Impact of dynamical tides on the measurement accuracy of NS tidal parameter

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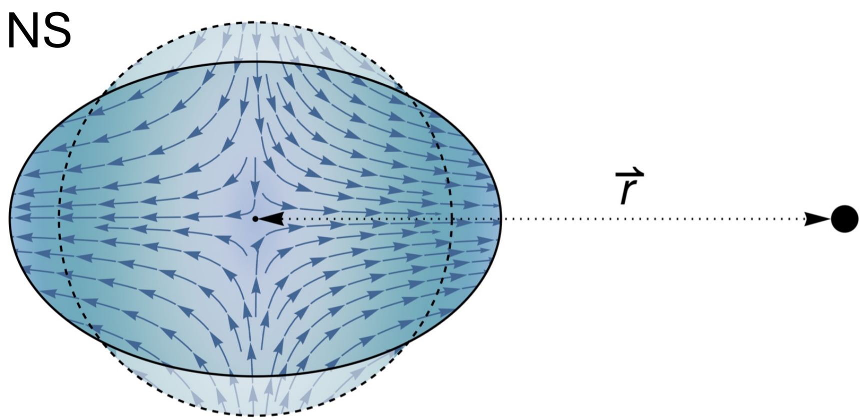
Tidal deformability of Neutron star(NS)



Thesis, Tiziano Abdelsalhin (2019)

$$\lambda_l = -\frac{Q_{ij}}{\mathcal{E}_{ij}}$$

Q_{ij} : mass multipole moments

$\mathcal{E}_{ij}$: external tidal field

λ_l : tidal deformability

$$\lambda_l = \frac{2}{(2l-1)!!} R^{2l+1} k_l$$

k_l : Tidal Love number

In early inspiral, companion's tidal force varies much slower than construction of internal equilibrium

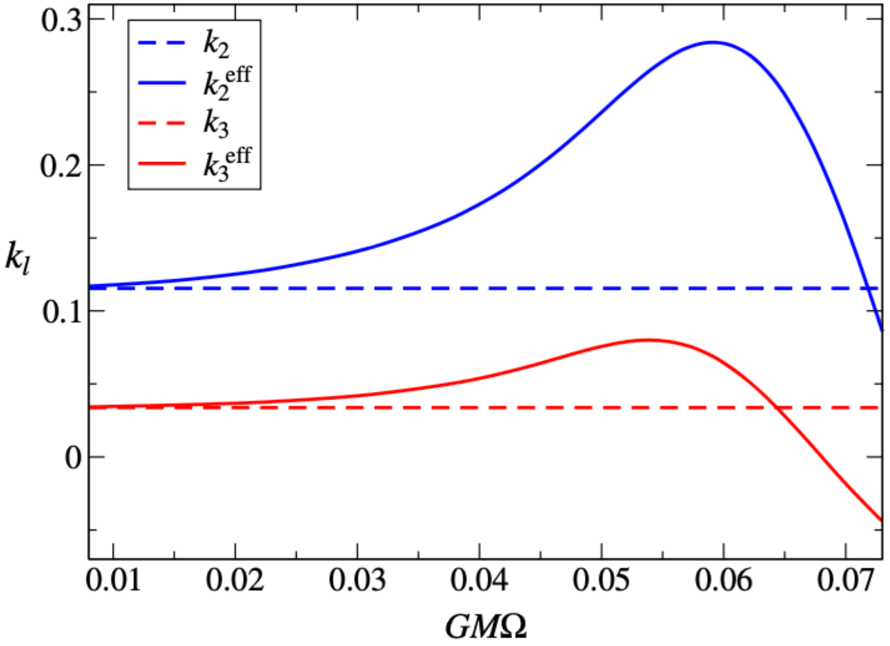
→ The extent to be tidally deformed is consistent.

→ Tidal deformability (λ_l) Love number (k_l) are constant.

[Adiabatic Tide(AT), Adiabatic approximation]

$c = G = 1$

Dynamical Tides & NS f-mode



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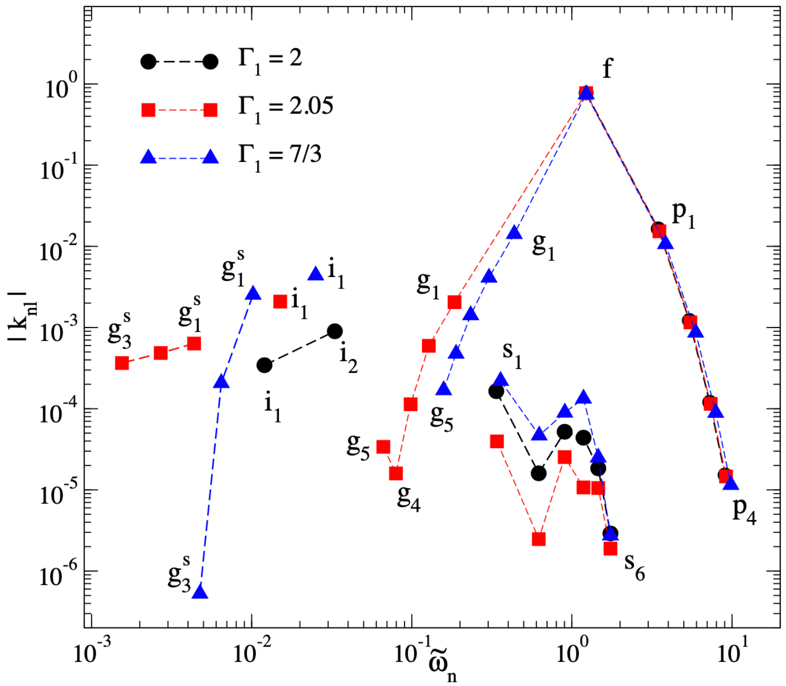

MNRAS 504, 1273-1293 (2021)

- Near the merger, adiabatic approximation breaks and time-varying driving force should be considered.
- Driving force by companion's tidal force makes NS's oscillation resonance.
- Among several mode, f - mode gives the dominant contribution to the Love number.

Tidal effect in GW phase

- The TaylorF2 waveform which is based on post-Newtonian is used.
- Binary neutron stars' spins are aligned with orbital angular momentum.

pN order	Adiabatic Tide (AT)				Dynamical Tide (DT)
	Mass Quadrupole	tidal-spin coupling	tail effect	Mass Octupole	f - mode
5 pN	LO				
6 pN	NLO				
6.5 pN		LO	LO		
7 pN	NNLO	…		LO	
7.5 pN	…		NLO	…	
8 pN			…		LO ($l = 2$)
10 pN					NLO ($l = 3$)
…					…

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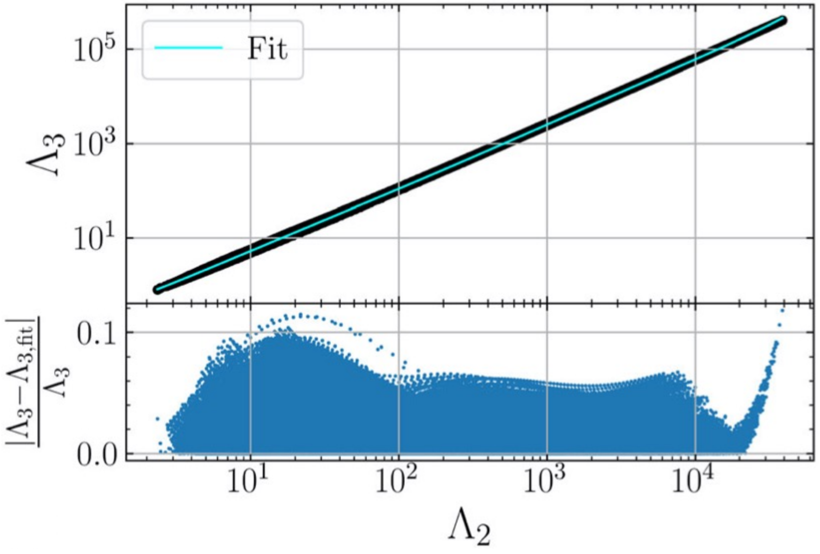
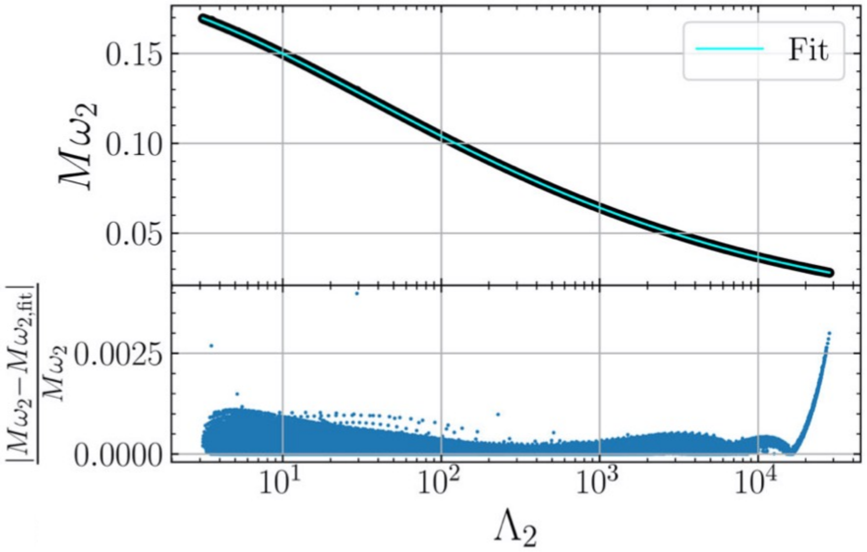
Method

- *Universal Relations*
- *Fisher Matrix Method*

Universal Relations(URs)





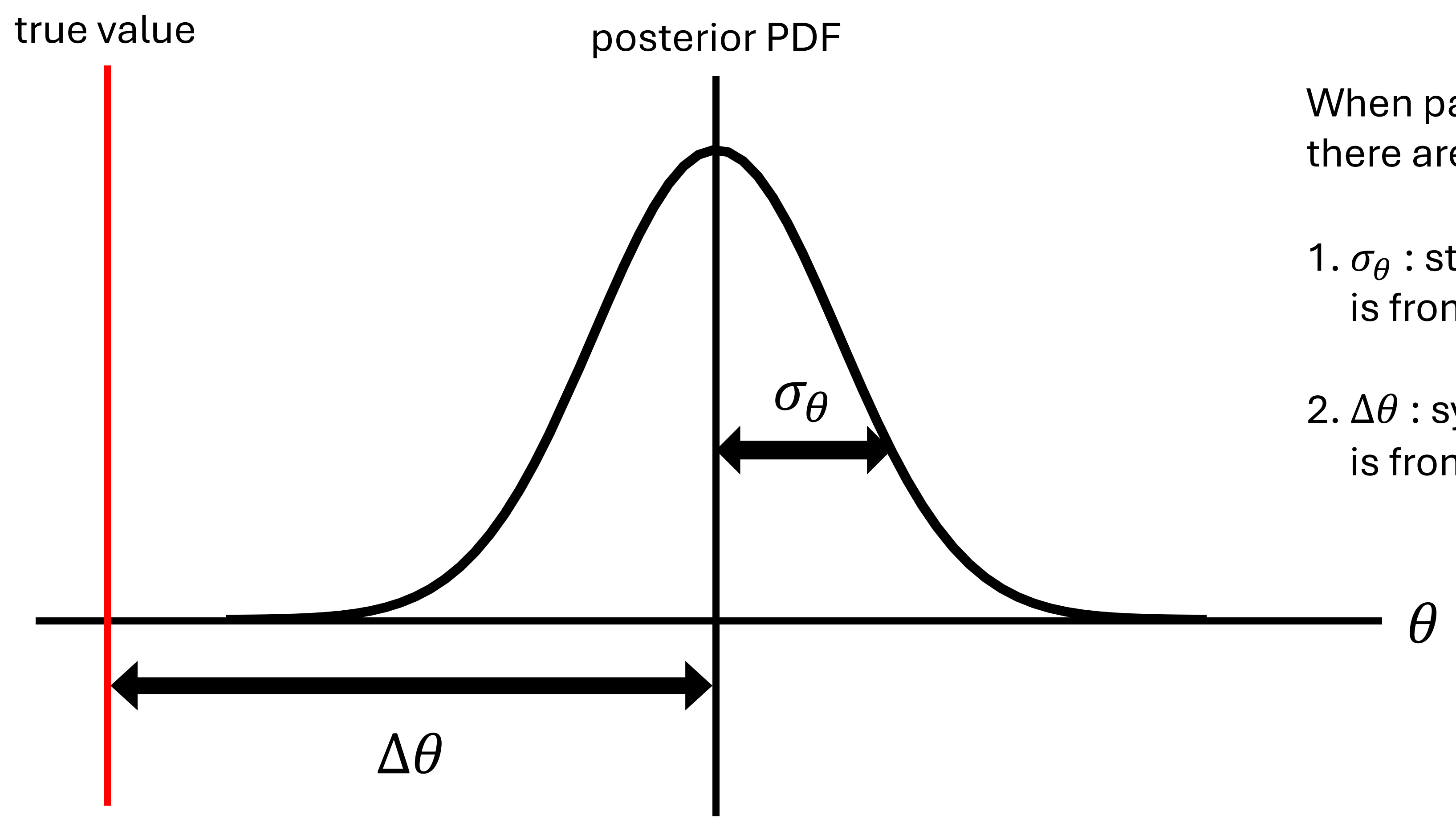
PHYS. REV. D 107, 023010 (2023)

$$\ln \Lambda_3 = \sum_{k=0}^{6} a_k \, [\, \ln \Lambda_2 \,]^k$$

$$\ln \Omega_2 = \sum_{k=0}^{6} a_k \, [\, \ln \Lambda_2 \,]^k$$

- Universal Relations are an empirical, approximate relations between parameters of NS.
- It is easy to use URs since URs are independent of EoS.
- URs help us effectively reduce the dimensionality of the parameter space.

Statistical & Systematic error



When parameters are estimated, there are two kinds of error.

1. σ_θ : statistical error (measurement error) is from the noise of the detector

2. $\Delta\theta$: systematic error (bias) is from the waveform mismodeling

Fisher Matrix Method

Bayesian inference $p(\theta|x) \propto p(\theta)\, L(x|\theta)$

Assumption ① Gaussian noise

Likelihood $L(x|\theta) \propto \exp[-\frac{1}{2}\langle x - h(\theta)|x - h(\theta)\rangle]$ where $\langle h|g\rangle \equiv \int_{f_{min}}^{f_{max}} \frac{\tilde{h}^*(f)\,\tilde{g}(f)}{S_n(f)} df$

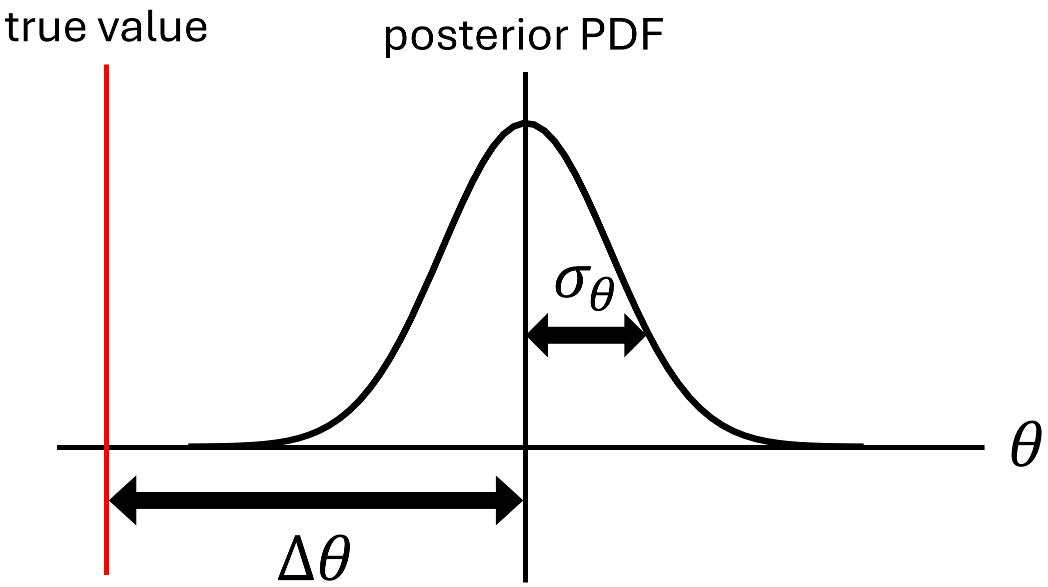
x : strain, h : waveform

Assumption ② high SNR(Signal to Noise Ratio)

$$\Delta\theta^i = \theta^i{}_{MLE} - \theta^i{}_0$$

$$h(\theta) \simeq h_0 + \left.\frac{\partial h}{\partial\theta^i}\right|_{\theta_0} \Delta\theta^i + \frac{1}{2}\left.\frac{\partial^2 h}{\partial\theta^i\partial\theta^j}\right|_{\theta_0} \Delta\theta^i\Delta\theta^j + \cdots$$

Fisher Matrix Method



posterior PDFs are Gaussian distribution.

And statistical & systematic errors can be easily computed.

$$\Gamma^{ij} = \left\langle \frac{\partial h}{\partial \theta^i} \middle| \frac{\partial h}{\partial \theta^j} \right\rangle$$ Fisher matrix

$$(\Gamma^{-1})^{ij} = \Sigma^{ij}$$ Covariance matrix

$$\sigma_{\theta^i} = \Sigma^{ii}$$ Standard deviation

$$\Delta\theta^i = \theta^i{}_{recover} - \theta^i{}_0$$ Systematic bias

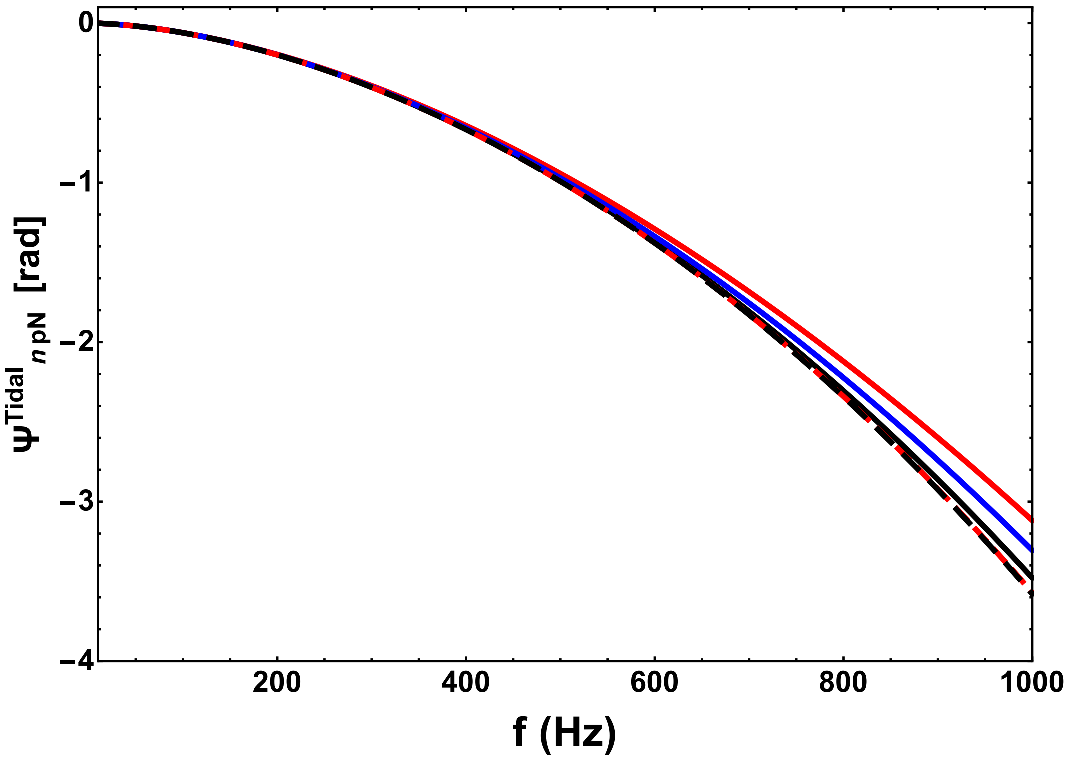
$$= \Sigma_{ij} \left\langle \frac{\partial h_{AP}}{\partial \theta^j} \middle| h_{True} - h_{AP} \right\rangle$$

Result

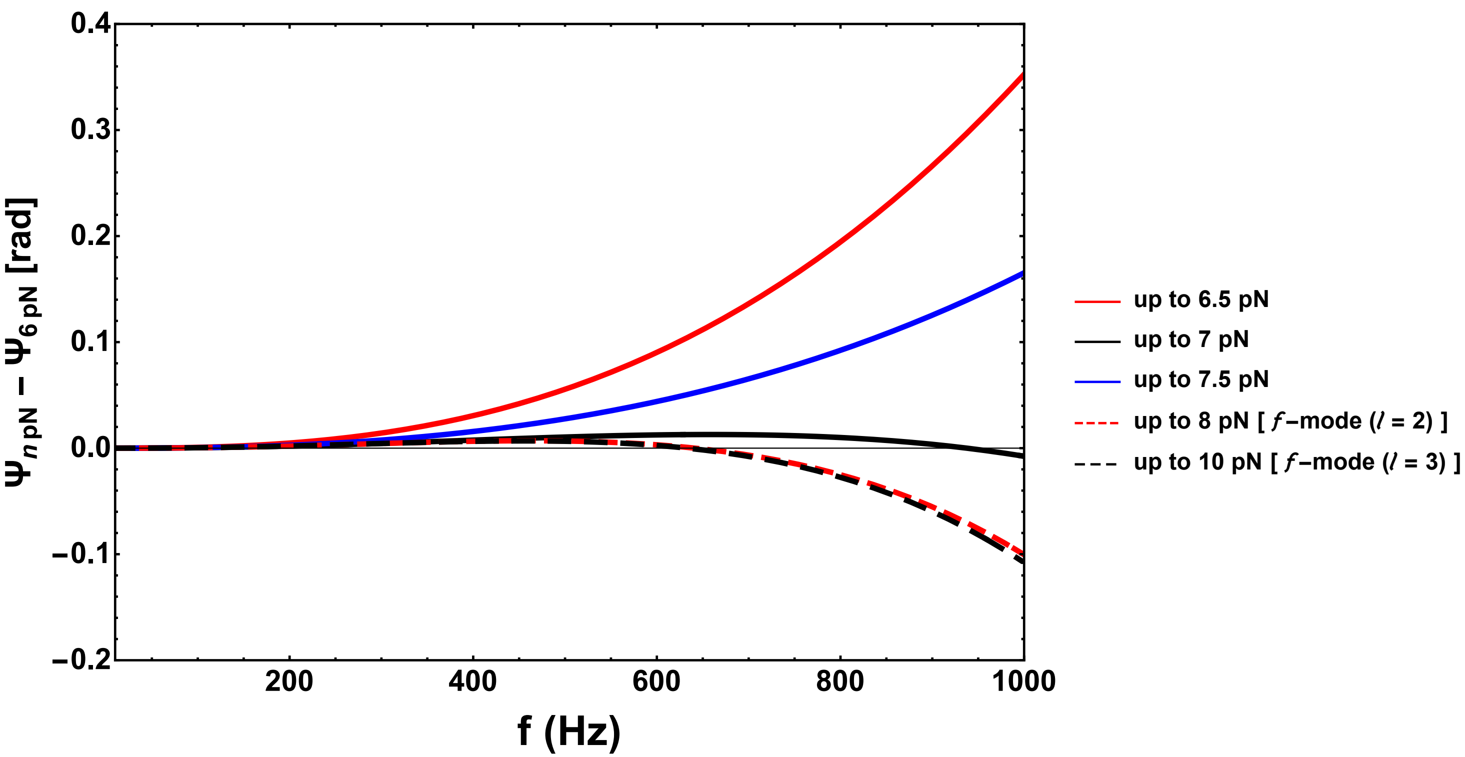
- *Accumulated GW phase due to tidal effect*
- *Depedence of statistical & systematic error on the f - mode*
- *effects of Λ - q^6 relation*

Accumulated GW phase due to tidal effect($\Delta\Psi$)

accumulated phase from tidal effect

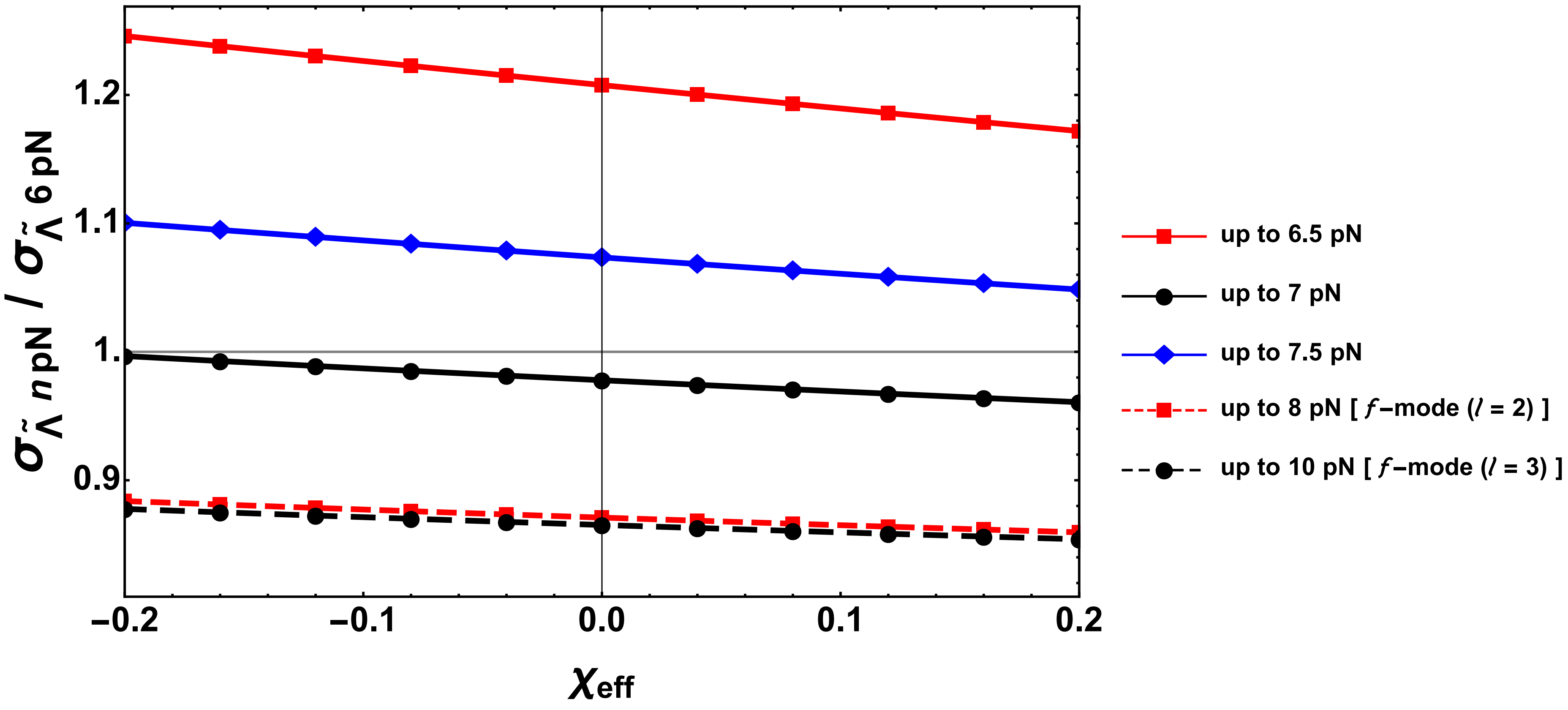


phase difference to tidal 6 pN



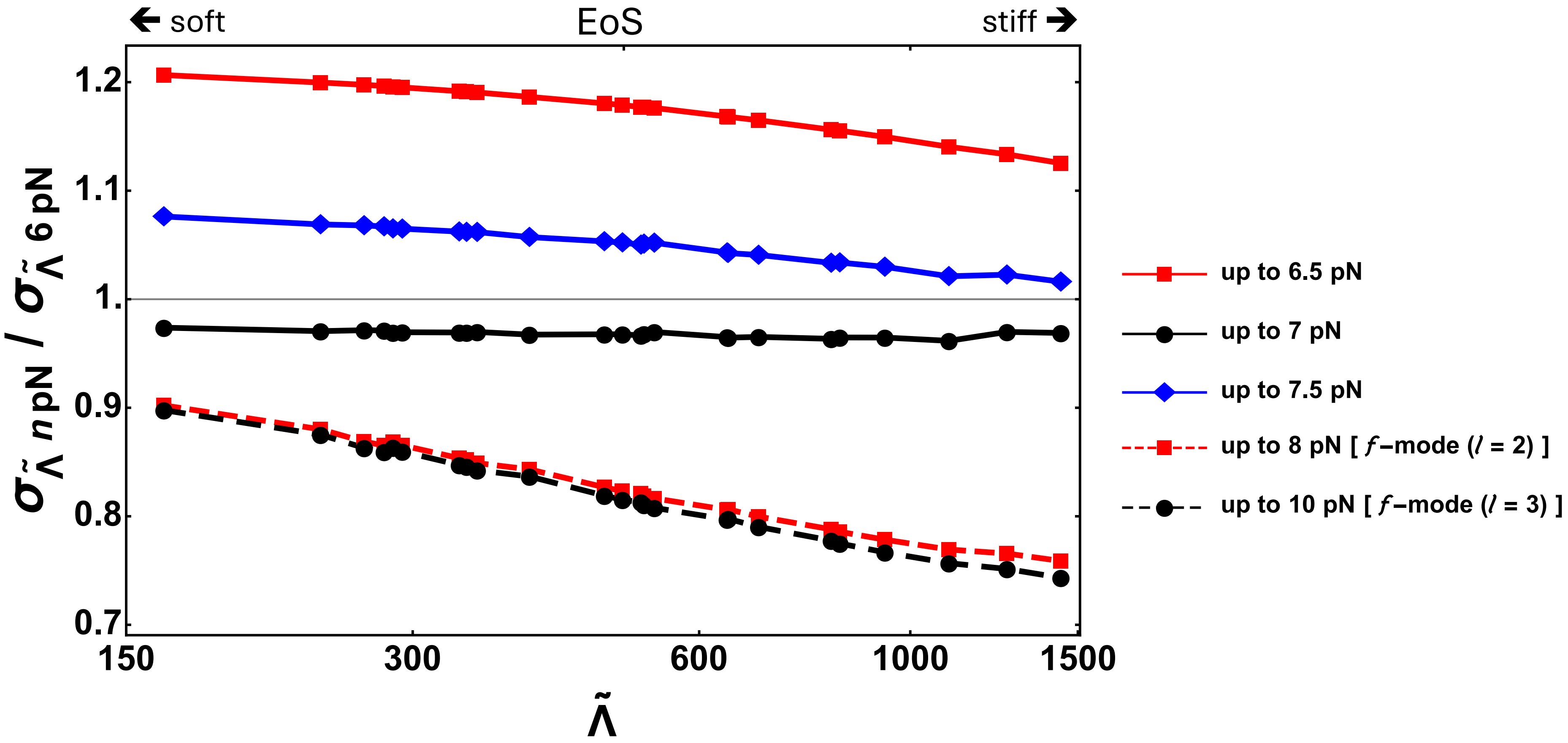
$m_1 = 1.6\, M_\odot,\ m_2 = 1.2\, M_\odot,\ \chi = 0.05,\ \tilde{\Lambda} = 267,\ \delta\tilde{\Lambda} = 57$ (APR4)

Result



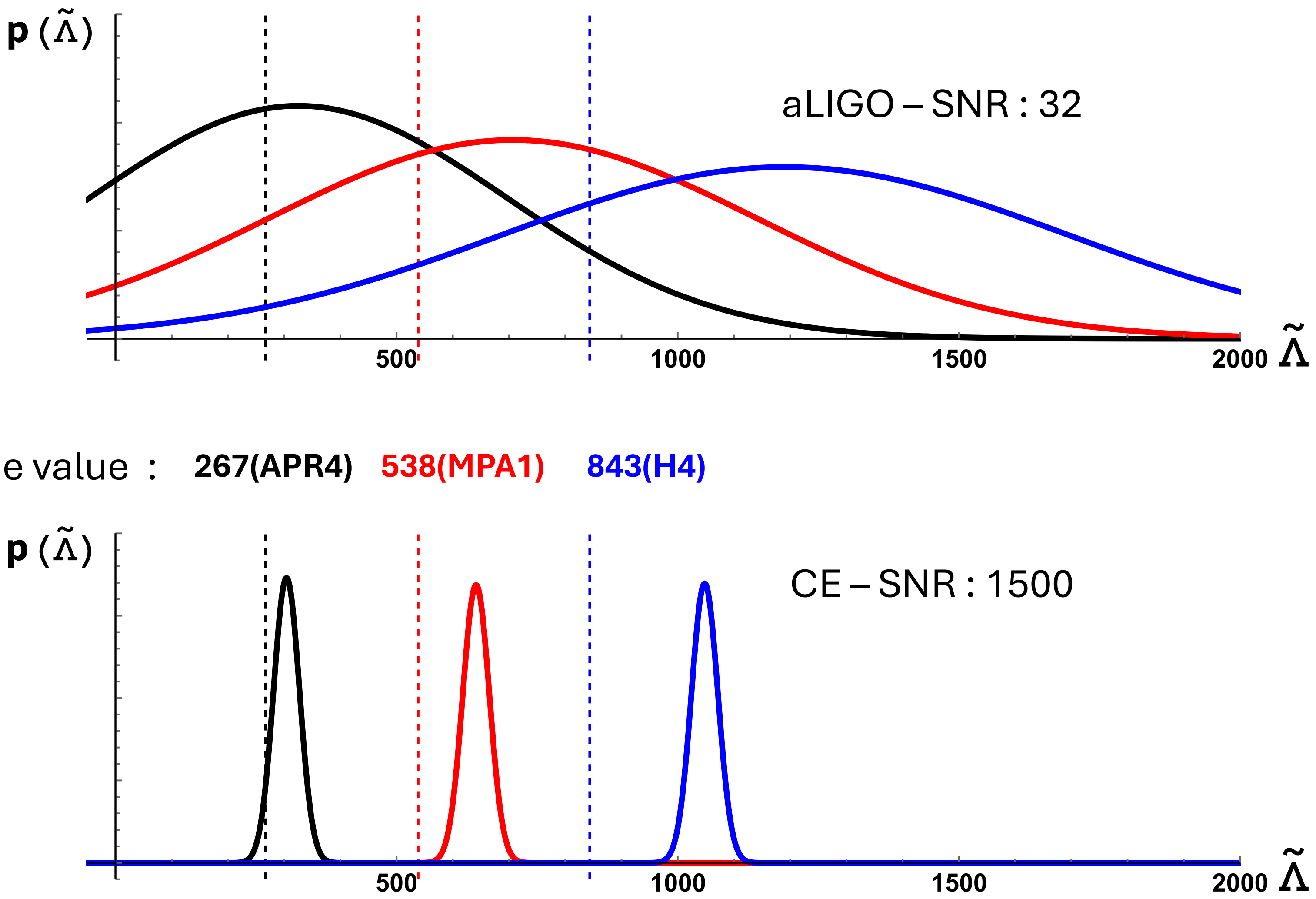
- Compared to 6 pN, the waveforms including higher orders have stronger spin dependence since tidal - spin coupling enters at 6.5 pN.

Result



- ATs have similar tidal deformability dependence regardless of the pN order.
- f – mode(l = 2) at 8 pN meaningfully reduces the error.

Result



true value : **267(APR4)** **538(MPA1)** **843(H4)**

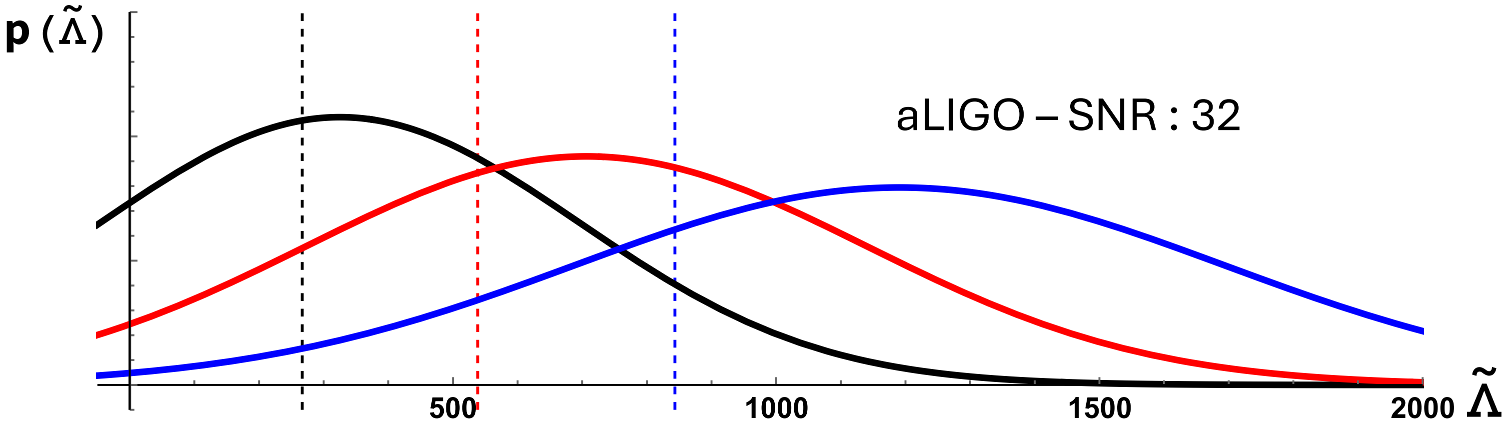
injection

: AT + f-mode(l = 2, 3)

recovery

: AT

Result



injection

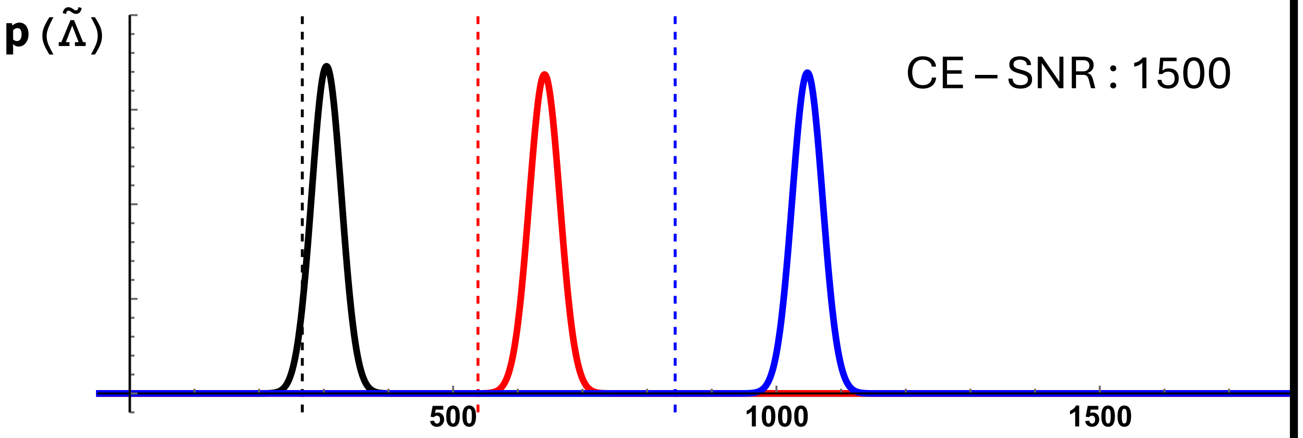
: AT + f-mode(l = 2, 3)

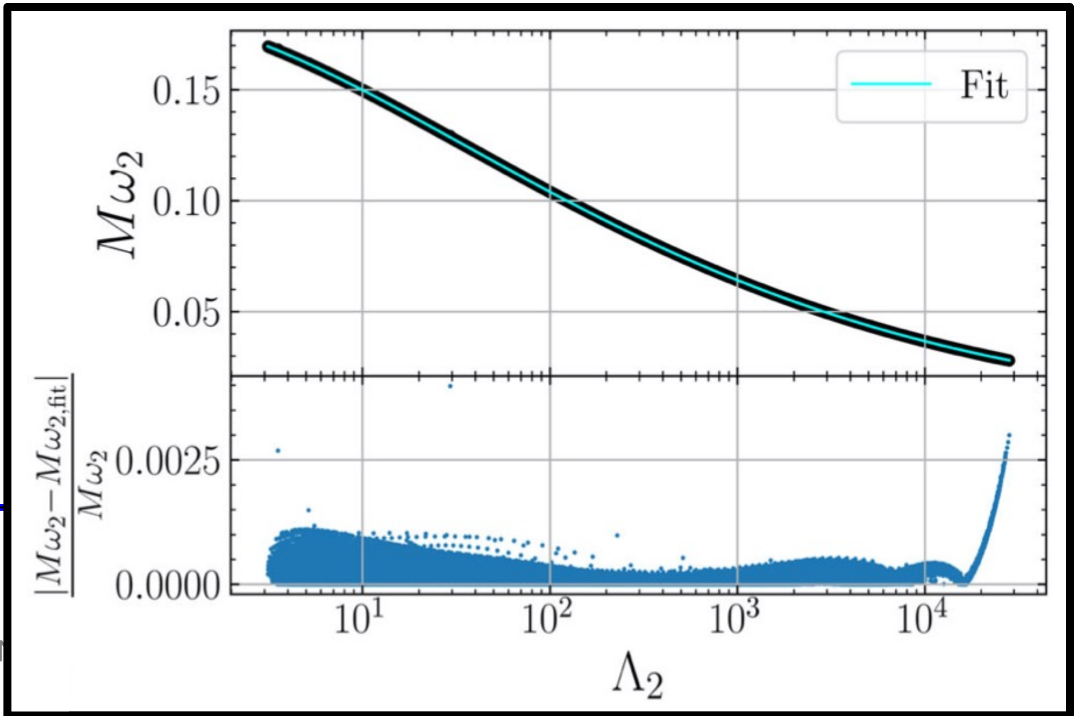
recovery

: AT

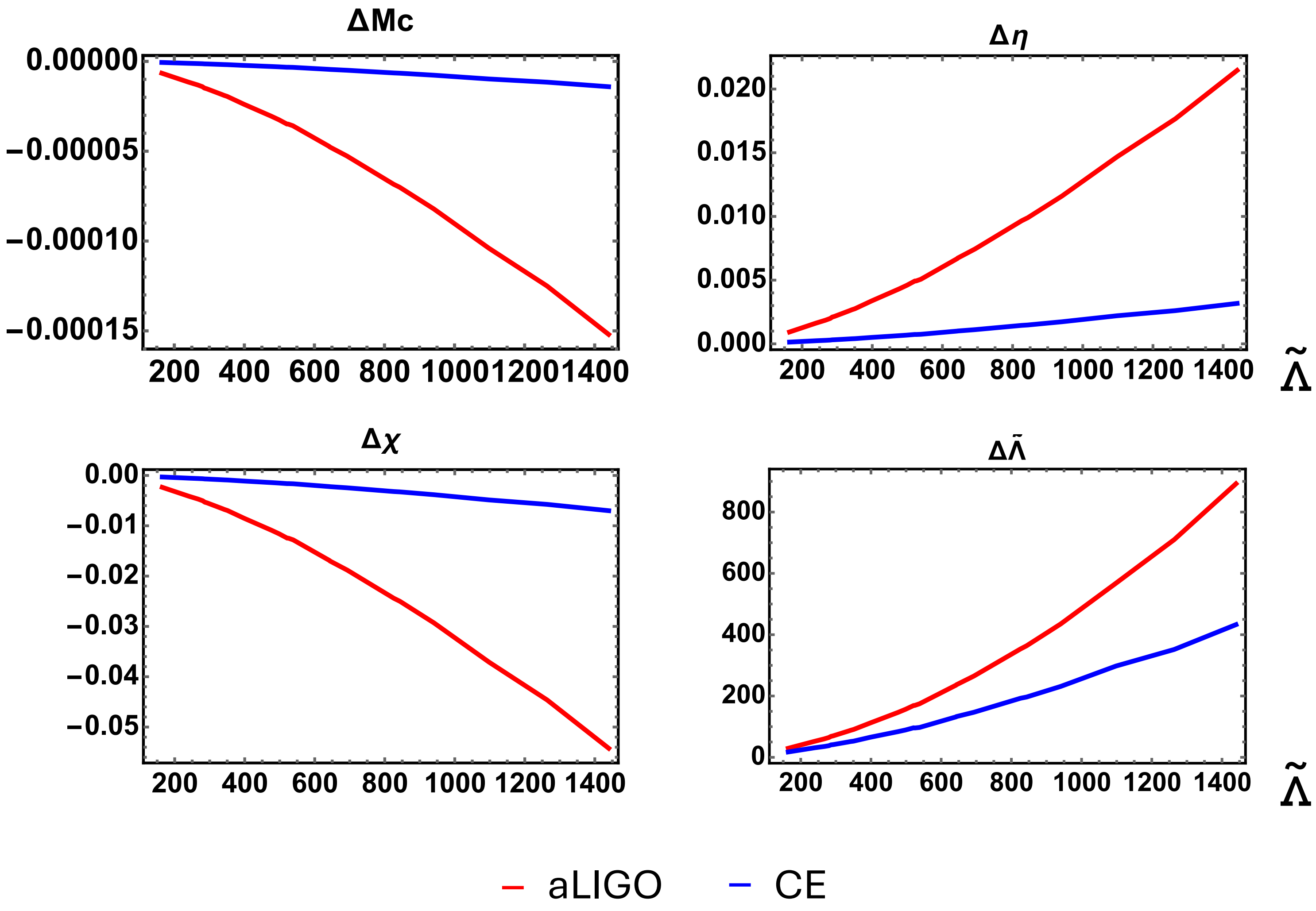
$$\psi_{f-mode} \propto \frac{\Lambda}{(\Omega)^2}$$

true value : **267(APR4)** **538(MPA1)** **843(H4)**





Result

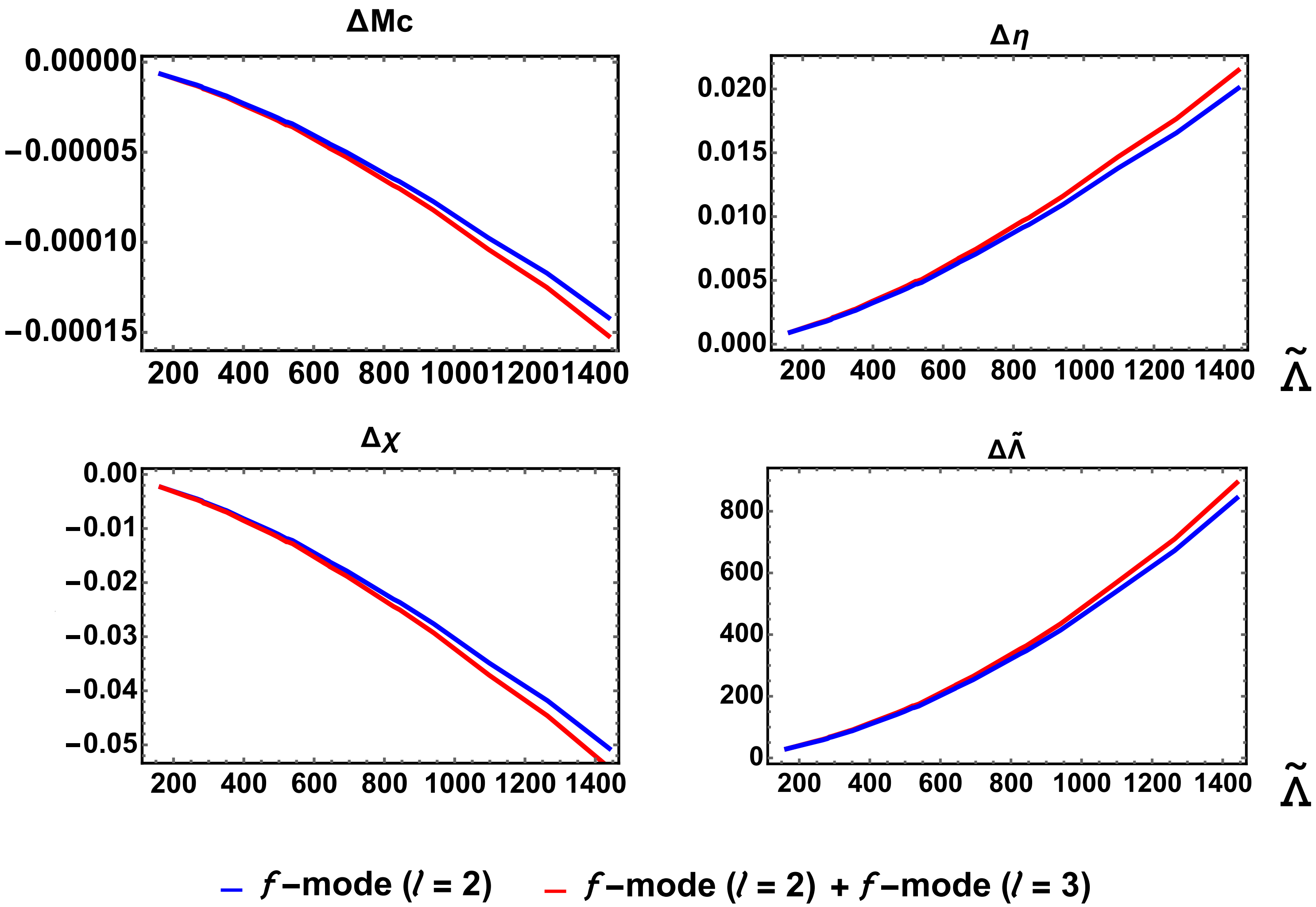


With next generation detector CE,

bias due to the f-modes as well as the error

can be reduced than the aLIGO

Result

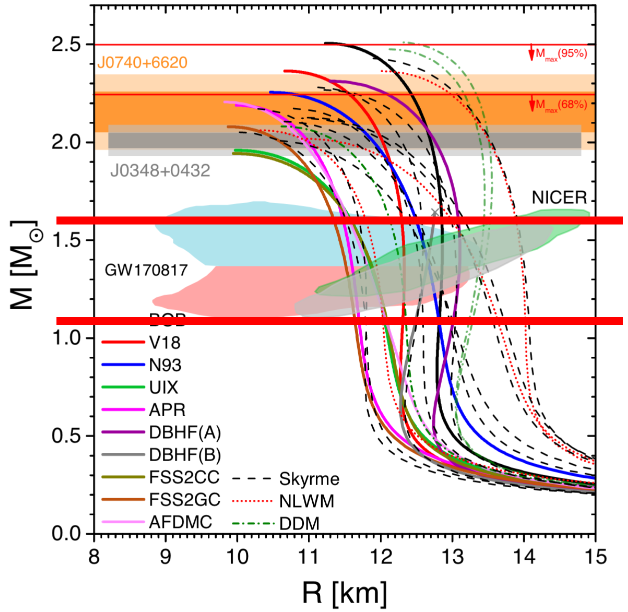


Compared to f-mode(l = 3),

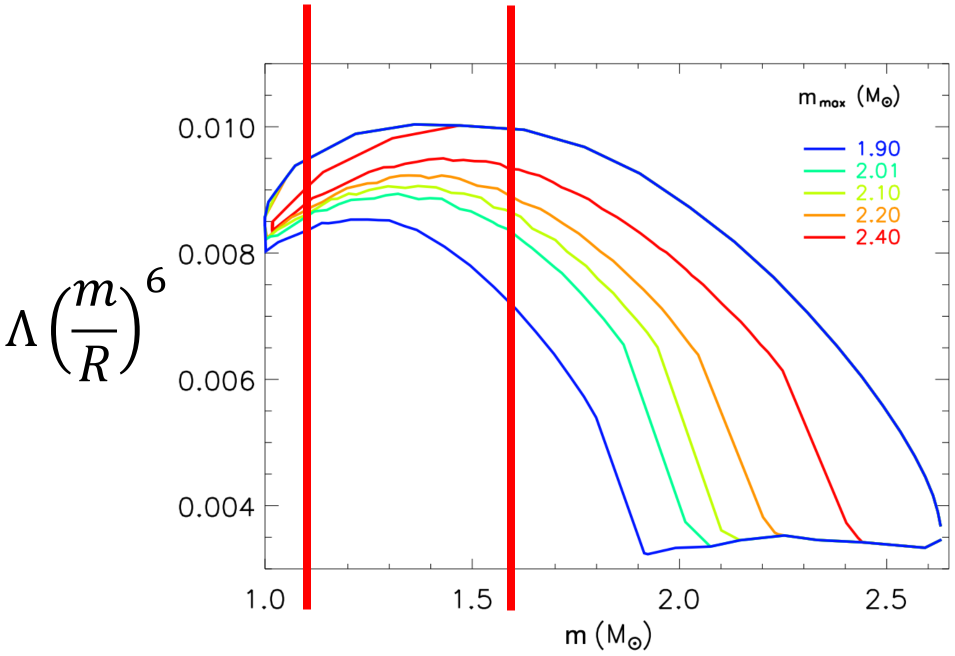
absence of the f-mode(l = 2)

makes the bias dominantly

effects of Λ - q^6 Relation



Klara Olofsson (2022)



PHYS. REV. LETT 121, 091102 (2018)

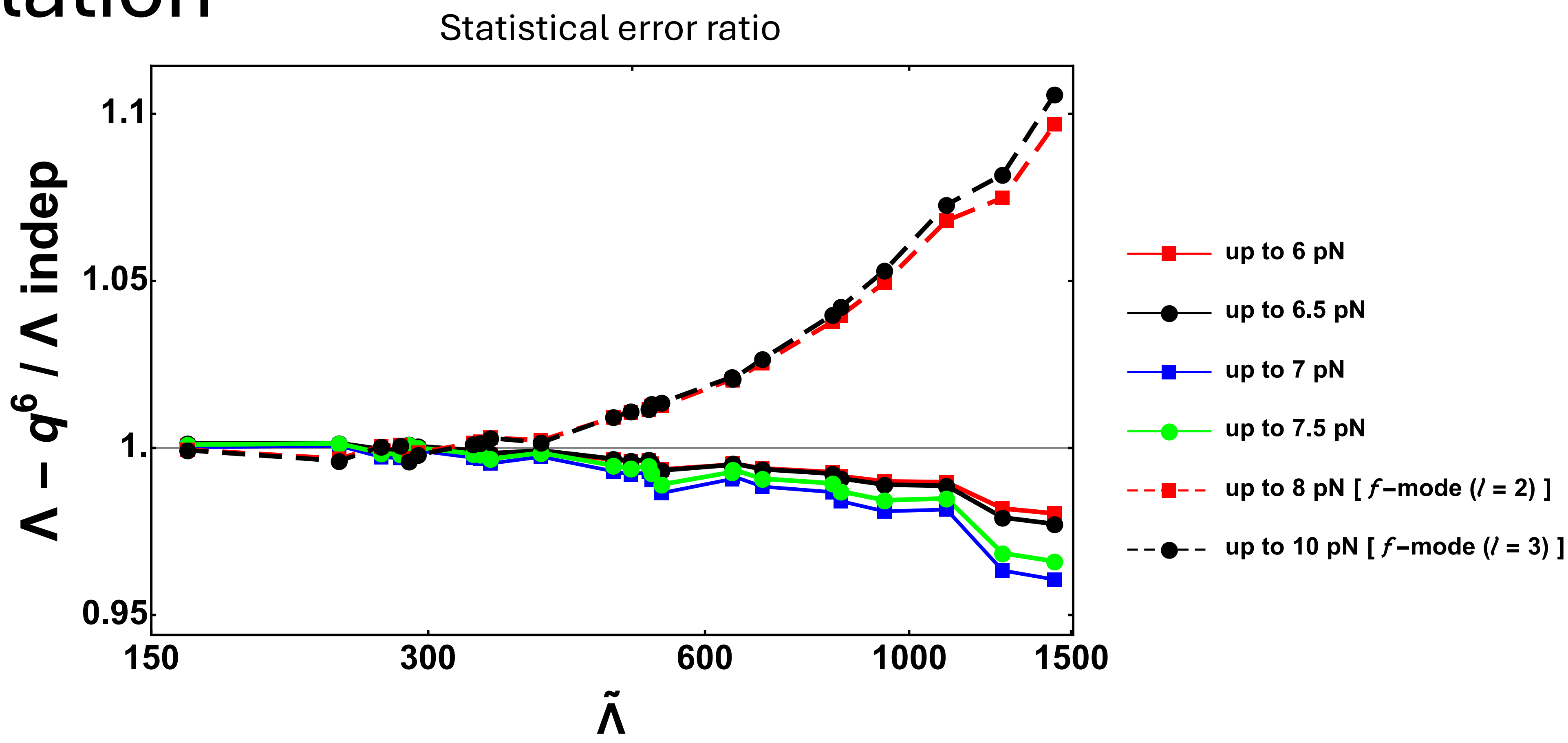
$$\Lambda = \frac{2}{3} k_2 \left(\frac{R}{m}\right)^5 \simeq a \left(\frac{R}{m}\right)^{-6}$$

in $1.1\, M_\odot \le m \le 1.6\, M_\odot$ range → NS's radii are almost constant

→ $\Lambda \propto m^{-6}$ → $\Lambda_2 = q^{-6} \Lambda_1$

$c = G = 1$

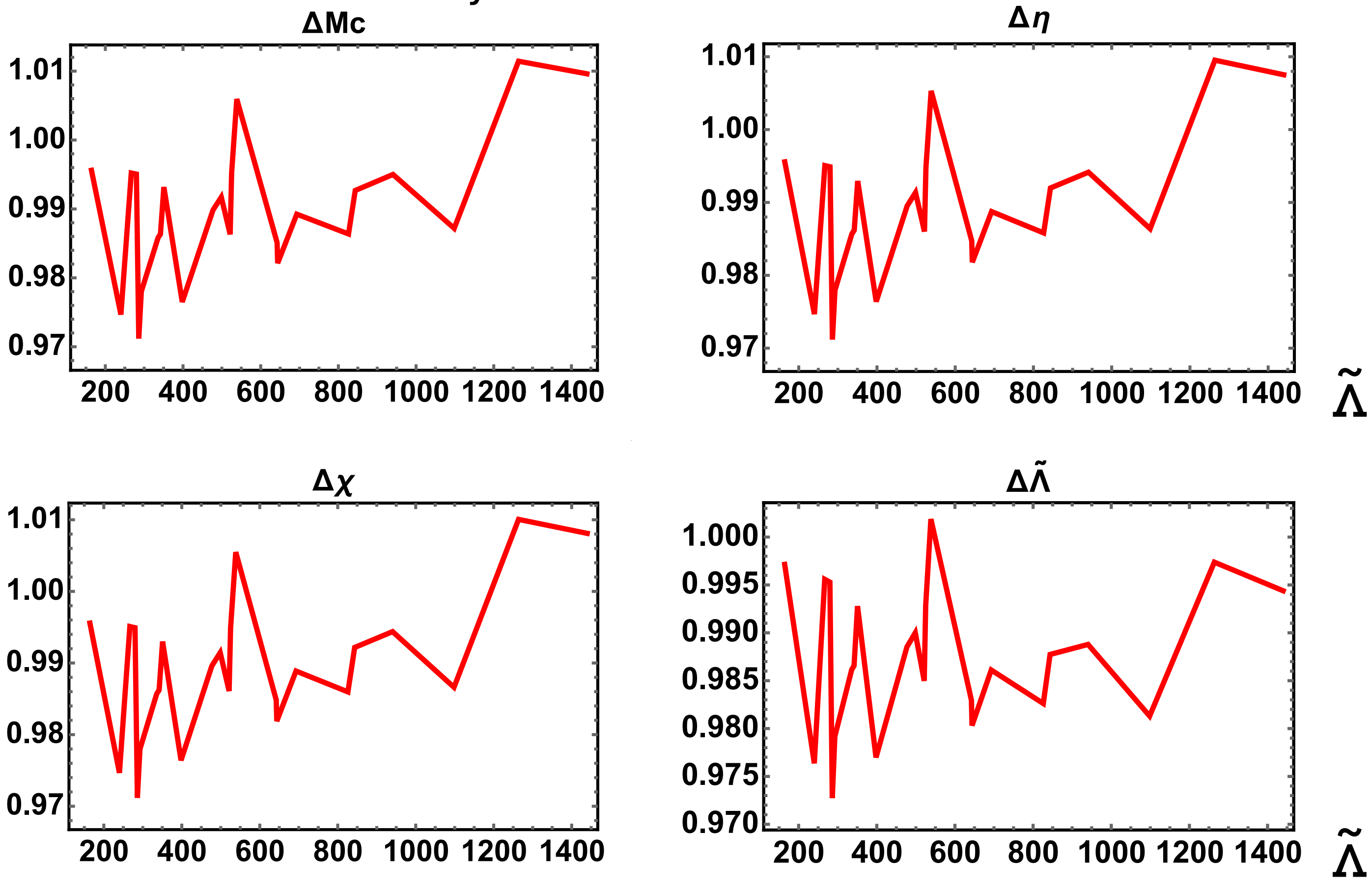
Λ - q^6 Relation



- Using q^6 relation improves the measurement error when only ATs are considered while it deteriorate the the measurement error when DTs are also considered.

Λ - q^6 Relation

Systematic bias ratio



Unlike the statistical error,

using Λ - q^6 relation don't affect

the bias of parameters

injection= : AT + f-mode(l = 2, 3) // recovery : AT

Summary

- We investigate the impact of the higher-order tidal terms beyond 6 pN, in particular, include f – mode resonance which is dominant among the NS oscillation modes.

- We find that there is no monotonically decreasing tendency with increasing pN order in AT, and the measurement error is reduced by about 20% by introducing the f – mode resonance due to DT.

- Also, the absence of the f – mode resonance makes the large systematic bias to parameters which is significant in next generation detector CE.

- Λ - q^6 relation influences to the measurement error while it doesn't affect to systematic bias when f – mode resonance is considered.

Thanks you for your attention.