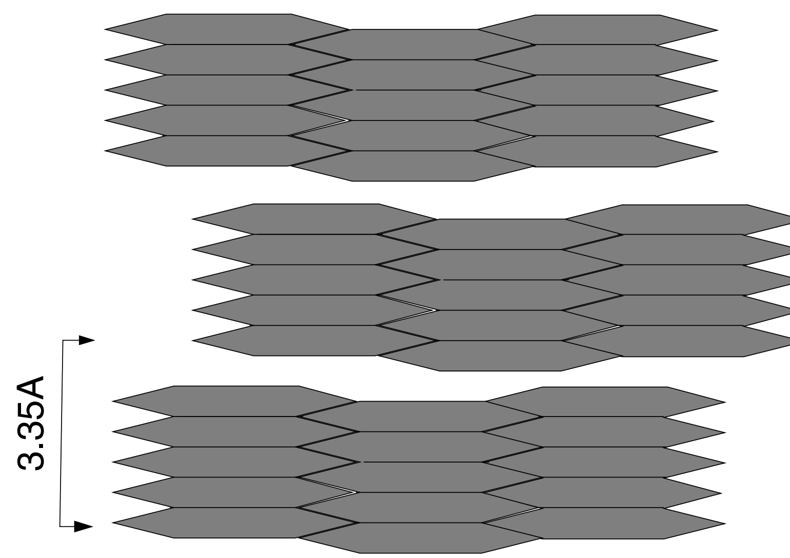
The IR spectrum of the reflection nebula NGC 7023 (Cesarsky et al. 1996)

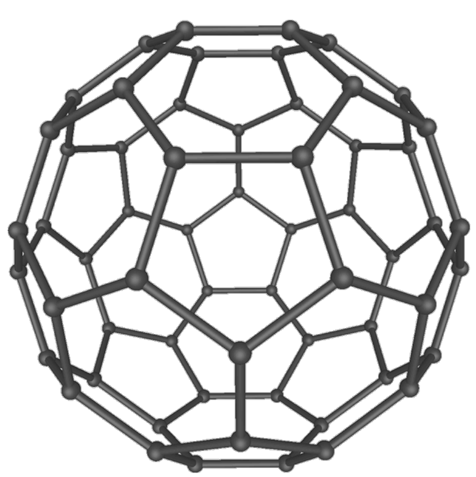


Structure of four PAHs.
[Fig 23.9 in Draine]

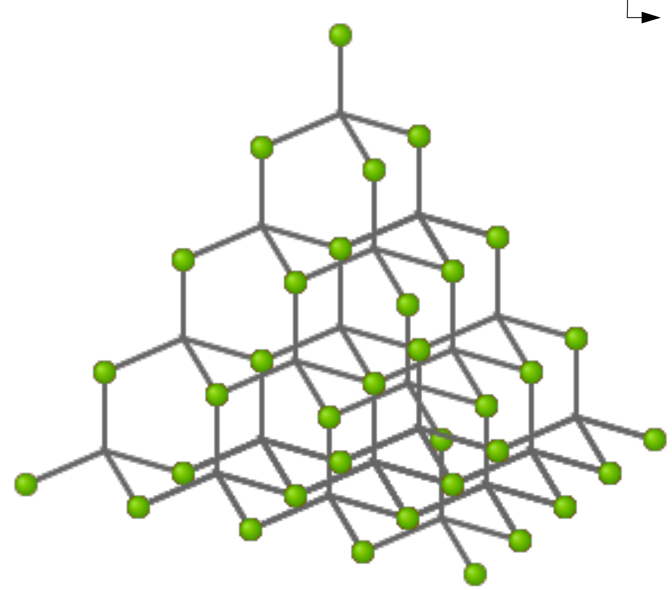
- Graphite (흑연)
    - Graphite is the most stable form of carbon (at low pressure), consisting of infinite parallel sheets of sp^2-bonded carbon.
- Nanodiamond
    - Diamond consists of sp^3-bonded carbon atoms, with each carbon bonded to four equidistant nearest neighbors (enclosed angles are 109.47^o).
- Armorphous carbon
- Hydrogenated amorphous carbon (HAC)
- Fullerenes

graphite sheets





Buckminsterfullerene (C_{60})

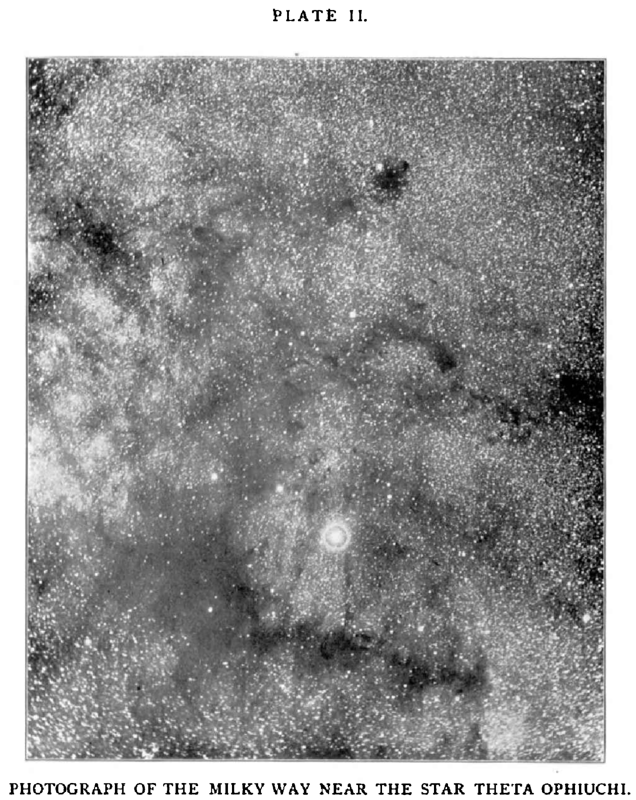


Structure of diamond.

RT effects in Dust

- **Extinction = Absorption + Scattering**
  - Dust particles can scatter light, changing its direction of propagation. When we look at a reflection nebula, like that surrounding the Pleiades, we are seeing light from the central stars that has been scattered by dust into our line of sight.
  - Dust particles can also absorb light. The relative amount of scattering and absorbing depends on the properties of the dust grains.
- **Thermal radiation from Dust**
  - When dust absorbs light, it becomes warmer, so dust grains can emit light in the form of thermal radiation. Most of this emission is at wavelengths from a few microns (near IR) to the sub-mm range (Far-IR).
- **Polarization**
  - The polarization of starlight was discovered in 1949 (Hall 1949).
  - The degree of polarization tends to be larger for stars with greater reddening, and stars in a given region of the sky tends to have similar polarization directions.

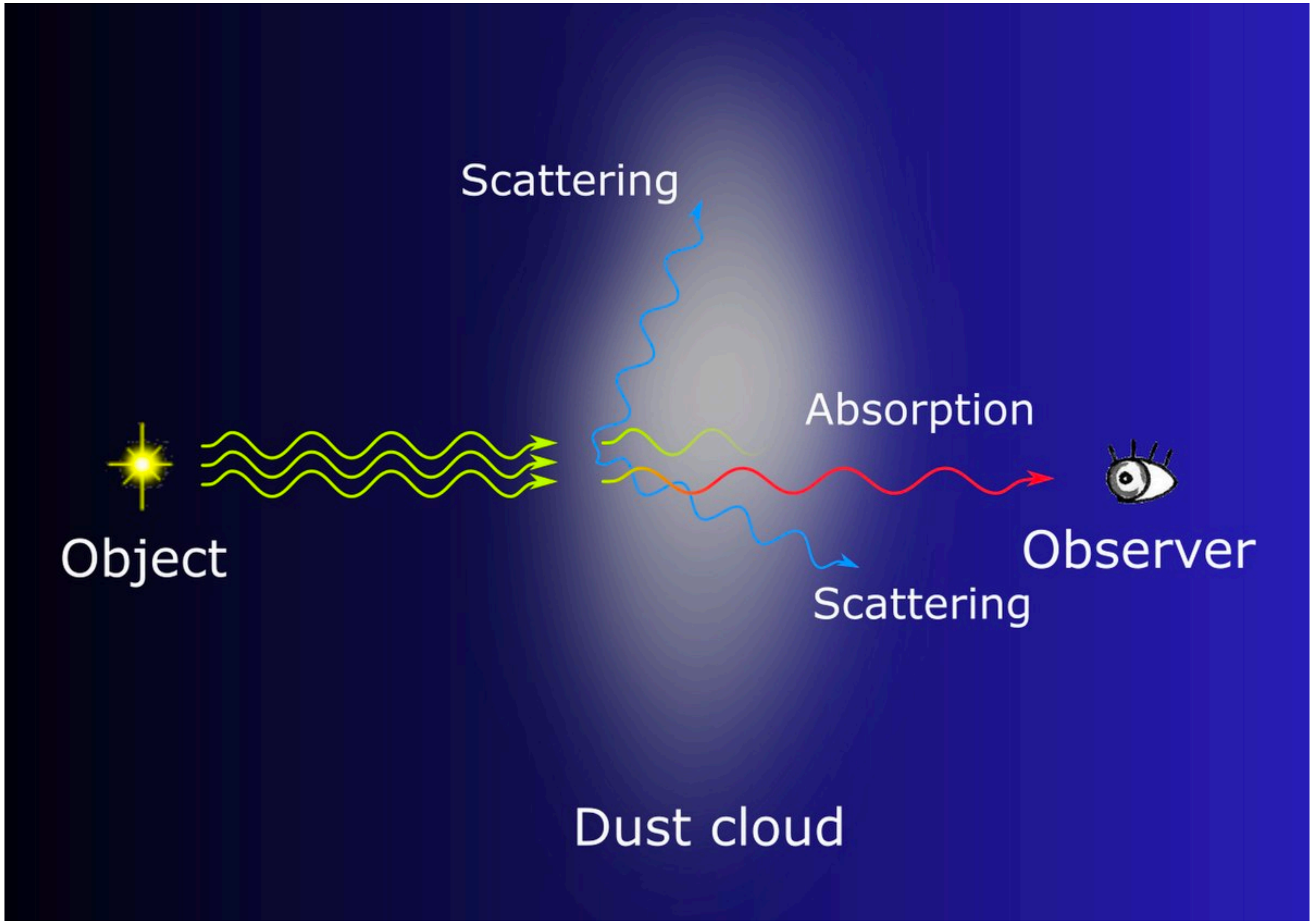
The Pleiades cluster and surrounding reflection nebulae (Fig. 6.3, Ryden)



The dark structures near θ Ophiuchi (Barnar 1899; Fig. 6.1, Ryden)

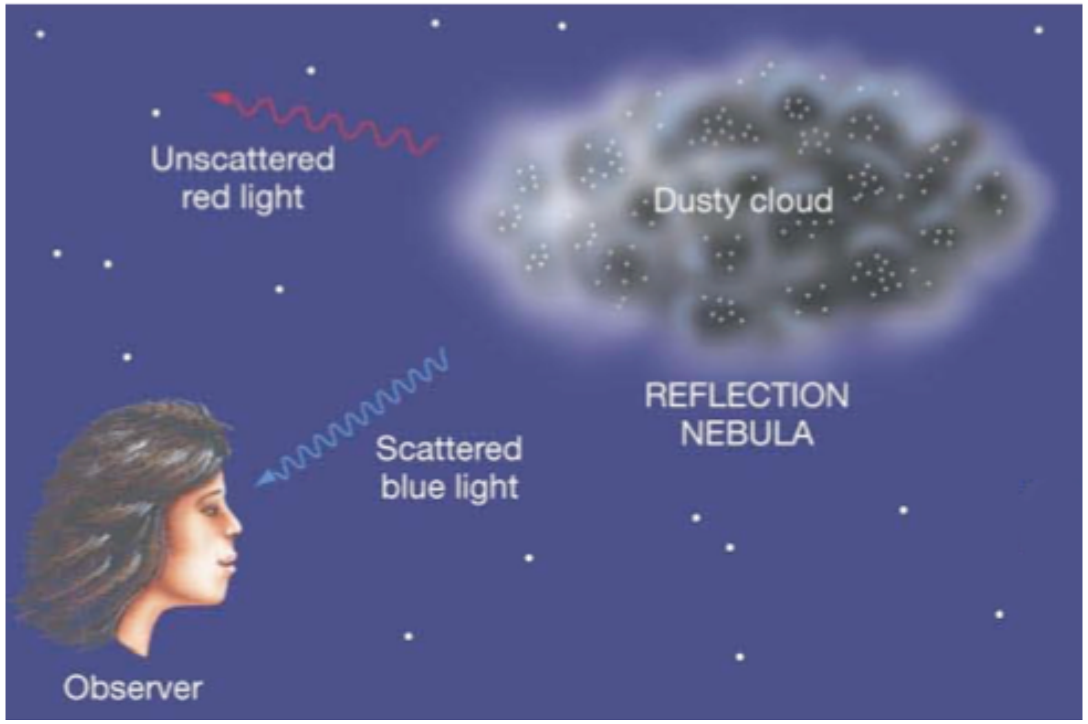
RT effects in Dust: Reddening

- ***Reddening***
  - Reddening is the phenomena of the color of a visible astronomical object (e.g., star) appearing more red from a distance than from nearby.
  - Within the visible wavelength range, **the absorption/scattering cross-section by dust increases with frequency, absorbing/scattering more of the blue light than red.**
  - This effect leads to the reddening.



RT effects in Dust: Blue Reflection Nebulae

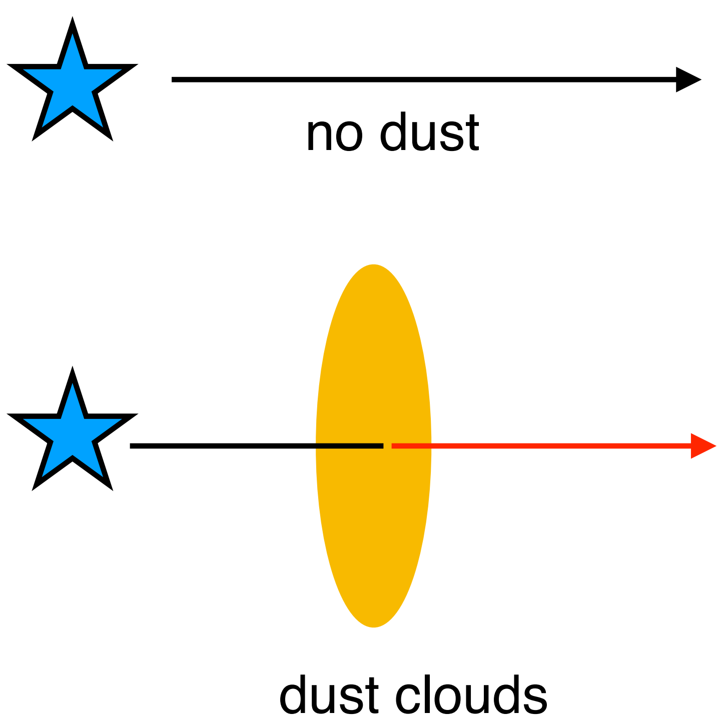
- ***Scattering of Starlight***
  - When an interstellar cloud happens to be unusually near one or more bright stars, we have a reflection nebula, where we see starlight photons that have been scattered by the dust in the cloud.
  - ***The spectrum of the light coming from from the cloud surface shows the stellar absorption lines***, thus demonstrating that scattering rather than some emission process is responsible.
  - Given the typical size of interstellar dust grains, blue light is scattered more than red light. ***A reflection nebulae is typically blue*** (so for the same reason that the sky is blue, except it's scattering by dust (for the reflection nebula) vs by molecules (for the earth's atmosphere)).



How to measure the Interstellar Extinction

- Pair method
  - Trumpler (1930) compared the spectra of pairs of stars with identical (or similar) spectral type, one with negligible obscuration and the other extinguished by dust along the line of sight. This method remains our most direct way to study the "selective extinction" or "reddening" of starlight by the interstellar dust.

Compare two stars
with the same spectral type

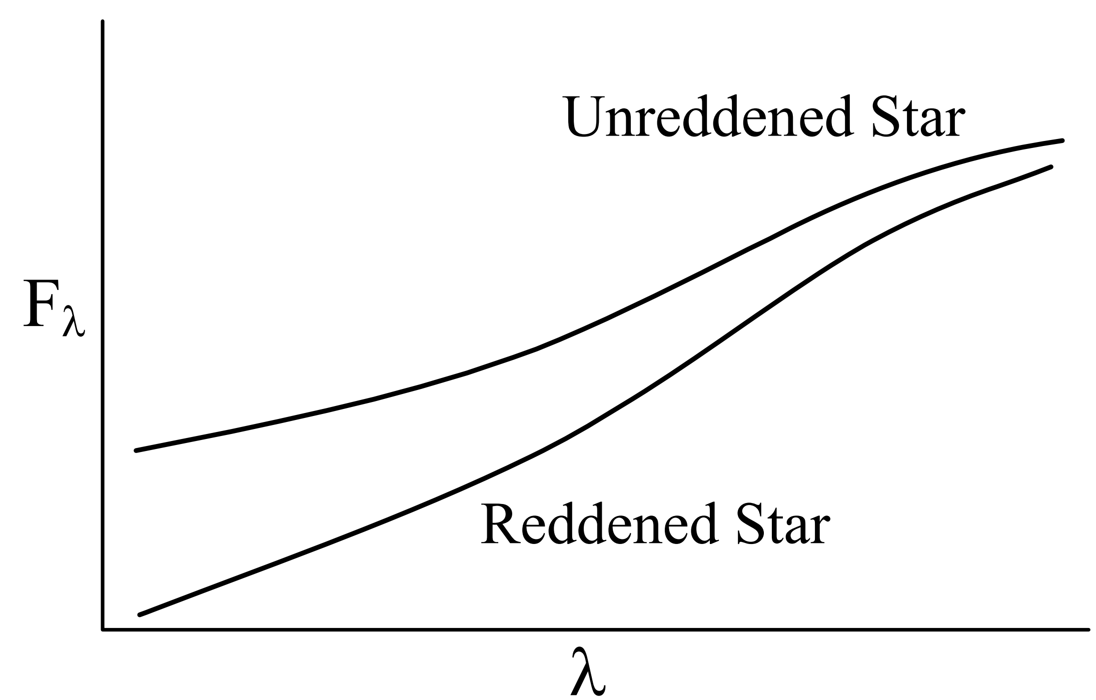


unreddened star

$$F(\lambda) = \frac{F_0(\lambda)}{4\pi d_1^2}$$

reddened star

$$F(\lambda) = \frac{F_0(\lambda)}{4\pi d_2^2} e^{-\tau_\lambda}$$



Application of Kirchhoff's Law: Dust Emission

- Consider a dusty cloud with a small volume V through which external radiation passes.
  1. (1) We first need to know the absorption coefficient $\alpha_\nu = \rho\kappa_\nu$.
  2. (2) Calculate the total absorbed energy

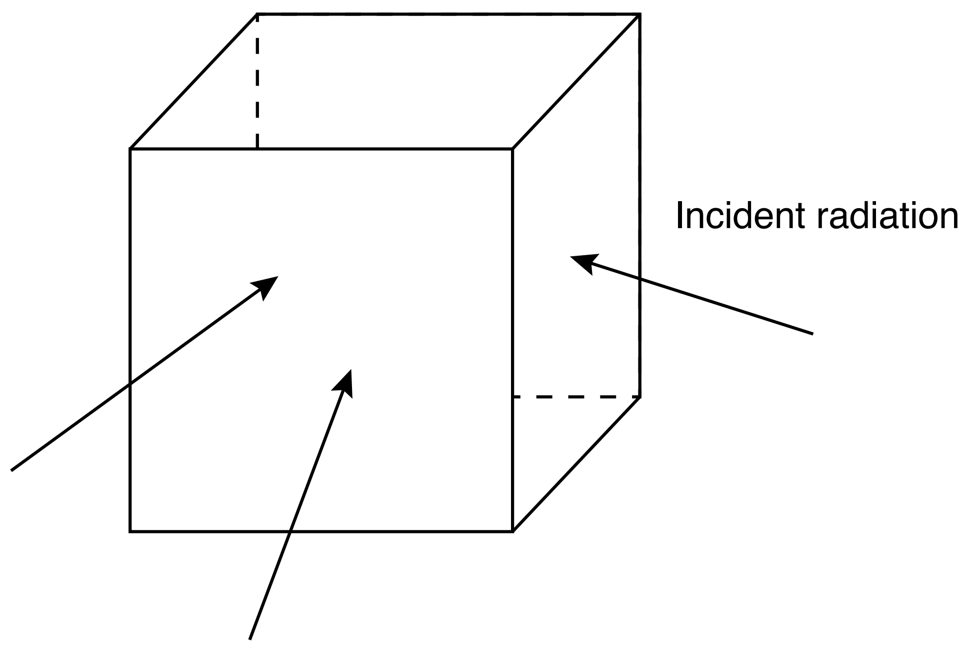
$$\boxed{\int L_\nu^{\rm abs} d\nu}$$

  3. (3) The total absorbed energy should be balanced by the energy emitted by dust grains.

$$\int L_\nu^{\rm abs} d\nu = \int L_\nu^{\rm em} d\nu = 4\pi V \int j_\nu^{\rm em} d\nu$$

we then calculate the temperature of dust, using Kirchhoff's law [$j_\nu^{\rm em} = \rho\kappa_\nu^{\rm abs} B_\nu(T)$]:

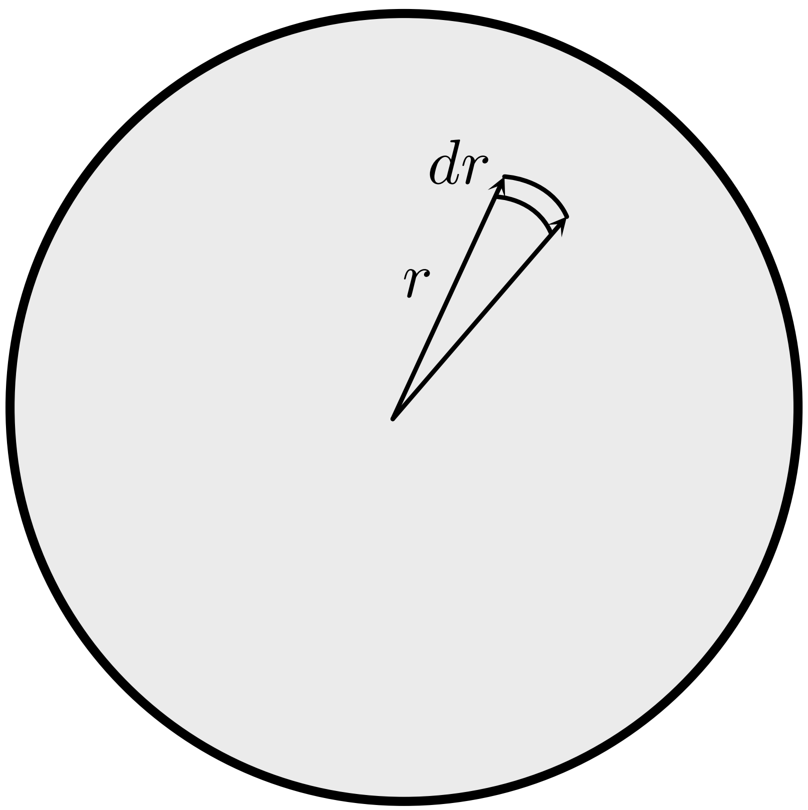
$$\boxed{\int L_\nu^{\rm abs} d\nu = 4\pi\rho V \int \kappa_\nu^{\rm abs} B_\nu(T) d\nu}$$



(4) The emission spectrum can be then obtained by

$$\boxed{L_\nu^{\rm em} = 4\pi\rho V \kappa_\nu^{\rm abs} B_\nu(T)}$$

Example: Uniform dusty cloud with a central source



L_ν = luminosity of the central source.

(1) Energy absorbed by a volume element $\Delta V = r^2 \Delta r \Delta\Omega$

$$\Delta L_\nu^{\mathrm{abs}} = L_\nu \frac{r^2 \Delta\Omega}{4\pi r^2} e^{-\tau_\nu} \left(1 - e^{-\Delta\tau_\nu}\right) = L_\nu \frac{r^2 \Delta\Omega}{4\pi r^2} e^{-\tau_\nu} \Delta\tau_\nu$$

where $\tau_\nu = \alpha_\nu^{\mathrm{abs}} r$

$\Delta\tau_\nu = \alpha_\nu^{\mathrm{abs}} \Delta r$

(2) Energy emitted by the volume element if it has a temperature T.

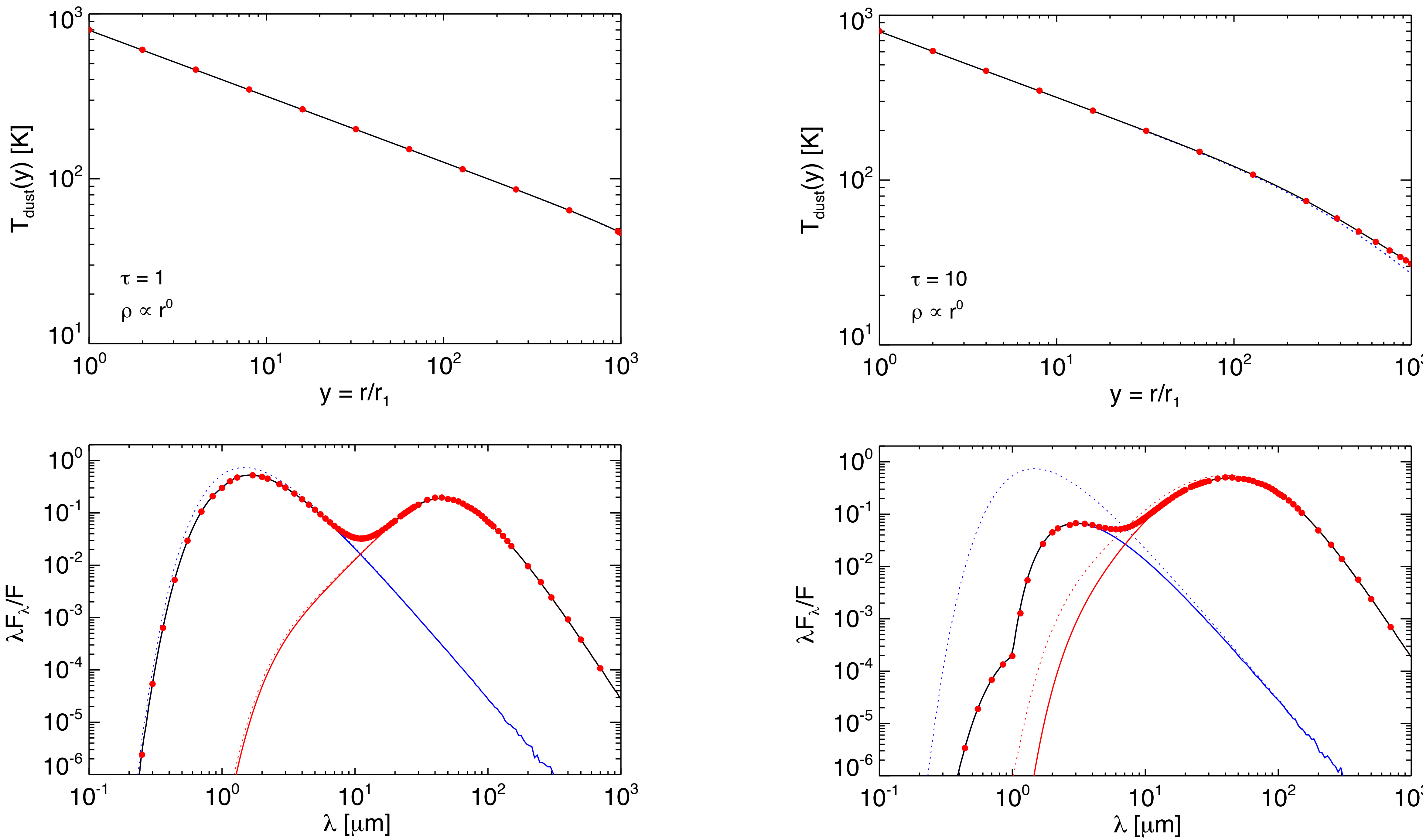
$$\Delta L_\nu^{\mathrm{em}} = 4\pi j_\nu r^2 \Delta\Omega \Delta r = 4\pi \alpha_\nu^{\mathrm{abs}} B_\nu(T) r^2 \Delta\Omega \Delta r$$

(3) Energy balance $\displaystyle \int \Delta L_\nu^{\mathrm{em}} d\nu = \int \Delta L_\nu^{\mathrm{abs}} d\nu \Rightarrow \boxed{\frac{1}{(4\pi)^2 r^2} \int L_\nu e^{-\tau_\nu} \alpha_\nu^{\mathrm{abs}} d\nu = \int \alpha_\nu^{\mathrm{abs}} B_\nu(T) d\nu}$

Calculate the temperature, $T(r)$, at each radius r from the above equation.

(4) Dust emission $\displaystyle \boxed{L_\nu = (4\pi)^2 \int \alpha_\nu^{\mathrm{abs}} B_\nu(T(r)) r^2 dr}$

- Monte Carlo RT simulations
  - Dust emission is also absorbed, so iterations are required until the solution converges.



Here, τ is the optical depth at 1 μm.

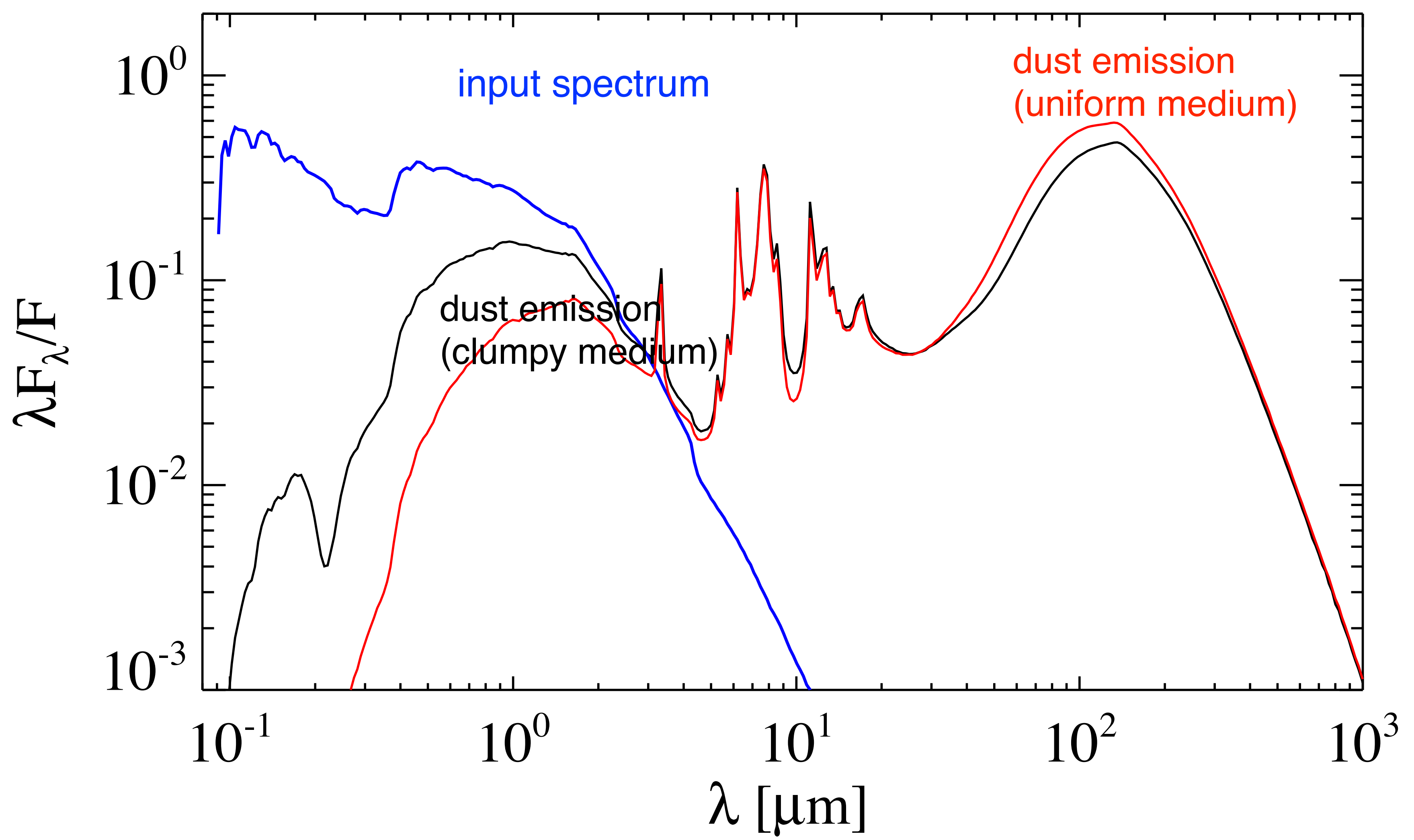
Blue curve: attenuated stellar spectrum
Red curve : dust emission

- Spectral Energy Distribution

Blue spectrum is the input stellar spectrum from a typical spiral galaxy.

Dust emission was calculated using the Kirchhoff's law.

This shows a typical SED shape of galaxies.



Please please note that α_ν or κ_ν is not the emissivity (nor emission coefficient), as often wrongly referred to in the literature of the external galaxies community.

$$\kappa_\nu = \kappa_0 \nu^\beta$$
$$\kappa_\lambda = \kappa_0 \lambda^{-\beta}$$

for dust absorption in Far-IR wavelengths

$$j_\lambda = \kappa_0 \lambda^{-\beta} B_\lambda(T) \quad (\beta \approx 1-2)$$

is referred to as the modified blackbody.

Application of Kirchhoff's Law: Thermal Bremsstrahlung

[Free-free emission]

The volume emissivity ($\varepsilon_\nu^{\rm ff} = 4\pi j_\nu^{\rm ff}$) of the thermal bremsstrahlung is

$$\boxed{\varepsilon_\nu^{\rm ff} = 6.8 \times 10^{-38} n_i n_e Z^2 T^{-1/2} e^{-h\nu/kT} \overline{g_{\rm ff}} \quad ({\rm erg\,s^{-1}\,cm^{-3}\,Hz^{-1}})}$$

$$\overline{g_{\rm ff}} \equiv \int_0^\infty g_{\rm ff}(v,\omega) e^{-u} du$$

$$g_{\rm ff}(v,\omega) = (\sqrt{3}/\pi) \ln (b_{\rm max}/b_{\rm min})$$

where $\overline{g_{\rm ff}}$ is the velocity-averaged free-free Gaunt factor.

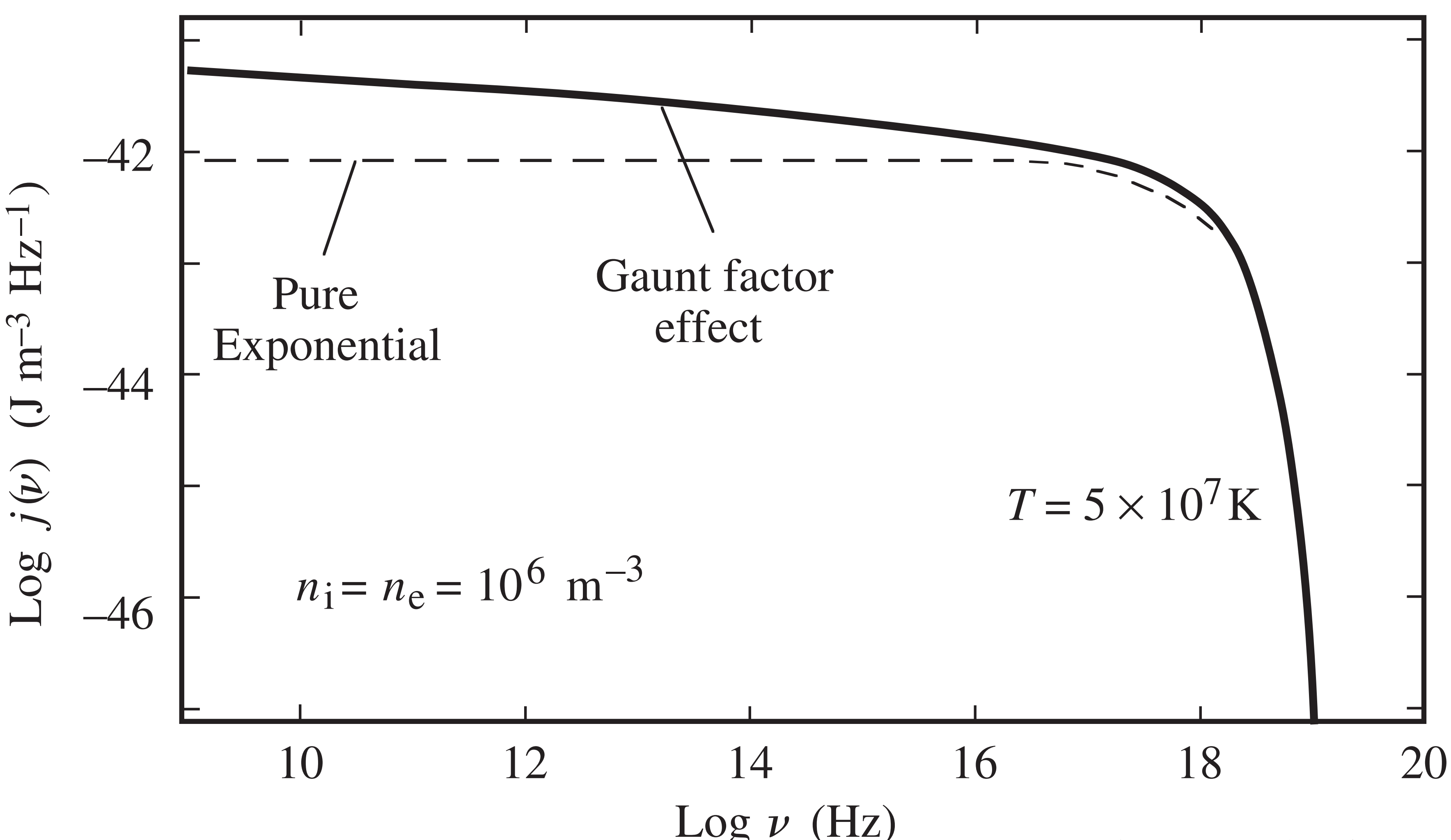
Summing over all ion species gives the emissivity:

$$\boxed{\varepsilon_\nu^{\rm ff} = 6.8 \times 10^{-38} \sum_i n_i n_e Z_i^2 T^{-1/2} e^{-h\nu/kT} \overline{g_{\rm ff}} \quad ({\rm erg\ s^{-1}\ cm^{-3}\ Hz^{-1}})}$$

Note that main frequency dependence is $\varepsilon_\nu^{\mathrm{ff}} \propto \exp\left(-h\nu/kT\right)$, which shows a "flat spectrum" with a cut off at $\nu \sim kT/h$. The cut-off of the spectrum can be used to determine the temperature of hot plasma.

For a hydrogen plasma ($Z = 1$) with $T > 3 \times 10^5$ K at low frequencies ($h\nu \ll kT$) Gaunt factor is given by

$$\overline{g_{\mathrm{ff}}} = \frac{\sqrt{3}}{\pi} \ln\left(\frac{2.25kT}{h\nu}\right)$$



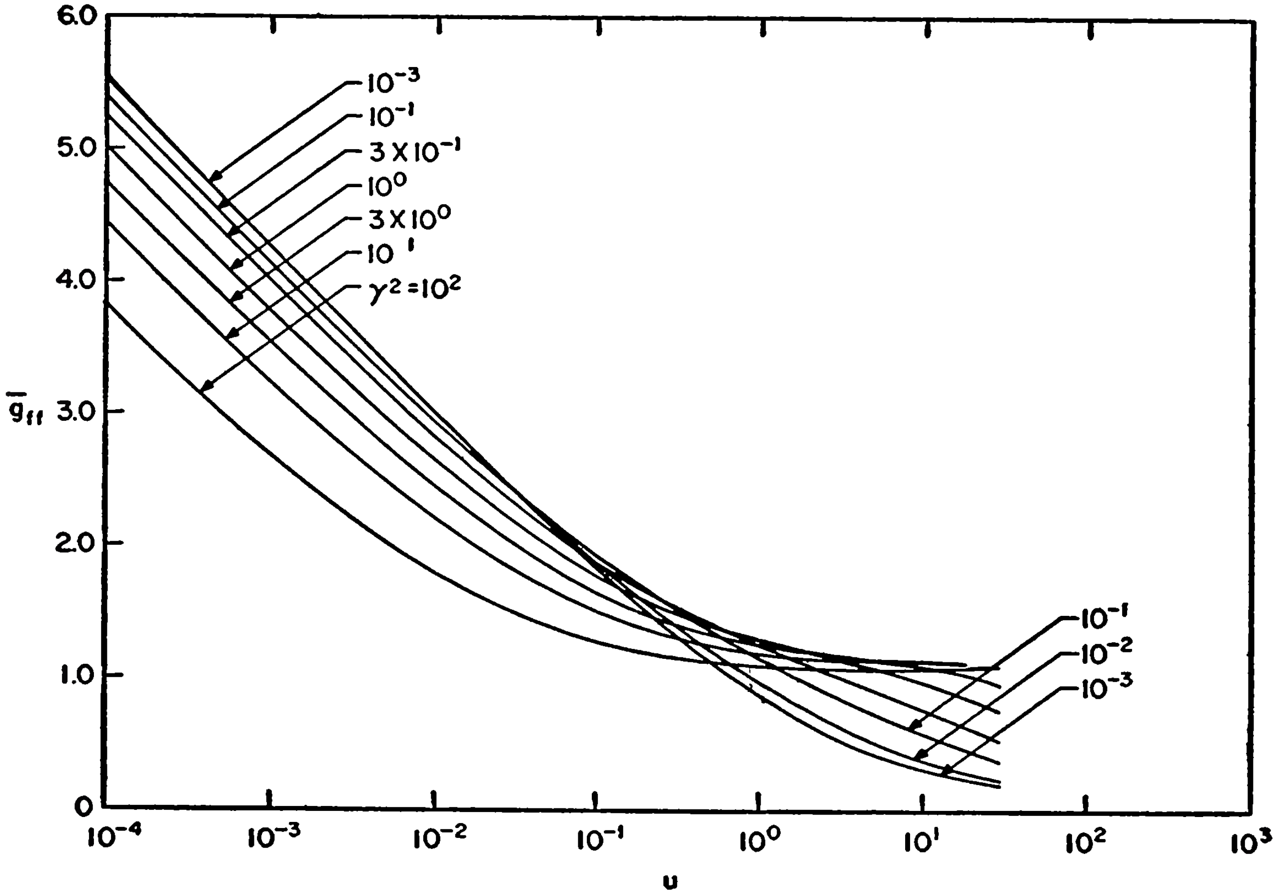
[Bradt, Astrophysics Processes]

- Gaunt Factor

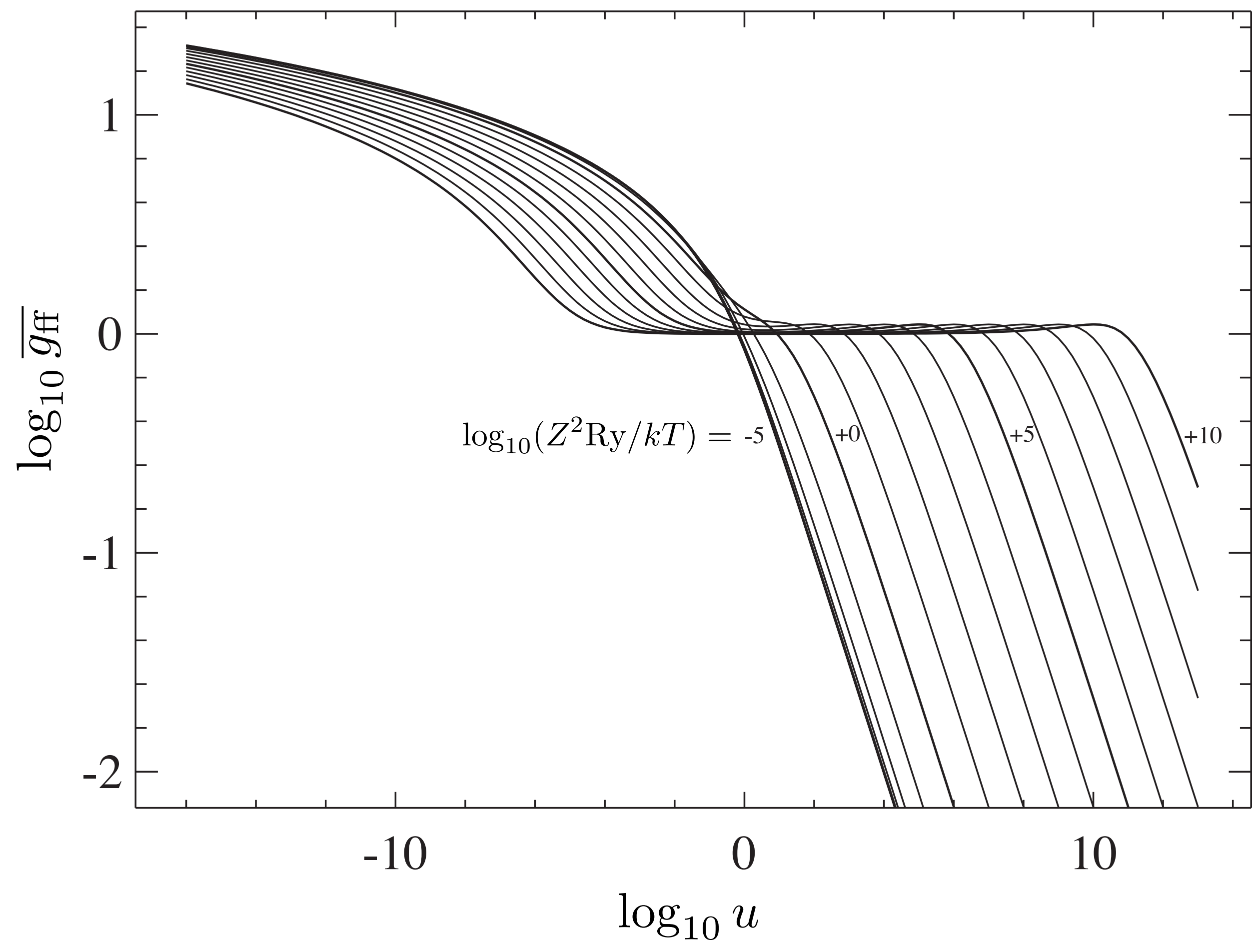
- Note that the values of Gaunt factor for $u = h\nu/kT \gg 1$ are not important, since the spectrum cuts off for these values.

$$\overline{g_{\rm ff}} \sim \begin{cases} 1 & \text{for } u \sim 1 \\ 1-5 & \text{for } 10^{-4} < u < 1 \end{cases}$$

Karzas & Latter (1961, ApJS, 6, 167)



van Hoof et al. (2014, MNRAS, 444, 420)



$$u = h\nu/kT = 4.8\times10^{11}\nu/T$$

$$\gamma^2 = Z^2{\rm Ry}/kT = 1.58\times10^5 Z^2/T$$

- Integrated Bremsstrahlung emission per unit volume:

$$\varepsilon^{\rm ff} \equiv \int \varepsilon^{\rm ff}(\nu) d\nu = \frac{2^5 \pi e^6}{3 m_e c^3} \left(\frac{2\pi}{3 k m_e}\right)^{1/2} T^{-1/2} n_i n_e Z^2 \int e^{-h\nu/kT} \overline{g_{\rm ff}} d\nu$$
$$= \frac{2^5 \pi e^6}{3 m_e c^3} \left(\frac{2\pi}{3 k m_e}\right)^{1/2} \left(\frac{k T^{1/2}}{h}\right) n_i n_e Z^2 \int_0^\infty e^{-u} \overline{g_{\rm ff}} du$$
$$= \left(\frac{2\pi k T}{3 m_e}\right)^{1/2} \frac{2^5 \pi e^6}{3 h m_e c^3} n_i n_e Z^2 \overline{g_{\rm B}}$$

where frequency average of the velocity averaged Gaunt factor:

$$\boxed{\varepsilon^{\rm ff} \left(\equiv \frac{dW}{dt dV}\right) = 1.42 \times 10^{-27} n_i n_e Z^2 T^{1/2} \overline{g_{\rm B}} \text{ erg cm}^{-3} \text{ s}^{-1} \longrightarrow \varepsilon_{\rm ff} \propto T^{1/2}}$$

$$\overline{g_{\rm B}} = \int_0^\infty e^{-u} \overline{g_{\rm ff}} du \quad (u = h\nu/kT)$$
$$= 1.3 \pm 0.2$$

$$\overline{g_{\rm B}} \approx 1 + \frac{0.44}{1 + 0.058 \left[\ln(T/10^{5.4} Z^2 K)\right]^2}$$

for $10^{4.2}$ K $\leq T/Z^2 \leq 10^{8.2}$ K, Draine (2011)

[Thermal Bremsstrahlung (free-free) Absorption]

- ***Free-free absorption is an inverse process of the free-free emission***, which we get by "running the film backward."
- Absorption of radiation by free electrons moving in the field of ions:

For thermal system, Kirchoff's law says:

$$\frac{1}{4\pi}\frac{dW}{dV\,dt\,d\nu} = j_\nu^{\rm ff} = \alpha_\nu^{\rm ff} B_\nu(T) \qquad B_\nu(T) = \left(2h\nu^3/c^2\right)\left[\exp(h\nu/kT) - 1\right]^{-1}$$

We have then

$$\boxed{\begin{aligned}\alpha_\nu^{\rm ff} &= \frac{4e^6}{3m_e hc}\left(\frac{2\pi}{3km_e}\right)^{1/2} n_i n_e Z^2 T^{-1/2}\nu^{-3}\left(1 - e^{-h\nu/kT}\right)\overline{g_{\rm ff}} \\ &= 3.7\times 10^8\ n_i n_e Z^2 T^{-1/2}\nu^{-3}\left(1 - e^{-h\nu/kT}\right)\overline{g_{\rm ff}}\quad ({\rm cm}^{-1})\end{aligned}}$$

For $h\nu \gg kT$, $\alpha_\nu^{\rm ff} = 3.7\times 10^8 n_i n_e Z^2 T^{-1/2}\nu^{-3}\overline{g_{\rm ff}}\quad ({\rm cm}^{-1})$ $\longrightarrow$ $\tau_\nu \propto \alpha_\nu^{\rm ff} \propto \nu^{-3}$ for $h\nu \gg kT$

For $h\nu \ll kT$, $\alpha_\nu^{\rm ff} = \dfrac{4e^6}{3m_e kc}\left(\dfrac{2\pi}{3km_e}\right)^{1/2} n_i n_e Z^2 T^{-3/2}\nu^{-2}\overline{g_{\rm ff}}$ $\longrightarrow$ $\tau_\nu \propto \alpha_\nu^{\rm ff} \propto \nu^{-2}$ for $h\nu \ll kT$

$$= 0.018 n_i n_e Z^2 T^{-3/2}\nu^{-2}\overline{g_{\rm ff}}$$

$1 - e^{-h\nu/kT} \approx 1 - (1 - h\nu/kT) = h\nu/kT$

Bremsstrahlung self-absorption: **The medium becomes always optically thick at sufficiently small frequency.** Therefore, the free-free emission is absorbed inside plasma at small frequencies.

[Overall Spectral Shape]

- An approximate formula for the free-free Gaunt factor is given by Draine (2011).

$$\overline{g_{\rm ff}} \approx 6.155(Z\nu_9)^{-0.118}T_4^{0.177} \quad (0.14 < Z\nu_9/T_4^{3/2} < 250) \quad \text{where } \nu_9 = \nu/10^9 \text{ Hz}, \quad T_4 = T/10^4 \text{ K}$$

- Emission and absorption coefficients:

$$\boxed{\begin{aligned} j_\nu &= \frac{1}{4\pi}\varepsilon_\nu \approx 3.35\times 10^{-40} n_i n_e Z^{1.882} T_4^{-0.323}\nu_9^{-0.118} \text{ erg cm}^{-3}\text{ s}^{-1}\text{ Hz}^{-1}\text{ sr}^{-1} \\ \alpha_\nu &= \frac{j_\nu}{B_\nu} \approx 3.37\times 10^{-7} n_i n_e Z^{1.882} T_4^{-1.323}\nu_9^{-2.118} \text{ pc}^{-1} \end{aligned}}$$

- Optical depth:

$$\tau_\nu = \int \alpha_\nu ds \approx 3.37\times 10^{-7} Z^{1.882} T_4^{-1.323}\nu_9^{-2.118}\left(\frac{n_i}{n_p}\right)\left[\frac{\rm EM}{\rm cm^{-6}pc}\right] \quad \text{where } \boxed{\begin{array}{c}\text{emission measure} \\ {\rm EM} \equiv \int n_e n_p ds\end{array}}$$

- SED (Spectral Energy Density) from a uniform sphere

$$\text{for } \tau_\nu \gg 1,\ h\nu \ll kT \longrightarrow I_\nu = S_\nu = B_\nu \qquad F_\nu = \pi B_\nu \left(\frac{R}{d}\right)^2 \propto \nu^2 \quad \text{(Rayleigh-Jeans Law)}$$

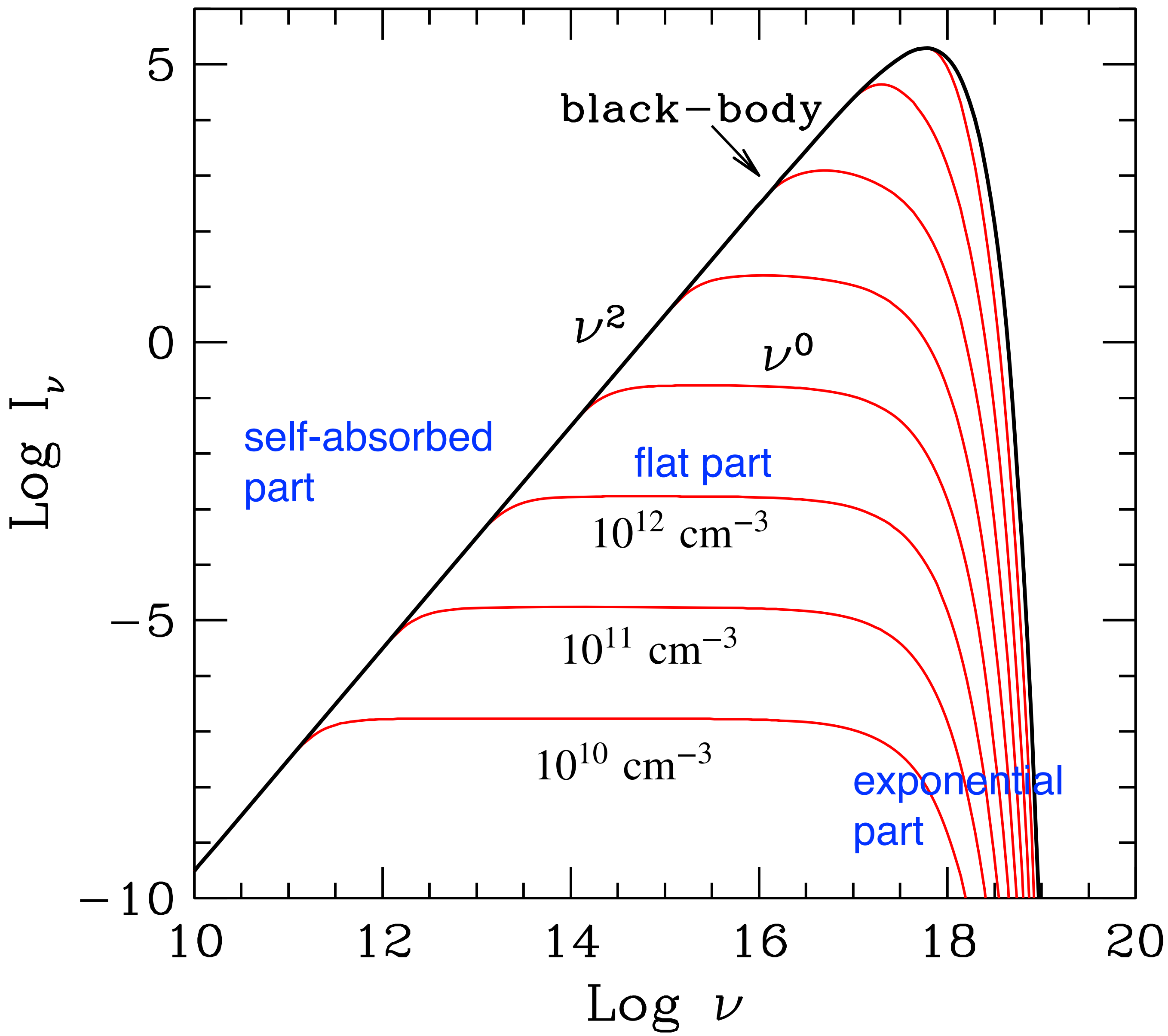
$$\text{for } \tau_\nu \ll 1,\ h\nu \gg kT \longrightarrow I_\nu = \int j_\nu ds \qquad F_\nu = 4\pi j_\nu \left(\frac{4\pi R^3}{3}\right)\frac{1}{4\pi d^2} \propto \nu^{-0.1}$$

- Spectral shape
  - At low frequencies (optically thick emission),

$$I_\nu = S_\nu = B_\nu \propto \nu^2$$

  - At high frequencies (optically thin emission),

$$I_\nu = \int j_\nu ds \propto e^{-h\nu/kT}$$
$$= \text{constant if } h\nu \ll kT$$

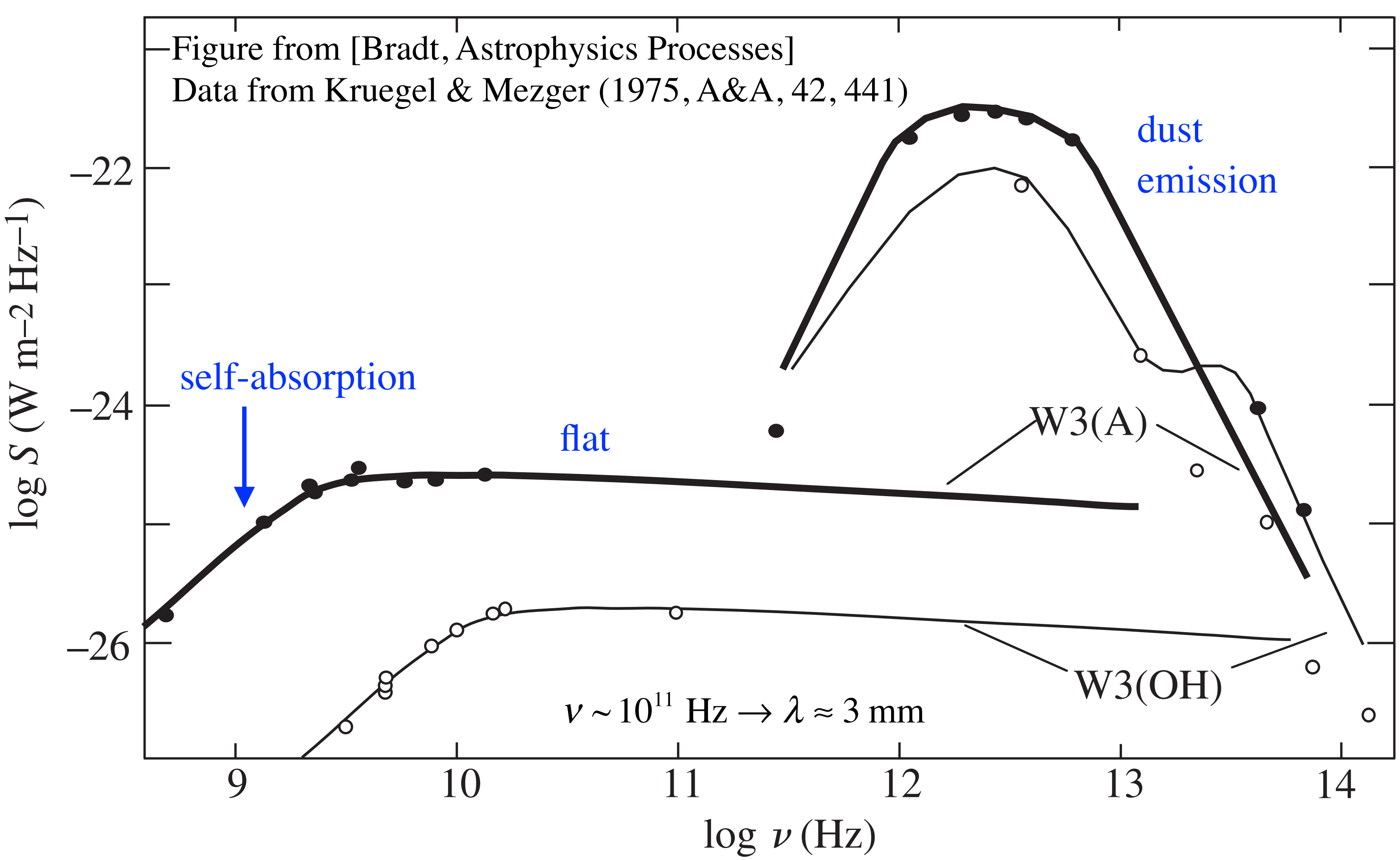


- This spectrum shows the bremsstrahlung intensity from a source of radius $R = 10^{15}$ cm and temperature $T = 10^7\ K$.
  - The Gaunt factor is set to unity for simplicity.
  - The density $n_e = n_p$ varies from 10^{10} cm^{-3} to 10^{18} cm^{-3} increasing by a factor 10 for each curve.
  - As the density increases, the optical depth also increases and the spectrum approaches the blackbody one.

Astronomical Examples - H II regions

- The radio spectra of H II regions clearly show the flat spectrum of an optically thin thermal source. The bright stars in the H II regions emit copiously in the UV and thus ionize the hydrogen gas.
- Continuum spectra of two H II regions, W3(A) and W3(OH):

  Note a flat thermal bremsstrahlung (radio), a low-frequency cutoff (radio, self absorption), and a large peak at high frequency (infrared, $10^{12} - 10^{13}$ Hz, 300-30 μm) due to heated, but still "cold" dust grains in the nebula.



The term H II is pronounced "H two" by astronomers. "H" indicates hydrogen, and "II" is the Roman numeral for 2.

Astronomers use "I" for neutral atoms, "II" for singly-ionized, "III" for doubly-ionized, etc.

An H II region in the Large Magellanic Cloud (observed with MUSE, VLT)

Dust Scattering: Iteration Method

$$\frac{dI(s,\Omega)}{ds} = -\alpha_{\rm ext} I(s,\Omega) + \alpha_{\rm scatt} \int \Phi(\Omega,\Omega') I(s,\Omega') d\Omega' + j_{\rm em}(s)$$

$$\frac{dI(\tau,\Omega)}{d\tau} = -I(\tau,\Omega) + a \int \Phi(\Omega,\Omega') I(\tau,\Omega') d\Omega' + \mathcal{S}_0(\tau) \qquad \left[d\tau \equiv \alpha_{\rm ext} ds, \ \mathcal{S}_0(\tau) \equiv \frac{j_{\rm em}(\tau)}{\alpha_{\rm ext}(\tau)} \right]$$

- Let I_0 be the intensity of photons that come directly from the source, I_1 the intensity of photons that have been scattered once by dust, and I_n the intensity after n scatterings. Then,

$$I(s,\Omega) = \sum_{n=0}^{\infty} I_n(s,\Omega)$$

$$\boxed{\begin{aligned} \mathcal{S}_0(\tau) &\equiv \frac{j_{\rm em}(\tau)}{\alpha_{\rm ext}(\tau)} \\ \mathcal{S}_n(\tau,\Omega) &\equiv a \int \Phi(\Omega,\Omega') I_{n-1}(\tau,\Omega') d\Omega' \end{aligned}}$$

- The intensities I_n satisfy the equations.

$$\frac{dI_0(\tau,\Omega)}{d\tau} = -I_0(\tau,\Omega) + \mathcal{S}_0(\tau)$$

$$\begin{aligned} \frac{dI_n(\tau,\Omega)}{d\tau} &= -I_n(\tau,\Omega) + a \int \Phi(\Omega,\Omega') I_{n-1}(\tau,\Omega') d\Omega' \\ &= -I_n(\tau,\Omega) + \mathcal{S}_n(\tau,\Omega) \end{aligned}$$

- Then, the formal solutions are:

$$\int_0^s e^{-[\tau(s)-\tau'(s')]} j_{\rm em}(s') ds'$$

$$\Rightarrow \quad \begin{aligned} I_0(\tau,\Omega) &= e^{-\tau} I_0(0,\Omega) + \int_0^{\tau} e^{-(\tau-\tau')} \mathcal{S}_0(\tau') d\tau' \\ I_n(\tau,\Omega) &= e^{-\tau} I_n(0,\Omega) + \int_0^{\tau} e^{-(\tau-\tau')} \mathcal{S}_n(\tau',\Omega) d\tau' \qquad I_n(0,\Omega) = 0 \ \text{ for } \ n \geq 1 \end{aligned}$$

Approximation: application to the edge-on galaxies

- The solution can be further simplified by assuming that

$$\frac{I_n}{I_{n-1}} \approx \frac{I_1}{I_0} \quad (n \geq 2)$$

- Then, the infinite series becomes

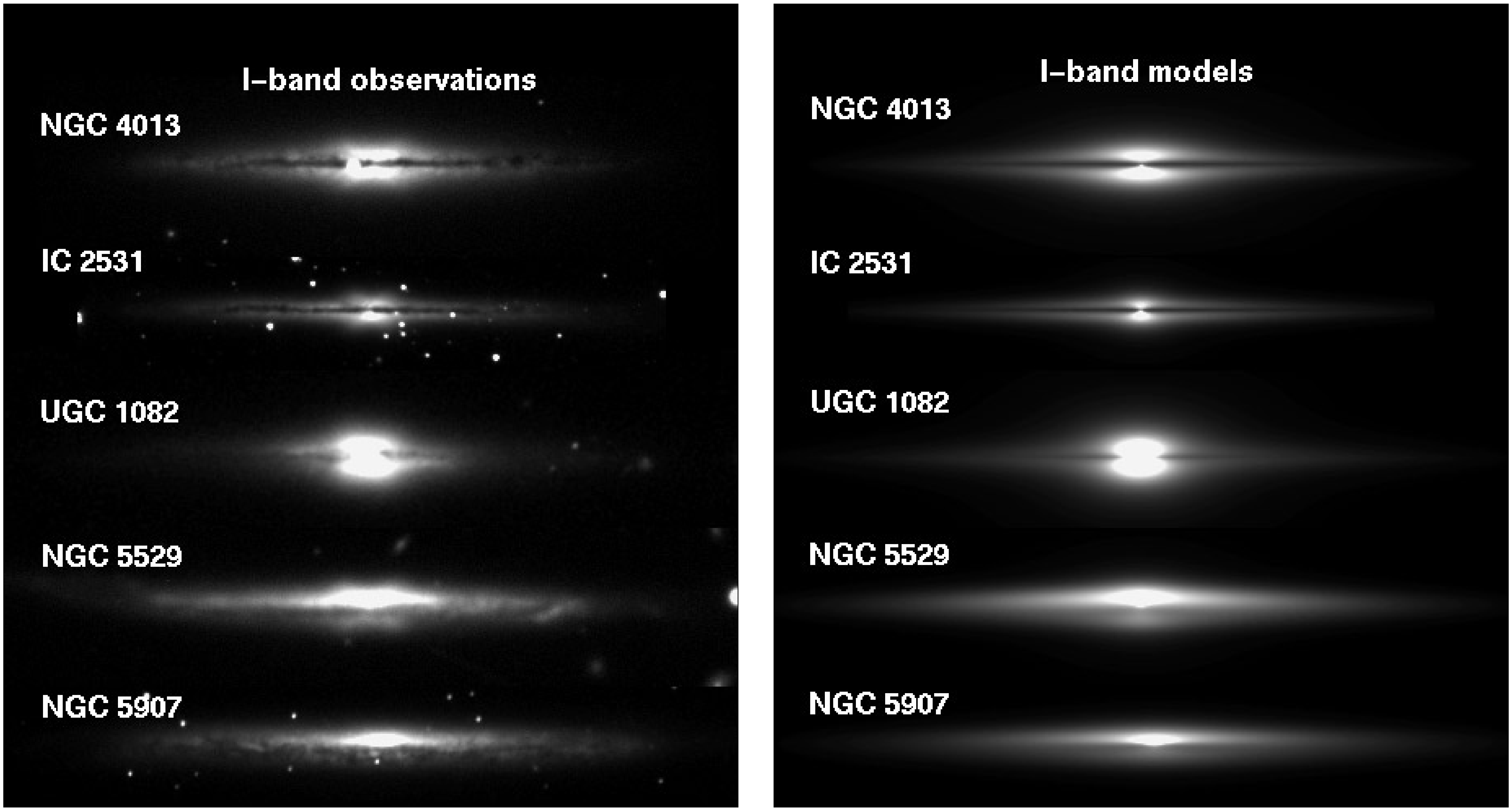
$$I = \sum_{n=0}^{\infty} I_n$$

$$I_n = \frac{I_n}{I_{n-1}} I_{n-1} \approx \left(\frac{I_1}{I_0}\right)^n$$

$$\boxed{I \approx I_0 \sum_{n=0}^{\infty} \left(\frac{I_1}{I_0}\right)^n = \frac{I_0}{1 - I_1/I_0}}$$

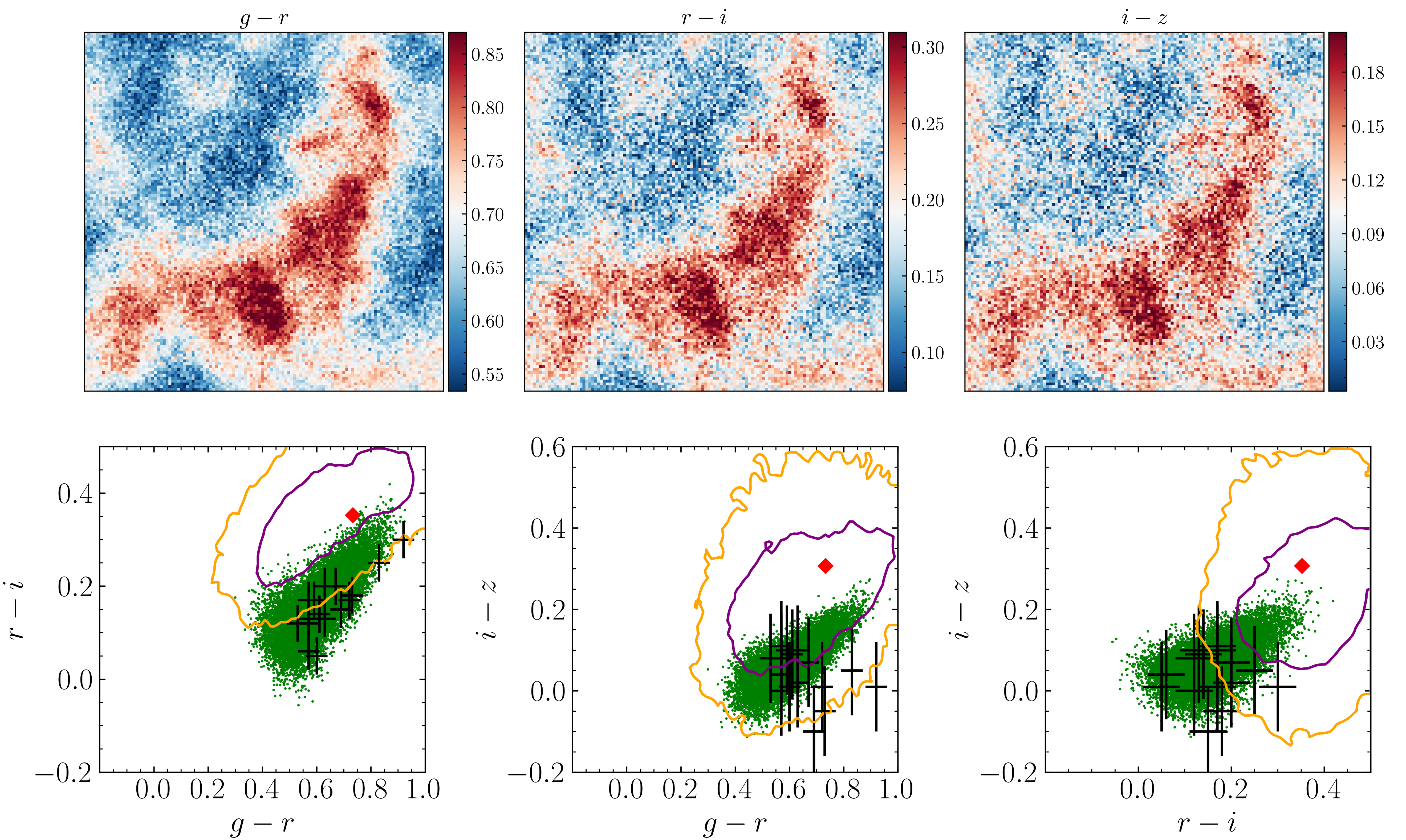
$$\sum_{n=0}^{\infty} \beta^n = \frac{1}{1-\beta} \quad \text{for} \quad \beta < 1$$

- Kylafis & Bahcall (1987) and Xilouris et al. (1997, 1998, 1999) applied this approximation to model the dust radiative transfer process in edge-on galaxies.



Xilouris et al. (1999)

Colors of a dust cloud at high Galactic latitudes



contours : external galaxies
red symbol: initial colors of the interstellar radiation field.
crosses: observational data
green dots: RT simulation

Atomic Processes

- **Excitation and de-excitation (Transition)**
  - Radiative excitation (photoexcitation; photoabsorption)
  - Radiative de-excitation (spontaneous emission and stimulated emission)
  - Collisional excitation
  - Collisional de-excitation

- **Emission Line**
  - Collisionally-excited emission lines
  - Recombination lines (recombination following photoionization or collisional ionization)

- **Ionization**
  - Photoionization and Auger-ionization
  - Collisional Ionization (Direct ionization and Excitation-autoionization)
- **Recombination**
  - Radiative recombination <=> Photoionization
  - Dielectronic Recombination (not dielectric!)
  - Three-body recombination <=> Direct collisional ionization
- **Charge exchange**

- Resonance Line

A spectral line caused by an electron jumping **between the ground state and the first energy level** in an atom or ion. It is the longest wavelength line produced by a jump to or from the ground state.

Because ***the majority of electrons are in the ground state in many astrophysical environments***, and because the energy required to reach the first level is the least needed for any transition, resonance lines are usually the strongest lines in the spectrum for any given atom or ion.

Draine, Physics of the interstellar and intergalactic medium

Table 9.4 Selected Resonance Lines[a] with $\lambda < 3000$ Å

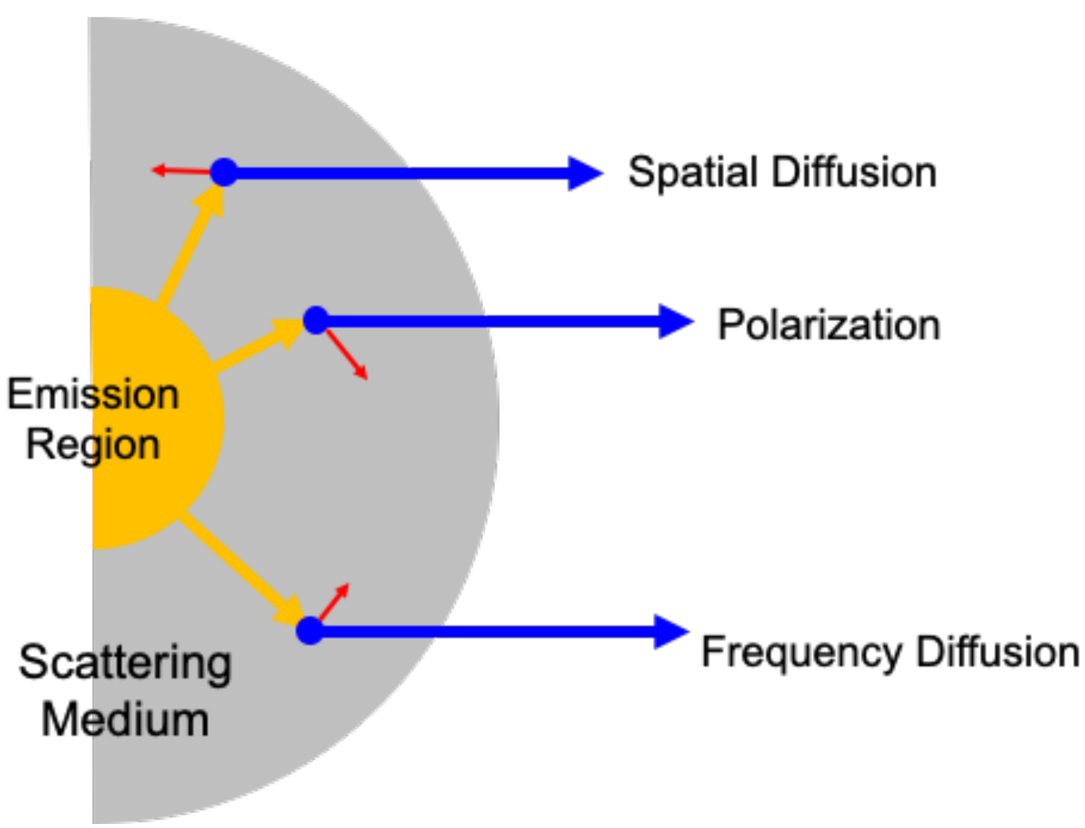
	Configurations	ℓ	u	$E_\ell/hc(\mathrm{cm}^{-1})$	λ_{vac}(Å)	$f_{\ell u}$
C IV	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	0	1550.772	0.0962
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	0	1548.202	0.190
N V	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	0	1242.804	0.0780
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	0	1242.821	0.156
O VI	$1s^22s - 1s^22p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	0	1037.613	0.066
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	0	1037.921	0.133
C III	$2s^2 - 2s2p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^{\mathrm{o}}_1$	0	977.02	0.7586
C II	$2s^22p - 2s2p^2$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{D}^{\mathrm{o}}_{3/2}$	0	1334.532	0.127
		$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{D}^{\mathrm{o}}_{5/2}$	63.42	1335.708	0.114
N III	$2s^22p - 2s2p^2$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{D}^{\mathrm{o}}_{3/2}$	0	989.790	0.123
		$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{D}^{\mathrm{o}}_{5/2}$	174.4	991.577	0.110
C I	$2s^22p^2 - 2s^22p3s$	$^3\mathrm{P}_0$	$^3\mathrm{P}^{\mathrm{o}}_1$	0	1656.928	0.140
		$^3\mathrm{P}_1$	$^3\mathrm{P}^{\mathrm{o}}_2$	16.40	1656.267	0.0588
		$^3\mathrm{P}_2$	$^3\mathrm{P}^{\mathrm{o}}_2$	43.40	1657.008	0.104
N II	$2s^22p^2 - 2s2p^3$	$^3\mathrm{P}_0$	$^3\mathrm{D}^{\mathrm{o}}_1$	0	1083.990	0.115
		$^3\mathrm{P}_1$	$^3\mathrm{D}^{\mathrm{o}}_2$	48.7	1084.580	0.0861
		$^3\mathrm{P}_2$	$^3\mathrm{D}^{\mathrm{o}}_3$	130.8	1085.701	0.0957
N I	$2s^22p^3 - 2s^22p^23s$	$^4\mathrm{S}^{\mathrm{o}}_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1199.550	0.130
		$^4\mathrm{S}^{\mathrm{o}}_{3/2}$	$^4\mathrm{P}_{3/2}$	0	1200.223	0.0862
O I	$2s^22p^4 - 2s^22p^33s$	$^3\mathrm{P}_2$	$^3\mathrm{S}^{\mathrm{o}}_1$	0	1302.168	0.0520
		$^3\mathrm{P}_1$	$^3\mathrm{S}^{\mathrm{o}}_1$	158.265	1304.858	0.0518
		$^3\mathrm{P}_0$	$^3\mathrm{S}^{\mathrm{o}}_1$	226.977	1306.029	0.0519
Mg II	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	0	2803.531	0.303
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	0	2796.352	0.608
Al III	$2p^63s - 2p^63p$	$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	0	1862.790	0.277
		$^2\mathrm{S}_{1/2}$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	0	1854.716	0.557

Table 9.4 contd.

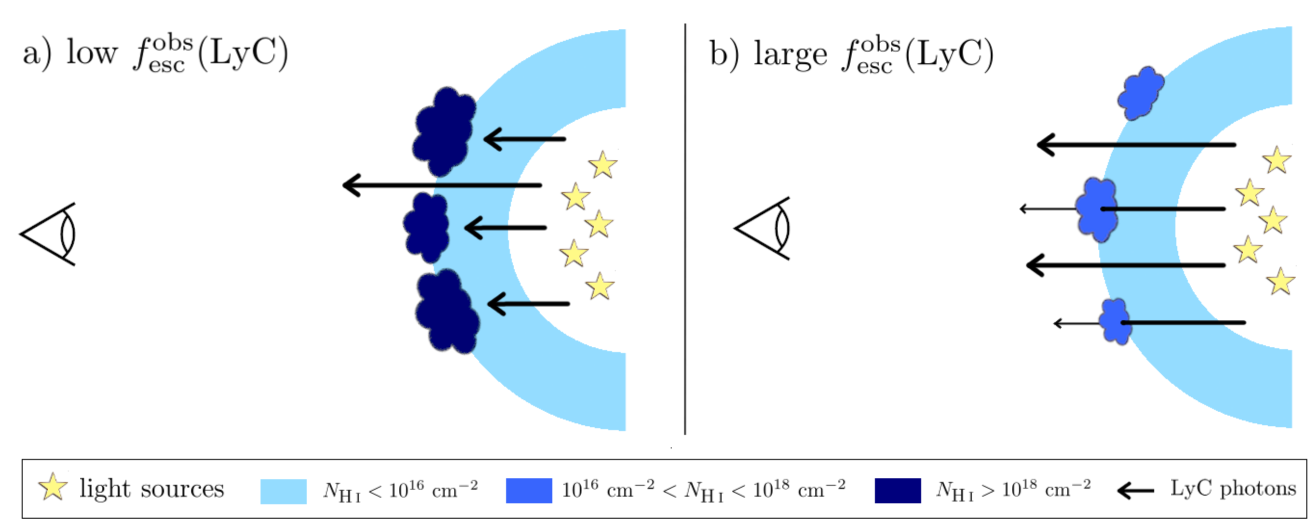
	Configurations	ℓ	u	$E_\ell/hc\,(\mathrm{cm}^{-1})$	λ_{vac}(Å)	$f_{\ell u}$
Mg I	$2p^63s^2 - 2p^63s3p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^{\mathrm{o}}_1$	0	2852.964	1.80
Al II	$2p^63s^2 - 2p^63s3p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^{\mathrm{o}}_1$	0	1670.787	1.83
Si III	$2p^63s^2 - 2p^63s3p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^{\mathrm{o}}_1$	0	1206.51	1.67
P IV	$2p^63s^2 - 2p^63s3p$	$^1\mathrm{S}_0$	$^1\mathrm{P}^{\mathrm{o}}_1$	0	950.655	1.60
Si II	$3s^23p - 3s^24s$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{S}_{1/2}$	0	1526.72	0.133
		$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{S}_{1/2}$	287.24	1533.45	0.133
P III	$3s^23p - 3s3p^2$	$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{D}_{3/2}$	0	1334.808	0.029
		$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{D}_{5/2}$	559.14	1344.327	0.026
Si I	$3s^23p^2 - 3s^23p4s$	$^3\mathrm{P}_0$	$^3\mathrm{P}^{\mathrm{o}}_1$	0	2515.08	0.17
		$^3\mathrm{P}_1$	$^3\mathrm{P}^{\mathrm{o}}_2$	77.115	2507.652	0.0732
		$^3\mathrm{P}_2$	$^3\mathrm{P}^{\mathrm{o}}_2$	223.157	2516.870	0.115
P II	$3s^23p^2 - 3s3p^3$	$^3\mathrm{P}_0$	$^3\mathrm{P}^{\mathrm{o}}_1$	0	1301.87	0.038
		$^3\mathrm{P}_1$	$^3\mathrm{P}^{\mathrm{o}}_2$	164.9	1305.48	0.016
		$^3\mathrm{P}_2$	$^3\mathrm{P}^{\mathrm{o}}_2$	469.12	1310.70	0.115
S III	$3s^23p^2 - 3s3p^3$	$^3\mathrm{P}_0$	$^3\mathrm{D}^{\mathrm{o}}_1$	0	1190.206	0.61
		$^3\mathrm{P}_1$	$^3\mathrm{D}^{\mathrm{o}}_2$	298.69	1194.061	0.46
		$^3\mathrm{P}_2$	$^3\mathrm{D}^{\mathrm{o}}_3$	833.08	1200.07	0.51
Cl IV	$3s^23p^2 - 3s3p^3$	$^3\mathrm{P}_0$	$^3\mathrm{D}^{\mathrm{o}}_1$	0	973.21	0.55
		$^3\mathrm{P}_1$	$^3\mathrm{D}^{\mathrm{o}}_2$	492.0	977.56	0.41
		$^3\mathrm{P}_2$	$^3\mathrm{D}^{\mathrm{o}}_3$	1341.9	984.95	0.47
P I	$3s^23p^3 - 3s^23p^24s$	$^4\mathrm{S}^{\mathrm{o}}_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1774.951	0.154
S II	$3s^23p^3 - 3s^23p^24s$	$^4\mathrm{S}^{\mathrm{o}}_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1259.518	0.12
Cl III	$3s^23p^3 - 3s^23p^24s$	$^4\mathrm{S}^{\mathrm{o}}_{3/2}$	$^4\mathrm{P}_{5/2}$	0	1015.019	0.58
S I	$3s^23p^4 - 3s^23p^34s$	$^3\mathrm{P}_2$	$^3\mathrm{S}^{\mathrm{o}}_1$	0	1807.311	0.11
		$^3\mathrm{P}_1$	$^3\mathrm{S}^{\mathrm{o}}_1$	396.055	1820.343	0.11
		$^3\mathrm{P}_0$	$^3\mathrm{S}^{\mathrm{o}}_1$	573.640	1826.245	0.11
Cl II	$3s^23p^4 - 3s3p^5$	$^3\mathrm{P}_2$	$^3\mathrm{P}^{\mathrm{o}}_2$	0	1071.036	0.014
		$^3\mathrm{P}_1$	$^3\mathrm{P}^{\mathrm{o}}_2$	696.00	1079.080	0.00793
		$^3\mathrm{P}_0$	$^3\mathrm{P}^{\mathrm{o}}_1$	996.47	1075.230	0.019
Cl I	$3s^23p^5 - 3s^23p^44s$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{P}_{3/2}$	0	1347.240	0.114
		$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{P}_{3/2}$	882.352	1351.657	0.0885
Ar II	$3s^23p^5 - 3s3p^6$	$^2\mathrm{P}^{\mathrm{o}}_{3/2}$	$^2\mathrm{S}_{1/2}$	0	919.781	0.0089
		$^2\mathrm{P}^{\mathrm{o}}_{1/2}$	$^2\mathrm{S}_{1/2}$	1431.583	932.054	0.0087
Ar I	$3p^6 - 3p^54s$	$^1\mathrm{S}_0$	$^2[1/2]^{\mathrm{o}}$	0	1048.220	0.25

[a] Transition data from NIST Atomic Spectra Database v4.0.0 (Ralchenko et al. 2010)

- A large number of ***resonance scatterings*** of resonance lines leads to
  - Diffusion in frequency and space
    - This property can be used to probe the properties (column density, porosity, clumpiness, and kinematics) of the surrounding/intervening media.

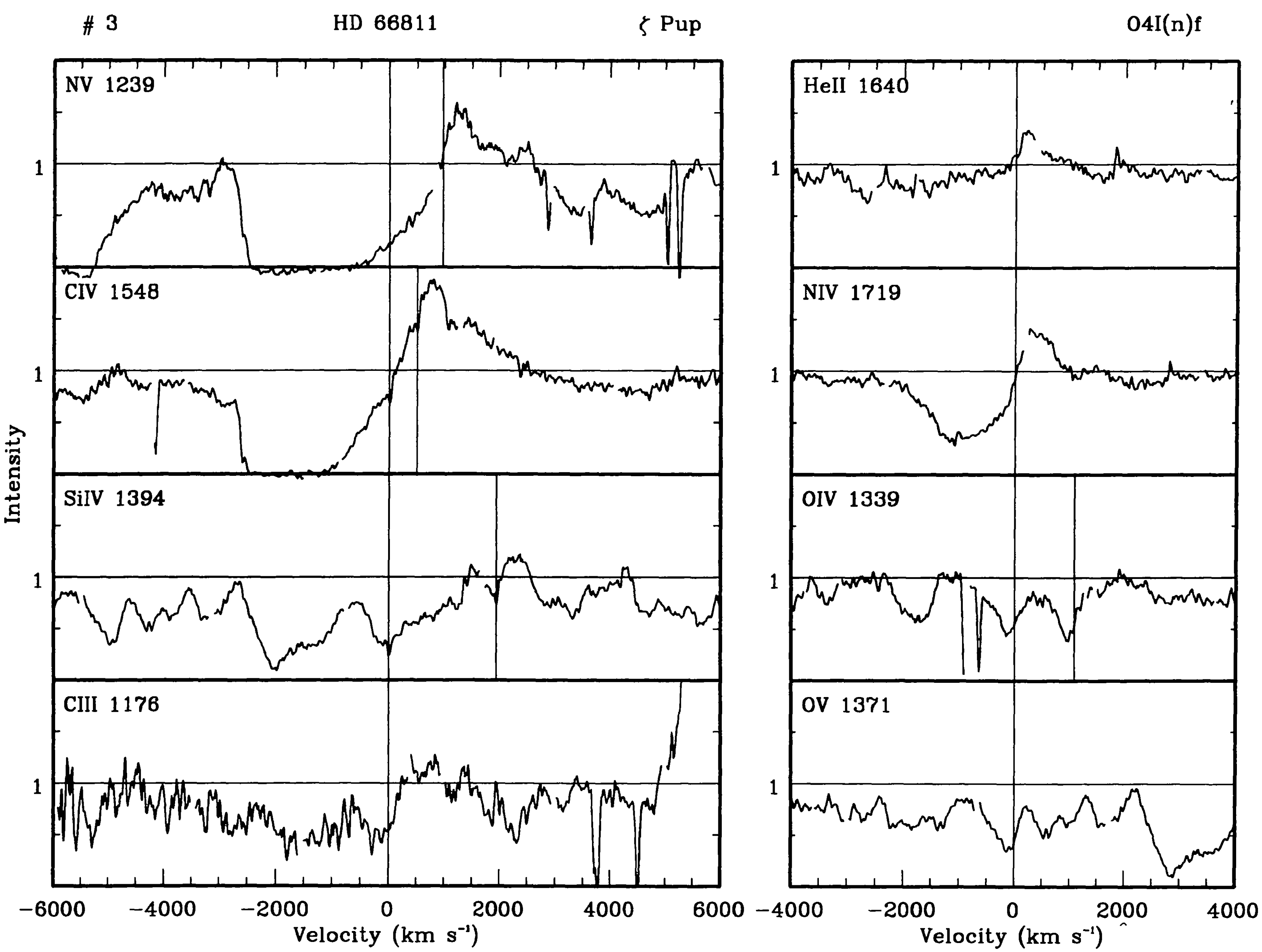


[Credit: Seok-Jun Chang]

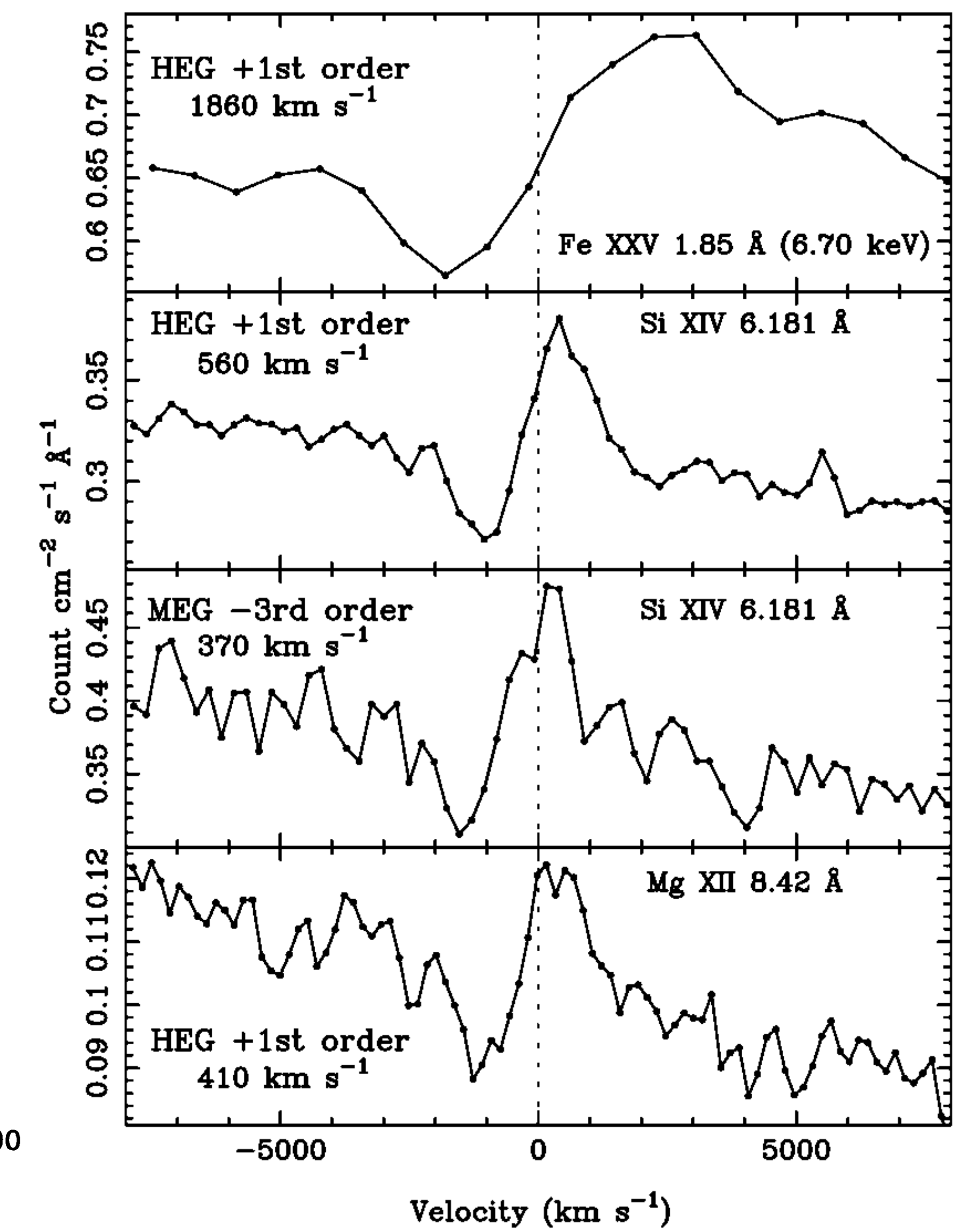


Gazagnes et al. (2020)

- The P Cygni profile is characterized by **strong (redshifted) emission lines** with corresponding **blueshifted absorption line**.

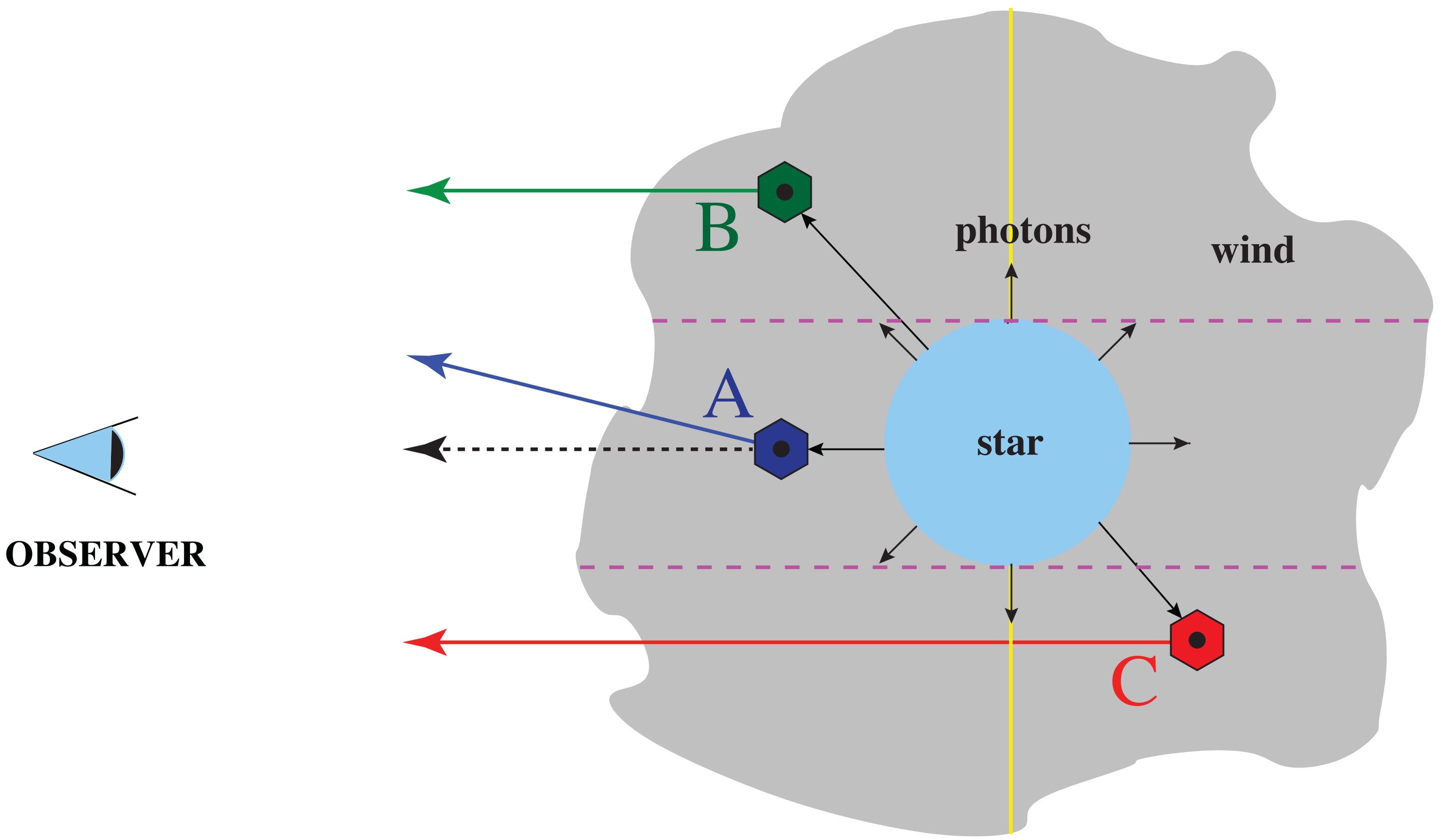


zeta Puppis (Snow et al., 1994, ApJS, 95, 163)

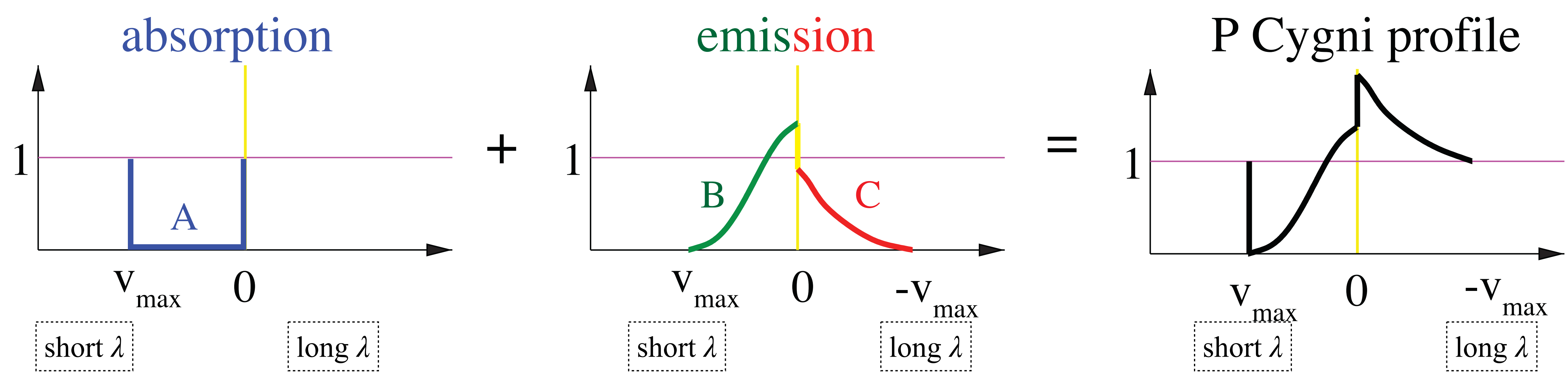


Circinus X-1
(Brandt & Schulz, 2000, ApJ, 544, L123)

- The blueshifted absorption line is produced by material moving away from the star and toward us, whereas the emission come from other parts of the expanding shell.



credit: Joachim Puls
slightly modified



- Spectral series of the H atom

The spectrum of H is divided into a number of series linking different upper levels n_2 with a single lower level n_1 value. ***Each series is denoted according to its n_1 value and is named after its discoverer.***

Within a given series, ***individual transitions are labelled by Greek letters.***

$$n_2 \longleftrightarrow n_1$$

n_1	Name	Symbol	Spectral region
1	Lyman	Ly	ultraviolet
2	Balmer	H	visible
3	Paschen	P	infrared
4	Brackett	Br	infrared
5	Pfund	Pf	infrared
6	Humphreys	Hu	infrared

$\Delta n \equiv n_2 - n_1$

$\Delta n = 1$ is α,

$\Delta n = 2$ is β,

$\Delta n = 3$ is γ,

$\Delta n = 4$ is δ,

$\Delta n = 5$ is ϵ.

Lyman series : $\mathrm{Ly}\alpha, \mathrm{Ly}\beta, \mathrm{Ly}\gamma, \cdots$
Balmer series : $\mathrm{H}\alpha, \mathrm{H}\beta, \mathrm{H}\gamma, \cdots$
Paschen series: $\mathrm{P}\alpha, \mathrm{P}\beta, \mathrm{P}\gamma, \cdots$
Brackett series : $\mathrm{Br}\alpha, \mathrm{Br}\beta, \mathrm{Br}\gamma, \cdots$

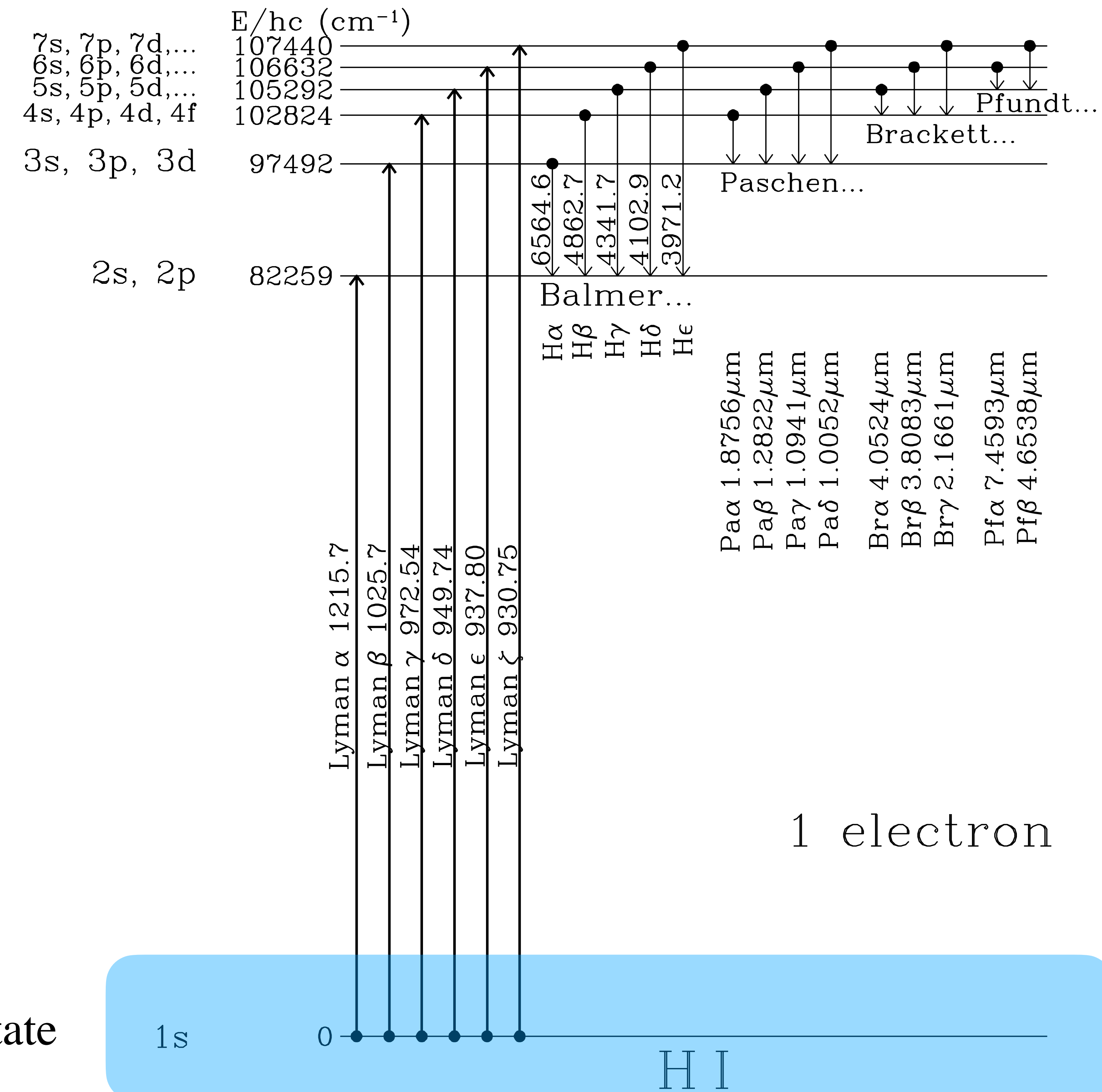
Transitions with high Δn are labelled by the n_2. Thus, $H15$ is the Balmer series transition between $n_1 = 2$ and $n_2 = 15$.

Paschen series
$Pa\alpha, Pa\beta, \ldots$

Balmer series
$H\alpha, H\beta, \ldots$

Lyman series
$Ly\alpha, Ly\beta, \ldots$

ground state



$n = \infty$
(free/unbound state)

Lyman continuum
LyC

Ionization Energy
of Hydrogen

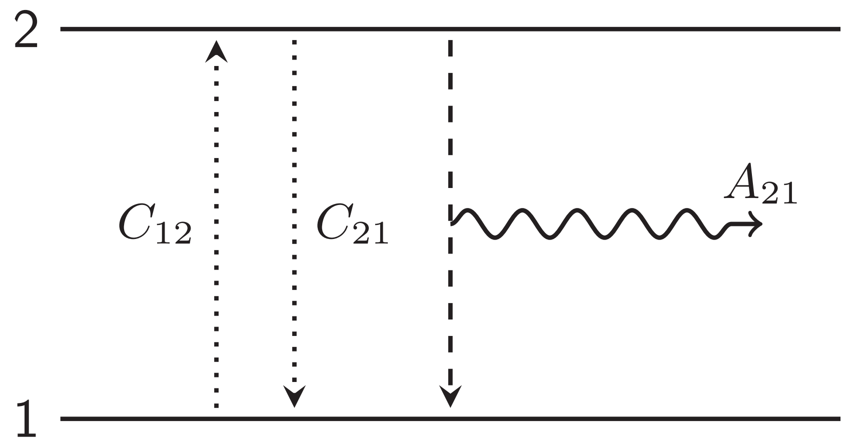
$\lambda = 912\text{Å}$
$E = 13.5\,\text{eV}$

$n = 1$
(ground state)

[Draine] Physics of the Interstellar and Intergalactic Medium

- **Collisional line**

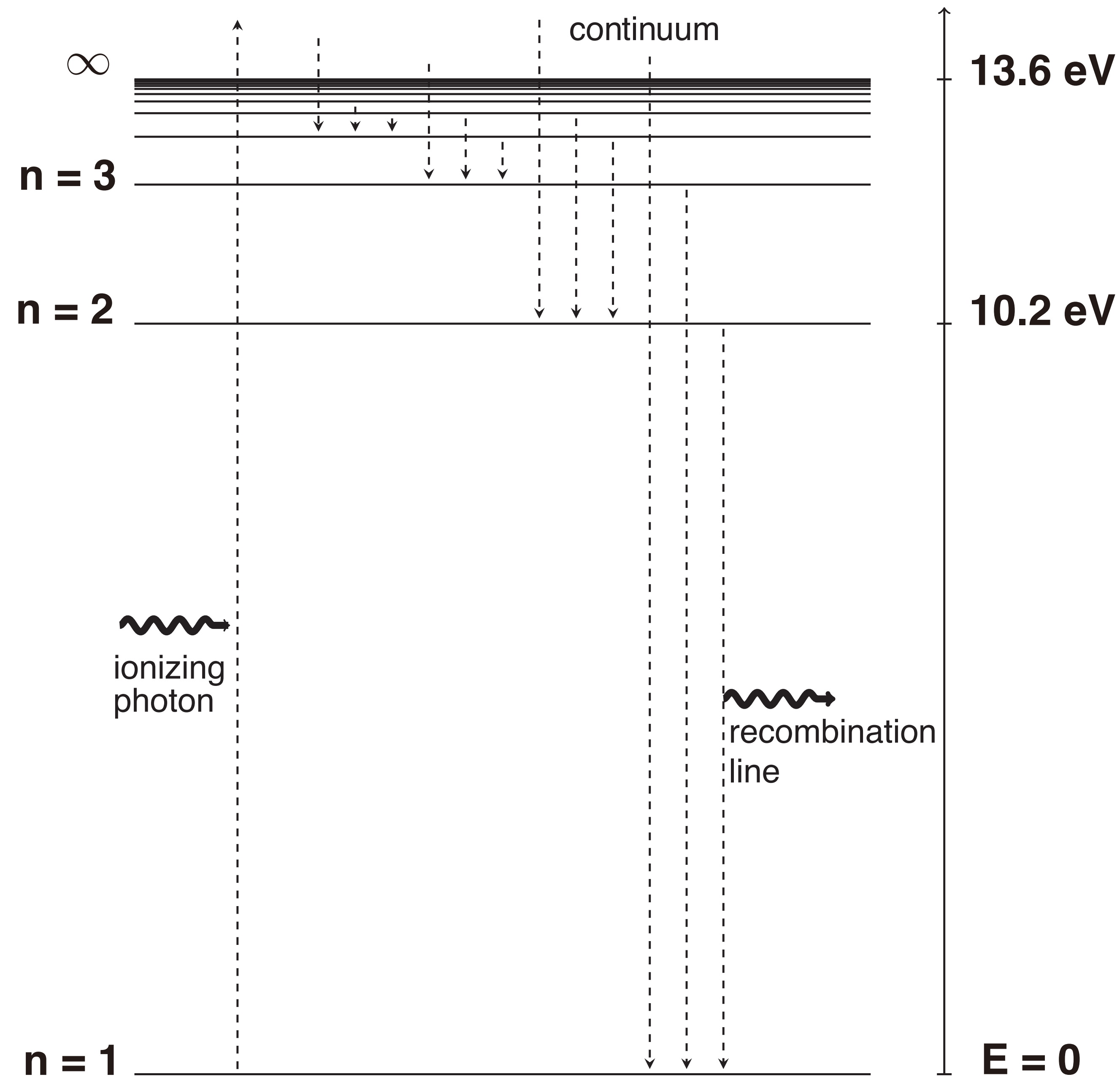
  - Hydrogen atoms are excited into the upper state through collisions with electrons. Then, the atoms decay spontaneously (radiatively) before collisional de-excitation occurs.



- **Recombination line**

  - The recombination of a free electron with a proton can occur to any of the energy levels.

  - Consider an electron recombining into a state n > 1. This produces a continuum photon and an excited hydrogen atom that will decay through either a Lyman α, β, γ, etc. line or a higher level series.



- ***In the interstellar / intergalactic medium, excited atoms are immediately cascade down to the ground level before they collide with other atoms/ electrons because the particle density is extremely low.***

  Electron-Hydrogen collision time scale

$$t_{\rm coll}(eH) \sim 4 \times 10^{8}\ {\rm sec} \left(\frac{n_e}{0.04\,{\rm cm}^{-3}}\right)^{-1} \left(\frac{\langle E \rangle}{1\,{\rm eV}}\right)$$

  Radiative Transition time scale = 1/Einstein A-coefficient ~ $2 \times 10^{-9}{\rm sec}$ ***for Ly***α

  ◉ ***Therefore, "almost" all atoms (including hydrogen) are in the ground state.***

- ***Resonance line*** is the allowed transition with the lowest energy, arising from the ground state of a particular atom.

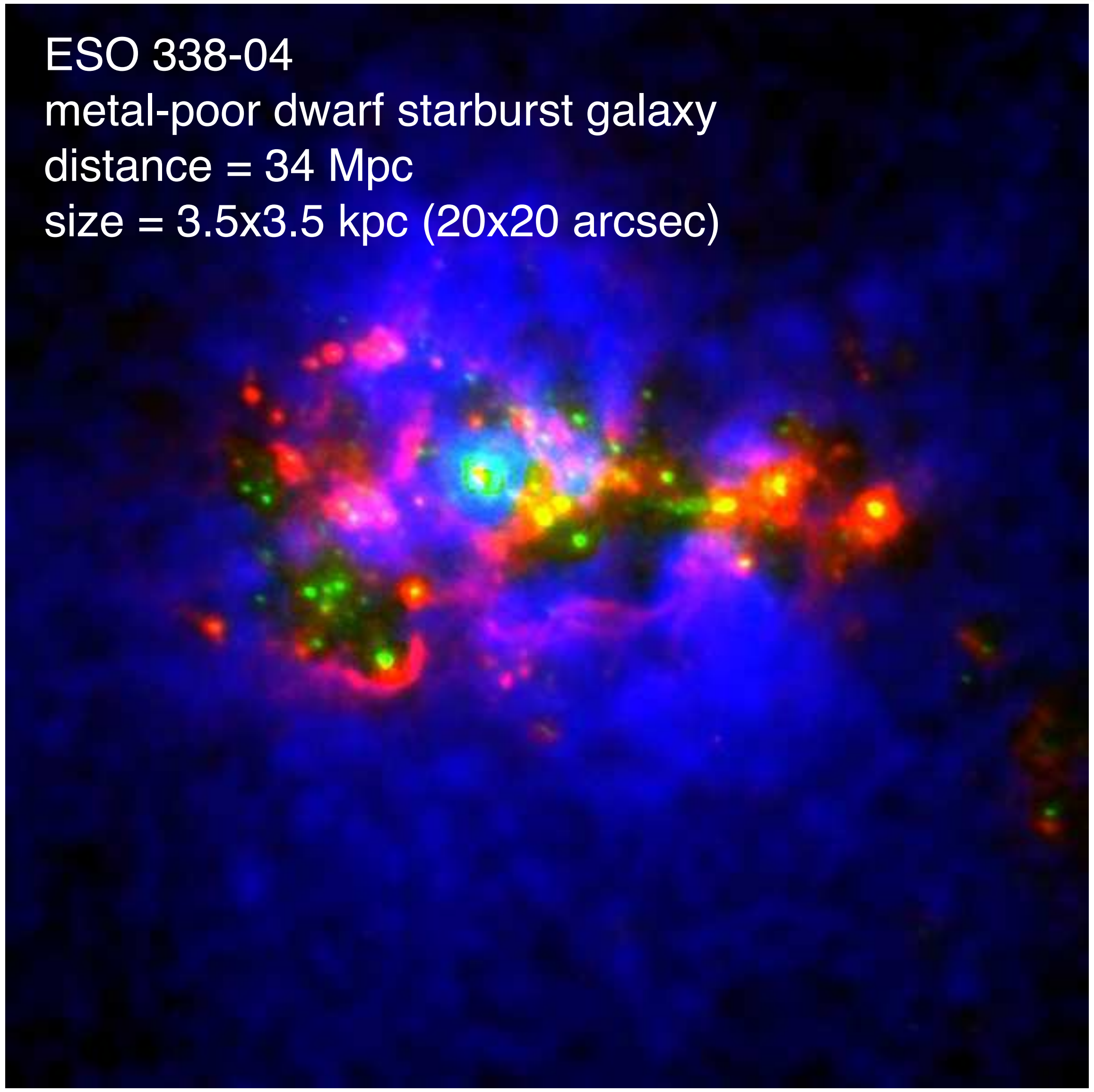
- ***Ly***α ***is the strongest line in the Universe*** because hydrogen is most abundant and Lyα is a resonance line.

- A LAE is a type of actively star-forming galaxy and often found to have a relatively low mass.

- Lyman Alpha Reference Sample (LARS): a project in which 14 nearby galaxies ($0.028 < z < 0.18$) and their Lyα emission are studied in detail using the HST.

- ***Lyα emission is not dominated by the bright super star clusters (traced by Hα) that dominate the production of ionizing photons.*** Towards most of them, Lyα is rather seen in absorption.

- ***Most of the escaping Lyα emission comes from a diffuse extended component*** where Lyα/Hα >> 10, that can only be produced by resonant scattering (Ostlin et al. 2009)



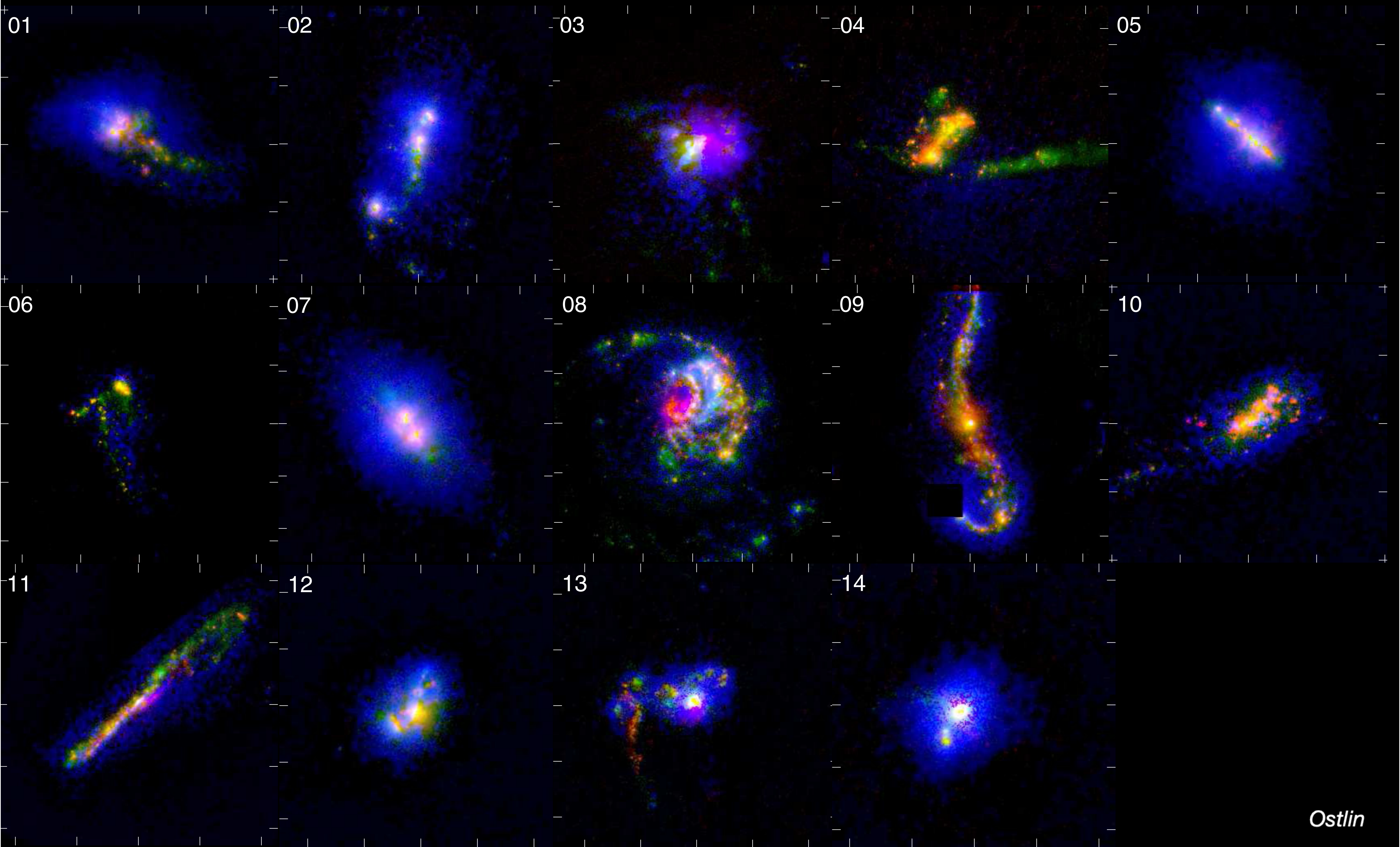
RGB composite
Hα (red), UV-continuum (green) and Lyα (blue)

Hα (red) is the most efficient tracer of bright stars.
Notice no clear correlation between Hα and Lyα.

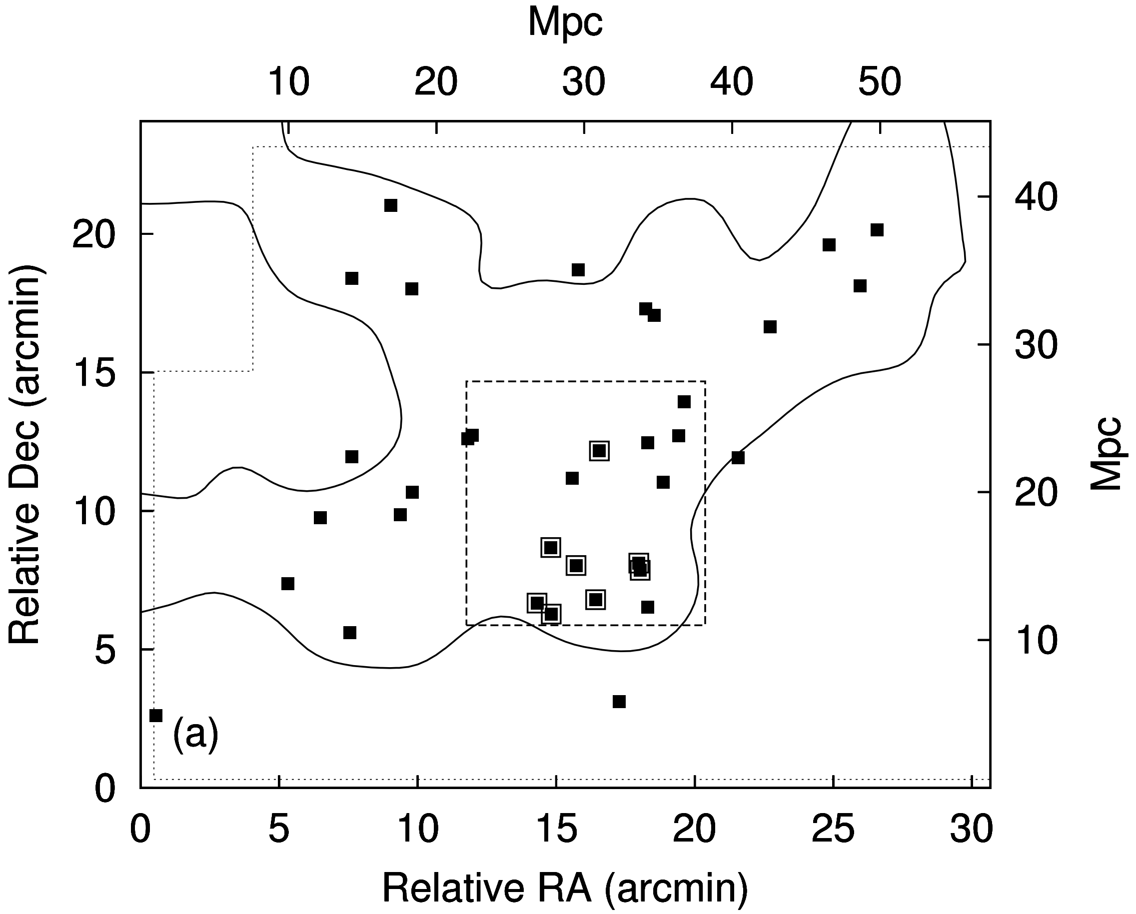
Lyman Alpha Reference Samples (0.028 < z < 0.18)

LARS sample as seen by HST. Blue = Lyα, Green = UV continuum, Red = Hα

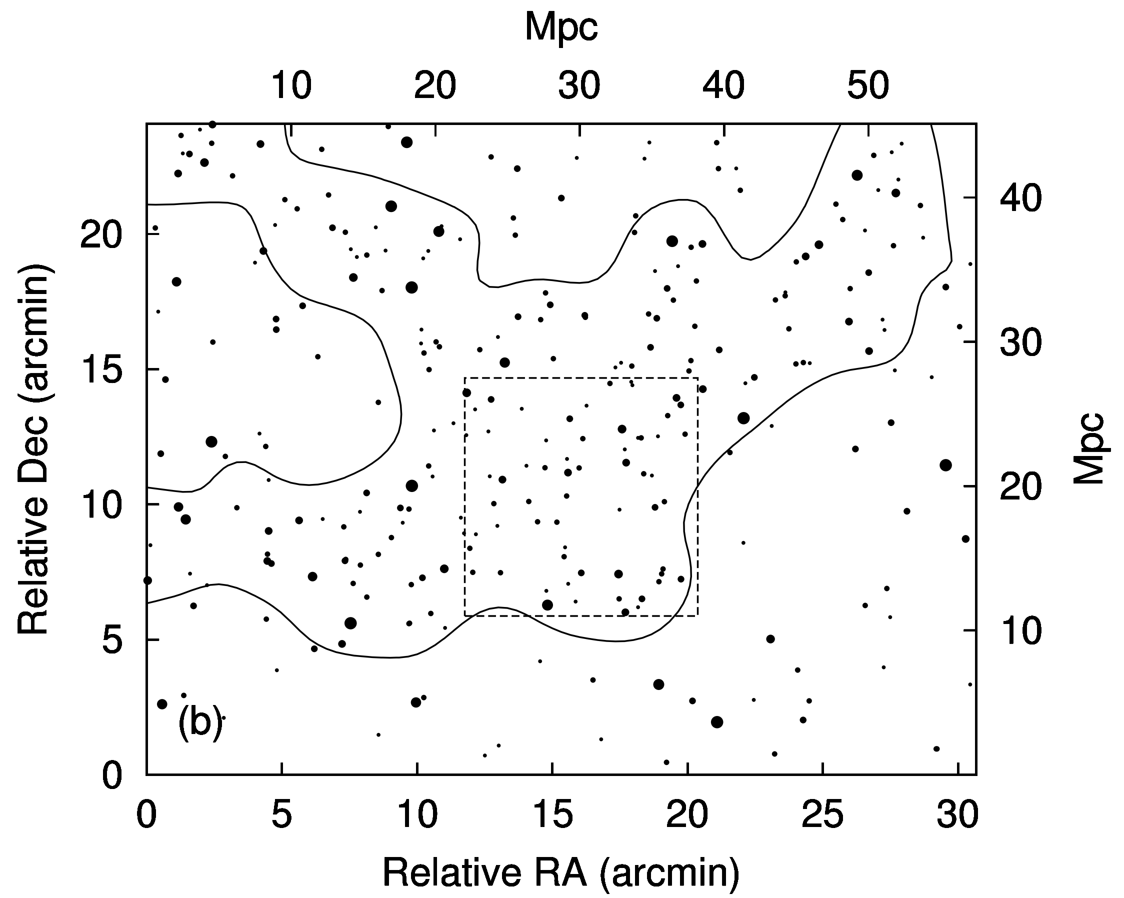
Due to scattering, Lyα is more extended and emerges in form of diffuse halo



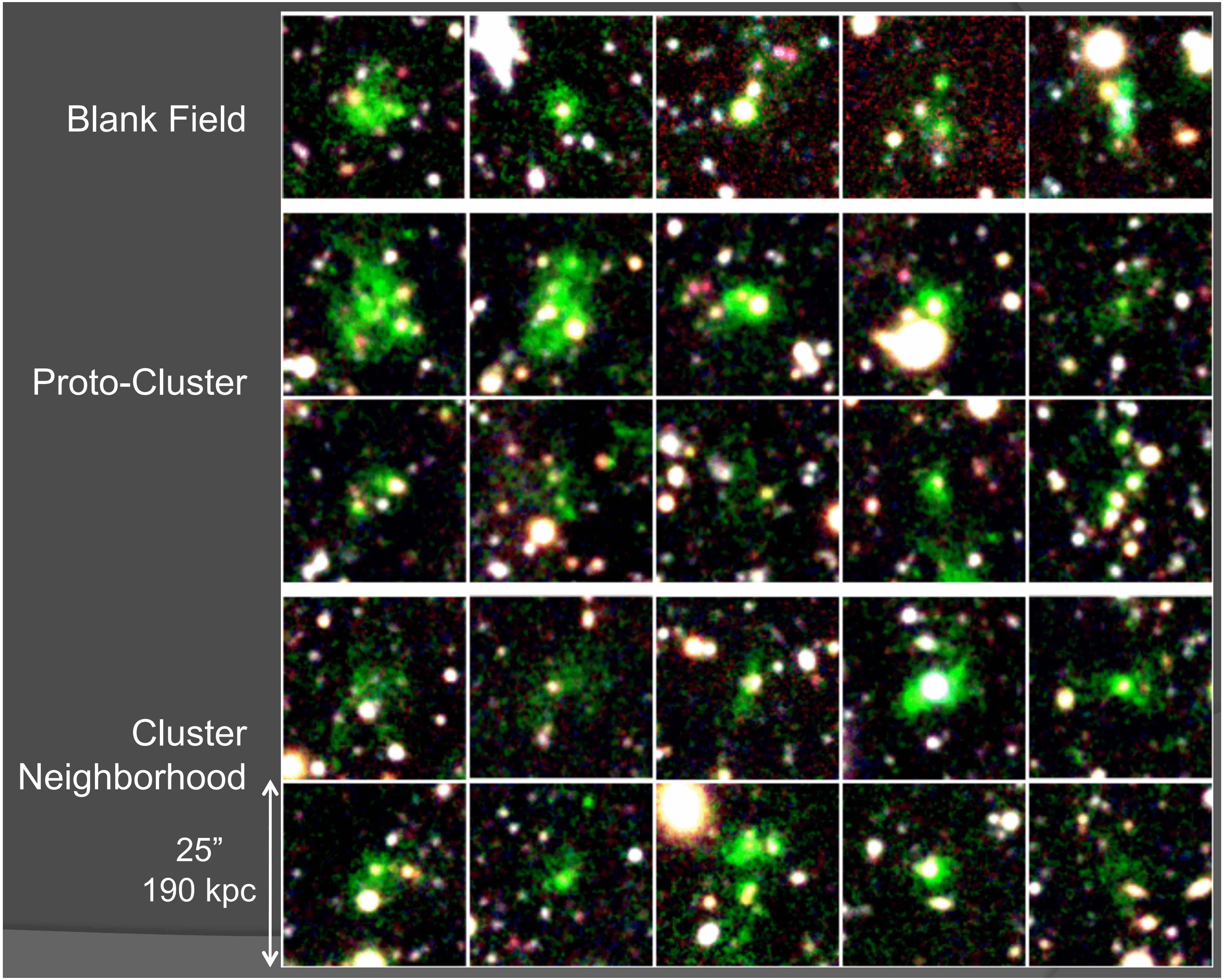
- LABs are one of ***the biggest objects in the Universe***: gigantic clouds of hydrogen gas.
- LABs preferentially observed in ***the high-density regions***.
- They are associated with matter density peaks in the universe and thus likely to ***evolve into the present-day groups and clusters of galaxies.***



35 LABs



283 LAEs (Lyman Alpha Emitters)



from Matsuda’s slide

- ***Lyα is a resonant line***.

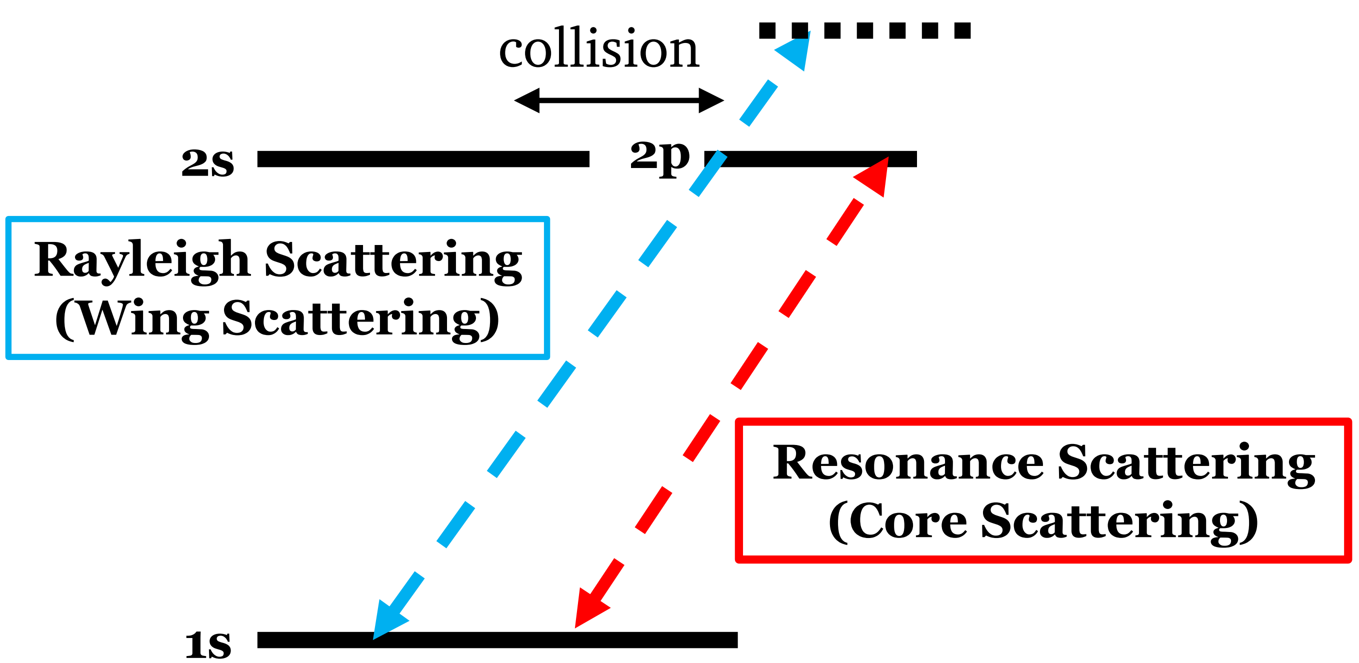
$$\tau \sim 1 \text{ at line center for } N_{\rm H} = 3 \times 10^{13} \text{ cm}^{-2}$$

Typical column densities are $N_{\rm H} \approx 10^{18} - 10^{22}\,\mathrm{cm}^{-2}$

Neutral hydrogen atom is in the ground state in the Universe. Lyα becomes optically thick even at modest column densities.

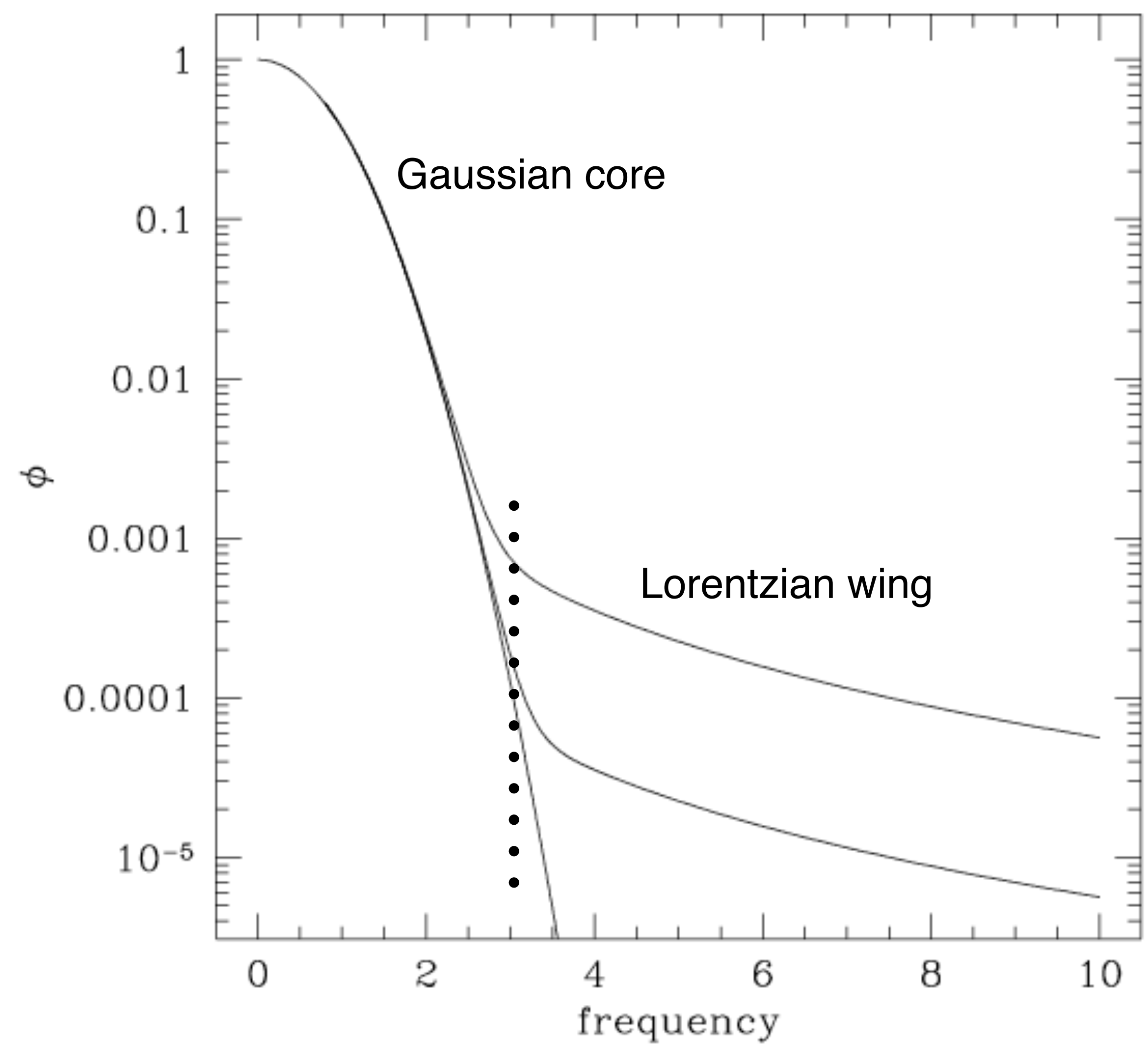
Lyα photons is absorbed immediately by another hydrogen atom in the ground state, emitted, absorbed, emitted, absorbed, emitted, absorbed,…

- ***Resonance trapping*** of Lyα photons increases the probability of being
  - absorbed by dust
  - shifted in frequency
  - converted to two-photon emission (in high density media)



Energy Level of Atomic Hydrogen

$$H(a,u) \equiv \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}\,dy}{(u-y)^2+a^2}$$



The Lyα frequency changes continuously due to the thermal motion of protons while the photon is scattered repeatedly.

When the photon frequency shifts significantly to the wing part, the cross-section becomes very low, and the photon can escape without further scattering.

(1) Initialize the photon frequency x and the propagation direction.

(2) Select an optical depth τ which the photon is allowed to travel.

(3) Select an interacting particle (dust or hydrogen).

(4) Select H or K.

(5) Select a velocity component of an atom parallel to the photon propagation direction, according to the distribution function.

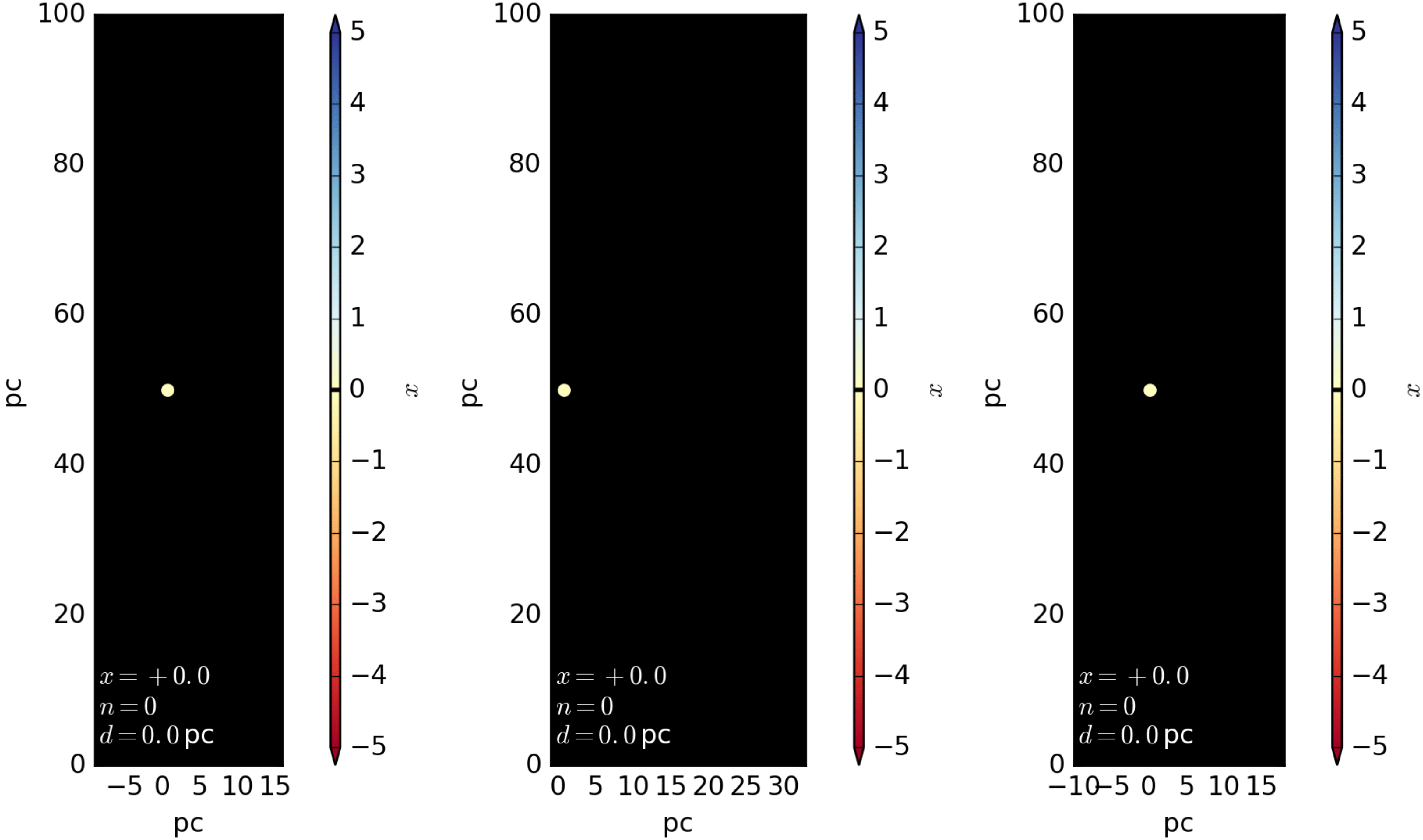
$$P(u|x,a) = \frac{1}{H(x,a)} \frac{a}{\pi} \frac{e^{-u^2}}{(x-u)^2 + a^2}$$

(5) Select two velocity components perpendicular to the photon direction, according to Maxwellian (Gaussian) distribution.

(6) Lorentz-transform the photon frequency into the atom's rest frame.

(7) Select scattering angle (θ, ϕ) according to the scattering phase matrix.

(8) Lorentz-transform the frequency into the lab frame. Go to (2) until the photon escapes the system.

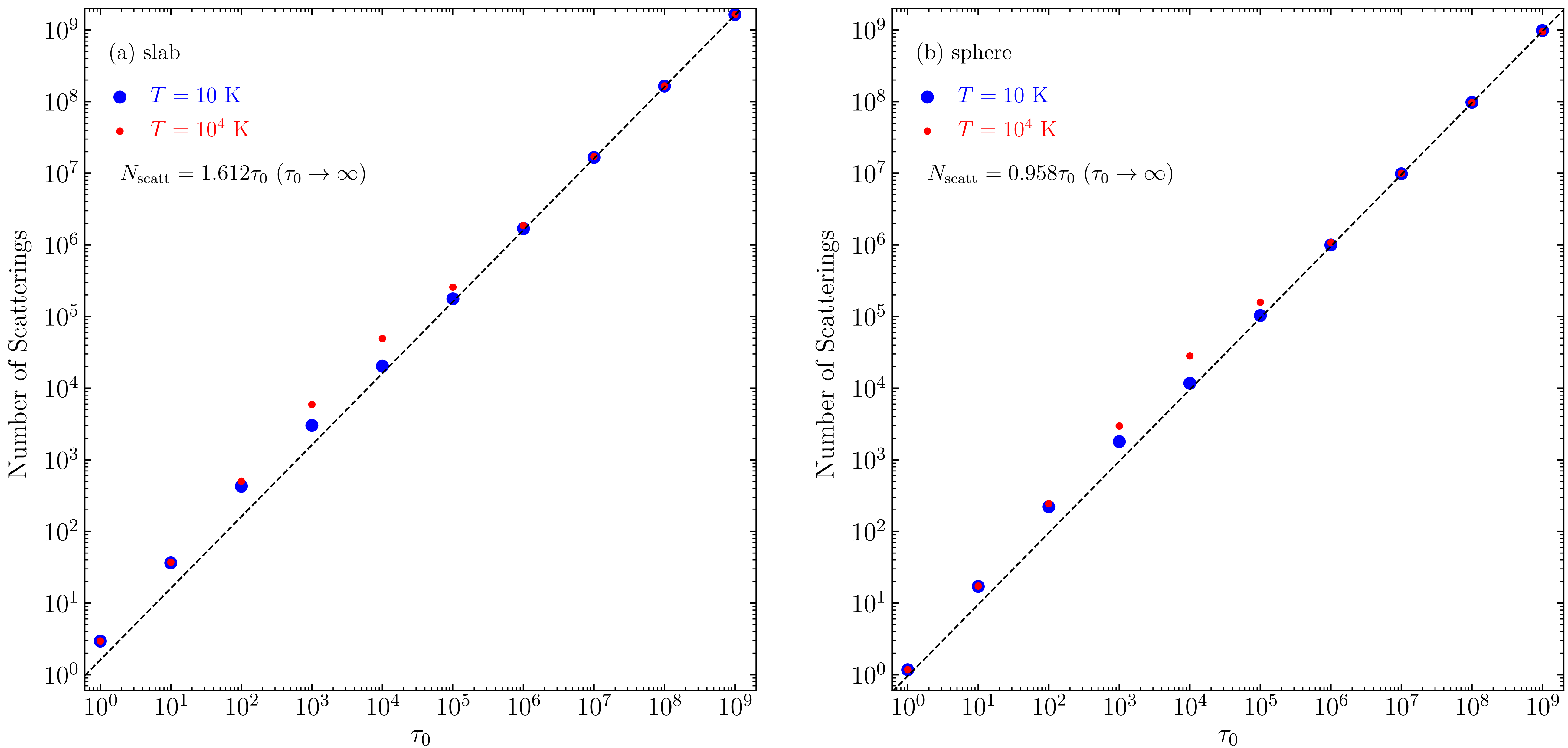


Gronke et al. (2017)

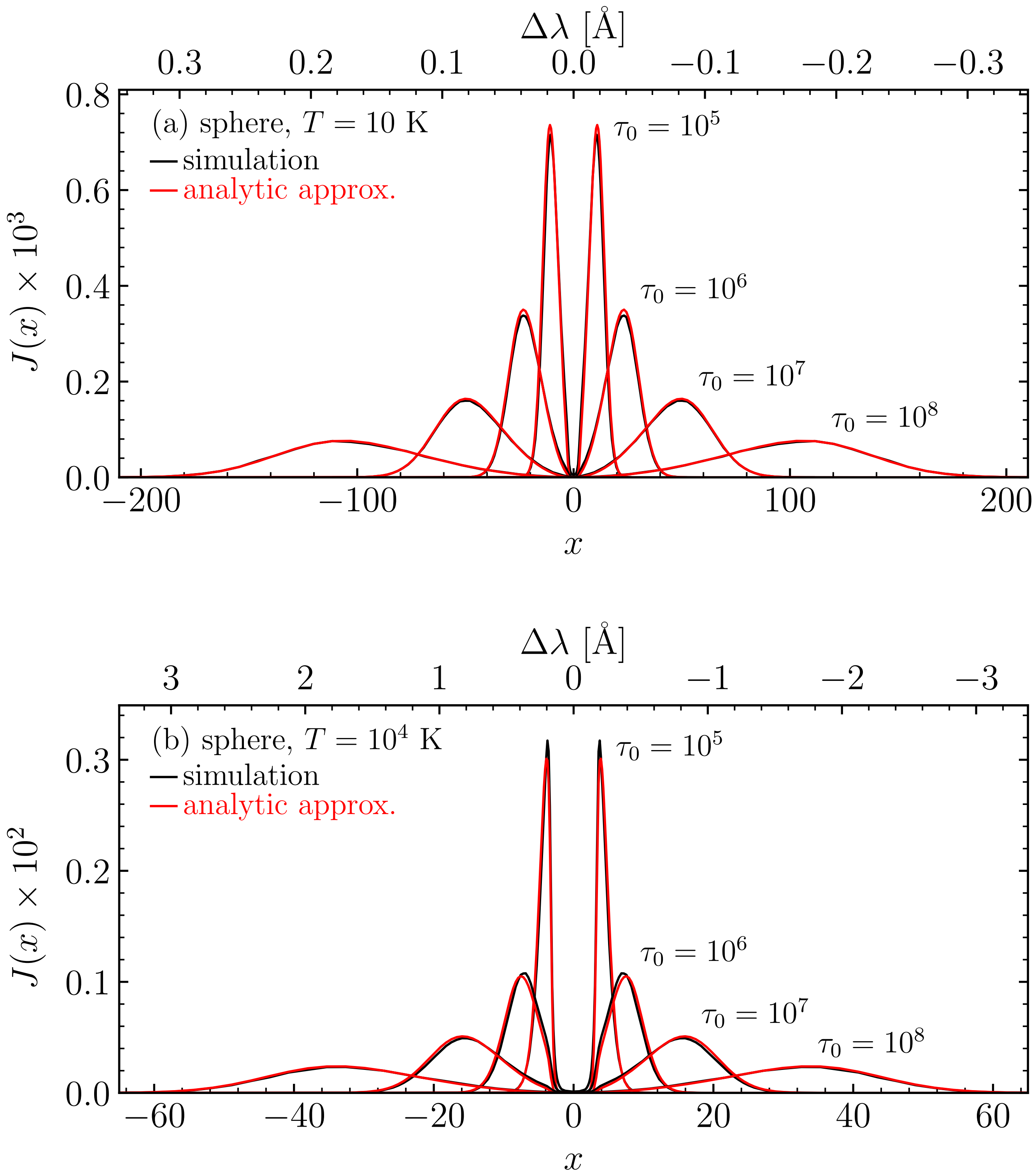
[Left] A photon escapes through the random walk from a static medium with $(N_{\mathrm{HI,cl}}, f_c) = (10^{20}\mathrm{cm}^{-2}, 100)$

[Center] A photon escapes in an excursion after a random walk $(N_{\mathrm{HI,cl}}, f_c) = (10^{17}\mathrm{cm}^{-2}, 100)$.

[Right] A photon escapes nearly directly through excursion/single flight due to movement of the clumps $(N_{\mathrm{HI,cl}}, f_c, \sigma_{\mathrm{cl}}) = (10^{17}\mathrm{cm}^{-2}, 100, 100\mathrm{kms}^{-1})$.



Seon & Kim (2020)



Seon & Kim (2020)

- Emergent Lyα spectra from a ***static, homogeneous sphere***

- Double peak with a deep central trough.

  This is because of the large cross-section at the line center. Thus, the Lyα photon cannot travel very far before it hits a hydrogen atom.

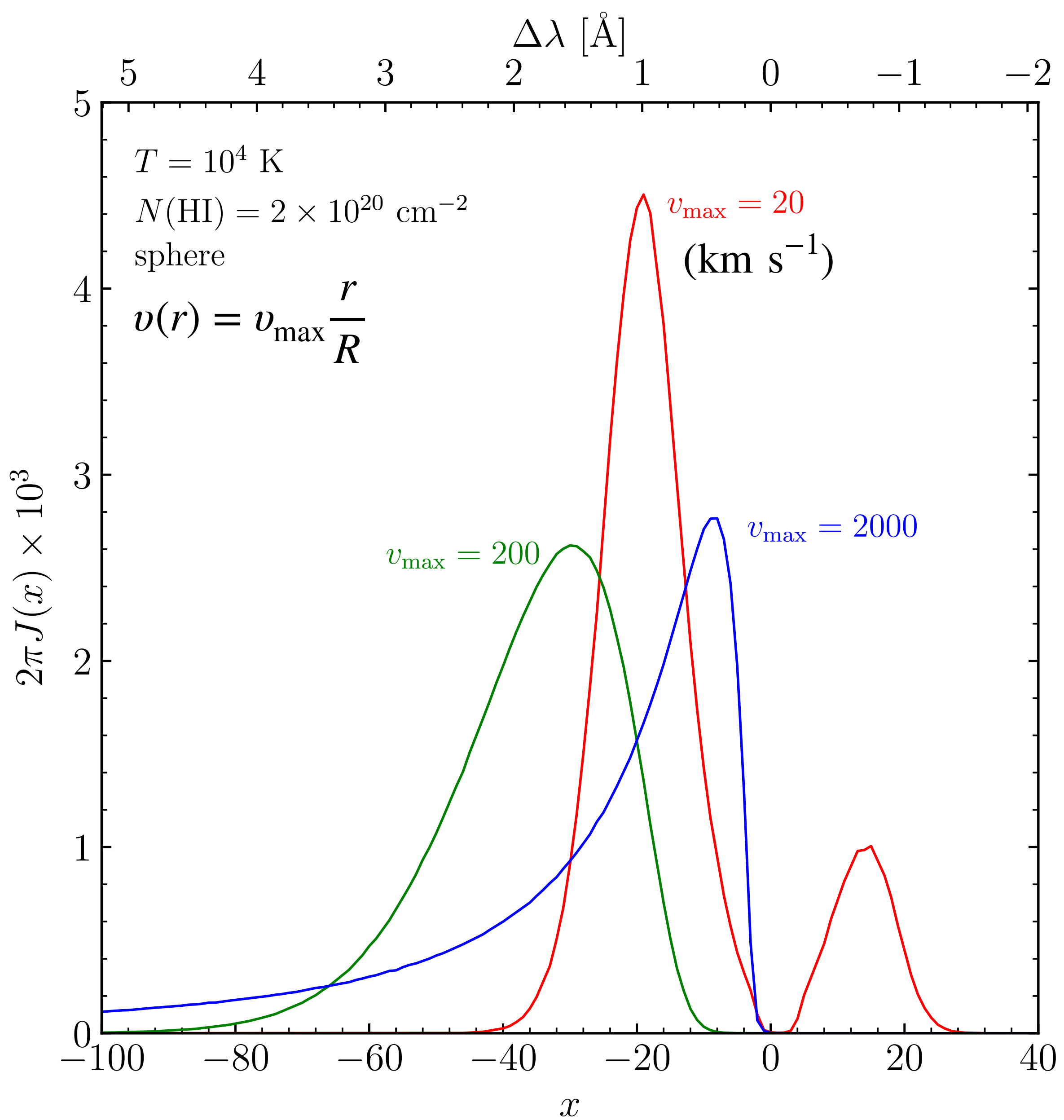
  Due to thermal motion of the atom, the photon picks up a small Doppler shift at each scattering, either to the blue or to the red.

  In this way the photons slowly diffuse not only in space but also in frequency.

  The diffusion makes the scattering cross-section decrease and thus it becomes easier to escape after having diffused either to the red or the blue side of the line center.

Isotropically expanding spherical cloud.

The expanding velocity profile is assumed to be $v(r) = v_{\max}\,(r/R)$, where R is the maximum radius.

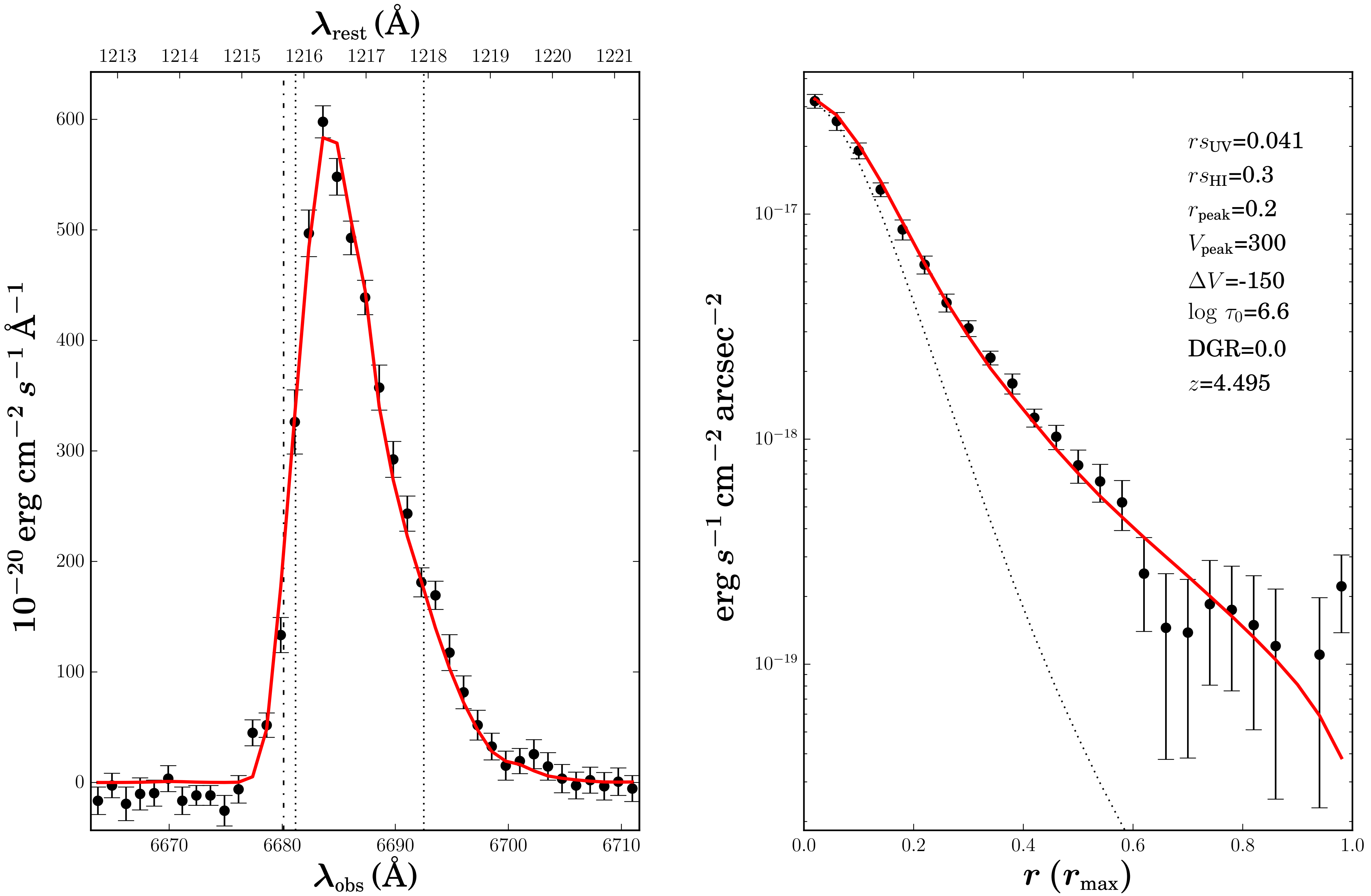


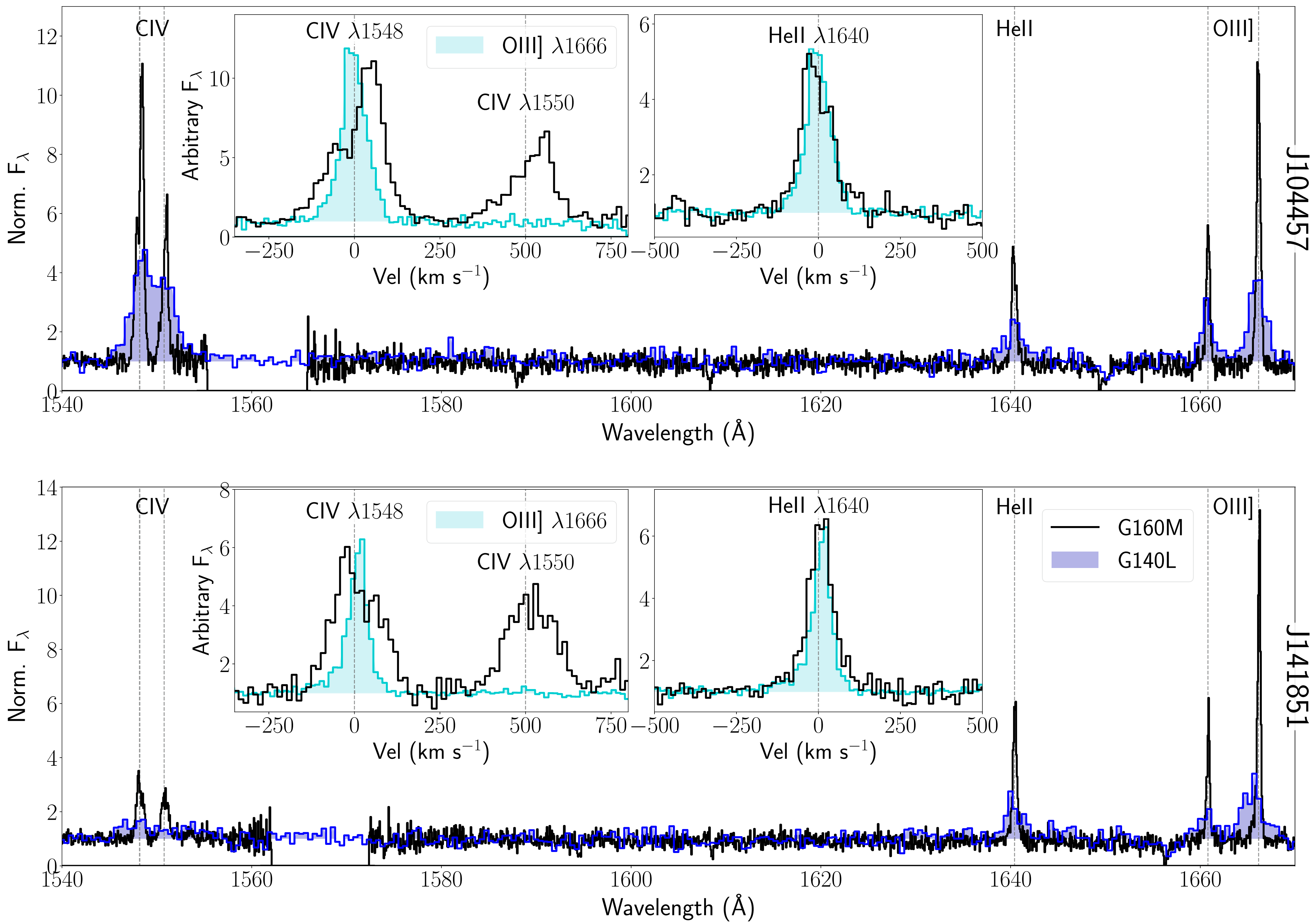
Seon & Kim (2020)

- If gas is outflowing, then in the reference frame of an outflowing shell of gas, the "blue" photons will be redshifted and thus be in the line center where the scattering cross-section is high. Hence they will be trapped in the gas.
- On the other hand, the "red" photons will be even redder in the reference frame of the outflowing gas, and pass through freely.
- Thus, outflows have the effect of erasing the blue peak by turning them into red photons.
- This is often the case in high-redshift galaxies due the enhanced star formation.

- Song & Seon et al. (2020)

Best-Fit Model for "SPECTRUM + SBP"





filled blue histogram - low resolution
black line - high resolution

Thank You!